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How Constant is Constant Elasticity of Substitution? Endogenous Substitution between Clean and Dirty Energy*

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Abstract

The degree of substitutability between clean and dirty energy plays a central role in leading economic analyses of optimal environmental policy. Despite the importance, a constant and exogenous elasticity of substitution has been a dominant theoretical approach. We challenge this assumption by developing a dynamic general equilibrium model with an endogenous elasticity of substitution that interacts with the relative share of clean inputs in the economy. We find strong dynamic feedback effects arising from endogenous substitution capacity that amplifies the impact of directed technical change and accelerates the transition to a green economy.

Keywords: Elasticity of substitution, directed technical change, climate change.
JEL Classification: Q40, Q55, Q54, O33.

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1 Introduction

In most economies, transition from fossil fuels to low-carbon renewable energy takes a central role in the policy visions to halt the progress of warming (IPCC, 2018). As a crucial parameter that governs the transition process, the degree of substitutability between clean and dirty energy has been shown to strongly influence the predictions for sustainable growth and optimal designs of climate policy. For example, Acemoglu et al. (2012) demonstrate that when clean and dirty inputs are weak substitutes or complements, a permanent carbon tax is necessary to avert an environmental disaster and to switch to clean production, while a much lower and temporary carbon tax suffices when the two inputs are strong substitutes. Further, Golosov et al. (2014) note from their calibrated model that a high degree of substitutability between different fuels induces the temperature to decline in the middle of the next century, while lower substitutability involves a continuous increase in the temperature even with optimal policy in place.

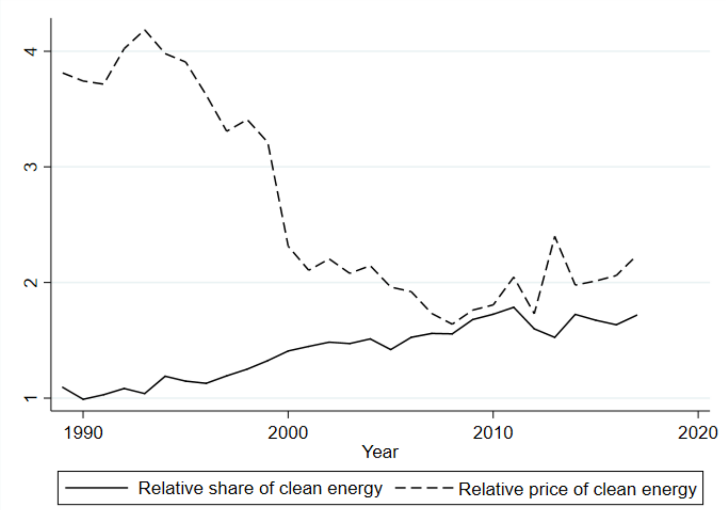
Despite its importance, most macroeconomic models as well as computable general equilibrium models that generate predictions over a very long period of time (beyond 2100) invariably assume a constant and exogenous elasticity of substitution between clean and dirty inputs. However, the growing integration of clean energy strongly suggests that this substitutability is likely to have improved over time. To illustrate, Figure 1 shows the penetration of clean energy in the economy with its relative price falling substantially over the last three decades in France, the world's 7th biggest economy.[1] These observations are also consistent with emerging policy initiatives around the world that have led to a sizeable expansion of the necessary technology and infrastructure for the use of clean energy (IEA, 2020).[2]

Accounting for these observations, this paper extends the important literature on endogenous growth and climate change in a novel way by developing a dynamic general equilibrium model with an *endogenous* elasticity of substitution that interacts with the relative share of clean inputs in the economy. Using micro data, we provide evidence for the empirical relevance of a variable elasticity of substitution between clean and dirty energy that supports our theoretical approach. Then, we calibrate the model to numerically explore how dynamic substitution capacity influences optimal climate policy. To the best of our knowledge, this paper is the first to make advances on this front by endogenizing the substitution elasticity and investigating its implications in the analyses of optimal environmental policy.

[1] Based on World Bank national accounts data in 2020.

[2] For instance, California passed a requirement for utilities to procure 1,300 MW of storage power capacity by 2020 with the goal of addressing the intermittency problem of renewables and thus facilitating the subsitution of fossil fuels by renewable energy.

Figure 1: Evolution of the relative use and price of clean energy in France



Notes: Authors' calculation of average relative prices and consumption based on the EACEI.

Our model builds on the macroeconomic literature investigating the possibilities of sustainable growth through directed technical change. The convention in this vast literature (see Fischer and Heutel (2013) or Hémous and Olsen (2021) for a review) is to model the production of a consumption good that combines two inputs, one clean and the other dirty, with an exogenous and constant elasticity of substitution (CES). Our innovation is to relax this assumption and to allow the substitution elasticity to endogenously evolve over time. To do so, we incorporate a Variable Elasticity of Substitution (VES) production function developed by Revankar (1971b) in the standard framework of directed technical change. The model highlights dynamic feedback effects arising from endogenous substitution capacity: an increase in demand for the clean input (given a climate policy) leads to an advance in clean technology, which lowers the price of the clean input. The lower price further increases its demand, which at the same time expands the substitution capacity and enables a larger increase in the demand for clean inputs, amplifying the effect of directed technical change and accelerating the switch to clean production and technology in a virtuous cycle.

The elasticity of substitution associated with the VES production function depends on the relative penetration of clean inputs in the economy through a parameter that directly regulates the strength of this relationship. This leads to one of the advantages of the VES technology, namely, that it allows a convenient testing of its empirical relevance. Revankar (1971b) and Karagiannis et al. (2005) demonstrate that this can be achieved by empirically examining the null hypothesis that this parameter is zero, in which case the elasticity of substitution is deemed not responsive to the input ratio.[3]

We follow the aforementioned studies and examine the empirical relevance of the VES technology in our model from micro data. For the purpose, we use plant-level data from the French manufacturing sector that includes information on energy consumption and expenditure by fuel from 1989–2017. Large variation in the demand and price of clean and dirty energy at the micro level facilitates the identification of the parameter of interest. Our analysis provides strong empirical evidence for the relevance of a variable elasticity of substitution between clean and dirty energy: we reject the null hypothesis that the parameter that captures the responsiveness of the elasticity of substitution to the relative input ratio is zero in a number of specifications that use instrumental variables. The analysis also provides empirical guidance for calibrating our theoretical model.

We perform two numerical exercises to explore the implications of endogenous substitution capacity in the analyses of climate policy. In both exercises, the economy begins from the same initial technology levels in the clean and dirty sector, but substitution capacity is endogenous and time-varying in the first and exogenous and fixed at an empirically plausible level in the second.[4] Thus, the comparison between the two economies highlights the impact of an endogenously evolving elasticity of substitution on optimal policy and transition dynamics.

There are two main findings. First, allowing for an endogenously evolving elasticity of substitution leads to substantially different optimal policy profiles. In the VES economy, the optimal carbon tax is much lower and only temporarily required, while the CES economy is associated with a higher and permanent carbon tax. The subsidy to clean innovation is also lower in the VES than in the CES case. Therefore, the profiles of optimal climate policy in the VES economy resemble those of a CES economy where a very high level of elasticity of substitution is fixed throughout the simulation period – for example, as in Acemoglu et al. (2012) where the parameter is set to 10 – that predicts a temporary carbon tax as optimal policy. Our finding demonstrates that such an optimistic policy recommendation need not necessarily come from a scenario where strong substitutability between clean and dirty inputs is assumed throughout. We show that relaxing the assumption of a fixed, exogenous elasticity of substitution and endogenizing the economy's input substitution capacity can lead to a more optimistic policy design *even when* the economy starts at a relative low, and empirically plausible, initial level of the substitution elasticity.[5]

[3]Their empirical exercises are in the context of labor-capital substitution and use macro data.

[4]In the second exercise, we set the elasticity of substitution to 3, given prior empirical studies that estimate the parameter to be around 2 or 3 (Papageorgiou et al., 2017; Jo, 2020).

[5]Given the data and the chosen parameters in our calibration, the initial elasticity of substitution in the VES economy is 1.9

Second, we find that the transition to clean production and technology occurs more quickly in the VES economy, where the substitution capacity expands as the relative use of clean inputs increases, despite the less stringent optimal policy in place. Due to the dynamic feedback effects revealed in our theoretical model – an increase in the relative use of clean inputs (in response to a climate policy) improves the substitutability between clean and dirty inputs, which enables a larger increase in the demand for clean inputs and further expands the substitution capacity and so on – the transition to clean production is strongly accelerated in the VES economy compared to the CES economy where the elasticity of substitution is kept constant.

Our paper builds on the vast literature of strong policy relevance on growth, directed technical change and the environment (e.g., Acemoglu et al., 2012; Gans, 2012; Golosov et al., 2014; Lemoine, 2017; Van den Bijgaart, 2017; Fried, 2018; Greaker et al., 2018; Hart, 2019). The dominant approach to model the production in this literature has been to adopt a CES production function with an exogenous and constant elasticity of substitution between clean and dirty inputs. We extend the literature by developing a dynamic general equilibrium model with an endogenous elasticity of substitution that flexibly interacts with the relative share of clean inputs in the economy. Our analysis yields new insights that even when the economy begins with a low, empirically plausible elasticity of substitution, the optimal environmental policy can be less stringent and more optimistic (a lower and temporary, rather than high and permanent, carbon tax) once we allow for the economy's input substitution capacity to evolve over time.

In addition, our analysis speaks to the literature investigating the role of the elasticity of substitution between labor and capital, a central parameter in many areas of economics (León-Ledesma et al., 2010). A number of studies have emphasized the elasticity of substitution between labor and capital as an engine of economic growth, a conjecture known as the de la Grandville hypothesis (de La Grandville, 1989; Yuhn, 1991; Klump and de La Grandville, 2000). Moving beyond the CES production function, other papers also considered an endogenous elasticity of substitution that changes in the process of economic development (Arrow et al., 1961; Revankar, 1971a,b; Miyagiwa and Papageorgiou, 2007). In a similar spirit, our paper endogenizes the substitution elasticity between clean and dirty inputs in the standard framework of directed climate change and finds that it is a strong driver behind energy transition that amplifies the effect of technical change.

The paper proceeds as follows. Section 2 explains the central role of the elasticity of substitution in the relevant literature. Section 3 presents the model. Section 4 provides empirical evidence for the relevance of the VES technology in the energy context. Section 5 presents numerical exercises and results. Section 6 concludes.

2 The role of clean-dirty substitutability in the literature

In most theoretical and numerical analyses investigating the possibility of sustainable growth, the predictions of green growth as well as optimal policy designs critically depend on the degree of substitutability between clean and dirty inputs (e.g., Acemoglu et al., 2012; Gans, 2012; Golosov et al., 2014; Lemoine, 2017; Van den Bijgaart, 2017; Fried, 2018; Greaker et al., 2018; Hart, 2019; Karydas and Zhang, 2019). The substitution elasticity reflects a change in relative factor demands in response to changing relative prices, which affects the growth (or de-growth) of fossil fuel consumption and therefore the progress of climate change and expected damage (Golosov et al., 2014; Hart, 2019). Furthermore, in the widely adopted framework of directed technical change, the parameter regulates the extent to which innovation efforts can be directed towards innovation in clean technologies (Acemoglu et al., 2012; Gans, 2012; Lemoine, 2017; Greaker et al., 2018). Environmental policy that increases the price of dirty energy spurs innovation in clean technology when clean and dirty energy are strong substitutes. On the other hand, it is harder to direct the course of innovation to clean technology with such price signals when the two inputs are weak substitutes or complements.

The significance of the substitution elasticity is further demonstrated by how sensitive quantitative predictions are to the value of this parameter in quantitative analyses. For instance, Acemoglu et al. (2012) provide a quantitative exercise that highlights the effects of different values of the elasticity of substitution (as well as discount rates) on the optimal policy designs. They consider a low and a high substitutability scenario (where the elasticity of substitution between clean and dirty inputs is 3 and 10, respectively) and find dramatic differences in optimal policy across the two scenarios. In the high-substitutability case, the optimal carbon tax is low and necessary only for a brief period of time because the switch to clean inputs follows swiftly. In the low-substitutability case, on the other hand, a much larger and permanent carbon tax is required to fully switch clean production. Similarly, the level of optimal subsidy to clean research is also lower and it is only temporarily needed in the high-substitutability case, while it is higher and lasts longer in the low-substitutability case because the switch to clean research also occurs much later.

Golosov et al. (2014) also discuss that the elasticity of substitution "does matter greatly for the quantity predictions." In the sensitivity analysis of their quantitative results, they consider an alternative case of high substitutability between fuels (the elasticity of substitution equal to 2) and find that the temperature starts to decline in the middle of the next century as a result of phasing out fossil fuel. This is in strong contrast to their baseline scenario with low substitutability (the elasticity of substitution equal to 0.95) where the temperature continuously increases even with optimal policy. They explain that when energy sources are highly substitutable, climate policy has a much stronger impact: the same amount of tax on fossil fuels leads to a larger shift towards non-fossil energy. Therefore, the social gains – or the costs of delay – from climate policy are much larger in the case of high substitutability.

Hart (2019) similarly observes from his calibrated model that the size of the substitution elasticity has substantial impacts on the results. In his model, carbon concentration in the atmosphere is almost 20 percent higher in the low-substitutability case than in the baseline with higher substitutability. The transition from dirty to clean energy is also much slower with very different implications for optimal subsidies for clean research.[6] Gans (2012) also explicitly discusses the cases of an elasticity of substitution between clean and dirty energy smaller and larger than one in studying how a tighter emissions cap would affect innovation. He finds that with a substitution elasticity below one, the emission cap would reduce innovation incentives for factor-augmenting technologies.

Although the sensitivity of the results (both theoretical and numerical) to the value the elasticity of substitution is widely recognized in the literature and the time frame of the analyses typically tends to be hundreds of years in previous work, the endogenous and variable nature of the substitution elasticity and its impact on climate policy has received very little attention. A few exceptions we are aware of include Gerlagh and Lise (2005) and Mattauch et al. (2015). Mattauch et al. (2015) consider an exogenous profile of linearly increasing elasticity of substitution and compare optimal policy response under the varying elasticity of substitution to that under scenarios with a low and a high elasticity fixed throughout.[7] Gerlagh and Lise (2005) study the role of induced innovation and learning-by-doing in emissions reduction in a model that features an endogenous elasticity of substitution between clean and dirty energy in the aggregate energy production function. In this paper, we attempt to address the gap in the literature by providing a dynamic general equilibrium model with an endogenous elasticity substitution that is designed to highlight the impact of an endogenous elasticity of substitution on the analysis of optimal climate policy compared to the CES approach standard in the literature.

[6] He considers a lower value of 2 compared to the baseline elasticity of substitution of 4.

[7] In the varying elasticity of substitution profile, it increases linearly from 3 to 10, the low and high benchmark value same as in Acemoglu et al. (2012), respectively.

3 Model

This section presents the model. We adapt the standard framework of directed technical change widely used in the literature with two inputs, clean and dirty. The model is designed to allow the elasticity of substitution between clean and dirty input to interact with the relative share of clean input and to investigate its impact on policy designs.

3.1 Preferences and final good technology

Our economy is in discrete time and inhabited by a continuum of households consisting of workers, entrepreneurs and scientists. The economy admits a representative household with the following preferences:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t}\, u(C_t, S_t), \tag{1}$$

where $u(C,S)$ is the instantaneous utility function. C_t is consumption of the unique final good at time t, S_t reflects the environmental quality at time t, and $\rho \geq 0$ is the discount rate. We assume that $u(C,S)$ is increasing in both C and S, twice differentiable, jointly concave in (C,S) and satisfies $\lim_{C\to 0} \frac{\partial u(C,S)}{\partial C} = \infty$, $\lim_{S\to 0} \frac{\partial u(C,S)}{\partial S} = \infty$, and $\lim_{S\to 0} u(C,S) = -\infty$. The quality of the environment is defined in the interval $(0, \bar{S})$ where $\bar{S}$ is the environmental quality absent any anthropogenic pollution. We assume $S_0 = \bar{S}$.

The unique final consumption good, Y is produced competitively from clean and dirty inputs, Y_c and Y_d, according to the Variable Elasticity of Substitution (VES) production function

$$Y_t = Y_{ct}^{\alpha}\,(Y_{dt} + \alpha\beta\, Y_{ct})^{(1-\alpha)}, \tag{2}$$

with the parameter constraints $0 < \alpha \leq 1$, $\beta > -1$, and $\frac{Y_{dt}}{Y_{ct}} \geq -\beta$ that ensure standard properties of a neoclassical production function are satisfied (Karagiannis et al., 2005).[8] Compared with the standard CES technology with a time-invariant and exogenous elasticity of substitution, the elasticity of substitution between clean and dirty inputs, σ_t, associated with the VES technology takes the following intuitive form:

$$\sigma_t = 1 + \beta\,\frac{Y_{ct}}{Y_{dt}}, \tag{3}$$

where $\sigma_t \lesseqgtr 1$ if $\beta \lesseqgtr 0$. The elasticity of substitution therefore interacts with the relative

[8] The limiting properties of (2) imply that, if $\beta > 0$, which is the empirically relevant case as will be shown in the next section, the marginal product of clean energy is bounded from below as the relative share of clean energy goes to infinity and therefore dirty energy is not essential in energy production in the long run.

share of clean input in the economy. It is also readily observed that β is a key parameter that defines the nature of the association between the two factors. More theoretical discussions on the properties of the VES function are found in Revankar (1971b) and Karagiannis et al. (2005).[9]

The quality of the environment S_t evolves according to the difference equation

$$S_{t+1} = -\xi Y_{dt} + (1+\delta)S_t, \tag{4}$$

where ξ captures the rate of environmental degradation caused by the dirty input and δ measures the rate of environmental regeneration. This specification captures the negative environmental externality caused by the production of the dirty input.

3.2 Intermediate input production

The two inputs, Y_c and Y_d, are produced competitively and sold at market prices to the final good producer. The production function for each input combines labor and machines in a constant returns to scale fashion:

$$Y_{ct} = L_{ct}^{1-\kappa} \int_0^1 A_{cit}^{1-\kappa} x_{cit}^{\kappa} di \quad \text{and} \quad Y_{dt} = L_{dt}^{1-\kappa} \int_0^1 A_{dit}^{1-\kappa} x_{dit}^{\kappa} di, \tag{5}$$

where $\kappa \in (0,1)$ and A_{jit} is the quality of machine used in sector $j \in \{c, d\}$ at time t and x_{jit} is the quantity of this machine. Labor L_{jt} is inelastically supplied and, with total labor supply normalized to 1, market clearing for labor requires

$$L_{ct} + L_{dt} \leq 1. \tag{6}$$

As is standard in the literature, machines x_{jit} are produced by monopolistically competitive firms. We assume that producing one unit of a machine takes ψ unit of the final good.

[9] Revankar (1971b) discusses other functional forms that allow a variable elasticity of substitution. For instance, one could think of the generalization of the CES to $Y_t = [\gamma {Y_{ct}}^{\eta} + (1-\gamma){Y_{ct}}^{m\eta}{Y_{dt}}^{(1-m)\eta}]^{\frac{1}{\eta}}$, which collapses to the CES when $m = 0$. However, unlike the function in (3), the elasticity of substitution of inputs from this specification takes a form that cannot be as easily interpreted. Moreover, the highly nonlinear nature of the CES generalization presents substantial econometric and numerical problems, which may explain the lack of empirical interest in this functional form (Revankar, 1971b; León-Ledesma et al., 2010; Genç and Bairam, 2018).

The market clearing condition for the final good reads

$$C_t = Y_t - \psi\left(\int_0^1 x_{cit}\, di + \int_0^1 x_{dit}\, di\right). \tag{7}$$

3.3 Innovation

In each period, scientists decide whether to direct their research to clean or dirty technology. They are then randomly allocated to at most one machine in that sector and are successful in innovation with probability $\eta_j \in (0,1)$ in sector $j \in \{c,d\}$. Successful innovation increases the quality of a machine by a factor of $1+\gamma$. The measure of scientists s is also normalized to 1 and we denote the mass of scientists working on machines in sector $j \in \{c,d\}$ at time t by s_{jt}. Market clearing for scientists then implies

$$s_{ct} + s_{dt} \leq 1. \tag{8}$$

Next, we define

$$A_{jt} = \int_0^1 A_{jit} di \tag{9}$$

as the average productivity in sector $j \in \{c,d\}$. Finally, taking into account all the elements comprising the innovation possibilities frontier explained above, the productivity in each sector j evolves according to

$$A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1}. \tag{10}$$

3.4 Equilibrium

The socially optimal equilibrium is given by a sequence of the final good production Y_t, consumption C_t, inputs productions Y_{jt}, machines productions x_{jit}, labor allocations L_{jt}, scientist allocations s_{jt}, environmental quality S_t, and qualities of machines A_{jit} such that the social planner maximizes the representative consumer's intertemporal utility (1) subject to (2), (4), (5), (6), (7), (8), and (10). It is assumed that both taxes on dirty input production τ_t and research subsidies for innovation in the clean sector q_t are available to the social planner. In addition, she will implement a subsidy on the use of all machines in order to remove the static monopoly distortion, which will allow more intensive use of existing machines. Following the literature, we assume that a socially optimal allocation of resources can be achieved by lump-sum taxes and transfers.

3.5 Discussion

The model embeds the VES technology in the standard framework of directed technical change in order to examine how an endogenously evolving elasticity of substitution affects the analyses of optimal environmental policy. It reveals dynamic feedback effects that arise from an endogenously evolving elasticity of substitution in the standard framework of directed technical change: an increase in demand for the clean input (given a climate policy) leads to higher demand for clean technology, which lowers the price of the clean input. The lower price of clean energy further increases its demand, which in turn improves the substitution capacity according to (3) and enables a larger increase in the demand for clean inputs. Thus, an endogenous elasticity of substitution that grows in the relative use of clean inputs amplifies the effect of directed technical change and accelerates the switch to clean production and technology in a virtuous cycle.

However, we note that the current framework is a conservative setting to study the dynamics induced by endogenous substitution capacity. To see this, the relationship between relative market shares and productivities is given by (see Appendix A for derivation)

$$\frac{L_{ct}}{L_{dt}} = \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\kappa)}}{\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{(1+\tau_t)}\left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\kappa)} - \beta}. \tag{11}$$

The expression suggests that an increase in the relative productivity of the clean sector will raise its relative market share; a higher carbon tax τ_t will also increase the market share of the clean input. It also implies that in our framework the relative productivity of the clean sector A_{ct}/A_{dt} has to be bounded from above in order for the market size of the dirty sector to remain non-negative; in other words, $\frac{A_{ct}}{A_{dt}} \leq \left(\frac{1-\alpha}{\alpha\beta}\right)^{\frac{1}{1-\kappa}}$.[10] This property puts an upper bound on the growth of the relative productivity of the clean sector induced by increasing penetration of clean inputs in production (see (A.7)). It is in contrast to the standard framework of directed technical change with a CES final good technology where the relative productivity of the clean sector can grow infinitely (e.g., Acemoglu et al., 2012). This feature of our model makes our framework a conservative setting for exploring the impact of dynamic substitution capacity on optimal designs of climate policy.

[10] Acemoglu et al. (2012) imposes a similar constraint but on the initial relative productivity of the clean sector, rather than on its growth. Assumption 1 in their set-up imposes that initially the productivity of the clean sector is sufficiently backward relative to the dirty sector. This assumption allows a strong contrast between an equilibrium where no climate policy is imposed and an environmental disaster arrives (because innovation only occurs in the dirty sector without climate policy) and a socially optimal equilibrium where optimal policies are implemented and an environmental disaster is averted by directing the path of innovation to clean technologies.

4 Empirical relevance of the VES technology

A novel feature of our theoretical model is the VES final good technology with an endogenous elasticity of substitution. In this section, we provide evidence for the empirical relevance of the VES technology and estimate a key parameter of our theoretical model, β, for subsequent numerical exercises. We first introduce the data used in the analysis, describe the estimation strategy and report the results.

4.1 Data

We use plant-level data on energy consumption and expenditure by fuel (Enquête sur les Consommations d'Énergie dans l'Industrie) provided by the French National Institute of Statistics and Economic Studies that covers a representative sample of manufacturing plants with at least 20 employees in France from 1990–2017. The survey provides information on energy consumption and expenditure by fuel at the plant level, which we exploit for identification. Our sample covers a total of 30,142 plants in 19 industries. Table A1 provides descriptive statistics of the key variables used in the analysis by industry.

Following Papageorgiou et al. (2017) and Jo (2020), we aggregate energy use by fuel to a clean and a dirty bundle for each plant by adding up electricity, steam and renewables into the clean aggregate and all other types (natural gas, petroleum products, etc) into the dirty aggregate.[11] The French context offers a conceptual advantage in classifying electricity as a clean energy source, given that approximately 80 percent of electricity is produced by nuclear power and greenhouse gas intensities of nuclear power generation tend to be considerably lower than those of fossil technologies.[12] We then construct plant-level unit prices of energy by dividing total expenditure by total consumption for each energy type (clean and dirty).[13] Plant-level variation in the unit price of energy largely comes from strong quantity discounts (Marin and Vona, 2021). Energy purchase prices are deflated by the GDP deflator to reflect real prices.

[11] Information on the use of renewable energy sources is included in the survey from 2005. Thus, up to 2004, only electricity and steam comprise the clean energy aggregate.

[12] Lenzen (2008) report that greenhouse gas intensity of nuclear power generation is between 10 and 130 g CO_2-e per kWh, with an average of 65 g CO_2-e per kWh, which are significantly lower than those of fossil technologies (typically 600–1200 g CO_2-e per kWh).

[13] For example, total expenditure for clean energy is the sum of expenditure on fuels in the clean energy bundle. This is divided by the corresponding consumption measure to obtain the unit price of clean energy at the plant level. The plant-level unit price of dirty energy is obtained similarly.

4.2 Estimation

One of the advantages of the VES functional form is that it allows a convenient testing of its empirical relevance (Revankar, 1971b; Karagiannis et al., 2005). To see this, the first-order conditions with respect to clean and dirty inputs from (2) are given by (with a plant subscript i)

$$\alpha \frac{Y_{it}}{Y_{cit}} + (1-\alpha)\alpha\beta \frac{Y_{it}}{Y_{dit} + \alpha\beta\, Y_{cit}} = p_{cit}, \tag{12a}$$

$$\frac{(1-\alpha)Y_{it}}{Y_{dit} + \alpha\beta\, Y_{cit}} = p_{dit}. \tag{12b}$$

Combining the two expressions and rearranging it yields

$$\frac{Y_{dit}}{Y_{cit}} = -\beta + \frac{(1-\alpha)}{\alpha}\frac{p_{cit}}{p_{dit}}. \tag{13}$$

Examining whether $\beta = 0$ in (13) provides a straightforward approach to testing the empirical relevance of a variable elasticity of substitution. In the case of $\beta = 0$, the elasticity of substitution under the VES technology in (3) would collapse to 1 as implied by the Cobb-Douglas functional form rather than interacting with the relative input ratio. We estimate the equation above to examine the empirical plausibility of a variable elasticity of substitution between the two types of energy.

Obtaining unbiased estimates from (13) requires that the relative price ratio be uncorrelated with the error term in the regression. However, it is plausible that there exists omitted variable bias such as productivity shocks that may affect plant-specific fuel prices and demands. That is, to the extent that plants take into account their factor-specific productivity when choosing inputs, it would affect their relative input demands as well as relative input prices through resulting quantity discounts.[14]

To account for such omitted variable bias, we follow the approach taken in earlier studies and develop instruments for the plant-specific price of clean and dirty energy that rely on national energy prices (Linn, 2008; Sato et al., 2019; Dussaux, 2020; Jo, 2020; Marin and Vona, 2021). Specifically, the instrument for the plant-level price of clean energy p^{IV}_{cit} is constructed as follows:

$$p^{IV}_{cit} = p_{cit_0} \times \prod_{j=1}^{t}(1 + G^{N}_{cj}), \tag{14}$$

[14] For example, a plant experiencing a positive productivity shock associated with clean energy might increase its demand for clean energy, which may lower its unit price through quantity discounts, leading to a change in the price ratio.

where p_{cit_0} is the unit price of clean energy of plant i in the pre-sample period (1989) and G^N_{ct} is the growth rate in the national average price of clean energy between years t and $t-1$.[15] Intuitively, the instrument for the clean energy price grows from the observed price in the pre-sample period ($t = t_0$) at the same rates as the national average price of clean energy in subsequent years. Since time variation only comes from changes in energy prices at the national level, the specification makes it unlikely that unobservable productivity shocks in a given plant are correlated with the instrument. The pre-sample unit price of energy provides information on the relative intensity of clean energy consumption (a lower unit price associated with higher consumption through larger quantity discounts) and makes the instrument sufficiently strong for the plant-level energy prices, avoiding a weak-instrument problem.[16] The same logic applies in constructing p^{IV}_{dit}. We construct the ratio from the instruments, $p^{IV}_{cit}/p^{IV}_{dit}$, and use it to instrument for the relative price ratio in (13). Standard errors are clustered at the plant level.

Finally, note that we do not include plant-fixed effects for two reasons. First, in fixed effects model, individual effects (for each plant) contain a constant term and consequently, the constant term cannot be separately estimated (Greene, 2000). Since the goal here is to estimate the significance and the sign of the constant term, we do not estimate fixed effects model. Furthermore, it is known that exploiting time-variation in time-series data or in panel data with fixed effects captures short-term substitution, while exploiting cross-sectional variation captures long-term substitution (Arnberg and Bjørner, 2007). Therefore, not including fixed effects allows us to interpret β as a long-run elasticity of substitution, or a key element that forms the parameter as in (3), which corresponds more closely to the theoretical literature.

Table 1: Tests of the empirical relevance of the Variable Elasticity of Substitution functional form

	Dependent variable: $\frac{Y_{dit}}{Y_{cit}}$			
	(1)	(2)	(3)	(4)
$\frac{p_{cit}}{p_{dit}}$	0.336***	1.826***	1.826***	2.162***
	(0.120)	(0.125)	(0.125)	(0.221)
t			0.028	0.039
			(0.017)	(0.031)
Constant	2.065***	-3.411***	-3.467***	-4.626***
	(0.554)	(0.616)	(0.641)	(1.180)
First stage F statistic		1157.98	1157.98	1247.25
Observations	169,182	169,182	169,182	169,182

Notes: Estimates from equation (13). Column (1) reports OLS estimates and column (2) reports IV estimates. Column (3) adds a time trend as a control in the IV specification. Column (4) weights the regression by total energy consumption. All specifications include year, sector, and region fixed effects. Standard errors are clustered at the plant level.

4.3 Empirical results

Table 1 reports the results from this exercise. Of interest are the significance and the sign of the constant, which corresponds to $-\beta$ in equation (13) – a key parameter of our theoretical model. Across all specifications, we reject the hypothesis that the constant is zero, which indicates that the VES technology where the elasticity of substitution interacts with the relative input ratio is empirically relevant. The positive sign of the OLS estimate in column (1) suggests that β is negative and the elasticity of substitution is decreasing in the ratio of clean to dirty energy. However, when we use the instruments to account for the potential endogeneity of the plant-level price ratio, the sign of the estimates turns negative

[15] Earlier studies have used a slightly different specification that weights national prices of different fuels using the pre-sample plant-specific fuel mix as weights ('shift-share' instruments). In both specifications, time variation comes from movements in energy prices at the national level. However, the shift-share instrument retains plant-level variation by fixing the plant-specific fuel mix, while our specification retains plant-level variation by fixing the initial plant-specific unit price of energy. This is because the plant-specific fuel mix does not provide as strong plant-level variation in the current setting, where energy is already partitioned into the clean and dirty bundle with the two most popular fuels, electricity and natural gas, belonging to each bundle (compared to other studies where energy is considered as a whole). In particular, the share of electricity in the clean bundle is very high (98 percent on average) and does not vary substantially across plants.

[16] At the same time, the validity of the instrument also assumes that the initial plant-level energy prices in $t = 0$ are not correlated with unobservable productivity shocks at the plant level in subsequent periods. This assumption of no serial correlation in idiosyncratic productivity shocks is common in the literature (e.g., Olley and Pakes, 1996).

in the next columns. With region and industry fixed effects in column (2), the constant is statistically significant at 1 percent level and negative. Including a time trend and weighting the regression by plants' total energy consumption continues to produce negative and statistically significant estimates of the constant. Table A2 reports additional IV specifications that use the two instruments separately, rather than as a ratio, which yields qualitatively similar results.

The negative sign of the estimates suggests that β is positive and therefore points to a positive association between the elasticity of substitution and the ratio of clean to dirty energy. The implied positive association is consistent the view that substitution possibilities between clean and dirty inputs are likely to improve as technology and infrastructure for the use of renewable energy sources expand over time (Mattauch et al., 2015; Kemp-Benedict, 2018). In addition to serving as strong evidence for the empirical relevance of the VES energy aggregate, these results provide us with empirical guidance for calibrating an important parameter of the theoretical model, β, in our numerical exercises.

In Appendix, we examine the possibility that the empirical support for the relevance of the VES is driven by the choice of the functional form in (2). We do so by estimating the elasticity of substitution from the standard CES function for each year between 1990 and 2010, thus estimating the parameter at each point in time and examining its evolution over time. The results from this additional exercise corroborate our findings in Table 1. Figure A1 shows a clear increasing trend in the cross-sectional estimates.

5 Calibration

Having established the empirical relevance of our theoretical model that features a variable elasticity of substitution, we now calibrate our model. The objective is to perform two exercises to numerically explore the implications of endogenous substitution capacity in the analyses of climate policy. In both exercises, the economy begins from the same initial technology levels in the clean and dirty sector, but substitution capacity is endogenous and time-varying in the first case and exogenous and fixed in the second case. For the second exercise, we simulate the socially optimal equilibrium in an economy with the following CES final good technology as in Acemoglu et al. (2012):

$$Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

with $\epsilon = 3$.

The set-up of intermediate input production and innovation is identical in both economies and consequently the numerical analysis below highlights the impact of an endogenously evolving elasticity of substitution.

5.1 Parameter choices

In choosing parameters, we attempt to remain as close as possible to previous work that adopts a standard CES technology in the directed technical change framework. One period in our model corresponds to 5 years. Following earlier studies (e.g., Acemoglu et al., 2012; Fried, 2018; Hart, 2019), we set $\eta_c = \eta_d = \eta = 0.02$ and $\gamma = 1$ and the machine share of intermediate input production κ equal to 1/3. We assume that there is initially no climate policy; however, the subsidy to machines is implemented throughout simulation in order to focus on the effects of environmental externalities. We compute the levels of clean and dirty technologies one period before the optimal climate policy is implemented, A_{c0} and A_{d0}, using data on the production of fossil and non-fossil energy in the world primary energy supply from 2010 – 2015.[17]

Next, we map changes in CO_2 emissions to the quality of environmental quality S_t by adopting the approximation used in Acemoglu et al. (2012)

$$\Delta \simeq 3 \log_2 \left(\frac{C_{CO_2}}{280} \right), \tag{15}$$

where Δ is the increase in the global mean temperature from the pre-industrial level in degrees Celsius and C_{CO_2} captures the atmospheric CO_2 concentration in parts per million (ppm). The mapping indicates that a doubling of atmospheric CO_2 concentration (from the pre-industrial level of 280 ppm) leads to a 3 degrees Celsius increase in the global mean temperature. Defining an increase of 6 degrees Celsius as an environmental disaster, Δ_d, and $C_{CO_2,d}$ as the corresponding level of CO_2 concentration in the atmosphere, we set $S_t = C_{CO_2,d} - \max\{C_{CO_2}, 280\}$. To pin down the initial environmental quality S_0, we use the average atmospheric concentration of 393 ppm between 2010 - 2015 for simulation forward.[18]

We compute ξ from the observed value of Y_d and emissions between 2010 and 2015 and set δ such that only half of the emitted CO_2 emissions contributes to increasing the stock of atmospheric concentration of emissions and the other half is offset by environmental regeneration, which leads to a value of $\delta = 0.014$.

[17] Although we estimate a key parameter of our model from French micro data, our numerical analysis uses data from the world primary energy supply in order to be informative in a broader context and compatible with earlier studies such as Acemoglu et al. (2012) that calibrate their model to world data.

[18] The data comes from the Environmental Protection Agency (EPA) and is available at www.epa.gov/climate-indicators.

Table 2: Parameter values

Parameter	Description	Value
ρ	Discount rate	0.015
σ	Risk aversion	2
λ	Damage parameter	0.144
ξ	Environmental degradation parameter	0.002
δ	Environmental regeneration parameter	0.014
α	Distribution parameter	0.163
β	Substitution capacity	4
κ	Machine share in intermediate goods	0.333
γ	Gain in productivity from innovation	1
η	Probability of successful innovation	0.02

The utility function takes the following CRRA form

$$u(C_t, S_t) = \frac{(\phi(S_t)C_t)^{1-\sigma}}{1-\sigma}, \tag{16}$$

where σ is set to 2. Further, we adopt the following function used in Acemoglu et al. (2012) that relates the deteriorating environmental quality to economic costs:

$$\phi(S) = \frac{(\Delta_d - \Delta(S))^{\lambda} - \lambda \Delta_d^{\lambda-1}(\Delta_d - \Delta(S))}{(1-\lambda)\Delta_d^{\lambda}}, \tag{17}$$

where λ is calibrated to match Nordhous's damage function over the range of temperature increases up to 3 degrees Celsius, which leads to $\lambda = 0.1443$. Finally, we use Nordhaus's discount rate of $\rho = 0.015$ in our baseline analysis (Nordhaus, 2007). Given the documented influence of the discount rate on the form of optimal climate policy (e.g., Heal and Millner, 2014), we also try different values in the sensitivity analysis in Section 6.2.

Our framework has two key parameters relevant for the VES technology, namely, α and β. For β that directly regulates the extent to which the elasticity of substitution responds to the relative use of clean inputs in the economy, we choose a value of 4 in our baseline analysis based on the empirical analysis in the previous section. Section 6.2 explores the sensitivity of the results to varying values of β. To calibrate α, we note that the goal of our numerical exercise is to examine the impact of having an endogenous elasticity of substitution compared to the standard constant elasticity of substitution on the optimal policy. Thus, we calibrate α such that the initial technology gap, A_{c0}/A_{d0}, implied by our choice of α (given $\beta = 4$) is equal to the initial technology levels implied by the benchmark CES case with $\epsilon = 3$. This approach ensures that our simulation of the two economies (one with VES and the other with CES) begins from the same starting point.[19] The resulting value of α is equal to 0.163. Table 2 collates the parameters of the model and their values.

6 Results

6.1 The effect of an endogenous elasticity of substitution

Panel A in Figure 2 shows the profile of an optimal carbon tax in both economies. In the VES case, the carbon tax starts increasing initially but is no longer required after around 150 years because the transition to clean production and technology occurs earlier in the VES economy (Panel C and F). In contrast, the carbon tax in the CES case increases continuously beyond 200 years and remains at a very high level. This shows the strong implications of allowing for an endogenous elasticity of substitution: although the two economies begin from the same initial technology gap, the optimal carbon tax required to induce the switch to clean production and technology in the VES case is much lower and temporary, while the tax in the CES case is permanent.

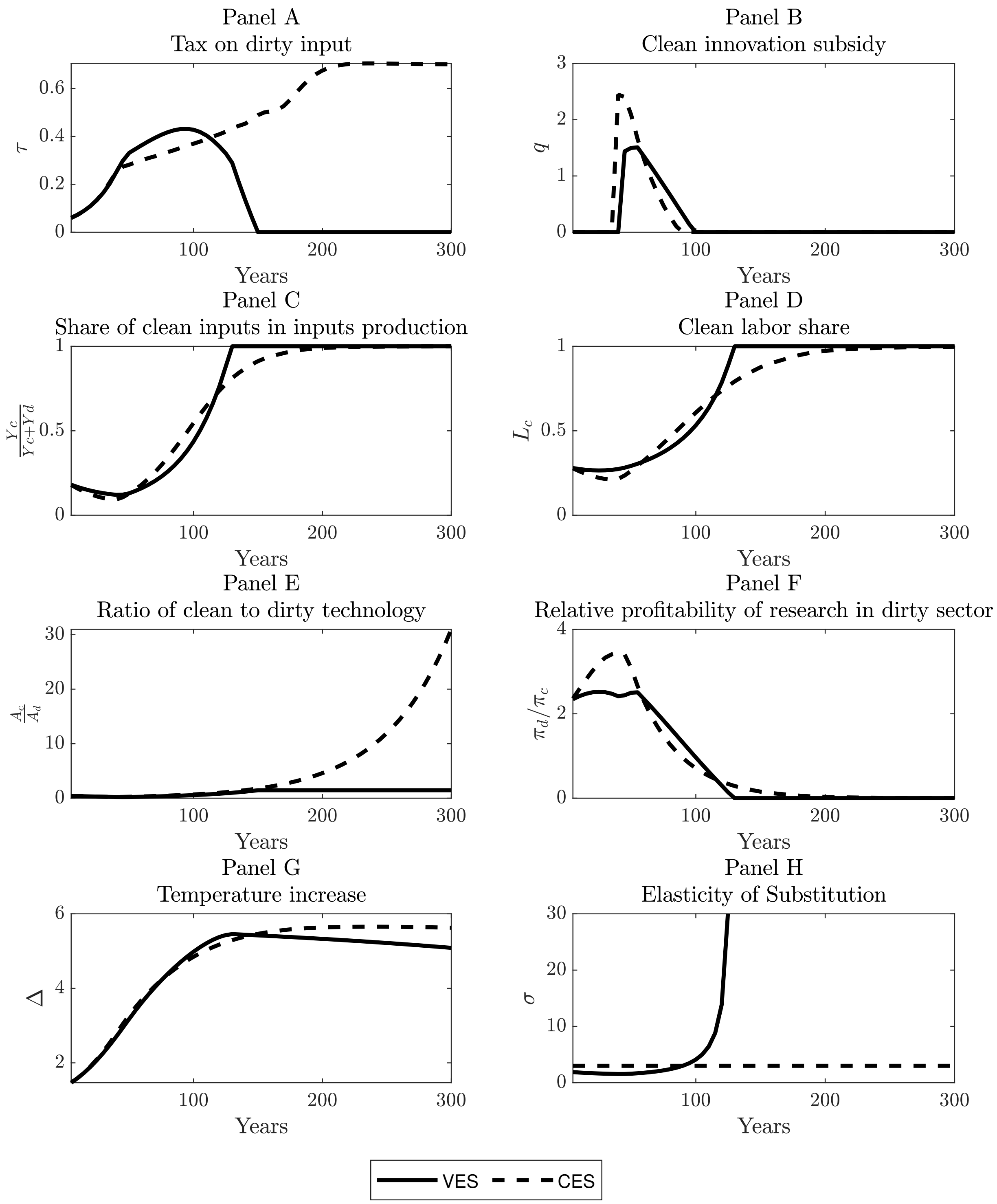
Therefore, the tax profile in the VES economy resembles that of a CES economy with a very high level of elasticity of substitution – for example, $\epsilon = 10$ in Acemoglu et al. (2012) – that predicts a temporary carbon tax as optimal policy. Our finding demonstrates that such an optimistic policy recommendation is not restricted to a scenario where strong substitutability between clean and dirty inputs is assumed throughout. In other words, relaxing the assumption of a fixed, exogenous elasticity of substitution and endogenizing the economy's input substitution capacity can lead to a more optimistic policy design *even when* the economy starts from a relative low, and empirically plausible, initial level of the substitution elasticity.[20] Panel H depicts that the elasticity of substitution in the VES, although initially lower than the chosen value of the parameter for the CES economy, rapidly increases after about 100 years as the transition to clean production kicks off around the same time (Panel C).

Panel B shows that the optimal subsidy to clean research is temporary in both VES and CES economies but lower in the VES case. This is due to the delay in the switch to

[19] It is also possible to obtain α from the coefficient on the relative price of clean input, p_{cit}/p_{dit}, from Table 1, which leads to a value around 0.3. However, fixing both α and β from Table 1 comes at the cost of not being able to ensure the same starting point in the VES and CES economies for simulation. Thus, we choose to use an empirically informed β, which is a key parameter that directly regulates the relationship between the elasticity of substitution and the relative use of clean inputs, and to calibrate α in a way that ensures the same starting point for a meaningful comparison of the two simulated economies.

[20] Given the data and the chosen parameter ($\beta = 4$), the initial elasticity of substitution in the VES economy is 1.9 according to (3). Prior empirical studies estimate the parameter to be around 2 or 3 (Papageorgiou et al., 2017; Jo, 2020).

Figure 2: Optimal climate policy in the VES and CES economies



clean innovation by almost 100 years in the CES case, although the relative profitability of research in the dirty (or clean) sector is initially the same in both VES and CES case (Panel F), which is ensured by our calibration strategy.[21]

We observe from Panel E the difference in the relative productivity of the clean sector between the VES and CES frameworks. In the CES case, the relative technology grows continuously, while the technical constraint on the growth of the relative productivity of the clean sector in the VES economy starts to bind in the middle of the simulation period. As discussed in Section 3.5, the constraint is precisely defined by our parameters, i.e., $(\frac{1-\alpha}{\alpha\beta})^{\frac{1}{1-\kappa}}$, and makes the VES framework a conservative setting for studying how the speed of energy transition is affected by an endogenous elasticity of substitution by restricting the range of growth in the relative productivity of the clean sector. Despite the setup being conservative, the VES economy nonetheless fully switches to clean production (Panel C and D) a couple of decades before the relative productivity of the clean sector hits the upper bound and also before the CES economy does so. This is achieved by the rapidly expanding elasticity of substitution (Panel H) that amplifies the effects of directed technical change and accelerates the process of the transition to green economy.

Finally, Panel G shows that temperature continues to increase in the CES case for about 250 years and remains fairly close to the disaster level of a 6 degrees Celsius increase. On the other hand, temperature in the VES case starts to decrease after around 130 years after exhibiting a similarly strong increase in temperature as in the CES case up to that point. To sum up, the results point to strong implications of incorporating endogenous input substitution in the analyses of optimal environmental policy: allowing for the substitution elasticity to evolve over time leads to a lower optimal carbon tax implemented for a shorter duration, a lower subsidy to clean innovation, and a more swift transition to clean production and technology, compared to the case where the elasticity of substitution is fixed at a constant level.

6.2 Sensitivity analysis

Here we examine the sensitivity of the optimal policies and transition dynamics to the model's two parameters – the discount rate, ρ, and the strength of relationship between the input

[21] This is in contrast to the comparison between two CES economies characterized by different levels of elasticities of substitution where the initial technology gaps adjust to the values of the parameter: a higher elasticity of substitution is associated with a smaller gap between the initial technology levels in the clean and dirty sector. As a result, a quicker switch to clean production and technology in the high substitutability case is partly driven by the smaller gap for the clean sector to catch up, compared to the low-substitutability case. Our calibration strategy that ensures both economies begin from the same technology gaps makes it straightforward to compare the transition dynamics across the VES and CES frameworks.

ratio and the substitution elasticity, β.

We try different discount rates ranging from 0.1% to 3% and find that the dynamics are qualitatively similar to our main results.[22] All else equal, economies with lower discount rates tend to undergo a more rapid transition to clean production, while the transition is delayed by a couple of decades in the economy with the highest discount rate ($\rho = 0.03$). The optimal tax tends to be lower and lasts for a shorter period of time when discount rates are lower. This is because of the faster switch to clean inputs associated with lower discount rates. Further, subsidies kick in earlier with lower discount rates than the cases with higher discount rates, leading to a rapid transition to clean research. Figure A2 reports the detailed results.

Our main results are also robust to the varying strength of the relationship between the input ratio and the elasticity of substitution or, put differently, how responsive the substitution elasticity is to the change in the input ratio.[23] Figure A3 shows that even if substitution capacity does not improve as much when the relative use of clean inputs increases (for example, $\beta = 2$ compared to 4 in the baseline calibration), the transition to clean production can still be achieved with a delay of 20 years. However, intuitively, a stronger feedback between the elasticity of substitution and the penetration of clean inputs is associated with a lower optimal tax that also lasts for a shorter period of time. This is in line with the rapidly increasing elasticity of substitution (due to its stronger responsiveness to the increasing relative use of clean inputs) that induces a faster transition to clean production and renders the carbon tax unnecessary. Further, we observe that a higher β is associated with higher subsidies that last for a shorter period of time, although the differences in subsidy profiles across different values of β are small. We conjecture that with a more responsive substitutability, the transition to clean research is achieved by a brief yet stronger push, rather than a weaker one over a longer period.

6.3 Endogenous substitution in a second best scenario

Next, we test the strength of the endogenous substitution elasticity on the transition to a clean economy by examining a second best scenario with no carbon tax available. A carbon tax directly increases the substitutability by affecting the relative input ratio per (3) as well as indirectly by inducing directed technical change. On the other hand, the subsidy for clean research affects the degree of substitutability only through the channel of directed technical

[22] This range spans the interval between the two extreme views of Stern (Stern et al., 2006) and Nordhaus (Nordhaus, 2007). It also encompasses the majority of the preferred values reported in the expert survey by Drupp et al. (2018) (they report 0.5% and 1.1% as the corresponding mean and median of preferred values).

[23] Note that varying β implies recalibrating α such that the initial technology gap is maintained. Hence Figure A3 shows the sensitivity to the joint changes in $\{\alpha, \beta\}$.

change. Thus, the goal is to explore whether an endogenously evolving substitutability between clean and dirty inputs can still accelerate the transition as we have seen in Section 6.1 even if a carbon tax that directly expands the substitution capacity is unavailable to the social planner.

Table 3: Difference in subsidies between the baseline and only-subsidy case

	Initial subsidy	Peak subsidy	Duration
Baseline	1.44	1.51	55 years
Only-subsidy case	2.24	2.65	80 years

Note: Optimal subsidies for clean research in the baseline with both carbon tax and subsidies and in the only-subsidy case.

We find that the effect of an endogenous elasticity of substitution remains strong without direct impacts of a carbon tax: the switch to clean production and research occurs virtually at the same time as in the first best with both policy instruments (Figure A4). Intuitively, this is achieved by higher optimal subsidies for clean research that also last a longer period of time compared to those in the first best. Yet, the difference in the subsidies across the two cases is not large. Table 3 shows that the peak subsidy in the second best is 1.8 times larger relative to the first best and lasts 25 years longer. It is noteworthy that the peak subsidy in the second best of the VES economy is only as high as that in the first best of the CES economy (Panel B in Figure 2).

7 Conclusion

In this paper, we made an attempt to extend the important literature on endogenous growth and climate change in a novel way by developing a dynamic general equilibrium model with an endogenous elasticity of substitution that flexibly interacts with the relative share of clean inputs in the economy. The model highlights dynamic feedback effects arising from endogenous substitution capacity: an increase in demand for the clean input (given a climate policy) leads to higher demand for clean technology, which lowers the price of the clean input. The lower price further increases its demand, which at the same time expands the substitution capacity and enables a larger increase in the demand for clean inputs, amplifying the effect of directed technical change and accelerating the switch to clean production and technology in a virtuous cycle.

The dynamic feedback effects lead to substantial differences in the design of optimal environmental policy. We find that the optimal carbon tax in the VES economy is much lower and only temporarily required even when the economy starts at a relative low, and empirically plausible, initial level of the substitution elasticity, while the CES economy is associated with a higher and permanent carbon tax. The subsidy to clean innovation is also lower in the VES than in the CES case. This result demonstrates that such an optimistic policy recommendation need not necessarily come from a scenario where strong substitutability between clean and dirty inputs is assumed throughout. We show that relaxing the assumption of a fixed, exogenous elasticity of substitution and endogenizing the economy's input substitution capacity can lead to a more optimistic policy design.

We believe our analysis opens new venues for future research. There are many questions to ask and answer: are there alternative modelling approaches that allow an endogenous elasticity of substitution other than the approach we adopted in this paper? What are the mechanisms at the micro level behind the expanding substitution capacity? We believe future research along the lines of these questions will deepen our understanding of sustainable growth and facilitate the economic analyses of optimal environmental policy.

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Appendix

A Solving for the socially optimal equilibrium

The social planner maximizes the representative consumer's intertemporal utility (1) subject to (2), (4), (5), (6), (7), (8), and (10). The shadow price of input j in time t denoted by $\hat{p}_{jt}$ can be derived from the first order conditions with respect to Y_{ct} and Y_{dt}

$$\alpha \frac{Y_t}{Y_{ct}} + (1-\alpha)\alpha\beta \frac{Y_t}{Y_{dt} + \alpha\beta\, Y_{ct}} = \frac{\lambda_{ct}}{\lambda_t} \equiv \hat{p}_{ct}, \tag{A.1a}$$

$$\frac{(1-\alpha)Y_t}{Y_{dt} + \alpha\beta\, Y_{ct}} - \frac{\omega_{t+1}\xi}{\lambda_t} = \frac{\lambda_{dt}}{\lambda_t} \equiv \hat{p}_{dt}, \tag{A.1b}$$

where λ_t, λ_{jt}, and ω_t are the Lagrangian multipliers for (2), (5) and (4). The shadow price of the clean input is equal to its marginal product. In contrast, the shadow price of the dirty input takes into account the environmental damage associated with the additional unit of dirty input production, $\omega_{t+1}\xi/\lambda_t$. This is equivalent to a tax on the use of dirty input by the final good producer $\tau_t = \omega_{t+1}\xi/\lambda_t\hat{p}_{dt}$. Combining the two expressions, the relative price of the two inputs implies

$$\frac{\alpha}{1-\alpha}\left(\frac{Y_{dt}}{Y_{ct}} + \beta\right) = \frac{\hat{p}_{ct}}{\hat{p}_{dt}(1+\tau_t)}. \tag{A.2}$$

The equation formalizes the intuition that the relative price of clean inputs decreases in their relative supply. With the price of the final good normalized to one, the price index of the clean and dirty inputs (including the tax) is given by

$$(\hat{p}_{ct} - \alpha\beta\hat{p}_{dt}(1+\tau_t))\left(\frac{\hat{p}_{dt}(1+\tau_t)}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}} + (1-\alpha) = 1. \tag{A.3}$$

The social planner also corrects for the monopoly distortion by providing a subsidy for the use of machines. The price of machines in the monopolistic competitive market involves a constant markup $1/\kappa$ above the marginal cost of producing a machine, ψ. The subsidy of $1-\kappa$ equates their price to the marginal cost, i.e., $(1-(1-\kappa))\psi/\kappa = \psi$. Given the price of machines, the demand for machines from intermediate input producers in sector $j \in \{c,d\}$ gives

$$x_{jit} = \left(\frac{\kappa}{\psi}\hat{p}_{jt}\right)^{1/(1-\kappa)} A_{jit}L_{jt}. \tag{A.4}$$

Combining (A.4) with (5), we derive the production of intermediate input $j \in \{c,d\}$

$$Y_{jt} = \left(\frac{\kappa}{\psi}\hat{p}_{jt}\right)^{\kappa/(1-\kappa)} A_{jt}L_{jt}. \tag{A.5}$$

Next, we note that in the socially optimal equilibrium, the relative price of the clean and dirty inputs satisfies

$$\frac{\hat{p}_{ct}}{\hat{p}_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\kappa)}, \tag{A.6}$$

which implies that the input produced with less productive machines is relatively more expensive.[24] Combining (A.2) and (A.6) yields the following relationship between the relative input share and the relative productivities:

$$\left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\kappa)} = \left(\beta + \frac{Y_{dit}}{Y_{cit}}\right)\frac{\alpha}{(1-\alpha)}(1+\tau_t), \tag{A.7}$$

which shows that an increase in the relative use of the clean input increases the relative productivity in the clean sector (a decrease in the relative use of the dirty input decreases the relative productivity in the dirty sector). Equation (11) is obtained by combining (A.5), (A.6) and (A.2).

The equilibrium thus makes clear the dynamic feedback effects that arise from allowing for an endogenous elasticity of substitution in the standard framework of directed technical change: an increase in demand for the clean input (given a climate policy) leads to higher demand for clean technology (by (A.7)), which lowers the price of the clean input (by (A.6)). The lower price of clean energy further increases its demand (by (A.2)), which in turn improves the substitution capacity according to (3), amplifying the effect of directed technical change and accelerating the switch to clean production and technology in a virtuous cycle.

Finally, we characterize a subsidy q_t to clean research in the socially optimal allocation. Given that pretax profits of machine producers are $\pi_{jit} = (1-\kappa)(\kappa/\psi)^{\kappa/(1-\kappa)}\hat{p}_{jt}^{1/(1-\kappa)}A_{jit}L_{jt}$, the ratio of the expected profit from innovation in sector c relative to sector d with subsidy q_t is given by (using (10), (A.6), and (11))

$$\frac{\Pi_{ct}}{\Pi_{dt}} = (1+q_t)\frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_d s_{dt}}{1+\gamma\eta_c s_{ct}}\right)\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{1+\tau_t} - \beta\left\{\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)\frac{A_{ct-1}}{A_{dt-1}}\right\}^{(1-\kappa)}\right]^{-1}. \tag{A.8}$$

The social planner will choose q_t such that this ratio is greater than 1 when the optimal allocation involves $s_{ct} = 1$. When the optimal allocation involves $s_{ct} \in (0,1)$, then q_t is set to satisfy $\Pi_{ct}/\Pi_{dt} = 1$.

[24] This expression is derived by setting the wage for labor equal across the two sectors and using (A.4).

B Detailed description of micro data

The EACEI (Enquête sur les Consommations d'Énergie dans l'Industrie) provided by the French National Institute of Statistics and Economic Studies that covers a representative sample of manufacturing plants with at least 20 employees in France. Below we define the variables used in the main analysis.

- Clean energy consumption (Y_{cit}): Amount of electricity, steam and renewables consumed in the calendar year in tonne of oil equivalent (TOE).

- Dirty energy consumption (Y_{dit}): Amount of natural gas, other types of gas, coal, lignite, coke, propane, butane, heavy fuel oil, heating oil and other petroleum products consumed in the calendar year in TOE.

- Unit price of clean energy (p_{cit}): Expenditure on clean energy purchase (electricity, steam and renewables) in the calendar year deflated by GDP deflator and divided by clean energy consumption (Y_{cit}). Thus, using self-generated electricity or steam (not purchased) lowers the firm's unit price of energy.

- Unit price of dirty energy (p_{dit}): Expenditure on dirty energy purchase in the calendar year deflated by GDP deflator and divided by dirty energy consumption (Y_{dit}).

- Weights: EACEI sample weights are used in the baseline regressions and EACEI sample weights multiplied by the total energy consumption are used in the regressions weighted by total energy consumption (Y_{it}).

Table A1 reports descriptive statistics of variables by industry. The growth of the relative share of clean energy ranges from -1.7% to 10.7% and is positive in most industries (column (1)). This observation is consistent with the decreasing relative price of clean to dirty energy over time in all industries (column (2)).

Table A1: Descriptive statistics

	Plants	Obs	(1) Y_c/Y_d	(2) p_c/p_d	(3) Y_c/Y	(4) Y_d/Y	(5) p_c	(6) p_d
Steel	58	432	0.064	-0.027	0.01	-0.008	-0.009	0.018
Metals	292	2,462	0.051	0.024	0.011	-0.01	-0.01	0.009
Minerals	206	1,313	0.007	-0.028	0.007	-0.004	-0.008	0.02
Cement	140	968	0.035	-0.03	0.03	-0.017	-0.014	0.025
Ceramic	2,265	14,909	0.022	-0.025	0.012	-0.007	-0.008	0.023
Glass	435	3,434	0.075	-0.022	0.015	-0.015	-0.009	0.022
Fertilizer	128	1,020	0.011	-0.034	0.009	-0.006	-0.014	0.02
Other minerals	229	1,758	-0.017	-0.025	0.006	-0.005	-0.007	0.024
Plastic	143	1,466	0.107	-0.024	0.009	-0.011	-0.005	0.018
Pharmaceutical	1,474	9,732	0.071	-0.021	0.016	-0.013	-0.01	0.014
Steel processing	6,189	32,742	0.065	-0.014	0.011	-0.011	-0.01	0.015
Machinery	4,450	23,011	0.038	-0.019	0.011	-0.009	-0.011	0.009
Electronics	2,968	18,021	0.039	-0.018	0.008	-0.01	-0.009	0.009
Transport equipment	1,410	9,094	0.097	-0.022	0.011	-0.011	-0.012	0.011
Shipbuilding	679	4,888	0.031	-0.018	0.007	-0.008	-0.009	0.009
Textile	4,908	24,533	0.018	-0.018	0.008	-0.006	-0.009	0.013
Paper	1,408	10,772	0.089	-0.022	0.011	-0.009	-0.009	0.013
Rubber products	371	2,760	0.067	-0.027	0.012	-0.012	-0.012	0.015
Plastic products	2,389	14,975	0.08	-0.015	0.003	-0.006	-0.005	0.02

Sources: EACEI, 1989-2017.

C Additional empirical results

We examine the possibility that the empirical support for the relevance of the VES is driven by the choice of the functional form in (2), where the elasticity of substitution is modelled to change over time. We do so by estimating the elasticity of substitution from the standard CES function for each year between 1990 and 2010, thus estimating the parameter at each point in time and examining its evolution. To begin, we consider a standard CES energy aggregate that combines clean and dirty energy,

$$Y_{it} = \left(Y_{cit}^{\frac{\sigma-1}{\sigma}} + Y_{dit}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

and derive our estimating equation by combining the first-order condition with respect to each input:

$$\log\left(\frac{Y_{dit}}{Y_{cit}}\right) = \alpha + \sigma \log\left(\frac{p_{cit}}{p_{dit}}\right) + \epsilon_{it}. \tag{A.9}$$

To examine potential time variation in the estimates of σ, we estimate (A.9) for each year between 1990 and 2017, thus estimating the parameter at each point in time and observing its evolution. We use the same instruments developed in the main text to instrument for $\log(p_{cit}/p_{dit})$. Figure A1 graphically reports the results from this exercise and shows a clear increasing trend in the cross-sectional estimates. The findings add confidence to our theoretical approach that allows the elasticity of substitution to vary over time.

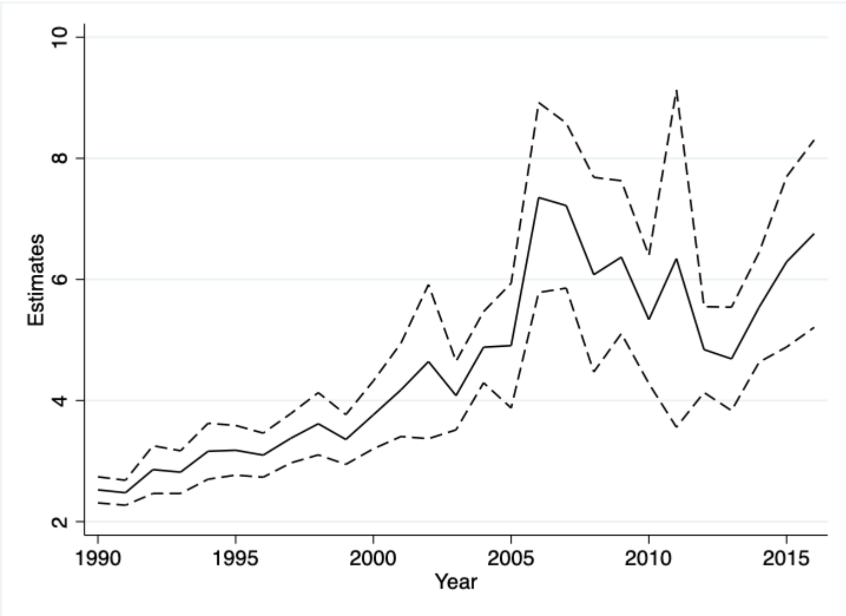
D Parameter sensitivity analysis

Figures A2 and A3 report the detailed results of the sensitivity analysis in Section 6.2 with respect to the pure rate of time preference, ρ, and the strength of relationship between the inputs ratio and the substitution elasticity, β, respectively.

E Second-best policy with only subsidy

Figure A4 reports the detailed results when only subsidy is available as a policy instrument.

Figure A1: Evolution of the elasticity of substitution: Cross-sectional estimates for the CES specification



Notes: Cross-sectional estimates of the elasticity of substitution between clean and dirty energy with 95 percent confidence intervals.

Table A2: Tests of the empirical relevance of the Variable Elasticity of Substitution functional form: Alternative IV specifications

	Dependent variable: $\frac{Y_{dit}}{Y_{cit}}$		
	(1)	(2)	(3)
$\frac{p_{cit}}{p_{dit}}$	2.493***	2.493***	5.470***
	(0.563)	(0.563)	(1.181)
t		0.059	0.225***
		(0.036)	(0.080)
Constant	-5.861***	-5.979***	-17.473***
	(2.168)	(2.236)	(4.810)
First stage F stat	625.26	625.26	501.31
Observations	169,182	169,182	169,182

Notes: Estimates from equation (13) using the two instruments separately rather than as a ratio. Column (1) reports OLS estimates and column (2) reports IV estimates. Column (3) weights the regression by total energy consumption. All specifications include year, sector and region fixed effects. Standard errors are clustered at the plant level.

Figure A2: Optimal climate policy in the VES economy under various discount rates

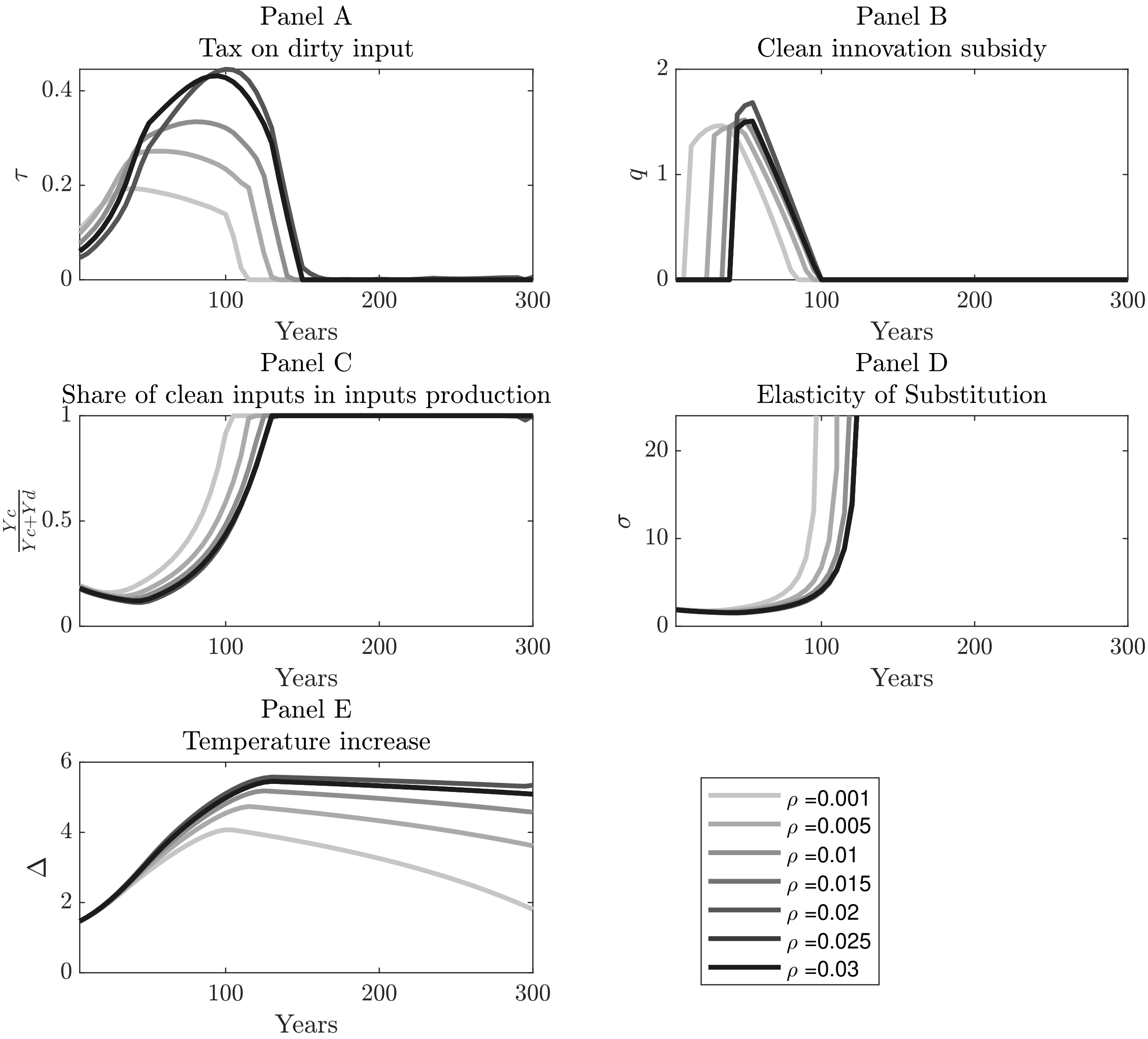


Figure A3: Optimal climate policy in the VES economy under various values for β

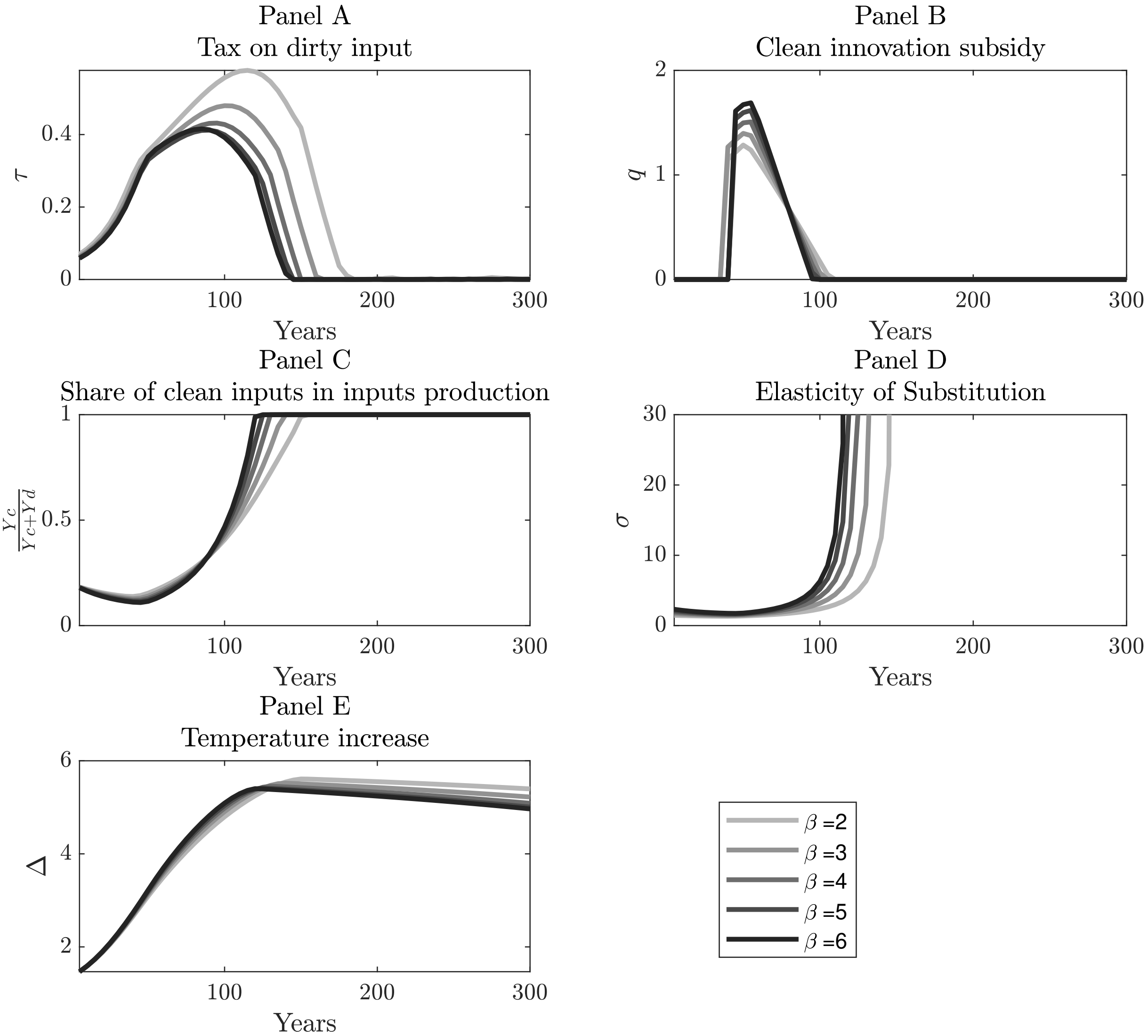
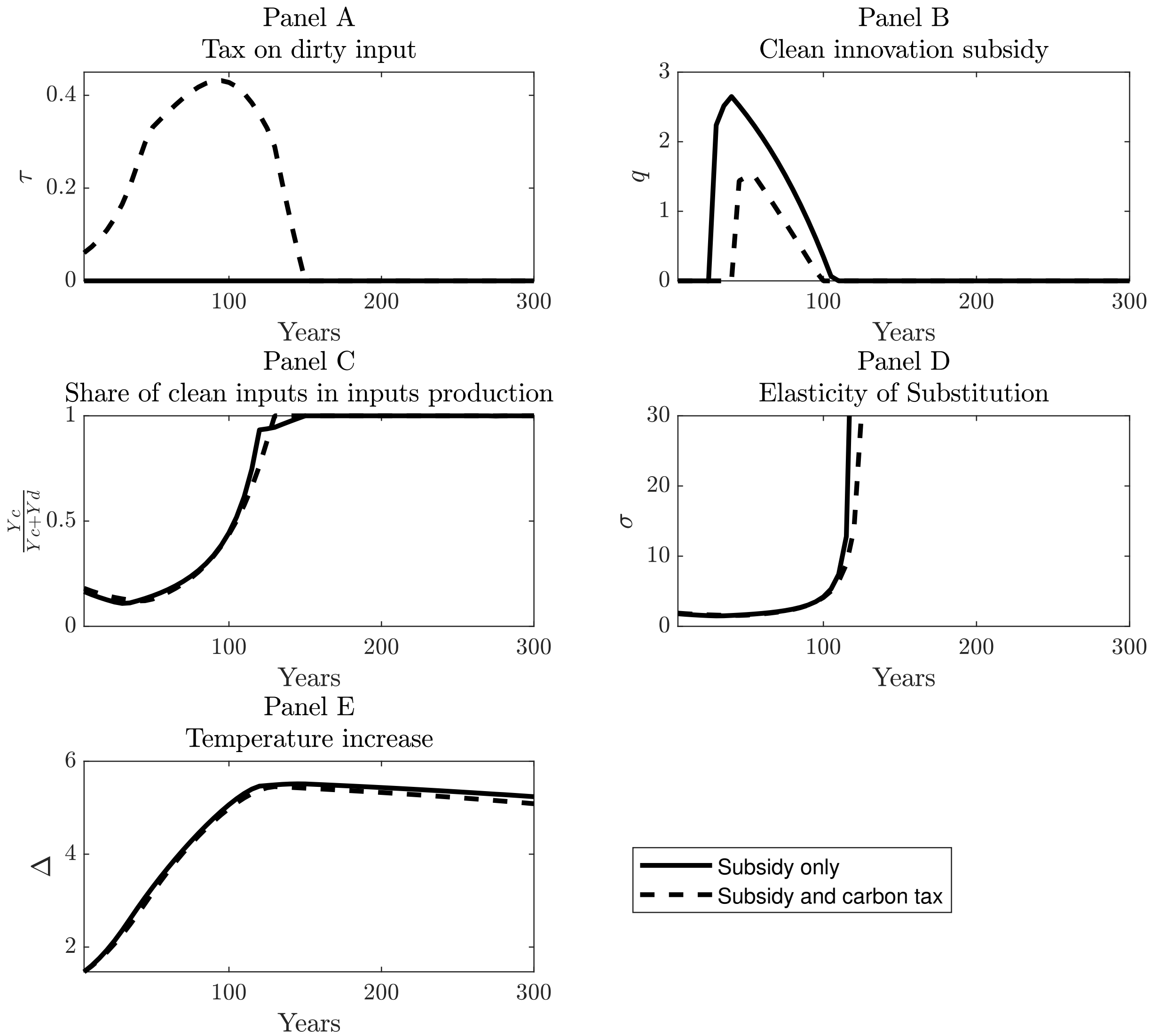


Figure A4: Optimal climate policy under first-best and second-best scenarios



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