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MINIMUM MASS LAMINATE DESIGN FOR UNCERTAIN IN-PLANE LOADING

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ABSTRACT

Balanced laminates, with zero in-plane to out-of-plane stiffness coupling are optimised over a range of tri-axial (N_x , N_y and N_{xy}) critical design loadings for minimum normalised elastic energy; equivalent to optimising for minimum mass in the absence of matrix failure. Laminates comprising standard angle plies (0° , $\pm 45^\circ$ and 90°) are designed for either a fixed design loading with a 10% minimum ply percentage rule (current practice) or directly for an uncertain design loading. Results show that the 10% rule performs well for the majority of design loadings. Nevertheless, for 8% of design loads considered, significantly lower mass ($>10\%$) designs are achieved with standard angle plies when designing directly for uncertain loading. Expansion of the ply envelope to include designs with continuous angles ($0^\circ \leq \theta \leq 180^\circ$) under an uncertain loading allows maximum mass savings up to 16% over current design practice. However, when designing directly for an uncertain loading, no significant mass reductions are achieved through the use of continuous angles.

1 INTRODUCTION

Minimum mass aerospace laminate design is a multi-constraint problem. All relevant failure modes must be considered in order to produce a minimum mass design that delivers the required performance. However, in this paper, only in-plane strength is considered, and thus results are only applicable to thick pristine laminates where buckling and other geometry-driven failure is non-critical. Netting analysis, which ignores the support of the resin matrix and aligns fibres in principal directions to carry principal stresses, leads to laminate designs in which the stresses in fibres are limited to some value associated with failure i.e. fully-stressed fibre design. Verchery [1] has shown that Netting analysis, can be treated as a limiting case of Classical Laminate Theory. His approach indicates that designs with fewer than three fibre directions produce mechanisms when subject to small disturbances in loading. This reveals the reasoning behind established aerospace laminate design practice of using four standard angles (SAs) (0° , $+45^\circ$, -45° and 90°) and a design rule of a 10% minimum ply percentage to provide a level of redundancy against loading uncertainty [2]. In contrast, continuous angle (CA) designs permit the use of all possible fibre angles ($0^\circ \leq \theta \leq 180^\circ$) providing greater scope for stiffness tailoring. However, a lack of specific design rules for CA laminates can lead to optimum designs for specific loadings that, in a Netting analysis regime, form mechanisms. Such laminates rely on the weak resin matrix to prevent collapse if the load state is varied. Here, to avoid this problem, both CA and SA laminates are designed considering an uncertain loading. This has the potential to remove the requirement for a 10% minimum ply percentage rule in SA designs and allows the use of CAs without drawbacks.

In order to compare design approaches that use SAs and CAs, laminate in-plane elastic energy under tri-axial loading is used to assess minimum mass capability. Elastic energy minimisation or compliance energy minimisation is a computationally efficient technique that uses either topology or orientation of materials with directional properties, to produce the most efficient structure. Structures with optimum efficiency take advantage of directional material stiffness properties to produce a minimum global strain state. This requires the structure to have the greatest global stiffness for a given volume of material. Prager and Taylor [3] first outlined optimality criteria justifying the technique of

minimisation of elastic energy (subject to given loads) to produce a structure with optimal efficiency. Pedersen [4] subsequently applied this technique to composite materials to find analytical solutions for orientation of a single ply angle subject to in-plane loading. In this paper, solutions for multi-layered anisotropic laminates are provided for tri-axial design loadings. Minimisation of elastic energy in laminate design does not however directly imply maximisation of in-plane strength of a composite material. Nevertheless, it is assumed to be sufficient to capture the in-plane strength relationship as fibres are aligned to best carry the applied tri-axial forces, thereby coinciding with the maximum in-plane strength design in a Netting analysis regime [5, 6].

In this paper, in-plane-strength limiting minimum-mass capabilities of design approaches using SAs and CAs are compared under uncertain critical design loadings. Design approaches optimise for minimum normalised elastic energy under a fixed design loading (current practice) or a worst case loading based on an uncertainty in secondary loading. The advantage of designing for an uncertain loading using both SAs and CAs is investigated and load combinations for which current laminate design practice can be improved are identified.

2 MINIMUM MASS LAMINATE DESIGN

In this section, optimum distributions of standard and continuous plies for maximum in-plane strength are identified through minimisation of elastic energy under fixed and uncertain in-plane tri-axial loadings (axial, transverse, shear). Design constraints, in the form of stacking sequence rules, are also derived.

2.1 IN-PLANE ELASTIC ENERGY

The efficiency of a given volume of fibrous layers, is assumed to be given by its in-plane Hookean strain energy or elastic energy. Assuming a panel has fixed length-width dimensions, the magnitude of energy stored depends on; (i) the ratios of the in-plane loads N , (ii) their magnitudes, (iii) the in-plane laminate stiffnesses Q , and (iv) the laminate thickness T . In order to isolate the effect of the laminate design (e.g. angles and ply percentages) and load ratios on the laminate in-plane elastic energy, the effect of the magnitude of the loads and thickness of the laminate is removed.

Q is independent of laminate dimensions. Assuming balanced laminates with zero in-plane to out-of-plane coupling (i.e. $Q_{13}, Q_{23} = 0$ and $B_{ij} = \frac{1}{2} \sum_{k=1}^n (Q_{ij})_k (h_k^2 - h_{k-1}^2) = 0$, where n is the number of plies and k is the ply number) and a state of plane stress ($\sigma_z, \tau_{xz}, \tau_{yz} = 0$), Classical Laminate Theory gives the in-plane stress-strain relationship as

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \mathbf{N}/T = \frac{1}{T} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \mathbf{Q}\boldsymbol{\varepsilon} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (1)$$

where $\sigma_x, \sigma_y, \tau_{xy}$ and $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ are the applied in-plane axial, transverse and shear stresses and corresponding laminate strains, respectively. Q_{ij} terms represent the individual in-plane stiffnesses.

Given that the elastic energy for a linear elastic solid is

$$U = \frac{1}{2} \int \boldsymbol{\sigma}^T \boldsymbol{\varepsilon} dV \quad (2)$$

Eq. (1) then implies

$$U = \frac{1}{2} \int \boldsymbol{\sigma}^T \mathbf{Q}^{-1} \boldsymbol{\sigma} dV \quad (3)$$

Working per unit volume allows for laminate geometry (thickness) to be ignored which further implies

$$U_V = \frac{1}{2} \boldsymbol{\sigma}^T \mathbf{Q}^{-1} \boldsymbol{\sigma} = \frac{1}{2} (q_{11}\sigma_x^2 + 2q_{12}\sigma_x\sigma_y + q_{22}\sigma_y^2 + q_{33}\tau_{xy}^2) \quad (4)$$

Division of Eq. (4) by the sum of the squares of the principal stresses normalises U_V , removing the effect of the magnitudes of the loads/stresses, and allows for an equal comparison between loading states of the same magnitude i.e.

$$\bar{U}_V = \frac{q_{11}\sigma_x^2 + 2q_{12}\sigma_x\sigma_y + q_{22}\sigma_y^2 + q_{33}\tau_{xy}^2}{2(\sigma_1^2 + \sigma_2^2)} \quad (5)$$

Where

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (6)$$

Substitution of Eq. (6) into Eq. (5) gives

$$\bar{U}_V = \frac{q_{11}\sigma_x^2 + 2q_{12}\sigma_x\sigma_y + q_{22}\sigma_y^2 + q_{33}\tau_{xy}^2}{2(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2)} \quad (7)$$

Eq. (7) gives an expression for normalised elastic energy (describing the efficiency of a design) in terms of applied stresses and compliance variables that is independent of the load magnitudes and the laminate thickness. A lower value of $\bar{U}_V$ relates to greater in-plane efficiency/strength and thus a lower amount of material required to support a certain load [5, 6]. If the length and width of the laminate is fixed, then since U is now proportional to $1/T$, the elastic energy stored within the laminate (Eq. (3)) scales linearly with laminate thickness i.e. $\sigma = N/T$ and $Q^{-1}\sigma = TQ^{-1}N$ and $dV = TdA$. Therefore, assuming a certain magnitude of in-plane loadings (N_x , N_y and N_{xy}) is to be carried and a certain magnitude of in-plane elastic energy is associated with failure, the most efficient design, corresponding to the optimum orientation and proportion of plies, would reach the in-plane elastic energy associated with failure whilst having a lower thickness than any other design. Thus a comparison of normalised elastic energies between designs reveals the lowest mass design.

2.2 OPTIMISATION

2.2.1 Problem Description

The problem of designing thick, membrane failure dependent, laminates for minimum mass aerospace applications is considered here although other applications are conceivable. Current design techniques consider a fixed critical loading condition, relating to some worst case from various critical loads that could be applied to the aircraft structure. Any uncertainty in secondary loading is considered to be negated by enforcement of a 10% minimum ply percentage rule [2]. The new design strategy proposed here, optimises directly for maximum in-plane strength performance for a critical design load case in which secondary loadings are uncertain. To provide a comparison of the current and proposed design practices, laminates are optimised for minimum normalised elastic energy under either a fixed loading (current practice, 10% rule) or uncertain loading (proposed strategy). Strategies are compared under a worst case loading derived from the envelope of loadings created by a deviation in secondary loads of up to $\pm 10\%$ of the primary load. Laminate performance is assumed to be represented by $\bar{U}_V$, the normalised elastic energy (Eq. (7)).

2.2.2 Laminate designs

Laminates to be optimised contain either SAs or CAs, and are designed for a fixed or uncertain design loading as shown in Table 1. SA laminates are assumed to be balanced and symmetric and are described by two independent (and one dependent) ply percentage variables. To reduce the computation required in the optimisation of CA designs to a manageable level, discrete ply stacking sequences are discarded in favour of three integer angle variables ($0 \leq \theta \leq 30$, $31 \leq \alpha \leq 60$, $61 \leq \varphi \leq 90$) and two independent ply percentage variables ($\%0$, $\%\alpha$) shown in Table 1. This has the advantage that all possible designs are assessed and a global picture of preferred design strategy is produced. However, there is a disadvantage in that some designs will not be discretisable into standard ply

thicknesses (depending on total laminate thickness) and thus results will be inconclusive for load cases where competing design strategies produce laminates with little difference in performance. Each of the three main angles (θ , α , φ) within a CA laminate is assumed to be made up of an infinite number of Winckler blocks [7]. Each block $[+\theta/-\theta/-\theta/+ \theta/-\theta/+ \theta/+ \theta/-\theta]$ is divided in half about the laminate mid-plane creating asymmetry. This ensures $Q_{13} = Q_{23} = B_{ij} = 0$ but is not a requirement as long as other stacking techniques can be used to maintain this condition. To further aid computational efficiency, CA ply angles are given integer values (in degrees). Note that variables are defined such that SA designs are a subset of CA designs and so any SA design is achievable with a CA design strategy if required.

Design strategy ID	Load designed for	Angles (integer °)	Ply %'s	10% Rule	Ply Unblocking Rule
SA1	Fixed	0, ± 45 , 90	%0, % ± 45 , %90	✓	✓
CA1	Fixed	$\pm\theta$, $\pm\alpha$, $\pm\varphi$ ($\pm\beta$)	% θ , % α , % φ	✗	✓
SA2	Uncertain	0, ± 45 , 90	%0, % ± 45 , %90	✗	✓
CA2	Uncertain	$\pm\theta$, $\pm\alpha$, $\pm\varphi$ ($\pm\beta$)	% θ , % α , % φ	✗	✓

Table 1. Details of four design strategies considered in the laminate design optimisation, angle variables, ply percentages variables and active design rules. β is an optional ply unblocking angle explained in Section 2.2.3.

2.2.3 Laminate design rules

Design rules are applied to laminate design in order to account for non-design failure mechanisms and to ensure favourable deformation which may not be taken care of during the design optimisation [2]. The two extra design rules, on top of the requirement for $Q_{13} = Q_{23} = B_{ij} = 0$ (considered in the optimisation), are:

(1) *Ply unblocking*: To prevent the formation of large inter-laminar shear stresses that may drive free edge failure, and thermal stresses that could cause premature failure, a maximum of four contiguous plies of the same/similar orientation is allowed [2].

In SA designs, compliance is ensured by requiring that the ply percentages of the non-dominant perpendicular plies (0° , 90°) and the $\pm 45^\circ$ plies sum to at least one quarter of the dominant 0° or 90° ply percentage, e.g. for every four dominant plies there is one ply that differs by at least 45° .

In CA designs, ply groups of similar angles ($\pm\theta < 20^\circ$ or $> 70^\circ$) are assumed to be sufficiently separated by an angular separation of at least 20° . The optimisation is able to choose the superior option between two techniques:

(i) Use of higher angles ($> 20^\circ$) to unblock dominant lower angles ($< 20^\circ$) and vice versa ($< 70^\circ$ unblock $> 70^\circ$). As for SAs, a 4:1 ratio of dominant and non-dominant ply angles is maintained. The angles used to unblock are assumed to be placed in a fashion that is symmetric and balanced, maintaining $Q_{13} = Q_{23} = B_{ij} = 0$.

(ii) The fixed ply unblocking angle β is considered where ply percentages of the two non-dominant angles are too low to implement unblocking as per (i). $\beta = 20^\circ$ and 70° unblocks ply groups containing angles $< 20^\circ$ and $> 70^\circ$, respectively, e.g. for a set of dominant plies, one fifth of the ply percentage is switched out for β in a symmetric and balanced fashion. For instance $[\pm 5/\mp 5/\mp 5/\pm 5]$ becomes $[\pm 5/\mp 5/20/\mp 5/\pm 5/-20]$.

(2) *10% minimum ply percentage*: In current design practice a 10% minimum of each of 0°, ±45° and 90° ply angles safeguards against uncertainty in loading in SA laminates (SA1, Table 1). No convention for enforcing rule (2) on CA designs exists. Instead the optimisation procedure in Section 2.2.4 designs directly for uncertainty in loading. This procedure is applied in both SA and CA design strategies, see SA2 and CA2 in Table 1.

2.2.4 Designing for an uncertain loading

An uncertainty in secondary loads of up to ±10% of the magnitude of the primary loading is assumed. For example, for a wing skin with critical design loadings $N_x/N_y = 9$ and $N_x/N_{xy} = 2$ (N_x dominant), possible loading scenarios are: $N_x = 9$, $N_y = 0.1-1.9$ and $N_{xy} = 3.6-5.4$, where the magnitudes of the loads have no effect on the value of $\bar{U}_V$. Laminate failure depends on the worst loading case that could be applied. Hence, the design concept is to achieve maximum performance under the worst case loading taken from the loading envelope defined by the ±10% uncertainty.

The worst loading case corresponds to the loading where a laminate has the highest normalised elastic energy, i.e. maximum mass/worst performance. However, this is dependent on the range of loadings and the laminate stacking sequence being considered. Individual worst loading cases within the uncertainty envelope and the value of normalised elastic energy associated with them, $\bar{U}_{V,WC}$, can be determined using the Extreme Value theorem for two variables. By keeping the value of primary loading fixed (e.g. σ_y), a three dimensional normalised elastic energy surface with the two secondary loading variables (e.g. σ_x and τ_{xy}) exists for each individual laminate design (fixed q_{ij}) and is described by Eq. (7). By finding the maxima of Eq. (7) (with fixed q_{ij}) both within the interior of the allowed range of secondary loads and on its boundaries, values of normalised elastic energy can be compared and the worst loading and corresponding normalised elastic energy ($\bar{U}_{V,WC}$) can be found.

2.2.5 Optimisation

Laminated aerospace structures are, in general, subject to a combination of in-plane axial, transverse and shear load. As described in Section 2.1, optimal laminate designs will distribute fibres to meet these loads producing a laminate with minimum elastic energy. In order to find optimal laminate designs for each of the four design approaches in Table 1 and subject to design constraints in Section 2.2.3, a Matlab Genetic Algorithm (GA) function ‘ga’ [8] is used to manipulate ply angle and percentage to minimise the objective function $\bar{U}_V$ given by Eq. (7). The genetic algorithm optimises either two or five variables, describing the SA and CA designs (see Table 1) respectively, to produce minimum normalised elastic energy ($\bar{U}_{V,min}$) under multiple fixed and uncertain tri-axial design loadings. The optimisation processes for uncertain and fixed design loadings are described in the flow charts of Figs. 1(a) and (b), respectively. The GA creates an initial random population of candidate design variables, calculating a scored fitness value for each. The most energy efficient designs are chosen and used to determine the next generation/population of design variables. (Eliteness, crossover and mutation all feature in *ga* [8]. Iteration continues until the stopping criteria is reached.) For SA designs only, a nonlinear constraint algorithm is employed to ensure that each new generation of variables meets the set of design rules. In the results that follow material properties of $E_{11} = 119$ GPa, $E_{22} = 8.9$ GPa, $G_{12} = 4.5$ GPa, $\nu_{12} = 0.35$ for M21/T700GC are assumed.

3 RESULTS

The tri-axial design loadings defining the horizontal and vertical axes of Figs. 2(a)-(d) are described by the two load ratios N_x/N_y and N_x/N_{xy} . In Figs. 2(a)-(d), there are 34×33 ($N_x/N_y \times N_x/N_{xy}$) points. Individual points are coloured based on the magnitude of the normalised elastic energy achieved through optimisation of laminate design variables, under a worst case loading (within bounds determined by the loading uncertainty). Figures 2(a), 2(b), 2(c) and 2(d) provide such results when design strategies SA1, CA1, SA2 and CA2 (see Table 1) are employed respectively.

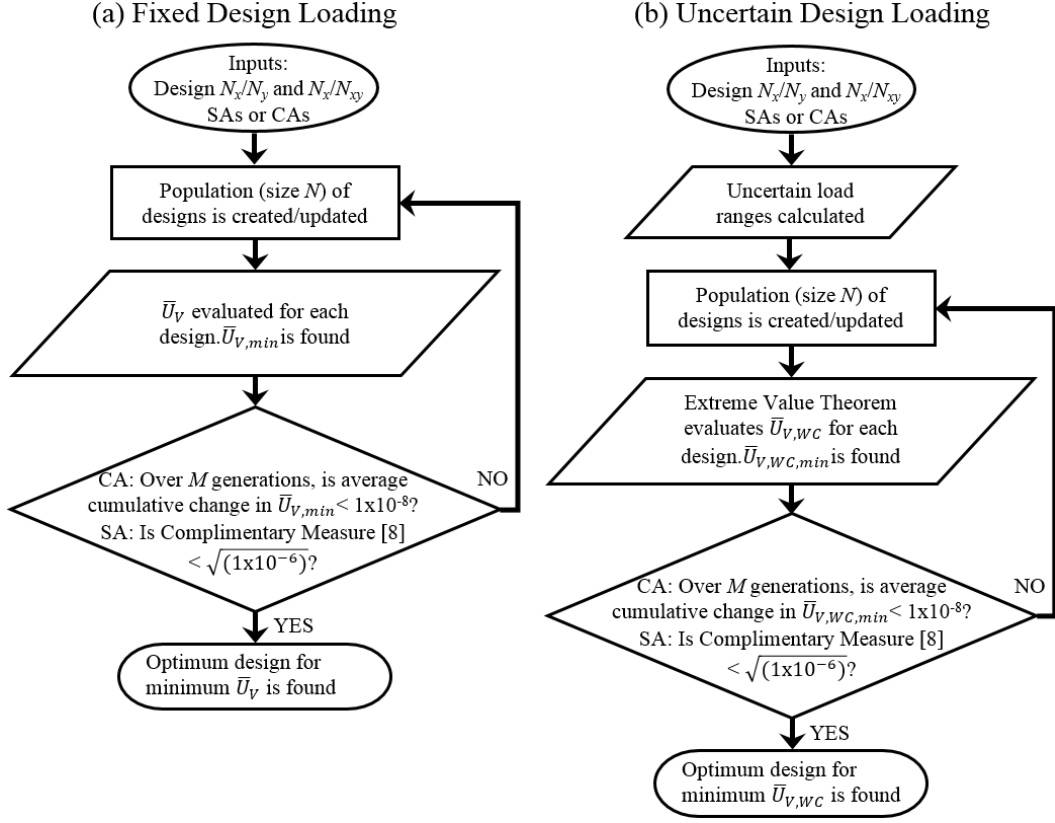


Figure 1: Optimisation procedure for design of a laminate for minimum elastic energy under (a) a fixed design loading and (b) an uncertain design loading.

Positive N_x/N_y load ratios represent tension-tension or compression-compression, and negative, tension-compression or compression-tension. In Figs. 2-5, only $|N_x/N_y| \leq 1$ is plotted (i.e. N_y is dominant) as $|N_x/N_y| > 1$, the inverse load ratio, constitutes an identical problem. Positive N_x/N_{xy} represents tension-shear or compression-shear which in Eq. (7), produces identical results, thus results for $N_x/N_{xy} < 0$ are not provided. Eq. (7) will, however, indicate any performance difference due to the relative sign of each of the loads. Log scales in Figs. 2-5 create nonlinear axes which equally emphasize the importance of load ratios above and below $|N_x/N_y| = |N_x/N_{xy}| = 1$. Load ratios below $|0.1|$ (indicated by a white strip for N_x/N_y) and above 10 are not provided as it is assumed a single load component dominates, resulting in systems that are physically very similar. To guide the reader through the process for determining the value of individual points in Figs. 2(a)-(d) an example for determining a wing skin panel design is provided:

The limiting design loading for the in-plane strength of a wing skin panel is assumed to be when $N_x/N_y = 9$ and $N_x/N_{xy} = 2$ (N_x dominant), which is identical to the design point $N_x/N_y = 1/9$ and $N_x/N_{xy} = 1/4.5$ (N_y dominant), with N_x and N_y switched. The latter is used to allow brevity in plotting. A $\pm 10\%$ uncertainty in the secondary loadings (as per Section 2.2.4) produces a possible loading scenario of arbitrary magnitude where $N_x = 0.1$ -1.9, $N_y = 9$ and $N_{xy} = 3.6$ -5.4. (The single point, X_1 , for this design loading can be seen in Figs. 2-5). Under design loading (SA1, CA1) the original design load is applied to each candidate laminate; the laminate with the lowest energy, after one GA run, being optimal. However, the elastic energy used for comparison (and recorded in Table 2) is based on the load case (within the bounds defined by uncertainty) that produces maximum elastic energy in the optimal laminate e.g. an unexpected worst loading is assumed to occur. This is found using the Extreme Value theorem. In the case of direct optimisation for uncertain loading (SA2, CA2), for each candidate stacking sequence defined by the GA the Extreme Value theorem is employed to determine the loading (within the specified bounds) that produces the maximum elastic energy for that candidate laminate. The laminate design that produces the lowest of these maximum energies is considered

optimal. The specific laminate designs and normalised elastic energies achieved by each strategy from Table 1 for this example design case, and two other design cases (X_2 and X_3), are shown in Table 2.

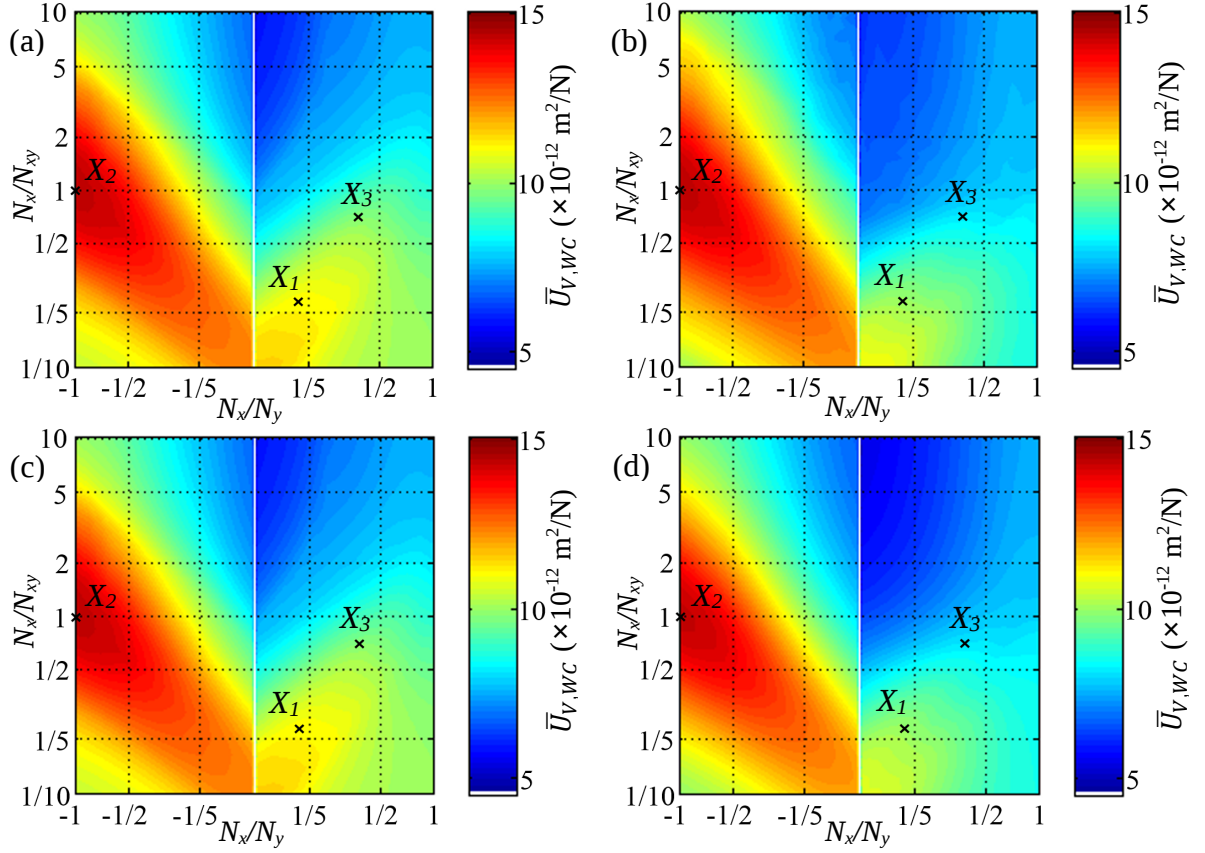


Figure 2: Laminate normalised elastic energies (Eq. (7)) under worst case loading for a range of tri-axial design loadings, (described by N_x/N_y and N_x/N_{xy}). (a) SA1: 10% minimum ply percentage rule accounts for load uncertainty. (b) CA1: no load uncertainty considered. (c) SA2: no 10% rule, designed directly for a load uncertainty. (d) CA2: designed directly for a load uncertainty.

Figure 2 shows plots of normalised elastic energy, for each design strategy (see Table 1), when laminates are subject to their individual worst case loadings. In all plots greater normalised elastic energies generally occur when $N_x/N_y < 0$. An individual peak (worst performance) occurs where $N_x = -N_y = N_{xy}$ (design point X_2). For $N_x/N_y > 0$, lower N_x/N_{xy} generally indicates greater normalised elastic energy. As elastic energy scales linearly with laminate thickness, the difference in magnitude of the normalised elastic energies between designs can be used to calculate a percentage mass saving, see Column 7 Table 2. Figures 3, 4 and 5 compare the potential of the various design strategies for mass reduction. In each figure, blue indicates mass saving is possible and green indicates little difference.

Figure 3 compares SA1, the current industry design strategy, to the SA2 design strategy. SA2 designs provide a significant mass saving in areas near $N_x/N_y = 1$ and $N_x/N_{xy} = 0-1$, and near $N_x/N_y = 0.1-0.25$, $N_x/N_{xy} = 1-4$. Figure 4 compares strategies SA1 to CA2, showing CA2 designs provide a significant mass saving in the same areas as Fig. 3 but also in an additional area peaking at $N_x/N_y = 0.4$, $N_x/N_{xy} = 0.7$ (design point X_3). The values in Fig. 5 show the percentage mass saving when using the CA2 design strategy compared to the SA2 strategy. CA2 designs produce a small mass saving at load ratios in the region $N_x/N_y = 0.4$, $N_x/N_{xy} = 0.7$ only. The majority of all other load ratios have insignificant mass saving.

Design Point	Design Strategy I.D.		θ	α	φ	$\bar{U}_{V,WC}$ ($\times 10^{-12} \text{ m}^2/\text{N}$)	Mass Saving
Wing Skin: $N_x/N_y = 1/9$ $N_x/N_{xy} = 1/4.5$ X_1	SA1	Angle ($^\circ$)	0	45	90	10.72	Datum
		Ply %	41	49	10		
	CA1	Angle ($^\circ$)	24	N/A	N/A	9.98	6.9%
		Ply %	100	0	0		
	SA2	Angle ($^\circ$)	0	45	90	10.09	5.9%
		Ply %	45	55	0		
	CA2	Angle ($^\circ$)	22	36	72	9.89	7.7%
		Ply %	76	21	3		
$N_x/N_y = -1$ $N_x/N_{xy} = 1$ X_2	SA1	Angle ($^\circ$)	0	45	90	14.21	Datum
		Ply %	25	50	25		
	CA1	Angle ($^\circ$)	8	48	73	14.21	0%
		Ply %	31	38	31		
	SA2	Angle ($^\circ$)	0	45	90	14.21	0%
		Ply %	25	50	25		
	CA2	Angle ($^\circ$)	21	47	73	14.21	0%
		Ply %	43	19	38		
$N_x/N_y = 1/2.5$ $N_x/N_{xy} = 1/1.4$ X_3	SA1	Angle ($^\circ$)	0	45	90	9.74	Datum
		Ply %	10	61	29		
	CA1	Angle ($^\circ$)	N/A	58	N/A	8.18	16.0%
		Ply %	0	100	0		
	SA2	Angle ($^\circ$)	0	45	90	8.93	8.2%
		Ply %	0	69	31		
	CA2	Angle ($^\circ$)	N/A	59	61	8.12	16.6%
		Ply %	0	97	3		

Table 2. Angles and ply percentages of laminate designs for three design loading cases, optimised using the four strategies in Table 1. Normalised elastic energies and percentage mass saving based on worst case loading are presented.

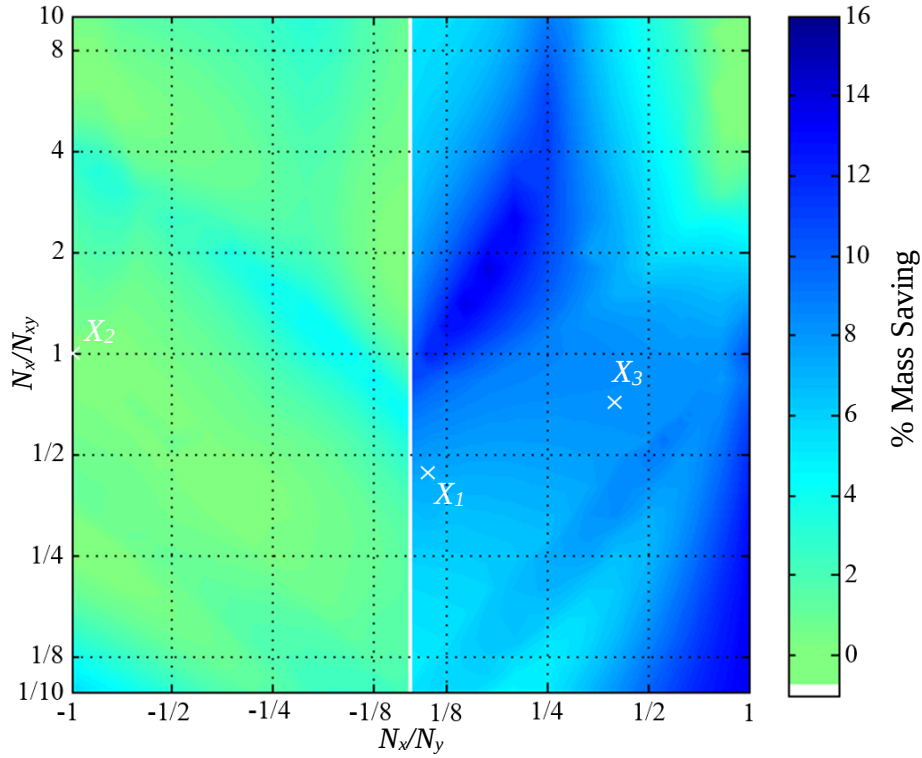


Figure 3: Percentage mass saving under design loadings described by the load ratios N_x/N_y and N_x/N_{xy} , when using SA designs optimised directly for an uncertain loading (SA2) compared to SA designs optimised for a fixed design load with a 10% rule (SA1). Energies are based on the application of worst case loading.

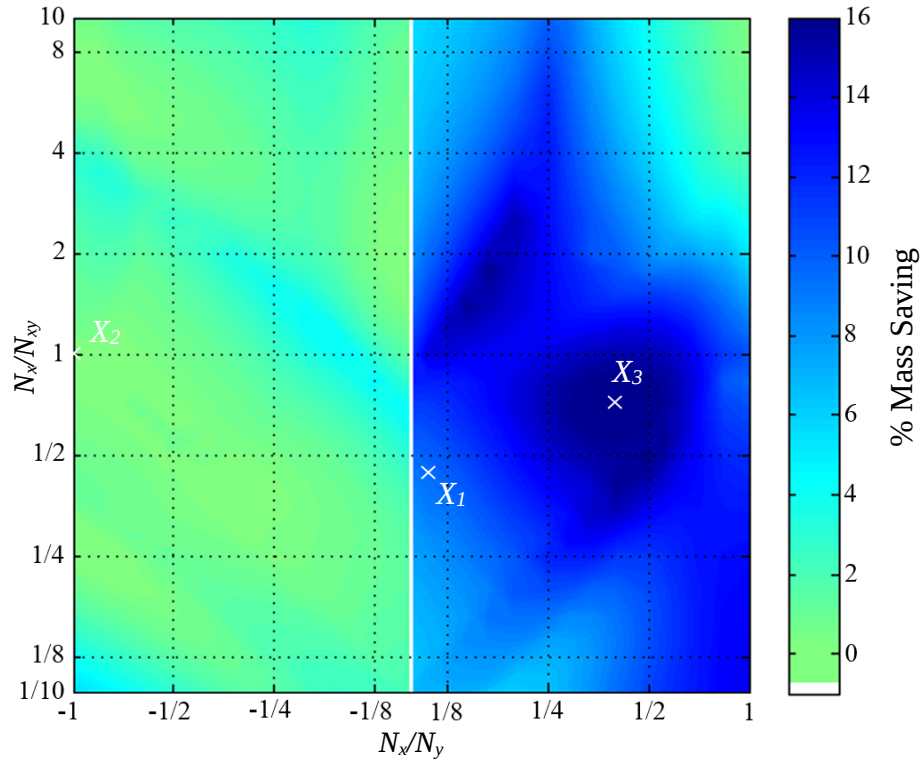


Figure 4: Percentage mass saving under design loadings described by the load ratios N_x/N_y and N_x/N_{xy} , when using CA designs optimised directly for an uncertain loading (CA2) compared to SA designs optimised for a fixed design load with a 10% rule (SA1). Energies are based on the application of worst case loading.

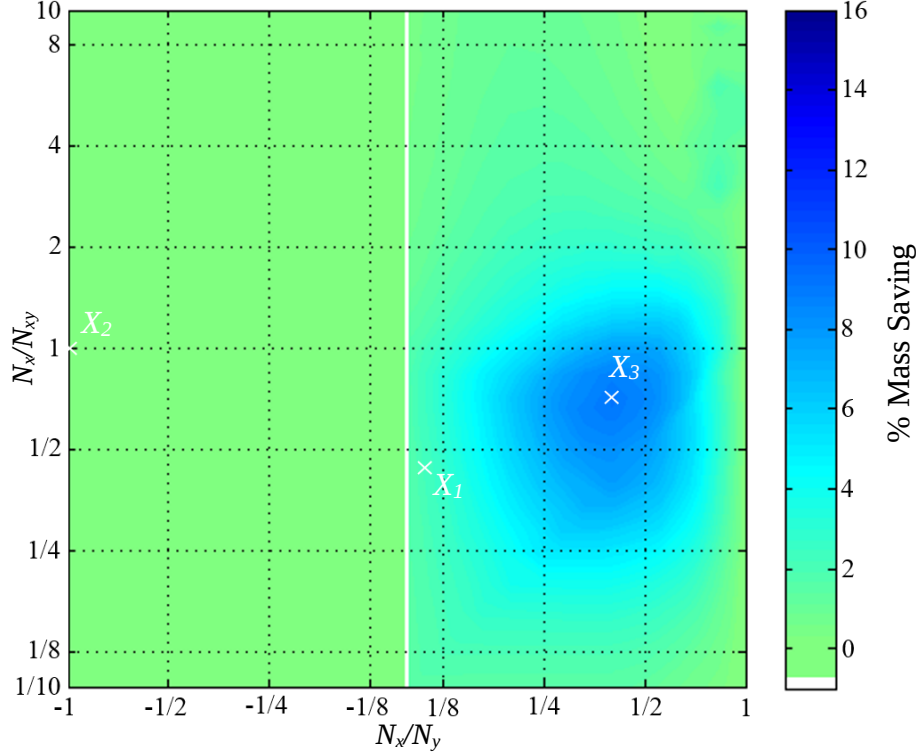


Figure 5: Percentage mass saving under design loadings described by the load ratios N_x/N_y and N_x/N_{xy} , when using CA designs optimised directly for an uncertain loading (CA2) compared to an SA strategy also optimising directly for an uncertain loading (SA2). Energies are based on the application of worst case loading.

4 DISCUSSION

Figure 2 shows the relationship between design loading and the minimum mass capabilities for all laminate design strategies considered for worst case normalised elastic energies in the range $5 - 15 \times 10^{-12} \text{ m}^2/\text{N}$. Normalised elastic energy, and therefore mass, are larger when $N_x/N_y < 0$, i.e. when the axial and transverse loads have opposite sign. This is because axial and transverse loads are applied in the same direction as the opposite load's Poisson's deformation. Thus greater straining is created within the laminates and greater normalised elastic energies are produced. The worst case normalised elastic energy peaks when $N_x = -N_y = N_{xy}$, i.e. where all design loads are the same magnitude and the axial and transverse loads have opposite sign. At this design point (X_2) different design traits are required to efficiently carry all three loads equally (SA1, CA1) or close to equally (SA2, CA2) depending on the worst case loading designed for from the uncertainty. For example, in SA laminates, 0° and 90° will best reduce deformation from axial-transverse loads of opposite sign (shear loading rotated by 45°), and $\pm 45^\circ$ will best reduce the deformation from shear. Therefore applying a worst case loading to a laminate which has been designed for neither of the loadings fully, produces the largest normalised elastic energy. SA and CA designs for this design point, X_2 , are shown in Table 2, where all designs are shown to perform just as poorly. For $N_x/N_y > 0$, the normalised elastic energies are large where there is a design loading with significant shear. In a Netting regime, a reduction in performance due to shear loading occurs because $\pm 45^\circ$ fibres carry shear loading with two times less efficiency than 0° fibres carry the same magnitude of axial loading.

Figure 3 shows that, SA1, the current design practice of designing for a fixed design loading using SAs with the 10% minimum ply percentage rule (to account for load uncertainty), has merit, even when subject to an uncertain critical loading. Designing directly for an uncertain loading with SAs (SA2) over design loadings of $N_x/N_y < 0$ provides mass savings of less than 5%. The potential for mass saving using an SA2 design strategy is only realised when $N_x/N_y > 0$, but is only significant ($>10\%$)

when $N_x/N_y = 1$, $N_x/N_{xy} = 0-1$, and $N_x/N_y = 0.1-0.25$, $N_x/N_{xy} = 1-4$, where 8% of designs can achieve greater than 10% mass saving, with a maximum of 15% mass saving possible.

In Fig. 4 CA laminates are designed for an uncertain loading (CA2) and significant mass saving over the current practice is again only seen at design load ratios of $N_x/N_y > 0$. However, the greater capacity for tailoring with CAs allows a greater range of loadings over which mass savings are significant (28% of designs) compared to the SA2 design strategy (8%). Peak savings of 16% occur near $N_x/N_y = 0.4$, $N_x/N_{xy} = 0.7$ (design point X_3). The CA2 design strategy provides the greatest mass saving over current practice (SA1). However, Fig. 5 shows that the mass saving from using CAs compared to SAs when designing directly for uncertainty in both, is less than 10%. The green areas in Fig. 5 show that over all design loadings, there is little or no advantage to using CAs. A switch to use of CAs may therefore be considered unwarranted as the considerable cost from change and development of manufacturing processes would not be worthwhile. As there is no risk of increase in laminate mass when designing directly for an uncertain design loading with SAs, it is suggested this design strategy (SA2) could be employed in order to produce a mass saving.

In Table 2 the CA2 strategy produces lower mass designs than a CA1 strategy. This is because CA2 laminates are designed to carry the worst case loading and CA1 only the design loading. However, despite not being designed for uncertainty CA1 designs still provide lower mass than SA2 designs (although this mass saving is only significant for design point X_3). The CA designs at design point X_3 are orientated at approximately $\pm 60^\circ$, which suggests a mechanism failure would need to be prevented by the resin matrix. This contradicts the original objective, as stated in the introduction, and so the stability of designs CA1 and CA2 for this design loading is a matter for further investigation.

The following caveats to the discussion above should also be noted:

As a consequence of the implementation of the 10% rule in current designs, fibres are spaced every 45° . This prevents large deformations in all directions and reduces laminate Poisson's ratio [2]. The Poisson's ratios of the optimum SA and CA designs (without the 10% minimum rule) presented may be large. This makes compatibility between laminates forming different aircraft components potentially problematic. However, a constraint could be added to the optimisation to ensure the Poisson's ratio is within reasonable limits.

Results are based on a variation in secondary load of $\pm 10\%$ of the primary loading. This is thought to be realistic but mass savings will vary if percentage uncertainty is altered.

It is noted that the minimum energy approach taken here is only applicable to thick laminates that would fail via a fibre based material failure mechanism. No attempt is made to account for geometric failures such as buckling, nor for damaged based failures, although both could be incorporated as constraints on the optimisation procedure. It is noted that CAs are likely to be more beneficial when buckling failure and damage tolerance are more critical.

The continuous nature of the ply percentage variables chosen for the optimisation essentially means laminates with an infinite number of plies are being compared. As such, conclusions regarding the optimal design strategy hold more strongly for thicker laminates where ratios of ply percentages are more achievable. This is compounded for CA designs which rely on a Winckler laminate structure that requires at least eight plies of the same magnitude of angle to be stacked in a certain pattern in order to remove all relevant coupling terms [7]. However, the advent of thin-ply technologies will allow for greater stacking sequence design freedom in thinner laminates.

Both SA and CA designs are assumed to feature ply unblocking that is symmetric about the mid-plane. If designs are to be discretised large separations between the + and - angles used to unblock may occur. This could produce large bend-twist stiffness coupling terms (D_{13} and D_{23}) that may have detrimental effects on buckling capacity and laminate strength under bending.

5 CONCLUSIONS AND FUTURE WORK

Normalised elastic energy of laminates subject to a worst case loading, within the bounds of uncertainty, is used to compare design strategies for minimum mass. The current use of the 10% minimum ply percentage rule in standard angle designs (0° , $\pm 45^\circ$ and 90°) is compared to designing directly for an uncertainty in secondary loads of up to $\pm 10\%$ of the primary load. The use of continuous angles ($0 \leq \theta \leq 180^\circ$) is compared to the current use of standard angles.

It is shown that designing with continuous angles for an uncertain critical design loading offers up to 16% reduction in mass over the current industry design practice; greater than 10% for 28% of design loadings considered. Where the latter designs for a fixed critical loading using standard angles whilst applying a 10% minimum ply percentage rule to account for load uncertainty. However, if optimising with standard angles for an uncertain critical design loading then mass reduction through the use of continuous angles is limited to less than 10%. Hence, given the additional cost and complexity of manufacture, in most cases a continuous angle design is unwarranted. Nevertheless, direct design for an uncertain critical design loading offers scope for improvement over current practice and the possibility of removal of the 10% minimum ply percentage design rule.

The conclusions above only hold with certainty for laminates where laminate strength is not limited by resin dominated failure. Designs presented are formed from continuous ply percentages, however, realistic minimum mass comparisons will depend on discrete ply stacking sequences. Nevertheless, the methodology does give an indication of optimum fibre angle distribution.

Future work will include incorporating constraints that account for geometry dependent failure modes such as buckling and impact damage driven failures. Maximum Poisson's ratio and thermal stress constraints, associated with curing, will also be considered.

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