Thermodynamics Basics, Heat Energy and Power

Course No: M06-030

Credit: 6 PDH

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Thermodynamics Fundamentals Series ©

Preface

As the adage goes, "a picture is worth a thousand words;" this text maximizes the utilization of diagram, graphs and flow charts to facilitate quick and effective comprehension of the concepts of thermodynamics by the reader.

This text is designed to serve as a tool for building basic engineering skills in the filed of thermodynamics.

If your objective as a reader is limited to the acquisition of basic knowledge in thermodynamics, then the material in this text should suffice. If, however, the reader wishes to progress their knowledge and skills in thermodynamics to intermediate or advanced level, this text could serve as a useful stepping stone.

In this text, the study of thermodynamics concepts, principles and analysis techniques is made relatively easy for the reader by inclusion of most of the reference data, in the form of excerpts, within the discussion of each case study, exercise and self assessment problem solutions. This is in an effort to facilitate quick study and comprehension of the material without repetitive search for reference data in other parts of the text.

Certain thermodynamic concepts and terms are explained more than once as these concepts appear in different segments of this text; often with a slightly different perspective. This approach is a deliberate attempt to make the study of some of the more abstract thermodynamics topics more fluid; allowing the reader continuity, and precluding the need for pausing and referring to segments where those specific topics were first introduced.

Due to the level of explanation and detail included for most thermodynamics concepts, principles, computational techniques and analyses methods, this text is a tool for those energy engineers, other engineers and non-engineers, who are not current on the subject of thermodynamics.

The solutions for end of the segment self-assessment problems are explained in just as much detail as the case studies and sample problem in the pertaining segments. This approach has been adopted so that this text can serve as a thermodynamics skill building resource for not just energy engineers but engineers of all disciplines. Since all segments and topics begin with the

introduction of important fundamental concepts and principles, this text can serve as a "brush-up" or review tool for even mechanical engineers whose current area of engineering specialty does not afford them the opportunity to keep their thermodynamics knowledge current.

In an effort to clarify some of the thermodynamic concepts effectively for energy engineers whose engineering education focus does not include thermodynamics, analogies are drawn from non-mechanical engineering realms, on certain complex topics, to facilitate comprehension of the relatively abstract thermodynamic concepts and principles.

Each segment in this text concludes with a list of questions or problems, for self-assessment, skill building and knowledge affirmation purposes. The reader is encouraged to attempt these problems and questions. The answers and solutions, for the questions and problems, are included under Appendix A of this text.

For reference and computational purposes, steam tables and Mollier (Enthalpy-Entropy) diagrams are included in Appendix B.

Most engineers understand the role units play in definition and verification of the engineering concepts, principles, equations and analytical techniques. Therefore, most thermodynamic concepts, principles and computational procedures covered in this text are punctuated with proper units. In addition, for the reader's convenience, units for commonly used thermodynamic entities and some conversion factors are listed under Appendix C.

Most thermodynamic concepts, principles, tables, graphs, and computational procedures covered in this text are premised on US/Imperial Units as well as SI/Metric Units. Certain numerical examples, case studies or self-assessment problems in this text are premised on *either* the SI unit realm *or* the US unit system. When the problems or numerical analysis are based on only one of the two unit systems, the given data and the final results can be transformed into the desired unit system through the use of unit conversion factors in Appendix C.

Some of the Greek symbols, used in the realm of thermodynamics, are listed in Appendix D, for reference.

What readers can gain from this text?

- Better understanding of thermodynamics terms, concepts, principles, laws, analysis methods, solution strategies and computational techniques.
- Greater confidence in interactions with thermodynamics design engineers and thermodynamics experts.
- Skills and preparation necessary for succeeding in thermodynamics portion of various certification and licensure exams, i.e. CEM, FE, PE, and many other trade certification tests.
- A better understanding of the thermodynamics component of heat related energy projects.

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Segment 1

Introduction to Energy, Heat and Thermodynamics

Topics:

- Concepts, terms, principles, units, laws and equations pertaining to energy.
- Heat and energy conversion.
- Specific heat.
- Energy transformation and associated case studies.

Introduction

The term "thermodynamics" comes from two root words: "**thermo,**" which means heat, and "**dynamic,**" meaning energy in motion, or power. This also explains why the Laws of Thermodynamics are sometimes viewed as Laws of "Heat Power."

Since heat is simply thermal energy, in this segment, we will review energy basics and lay the foundation in depth for discussion on heat energy and set the tone for discussion on more complex topics in thermodynamics.

Energy

The capacity of an, object, entity or a system to perform work is called energy. Energy is a scalar physical quantity. In the International System of Units (SI), energy is measured in Newton-meters (N-m) or Joules, while in the US system of units, energy is measured in ft-lbf, BTU's, therms or calories. In the field of electricity, energy is measured in watt-hours, (Wh), kilowatt-hours (kWh), Gigawatt-hours (GWh), Terawatt-hours (TWh), etc. Units for energy, such as ft-lbs and N-m, point to the equivalence of **energy** with **torque** (moment) and **work**. This point will be discussed later in this segment.

Energy exists in many forms. Some of the more common forms of energy, and associated units, are as follows:

- 1) **Kinetic Energy**¹; measured in ft-lbf, BTUs, Joules, N-m (1 N-m = 1 Joule), etc. Where, BTU stands for British Thermal Units
- 2) **Potential Energy**¹; measured in ft-lbf, BTUs, Joules, N-m, etc.
- 3) **Thermal Energy**¹, or heat (Q); commonly measured in Calories, BTUs, Joules, therms, etc.
- 4) **Internal Energy**¹, (U); commonly measured in BTU's, calories or Joules.
- 5) **Electrical Energy**; measured in Watt-hours (Wh), killowatt-hours (kWh) and horsepower-hours (hp-hrs), etc.
- 6) **Gravitational Energy**; measured in ft-lbf, Joules, N-m, etc.
- 7) **Sound Energy**; measured in Joules.
- 8) Light Energy; measured in Joules.
- 9) Elastic Energy; measured in ft-lbf, BTUs, Joules, N-m, etc.
- 10) **Electromagnetic Energy**; measured in Joules.
- 11) **Pressure Energy**¹; measured in ft-lbf, BTUs, Joules, N-m, etc.

¹ **Note:** These forms of energy are discussed in greater detail later in this segment.

Root Concepts and Terms that Contribute toward the Production or Transformation of Energy:

Force and Mass

Force or weight, in US or imperial units is measured in lbf. While, mass is measured in lbm. Mass, in lbm, can be converted to weight, in lbf, through multiplication by factor g/g_c , as follows:

```
Given: An object of mass, \mathbf{m} = 1 lbm.

Weight = Force = m. (g/g_c)

= (1-\text{lbm}) \cdot (g/g_c)

= (1-\text{lbm}) \cdot (32.2 \text{ ft/sec}^2)/(32.2 \text{ lbm-ft/lbf-sec}^2).

= 1-\text{lbf}
```

Force or weight, in SI, or metric units, is measured in Newton's, or "N". Mass, in SI units, is measured in **kg.** Mass, available in kg, can be converted to weight or force, in Newton's, through multiplication by, simply, the gravitational acceleration "g," which is equal to 9.81 m/sec². Conversion of 1 kg of mass to its corresponding weight would be as follows:

Given: An object of mass, m = 1 kg.

```
Weight = Force = m.g

= (1-kg) . (9.81 m/sec<sup>2</sup>)

= 9.81 kg-m/sec<sup>2</sup>

Since,

1 N = 1 kg-m/sec<sup>2</sup>

Weight = 9.81 kg-m/sec<sup>2</sup> = 9.81 N
```

Density and Weight Density

Density is defined as mass per unit volume. The symbol for density is: ρ (Rho)

Density of water, in the SI (Metric) realm, under STP conditions, is **1000** kg/m^3 . This density can be converted to **specific weight**, γ (gamma), or weight density, as follows:

$$\gamma = \rho$$
. (g) Eq. 1.1
= (1000 kg/m^3) . (9.81 m/sec^2)
= 9810 N/m^3

Density ρ of water, in the US (Imperial) realm, under STP conditions, is **62.4** lbm/ft³. The density can be converted to specific weight, γ , or weight density, as follows:

$$\gamma = \rho$$
. (g) = (62.4 lbm/ft³). (g/g_c)
= (62.4 lbm/ft³). (32.2 ft/sec²)/(32.2 lbm-ft/lbf-sec²)
= 62.4 lbf/ ft³

Specific Volume

Specific Volume is the *inverse of density*. The symbol for specific volume is "v," (upsilon). The formula for specific volume, v, is:

$$v = 1/\rho$$
 Eq. 1.2

The units for specific volume are as follows:

- US (Imperial) Units: v is measured in **cu-ft/lbm or ft**³/**lbm**.
- SI (Metric) Units: v is measured in m^3/kg .

Pressure

Pressure is defined as force applied per unit area. The symbol used for pressure is **p.** The formula for pressure is:

$$\mathbf{p} = \mathbf{F/A}$$
 Eq. 1.3

Where,

 $\mathbf{F} = \mathbf{Applied}$ force

A = Area over which the force is applied

There are a number of units used, customarily, for measuring pressure. Common units for pressure, in the SI, or Metric system are N/m², Pa (Pascals), kPa, or MPa. **One Pascal** is equal to **1-N/m²**. Common units for pressure, in the US, or Imperial, system are **psi**, or lbf/in², **psia** (psi-absolute) or **psig** (psi-gage). Other units utilized for pressure, and corresponding conversions are shown in **Table 1.1** below.

Units for Pressure					
	Pascal	bar	atmos.	torr	psi
	(Pa)	(bar)	(atm)	(Torr)	(psi)
1 Pa	1 N/m2	10 ⁻⁵	1.0197×10 ⁻⁵	7.5006×10^{-3}	145.04×10 ⁻⁶
1 bar	100,000	Approx. 106 dyne/cm2	1.0197	750.06	14.50377
1 atm	98,066.50	0.980665	Approx.1 kgf/cm2	735.56	14.223
1 atm	101,325	1.01325	1.0332	760	14.696
1 torr	133.322	1.3332×10 ⁻³	1.3595×10 ⁻³	Approx. 1 Torr; or 1 mmHg	19.337×10 ⁻³
1 psi	6.894×10 ³	68.948×10 ⁻³	70.307×10^{-3}	51.715	1 lbf/in2

Table 1.1; Units for Pressure and Associated Conversion Factors

Temperature

Temperature can be defined as a measure of the average kinetic energy of the particles in a substance, where such energy is directly proportional to the degree of hotness or coldness of the substance.

While temperature is one of the principal parameters of thermodynamics, it must be clear that temperature is *not a direct* measurement of heat, Q. Temperature, however, is a parameter that is instrumental in determining the direction of flow of heat, Q. In that, heat travels from bodies at higher temperature to bodies at lower temperature. This role of temperature comports with the laws of thermodynamics.

From physics perspective, temperature is an indicator of the level of kinetic energy possessed by atoms and molecules in substances. In **solids**, at higher temperature, the atoms oscillate or vibrate at higher frequency. In atomic **gases**, the atoms, at higher temperatures, tend to exhibit faster translational movement. In molecular gases, the molecules, at higher temperatures, tend to exhibit higher rates of vibrational and rotational movement.

Even though, for a system in thermal equilibrium at a constant volume, temperature is thermodynamically defined in terms of its energy (E) and entropy (S), as shown in **Eq. 1.4** below. Unlike pressure, temperature is not commonly recognized as a derivative entity and, therefore, the units for temperature are not derived from the units of other independent entities.

$$T \equiv \frac{\partial E}{\partial S}$$
 Eq. 1.4

The universal symbol for temperature is: **T.** The unit for temperature, in the SI, or metric, realm is °C. In the Celsius temperature scale system, **0**°C represents the freezing point of water. The unit for temperature, in the US, or imperial, realm is °F. On the Fahrenheit temperature scale system, **32**°F represents the freezing point of water. The formulas used for conversion of temperature from metric to US realm, and vice and versa, are as follows:

$$(^{\circ}C \times ^{9}/_{5}) + 32 = ^{\circ}F$$
 Eq. 1.5

$$(^{\circ}F - 32) \times ^{5}/_{9} = ^{\circ}C$$
 Eq. 1.6

Absolute Temperature

Unlike the Celsius temperature scale system, where 0°C represents the freezing point of water, the absolute temperature scale defines temperature independent of the properties of any specific substance. According to the laws of thermodynamics, absolute zero cannot be reached because this would require a thermodynamic system to be fully removed from the rest of the universe. **Absolute zero** is the theoretical temperature at which entropy would reach its **minimum value**. Absolute zero is defined as **0°K** on the Kelvin scale and as **-273.15°C** on the Celsius scale. This equates to **-459.67°F** on the Fahrenheit scale.

It is postulated that a system at absolute zero would possess finite quantum, mechanical, zero-point energy. In other words, while molecular motion would not cease entirely at absolute zero, the system would lack enough energy to initiate or sustain transference of energy to other systems. It would, therefore, be more accurate to state that *molecular kinetic energy is minimal at absolute zero*.

According to the **Second Law of Thermodynamics**, at temperatures approaching the absolute zero, the *change in entropy approaches zero*. This comports with the stipulation that as temperatures of systems or bodies approach absolute zero, the transference of heat energy diminishes. **Equation 1.7** is mathematical statement of the Second Law of Thermodynamics.

$$\lim_{T \to 0} \Delta S = 0$$
 Eq. 1.7

Scientists, under laboratory conditions, have achieved temperatures approaching absolute zero. As temperature approaches absolute zero, matter exhibits quantum effects such as **superconductivity and superfluidity**. A substance in a state of superconductivity has *electrical resistance approaching zero*. In superfluidity state, viscosity of a fluid approaches zero.

Table 1.2 shows factors for conversion of temperatures between Kelvin, Celsius, Fahrenheit and Rankin scales. This table also shows absolute temperature, freezing point of water, triple point for water and the boiling point of water. **Tables 1.3 and 1.4** list *formulas* for *conversion of temperatures* between Kelvin, Celsius, Fahrenheit and Rankin scales.

Important Temperatures and Conversion Table				
	Kelvin	Celsius	Fahrenheit	Rankin
Absolute Zero	0 °K	−273.15 °C	−459.67 °F	0 °R
Freezing Point of Water	273.15 °K	0 °C	32 °F	491.67 °R
Triple Point of Water	273.16 °K	0.01 °C	32.0 °F	491.69 °R
Boiling Point of Water	373.13 °K	99.98 °C	211.97 °F	671.64 °R

Table 1.2; Important Temperatures and Associated Conversion Factors

In the metric or SI system, the absolute temperature is measured in °K. The relationship between °C and °K is as follows:

$$T_{^{\circ}K} = T_{^{\circ}C} + 273^{\circ}$$

and, $\Delta T_{^{\circ}K} = \Delta T_{^{\circ}C}$

In the US system, the absolute temperature is measured in ${}^{\circ}R$. The relationship between ${}^{\circ}F$ and ${}^{\circ}R$ is as follows:

$$T_{^{\circ}R} = T_{^{\circ}F} + 460^{\circ}$$

and, $\Delta T_{^{\circ}R} = \Delta T_{^{\circ}F}$

The absolute temperature system should be used for all thermodynamics calculations, unless otherwise required.

Rankin Temperature Conversion Formulas				
	From Rankin	To Rankin		
Celsius	$[^{\circ}C] = ([^{\circ}R] - 492) \times \frac{5}{9}$	$[^{\circ}R] = ([^{\circ}C] + 273) \times \frac{9}{5}$		
Fahrenheit	$[^{\circ}F] = [^{\circ}R] - 460$	$[{}^{\circ}R] = [{}^{\circ}F] + 460$		
Kelvin	$[K] = [^{\circ}R] \times \frac{5}{9}$	$[^{\circ}R] = [K] \times \frac{9}{5}$		

Table 1.3; Rankin Temperature Conversion Formulas

Kelvin Temperature Conversion Formulas			
From Kelvin To Kelvin			
Celsius	$[^{\circ}C] = [K] - 273$	$[K] = [^{\circ}C] + 273$	
Fahrenheit	$[^{\circ}F] = [K] \times \frac{9}{5} - 460$	$[K] = ([°F] + 460) \times \frac{5}{9}$	
Rankin	$[^{\circ}R] = [K] \times \frac{9}{5}$	$[K] = [^{\circ}R] \times \frac{5}{9}$	

Table 1.4; Kelvin Temperature Conversion Factors

Law of Conservation of Energy:

The law of conservation of energy states that energy can be converted from one form to another but *cannot be created or destroyed*. This can be expressed, mathematically, as:

$$\sum \mathbf{E} = \sum \mathbf{Energy} = \mathbf{Constant}$$

Forms of Energy in Mechanical and Thermodynamic Systems:

Potential Energy

Potential energy is defined as energy possessed by an object by virtue of its height or elevation. Potential energy can be defined, mathematically, as follows:

$E_{potential} = m.g.h,$	{SI Units}	Eq. 1.8
$E_{potential} = m.(g/g_c).h,$	{US Units}	Eq. 1.8a

When the change in potential energy is achieved through performance of work, W:

$$W = \Delta E_{potential}$$
 Eq. 1.9

Kinetic Energy

Kinetic energy is defined as energy possessed by an object by virtue of its motion. Kinetic energy can be defined, mathematically, as follows:

$$\mathbf{E}_{\text{kinetic}} = \frac{1}{2} \cdot \mathbf{m} \cdot \mathbf{v}^2 \qquad \{ \text{SI Units} \} \qquad \mathbf{Eq. 1.10}$$

$$E_{kinetic} = \frac{1}{2}$$
. (m/g_c). v² {US Units} Eq. 1.10a

Where,

 $\mathbf{m} = \text{mass of the object in motion}$

 \mathbf{v} = velocity of the object in motion

$$g_c = 32 lbm-ft/lbf-s^2$$

When the change in kinetic energy is achieved through performance of work, **W**:

$$W = \Delta E_{kinetic}$$
 Eq. 1.11

Energy Stored in a Spring²

Potential energy can be stored in a spring - or in any elastic object - by compression or extension of the spring. Potential energy stored in a spring can be expressed, mathematically, as follows:

$$E_{\text{spring}} = \frac{1}{2} k.x^2$$
 Eq. 1.12

And,

$$W_{spring} = \Delta E_{spring}$$
 Eq. 1.13

Where,

 \mathbf{k} = The spring constant

 \mathbf{x} = The contraction or expansion of the spring

² Note: In steel beam systems, beams act as springs, when loaded, to a certain degree. The deflection of a beam would represent the "x," in Eq. 1.12.

Pressure Energy

Energy stored in a system in the form of pressure is referred to as pressure energy. For instance, energy stored in a compressed air tank is pressure energy. Pressure energy can be expressed, mathematically, as follows:

$$E_{\text{pressure}} = E_{\text{flow}} = m \cdot p \cdot v$$
 Eq. 1.14

Where,

m = mass of the pressurized system; this would be compressed air in a compressed air system

 \mathbf{p} = pressure in the system

v = is the specific volume

Heat and Internal Energy of a System

If heat "Q" is added or removed from a system, in the absence of net work performed by or on the system, change in the internal energy "U "of a system would be:

$$U_f - U_i = \Delta Q Eq. 1.15$$

Where \mathbf{Q} is positive when heat flows into a thermodynamic system, and negative when heat exits a system.

Specific internal energy "**u**" is defined as *internal energy per unit mass*. The units for internal energy are BTU/lbm, in the US System, and are kJ/kg, in the Metric or SI System.

Unit Conversions³ **Associated with Heat Energy:**

Some of the common heat energy units and unit conversion formulas are listed below:

- Conversion of heat energy measured in MMBTU's to BTU's:
 - 1 MMBTU x (1000,000 BTU/MMBTU) = 10^6 MMBTU's
- Conversion of heat energy measured in BTU's to tons and tons to BTU's:
 - 1 BTU x (8.333x 10.5 tons/BTU) = 0.00008333 tons
 - -1 ton x (12,000 BTU/ton) = 12,000 BTU's
- Conversion of heat energy measured in **Deca Therms to BTU's**:
 - 1 dT x (1,000,000 BTU/dT) = 1,000,000 BTU's or 1MMBTU
- Conversion of heat energy measured in BTU's to kWh and KWh to BTU's:
 - -1 BTU x (2.928 x 10⁻⁴ kWh/BTU) = 0.0002928 kWh
 - -1 kWh x (3413 BTU/ kWh) = 3,413 BTU's

³ **Note:** These heat energy conversion formulas will be used in various analysis and example problems through this text.

Molar Internal Energy

Molar internal energy "U" is defined as internal energy per mole. The units for internal energy U are BTU/lbmole, in the US system, and are kJ/kmole, in the Metric or SI System.

Case Study 1.1: Energy and Energy Unit Conversion

As an energy engineer, you are to analyze substitution of coal, as heating fuel, in lieu of nuclear energy derived from complete conversion, of **2.5 grams** of a certain mass. The nuclear reaction is similar to the Uranium fission reaction shown in Figure 1.1 below. If the heating value of coal is **13,000 BTU/lbm**, how many U.S. tons of coal must be burned in order to derive the same amount of energy?

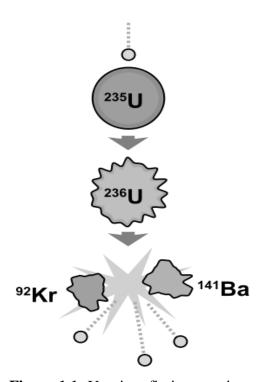


Figure 1.1; Uranium fission reaction

Solution:

Given or known:

 $c = Speed of light = 3 \times 10^8 \text{ m/s}$

m = Mass of material to be converted to Energy: 2.5 g, or 0.0025kg

Energy content of coal: 1 lbm of coal contains **13,000 BTU's** of energy

Mass conversion factor; lbm to US tons, and vice and versa: 2000 lbm/ton

Energy unit conversion between Joules and BTU's: 1055 Joules/BTU

According to Einstein's Equation:

$$E = m. c^2$$
 Eq. 1.16

By applying **Eq. 1.16**, Energy derived from 2 grams of given mass would be:

$$E = m. c^2 = (0.0025 \text{ kg}) \times (3 \times 10^8 \text{ m/s})^2 = 2.25 \times 10^{14} \text{ Joules}$$

This energy can be converted into BTU's as follows:

$$E = (2.25 \times 10^{14} \text{ Joules}) / (1055 \text{ Joules/BTU})$$

= 2.13x 10^{11 BTU's}.

Since 1 lbm of coal contains 13,000 BTU's of heat, the number of lbs of coal required to obtain $2.13x\ 10^{11}\ ^{BTU's}$ of heat energy would be:

E =
$$(2.13 \times 10^{11 \text{ BTU's}}) / (13,000 \text{ BTU's/lbm})$$

= $1.64 \times 10^{07} \text{ lbm}$

Since there are 2000 lbm per ton:

$$E = (1.64x \ 10^{07} \ lbm \)/(2000 \ lbm/ton)$$
 = 8,203 US tons of coal.

Conclusion: Energy derived from fission of **2.5 grams** of fissile material is equivalent to the energy derived from **8,203 tons of coal**.

Work

As we will see, through the exploration of various topics in this text, work can be viewed as a vehicle for converting energy contained in various types of fuels to mechanical or electrical energy. In this section, we will elaborate on aspects of work that will be applied in the discussion and analysis of thermodynamic systems.

Work in a Mechanical System:

In a mechanical system, work performed by an external force is referred to as **external work**. While, work performed by an internal force is referred to as **internal work**. Units such as BTU's or kilocalories are not, customarily, used to measure mechanical work.

In a mechanical system, **work is positive** when it is the result of force acting in the direction of motion. **Work is negative** when it is the result of a force opposing motion. Work attributed to friction is an example of negative work. Where, friction " $\mathbf{F}_{\mathbf{f}}$ " is defined, mathematically, as:

$$F_f = \mu_f . N$$
 Eq. 1.17

Where,

 μ_f = Coefficient of friction

N= Normal force applied by the surface against the object

Frictional force, $\mathbf{F_f}$, can either be **static frictional force** or **dynamic frictional force**. As stipulated by **Eq. 1.17**, $\mathbf{F_f}$ is directly proportional to the normal force \mathbf{N} . The coefficient of friction for static friction is, typically, higher as compared to the coefficient of friction for dynamic friction. This is ostensible from the fact that greater force is required to set an object in motion as opposed to the force required to maintain the object in motion.

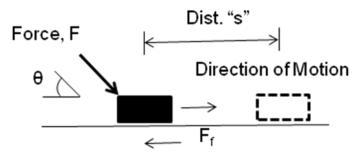


Figure 1.2, Illustration of mechanical work, in a system with friction

Mathematical Equations for Work

Work can be performed, defined and computed in several ways. Some of the diferences stem from the realm or frame of reference that work occurs in. Listed below are some of the scenarios in which work can occur, and pertinent formulas:

In a rotational system, with *variable torque*:

$$\mathbf{W}_{\text{variable torque}} = \int T \cdot d\theta \qquad \qquad \mathbf{Eq. 1.18}$$

In a rotational system, with *constant torque*:

$$W_{constant torque} = \tau \cdot \theta$$
 Eq. 1.19

Where,

 $T = \tau = Torque$

 θ = Angular distance traversed in the same direction as the torque, τ .

In a linear system, with variable force:

$$\mathbf{W_{variable force}} = \int \mathbf{F}.ds \qquad \qquad \mathbf{Eq. 1.20}$$

In a linear system, with *constant force*, where, **f**orce and distance are *colinear*:

$$W_{constant force} = F \cdot s.$$
 Eq. 1.21

General equation for work performed by a constant frictional force where, force and distance are colinear:

$$W_{friction} = F_f \cdot s$$
 Eq. 1.22

When work is performed by a force, \mathbf{F} , which is applied at an angle, $\mathbf{\theta}$, with respect to the direction of motion – as shown in Figure 1.2 - it can be defined, mathematically, as follows:

$$W = Work = (F.Cos\theta - F_f) \cdot s$$
 Eq. 1.23

Or,

$$W = (F.Cos\theta) \cdot s - F_f \cdot s$$
 Eq. 1.24

Where,

s = Distance over which the force is applied

In Equations 1.23 and 1.24, the mathematical term " $(\mathbf{F.Cos\theta})$. s" constitutes *positive work* performed by the force \mathbf{F} in the direction of motion, and $(\mathbf{F_f.s})$ constitutes *negative work*, performed against the direction of motion. Note that component " $\mathbf{F.Cos\theta}$," in Equations 1.23 and 1.24, represents the *horizontal component of force* contributed by the diagonally applied force \mathbf{F} .

Work performed by **gravitational force** is defined, mathematically, as:

$$W_{g(SD)} = m. g. (h_f - h_i)$$
, in the Metric Unit Systems Eq. 1.25

$$W_{g(US)} = m. \ (g/g_c) \ . \ (h_f \ - h_i), \ \ \text{in the US Unit Systems}$$
 Eq. 1.26 Where,

 $\mathbf{h_f}$ = The final elevation of the object

 $\mathbf{h_i}$ = The initial elevation of the object

Work performed in the case of a linear spring **expansion or contraction** is represented, mathematically, as:

$$W_{\text{spring}} = \frac{1}{2} \cdot k \cdot (x_f - x_i)^2$$
 Eq. 1.27

Where,

 \mathbf{k} = The spring constant

 $\mathbf{x_i}$ = The initial length of the spring

 $\mathbf{x_f}$ = The final length of the spring

Work Performed in a Thermodynamic System

In the thermodynamics domain, work constitutes the phenomenon of changing the energy level of an object or a system.

The term "**system**," in thermodynamics, is often used interchangeably with the term **medium**. For instance, in the case of an open thermodynamic system - such as steam powered turbine - steam is considered as a system performing work on the surroundings, i.e., the turbine.

In a thermodynamic system, **work is positive** when an object or system performs work on the surroundings. Example: If the vanes of an air compressor are considered to constitute the system, then the work performed on air, by the vanes in an air compressor, would be positive. **Work is negative** when the surroundings perform work on the object. Inflating of a raft or an inner tube constitutes negative work as the air (environment or

surrounding) performs work on the walls of the raft or tube (the system) during the inflation process.

Specific Heat

Specific heat is defined as the amount of the heat, \mathbf{Q} , required to change the temperature of mass " \mathbf{m} " of a substance by $\Delta \mathbf{T}$. The symbol for specific heat is " \mathbf{c} ."

The mathematical formula for specific heat of solids and liquids is:

$$c = Q/(m. \Delta T)$$
 Eq. 1.28

Or,

$$Q = m. c. \Delta T$$
 Eq. 1.29

Where,

m = Mass of the substance; measured in **kg**, in the SI system, or in **lbm** in the US system

Q = The heat added or removed; measured in **Joules** or **kJ** in the SI System, or in **BTU's** in the US system

 ΔT = The change in temperature, measured in ${}^{\circ}K$ in the SI Systems, or in ${}^{\circ}R$ in the US System

The units for c are kJ/(kg. °K), kJ/(kg. °C), BTU/(lbm. °F) or BTU/(lbm. °R).

The thermodynamic equation involving **specific heats of gases** are as follows:

$$Q = m. c_v. \Delta T$$
, when volume is held constant. Eq. 1.30

$$Q = m. c_p. \Delta T$$
, when pressure is held constant. Eq. 1.31

Approximate specific heat, c_p , for selected liquids and solids are listed in **Table 1.5**, below.

Substance	c _p in kJ/kg °K	c _p in kcal/kg °K or BTU/lbm °F	Molar C _p J/mol °K
Aluminum	0.9	0.215	24.3
Bismuth	0.123	0.0294	25.7
Copper	0.386	0.0923	24.5
Brass	0.38	0.092	N/A
Gold	0.126	0.0301	25.6
Lead	0.128	0.0305	26.4
Iron	0.460	0.11	N/A
Silver	0.233	0.0558	24.9
Tungsten	0.134	0.0321	24.8
Zinc	0.387	0.0925	25.2
Mercury	0.14	0.033	28.3
Ethyl Alcohol	2.4	0.58	111
Water	4.186	1	75.2
Ice at -10 °C	2.05	0.49	36.9
Granite	0.79	0.19	N/A
Glass	0.84	0.2	N/A

 $Table~1.5~-~Approximate~Specific~Heat,~c_{p,}~for~Selected~Liquids~and~Solids,~in~kJ/kg~^cK,~cal/gm~^cK,~BTU/lbm~^cF,~J/mol~^cK$

Densities of Common Materials				
Metal	g/cm³	lb/in³	lb/ft³	lb/gal
Water	1.00	0.036	62	8.35
Aluminum	2.7	0.098	169	22.53
Zinc	7.13	0.258	445	59.5
Iron	7.87	0.284	491	65.68
Copper	8.96	0.324	559	74.78
Silver	10.49	0.379	655	87.54
Lead	11.36	0.41	709	94.8
Mercury	13.55	0.49	846	113.08
Gold	19.32	0.698	1206	161.23

Table 1.6 – Densities of Common Materials

The next case study, **Case Study 1.2**, is designed to expand our exploration of energy related analysis methods and computational techniques. Some of the energy, work and heat considerations involved in this case study lay a foundation for more complex energy work and thermodynamic topics that lie ahead in this text. This case study also provides us an opportunity to experience the translation between the SI (Metric) unit system and the US (Imperial) unit system. As we compare the solutions for this case study in the US and SI unit systems, we see that choosing one unit system versus another, in some cases does involve the *use of different formulas*. This difference in formulas for different unit systems is evidenced in the potential and kinetic energy components of the energy conservation equations.

Furthermore, this case study helps us understand the vital and integrated role that work, kinetic energy and potential energy play in the application of law of conservation of energy in thermodynamic system analysis. In this case study we start off with energy and work considerations and conclude our analysis with the quantitative assessment of *thermodynamic impact* on *steel, air* and *water*, and other key substances involved in the overall process.

Case Study 1.2: Energy Conservation, Energy Conversion and Thermodynamics

At a foundry, a solid rectangular block of carbon steel, density 7,850 kg/m³ (491 lbm/ft³), from Table 1.6, is released to a downward inclined ramp as shown in Figure 1.3. The volume of the block is 1.0 m³ (35.32 ft3) and its release velocity, at the top of the ramp is 1.5 m/s (4.92 ft/s). The force of friction between the block, the inclined surface and the flat conveyor bed is 400 N (89.924 lbf). The block is stopped on the flat section through compression of a shock absorbing spring system before it settles on a roller conveyor operating at a linear speed of 2 m/s (6.562 ft/s). Assume that the frictional force stays constant through the entire path of the block.

- a) Determine the velocity of the steel block when it enters the horizontal segment of the travel, i.e. point "w."
- b) Employing the law of energy conservation and principles of energy conversion, calculate the value of the spring constant for the shock absorbing spring system.
- c) How much energy is stored in the shock absorbing spring upon complete compression?
- d) What would the steady state speed of the block be after it settles onto the roller conveyor?
- e) If the spring type shock absorbing system is replaced by a compressed air cylinder of 1.0 m³ (35.32 ft³) uncompressed volume, at room temperature of 20°C (68 °F) and standard atmospheric pressure, what would be the rise in temperature of the cylinder air immediately after the steel block's impact? The final, compressed, volume is 0.75 m³ (26.49 ft³), and the pressure gage on the cylinder reads 2 bar (29 psia).
- f) If the conveyor, at the bottom of the incline, is a belt driven roller conveyor and the rate of flow of blocks onto the conveyor is one per 10 seconds, determine the horsepower rating of the conveyor motor.
 Assume the conveyor belt to be directly driven off the conveyor motor

- shaft and that there is no slip between the belt and the rollers. Assume the motor efficiency to be 90%.
- g) The conveyor transports the blocks to a cooling/quenching tank. The temperature of the blocks, when they are dropped into the quenching tank, is 100°C (212 °F). The initial temperature of the water in the quench tank is 20°C (68 °F) and volume of water is 6.038 m³ (213.23 ft³). The final, equilibrium, temperature of the water and the block is 30°C (86 °F). Determine the amount of heat extracted by the quench water per block.

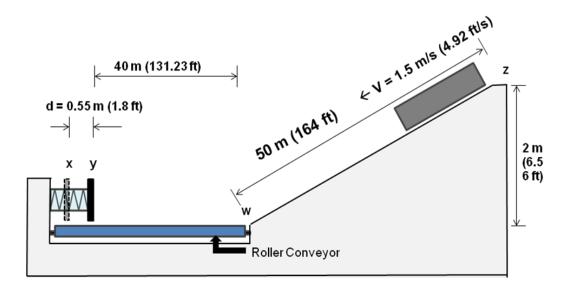


Figure 1.3, Case Study 1.2

Solution, Case Study 1.2 – SI (Metric) version

Given, known or derived values:

```
Volume of the steel block = V_{steel\ block} = 1.0\ m^3
Density of the steel block = \rho_{steel} = 7850\ kg/m^3
Mass of the steel block = m_{steel\ block}
= \rho_{steel\ block}
= \rho_{steel\ block}
= (7850\ kg/m^3)\ x\ (1.0\ m^3)
\therefore m_{steel\ block} = 7850\ kg
Velocity of block at point "z" = V_z = 1.5\ m/s
Elevation at point "z" = h_z = 2.0\ m
```

```
Spring constant for the shock absorbing spring = k Frictional force, F_f = N. \mu_f = 400 \ N Density of air at STP: 1.29 kg/ m^3 At STP:

Temperature = 0 °C,

Pressure = 1 bar, 101.33kPa, or 101,330 Pa
```

a) Determine the velocity, $\mathbf{v}_{\mathbf{w}}$, of the steel block when it enters the horizontal segment of the travel, i.e. point \mathbf{w} :

Solution Strategy: The velocity variable $\mathbf{v_w}$ is embedded in the kinetic energy of the block, $1/2 \cdot \mathbf{m} \cdot \mathbf{v_w}^2 \cdot$ So, if we can determine the amount of **kinetic energy** possessed by the block at point \mathbf{w} , we can derive the required velocity $\mathbf{v_w} \cdot$ To find the kinetic energy at point \mathbf{w} , we can apply the law of conservation of energy at points \mathbf{z} and \mathbf{w} as shown below:

Let total energy at point $z = E_{z-total}$

Then,

$$\begin{split} E_{z\text{-total}} &= E_{z\text{-kinetic}} + E_{z\text{-potential}} \\ E_{z\text{-total}} &= 1/2 \text{ . m. } v_z^2 + m \text{ . g. } h_z \\ E_{z\text{-total}} &= 1/2 \text{ . } (7850 \text{ kg}) \text{ . } (1.5 \text{ m/s})^2 + (7850 \text{ kg}) \text{ . } (9.81 \text{ m/s}^2) \text{ . } (2 \text{ m}) \\ &\therefore E_{z\text{-total}} &= 162,\!848 \text{ J} \end{split}$$

The energy lost in the work performed against friction, during the block's travel from **z** to **w**, is accounted for as follows:

$$W_{f-wz}$$
 = Work performed against friction
= (Dist. w-z) . (F_f)
= (50 m) . (400 N) = 20,000 J

Therefore, the energy left in the block when it arrives at point \mathbf{w} , at the bottom of the ramp, would be as follows:

$$E_{w-total} = E_{z-total} - W_{f-wz}$$

= 162,848 J - 20,000 J
= 142,848 J

Since the block is at "ground" elevation when it arrives at point **w**, the potential energy at point **w** would be "**zero**."

$$\therefore E_{w-total} = 1/2 \cdot m \cdot v_w^2$$
Or,
$$v_w = \{2 \cdot (E_{w-total}) / m \}^{1/2}$$

$$= \{2 \cdot (142,848 \text{ J}) / 7850 \text{ kg }\}^{1/2}$$

$$= 6.03 \text{ m/s}$$

b) Employing the law of energy conservation and principles of energy conversion, calculate the value of the spring constant for the shock absorbing spring system.

Solution Strategy: The unknown constant \mathbf{k} is embedded in the formula for the **potential energy stored** in the spring after it has been fully compressed, upon stopping of the block. This potential energy is equal to the work performed on the spring, i.e., $\mathbf{W}_{\text{spring}} = (1/2) \cdot \mathbf{k} \cdot \mathbf{x}^2$. So, if we can determine the amount of **work performed on the spring**, during the compression of the spring, we can derive the required value of \mathbf{k} .

To derive the value of W_{spring} , we will apply the law of conservation of energy to the travel of the block from point z to point x.

Based on the dimensions in Figure 1.3:

Distance x-z =
$$(0.55m + 50 m + 40m)$$

= $90.55 m$
W_{f-xz} = Work performed against friction over Dist. x-z
= $(Dist. x-z) \cdot F_f$
= $(90.55 m) \cdot (400 N)$
= $36,220 J$

Therefore, the total energy at point z would be:

$$E_{z-total} = W_{spring} + W_{f-xz}$$

Or,
 $W_{spring} = E_{z-total} - W_{f-xz}$
= 162,848 J - 36,220 J
= **126,628 J**

Energy stored in the spring is quantified as:

$$W_{\text{spring}} = 1/2 \cdot k \cdot x^2$$

Since W spring has beed determined to be equal to 126, 628 J,

126, 628
$$J = \frac{1}{2}$$
. k. $(0.55)^2$

Therefore,

$$k = 2 \cdot W_{spring} / x^2$$

= 2 \cdot W_{spring} / x^2 = 2 \cdot (126,628 J) / (0.55)^2
= 837,212 N/m

c) How much energy is stored in the shock absorbing spring upon complete compression?

Solution:

Energy stored in the spring is equal to the work performed on the spring. The work performed on the spring, as computed in part (b) above, is:

$$W_{spring} = E_{z-total} - W_{f-xz}$$

= 162,848 - 36,220
= 126,628 J

d) What would the steady state speed of the block be after it settles onto the roller conveyor?

Solution/answer:

After the block settles into a steady state condition on the conveyor, it assumes the speed of the conveyor, i.e. 2 m/s.

e) Rise in the temperature of the compressed air in the shock absorbing cylinder:

Solution:

The rise in the cylinder's air temperature can be determined after calculating the final temperature of the air through the application of the ideal gas law. Ideal gas laws can be applied in this case because air, for most practical purposes, is assumed to act as an ideal gas.

According to ideal gas law:

$$(P_1. V_1)/T_1 = (P_2. V_2)/T_2$$
 Eq. 1.32

Or, through rearrangement of Eq. 1.32:

$$T_2 = (P_2 \cdot V_2 \cdot T_1)/(P_1 \cdot V_1)$$
 Eq. 1.33

Given or known:

$$P_1 = 1 \text{ Bar} = 101.33 \text{ kPa}$$
 $V_1 = 1.0 \text{ m}^3$
 $T_1 = 20 \text{ °C} \Rightarrow 273 + 20 \text{ °C} = 293 \text{ °K}$
 $P_2 = 2 \text{ bar or } 202.66 \text{ kPa}$
 $V_2 = 0.75 \text{ m}^3$

Then, by applying Eq. 1.33:

$$T_2 = \{(202.66 \text{ kPa}) \cdot (0.75 \text{ m}^3) \cdot (293 \text{ °K})\}/\{(101.33 \text{ kPa}) \cdot (1.0 \text{ m}^3)\}\$$

 $T_2 = 439.5 \text{ °K}$ i.e. 166.5 °C;

Therefore, the rise in the cylinder air temperature would be:

f) If the conveyor, at the bottom of the incline, is a belt driven roller conveyor and the rate of flow of blocks onto the conveyor is **one per 10 seconds**, determine the horsepower rating of the conveyor motor. Assume the conveyor belt to be directly driven off the conveyor motor shaft and that there is no slip between the belt and the rollers. Assume the motor efficiency to be 90%.

Solution:

Apply the **power**, **velocity** and **force** formula to determine the power requirment, as follows:

$$P = F \cdot v$$
 Eq. 1.34

Where,

P = Power Required to move the steel blocks

 \mathbf{F} = Force required to move the block or the force required to move the conveyor belt with the block on the rollers

 $\mathbf{v} = \text{Velocity of the belt; i.e. } \mathbf{2.0} \text{ m/s, as given.}$

While the velocity \mathbf{v} is given, the force \mathbf{F} is unknown and must be derived. Force can be defined in terms of mass flow rate $\dot{\mathbf{m}}$ and the change in velocity $\Delta \mathbf{v}$, as stated in **Eq. 1.35** below:

$$\mathbf{F} = \dot{\mathbf{m}} \cdot \Delta \mathbf{v}$$
 Eq. 1.35

Based on the derived mass of the block as 7850 kg and the fact that 1 block is moved ecery 10 seconds:

$$\dot{m}$$
 = mass flow rate = 7850 kg / 10 secs = 785 kg/s

And, based on the given conveyor speed of 2.0 m/s:

$$\Delta v =$$
Change in the velocity of the block
= $v_f \cdot v_f$
= 2.0 m/s - 0

 $\therefore \Delta v = 2.0 \text{ m/s}$ {Note: This change in the velocity is in the direction of the roller conveyor}

Therefore, applying **Eq. 1.35**, the force required to move the block would be:

$$F = \dot{\mathbf{m}} \cdot \Delta \mathbf{v}$$
= (785 kg/s) \cdot (2.0 m/s - 0)
= 1570 N {Note: (kg/s \cdot m/s) => kg \cdot m/s² => m \cdot a}

Then, by applying **Eq. 1.34**:

Since there are 746 watts per hp, the computed power of **3140 W**, in hp, would be:

$$P = (3140 \text{ W}) / (746 \text{ W/hp})$$

= 4.21 hp

Therefore, choose a standard **5 hp** motor.

Note: The efficiency of the motor is not needed in the motor size determination. Motor is specified on the basis of the brake horsepower required by the load; which in this case, is 4.21hp.

g) The conveyor transports the blocks to a cooling/quenching tank. The temperature of the blocks, when they are dropped into the quenching tank, is 100° C. The initial temperature of the water in the quench tank is 20° C and volume of water is 6.038 m³. The final, equilibrium, temperature of the water and the block is 30° C. How much heat is extracted by the quench water per block?

Solution:

Given or known:

```
c cast iron = 0.460 kJ/kg. °K {From Table 1.5}
m_{steel \ block} = 7850 \ kg, as determined earlier
T_{block - i} = 100 \ ^{\circ}C = 273 + 100 \ ^{\circ}C = 373 \ ^{\circ}K
T_{block - f} = 30 \ ^{\circ}C = 273 + 30 \ ^{\circ}C = 303 \ ^{\circ}K
\therefore \Delta T_{block} = 303 \ ^{\circ}K - 373 \ ^{\circ}K = -70 \ ^{\circ}K
According to Eq. 1.29:
Q = m. \ c. \Delta T
```

Therefore,

Q lost by the block =
$$(\mathbf{m}_{block})$$
. (c). (ΔT_{block})
Q lost by the block = (7850 kg) . $(0.460 \text{ kJ/kg.} ^{\circ}\text{K})$. (- 70°K)
= - 252,770 kJ

Since,

$$Q_{absorbed by water} = -Q_{lost by the block},$$

$$Q_{absorbed by water} = -(-252,770 \text{ kJ})$$

$$= +252,770 \text{ kJ}$$

Solution, Case Study 1.2 – US (Imperial) Unit Version

Given, known or derived values:

Volume of the steel block = $V_{steel\ block}$ = 35.32 ft³

Density of the steel block = ρ_{steel} = 491 lbm/ft³

Mass of the steel block = $m_{steel\ block}$ = $\rho_{steel} \cdot V_{steel\ block}$ = $(491\ lbm/ft^3) \times (35.32\ ft^3)$ $\therefore m_{steel\ block} = 17,342\ lbm$ Velocity of block at point "z" = V_z = 4.92 ft/s

Elevation at point "z" = h_z = 6.56 ft

Spring constant for the shock absorbing spring = V_z Frictional force, V_z = V_z = 4.92 lbf

Density of air at STP: 0.0805 lbm/ft³, at STP: Temperature = 32 °F, Pressure = 1 atm or 14.7 psia

a) Determine the velocity, v_w , of the steel block when it enters the horizontal segment of the travel, i.e. point w:

Solution Strategy: The velocity variable $\mathbf{v_w}$ is embedded in the kinetic energy of the block: $1/2 \cdot \mathbf{m}/\mathbf{g_c} \cdot \mathbf{v_w}^2$. So, if we can determine the amount of **kinetic energy** possessed by the block at point \mathbf{w} , we can derive the required velocity $\mathbf{v_w}$. To find the kinetic energy at point \mathbf{w} , we can apply the law of conservation of energy at points \mathbf{z} and \mathbf{w} as shown below:

Let total energy at point $z = E_{z-total}$

Then,

$$\begin{split} \mathbf{E}_{z\text{-total}} &= \mathbf{E}_{z\text{-kinetic}} + \mathbf{E}_{z\text{-potential}} \\ \mathbf{E}_{z\text{-total}} &= 1/2 \cdot (m/g_c) \cdot v_z^2 + m \cdot (g/g_c) \cdot h_z \\ \mathbf{E}_{z\text{-total}} &= 1/2 \cdot \{(17,342 \text{ lbm/}(32 \text{ lbm-ft/lbf-s}^2)\} \cdot (4.92 \text{ ft/s})^2 + (17,342 \text{ bm}) \cdot (32 \text{ ft/s}^2/32 \text{ lbm-ft/lbf-s}^2) \cdot (6.56 \text{ ft}) \\ &\therefore \mathbf{E}_{z\text{-total}} = 120,282 \text{ ft-lbf} \end{split}$$

The energy lost in the work performed against friction, during the block's travel from z to w, is accounted for as follows:

W_{f-wz} = Work performed against friction

= (Dist. w-z). (
$$F_f$$
)
= (164 ft). (89.92 lbf)
= 14,747 ft-lbf

Therefore, the energy left in the block when it arrives at point \mathbf{w} , at the bottom of the ramp, would be as follows:

$$E_{w-total} = E_{z-total} - W_{f-wz}$$

= 120,282 ft-lbf - 14,747 ft-lbf
= **105,535 ft-lbf**

Since the block is at "ground" elevation when it arrives at point **w**, the potential energy at point **w** would be "**zero**."

$$\therefore E_{w-total} = 1/2 \cdot (m/g_c) \cdot v_w^2$$

$$\text{Eq. 1.37}$$
Or,
$$v_w = \{2 \cdot (g_c) \cdot (E_{w-total}) / m \}^{1/2}$$

$$= \{2 \cdot (32 \text{ lbm-ft/lbf-s}^2) \cdot (105,535 \text{ ft-lbf}) / 17,342 \text{ lbm } \}^{1/2}$$

$$= 19.74 \text{ ft/s}$$

b) Employing the law of energy conservation and principles of energy conversion, calculate the value of the spring constant for the shock absorbing spring system.

Solution Strategy: The unknown constant \mathbf{k} is embedded in the formula for the **potential energy stored** in the spring after it has been fully compressed, upon stopping of the block. This potential energy is equal to the work performed on the spring, i.e., $\mathbf{W}_{\text{spring}} = 1/2 \cdot \mathbf{k} \cdot \mathbf{x}^2$. So, if we can determine the amount of **work performed on the spring**, during the compression of the spring, we can derive the required value of \mathbf{k} .

To derive the value of W_{spring} , we will apply the law of conservation of energy to the travel of the block from point z to point x.

Based on the dimensions in **Figure 1.3**:

Distance
$$x-z = (1.8 \text{ ft} + 131.23 \text{ ft} + 164 \text{ ft})$$

= 297 ft

 W_{f-xz} = Work performed against friction over Dist. x-z

Therefore, the total energy at point **z** would be:

$$\begin{aligned} \mathbf{E_{z\text{-total}}} &= \mathbf{W_{spring}} + \mathbf{W_{f - xz}} \\ \text{Or,} \\ \mathbf{W_{spring}} &= \mathbf{E_{z\text{-total}}} - \mathbf{W_{f - xz}} \\ &= 120,282 \text{ ft-lbf} - 26,706 \text{ ft-lbf} \\ &= \mathbf{93,576 \text{ ft-lbf}} \end{aligned}$$

Energy stored in the spring is quantified as:

$$W_{spring} = 1/2 \cdot k \cdot x^2$$

Since W spring has beed determined to be equal to 93,576 ft-lbf,

93,576 ft- lbf =
$$\frac{1}{2}$$
 . k . (1.8 ft)²

Therefore,

$$k = 2 \cdot W_{spring} / x^2$$

= 2 · W spring / x^2
= 2 · (93,576 ft- lbf) / (1.8 ft) ²
= 57,763 lbf/ft

Ancillary Exercise: This value of $\mathbf{k} = 57,763$ lbf/ft, in US units, is within 0.7% of the value of $\mathbf{k} = 837,212$ N/m, derived in SI units. The reader is encouraged to perform the unit conversions necessary to prove the practical equivalence between the \mathbf{k} values calculated in US and SI units.

c) How much energy is stored in the shock absorbing spring upon complete compression?

Solution:

Energy stored in the spring is equal to the work performed on the spring. The work performed on the spring, as computed in part (b) above, is:

$$W_{spring} = E_{z-total} - W_{f-xz}$$
$$= 120,282 \text{ ft-lbf} - 26,706 \text{ ft-lbf}$$

$$= 93.576 \text{ ft-lbf}$$

d) What would the steady state speed of the block be after it settles onto the roller conveyor?

Solution/answer:

After the block settles into a steady state condition on the conveyor, it assumes the speed of the conveyor, i.e. **6.56 ft/s**.

e) Rise in the temperature of the compressed air in the shock absorbing cylinder:

Solution:

The rise in the cylinder's air temperature can be determined after calculating the final temperature of the air through the application of the ideal gas law. Ideal gas laws can be applied in this case because air, for most practical purposes, is assumed to act as an ideal gas.

According to ideal gas law:

$$(P_1. V_1)/T_1 = (P_2. V_2)/T_2$$
 Eq. 1.32

Or, through rearrangement of Eq. 1.32:

$$T_2 = (P_2 \cdot V_2 \cdot T_1)/(P_1 \cdot V_1)$$
 Eq. 1.33

Given or known:

$$P_1 = 1 \text{ Atm} = 14.7 \text{ psia}$$

 $V_1 = 35.32 \text{ ft}^3$
 $T_1 = = > 461 + 68 \text{ °F} = 529 \text{ °R}$
 $P_2 = 2 \text{ Atm.} = 29 \text{ psia}$
 $V_2 = 26.49 \text{ ft}^3$

Then, by applying **Eq. 1.33**:

$$T_2 = \{(29 \text{ psia}) \cdot (26.49 \text{ ft}^3) \cdot (529 \text{ }^{\circ}\text{R})\} / \{(14.7 \text{ psia}) \cdot (35.32 \text{ ft}^3)\}$$
 $T_2 = 782.7 \text{ }^{\circ}\text{R}$
Or,
 $T_2 = 793.5 \text{ }^{\circ}\text{R} - 461 = 332.5 \text{ }^{\circ}\text{F}$

Therefore, the rise in the cylinder air temperature would be:

$$= 332.5 \, ^{\circ}\text{F} - 68 \, ^{\circ}\text{F}$$

 $= 264.5 \, ^{\circ}F$

f) If the conveyor, at the bottom of the incline, is a belt driven roller conveyor and the flow of blocks onto the conveyor is one per 10 seconds, determine the horsepower rating of the conveyor motor. Assume the conveyor belt to be directly driven off the conveyor motor shaft and that there is no slip between the belt and the rollers. Assume the motor efficiency to be 90%.

Solution:

Apply the **power**, **velocity** and **force** formula to determine the power requirment, as follows:

$$P = F \cdot v \qquad Eq. 1.34$$

Where.

P = Power Required to move the steel blocks

 \mathbf{F} = Force required to move the block or the force required to move the conveyor belt with the block on the rollers

 $\mathbf{v} = \text{Velocity of the belt; i.e. } \mathbf{6.562 \text{ ft/s}}, \text{ as given.}$

While the velocity \mathbf{v} is given, the force \mathbf{F} is unknown and must be derived. Force can be defined in terms of mass flow rate $\dot{\mathbf{m}}$ and the change in velocity $\Delta \mathbf{v}$, as stated in Eq. 1.38 below:

$$\mathbf{F} = (\dot{\mathbf{m}}/\mathbf{g}_c) \cdot \Delta \mathbf{v}$$
 Eq. 1.38

Based on the derived mass of the block as 7850 kg and the fact that 1 block is moved ecery 10 seconds:

$$\dot{m}$$
 = mass flow rate = 17,342 lbm / 10 secs = 1,734 lbm/sec

And, based on the given conveyor speed of 6.562 ft/s:

Then, by applying Eq. 1.38, the force required to move the block would be:

$$\mathbf{F} = (\dot{\mathbf{m}} / \mathbf{g_c}) \cdot \Delta \mathbf{v}$$

= {(1,734 lbm/sec)/(32 lbm-ft/lbf-s²)} . (6.562 ft/s)

 $= 356 \, lbf$

Then, by applying **Eq. 1.34**:

In hp, the computed power of 2336 ft-lbf/s would be:

Therefore, choose a standard **5 hp** motor.

Note: The efficiency of the motor is not needed in the motor size determination. Motor is specified on the basis of the brake horsepower required by the load; which in this case, is 4.21hp.

g) The conveyor transports the blocks to a cooling/quenching tank. The temperature of the blocks, when they are dropped into the quenching tank, is **212** °F. The initial temperature of the water in the quench tank is **68** °F and volume of water is **213.23** ft³. The final, equilibrium, temperature of the water and the block is **86** °F. Determine the amount of heat extracted by the quench water per block?

Solution:

Given or known:

$$\begin{array}{lll} c_{\; cast\; iron} = & 0.11\; BTU/lbm\; ^{\circ}F & \text{or } 0.11\; BTU/lbm\; ^{\circ}R \; \; \{ \text{From Table 1.5} \} \\ m_{steel\; block} = & 17,342\; lbm, \text{ as determined earlier.} \\ T_{block - i} = & 212\; ^{\circ}F \; => 461 + 212\; ^{\circ}F \; = 673\; ^{\circ}R \\ T_{block - f} = & 86\; ^{\circ}F \; => 461 + 86\; ^{\circ}F \; = 547\; ^{\circ}R \\ \therefore \Delta T_{block} = & 673\; ^{\circ}R \; - \; 547\; ^{\circ}R = 126\; ^{\circ}R \end{array}$$

And,

$$\Delta T_{block} = 212 \text{ }^{\circ}\text{F} - 86 \text{ }^{\circ}\text{F} = 126 \text{ }^{\circ}\text{F}$$

According to Eq. 1.29:

 $Q = m. c. \Delta T$ Eq. 1.29

Therefore,

$$\begin{array}{l} Q_{\ lost\ by\ the\ block} \ = \ (m_{\ block})\ .\ (c\)\ .\ (\Delta T_{block}) \\ Q_{\ lost\ by\ the\ block} \ = \ (17,342\ lbm)\ .(0.11BTU/lbm\ ^{\circ}R\)\ .\ (-126\ ^{\circ}R) \\ = \ -\ 240,360\ BTU \end{array}$$

Since,

Q absorbed by water = - Q lost by the block, Q absorbed by water = - (- 240,360 BTU) = + 240,360 BTU

Segment 1 Self-Assessment Problem & Question

1. Determine the amount of heat extracted by the quench water, per block, in Case Study 1.2, *using the temperature rise of the water* when the steel block is dropped into the quenching tank. The temperature of the block is $100^{\circ}C$ when it enters the quench water. The initial temperature of the water in the quench tank is $20^{\circ}C$ and volume of water is $6.038~m^3$. The final, equilibrium, temperature of the water and the block is $30^{\circ}C$.

Segment 2

Thermodynamics and Power Topics

- Concepts of power, power conversion, and efficiency
- Steam to wire power and energy transformations
- Steam to electrical power case study

Introduction

This segment focuses not only on the important concepts, theories, principles and analyses techniques associated with thermodynamics but also demonstrates their practical applications through case studies that illustrate the flow of energy from thermal form to utilities such as electricity. In order to be able to understand practical thermodynamic systems in a comprehensive fashion in advanced thermodynamics topics, in this segment, we will examine what happens to the energy after it is transformed from enthalpy to work performed by the turbine. Since thermodynamic systems are constructed and installed for applications and purposes that extend beyond the boilers and turbines, knowledge and appreciation of flow of energy downstream of the turbines is essential for ensuring that investment in comprehensive power generating systems, as a whole, is productive and effective.

This brief segment prepares us to better understand the flow of energy beyond the turbines, through examination of energy as it transforms and flows from superheated steam enthalpy form to electrical power delivered onto to the electrical power grid.

Before we embark on the exploration of flow of energy, let's review the concepts of power and efficiency.

Power and Efficiency:

Power: The concept of power was introduced briefly in the last segment. We introduced the fact that *power is rate of performance of work*, or **P** = **Work**/ **Elapsed Time** and that one of the ways power can be calculated is through the mathematical relationship **Power** = **Force x Velocity.** The counterpart of the last power formula, in the *rotational motion* realm, would be **Power** =

Torque x Rotational Velocity. Other aspects of power, forms of power and formulas for power will be introduced and discussed, in depth, in subsequent segments of this text.

Units for Power:

US/Imperial Unit System: hp, ft-lbf/sec, ft-lbf/min, BTU/sec

SI or Metric: Watts, kW, MW, GW, TW (10^{12} W)

Common Power Conversion Factors in the SI System:

```
1 \text{ J/s} = 1 \text{ N-m/s} = 1 \text{ W}

1 \text{ kJ/s} = 1 \text{ kW}

1000 \text{ kW} = 1 \text{ MW}
```

1.055 kJ/s = 1 BTU/s

One hp = 746 Watts = 0.746 kW = 550 ft-lbf/sec

Since the units for power and energy are often confused, let's also examine common units for energy so that the similarities and differences between the units for power and energy can be observed and noted.

Units for Energy:

US/Imperial Unit System: ft-lbf, BTU

SI or Metric Unit System: N-m, Joules or J, Wh, kWh, MWh, GWh, TWh (10 ¹² Wh)

Common Energy Conversion Factors:

1 J = 1 N-m

1 W x 1h = 1 Wh 1 kW x 1h = 1 kWh1000 kW x 1h = 1 MWh

1 BTU = 1055 J = 1.055 kJ 1 BTU = 778 ft-lbf 1 hp x 1 hour = 1 hp-hour

Efficiency: Efficiency is defined, generally, as the ratio of output to input. The output and input could be in form of power, energy or work. Efficiency assumes a more specific definition when considered in the context of a specific form of energy, work or power. The concept of efficiency, when applied in the thermodynamics domain, can involve power, energy or work.

In thermodynamics, when *power* is the subject of analysis, efficiency is defined as follows:

Efficiency = η = (Output Power)/(Input Power)

Efficiency in percent = η = (Output Power)/(Input Power) x 100

Where,

 η (Eta) is a universal symbol for efficiency

Also, in thermodynamics, when energy is the subject of analysis, efficiency is defined as follows:

Efficiency = η = (Output Energy)/(Input Energy)

Efficiency in percent = η = (Output Energy)/(Input Energy) x 100

Although work is not used as commonly in the computation of efficiency, in thermodynamics, where applicable, the efficiency calculation based on work would be as follows:

Efficiency = η = (Work Performed by the System)/(Work Performed on System)

Efficiency = η = (Work Performed by the System)/(Work Performed on System) x 100

As obvious from the definitions of efficiency above, since energy cannot be created, efficiency is always less than 1, or less than 100%. The decimal result for efficiency is often converted to, and stated as, a percentage value.

In the following section, we will explore the relationship between power and efficiency in steam, mechanical and electrical systems, and develop better understanding of the flow of power in steam type electrical power generating systems.

Power - Steam, Mechanical and Electrical

The power delivered by steam to the turbine blades, P_{steam} , in a simplified - no heat loss, no kinetic head loss, no potential head loss and zero frictional head loss - scenario can be represented by the mathematical relationship stated in form of Eq. 2.1. In the context of flow of energy from steam to electricity, functional relationship between electrical power, $P_{Electrical}$, generator efficiency $\eta_{Generator}$, steam turbine efficiency $\eta_{Turbine}$, and P_{steam} can be expressed in the form of Eq. 2.2.

$$P_{steam} = (h_i - h_f) \cdot \dot{m}$$
 Eq. 2.1

$$P_{Electrical} = (P_{steam}) \cdot (\eta_{Turbine}) \cdot (\eta_{Generator})$$
 Eq. 2.2

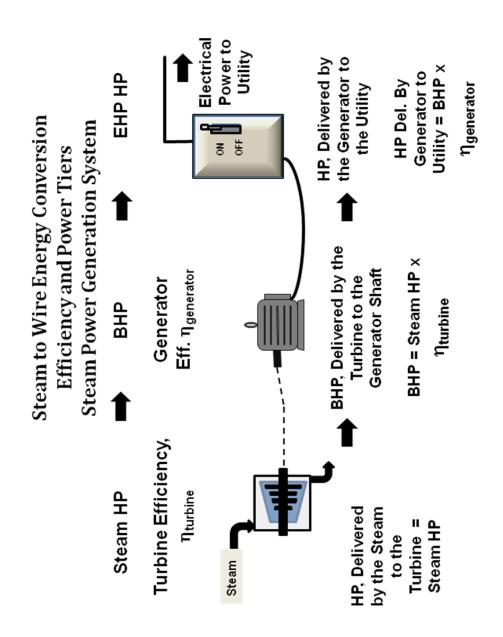


Figure 2.1 Steam to Wire Power Flow – Steam Power Generation System

The flow of power and energy from steam to electricity is depicted, in a power flow diagram, in **Figure 2.1**. This diagram is, essentially a pictorial illustration of **Eq. 2.2**. The power flow diagram in Figure 2.1 also forms the crux of the scenario analyzed in Case Study 2.1, below.

Case Study 2.1; Steam to Electricity Conversion

As an energy engineer, you are charged with the task to estimate the heat content or enthalpy, h_i , of the superheated steam that must be fed to a steam turbine in order to supply 10~MW (Mega Watt) of electrical power to the electrical grid. Assume that there is no heat loss in the turbine system and that difference between the enthalpies on the entrance and exit ends of the turbine is converted completely into work, minus the inefficiency of the turbine. All of the data available and pertinent to this project is listed below:

- Electrical Power Generator Efficiency: 90%
- Steam Turbine Efficiency: 70%
- Mass flow rate for steam, **m**: 25 kg/s (55 lbm/s)
- Estimated exit enthalpy, h_f, of the steam: 2875 kJ/kg (1239 BTU/lbm)

Solution:

Solution Strategy: In order to determine the estimated enthalpy, h_i , of the incoming steam, we need to start with the stated output (10 MW) of the generator and work our way upstream to derive the energy delivered to the vanes of the turbine. The assumption that there is no heat loss in the turbine system and that the difference between the enthalpies on the entrance and exit ends of the turbine is converted completely into work, minus the inefficiency of the turbine, implies that the energy delivered by the steam is equal to the net energy delivered to the turbine vanes. Also, note that net energy delivered to the turbine vanes is reduced or derated according to the given efficiency of the turbine.

Solution in SI/Metric Units

Since, 1J/s = 1W and 1 kJ/s = 1kW,

Power output of the generator = 10 MW = 10,000 kW= 10,000 kJ/s

Brake horsepower delivered by the turbine to the generator, through the turbine shaft is determined as follows:

Power delivered by the steam to the turbine vanes is determined as follows:

$$P_{steam}$$
 = BHP/Turbine Efficiency
= $(1.11 \times 10^4 \text{ kJ/s})/0.7$
= 1.5873 x 10⁴ kJ/s or 15,873 kJ/s

Of course, we could obtain the same result, in one step, by rearranging and applying Eq. 2.2 as follows:

$$\begin{split} P_{Electrical} &= (P_{steam}) \cdot (\eta_{Turbine}) \cdot (\eta_{Generator}) \\ P_{steam} &= P_{Electrical} / \{ (\eta_{Turbine}) \cdot (\eta_{Generator}) \} \\ P_{steam} &= (10,000 \text{ kJ/s}) / \{ (0.9) \cdot (0.7) \} \\ &= 15,873 \text{ kJ/s} \end{split}$$

Since the difference in the turbine entrance and exit enthalpies, in this scenario, is equal to the energy delivered to the turbine vanes:

$$\begin{aligned} & \textbf{P}_{steam} = \ (\textbf{h}_i - \textbf{h}_f) \ . \ \dot{\textbf{m}} \\ & 15,873 \ kJ/s = (\textbf{h}_i - 2875 \ kJ/kg) \ . \ 25 \ kg/s \\ & \textbf{h}_i = (15,873 \ kJ/s)/(25 \ kg/s) + 2875 \ kJ/kg \\ & \textbf{h}_i = \textbf{3,509} \ kJ/kg \end{aligned}$$

Solution in US/Imperial Units

Since,
$$1J/s = 1W$$
 and $1 kJ/s = 1kW$,
Power output of the generator = $10 MW = 10,000kW$
= $10,000kJ/s$

Since 1.055 kJ = 1.0 BTU,

Power output of the generator =
$$(10,000 \text{kJ/s}) \cdot (1/1.055 \text{kJ/BTU})$$

= 9,479 BTU/s

Brake horsepower delivered by the turbine to the generator, through the turbine shaft, is determined as follows:

BHP = Generator Output/Generator Efficiency = (9,479 BTU/s) /0.9 = 10,532 BTU/s

Power delivered by the steam to the turbine vanes is determined as follows:

Alternatively, we could obtain the same result, in one step, by rearranging and applying Eq. 2.2 as follows:

$$\begin{split} P_{Electrical} &= (P_{steam}) \cdot (\eta_{Turbine}) \cdot (\eta_{Generator}) \\ P_{steam} &= P_{Electrical} / \{ (\eta_{Turbine}) \cdot (\eta_{Generator}) \} \\ P_{steam} &= (9,479 \text{ BTU/s}) / \{ (0.9) \cdot (0.7) \} \\ &= 15,046 \text{ BTU/s} \end{split}$$

Since the difference in the turbine entrance and exit enthalpies, in this scenario, is equal to the energy delivered to the turbine vanes:

$$\begin{aligned} & \textbf{P}_{steam} = \ (\textbf{h}_{i} - \textbf{h}_{f}) \ . \ \dot{\textbf{m}} & \textbf{Eq. 2.1} \\ & 15,046 \ BTU/s = (\textbf{h}_{i} - 1239 \ BTU/lbm) \ . \ (55 \ lbm/s) \\ & \textbf{h}_{i} = (15,046 \ BTU/s)/(\ 55 \ lbm/s) + 1239 \ BTU/lbm \\ & \textbf{h}_{i} = \textbf{1512} \ BTU/lbm \end{aligned}$$

Self-Assessment Problems & Questions – Segment 2

1. As an energy engineer, you are charged with the task to estimate the amount of electrical power produced, in MW, by a steam based power generating plant. Assume that there is no heat loss in the turbine system and that difference between the enthalpies on the entrance and exit ends of the turbine is completely converted into work, minus the inefficiency of the turbine. All of the data available, pertinent to this project, is listed below:

- Electrical Power Generator Efficiency: 87%

- Steam Turbine Efficiency: **67%**

- Mass flow rate for steam, **m**: 20 kg/s (44 lbm/s)

- Exit enthalpy, h_f, of the steam: 2900 kJ/kg (1249 BTU/lbm)
- Incoming superheated steam enthalpy, h_i : 3586 kJ/kg (1545 BTU/lbm)
- 2. Consider the scenario described in Problem (1). Your client has informed you that the power generating plant output requirement has now **doubled**. Based on the concepts and principles learned in Segment 2, what is the most suitable alternative for doubling the power output if the exit enthalpy, h_f , of the steam must be kept constant at the original 2900 kJ/kg (1249 BTU/lbm) level?
 - A. Double the mass flow rate, **m**, only.
 - B. Double the incoming superheated steam enthalpy, $\mathbf{h_i}$ only.
 - C. Double the efficiency of the turbine.
 - D. Double the efficiency of the generator.
 - E. Increase mass flow rate, $\dot{\mathbf{m}}$, incoming superheated steam enthalpy, $\mathbf{h_i}$ and increase the efficiency specification on the turbine.

APPENDICES

Appendix A

This appendix includes the solutions and answers to end of segment self-assessment problems and questions.

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Appendix B Steam Tables

These steam tables, copyright ASME, published with ASME permission, do not include the heat of evaporation value, $\mathbf{h_{fg}}$, values for the saturation temperature and pressures. The saturated steam tables presented in this text are the compact version. However, the $\mathbf{h_{fg}}$ values can be derived by simply subtracting the available values of $\mathbf{h_L}$ from $\mathbf{h_v}$, for the respective saturation pressures and temperatures. In other words:

$$h_{fg} = h_v - h_L$$

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Appendix C

Common Units and Unit Conversion Factors

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Appendix D
Common Symbols

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