Electrical Engineering Fundamentals:
DC Circuit Analysis

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Electrical Engineering Fundamentals and Direct Current ©

Electrical Engineering for Non-Electrical Engineers Series ©

By

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Preface

Many Non-engineering professionals as well as engineers who are not electrical engineers tend to have a phobia related to electrical engineering. One reason for this apprehensiveness about electrical engineering is due to the fact that electrical engineering is premised concepts, methods and mathematical techniques that are somewhat more abstract than those employed in other disciplines, such as civil, mechanical, environmental and industrial engineering. Yet, because of the prevalence and ubiquitous nature of the electrical equipment, appliances, and the role electricity plays in our daily lives, the non-electrical professionals find themselves interfacing with systems and dealing with matters that broach into the electrical realm. Therein rests the purpose and objective of this text.

This text is designed to serve as a resource for exploring and understanding basic electrical engineering concepts, principles, analytical strategies and mathematical strategies.

If your objective as a reader is limited to the acquisition of basic knowledge in electrical engineering, then the material in this text should suffice. If, however, the reader wishes to progress their electrical engineering knowledge to intermediate or advanced level, this text could serve as a useful platform.

As the adage goes, "a picture is worth a thousand words;" this text maximizes the utilization of diagram, graphs, pictures and flow charts to facilitate quick and effective comprehension of the concepts of electrical engineering.

In this text, the study of electrical engineering concepts, principles and analysis techniques is made relatively easy for the reader by inclusion of most of the reference data, in form of excerpts from different parts of the text, within the discussion of each case study, exercise and self-assessment

problem solutions. This is in an effort to facilitate quick study and comprehension of the material without repetitive search for reference data in other parts of the text.

Due to the level of explanation and detail included for most electrical engineering concepts, principles, computational techniques and analyses methods, this text is a tool for those engineers and non-engineers, who are not current on the subject of electrical engineering.

The solutions for end of the segment self-assessment problems are explained in just as much detail as the case studies and sample problem in the pertaining segments. This approach has been adopted so that this text can serve as an electrical engineering skill building resource for engineers of all disciplines. Since all segments and topics begin with the introduction of important fundamental concepts and principles, this text can serve as a "brush-up," refresher or review tool for even electrical engineers whose current area of engineering specialty does not afford them the opportunity to keep their electrical engineering knowledge current.

In an effort to clarify some of the electrical engineering concepts effectively for energy engineers whose engineering education focus does not include electrical engineering, analogies are drawn from non-electrical engineering realms, on certain complex topics, to facilitate comprehension of the relatively abstract electrical engineering concepts and principles.

Each segment in this text concludes with a list of questions or problems, for self-assessment, skill building and knowledge affirmation purposes. The reader is encouraged to attempt these problems and questions. The answers and solutions, for the questions and problems, are included under Appendix A of this text.

Most engineers understand the role units play in definition and verification of the engineering concepts, principles, equations, and analytical techniques. Therefore, most electrical engineering concepts, principles and computational procedures covered in this text are punctuated with proper units. In addition, for the reader's convenience, units for commonly used electrical engineering entities, and some conversion factors are listed under Appendix C.

Most electrical engineering concepts, principles, tables, graphs, and computational procedures covered in this text are premised on SI/Metric Units. However, US/Imperial Units are utilized where appropriate and conventional. When the problems or numerical analysis are based on only one of the two unit systems, the given data and the final results can – in most cases - be transformed into the desired unit system through the use of unit conversion factors in Appendix B.

Some of the Greek symbols, used in the realm of electrical engineering, are listed in Appendix C, for reference.

What readers can gain from this text:

- Better understanding of some of the electrical engineering terms, concepts, principles, laws, analysis methods, solution strategies and computational techniques.

- Greater confidence in interactions with electrical engineering design engineers, electricians, controls engineers and electrical engineering experts.

- A number of skills necessary for succeeding in electrical engineering portion of various certification and licensure exams, i.e. CEM, Certified Energy Manager, FE, Fundamentals of Engineering (also known as EIT, or Engineer in Training), PE, Professional Engineering and many other trade certification tests.

An epistemic advice to the reader: if you don't understand some of the abstract concepts the first time, don't give up. Read it again! Such is the nature, intrigue and challenge of engineering, physics, science and other subjects that require thinking, reflection and rumination.

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Segment 1

Electrical Engineering Basics

Introduction

In this first segment of the Electrical Engineering for Non-Electrical Engineers text, we will explore the basics of electrical engineering terms concepts, principles, and analytical techniques. Many readers who embark on investing time and effort in studying this text are likely to do so for the key purpose of gaining an introduction into the field of electricity. Many others, on the other hand, might be interested in refurbishing prior knowledge of electrical engineering terms, concepts, principles and basic analytical techniques. Regardless of whether you belong to one of these two groups – or are simply in pursuit of electrical engineering at the intermediate or associates degree level – in this this segment we will lay the foundations in the electrical engineering realm by covering basic electrical engineering terms, concepts and principles, without the understanding of which, discussion and study of terms that bear important practical significance, such as power factor, real power, reactive power, apparent power, load factor, etc. would not be feasible.

Most of the material in this segment pertains to DC, or Direct Current, electricity. However, some entities we will discuss in this segment, such as, capacitive reactance, inductive reactance and impedance are fundamentally premised in the AC, alternating current, realm.

Electrical engineering is rooted in the field of physics. Physics, and electrical engineering, as most other fields in science, depend on empirical proof of principles and theories. Empirical analysis and verification requires measurement tools or instrumentation. So, after gaining a better understanding of the basic electrical concepts, we will conclude this segment with an introduction to two of the most common and basic electrical instruments, i.e. multi-meter and clamp-on ammeter.

Voltage

Voltage is defined as an electromotive that moves or pushes electrically charged particles like electrons, holes, negatively charged ions or positively charged ions. The term "electromotive" force stems from the early recognition of electricity as something that consisted, strictly, of the movement of "electrons." Nowadays, however, with the more recent breakthroughs in the renewable and non-traditional electrical power generating methods and systems like microbial fuel cells, and hydrocarbon fuel cells, electrical power is being harnessed, more and more, in form of charged particles that may not be electrons.

Two, relatively putative, analogies for voltage in the mechanical and civil engineering disciplines are pressure and elevation. In the mechanical realm – or more specifically in the fluid and hydraulic systems – high pressure or pressure differential pushes fluid from one point to another and performs mechanical work. Similarly, voltage – in form of voltage difference between two points, as with the positive and negative terminals of an automobile battery – moves electrons or charged particles through loads such as motors, coils, resistive elements, lamps, etc. As electrons or charged particles are pushed through loads like motors, coils, resistive elements, light filaments, etc., electrical energy is converted into mechanical energy, heat energy or light energy. In equipment like rechargeable batteries, during the charging process, applied voltage can push ions from one electrode (or terminal) to another and thereby "charge" the battery. Charging of a battery, essentially, amounts to the restoration of battery terminals' or plates' chemical composition to "full strength." So, in essence, the charging of a battery could be viewed as the "charging" of an electrochemical "engine." Once charged, a chemical or electrochemical engine, when presented with an electrical load, initiates and sustains the flow of electrical current, and performs mechanical work through electrical machines.

Common symbols for voltage are: **E, V, V_{DC}, V_{AC}, V_{P}, V_{M}, V_{Eff}, V_{RMS}.** Symbols "**E**" and "**V**" are synonymous, and both represent voltage. The symbol **E** stands for electromotive force, while **V**, simply, denotes voltage. In the absence of further specification through a subscript, these symbols can be somewhat ambiguous, in that, they could be construed to represent either AC or DC voltage.

The symbol $\mathbf{V_{AC}}$ represents AC voltage. When dealing with AC voltage, one needs to be specific about whether one is referring to "**peak**" voltage, $\mathbf{V_P}$, or **RMS** voltage, $\mathbf{V_{RMS}}$. Note that peak voltage, $\mathbf{V_P}$, is synonymous with maximum voltage $\mathbf{V_M}$. In addition, $\mathbf{V_{Eff}}$, the effective (AC) voltage, is the same as, RMS voltage, $\mathbf{V_{RMS}}$. The term RMS stands for **R**oot **M**ean **S**quare Value of AC Voltage. The RMS or effective value of AC voltage is the work producing portion of the AC voltage. This implies that AC voltage, current and power, all, have the "work producing" components and the" non-work producing" components. The work producing components, in essence, transform into - or contribute toward the production of – various forms of energy, mechanical work and break horsepower – or, to be more accurate, ***break horsepower-hour***.

Even though detail discussion on the mathematical composition of RMS voltage is outside the scope of this text, the formula for RMS voltage is as follows:

$$V_{RMS} = V_{EFF} = \sqrt{\frac{1}{T}\int_0^T V^2(t).\, d(t)} \qquad \textbf{Eq. 1.1}$$

AC voltage $\mathbf{V_{RMS}}$, $\mathbf{V_{Eff}}$, $\mathbf{V_P}$, and $\mathbf{V_M}$ are inter-related through the following equations:

$$V_p = V_m = \sqrt{2}V_{RMS} = \sqrt{2}V_{EFF} \qquad \textbf{Eq. 1.2}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}} = \frac{V_m}{\sqrt{2}} \qquad \textbf{Eq. 1.3}$$

Voltage is measured in volts, or V's; named after the Italian physicist Alessandro Volta (1745–1827), who invented the first chemical battery.

Current

Current consists of movement of electrons, ions, or simply charged particles. Movement of electrons can be oscillatory, vibratory or linear. When electrons vibrate or oscillate, the resulting current is AC current. AC current is established and sustained by AC voltage. When DC voltage is applied in an electrical circuit, electrons, ions or charged particles move in one direction. Such linear, unidirectional, movement of charged particles or electrons is DC

current. DC electrical current is analogous to fluid flow in mechanical or hydraulic systems. Just as pressure, or pressure differential, cause fluid to flow from point A to point B, DC voltage drives electrically charged particles to move from one point to another.

Common symbols for current are: **I, I_{DC}, I_{AC}, I_P, I_M, I_{Eff},** and **I_{RMS}**. Similar to the voltage symbols, the symbols of current assume a more specific meaning through associated subscripts.

I_{AC} represents AC current. When dealing with AC current, one needs to be specific about whether one is referring to "peak" current, **I_P**, or RMS current, **I_{RMS}**. Note that peak current, **I_P**, is synonymous with maximum current **I_M.** In addition, **I_{Eff}**, the effective (AC) current, is the same as, RMS current, **I_{RMS}**. The term RMS current stands for **R**oot **M**ean **S**quare Value of AC current. The RMS or effective value of AC current is the work producing portion of the AC current. AC current, like AC voltage and AC power, has a "work producing" component and a "non-work producing" component. The work producing component of AC current contributes toward the production of mechanical work and break horsepower.

The formula for RMS current is as follows:

$$I_{RMS} = I_{EFF} = \sqrt{\frac{1}{T}\int_0^T I^2(t).\, d(t)} \qquad \textbf{Eq. 1.4}$$

AC Current **I_{RMS}**, **I_{Eff}**, **I_P**, and **I_M** are inter-related through the following equations:

$$I_p = I_m = \sqrt{2} I_{RMS} = \sqrt{2} I_{EFF} \qquad \textbf{Eq. 1.5}$$

$$I_{RMS} = \frac{I_p}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \qquad \textbf{Eq. 1.6}$$

The unit for current is Ampere named after André-Marie Ampère (1775–1836), a French mathematician and physicist. André-Marie Ampère is revered as the father of electrodynamics. One amp of current is said to flow when electrical charge is flowing at the rate of ***one Coulomb per second***. This leads to the following mathematical definition:

$$1 \text{ Amp of Current} = \frac{1 \text{ - Coulomb of Charge}}{\text{Second}} \qquad \textbf{Eq. 1.7}$$

This definition of current is analogous to flow rate of fluid quantified in terms of mass flow rate $\dot{m}$ or volumetric flow rate $\dot{Q}$.

Example 1.1
In an AC system, a voltage source **V(t) = 156Sin(377t + 0°)** volts sets up a current of **I(t) = 15Sin(377t + 45°) amps**. The peak voltage and the peak current, in accordance with convention, are **156 V and 15 A**, respectively. Calculate the RMS values of voltage and current.

Solution:
According the Eq. 1.3:

$$V_{RMS} = \frac{V_p}{\sqrt{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore \; V_{RMS} = \frac{V_p}{\sqrt{2}} = \frac{156}{\sqrt{2}} = 110 \; V_{rms}$$

Note: This is the voltage indicated by a true RMS voltmeter when measuring the AC voltage at a typical household or workplace wall receptacle. See more discussion toward the end of this segment.

According the Eq. 1.6:

$$I_{RMS} = \frac{I_p}{\sqrt{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \; I_{RMS} = \frac{15}{\sqrt{2}} = 10.6 \; A_{RMS}$$

Note: This is the current indicated by a true RMS clamp-on ammeter when measuring AC current. See more discussion toward the end of this segment.

Resistance

Property of a material that opposes or resists the flow of current is known as electrical resistance, or simply, resistance. Electrical resistance is analogous to friction in mechanical systems; for instance, friction between the surfaces of two objects that slide against each other. In a fluid flow scenario, electrical resistance is analogous to friction between the fluid and the walls of the pipe. In electrical systems, resistance in conductors (wires) is an undesirable characteristic and results in wasted heat or heat losses. This is not unlike frictional head losses in fluid systems – frictional head losses governed by Darcy's equation and Hazen-William's equations. The symbol for electrical resistance is "**R**." Resistance is measured in ohms, or simply, **Ωs**. The ohm symbol **Ω** is often prefixed with letters, such as, **k** for kilo or **M** for Mega. Where, **1kΩ** would represent 1,000 Ohms and **1MΩ** would represent 1,000,000 ohms.

$$R = \frac{V}{I} \text{ or, } V = I.R \qquad \textbf{Eq. 1.8}$$

An ohm can also be defined on the basis of the Ohm's Law. The Ohm's Law and its application in AC and DC systems are discussed in greater depth in Segment 2. At this point, upon examination of Ohm's Law, in form of Eq. 1.8, we can define **1 ohm** as the amount of resistance that would permit the flow of only **1-amp** of current when a voltage of **1-volt** is applied across a specific length of that conductor. In other words:

$$1\Omega = \frac{\text{1-volt}}{\text{1-amp}}$$

The Ohm's Law equation stated above also stipulates that as the increase in electrical "demand" manifests itself in form of an increase in current, **I**, and as resistance, **R,** of the conductor increases, the voltage – or voltage drop – across the conductor increases; resulting in lower voltage at the point of delivery of power to the consumers. This is similar to the ***pressure drop***, ***or loss***, experienced in a long compressed air pipe or header due to frictional head loss.

From physical characteristic and physical composition point of view, we could define resistance as being directly proportional to the **length** of the conductor and inversely proportional to the **area of cross-section** of the

conductor. This proportional relationship can be transformed into a mathematical relationship or equation as follows:

$$R = \rho.\frac{L}{A} \qquad \textbf{Eq. 1.9}$$

Where, ρ = Resistivity of the Conductor,

L = Length of the Conductor and

A = Area of Cross-section of the Conductor.

ρ_{copper} = **17.2 n Ωm**; where, **n** = Nano = 10^{-9}

$\rho_{aluminum}$ = **28.2 n Ωm**

Example 1.2

A cubic block of electrically conductive material measures 0.02 on each side. The resistivity of this material is 0.01 Ω˙m. What is the resistance between opposite sides of this block?

Solution:

$$R = \frac{\rho L}{A} = \frac{(0.01\,\Omega\text{-m})(0.02\text{ m})}{(0.02\text{ m})^2} = 0.5\,\Omega$$

Example 1.3

A phase conductor of a power distribution line spans, approximately, 500 ft and has a diameter of 1.5 inch. The conductor is composed of copper. Calculate the electrical resistance of this conductor, per phase.

Solution:

Solution Strategy: Since the resistivity value of copper, as stated above, is in metric or SI unit system, the length and diameter specifications stated in this problem must be streamlined in metric units before application of Eq. 1.9 for determination of resistance in ohms (Ωs).

L = 500 ft = 152.4 m

Diameter = 1.5 inch = 0.0381m; ∴**R** = Radius = D/2 = 0.019 m

A = Area of cross-section = $\boldsymbol{\pi.R^2}$ = (3.14)(0.019) ^{2} = 0.00113m^2

ρ_{copper} = 17.2 n Ωm = 17.2x10^{-9} Ωm

$$R = \rho.\frac{L}{A}$$

$$\therefore\ R = \rho.\frac{L}{A} = 17.2x10^{-9}\left(\frac{152.4}{0.00113}\right) = 0.00232\Omega$$

As described earlier, in electrical systems, resistance in conductors (wires) is an undesirable characteristic and results in wasted heat or heat losses. This energy lost, as heat, as the current flows through a conductor can be quantified through Eq. 1.10 below.

$$\text{Heat Loss} = P = VI = \frac{V^2}{R} = I^2R, \text{ in Watts} \qquad \textbf{Eq. 1.10}$$

In an effort to illustrate the substantial impact of resistance on heat losses and design challenges associated with power transmission, let's continue examination of long transmission lines used to transport electrical power from power generating plants to power consumers. On a hot summer afternoon, those power transmission lines that we notice crisscrossing the country side are not only carrying higher currents due to the higher air conditioning loads, but experience an increase in resistance due to increases in resistivity, $\boldsymbol{\rho}$, of the conductor in accordance with Eq. 1.11. In other words, the resistivity of aluminum, stated above as $\rho_{aluminum}$ = **28.2 n Ωm,** serves as a ***constant only at standard temperature of 20°C or 68°F.***

$$\rho = \rho_0\left(1+\alpha\left(T-T_0\right)\right) = \rho_0\left(1+\alpha\Delta T\right) \qquad \textbf{Eq. 1.11}$$

In Eq. 1.11:

$\boldsymbol{\rho}$ = Resistivity at current temperature "**T**"
$\boldsymbol{\rho_o}$ = Resistivity at standard temperature "$\mathbf{T_o}$"
$\mathbf{T_o}$ **=** Standard temperature in °C
T = Current temperature °C
$\boldsymbol{\alpha}$ = Thermal coefficient of resistance, in 1/°C

In addition, it's common knowledge that as temperature rises, most metals or conductors expand. So, with exposure to elevated summer season solar radiation, during those hot afternoons, the transmission lines elongate to a certain extent, resulting in higher "**L**." As those transmission lines elongate, the diameter of the conductors drops, resulting in lower area of cross-section **A**. So, the perceptible "sag" in the transmission lines during the hot summer afternoons is not simply an optical illusion or a myth.

As we account for the increase in **L**, reduction in **A** and the rise in **ρ** collectively, in accordance with Eq. 1.9, we see that all of these factors result in escalation of resistance. Furthermore, if we consider resistive heat loss equation, **Eq. 1.10**, the ***exponential*** effect of the rise in load current **I** and the increase in **R** precipitate in a "cascading," unfavorable, physical and electrical impact on the transmission line conductors.

Resistors in series

When electrical circuits, AC or DC, consist of multiple resistors, circuit analyses require simplification of such network of resistors into one, ***equivalent***, resistor or resistance, $\mathbf{R_{EQ}}$. Often, this equivalent, resistance is referred to as a "***total***" resistance. When "**n**" number of resistors are connected in a "daisy chained" or concatenated fashion as shown in Figure 1.1, they are said to be connected in series.

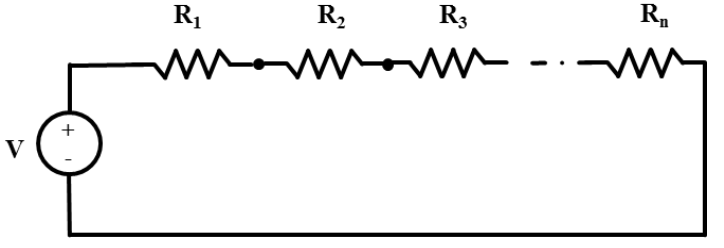


Figure 1.1: n –Resistors in series

When resistors are connected in series, they can be combined in a "linear addition" format, as stipulated in Eq. 1.12 below, for "**n**" number of resistors.

$$R_{EQ} = R_T = R_1 + R_2 + R_3 + \ldots + R_n \qquad \textbf{Eq. 1.12}$$

When multiple resistors are combined into an equivalent resistor, with resistance value $\mathbf{R_{EQ}}$, the simplified version of the original series circuit would appear as pictured in Figure 1.2.

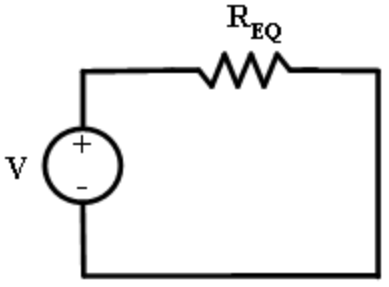


Figure 1.2: Equivalent resistance for **n** –resistors in series

Resistors in Parallel

When electrical circuits, AC or DC, consist of multiple resistors, connected in a parallel fashion, as shown in Figure 1.3, circuit analysis would require simplification of the parallel network of resistors into one, equivalent resistor $\mathbf{R_{EQ}}$. Figure 1.3 shows "**n**" number of resistors connected such that the "heads" of all resistors are "bonded" are connected together, with and electrical connection to the anode (positive terminal) of the DC power supply. In addition, the "tails" of all resistors are connected together and to the cathode of the DC power source.

When multiple parallel resistors are combined into an equivalent resistor, with resistance value $\mathbf{R_{EQ}}$, the simplified version of the original series circuit would appear as pictured in Figure 1.4. In parallel resistor networks, the calculation of $\mathbf{R_{EQ}}$ involves addition of the inverses of all resistors in the parallel network, and taking the inverse of the sum as stipulated in Eq. 1.13.

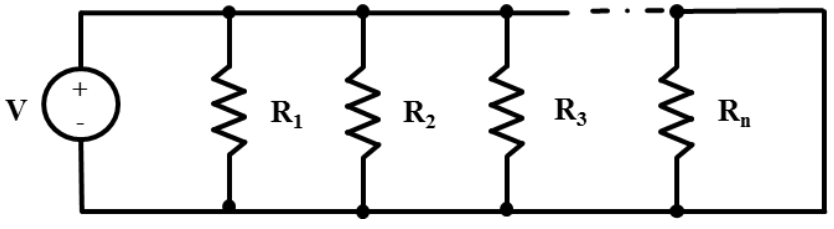


Figure 1.3: n –Resistors in parallel

$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}} \qquad \textbf{Eq. 1.13}$$

In case of a simple two parallel resistor circuit, the equivalent resistance $\mathbf{R_{EQ}}$ could be calculated using the simplified equation, Eq. 1.14.

$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2} \qquad \textbf{Eq. 1.14}$$

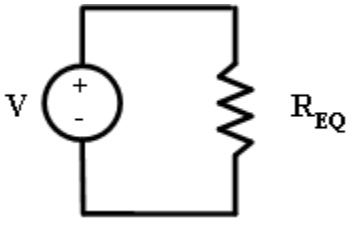


Figure 1.4: Equivalent resistance for **n** –resistors in parallel

As a special case, assume that the parallel resistor network shown in Figure 1.3 consists of "**n**" ***equal*** parallel resistors. Because, the resistors are assumed to be equal, calculation of $\mathbf{R_{EQ}}$, or $\mathbf{R_{EQ\text{-}n}}$, boils down to Eq. 1.15.

$$R_{EQ} = R_{EQ\text{-}n} = \frac{R}{n} \qquad \textbf{Eq. 1.15}$$

Electrical Short and Open Circuit

When a conductor, wire or bus-bar, is used to pass current between two or more points in an electrical circuit - with smallest resistance feasible - such a connection is referred to as a "**short.**" A short segment of wire, assembled with an "***alligator clip***" on each end, is used by electrician and electrical engineers to establish a temporary short circuit between points in an electrical circuit. Such a specially fabricated pieces of wire are sometimes referred to as a "***jumper leads***," in electrical jargon.

A short circuit between two points implies zero or negligible resistance. The opposite of an electrical short is an "**open circuit**." Examples of open circuits would be a switch that is open, a breaker that is turned off, or simply a wire that has been cut or "***clipped***." An open circuit between two points implies an infinite (∞) resistance. The concepts of open and short

circuits are illustrated in Figure 1.5. Figures 1.5 (a) and (b) represent electrical circuit segments between points A and B. When the switch between points (1) and (2) is open, as depicted in Figure (a), we have an open circuit, and no current flows between points **A** and **B**. However, closure of the same switch, as shown in Figure (b), constitutes a short circuit. The closed switch scenario depicted in Figure (b) also represents "**continuity.**" The term "continuity" is used commonly by electrical engineers and electricians during trouble shooting of equipment. When trouble shooting electrical or electronic equipment failures, engineers and technicians often perform continuity tests on fuses to determine if they have cleared or opened. The instrument used for performing continuity checks is a multi-meter. A continuous piece of wire, shown in Figure 1.5 (c), represents continuity between terminals **A** and **B**.

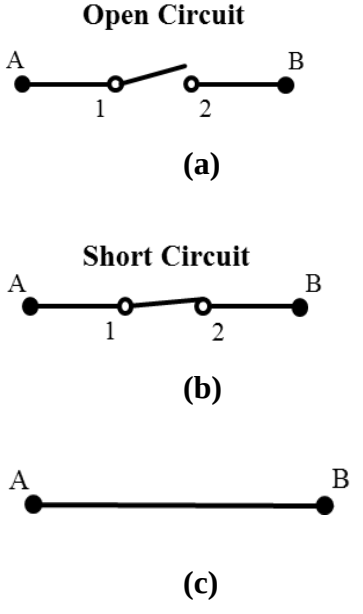
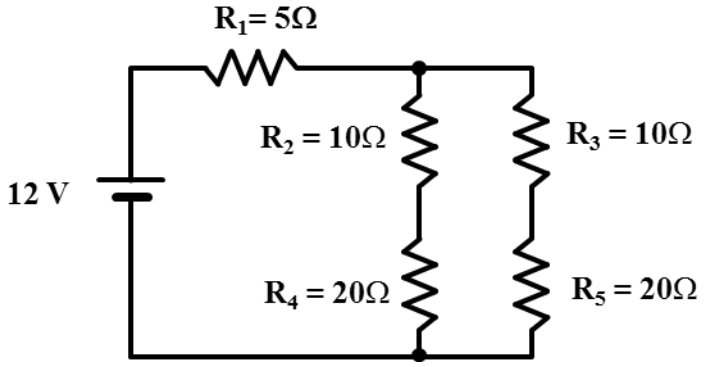


Figure 1.5: Open and short circuits

Example 1.4

Determine the equivalent resistance for the DC circuit shown below if R_1 = 5Ω, $R_2 = R_3$= 10Ω, and $R_4 = R_5$= 20Ω.



Solution:

The resistances need to be combined in a multistep process in order to determine R_{eq} for the overall circuit.

Combination of R_2 and $R_4 = R_{2,4} = R_2 + R_4 = 10\Omega + 20\Omega = 30\Omega$

Combination of R_3 and $R_5 = R_{3,5} = R_3 + R_5 = 10\Omega + 20\Omega = 30\Omega$

Combination of $R_{2,4}$ and $R_{3,5}$:

$$R_{2-5} = \frac{(30\Omega).(30\Omega)}{(30\Omega + 30\Omega)} = \frac{900}{60} = 15\Omega$$

$$\therefore\ R_{eq} = 15\Omega + 5\Omega = 20\Omega$$

Capacitor and Capacitance

A capacitor is a charge storage device, capable of storing charge in DC and AC applications. A charged capacitor stores electrical charge on two electrodes; one of the two electrodes is negative and the other one is positive. The negative electrode is called a ***cathode*** and the positive electrode is referred to as an ***anode***. This separation of charge and the quantity of charge separated determine the electrical potential - or voltage - developed across the electrodes of the capacitor. The electrical potential difference between the capacitor plates, or electrodes, signifies the storage of electrical energy in the capacitor. Therefore, as an energy storage device -with a potential difference – capacitors are analogous to air receivers and pneumatic cylinders that store

pressure energy - in form of higher pressure relative to the atmospheric pressure – which can be used to perform mechanical work.

Construction of a simple capacitor is depicted in Figure 1.6. As shown in Figure 1.6, a simple capacitor can be constructed with two parallel square plates, of equal size, separated by a dielectric substance like glass, mica, etc. The separation between the two plates (electrodes), "**r**," in conjunction with the area of the plates determines the "**capacitance**" of the capacitor. Capacitance, "**C**," of a capacitor is defined as the charge storage capacity of the capacitor.

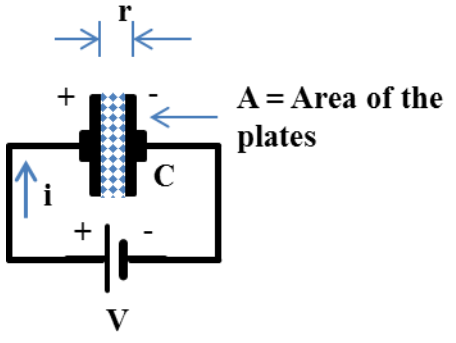


Figure 1.6: A simple parallel plate capacitor

Capacitance can be defined, mathematically, through Eq. 1.16, below.

$$C = \in \frac{A}{r} \qquad \textbf{Eq. 1.16}$$

Where,

C = Capacitance is quantified or specified in farads.

A = The area of cross-section – or simply area – of the capacitor electrode plates.

ϵ = Permittivity of the dielectric medium between the plates.

And, $\epsilon = \epsilon_r \cdot \epsilon_0$

Where, ϵ_r = Relative permittivity of a specific dielectric medium

And, ϵ_0 = permittivity in vacuum or in air = 8.854 x 10^{-12} farads per meter (F·m−1).

One farad is rather a large amount of capacitance for most common capacitor applications. Therefore, many capacitors – especially, at the circuit board level – are specified or labeled in terms of smaller units, such as, **mF (**milli-

Farad), **μF (**micro-Farad), or **nF (**nano-Farad). The capacitor shown in Figure 1.7 is rated 470**μF** and designed to operate at a maximum of 35V.

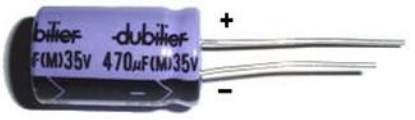


Figure 1.7: A cylindrical 470 Micro-Farad Capacitor

The mathematical relationship stated as Eq. 1.16 stipulates that capacitance is directly proportional to the area **A** of the capacitor plates and inversely proportional to the separation **r** between the plates. In other words, if larger capacitance or ***charge storage capacity*** is desired, one must increase the area of the plates and/or decrease the separation between the capacitor plates. In addition to serving as a "***constant of proportionality***" for the equation, permittivity **ϵ** injects the property or characteristic of the dielectric medium into the computation of capacitance through the dielectric medium's characteristic ϵ_r value. Although, for the sake of simplicity, the discussion on capacitance in this text is limited to flat plate capacitors, many capacitors have cylindrical construction such as the one shown in Figure 1.7.

Electrical energy stored in a capacitor can be determined through application of Eq. 1.17

$$\text{Energy (joules)} = \frac{1}{2}CV^2 \qquad \textbf{Eq. 1.17}$$

Charge storage characteristic of the capacitor should not be confused with the charge storage and power source function of a battery. One difference between capacitors and batteries is that when capacitors are charging or discharging, charge flows through a "dielectric" medium; while most batteries consist of electrolytes (i.e. sulfuric acid) that ionize readily and the ions sustain the flow of current. Capacitors allow the charge to move between electrode plates through a dielectric medium.

The dynamics of how a capacitor stores and dissipates charge are somewhat different between the DC and AC realms. Unlike resistive circuits, current and voltage associated with capacitors vary in a non-linear fashion. A common, series, RC circuit is shown in Figure 1.8, consisting of a capacitor, resistor and a switch that can be used to control the charging and discharging

of the capacitor. The graphs in Figure 1.9 through 1.11 below compare voltage and current responses in circuits that are purely resistive versus a series RC circuit, the type illustrated in Figure1.8. This "non-linear" charging and discharging of capacitors is referred to as ***transient behavior*** of RC circuits.

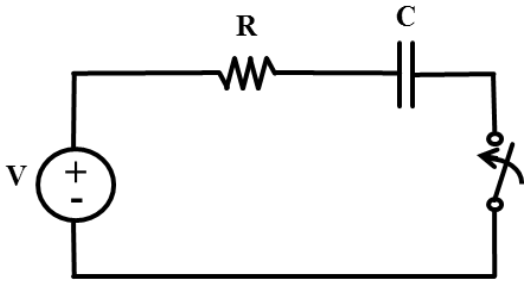


Figure 1.8: A series RC circuit

The straight line graph in Figure 1.9 illustrates and validates the Ohms law. This graph shows that voltage and current are directly proportional, with the "constant" resistor serving as the constant of proportionality. In a purely resistive circuit, the current would respond, instantaneously and linearly, to the application of voltage across a resistor.

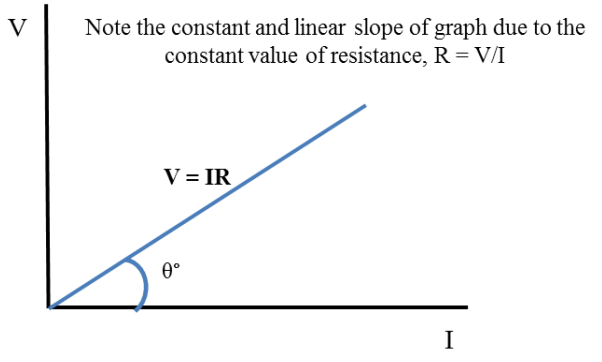


Figure 1.9: Linear, voltage vs. current, response in a purely resistive circuit.

Voltage vs. Time, transient response in series RC circuit: Contrary to linear and instantaneous response in a purely resistive circuit, the current and voltage response in a common RC circuit is non-linear and non-instantaneous.

$$v_c(t) = v_c(0)\, e^{-\frac{t}{RC}} + V\left(1 - e^{-\frac{t}{RC}}\right) \qquad \text{Eq. 1.18}$$

The voltage response - or voltage variation - of a capacitor can be predicted through Eq. 1.18.

Where,

R = Resistance in series with the capacitance
C = Capacitance
$v_c(0)$ = Voltage across the capacitor, at time t = 0
$v_c(t)$ = Voltage across the capacitor, at a given time **t**
V = Voltage of the power source
$RC = \tau$ = Time constant of an RC circuit

If the voltage variation or response of a capacitor were graphed, with respect to time, it would be resemble the v_c versus ***t*** graph shown in Figure 1.10.

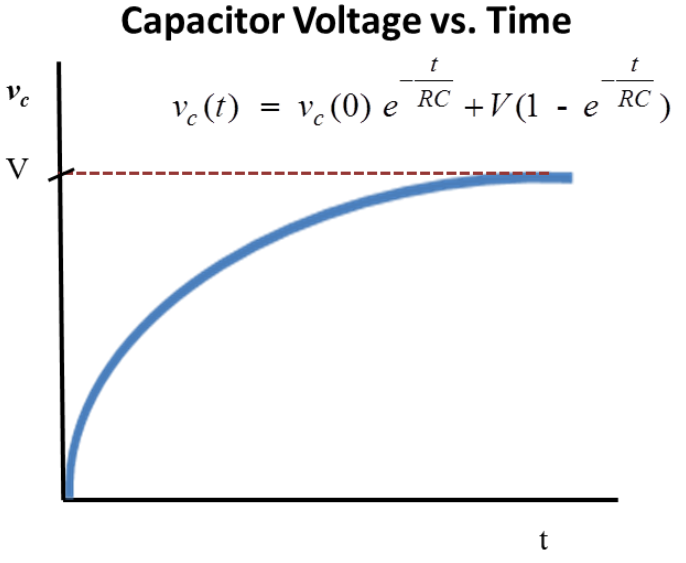


Figure 1.10: Non-linear, transient, voltage response in a circuit consisting of capacitance and resistance

Current vs. Time, transient, response in series RC circuit: The current response - or current variation - in a capacitor-resistor circuit can be predicted through Eq. 1.19.

$$i_c(t) = \left(\frac{V - v_c(0)}{R} \right) e^{-\frac{t}{RC}}$$ Eq. 1.19

Where,

R = Resistance in series with the capacitance

C = Capacitance, in farads

$\boldsymbol{v_c(0)}$ = Voltage across the capacitor, at time t = 0

$\boldsymbol{i_c(t)}$ = Current through the capacitor-resistor circuit, at a given time **t**

V = Voltage of the power source

If the current response of a capacitor were graphed, with respect to time, it would resemble the $\boldsymbol{i_c}$ versus ***t*** graph shown in Figure 1.11. Note that the capacitor current versus time curve in Figure 1.11 validates Eq. 1.19. For instance, the graph shows that at t = ∞, or when steady state is achieved, current $i_c(t)$ through the RC circuit diminishes to zero; and if you substitute t = ∞ in Eq. 1.19, $i_c(t)$ becomes zero. In other words, at steady state, the capacitor transforms into an open circuit and the current ceases to flow.

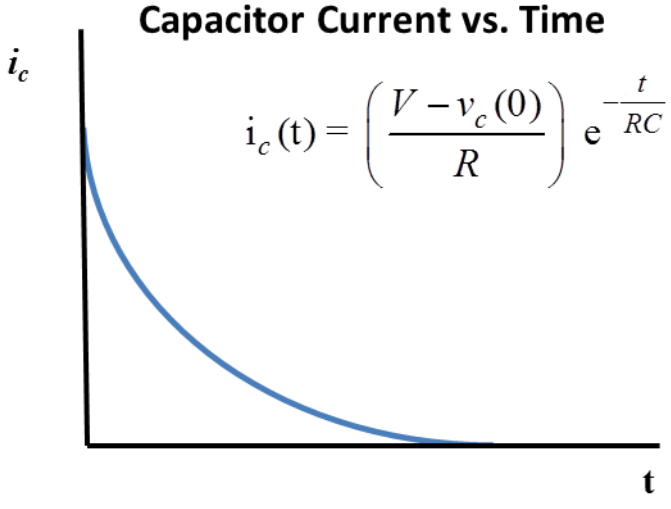


Figure 1.11: Non-linear, transient, current response in a circuit consisting of capacitance and resistance

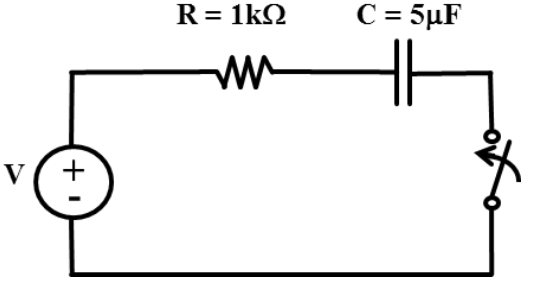
The presence of resistance **R** in capacitive circuits results in what is referred to as a time constant "**τ.**" The relationship between**τ, R and C** is stipulated by Eq. 1.20. The physical significance of time constant **τ** is that it represents the time it takes to charge a capacitor to **63.2% of the full value**, or

63.2% of the full voltage of the source. Time constant τ also represents the time it takes to discharge a given capacitor to **36.8% of the full voltage**, or the voltage of the source.

$$\tau = RC \qquad \textbf{Eq. 1.20}$$

Example 1.4

Consider the RC circuit shown in the diagram below. The source voltage is 12V. The capacitor is in a discharged state before the switch is closed. The switch is closed at t = 0. What would the capacitor voltage be at t = 2τ?



Solution:

This particular case represents a capacitor charging scenario. Given the value of **R, C**, $v_c(0)$ and the source voltage ***V***, Equation 1.18 allows us to calculate the voltage after and elapsed time "**t**," during the capacitor charging phase.

$$v_c(t) = v_c(0)\, e^{-\frac{t}{RC}} + V(1 - e^{-\frac{t}{RC}})$$

In this case,

R = 1 kΩ = 1,000 Ω

C = 5μF = $5x10^{-6}$ F

$v_c(0)$ = 0V = Voltage across the capacitor at t = 0

v_c**(t) =** Voltage across the capacitor, at a given time **t = ?**

V **=** Voltage of the power source = 12V

$RC = \tau =$ RC circuit time constant

$t = 2\tau = 2RC$

Substitution of the given values, stated above, Eq. 1.18 can be expanded and simplified as follows:

$$v_c(t) = v_c(0) \,.\, e^{-\frac{2\tau}{\tau}} + 12.(1 - e^{-\frac{2\tau}{\tau}})$$

$$= 0 + (12).(1 - e^{-2}) = 12(0.865)$$

$$= 10.38\text{V}$$

Capacitors in Series

When a number of capacitors are connected in a "daisy- chained," or concatenated fashion, as shown in fig.1.12, they are said to be connected in series. In Figure 1.12, "**n**" number of capacitors, $\mathbf{C_1}$ through $\mathbf{C_n}$, are shown connected in series.

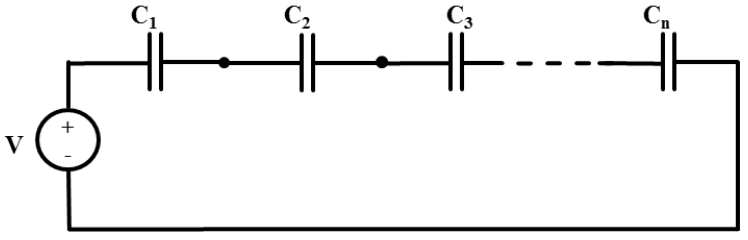


Figure 1.12: Capacitors in series

When electrical circuits, AC or DC, consist of multiple capacitors, circuit analyses require simplification – or combination - of such network of capacitors into one, ***equivalent***, capacitor or capacitance, $\mathbf{C_{EQ}}$. This equivalent, capacitance can also be referred to as a "***total***" capacitance.

Unlike series combination of resistors, when capacitors are connected in series, they can be combined in an "**addition of inverses**" format, as stipulated in Eq. 1.21 below, for "**n**" number of capacitors.

$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}} \qquad \textbf{Eq. 1.21}$$

When multiple capacitors are combined into an equivalent capacitor, with capacitance value $\mathbf{C_{EQ}}$, the simplified version of the original series circuit can be drawn as shown in Figure 1.13.

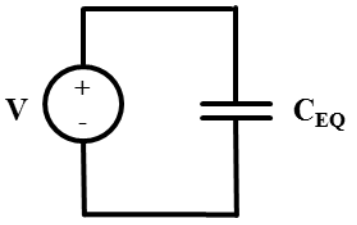


Figure 1.13: Equivalent capacitance for capacitors in series.

As a special case, suppose that the series capacitor network shown in Figure 1.12 consists of "**n**" series capacitors, with ***equal*** capacitance. Because the capacitors are assumed to be equal, calculation of $\mathbf{C_{EQ}}$, or $\mathbf{C_{EQ\text{-}n}}$, and application of Eq. 1.21 can be simplified to Eq. 1.22.

$$C_{EQ} = C_{EQ\text{-}n} = \frac{C}{n} \qquad \textbf{Eq. 1.22}$$

If a series capacitive circuit consists of only three capacitors, as shown in Figure 1.14, Eq. 1.21 can be reduced to Eq.1.23. Further simplification of Eq. 1.23 would result in a, simplified, three capacitor series equivalent capacitance equation, Eq. 1.24.

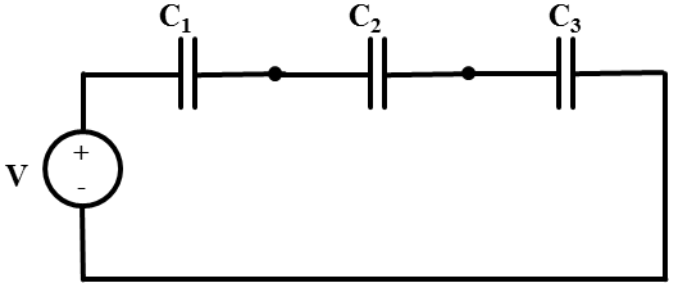


Figure 1.14: Equivalent capacitance for capacitors in series.

$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad \textbf{Eq. 1.23}$$

$$C_{EQ} = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_1 C_3} \qquad \textbf{Eq. 1.24}$$

Example 1.5
Determine the equivalent capacitance for the DC circuit shown Figure 1.14 if $C_1 = 5\mu F$, and $C_2 = C_3 = 10\mu F$.

Solution:
Application of Eq. 1.24 to the three series capacitor circuit shown in Figure 1.14 yields:

$$C_{EQ} = \frac{(5x10^{-6})(10x10^{-6})(10x10^{-6})}{(5x10^{-6})(10x10^{-6}) + (5x10^{-6})(10x10^{-6}) + (10x10^{-6})(10x10^{-6})}$$

$$C_{EQ} = 2.5\mu F$$

Capacitors in Parallel

When electrical circuits consist of capacitors connected in parallel, as shown in Figure 1.15, circuit analyses – as with a network of series connected capacitors - would require simplification of the parallel network of capacitors into one, equivalent capacitor $\mathbf{C_{EQ}}$. Figure 1.15 shows "**n**" number of capacitors connected such the "heads" of all capacitors are (electrically) "bonded" or are connected together, with an electrical connection to the anode (or positive terminal) of the DC power supply, and the "tails" of all capacitors are connected together to the cathode (or negative terminal) of the power source.

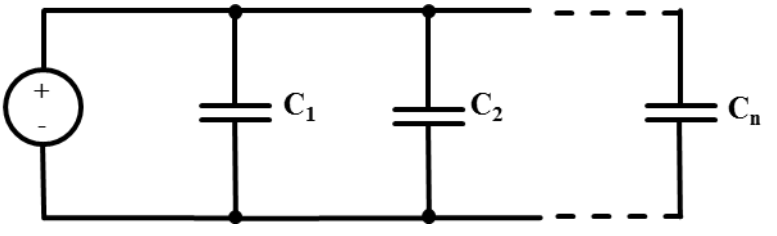


Figure 1.15: Equivalent capacitance for capacitors in series.

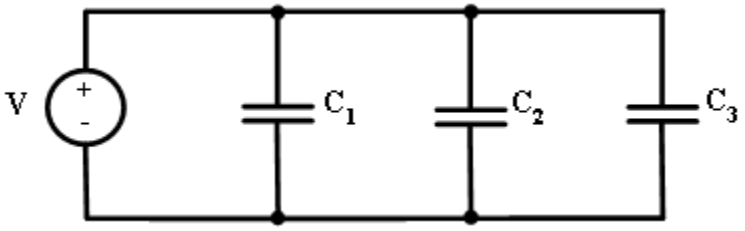
Simplification of a network of capacitors is parallel is similar to the approach utilized in the combination of ***resistors in series***. When multiple

parallel capacitors are combined into an equivalent capacitor, $\mathbf{C_{EQ}}$, the simplified or condensed equivalent of the original parallel circuit could also be represented by the equivalent circuit diagram in Figure 1.13.

For a parallel capacitor circuit consisting of "**n**" number of parallel capacitors, equivalent capacitance can be calculated by applying equation Eq. 1.18

$$C_{EQ} = C_1 + C_2 + C_3 + ... + C_n \qquad \textbf{Eq. 1.25}$$

Example 1.6
Determine the equivalent capacitance for the DC circuit shown below if C_1 = C_2 =5μF, and C_3 = 10μF.



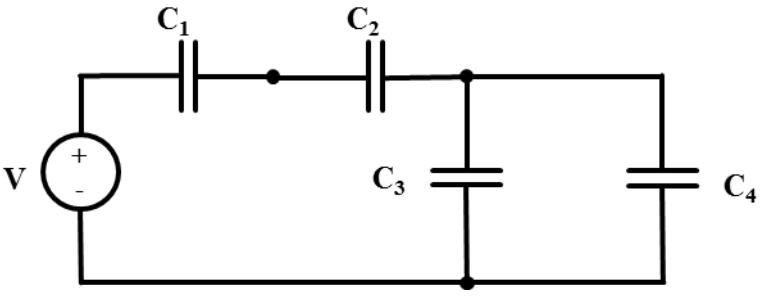
Solution:
Applying Eq. 1.25 to the three parallel capacitor circuit shown in diagram above yields:

$$C_{EQ} = C_1 + C_2 + C_3$$

$$C_{EQ} = (5x10^{-6})(10x10^{-6}) + (5x10^{-6})(10x10^{-6}) + (10x10^{-6})(10x10^{-6})$$

$$= 20x10^{-6} \text{ F, or } 20\mu\text{F}$$

Example 1.7
Determine the equivalent capacitance in series and parallel combination circuit shown below. The capacitance values are: C_1 = C_2 = 5μF, C_3 = C_4 = 10μF.



Solution:

Similar to the parallel and series combination approach described in the resistor section, our aim would be to simplify the circuit by first combining the linearly additive segment (or segments), followed by reciprocal combination of the remaining capacitors.

The capacitors in this circuit that lend themselves to linear combination are C_3 and C_4. Therefore, the combined capacitance, $C_{3\text{-}4}$, would be:

$$C_{3\text{-}4} = C_3 + C_4 = 10\mu F + 10\mu F = 20\mu F$$

Then, by applying Eq.1.24 to this special hybrid capacitor combination case, with the 3rd capacitance being C_{34} instead of C_3:

$$C_{EQ} = \frac{C_1 C_2 C_{3-4}}{C_1 C_2 + C_2 C_{3-4} + C_1 C_{3-4}}$$

$$C_{EQ} = \frac{(5x10^{-6})(5x10^{-6})(20x10^{-6})}{(5x10^{-6})(5x10^{-6}) + (5x10^{-6})(20x10^{-6}) + (5x10^{-6})(20x10^{-6})}$$

$$C_{EQ} = 2.22\mu F$$

Capacitive Reactance

When a capacitor is incorporated into an AC circuit, its impact in that circuit is quantified through an entity referred to as the capacitive reactance. The symbol for capacitive reactance is $\mathbf{X_c}$. Capacitive reactance can be defined, mathematically, as:

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad \textbf{Eq. 1.26}$$

Where,

f = frequency of the AC power source, i.e., 60 Hz in the US and 50 Hz in some other parts of the world.

ω = Rotational speed, in radians per second

C = Capacitance in farads

Capacitive reactance is measured in ohms, or, **Ω**'s. It is important to note that $\mathbf{X_c}$ is often misrepresented as $\mathbf{Z_c}$. To the contrary, as explained in the impedance section, $\mathbf{Z_c}$ is the ***impedance contribution*** by the capacitor, and is represented as:

$$\mathbf{Z_c = - jX_c},$$

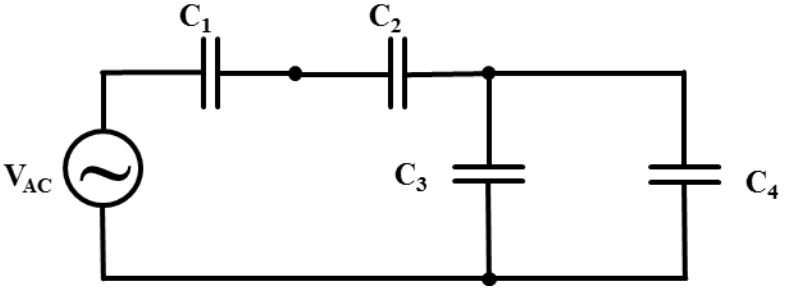
$$\therefore \ \mathbf{Z_c \neq X_c}$$

Example 1.8

Assume that the circuit in Example 1.7 is powered by a 60 Hz, AC source instead of the DC source. Determine the total capacitive reactance, X_c, seen by the AC source.

Solution:

If the DC source is replaced by an AC source, the circuit would appear as follows:



As computed in Example 1.4, the combined or net capacitance contributed to the circuit by the parallel and series network of capacitors is $C_{EQ} = 2.22\mu F$. Then, by applying Eq.1.26:

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2(3.14)(60)(2.22x10^{-6})} = 1194\Omega$$

Inductor and Inductance

Similar to capacitor, an inductor can be viewed as an energy storage device, which can be applied in DC or AC applications. Unlike a capacitor – where energy is stored in form of separation of charges, resulting in a potential difference - in an inductor, the energy is stored in the magnetic field that is produced through the flow of electric current. This phenomenon was first discovered by Michael Faraday in 1831. The magnetic field – which can be referred to as magnetic flux – is not established instantaneously upon flow of current through an inductor. Instead, much like a "time constant, τ" based charge build up in a capacitor, the current change and magnetic field build up in an inductor ramps up, or down, at a non-linear rate. This non-linear rate is a function of the inductor's time constant, τ. Inductor's time constant concept will be explained later in this segment.

Since the energy stored in the magnetic field of an inductor is due to the flow of current – and, ultimately, due to the movement of electrons – we could view an inductor as being analogous to a rotating "flywheel," in the mechanical realm – where the energy is stored in a rotating mass. As with a flywheel - where any attempt to stop the rotation of the flywheel is opposed by momentum and kinetic energy in the rotating mass of the flywheel - ***any attempt to change the flow of current in an inductor is opposed by the "inductance***" of the inductor. It is due to the inductance of an inductor that if you, for instance, have 10 amps flowing in an electrical circuit, and you try to "break" the circuit by opening a switch, current flow is maintained briefly through an ***electric arc***, where permitted, across the opening switch. Where the electric arc – which is, in essence, plasma or ionized air – serves as a temporary "channel" for the flow of current. Conversely, when a switch – in a de-energized inductive circuit - is closed to connect a power source to an inductive device, no current flows through the circuit, initially, as ***inductance*** of the inductor in the electrical circuit opposes the ***change*** in the flow of current from "***zero***" to some measurable "***non-zero***" level. Therefore, inductance is defined as the capacity or tendency of an inductor to resist the ***change in flow of current.***

In addition, it is worth noting that just like the kinetic energy stored in a rotating flywheel can be "tapped" to perform mechanical work, the energy stored in the ***magnetic field*** of a "charged" inductor can be released to push a ferromagnetic cylindrical core, in one direction or another, to open or close a

valve, or to open or close an electrical switch. The former application is an example of a **solenoid operated valve**, while the later represents the operation of a **contactor or a relay**.

The principle of inductance and physical aspects of inductors (or coils) are illustrated in Figure 1.16 (a) and (b). Basically, if you take a straight piece of wire, as shown in Figure 1.16 (a), and wind it around a cylindrical core, the final product would be a coil or an inductor. As shown in the Figure 1.16 (a), current flowing through a straight wire produces a "weak" magnetic field. While, current flowing through a "coiled" conductor produces a stronger and denser magnetic field, capable for conducting "work," such as, pushing of a "plunger" against the restraint of a spring' to open or close a valve or to open or close an electrical circuit in a relay or a contactor.

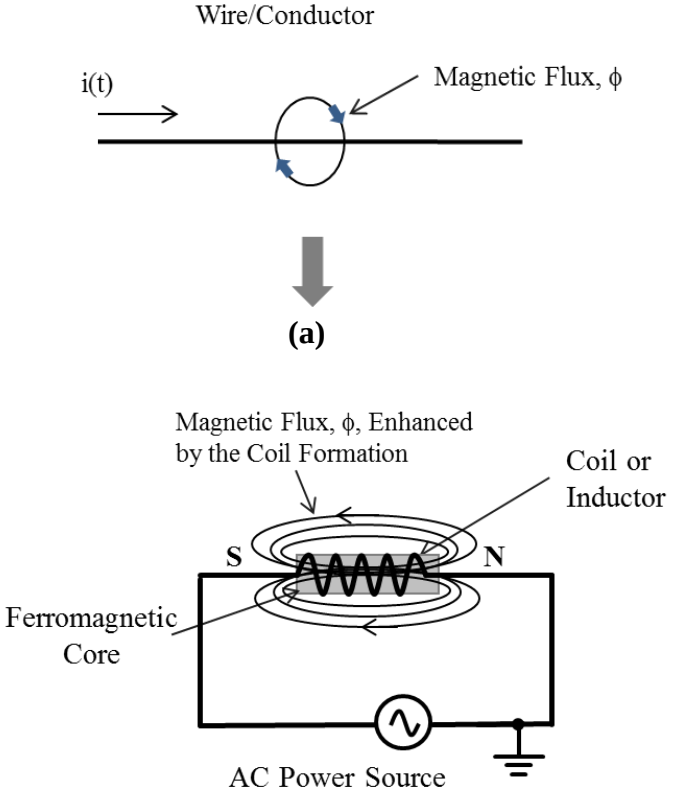


Figure 1.16: (a) Straight current carrying conductor (b) "Coiled" current carrying conductor.

Figure 1.16 (b) represents a conceptual view of a solenoid or a coil, illustrating the fundamental principle of electromagnetism. On the other hand, physical construction of a simple solenoid or inductor is depicted in Figure

1.17, in a diametrical cross-sectional view. When the coil of the solenoid is energized, the plunger – or core – responds to the magnetic flux by moving to the left. As the plunger moves to the left - against the spring - with a force that is proportional to the magnetic flux, the "pin" attached to the tip of the plunger pushes mechanical devices such as relay contacts or valves, etc., to change their state from open to closed, or vice and versa.

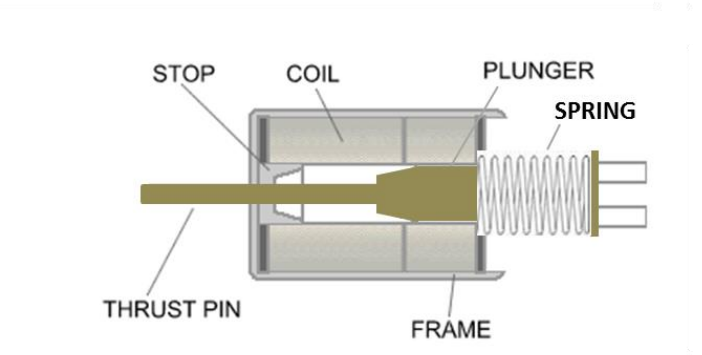


Figure 1.17: Construction of a solenoid.

Inductance is denoted by "**L**," and it can be defined, mathematically, through Eq. 1.27, below.

$$L = \frac{\mu . N^2 . A}{l} \qquad \textbf{Eq. 1.27}$$

Where,

μ = Permeability of the medium, in **H/m**
N = Number of turns of coil (unit-less)
A = Cross-sectional area of the core (in $\mathbf{m^2}$)
l = Mean length through the core (in **m**)
Unit for Inductance: **H** (**henry**)

The mathematical relationship stated as Eq. 1.27 stipulates that inductance is directly proportional to the area of cross-section "**A**" of the core. This equation also states that the inductance is directly proportional to "$\mathbf{N^2}$," the "square" of the number of turns in the coil, and is inversely proportional to the mean length "**l**" of the core. In other words, if larger inductance or ***energy storage capacity*** is desired, one must increase the area of cross-section of the core, increase the number of turns, or reduce the mean length of the core and the coil. Of course, the values of these variables can be increased or decreased, simultaneously, to achieve the desired results. In addition to

serving as a "***constant of proportionality***" for the equation, permittivity "**μ**" injects the physical characteristics of the core into the computation of inductance. So, if a material with higher relative permeability, μ_r, is chosen, the inductance of a coil would be greater. The permeability **μ** of a specific medium can be defined, mathematically as follows:

$$\mu = \mu_r \cdot \mu_o$$

Where,

μ_r = Relative permeability of the core material. Relative permeability of steel is 100 and that of an insulating materials like wood and Teflon is 1.0; which is the same as free space, vacuum or air.

μ_o = Permeability of free space or vacuum = $4\pi \times 10{-}7 = 1.257 \times 10^{-6}$ **H/m**

Electrical energy stored in an inductor can be determined through Eq. 1.28, below:

$$E_{\text{stored}} = \frac{1}{2} L I^2 \qquad \textbf{Eq. 1.28}$$

Where, the energy is measured in joules (or N-m), **L** in H (henry) and **I** in amps.

Most inductive electrical systems - that is electrical circuits with inductance - and inductive devices like motors and transformers, contain inductance **L**, as well as resistance, **R**. In order to understand the current and voltage response in inductive circuits – or to understand current and voltage variation in inductive systems – we will examine a simple series RL circuit diagram in Figure 1.18.

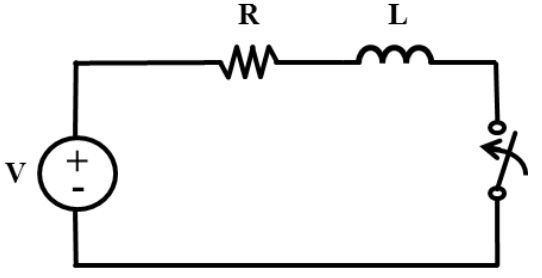


Figure 1.18: Series RL circuit

In a typical inductive-resistive circuit, as the one shown in Figure 1.18, the voltage and current relationships, as a function of time, are governed by Equations 1.29, 1.30 and 1.31.

$$v_L(\mathrm{t}) = L\frac{di_L}{dt} \qquad \textbf{Eq. 1.29}$$

$$v_L(t) = -i(0)Re^{-\frac{R}{L}t} + Ve^{-\frac{R}{L}t} \qquad \textbf{Eq. 1.30}$$

$$i_L(t) = i_R(t) = i(0)e^{-\frac{R}{L}t} + \frac{V}{R}(1 - e^{-\frac{R}{L}t}) \qquad \textbf{Eq. 1.31}$$

Equations 1.30 and 1.31 hold greater practical significance, in that, they can be used to predict the changes in voltage and current - with respect to time - for given values of source voltage, **V**, series resistance **R** and series inductance **L**. The relationship between resistance **R** and inductance **L**, or the ***relative*** size of **R** and **L**, determine the charging and discharging rate of the inductor via the time constant "**τ** ." The time constant **τ** can be defined, mathematically, in terms of **R** and **L**, in form of **Eq. 1.32**.

$$\tau = \frac{L}{R} \qquad \textbf{Eq. 1.32}$$

It is ostensible from examination of Eq. 1.30 and 1.31 that in circuits that consist of inductance and resistance, unlike purely resistive circuits - but similar to RC circuits - current and voltage associated with inductors vary in a non-linear fashion. This "non-linear" charging and discharging of an inductor in an RL circuit – similar to RC circuits - is referred to as ***transient behavior***. The graphs in Figures 1.19 and 1.20 compare voltage and current responses - on the basis of Eq. 1.30 and 1.31, respectively.

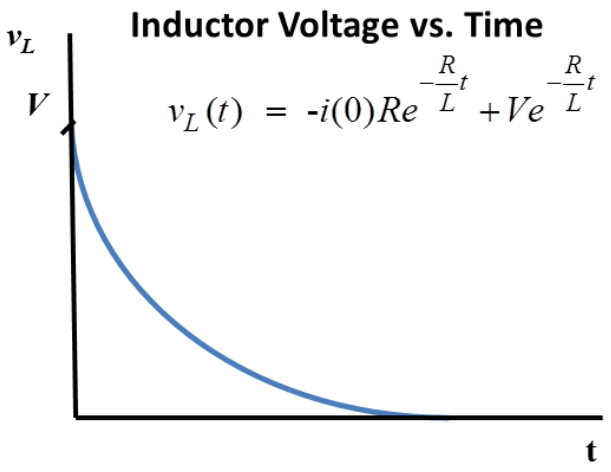


Figure 1.19: Non-linear ***voltage*** response in a circuit consisting of inductance and resistance.

Voltage vs. Time, transient, response in Series RL Circuit: Examination of Eq. 1.30 and Figures 1.18 and 1.19 reveals the following facts:

a) At time, t = 0, which is when the switch in the circuit is closed**, *i*(0) = 0**; so the first segment of Eq. 1.30 becomes zero and drops out. Also, at t = 0, the exponent of "e" becomes zero, which makes $\mathbf{e^{-Rt/L} = 1}$. This results in $\boldsymbol{v_L(t) = V}$**,** (i.e., the source voltage) at t = 0. In other words:

   $\boldsymbol{v_L(t) = V}$ at the instant the switch is closed in the series RL circuit.

b) On the other end of the time spectrum, where $t = \infty$, or when steady state condition has been achieved:

$$v_L(\infty) = -i(0)Re^{-\infty} + Ve^{-\infty}$$

$$= -(0).\left(\frac{1}{\infty}\right) + V\left(\frac{1}{\infty}\right) = 0$$

The analyses above support the following basic tenets of series RL inductive circuits:

i. The voltage across the inductor, at the instant the switch in a series RL circuit is closed, is the same as the source voltage, implying that no current flows through the inductor.

ii. The voltage across the inductor, after a large amount of time has elapsed, diminishes to zero. And, with voltage drop across the inductor zero, the inductor acts as a short in a series RL circuit under steady state conditions - with steady state current, V/R, flowing through it.

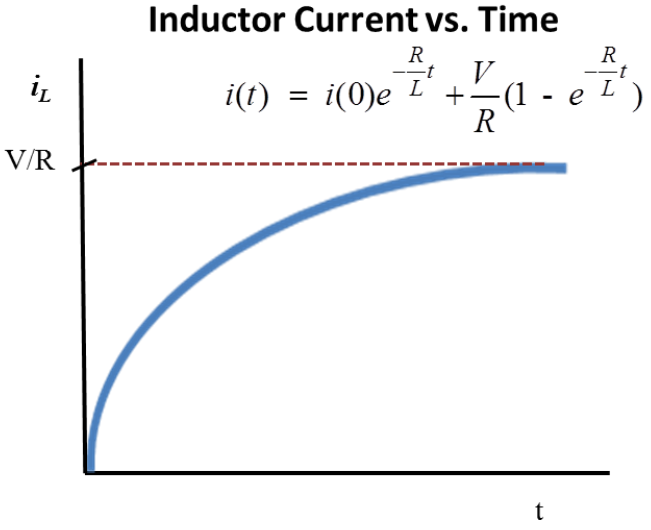


Figure 1.20: Non-linear ***current*** response in a circuit consisting of inductance and resistance

Current vs. Time, transient, response in series RL Circuit: Examination of Eq. 1.31 and Figures 1.18 and 1.20 reveals the following facts associated with current versus time response in a series RL circuit:

a) At time, t = 0, which is when the switch in the circuit is closed**, i(0) = 0**; so the first segment of Eq. 1.31 becomes zero and drops out. Also, at t = 0, the exponent of "**e**" becomes **zero**, which makes $\mathbf{e^{-Rt/L} = 1}$. This results in $\boldsymbol{i_L(t) = 0 + V/R(1-e^0) = 0}$, this means that at **t = 0**, or at the instant the switch is closed, the inductance of the inductor resists the initiation of current flow, hence, no current flows through the inductor at **t = 0.**

b) On the other end of the time spectrum, when t = ∞ , or when steady state condition has been achieved:

$$i_L(\infty) = i_R(\infty) = i(0)e^{-\frac{R}{L}t} + \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$

$$i_L(\infty) = i_R(\infty) = (0)(e^{-\infty}) + \frac{V}{R}(1 - e^{-\infty})$$

$$i_L(\infty) = i_R(\infty) = \frac{V}{R}(1 - 0) = \frac{V}{R}$$

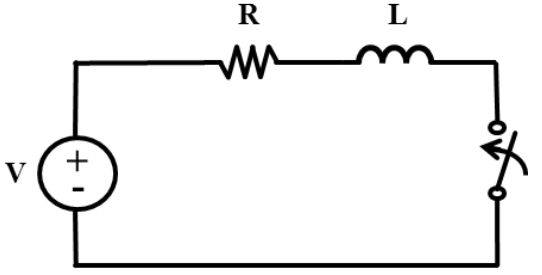
The analyses stated above support the following basic characteristics of inductive circuits:

i. No current flows through the inductor and resistor combination at the instant the switch is closed, or at t = 0, which is congruent with one of the fundamental characteristics of an inductor described earlier. In other words, at the outset, the inductance of the inductor, successfully, resists the rise of the current to a non-zero value.

ii. Current through the inductor and resistor combination develops to the maximum level after a long span of time. The maximum level of current in the inductor and resistor combination is equal to ***V*/R**. This also implies that the inductor acts as a "***short***" when steady state condition is achieved or after 10τ amount of time.

Example 1.9

Consider the series RL circuit shown in the diagram below. The source voltage is 12V and R = 10Ω. The switch is closed at t = 0. What would be magnitude of current flowing through this circuit at t = **τ**?



Solution:

In most series RL cases, the current value at a certain time "**t**" can be predicted through **Eq. 1.31**.

$$i_L(t) \;=\; i_R(t) \;= i(0)e^{-\frac{R}{L}t} + \frac{V}{R}(1 \;-\; e^{-\frac{R}{L}t})$$

Note: In this case, the value of **L** is not given, but the elapsed time is given as a function of time constant as, **"1τ."**

Since **τ = L/R**, **Eq. 1.31** can be rewritten, in **τ** form as:

$$i_L(t) \;=\; i_R(t) = i(0)e^{-\frac{t}{\tau}} + \frac{V}{R}(1 \;-\; e^{-\frac{t}{\tau}})$$

Then, by substituting t = τ, and given the fact that i(0) = 0, the $\boldsymbol{i_L(t)}$ equations simplifies into the following form:

$$i_L(t) \;=\; i_R(t) = i(0)e^{-\frac{\tau}{\tau}} + \frac{V}{R}(1 \;-\; e^{-\frac{\tau}{\tau}})$$

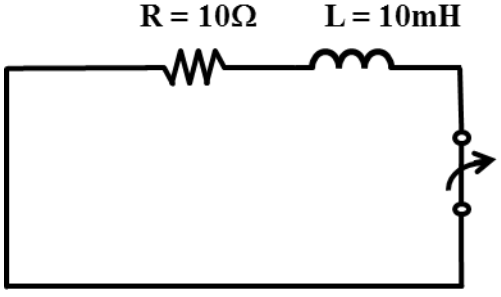
$$= \frac{V}{R}(1 \;-\; e^{-1})$$

$$= \frac{V}{R}(0.632) = \frac{12}{10}(0.632) = (1.2)(0.632) = 0.759 \text{ A}$$

This analysis of current response validates a characteristic fact about inductors: current develops to **63.2%** of its full potential in "one time constant" or **1τ** worth of time.

Example 1.10

Consider the series RL circuit given in Example 1.9, in discharge mode, with voltage source removed. Inductor L = 10mH. The switch has been closed for long period of time, such that the current has developed to the maximum or steady state level 1.2 A. How much time would need to elapse for the current to drop to 0.8A after the switch is opened.



Solution:

Apply series RL current equation, **Eq. 1.31**.

$$i_L(t) = i_R(t) = i(0)e^{-\frac{R}{L}t} + \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$

Given:

t = ?

L = 10 x 10^{-3}H

R = 10Ω

V = 0

i(0) = 1.2A

$i_L(t)$ = 0.8A

$$i_L(t) = (0.8) = (1.2)e^{-\frac{10}{0.01}t} + (0)(1 - e^{-\frac{10}{0.01}t})$$

$$0.8 = (1.2)e^{-\frac{10}{0.01}t}$$

$$0.667 = e^{-\frac{10}{0.01}t}$$

$$\ln(0.667) = \ln(e^{-\frac{10}{0.01}t})$$

$$-0.4055 = -1000t$$

t = 0.00041s or 0.41 ms

Series Inductor Combination

When analyzing dc circuits with inductors connected in series, derivation of equivalent inductance, $\mathbf{L_{eq}}$, can accomplished by, simply, adding the inductance values linearly as represented by Eq. 1.33. Figure 1.21 depicts "**n**" inductors, $\mathbf{L_1}$ through $\mathbf{L_n}$, connected in series.

$$L_{EQ} = L_1 + L_2 + L_3 \ ..... + L_n \qquad \textbf{Eq. 1.33}$$

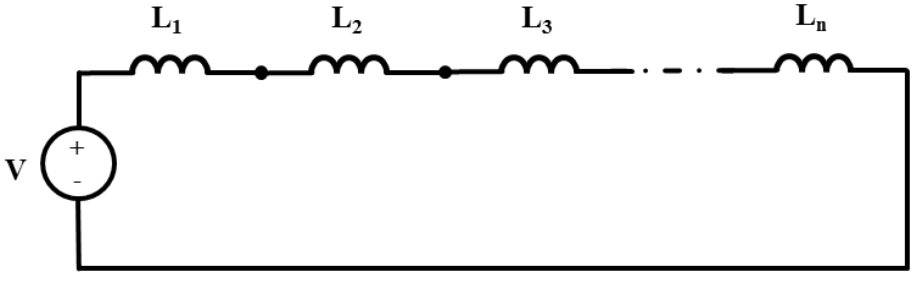


Figure 1.21: Series combination of "**n**" inductors

Derivation of the combined equivalent inductance $\mathbf{L_{EQ}}$, permits us to represent Figure 1.21 in form of a condensed version depicted in Figure 1.22.

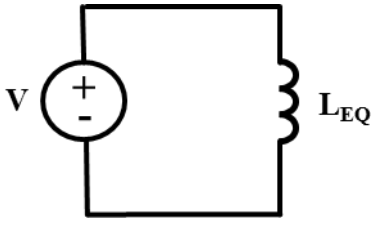
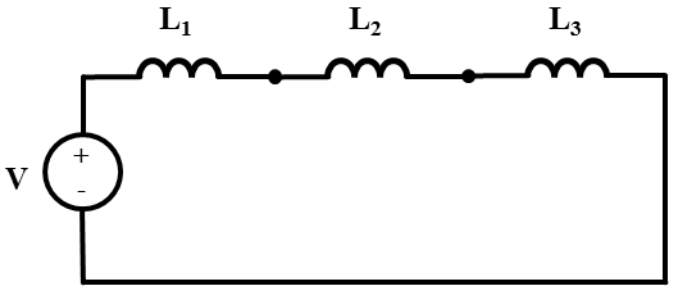


Figure 1.22: $\mathbf{L_{EQ}}$, equivalent inductance representing series combination of "**n**" inductors.

Example 1.11

Determine the equivalent inductance for three inductors connected in a series combination circuit shown below. The inductance values of the three inductors are: L_1 = 5mH, L_2 = 5mH, and L_3 = 10mH.



Solution:

Apply Eq. 1.33:

$$L_{EQ} = L_1 + L_2 + L_3$$

$$= 5mH + 5mH + 10\ mH$$

$$= 20mH$$

Parallel Combination of Inductors

The circuit depicted in Figure 1.23 shows "n" number of inductors connected in parallel. The formula for determining the equivalent inductance L_{eq} for this circuit is represented by Eq. 1.34.

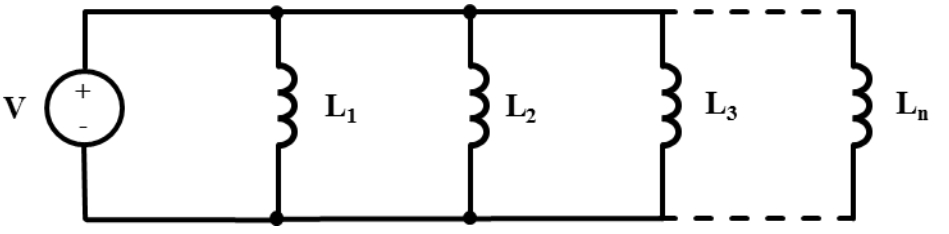


Figure 1.23: Parallel combination of "n" inductors

$$L_{EQ} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}} \qquad \textbf{Eq. 1.34}$$

If a parallel resistor circuit or network consists of only three inductors, the circuit would appear as shown in Figure 1.24 and the $\mathbf{L_{eq}}$ equation for this circuit would reduce to Eq. 1.36.

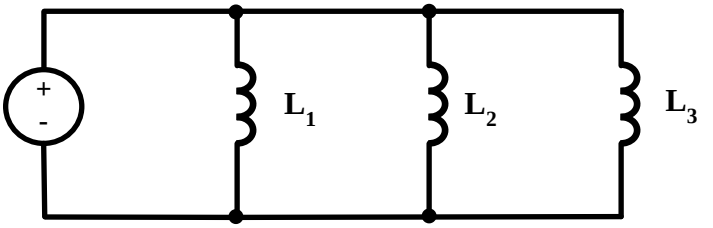


Figure 1.24: Combination of three parallel inductors

$$L_{EQ} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} \qquad \textbf{Eq. 1.35}$$

$$L_{EQ} = \frac{L_1 L_2 L_3}{L_1 L_2 + L_2 L_3 + L_1 L_3} \qquad \textbf{Eq. 1.36}$$

Example 1.12
Determine the equivalent inductance L_{EQ} for three parallel inductor DC circuit shown Figure 1.24 if L_1 = 1H, and L_2 = 5H and L_3= 10H.

Solution:
Apply Eq. 1.36 to compute L_{EQ} for the three parallel inductor circuit shown in Figure 1.24:

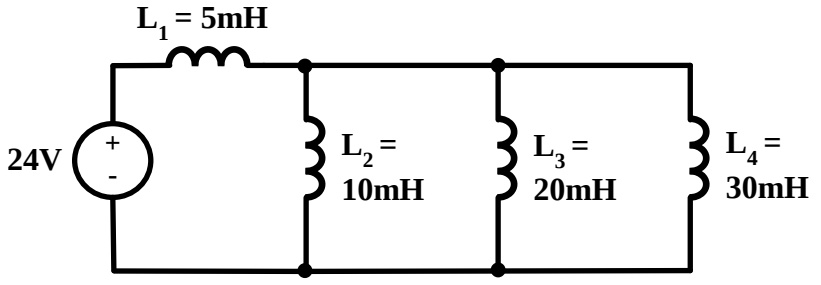
$$L_{EQ} = \frac{L_1 L_2 L_3}{L_1 L_2 + L_2 L_3 + L_1 L_3}$$

$$= \frac{(1H)(5H)(10H)}{(1H)(5H) + (5H)(10H) + (1H)(10H)}$$

$$= 0.77H$$

Example 1.13

Calculate the net or total inductance as seen from the 24V source vantage point in the circuit shown below.



Solution:

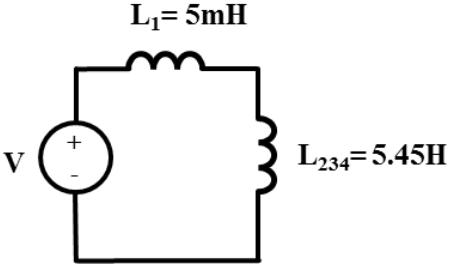
We need to focus on the parallel combination of **L_2, L_3, and L_4** first. Apply Eq. 1.36 to calculate the equivalent inductance **L_{234}** for the three parallel inductors:

$$L_{234} = \frac{L_2 L_3 L_4}{L_2 L_3 + L_3 L_4 + L_1 L_4}$$

$$= \frac{(10\text{mH})(20\text{mH})(30\text{mH})}{(10\text{mH})(20\text{mH}) + (20\text{mH})(30\text{mH}) + (10\text{mH})(30\text{mH})}$$

$$= 5.45\text{mH}$$

This reduces the circuit as shown below:



Inductors L_1 and L_{234}, in this reduced circuit, lend themselves to a linear combination. Therefore, the equivalent inductance **L_{EQ}** for the entire parallel and series inductor hybrid circuit would be:

$\mathbf{L_{EQ} = L_1 + L_{234}}$ = 5mH +5.45mH = **10.45mH**

Inductive Reactance

When an inductor is incorporated into an AC circuit, its impact in that circuit is quantified through an entity referred to as the ***inductive reactance***. The symbol for inductive reactance is $\mathbf{X_L}$. Inductive reactance can be defined, mathematically, as:

$$X_L = \omega L = 2\pi f L \qquad \textbf{Eq. 1.37}$$

Where,

f = frequency of the AC power source, i.e., 60 Hz in the US and 50 Hz in some other parts of the world.

ω = Rotational speed, in radians per second

L = Inductance in henry, or **H**.

Inductive reactance is measured in ohms, or, Ω's. It is important to note that, $\mathbf{X_L}$ is often misconstrued as $\mathbf{Z_L}$. To the contrary, as explained in the impedance section, $\mathbf{Z_L}$ is the ***impedance contribution by the inductor***, and is represented as:

$\mathbf{Z_L = jX_L}$,

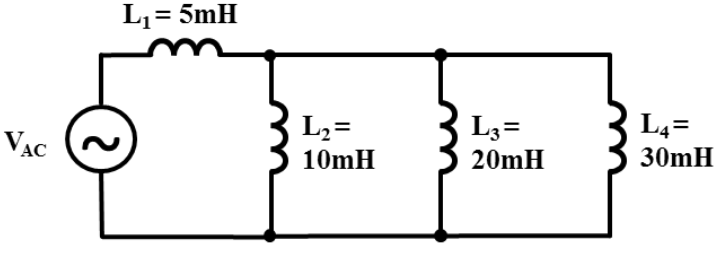
$\therefore \ \mathbf{Z_L \neq X_L}$

Example 1.14

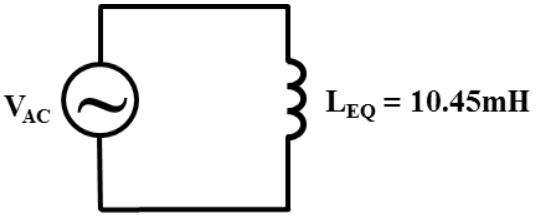
Assume that the circuit in Example 1.13 is powered by a 60 Hz AC source. Calculate the inductive reactance, X_L, as seen by the AC voltage source.

Solution:

If the DC source is replaced by an AC source, the circuit would appear as follows:



L_{EQ}, as seen by the AC voltage source, is shown in the simplified equivalent circuit below:



As computed in Example 1.13, the combined or net inductance contributed to the circuit by the parallel and series network of inductors is L_{EQ} = 10.45mH. Then, by applying Eq.1.37, the inductive reactance, $\mathbf{X_{L\text{-}EQ}}$ as seen by the AC voltage source V_{AC}, would be:

$$\mathbf{X_{L\text{-}EQ} = \omega.L = (2\pi f).L_{EQ} = 2(3.14)(60Hz)(10.45mH) = 3.94\ \Omega}$$

Impedance

The narrative definition of impedance would be that it is the current resisting and impeding characteristic of load or conductor, in an AC circuit. As implied in the definition of this term, impedance is an alternating current entity. While in DC circuits the factor that opposes the flow of DC current is resistance, the entity that influences the flow of AC current, in AC circuits, is impedance. Impedance, like AC current, voltage and power, is a vector entity. By definition, a vector can be completely defined by two key characteristics, namely, the ***magnitude*** and the ***direction***. Therefore, impedance and other entities in AC circuits, such as current, voltage and power, can be defined completely through specification of their magnitude and direction.

Symbol for impedance is **Z**. When typewritten, as with most AC entities, the symbol **Z** for impedance is represented in bold font. When handwritten, most AC entities are denoted by the respective symbol with a half arrow. So, impedance would be denoted by " $\bar{Z}$." The unit for impedance is **ohm,** or, **Ω**; similar to the unit for resistance, **R**, capacitive reactance $\mathbf{X_c}$ and Inductive reactance $\mathbf{X_L}$. Consistency of units between **R,** $\mathbf{X_L}$**,** $\mathbf{X_C}$ and **Z** is one justification for the following mathematical definition for **Z**:

$$\mathbf{Z = R + jX_l - jX_c} \qquad \textbf{Eq. 1.38}$$

Where,

$$jX_l = Z_l \qquad \text{Eq. 1.39}$$

= Impedance contribution by the inductance in the circuit.

And,

$$-jX_c = Z_c \qquad \text{Eq. 1.40}$$

= Impedance contribution by the capacitance in the circuit.

Magnetic Circuits vs. Electrical Circuits

The two diagrams shown in Figure 1.25 below illustrate similarities and differences between a basic electrical circuit and a magnetic circuit. In depth discussion on the subject of magnetism is beyond the scope of this text. Nevertheless, a contrast between the basic magnetic and electric circuits below will allow the reader an opportunity to gain basic understanding of magnetism, more specifically, electromagnetism.

The circuit shown in Figure 1.25 (a) represents a basic DC electrical circuit. This circuit consists of a DC voltage source labeled "**V**." As explained earlier, voltage is synonymous to the term electromotive force. So, we can explain the phenomenon in the electrical circuit as the electromotive force "**V**" driving DC current "**I**" through the load or resistor "**R**." Note the direction of the current is from the left to right, in a clockwise loop, emerging from the positive electrode of the DC voltage source and terminating into the negative electrode of the voltage source. This clockwise current flow is assigned on the basis of an electrical convention that stipulates that the current flow consists of "***holes***," or positively charged particles, being repelled or driven out of the positive terminal. This convention also affirms the fact that electrical current is not necessarily, always, due to the flow or movement of electrons. Electrical current, as explained in the section on the topic of current, can be due the flow or movement of negatively or positively charged particles. The positively or negatively charged particles, at the atomic or molecular level are referred to as ions.

The relationship between V, I and R in the electrical circuit is governed by the Ohms Law. The Ohms Law, in conjunction with other basic electrical laws - used to analyze electrical circuits – will be explained in more detail in Segment 2. For now, note that the Ohms Law is stated mathematically in the form of Eq. 1.41. In other words, according to Ohms Law, electromotive force is equal to the product of current and resistance.

Electromotive Force, V = I.R = (Current) x (Resistance) **Eq. 1.41**

The circuit shown in Figure 1.25 (b) represents a basic electromagnetic circuit. This circuit consists of a toroid or donut shaped core – typically constructed out of iron. In this magnetic circuit, a conductor, or wire, is wrapped in four turns around the left side of the toroid core. When current is passed through wound conductor, magnetic field is established in the core as represented by the dashed circular line, with an arrow pointing in clockwise direction. This magnetic field is referred to as magnetic flux, **ϕ**. Magnetic flux is measured in weber. The unit weber is named for the German physicist **Wilhelm Eduard Weber** (1804–1891). In the magnetic realm, the flux serves as a counterpart to the current, **I**, from the electrical realm. Just like the electromotive force, **EMF**, or voltage, drives the current through the resistor, **R**, the magnetomotive force (**MMF**), $\mathcal{F}$, drives the magnetic flux, **ϕ,** through the toroid magnetic core. Magnetomotive force is measured in ampere-turns. In electrical systems, load is represented by the resistor **R**. In the magnetic circuit, the flow of magnetic flux is opposed by reluctance, $\mathcal{R}$. Just as the Ohm's Law, represented by Eq. 1.41, governs the relationship between electromotive force (voltage), current and resistance in the electrical realm, **Eq. 1.42** represents the relationship between the magnetomotive force, $\mathcal{F}$, the magnetic flux, **ϕ,** and the reluctance, $\mathcal{R}$, in the magnetic domain.

$\mathcal{F} = \phi.\mathcal{R}$ = (Magnetic Flux) x (Reluctance) **Eq. 1.42**

Equation 1.42 is referred to as the ***Hopkinson's Law***.

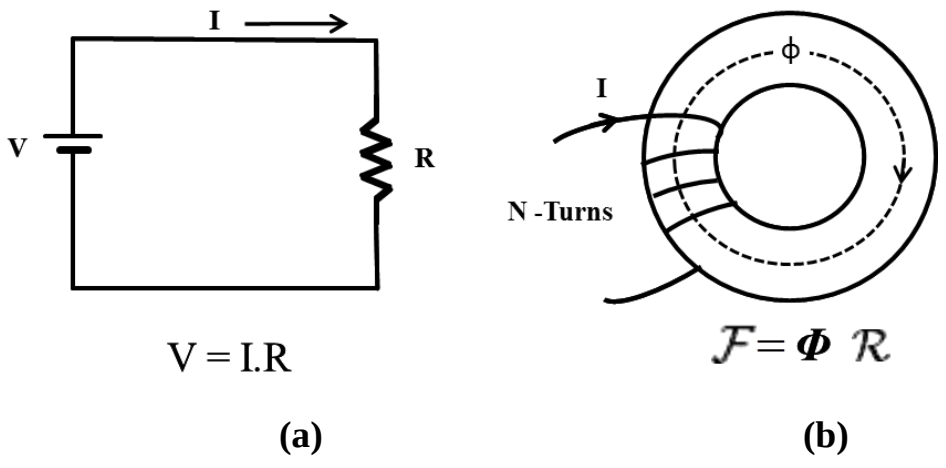


Figure 1.25: Magnetic and electrical circuit comparison

Magnetic reluctance can also be perceived as magnetic resistance; a resistance that opposes the flow of magnetic flux. Like resistance, reluctance is a scalar entity, but unlike electric resistance, it stores magnetic energy instead of dissipating it. Reluctance is measured in ampere-turns per weber, or turns per henry. Ferromagnetic substances such as iron have low reluctance while dielectric substances like air and vacuum offer high reluctance to magnetic flux. That is the reason why transformers, contactors, relays and other similar electromagnetic devices utilize iron – or iron alloy – cores.

Analogous to Eq. 1.1, which represents the relationship between resistance, resistivity, length and area of cross-section, the reluctance of a uniform magnetic circuit can be calculated as:

$$R = \left(\frac{1}{\mu_r \mu_o}\right).\frac{l}{A} \qquad \textbf{Eq. 1.43}$$

Or,

$$R = \left(\frac{1}{\mu}\right).\frac{l}{A} \qquad \textbf{Eq. 1.44}$$

Where,

l is the mean length of the circuit or core, in meters.

μ_o is the permeability of free space or vacuum = $\mathbf{4.\pi.10^{-7}}$ henry per meter.

μ_r is the relative magnetic permeability of the core material. This is a dimensionless number.

μ is the permeability of the core material in henry per meter.

A is the area of cross-section of the core or the magnetic circuit defined in $\mathbf{m^2}$.

Basic Electrical Instruments

Two of the most fundamental instruments used to make electrical measurements and perform electrical system trouble shooting are as follows:

I. Multi-meter, or a VOM, Volt-Ohm-Meter
II. Clamp-on Ammeter

Multi-meter

The modern multi-meter, sometimes just written as "multimeter," has evolved from its basic predecessor, the ***Ohm-meter.*** The original Ohm-meters ware designed to measure resistance of electrical components and to verify continuity and integrity of electrical or electronic circuits. Voltage measuring feature was later added to the basic resistance measuring function of the Ohm-meter in the form of a more versatile instrument called the ***Volt-Ohm-Meter***, or **VOM**. Due to the miniaturization of electronic components, additional functions were added to the basic VOM resulting in the contemporary multi-meter that transitioned from analog to digital format. See the diagram of a digital multi-meter in Figure 1.26. Some of the following features and functions are common among most multi-meters available on the market today:

a) Voltage measurement, AC and DC
b) Resistance measurement
c) Current measurement
d) Temperature measurement
e) Capacitor testing
f) Diode testing
g) Transistor testing

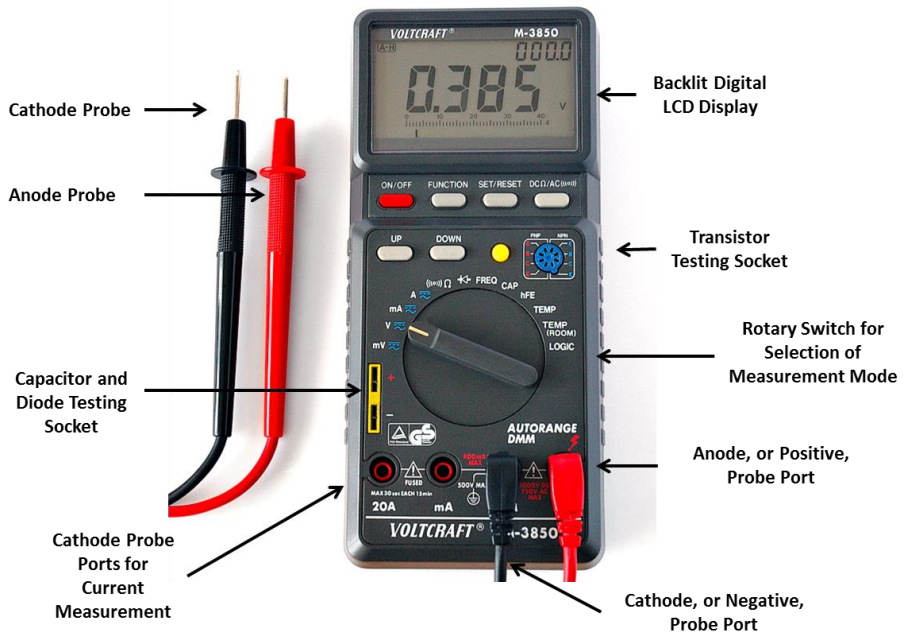


Figure 1.26: VOLTCRAFT ® Digital Multi-meter

Standard accuracy of a portable digital multi-meter is, approximately, 0.3%. Bench top and lab grade multi-meters are known to offer accuracy that is better than ±0.1%. Many multi-meters include the "***Peak-Hold***" feature, which allows one to capture peak reading when the measured parameter is not steady. Some of the more sophisticated multi-meters have the ability to interface with PC's for direct data transfer, plotting and storage.

When used for measuring voltage, multi-meters operate in "high impedance" mode. In other words, when multi-meters are used to measure voltage, the circuit being tested "sees' high impedance between the probes. This allows for the application of a multi-meter to be a ***non-invasive*** to the circuit being tested. This means that for most voltage measurements, a multi-meter can be applied to an electrical circuit without turning off the power to the system being tested. On the other hand, if a multi-meter is being used for measuring current, the circuit must be turned off, interrupted and severed at the point of measurement, and the meter must be injected in series to route the current through the multi-meter for measurement.

One must consider certain important safety measure associated with the application of multi-meters. Multi-meters, similar to other electrical

instrumentation, are designed to operate within specified voltage range. Most, commonly used, multi-meters are rated for ***low voltage*** applications. Low voltage category ranges from 0 to 600 VAC or VDC. Application of a low voltage instrument on higher voltages can result in a potentially catastrophic failure of the instrument and a potentially lethal arc flash incident. In addition, from safety point of view, it is important to inspect the instrument, periodically, for signs of mechanical or electrical damage. The probes must always be inspected for signs of wear, fraying, and signs of "***mechanical*** stress or strain." The outermost surface of the cathode and anode probes is, essentially, insulation. Therefore, if the insulation is damaged or atrophied, a person handling the probes could be exposed to an electrical shock hazard. In most cases, prudent and safe course of action would be to ***decommission and discard*** faulty probes, and replace them with, suitable, manufacturer recommended replacements.

The Clamp-on Ammeter

When current in an electrical or electronic circuit must be measured without interruption of an electrical or electronic circuit, a clamp-on ammeter, sometimes referred to as "clamp meter," can be used. Current measurement with a clamp-on ammeter, unlike a multi-meter, requires no probes. See Figure 1.27. When measuring current, the rotary selector switch on the face of the ammeter is switched to AC current measurement setting. The spring loaded "clamping" current transformer is opened by pressing the knob on one side of the ammeter. The current carrying conductor is surrounded by the open current transformer clamp and the current transformer clamp is allowed to close around the conductor. As the current transformer closes or loops around current carrying conductor, it develops magnetic flux that is proportional to the flow of current through the conductor. This magnetic flux is ***transduced*** into voltage that is subsequently scaled and displayed, digitally. The current indicated on the display of a typical AC ammeter is an RMS value. In single phase AC applications, the clamp-on ammeter should be clamped around the ungrounded or energized conductor. If the neutral and the energized conductors are clamped together, the currents through the two conductors cancel each other resulting in null or zero readout on the display. Similarly, while measuring line currents in multiphase AC systems, ***only one phase*** must be enclosed in the clamp-on ammeter current transformer. Some clamp-on ammeters are equipped with auxiliary features like voltage measurement; such is the case with the clamp-on ammeter shown in Figure 1.27. The ports for

connecting the voltage measuring anode and cathode probe leads are located at the bottom of the ammeter.

The voltage rating related safety precaution, advised in the use of multi-meters, applies to clamp-on ammeters, as well. The clamp-on ammeter depicted in Figure 1.27 is a low voltage (0 – 600 VAC) device. Use of low voltage clamp-on ammeters on **Medium or High Voltage** electrical circuits is unsafe and can result in catastrophic failures, such as, ***arc flash incidents***. Proper precaution must also be exercised concerning the maximum current rating of a clamp-on ammeter. Current and voltage ratings of a clamp-on ammeter must ***not*** be exceeded. Miss-application of test instrumentation can result in catastrophic faults or arc flash incidents.

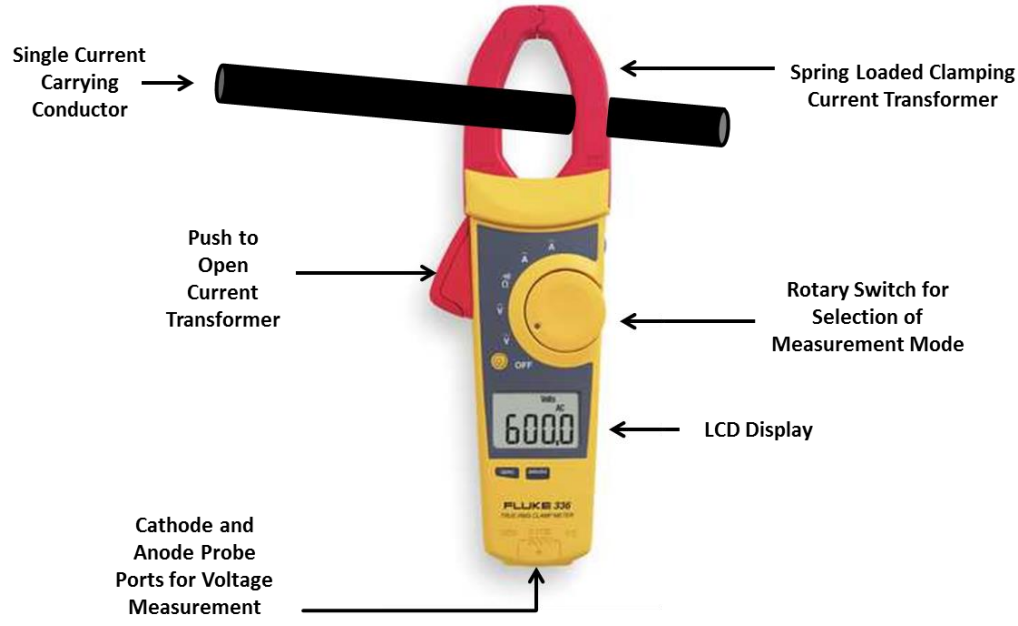


Figure 1.27: Fluke ® Digital Clamp-on Ammeter

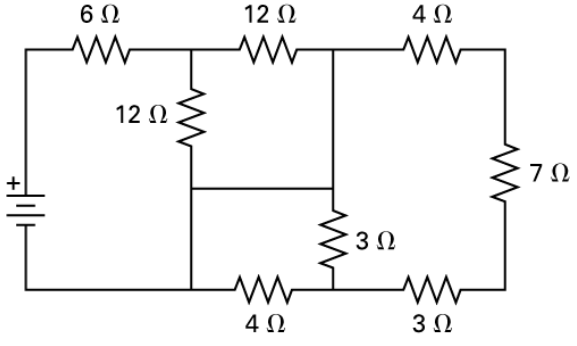
As noted earlier, most clamp-on ammeters used routinely by electrical engineers and electricians are designed to detect and measure AC current. Although uncommon, DC clamp-on ammeters, operating on **Hall Effect Principle**, are available for "non-invasive" DC current measurement.

Self-assessment Problems & Questions - Segment 1

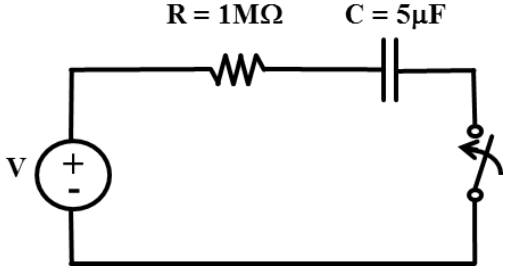
1. In an AC system, a voltage source **V(t) = 120Sin(377t + 0°)** volts, RMS, sets up a current of **I(t) = 5Sin(377t + 45°)** amps, rms. Calculate the ***maximum*** values of voltage and current in this case.

2. A phase conductor of a transmission line is one mile long and has a diameter of 1.5 inch. The conductor is composed of aluminum. Calculate the electrical resistance of this conductor.

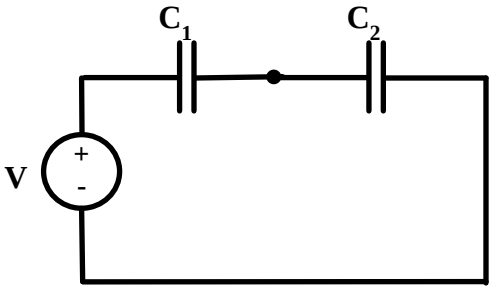
3. What is the resistance of the following circuit as seen from the battery?



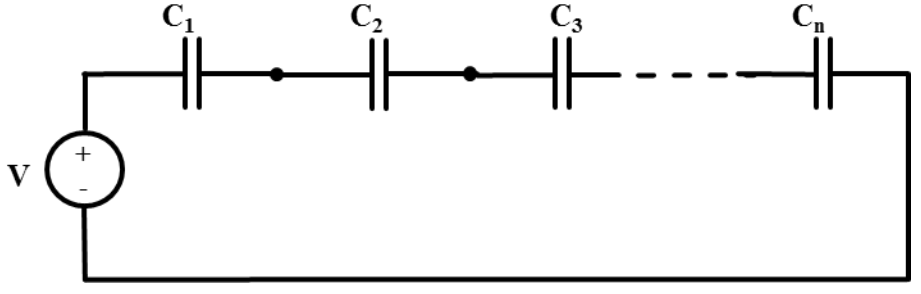
4. Consider the RC circuit shown in the diagram below. The source voltage is 12V. The capacitor voltage before the switch is closed is 2V. The switch is closed at t = 0. What would the capacitor voltage be at t = 5 sec?



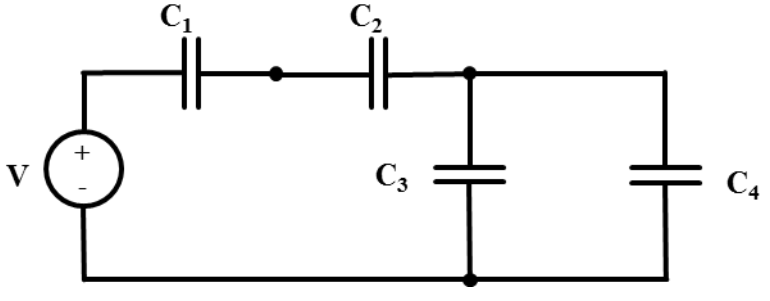
5. Determine the equivalent capacitance for the DC circuit shown in the circuit diagram below if $C_1 = 5\mu F$ and $C_2 = 10\mu F$.



6. Determine the equivalent capacitance for the DC circuit shown below if this circuit consists of twenty $100\mu F$ capacitors in series.

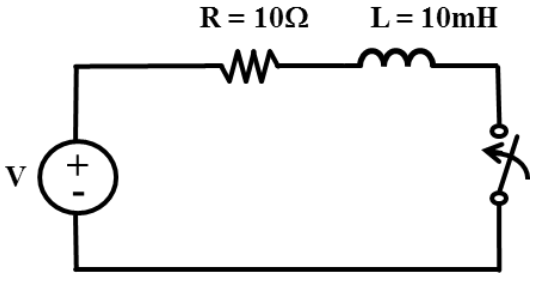


7. Determine the equivalent capacitance in series and parallel combination circuit shown below. The capacitance values are: $C_1 = 10\mu F$, $C_2 = 10\mu F$, $C_3 = 20\mu F$, $C_4 = 20\mu F$.

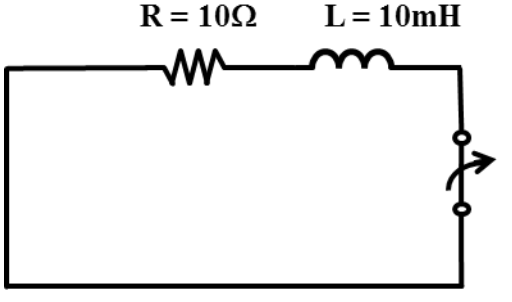


8. Assume that the circuit in problem 4 is powered by a 60 Hz AC source instead of the DC source. Determine the total capacitive reactance, X_c, seen by the AC source.

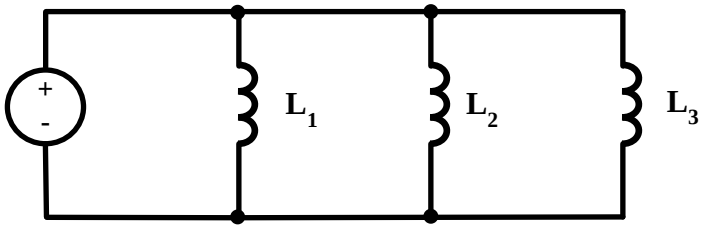
9. Consider the series RL circuit shown in the diagram below. The source voltage is 12V, R = 10Ω and L = 10mH. The switch is closed at t = 0. What would be magnitude of current flowing through this circuit at t = 2ms?



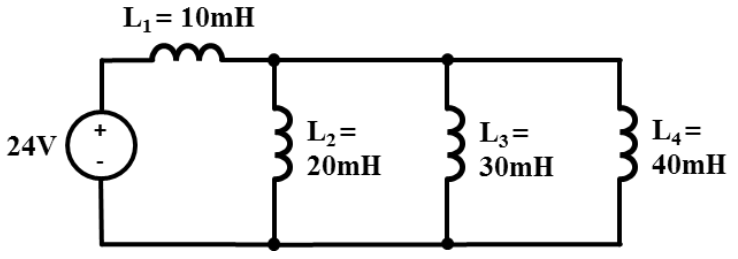
10. Consider the series RL circuit given in problem 9, in discharge mode, with voltage source removed. Parameters such as R = 10Ω and L = 10mH, are the same. The switch has been closed for long period of time, such that the current has developed to the maximum or steady state level 1.04 A. How much time would need to elapse for the current to drop to 0.5 A after the switch is opened.



11. Determine the equivalent inductance L_{EQ} for three parallel inductor DC circuit shown in the diagram below if L_1 = 2mH, and L_2 = 5mH and L_3= 20mH.



12. Calculate the net or total inductance as seen from the 24V source vantage point in the circuit shown below.



13. Assume that the circuit in Problem 12 is powered by a 60 Hz AC source. Calculate the inductive reactance, X_L, as seen by the AC voltage source.

Segment 2

DC Circuit Analysis and Basic Electronic Devices

Introduction

In this segment, we will review three fundamental laws of physics and electrical engineering, namely, (1) Ohms Law, (2) Kirchhoff's Voltage Law and (3) Kirchhoff's Current Law. These three laws are instrumental in performing basic circuit analysis. Laws serve a vital purpose in electrical engineering analysis and problem solving; in DC as well as AC domains. Most engineering problems entail determination of unknown values of key parameters, under certain known circumstances or conditions. Laws allow us the opportunity to formulate equations that can be solved for values of unknown parameters. Solving for ***one unknown*** variable requires a minimum of ***one equation***. Solution for determination of the values of ***two unknown variables*** requires ***two equations***, and so on. Therefore, laws afford us the opportunity to model an engineering problem or scenario in form of set of equations that can be solved to adequately define and identify unknowns. Similar to the three basic laws of static equilibrium, i.e. $\mathbf{\Sigma F_x = 0, \Sigma F_y = 0}$, and $\mathbf{\Sigma M = 0}$ **-** laws that a civil or mechanical engineer might employ to solve for unknown forces in a static equilibrium beam or truss scenario **-** the Ohms law, the Kirchhoff's voltage law and the Kirchhoff's current law are used to solve for important electrical parameters like current, voltage, resistance, power, energy, reactance, impedance, reactance, etc.

In addition to the basic laws, important electrical circuit principles and circuit simplification techniques, such as, voltage division and current division will be discussed and their application will be illustrated through sample problems in this segment. Note that there are many other principles and methods that are at electrical engineer's disposal to solve or analyze complex circuits, such as, the conversion of **Y** load (or resistor) configuration to **Delta** "Δ" load configuration, or vice and versa; conversion of complex circuits to Thevenin equivalent, Norton equivalent, two port networks, etc. However, these and many other advanced circuit analysis methods are outside the scope of this text.

We will introduce the reader to two basic electronic devices, namely, a diode and a transistor, their basic characteristics and some of their applications. We will conclude this segment with a pictorial "tour" of a typical

electronic printed circuit board to allow the reader an opportunity to gain a measure of familiarity with basic electronic devices.

Ohms Law

Ohm's law was introduced, briefly, in Segment 1. Ohm's law stipulates that voltage, or voltage drop, in a DC (or AC) circuit is equal to the product of current flowing in the circuit and the resistance (or impedance) in the electrical circuit. In other words, voltage or voltage drop in an electrical circuit is directly proportional to the resistance (or impedance) of the circuit and the current flowing through it. Ohm's law can, therefore, be stated mathematically as follows:

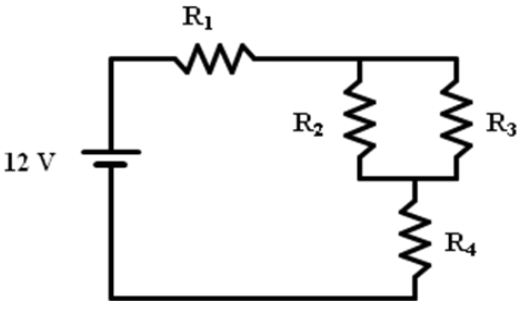
$$V = IR,\ I = \frac{V}{R},\ \text{or}\ R = \frac{V}{I} \quad \text{for DC Circuits} \qquad \textbf{Eq. 2.1}$$

$$\vec{V} = \vec{I}\vec{Z},\ \vec{I} = \frac{\vec{V}}{\vec{Z}},\ \text{or}\ \vec{Z} = \frac{\vec{V}}{\vec{I}} \quad \text{for AC Circuits} \qquad \textbf{Eq. 2.2}$$

The first mathematical representation of Ohm's law, in form of Eq. 2.1, pertains to DC circuits, and in this statement of Ohm's law, all three parameters – **V, I** and **R -** are scaler. On the other hand, mathematical representation of Ohm's law, in form of Eq. 2.2 applies to AC circuits, where voltage "**V"** and current **"I"** are vectors or complex entities and **R**, as always**,** is scalar. Note that the symbols with "half-arrows" above them denote complex AC, or vector, entities. These half arrows are shown here to introduce the reader to this method for representing vector or complex entities. Later in this text, vector or complex entities will be represented mostly in bold fonts. As one examines the statement of Ohm's law in form of Equations 2.1 and 2.2, it becomes obvious that the Ohm's law can be interpreted and applied in several ways. This fundamental aspect of Ohm's law, its versatility and wide application will become more evident through various circuit analysis problems in this segment, and others in this text, beginning with Example 2.1.

Example 2.1

The DC circuit shown below consists of a hybrid, parallel-series, network or resistors: $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 1\ \Omega$, and $R_4 = 10\ \Omega$. Calculate the following parameters in this circuit: (a) R_{eq} or R_{total} for the entire circuit. (b) The amount of current "i" flowing through resistor R_1.



Solution:

a) R_{eq} or R_{total} for the entire circuit:

$R_{eq} = R_1 + R_4 + (R_2 \,.\, R_3)/(R_2 + R_3)$

$= 10\ \Omega + 10\ \Omega + (5/6\ \Omega) = 20.833\ \Omega$

b) The amount of current "i" flowing through resistor R_1:

By Ohms Law: V = I .R, or, I = V/R.

Therefore, $i = 12V/R_{eq}$

$= 12V/20.833\ \Omega = 0.576$ Amps.

Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law stipulates that the algebraic sum of voltage drops around any closed path, within a circuit, is equal to the sum of voltages presented by all of the voltage sources. The mathematical representation of KVL is as follows:

$$\mathbf{\Sigma\ V_{Drops} = \Sigma\ V_{Source}} \qquad \textbf{Eq. 2.3}$$

Kirchhoff's voltage law can also be stated as:

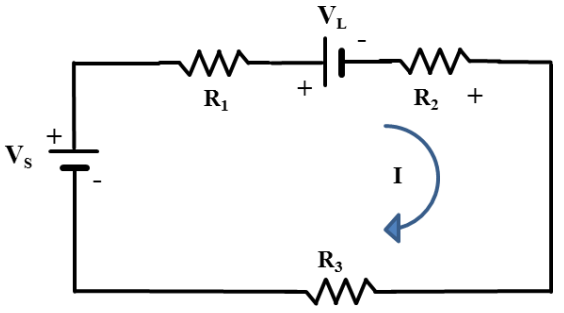
Sum of ALL voltages in a circuit loop = 0, or:

$$\mathbf{\Sigma\ V = 0} \qquad \textbf{Eq. 2.3a}$$

Some electrical engineers find the later representation of Kirchhoff's voltage law somewhat easier to apply when performing circuit analyses because, with this version, once the voltages and respective polarities have been identified, you simply sum up all the voltage values with appropriate signs, as observed, while going around the loop in the chosen direction. This importance of this approach, and alternative approaches, is illustrated through the Example 2.2 below.

Example 2.2 – KVL

Variation of current in the circuit shown below needs to be studied as a function of the three resistors and the voltage source, $\mathbf{V_s}$. Using the Ohm's Law and KVL, develop an equation that can be used to compute the value of current **I** for various values of $\mathbf{R_1}$, $\mathbf{R_2}$, $\mathbf{R_3}$ and $\mathbf{V_s}$.



Solution:

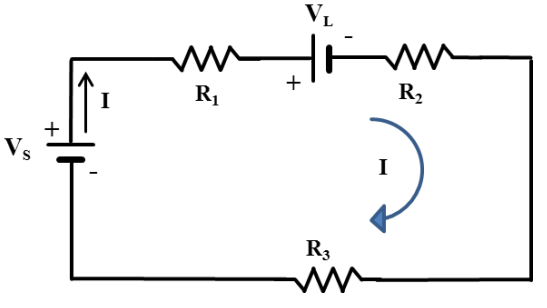
Similar to other engineering disciplines, in most electrical engineering problems, multiple methods can be employed to derive the solution. The suitability of one method over another depends on the known parameters and the complexity of the circuit.

Approach I:

Reduce or simplify the given circuit to a "net" voltage source and equivalent resistance $\mathbf{R_{eq}}$. Since $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$ are in series:

$$\mathbf{R_{eq} = R_1 + R_2 + R_3}$$

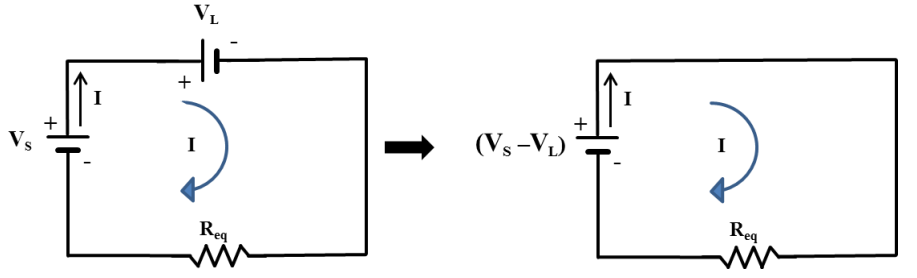
Based on the assumption that $\mathbf{V_s}$ is indeed the source driving this circuit, by electrical convention, the current in this circuit would flow "out" of the positive terminal, or anode, of the voltage source. Hence, the current would flow in the clockwise direction as shown in the diagram, below.



By the same token, application of the electrical current convention to the voltage device, or load voltage, $\mathbf{V_L}$ would mean that it would ***try*** to set up current in the counter-clockwise direction. However, because we assumed that voltage source $\mathbf{V_s}$ is driving the net flow of current through the circuit, its dominance over $\mathbf{V_L}$ is implied, and the net voltage in the circuit would be:

$$\mathbf{V_{Net} = V_S - V_L}$$

This results in the simplification of the circuit as depicted below:

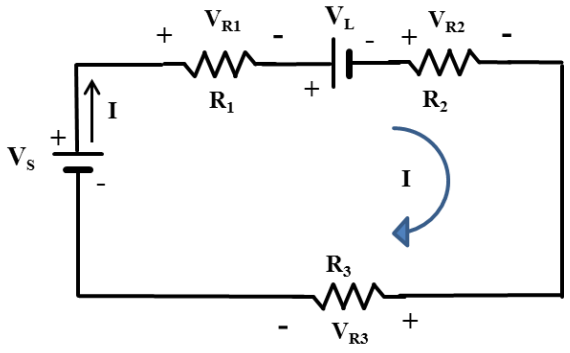


Then, application of Ohm's law yields:

$$I = \frac{(V_s - V_L)}{R_{eq}} \text{ or, } I = \frac{V_s - V_L}{R_1 + R_2 + R_3}$$

Approach II:
This approach is premised on the application of KVL to the given circuit after the circuit has been annotated with voltage designations, voltage polarities and current direction.

According to another electrical convention, voltage polarities are assigned such that the current enters the resistances (or loads, in general) on the ***positive*** side and exits from the ***negative*** side. The voltage sources, or existing "voltage loads" retain their stated polarities. The aforementioned steps result in the transformation of the original (given) circuit as follows:



Apply the Ohm's law to define the voltages, or voltage drops, across the three resistors.

$$V_{R1} = IR_1 \quad V_{R2} = IR_2 \quad V_{R3} = IR_3$$

With all voltages – voltage source, voltage load and voltage drops across the resistors – identified and their polarities noted, apply KVL by "walking" the annotated circuit beginning at the cathode or negative electrode of the voltage source, $\mathbf{V_s}$. Note the voltages and respective polarities as you make a complete loop around the circuit in the clockwise direction of the current. This results in the following equation:

$$\Sigma V = 0$$
$$- V_s + V_{R1} + V_L + V_{R2} + V_{R3} = 0$$

Expansion of this KVL based equation through substitution of the resistor voltage drop formulas, derived earlier, yields:

$$- V_s + IR_1 + V_L + IR_2 + IR_3 = 0$$

Further rearrangement and simplification results in:

$$I (R_1 + R_2 + R_3) = (V_s - V_L)$$
$$I = \frac{V_s - V_L}{R_1 + R_2 + R_3}$$

Kirchhoff's Current Law (KCL)

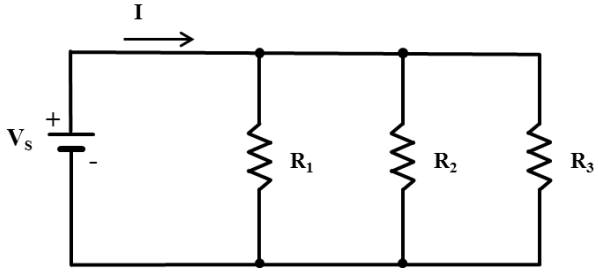
According to Kirchhoff's Current Law, total current flowing ***into*** a ***node*** is equal to the total current that flows ***out*** of the ***node***. The mathematical representation of KCL is as follows:

$$\Sigma\, i_{in} = \Sigma\, i_{out} \qquad \textbf{Eq. 2.4}$$

Proper identification of a ***node*** before application of KCL is pivotal in application of KCL. The concept of a node is not unique to electrical circuit. Intersections and junctions in fluid piping systems are sometimes referred to as nodes, by Mechanical Engineers and Technicians. The term finds its use in disciplines as diverse as human anatomical "lymph node" system and the algorithm nodes in computer systems. In the electrical realm, a node is sometimes construed as a point where two conductors merge or get connected. However, as illustrated through application of KCL in Example 2.3, a more meaningful definition of a node in electrical circuits is that it is a point where ***three or more conductors are electrically terminated or connected together***. Just as significance and effectiveness of KVL was illustrated through Example 2.2, we will demonstrate the utility of KCL, Kirchhoff's current law, and selection of a meaningful node in a "parallel" electrical circuit through Example 2.3 below.

Example 2.3

Determine the value of voltage source ***current*** in the parallel circuit below.



Solution:

Similar to Example 2.2, we will present two different approaches for determining the value of unknown source current. The first approach simply utilizes the Ohm's law and the parallel circuit simplification method. The second approach, on the other hand, utilizes KCL and "***nodal***" analysis technique.

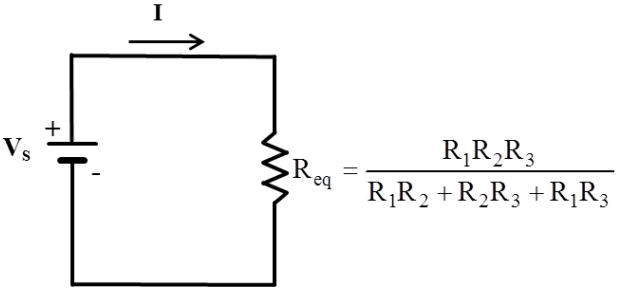
Approach I:

Reduce or simplify the given circuit to a voltage source and equivalent resistance $\mathbf{R_{eq}}$. Since $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$ are in ***parallel***, application of Eq. 1.13 yields***:***

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

This simplifies the given parallel DC circuit as follows:

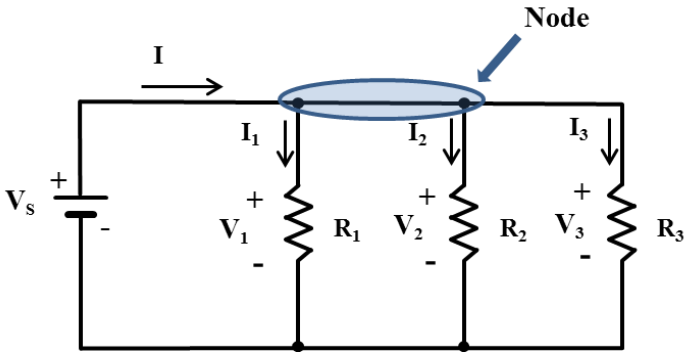


Next, Ohm's law is applied to determine the source current:

$$I = V_s \left(\frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1 R_2 R_3} \right)$$

Approach II:

This approach is premised on the application of KCL to the given circuit after the node had been identified, circuit has been annotated with voltage designation, voltage polarity, branch currents and current directions. See circuit diagram below:



Subscribing to the definition of a node as a point where three or more conductors merge, the shaded segment in the diagram above is designated as the node for this circuit. Next, before KCL can be applied to determine the source current, the individual currents, through each of the resistors, need to be defined – using the Ohm's law - in terms of the specific resistance values and the voltages around them:

$$I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_2}{R_2} \quad I_3 = \frac{V_3}{R_3}$$

Then, application of KCL at the designated node yields the following equation:

$$I = I_1 + I_2 + I_3$$

Substitution of the values of branch currents, as defined earlier, yields:

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

At this juncture, it is important to note that when circuit elements are in parallel – as is the case with $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$ – their voltages (or voltage drops around them) are equal. In fact, not only are the voltages around the parallel circuit elements equal to each other but they are the same as the source voltage, $\mathbf{V_s}$. In other words:

$$V_s = V_1 = V_2 = V_3$$

Therefore, the current equation can be rewritten as:

$$I = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

And the source current would be:

$$I = V_s\left(\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1R_2R_3}\right)$$

Which is the same as the answer derived through Approach 1.

Voltage Division

Voltage division is a shortcut for determination of voltage across a series resistor. According to the voltage division rule, the voltage across resistance **R**, in a DC circuit, with total resistance $\mathbf{R_{total}}$, and a voltage source **V**, can be determined through the following formula:

$$V_R = \frac{R}{R_{total}} V \qquad \textbf{Eq. 2.5}$$

For AC circuits, the voltage on impedance $\mathbf{Z_i}$, in a loop with total impedance $\boldsymbol{Z_{total}}$, with a voltage source **V,** would be:

$$V_i = \left(\frac{Z_i}{Z_{total}}\right).V \qquad \textbf{Eq. 2.6}$$

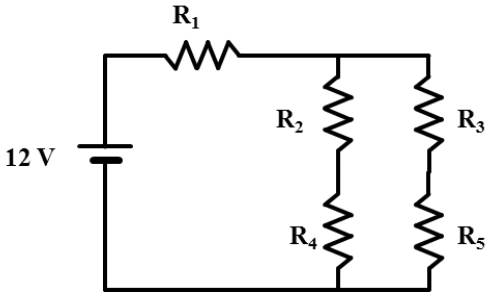
Example 2.4

Determine the following for the DC circuit shown below:

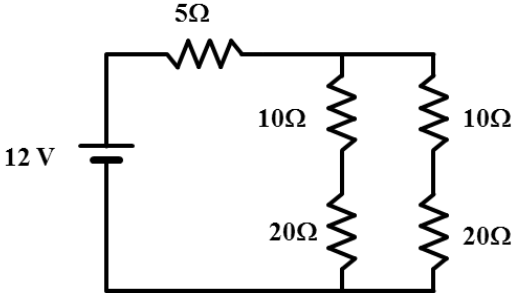
a) Equivalent resistance for the entire circuit, if $R_1 = 5\Omega$, $R_2 = R_3 = 10\Omega$, and $R_4 = R_5 = 20\Omega$

b) Current flowing through resistor R_1

c) Voltage across resistor R_5



Solution:



a) Combination of R_2 and R_4 = $R_{2,4}$ = $R_2 + R_4$ = 10Ω + 20Ω = 30Ω
Combination of R_3 and R_5 = $R_{3,5}$ = $R_3 + R_5$ = 10Ω + 20Ω = 30Ω
Combination of $R_{2,4}$ and $R_{3,5}$ =

$$R_{2-5} = \frac{(30\Omega).(30\Omega)}{(30\Omega + 30\Omega)} = \frac{900}{60} = 15\Omega$$

$$R_{eq} = R_1 + R_{2\text{-}5} = 5\Omega + 15\Omega = 20\Omega$$

b) Current through R_1 would be the same as the current through the 12V supply:

$$I = \frac{V}{R_{eq}} = \frac{12V}{20\Omega} = 0.6A$$

c) One method for determining $\mathbf{V_{R5}}$, voltage across $\mathbf{R_5}$, is to first calculate $\mathbf{V_{R2\text{-}5}}$, the voltage across the combined resistance of resistances $\mathbf{R_2}$, $\mathbf{R_3}$, $\mathbf{R_4}$, and $\mathbf{R_5}$. Then, by applying voltage division, calculate $\mathbf{V_{R5}}$ as follows:

According to Ohm's law,

$$\mathbf{V_{R2\text{-}5} = I.(R_{2\text{-}5}) = (0.6A).(15\Omega) = 9V}$$

Then, by applying the voltage division rule:

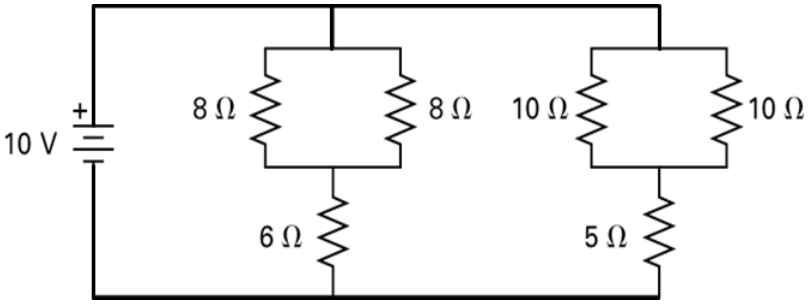
$$V_{R5} = (9V).(\frac{R_5}{R_5 + R_3})$$

$$= (9V).(\frac{20\Omega}{20\Omega + 10\Omega})$$

$$= (9V).(0.67) = 6V$$

Example 2.5

What is the voltage across the 6Ω resistor?



Solution:

The right hand side of the circuit, consisting of the two 10Ω resistors and the 5Ω resistor, is irrelevant insofar as the determination of voltage across the 6Ω resistor is concerned.

Parallel combination of the two 8 Ω resistors results in an equivalent resistance of 4 Ω as follows:

$$R_{8\Omega//8\Omega} = \frac{(8\Omega).(8\Omega)}{8\Omega + 8\Omega}$$

Using voltage division, the voltage across the 6 Ω resistor would be:

$$V_{6\Omega} = (10V).(\frac{6\Omega}{4\Omega + 6\Omega})$$

$$= 6V$$

Current Division

The current through a resistor **R** in parallel, or in **shunt**, with another resistance $\mathbf{R_{parallel}}$ and a current into the node of **I** is:

$$I_R = \left(\frac{R_{parallel}}{R_{total}} \right).I \qquad \textbf{Eq. 2.7}$$

Where,

$\mathbf{R_{total}}$ = The sum of the resistances in parallel (and **not** the parallel combination $\mathbf{R_{EQ}}$).

$\mathbf{R_{parallel}}$ = Resistance value of the resistor opposite the "subject" resistor.

I = Current through the source.

When current division is applied in **AC** circuits, the formula for current through an impedance **Z**, in parallel with another impedance $Z_{parallel}$, would be:

$$I_R = \left(\frac{Z_{parallel}}{Z_{total}} \right).I \qquad \textbf{Eq. 2.8}$$

Where,

I = Current, in its complex AC form, flowing into the node formed by the parallel impedances.

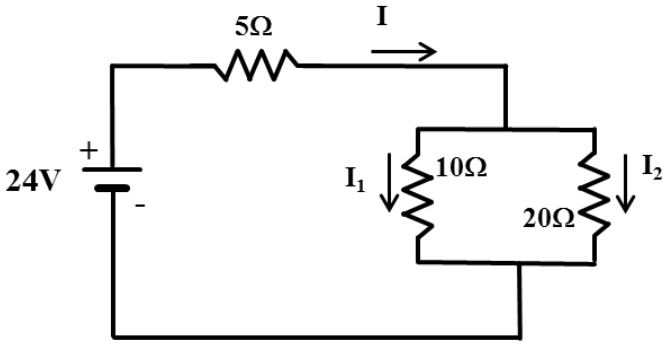
$\mathbf{Z_{total}}$ = The sum of the impedances in parallel (and ***not*** the parallel combination $\mathbf{Z_{EQ}}$).

$\mathbf{Z_{parallel}}$ = The impedance of the load opposite to the subject impedance.

I = AC current through the source.

Example 2.6

Determine the current flowing through the 10Ω resistance in the circuit shown below.



Solution:

We must to determine the value of source current **I,** first. In order to determine the value of current **I** flowing through the source and the 5Ω resistor we must consolidate all resistors into an equivalent resistance $\mathbf{R_{EQ}}$ and then apply the Ohm's law.

$$R_{EQ} = 5\Omega + \left(\frac{(10\Omega).(20\Omega)}{(10\Omega)+(20\Omega)} \right) = 5\Omega + 6.67\Omega = 11.67\Omega$$

$$I = \frac{V}{R_{EQ}} = \frac{24V}{11.67\Omega} = 2.06\ A$$

Apply current division formula in form of **Eq. 2.7**

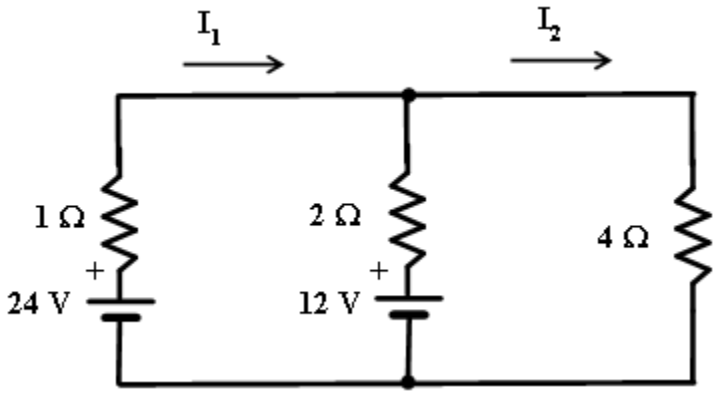
$$I_{10\Omega} = I_1 = \left(\frac{R_{parallel}}{R_{total}} \right) I = \left(\frac{20\Omega}{10\Omega + 20\Omega} \right) 2.06A = 1.36\ A$$

Multi-loop circuit analysis

As expected, analyses of circuits that consist of more than one current loop, tend to be more complex, require formulation of multiple equations and utilization of additional conventions and principles. Similar to single loop circuit analysis covered earlier, in most cases, there are multiple strategies and approaches available for solving multi-loop circuits. We will illustrate one approach through Example 2.7.

Example 2.7

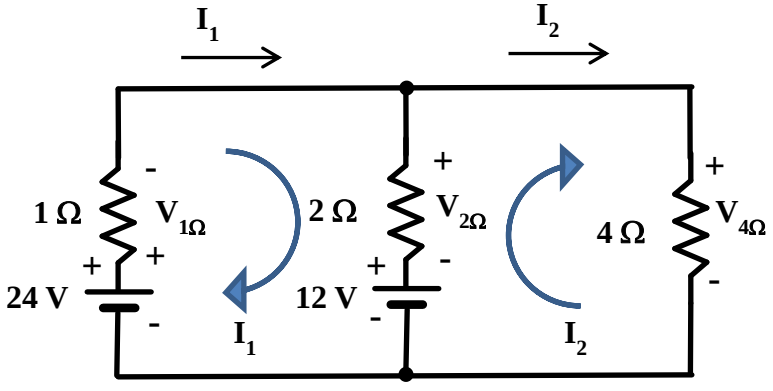
Using the KVL method, determine the current flows, $\mathbf{I_1}$ and $\mathbf{I_2}$ in the circuit below.



Solution:

Current $\mathbf{I_1}$ and $\mathbf{I_2}$ are two of the five unknown parameters in the circuit above. The other unknown parameters are, $\mathbf{V_{1\Omega}}$, $\mathbf{V_{2\Omega}}$ and $\mathbf{V_{4\Omega}}$. One approach for determining the values of $\mathbf{I_1}$ and $\mathbf{I_2}$ would be to formulate two equations, using the Kirchhoff's Voltage Law, such that each equation includes the same two unknown variables, $\mathbf{I_1}$ and $\mathbf{I_2}$. Then, by applying the simultaneous equation technique to the two equation system, with two unknowns, we can determine the values of $\mathbf{I_1}$ and $\mathbf{I_2}$.

Application of KVL requires that voltage around each circuit element be defined in terms of known values and the unknown variables. In addition, the sign or polarity of each voltage must be assigned. See the diagram below.



Rules for assuming the current direction and assigning voltage polarities to various loads (resistors) and sources (voltage sources) are as follows:

1) As shown in the figure above, by convention, the currents are assumed to be emanating from the positive pole (positive electrode or anode) of the voltage source and are assumed to be terminating into the negative pole (negative electrode or cathode) of the voltage source.

2) As shown in the figure above, the end or side of the resistor or load that the current enters from is labeled as positive.

3) The polarity for a voltage source is assumed as encountered in the direction of current flow.

Examination of the circuit above reveals that there are three loops in the given circuit. The left loop will be referred to as loop 1, the loop on the right segment of the circuit is loop 2. The third loop in this circuit is formed by the outer perimeter. We will focus on the first two loops to derive two equations for the determination of the two unknown currents. Assume that $\mathbf{I_1}$ is greater than $\mathbf{I_2}$. Conventionally, it is acceptable to make such assumptions as long as the assumptions are, strictly, adhered to in deriving all equations necessary for the solution.

"Walking" loop 1, beginning at the negative terminal of the 24 V dc source, yields the following equation:

$$-24V + V_{1\Omega} + V_{2\Omega} + 12V = 0 \qquad \text{Eq. 2.9}$$

"Walking" loop 2, beginning at the negative terminal of the 12 V dc source, yields the following equation:

$$-12V - V_{2\Omega} + V_{4\Omega} = 0 \qquad \text{Eq. 2.10}$$

Based on Ohm's Law:

$$V_{1\Omega} = (\mathbf{I_1}).(1\Omega) = \mathbf{I_1} \qquad \text{Eq. 2.11}$$

$$V_{4\Omega} = (\mathbf{I_2}).(4\Omega) = 4\mathbf{I_2} \qquad \text{Eq. 2.12}$$

$$V_{2\Omega} = (\mathbf{I_1} - \mathbf{I_2}).(2\Omega) = 2.(\mathbf{I_1} - \mathbf{I_2}) \qquad \text{Eq. 2.13}$$

Then, by substituting Eq. 2.11, Eq. 2.12 and Eq. 2.13 into Eq. 2.9 and Eq. 2.10, we get:

$$-24 + \mathbf{I_1} + 2.(\mathbf{I_1} - \mathbf{I_2}) + 12 = 0$$

$$3\mathbf{I_1} - 2\mathbf{I_2} = 12 \qquad \text{Eq. 2.14}$$

$$-12 - 2.(\mathbf{I_1} - \mathbf{I_2}) + 4\mathbf{I_2} = 0$$

$$-2\mathbf{I_1} + 6\mathbf{I_2} = 12 \qquad \text{Eq. 2.15}$$

Equations 2.14 and 2.15 represent the two simultaneous equations that were needed to solve for currents $\mathbf{I_1}$ and $\mathbf{I_2}$. These equations will be solved simultaneously to determine the values of $\mathbf{I_1}$ and $\mathbf{I_2}$.

$$3\mathbf{I_1} - 2\mathbf{I_2} = 12 \qquad \text{Eq. 2.14}$$

$$-2\mathbf{I_1} + 6\mathbf{I_2} = 12 \qquad \text{Eq. 2.15}$$

For simultaneous equation solution, multiply left hand side and the right hand side of Eq. 2.14 by 3 and add it to Eq. 2.15:

$$9\mathbf{I_1} - 6\mathbf{I_2} = 36$$

$$\underline{-2\mathbf{I_1} + 6\mathbf{I_2} = 12}$$

$$7\mathbf{I_1} = 48$$

$$\therefore \mathbf{I_1} = 6.86 \text{ Amps}$$

Then, by substituting this value of $\mathbf{I_1}$ into Eq. 2.15 yields:

$$-2\mathbf{(6.86)} + 6\mathbf{I_2} = 12$$

Or,

$$\mathbf{I_2} = 4.29 \text{ Amps}$$

Note: The values of unknown currents $\mathbf{I_1}$ and $\mathbf{I_2}$ can also be determined by applying Cramer's Rule to Eq. 2.14 and 2.15, in matrix format, and linear algebra.

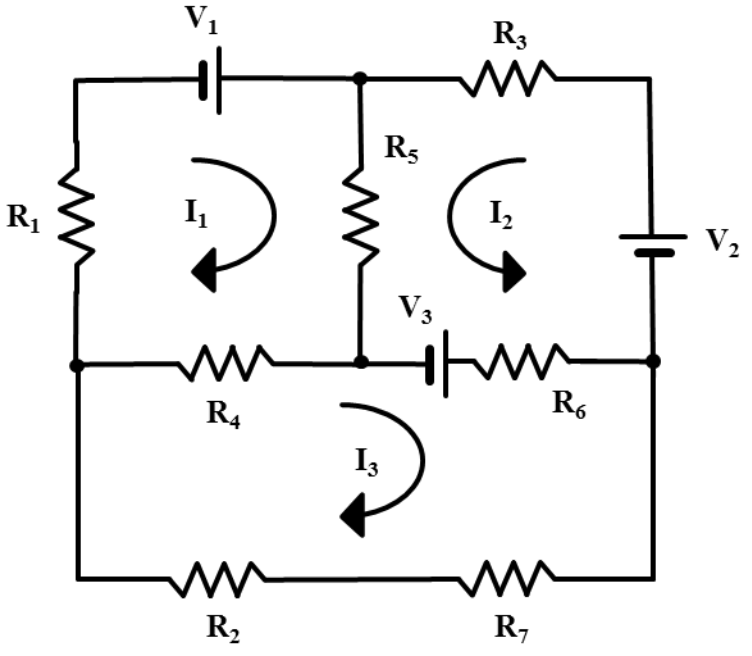
Circuit analysis using Cramer's Rule and Linear Algebra

Cramer's Rule can be applied to solve for unknowns, in lieu of simultaneous equations or substitution methods, after set of equations have been formulated using the loop analysis method described above. Since the

Cramer's Rule involves matrices and linear algebra, it is the method that electrical circuit analysis computer software is premised on. The application of Cramer's Rule is illustrated through Example 2.8 below.

Example 2.8

The values of all known parameters for the following multi-loop circuit are listed in the table below. Find the values of currents $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$.



R_1	10Ω
R_2	2Ω
R_3	3Ω
R_4	4Ω
R_5	7Ω
R_6	3Ω
R_7	5Ω
V_1	20
V_2	5
V_3	12

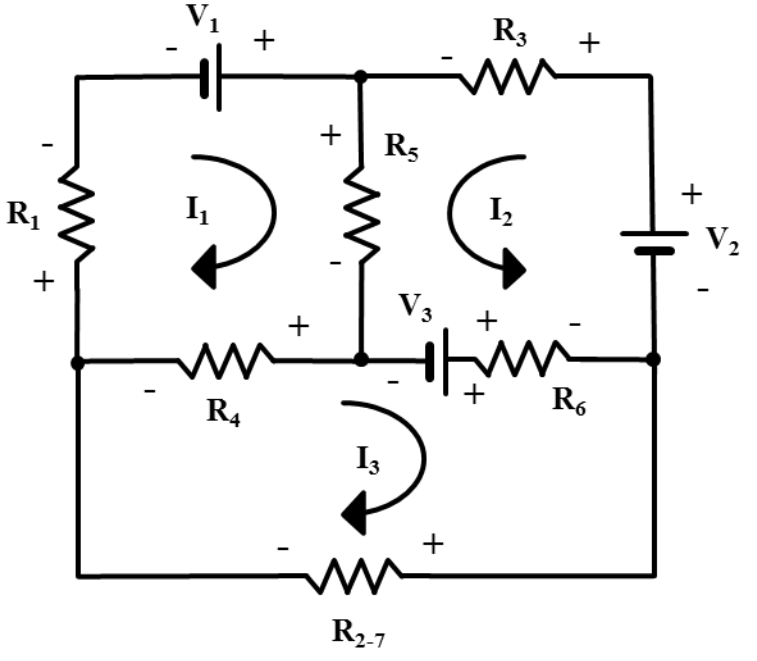
Solution:

In this example, the fundamental strategy, conventions and principles needed to calculate the three unknown currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$ would be the same as the ones described in Example 2.4. The obvious difference is that this circuit has a total of four loops – including the outermost loop – and has three unknown variables in form of currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$. Therefore, we will need a minimum of three equations. Those three equations, as before, are derived by applying the Kirchhoff's Voltage Law to each of the three inner loops. The voltage drops across each of the load components (or resistors) are defined on the basis of the Ohm's law, V = I.R.

Before we embark on the formulation of current computation equations, let's ensure that the circuit is in its most simplified form. In that vane, by inspection, we notice that the two series resistors in the bottom loop, R_2 and R_7, can be added together or combined as follows:

$$R_{2\text{-}7} = R_2 + R_7$$

This simplification and assignment of voltage drop polarities results in the following circuit schematic:



Application of KVL in loop 1, with the assumptions that $\mathbf{I_1} > \mathbf{I_2}$ and $\mathbf{I_3}$, and $\mathbf{I_2} > \mathbf{I_3}$ yields:

$$I_1R_1 - V_1 + (I_1 + I_2)\,R_5 + (I_1 - I_3)R_4 = 0$$
$$I_1R_1 - V_1 + I_1R_5 + I_2R_5 + I_1R_4 - I_3R_4 = 0$$
$$(R_1 + R_5 + R_4)\,I_1 + I_2R_5 - I_3R_4 = V_1 \qquad \textbf{Eq. 2.16}$$

Application of KVL in loop 2, maintaining the assumption that $I_1 > I_2$ and I_3, and $I_2 > I_3$ yields:

$$- V_2 + I_2R_3 + (I_1 + I_2)R_5 - V_3 + (I_2 + I_3)R_6 = 0$$
$$- V_2 + I_2R_3 + I_1 R_5 + I_2R_5 - V_3 + I_2R_6 + I_3R_6 = 0$$
$$R_5I_1 + (R_3 + R_5 + R_6)I_2 + I_3R_6 = V_2 + V_3 \qquad \textbf{Eq. 2.17}$$

Application of KVL in loop 3, maintaining the assumption that $I_1 > I_2$ and I_3, and $I_2 > I_3$ yields:

$$I_3R_{2\text{-}7} - (I_1 - I_3)R_4 - V_3 + (I_2 + I_3)R_6 = 0$$
$$I_3R_{2\text{-}7} - I_1R_4 + I_3R_4 - V_3 + I_2 R_6 + I_3R_6 = 0$$
$$- R_4 I_1 + R_6 I_2 + (R_4 + R_6 + R_{2\text{-}7})\,I_3 = V_3 \qquad \textbf{Eq. 2.18}$$

The three simultaneous equations thus derived are:

$$(R_1 + R_5 + R_4)I_1 + R_5I_2 - R_4I_3 = V_1 \qquad \textbf{Eq. 2.16}$$
$$R_5I_1 + (R_3 + R_5 + R6)I_2 + R_6I_3 = V_2 + V_3 \qquad \textbf{Eq. 2.17}$$
$$- R_4 I_1 + R_6I_2 + (R_4 + R_6 + R_{2\text{-}7})\,I_3 = V_3 \qquad \textbf{Eq. 2.18}$$

Substitution of the given resistor and voltage source values into Eq. 2.16, Eq. 2.17, and Eq. 2.18 yields the following simultaneous equations:

$$21I_1 + 7I_2 - 4I_3 = 20$$
$$7I_1 + 13I_2 + 3I_3 = 17$$
$$- 4\,I_1 + 3I_2 + 14\,I_3 = 12$$

Apply the Cramer's rule to solve for the three unknown currents I_1, I_2 and I_3. The augmented matrix thus developed would be:

$$\left|\begin{array}{ccc:c} 21 & 7 & -4 & 20 \\ 7 & 13 & 3 & 17 \\ -4 & 3 & 14 & 12 \end{array}\right|$$

The coefficient matrix, denoted as **A**, would be:

$$\begin{vmatrix} \mathbf{21} & \mathbf{7} & \mathbf{-4} \\ \mathbf{7} & \mathbf{13} & \mathbf{3} \\ \mathbf{-4} & \mathbf{3} & \mathbf{14} \end{vmatrix}$$

The determinant of the coefficient matrix, denoted as $|\mathbf{A}|$, would be:

$|\mathbf{A}| = 21\{(13x14) - (3x3)\} - 7\{(7x14) - (-4x3)\} - 4\{(7x3) - (-4x13)\} = \mathbf{2571}$

The determinant of the substitutional matrix, $\mathbf{A_1}$, for determining the value of I_1, is denoted as $|\mathbf{A_1}|$, and

$$\mathbf{A_1} = \begin{vmatrix} \mathbf{20} & \mathbf{7} & \mathbf{-4} \\ \mathbf{17} & \mathbf{13} & \mathbf{3} \\ \mathbf{12} & \mathbf{3} & \mathbf{14} \end{vmatrix}$$

$|\mathbf{A_1}| = 20\{(13x14) - (3x3)\} - 7\{(17x14) - (12x3)\} - 4\{(17x3) - (12x13)\} = \mathbf{2466}$

The determinant of the substitutional matrix, $\mathbf{A_2}$, for determining the value of I_2, is denoted as $|\mathbf{A_2}|$, and

$$\mathbf{A_2} = \begin{vmatrix} \mathbf{21} & \mathbf{20} & \mathbf{-4} \\ \mathbf{7} & \mathbf{17} & \mathbf{3} \\ \mathbf{-4} & \mathbf{12} & \mathbf{14} \end{vmatrix}$$

$|\mathbf{A_2}| = 21\{(17x14) - (12x3)\} - 20\{(7x14) - (-4x3)\} - 4\{(7x12) - (-4x17)\} = \mathbf{1434}$

The determinant of the substitutional matrix, $\mathbf{A_3}$, for determining the value of I_3, is denoted as $|\mathbf{A_3}|$, and

$$\mathbf{A_3} = \begin{vmatrix} \mathbf{21} & \mathbf{7} & \mathbf{20} \\ \mathbf{7} & \mathbf{13} & \mathbf{17} \\ \mathbf{-4} & \mathbf{3} & \mathbf{12} \end{vmatrix}$$

$|\mathbf{A_3}| = 21\{(13x12) - (3x17)\} - 7\{(7x12) - (-4x17)\} + 20\{(7x3) - (-4x13)\}$
$= \mathbf{2601}$

Applying the Cramer's rule, the unknown variables, currents $\mathbf{I_1}$, $\mathbf{I_2}$ and $\mathbf{I_3}$, can be calculated by dividing the determinants of substitutional matrices $\mathbf{A_1}$, $\mathbf{A_2}$ and $\mathbf{A_3}$, respectively, by the determinant of the coefficient matrix **A**.

Therefore,

$$I_1 = \frac{|A_1|}{|A|} = \frac{2466}{2571} = 0.959 \text{ A}$$

$$I_2 = \frac{|A_2|}{|A|} = \frac{1434}{2571} = 0.558 \text{ A}$$

$$I_3 = \frac{|A_3|}{|A|} = \frac{2601}{2571} = 1.012 \text{ A}$$

Diodes

A diode, unlike its "functional" predecessor - the vacuum tube rectifier - is constructed out of ***semiconductor*** materials such as silicone, germanium, gallium arsenide, etc. While a semiconductor is not a good conductor at room temperature, it doesn't fall distinctly in the category of insulators, such as, glass, ceramics, urethanes, plastics, PVC's, etc. A semiconductor can, however, be transformed into a "partially" or "selectively" conductive substance through a process called "**doping**." The term doping implies addition of "impurities" into a pure substance like silicone. If these impurities are added to create a region with a predominant concentration of negative charge carriers, or electrons, the end result would be the formation of a region called **n-type** semiconductor. At the same time, if impurities are added

adjacently, to create a region with predominant positive charge carriers, or holes, a **p-type** semiconductor is formed. The plane where the **p** and **n** doped materials interface with each other is called the **p-n junction**. A p-n junction is where the essential function of a diode takes place in response to the application of proper voltage.

A diode can be viewed as a device that functions as an electronic "check valve." As we know, the function of a check valve, in mechanical or hydraulic systems, is to permit the flow of fluids in one specific direction. In other words, an attempt by the fluid to move in the reverse direction is blocked by a check valve. A diode performs the same function in the flow of current. A diode permits the flow of current only from a higher voltage (or electrical potential) point in an electrical circuit to a lower voltage or ground potential point. This unidirectional behavior is called ***rectification***, and this function of a diode finds a common application in the conversion of alternating current (AC) to direct current (DC). Common application of diodes in rectifiers and other equipment are discussed later in this segment.

The symbol and drawing of common circuit board type diode are shown in Figure 2.1.

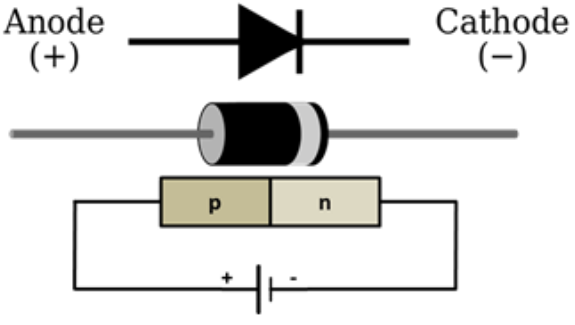


Figure 2.1: Symbol, diagram and schematic of a basic diode circuit.

The left side of the diode, labeled "Anode" is normally connected to the positive or higher voltage point in the circuit. The right side of the diode, labeled "cathode," on the other hand, is normally connected to the ground, negative, or lower potential point in the circuit. When a diode is connected in this manner, it is said to be ***forward biased***. As apparent from the diagram of a typical diode in Figure 2.1, a band on one side of the diode denotes the cathode side of the diode. If, however, the voltage is reversed such that the anode of the diode is connected to the negative voltage potential and the

cathode is connected to the positive potential, the diode is said to be ***reverse biased.*** The current response of a diode in forward and reverse bias modes of operation is depicted in Figure 2.2.

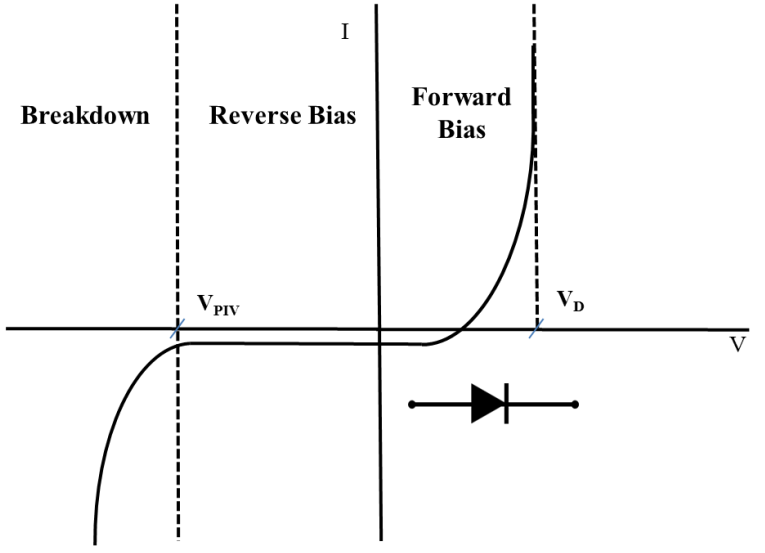


Figure 2.2: Current response of a diode in forward and reverse bias modes

In Figure 2.2, $\mathbf{V_D}$ represents the forward bias voltage. Forward bias voltage of a diode is the voltage at which the diode begins to conduct current; or is the voltage where the diode is said to be "turned on." A diode can be perceived as a "self-actuating" electronic switch, as well. The forward bias voltage, $\mathbf{V_D}$, is approximately, **0.7V** for silicone type diodes and **0.3 volts** for germanium diodes.

As depicted in Figure 2.2, when a diode is reverse biased the amount of current (or reverse current) it conducts is negligible, until the magnitude of the reverse bias voltage approach approaches $\mathbf{V_{BR}}$, breakdown voltage. The breakdown voltage is also called the $\mathbf{V_{PIV}}$, Peak Inverse Voltage or $\mathbf{V_{PRV}}$, Peak Reverse Voltage. The reverse voltage is considerably higher than the forward bias voltage; approximately ***ten*** times the magnitude of $\mathbf{V_D}$. Although, technically, all diodes are capable of rectifying AC, the term "**rectifier**" is essentially reserved for diodes designed to operate at currents in excess of 1A, and therefore, utilized frequently to convert AC into DC. Conventionally, the term diode refers to applications involving currents less than or equal to 1A.

Special Purpose Diodes

Two of the most common, special purpose, diodes that find applications in mainstream electrical or electronic equipment, i.e. Variable Frequency Drives, Inverters or Converters are SCR's and Zener Diodes.

SCR: The acronym "SCR" stands for "Silicone Controlled Rectifier," or "Semiconductor Controlled Rectifier." These specialty diodes are also referred to as "thyristors**.**" Schematic of an SCR is shown in Figure 2.3.

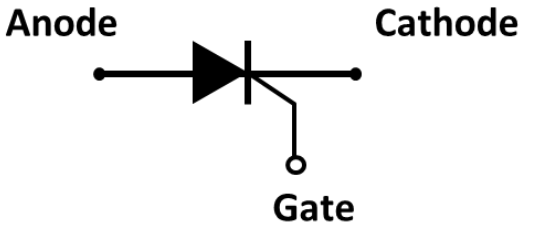


Figure 2.3: Symbol or diagram for an SCR or Thyristor

As apparent in Figure 2.3, an SCR differs from a regular diode, mainly, due to an added feature called the "gate." The gate serves as "trigger" or "firing" mechanism for an SCR. Specific voltage application at the gate triggers or "pulses" the SCR on, and allows the current to flow. The diode portion of the SCR continues to conduct after the gate voltage dissipates. The SCR, or diode portion of the SCR, stops conducting once the forward bias voltage drops below the threshold voltage, $\mathbf{V_D}$.

SCR's are mainly used in devices associated with the control of high power and high voltage. Their innate characteristic and mode of operation makes them suitable for use in medium to high-voltage AC power control applications, such as lamp dimming, regulators and motor control. Thyristors are also commonly used for rectification of high power AC in high-voltage direct current power transmission applications. They are also used in the control of welding machines.

Zener Diode: Zener diodes are similar, in construction, to basic diodes. The key difference is that Zener diodes, unlike regular diodes, are capable of recovering from avalanche reverse bias break down mode when the reverse bias is removed. Therefore, as a mechanical analogy, one could compare zener

diodes to “relief valves” that open under abnormally high pressure, and reinstate their normal blocking function once the pressure subsides to a nominal level. The Zener diode is named after Clarence Zener, who discovered its electrical property. The symbol or electronic representation of a Zener diode is depicted in Figure 2.4.

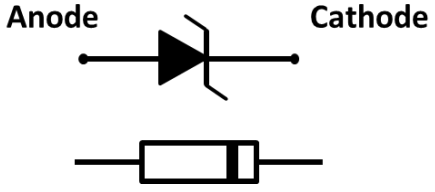


Figure 2.3: Symbol and diagram for Zener diode

A Zener diode allows current to flow in the forward direction in the same manner as an ideal diode, but will also permit it to flow in the reverse direction when the voltage is above the breakdown voltage. The breakdown voltage is also referred to as the "zener knee voltage," "zener voltage" or "avalanche point". Because of this basic characteristic of a Zener diode, it is commonly used to provide a reference voltage for voltage regulators, or to protect other semiconductor devices from momentary voltage pulses or excessive voltage “spikes.” The current response of a Zener diode is shown in Figure 2.4.

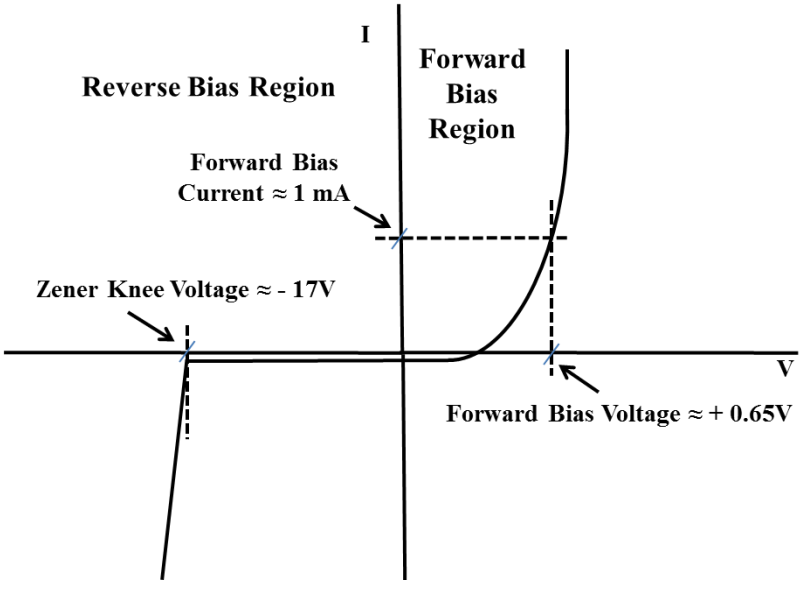


Figure 2.4: Current Response of a Zener diode

Common Applications of Diodes

Some common applications of diodes are as follows:

1) Half wave rectifier
2) Full wave rectifier
3) Clamping circuit
4) Base clipper
5) Peak clipper

Among the diode applications listed above, the two most common ones are (1) Half Wave Rectifier and (2) Full Wave Rectifier. These two applications are explored below.

Half Wave Rectifier

Half wave rectifier is a circuit consisting of two core components, a diode and resistor. See Figure 2.5(a). As shown in Figure 2.5(a), the sinusoidal AC waveform is applied to the input side of the diode-resistor circuit. The diode, as shown in Figure 2.5(b), is the core "active" component. A half wave rectifier circuit banks on the innate characteristic of a diode to allow current to flow only "one-way," in the forward biased direction. In other words, the diode permits the current to flow only during the positive half of the AC cycle. When the AC voltage "dives" into the negative realm - acting as a "check valve" - the diode shuts off the flow of current. This response of the diode is plotted graphically in Figure 2.5 (c), in form of a series of positive wave crests; average of which, represents the DC voltage produced.

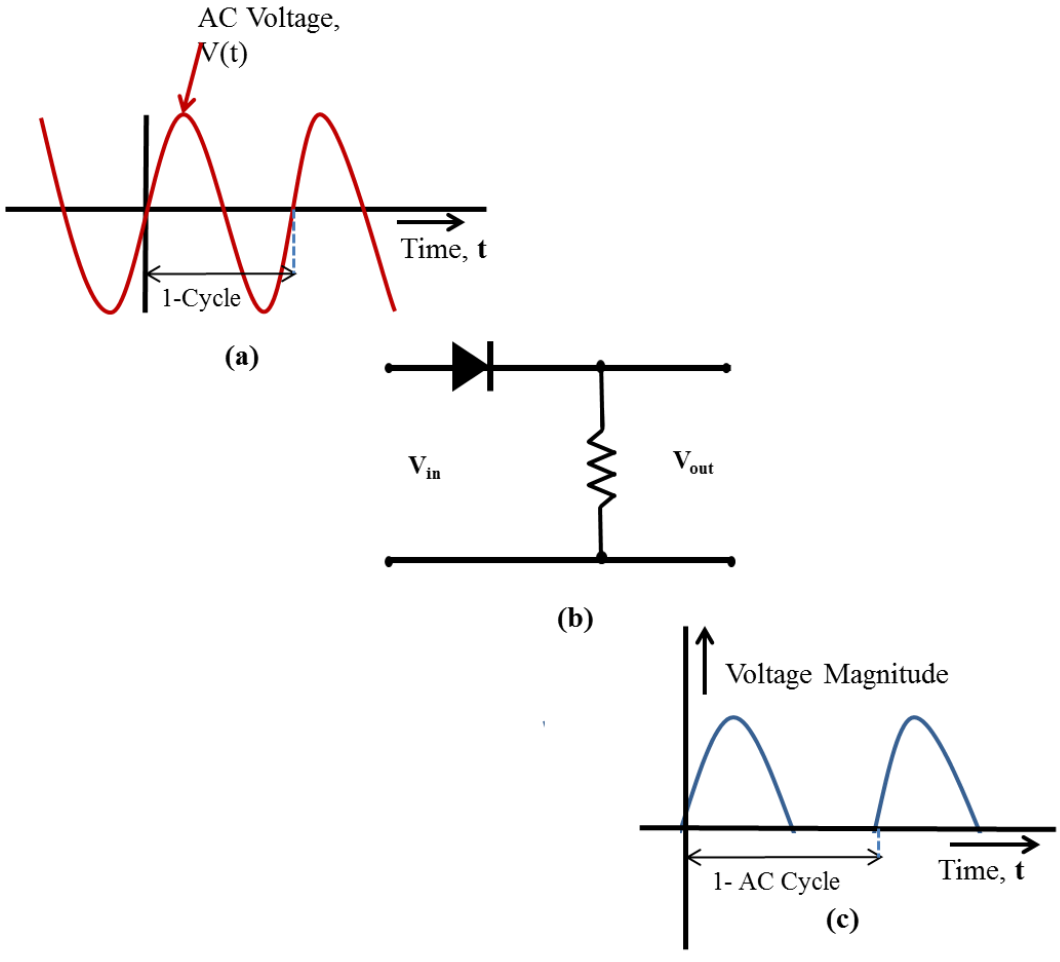


Figure 2.5: Half wave rectifier

Note, however, that half wave rectifiers only produce ***one*** positive DC crest per AC cycle. The DC output of a half wave rectifier can be computed through Eq. 2.19.

$$V_{dc} = \frac{V_p}{\pi} \qquad \textbf{Eq. 2.19}$$

Full Wave Rectifier

Common full wave rectifier is a circuit consisting of four diodes and a resistor. See Figure 2.6.The sinusoidal AC waveform applied to the input side of the ***four diode bridge*** and resistor circuit is shown in Figure 2.6(a). The four diode bridge configuration shown in Figure 2.6(b) is the essential power

conversion segment of the overall full wave bridge rectifier circuit. When AC voltage is applied between terminals **A** and **B** on the input side of the diode bridge, as $\mathbf{V_{in}}$, the positive crest of the sinusoidal waveform drives current through terminal **A**, diode $\mathbf{CR_1}$, resistor **R**, diode $\mathbf{CR_4}$ to terminal **B**. During this positive crest initiated flow of current, diodes $\mathbf{CR_1}$ and $\mathbf{CR_4}$ are forward biased. As the current assumes this course, it "drops" a positive "half" wave across the resistor, or terminals **C** and **D**. This positive DC crest is the first crest from the left, in Figure 2.6(c). During the positive half of the AC cycle, diodes $\mathbf{CR_2}$ and $\mathbf{CR_3}$ are reverse biased; therefore, they do not conduct, and ***no*** current flows through the $\mathbf{CR_2}$, **R** and $\mathbf{CR_3}$ path.

During the negative half of the AC cycle, diodes $\mathbf{CR_1}$ and $\mathbf{CR_4}$ are reverse biased and no current flow through them**.** However, because terminal **A** is negative during the negative half of the AC cycle, diodes $\mathbf{CR_2}$ and $\mathbf{CR_3}$ are forward biased and the current flow is driven from terminal **B**, through diode $\mathbf{CR_3}$, resistor **R**, diode $\mathbf{CR_2}$**.** This path of current flow also results in a positive voltage drop, or positive voltage crest across terminals **C** and **D**. This positive DC crest is represented by the ***second*** crest in Figure 2.6(c). Therefore, ***one*** AC cycle on the $\mathbf{V_{in}}$ side results in ***two*** positive crests on the $\mathbf{V_{out}}$ side. The average value of the positive crests shown in Figure 2.6(c) represents the DC output of this full wave bridge rectifier circuit. Of course, the DC output in most DC power supplies is refined or corrected into a straight line form through application of resistors, capacitors and inductors. The formula for full wave rectified DC voltage is represented by Eq. 2.20.

$$V_{DC} = 2.\left(\frac{V_p}{\pi}\right) \qquad \textbf{Eq. 2.20}$$

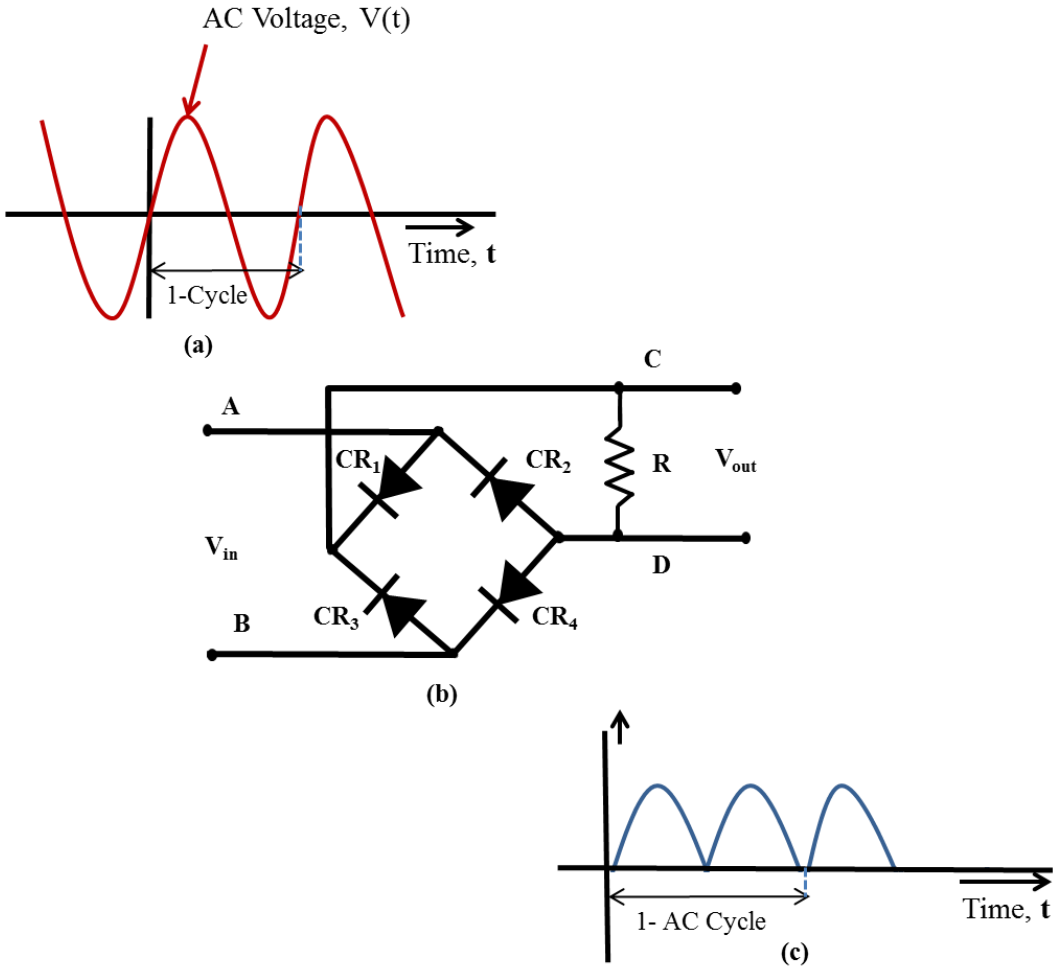


Figure 2.6: Full wave rectifier

Transistors:

Similar to diodes, transistors are semiconductor devices. The approach to constructing transistors is similar to the approach used for fabrication of diodes. The "n and "p" doping approach employed with the construction of transistors is shown in Figure 2.7. As somewhat evident from Figure 2.7, a transistor – functionally, and from a construction point of view – appears as a set of two diodes connected "back to back." There are many types of transistors. In this text, we will focus on the type of transistors referred to as the "bipolar" or FET, Field Effect Transistors. Within the bipolar junction transformer category, there are two sub-categories: The "**npn**" and the "**pnp**"

transistors. We will limit our discussion to the introduction of **npn** type transistors in this text.

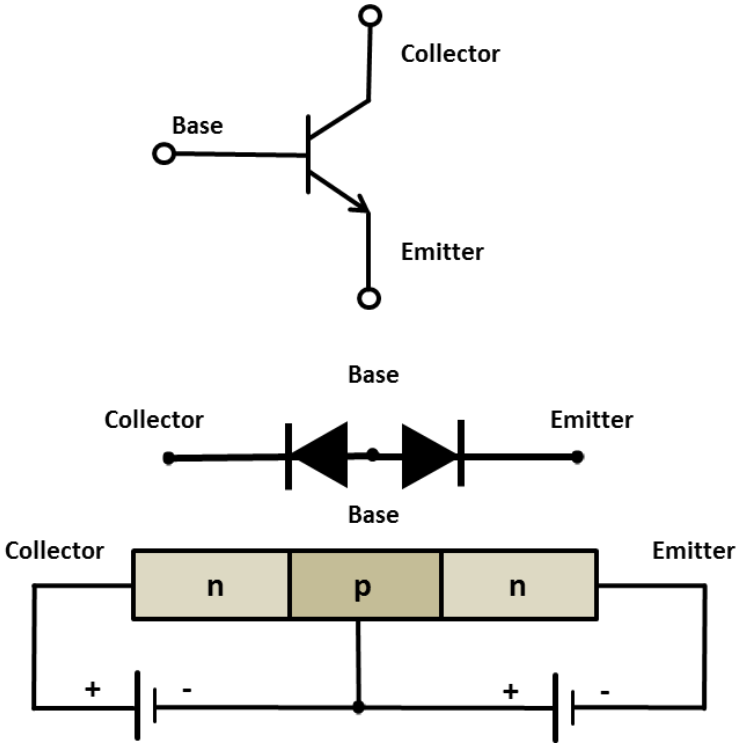


Figure 2.7: Bipolar (FET) junction, npn transistor

Unlike the diode, an npn transistor consists of three "doped" zones: (a) An n-doped segment that known as the "collector," (b) A center p-doped segment referred to as the "base," and (c) A second n-doped segment known as the "emitter." See the bottom section of Figure 2.7 for the relative location and the construction of an npn transistor. The interface lines where the collector base and the base to emitter regions meet are referred to as the n-p, collector to base junction, and the p-n, base to emitter junction, respectively. As shown in Figure 2.7, in a normal npn transistor application, the collector to base junction is reverse biased and the base to emitter junction is forward biased. As shown in Figure 2.7, common configuration and application of a transistor resembles a back to back connection of two diodes; the one on the left being reverse biased and the one on the right, forward biased.

The symbol used to represent a transistor in an electrical or electronic circuit is depicted in the top segment of Figure 2.7, with proper labeling of the three terminals of the transistors, in terms of collector, base and emitter.

Common application of FET, bipolar junction transistors

Application of a transistor in a typical "output module" of a PLC: A transistor can be applied as an "electronic switch" to execute a PLC microprocessor command in the field. This is illustrated in form of a transistor based PLC output, operating a solenoid valve, as shown in Figure 2.8. Applying ample positive bias to the left of R_1 will turn "on" the npn transistor, causing a "short" to ground. This permits the current to flow through the solenoid coil. Flow of current through the solenoid coil generates magnetic flux through the core of the coil, thus applying magnetic force on the "plunger" or "pin" in the core of the solenoid coil. Depending on the design of the solenoid, the magnetic force can either open or close the solenoid valve.

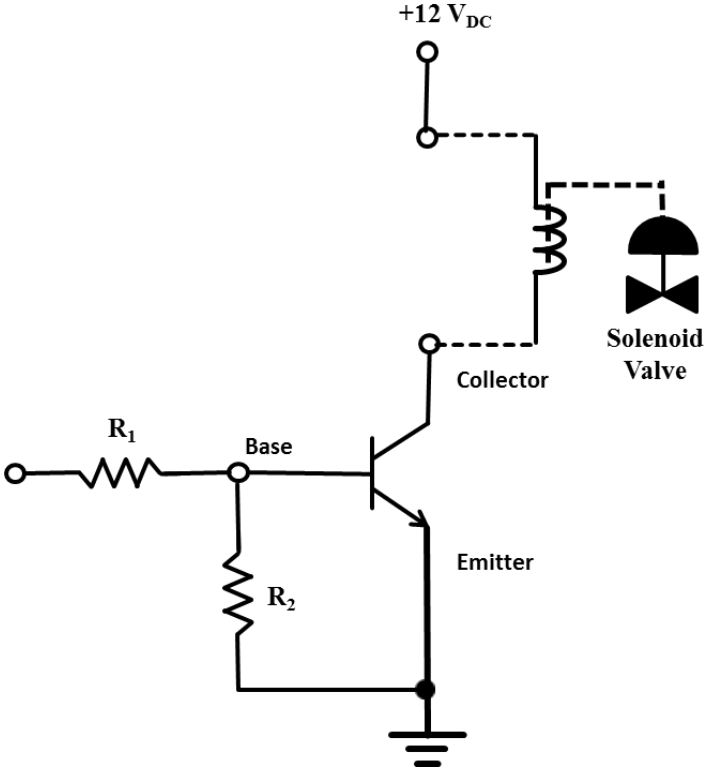


Figure 2.8: Bipolar (FET) junction application in a PLC output module

Electronic device applications on printed circuit boards

This section is designed to provide a brief introduction to a common printed circuit (PCB) board and an assortment of common electronic devices typically installed on a PCB to perform control, monitoring and data

collection functions. Some of the electronic and electrical devices identified on this board are explained in greater detail in the preceding and subsequent segments.

The devices pin pointed in Figure 2.9 are listed below, in clockwise order, beginning from the top right corner:

1. A toroid type ***inductor or coil*** applied for DC power refining and filtering purpose.
2. DC ***voltage regulator*** applied for voltage regulation purposes.
3. Solid state logic gate ***integrated circuit*** micro-chip applied to perform logic, algorithm and computations for control purposes. This type of IC consists of logic gates such as OR gates, NOR gates, Exclusive OR gates, AND gates, NAND gates, Flip Flops, etc.
4. A typical, low voltage electrical ***fuse*** for protection of the board and upstream power source against faults and shorts.
5. A ***transistor***, with collector, base and emitter pins visible to the right.
6. A ***diode***, labeled using the conventional "CR" prefix based nomenclature.
7. A typical "***proprietary" integrated circuit (IC)*** device. While the functional specifications of such devices are made available to electronic control engineers for application purposes, the contents and design of such devices are, typically, kept confidential and are not published.

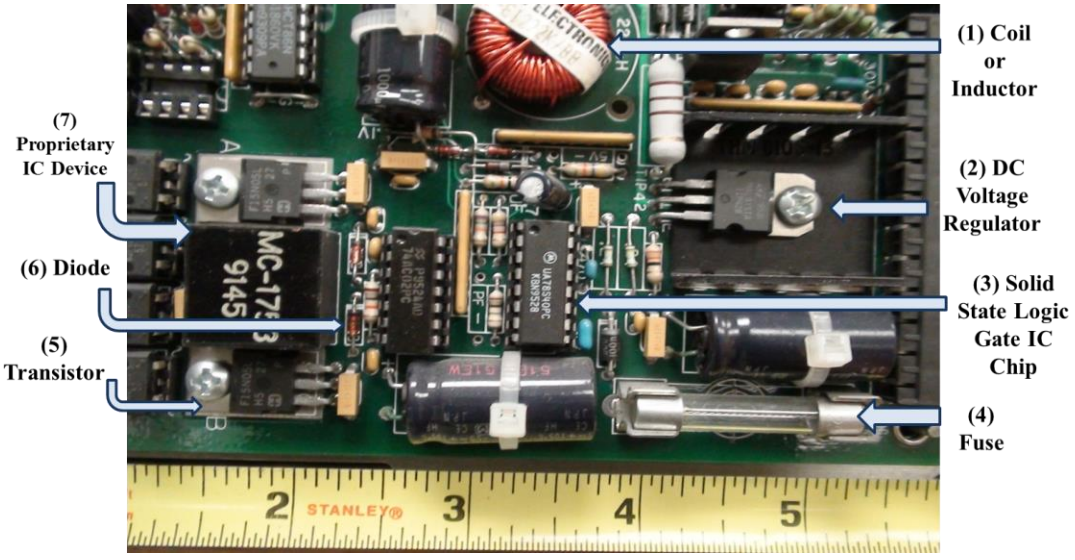


Figure 2.9: Electrical and electronic devices on a printed circuit board- I

Figure 2.10 depicts another segment of the same printed circuit control board. The devices noted in this segment of the control board, in clockwise order, beginning from the top right corner are:

1. Power transistor, npn (or NPN); with collector current rating of 30A at 40V, capable of switching at a maximum frequency 200kHz. Maximum power rating of 150W DC. The black heat dissipating fins surrounding the power transistor are designed to radiate waste heat and to protect the transistor against overheating.
2. A low wattage resistor with visible color coded bands identifying the resistance value and the tolerance specification of the resistor.
3. Higher wattage ceramic resistor. Notice the remarkable difference in the physical size and construction of this higher wattage and higher operating temperature ceramic resistor and its lower wattage counterpart described above. The ceramic resistor in Figure 2.10 is encased in a ceramic enclosure to withstand higher temperature and heat.
4. A capacitor.
5. A row of LED's designed to ***annunciate*** the ***state*** of various inputs **received** and outputs **transmitted** to electrical and electronic equipment in the field. Each of the output terminals represents a hardware based command from on-board controls out to the field.

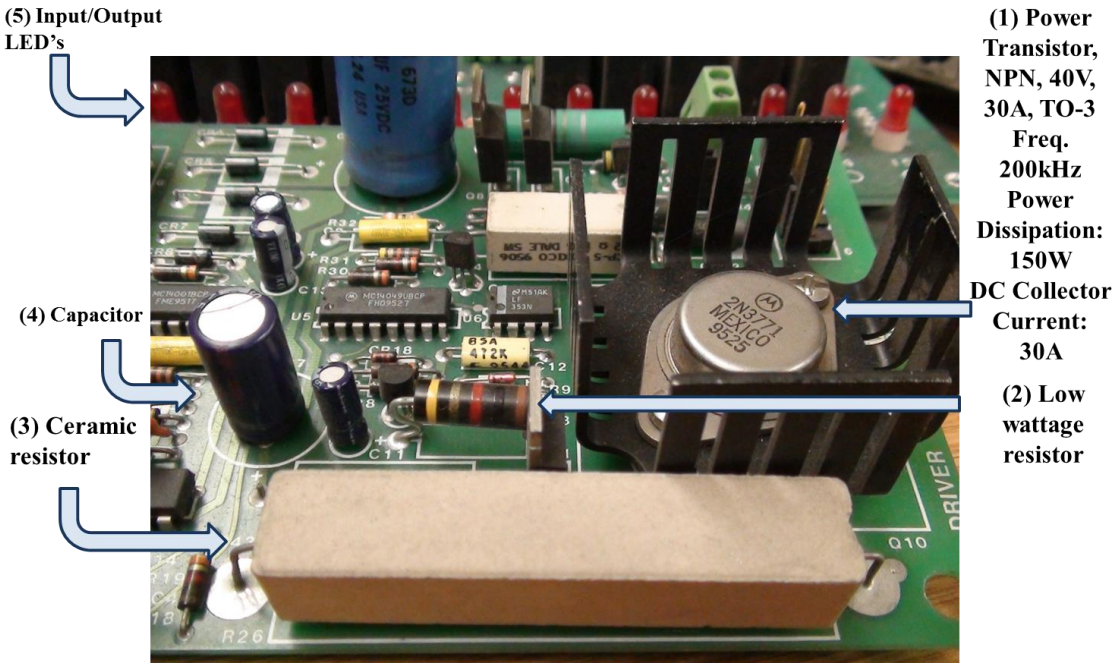


Figure 2.10: Electrical and electronic devices on a printed circuit board- II

An important feature that is instrumental in minimizing the time it takes to replace a control board, in case of board malfunction, is the "quick connect/disconnect" type terminal strip – labeled as (1) – shown on the right side to the control board (PCB), as shown in Figure 2.11. Prior to the advent of this quick connect/disconnect type terminal strip, board replacement required meticulous examination and care in reading the electrical drawings, identification of each wire and terminal, and finally, exercise of proper craftsmanship in termination of wires to ensure correct and reliable electrical connections. The lack of such diligence resulted in miswiring, electrical faults, extensive troubleshooting period, prolonged commissioning time, etc.

When disconnecting a control board, the quick connect/disconnect terminal strip, shown in Figure 2.11, allows technicians and engineers to simply pull off the connector, with reasonable certainty that wires will remain securely intact for quick reinstallation. Since all wires normally remain terminated in the connector, once the new (replacement) board is physically secured in place, the connector is pushed or plugged onto the connecting edge of the new board. Note that the circuit boards and respective connectors are, typically, equipped with an interlocking feature to prevent incorrect

orientation of the connector. As obvious, incorrect insertion of the connector onto the board can result in electrical faults and damage to the electrical devices.

The picture of the circuit board in Figure 2.11 also shows a set of two LED's, labeled as item (2). These LED's serve as indicators of certain control conditions or signals.

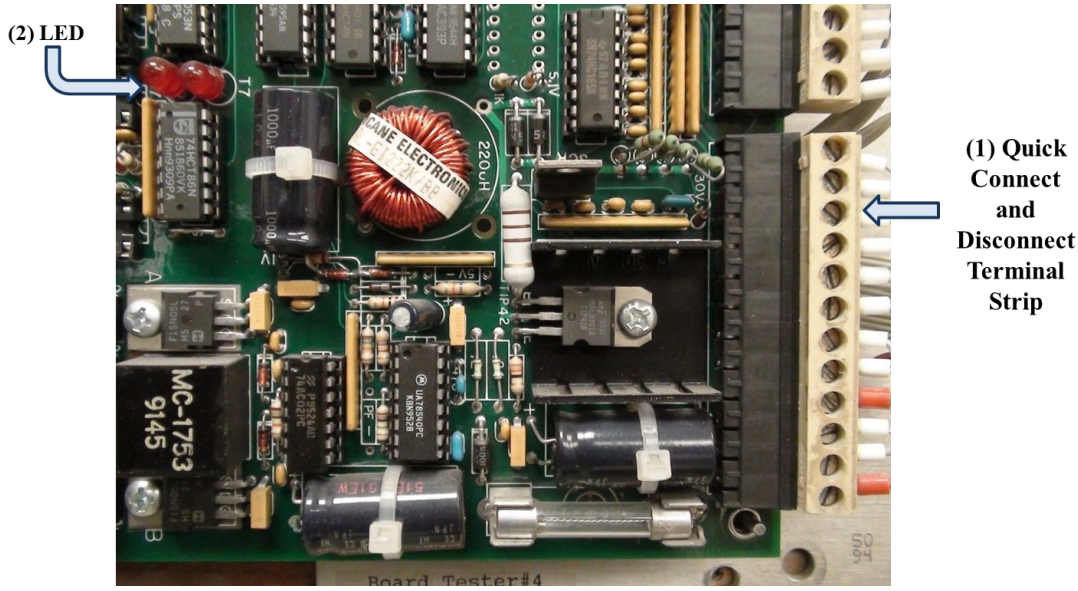


Figure 2.11: Electrical and electronic devices on a printed circuit board-III

Three devices pointed out on the circuit board pictured in Figure 2.12 are as follows:

1. A large size circuit board capacitor
2. Higher wattage ceramic resistor. Notice the legend on the right side of the resistor. Typically, this legend includes the resistance value and power capacity specifications.
3. Metal oxide varistor, rated 100J (100 Joules) and 1kVA. Metal oxide varistor, also referred to as MOV, serves as stray energy arresting device on electronic circuits and electrical systems, in general. In this capacity, an MOV serves to absorb voltage spikes and rogue energy that might otherwise spread around various parts of a circuit board or electrical

control system. Stray energy or voltage spikes, left unchecked, can damage integrated circuits, or IC semiconductor chips. IC's are relatively sensitive. They operate at low voltages, typically, at around 5 volts DC. The MOV in this particular case, as visible in the picture, is designed to absorb a maximum of 100 Joules of electrical energy. The "rate of absorption of this energy," or the power absorption capacity, is labeled on the MOV as 1kVA; to be exact, the apparent power (**S**) rating of this MOV is 1,000 volt amperes.

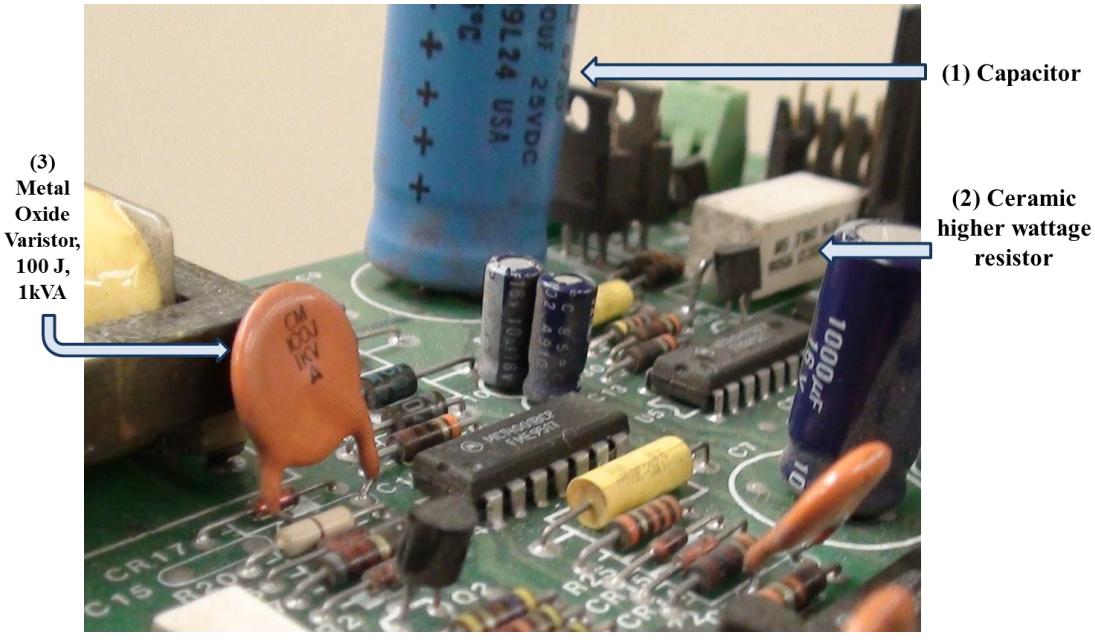
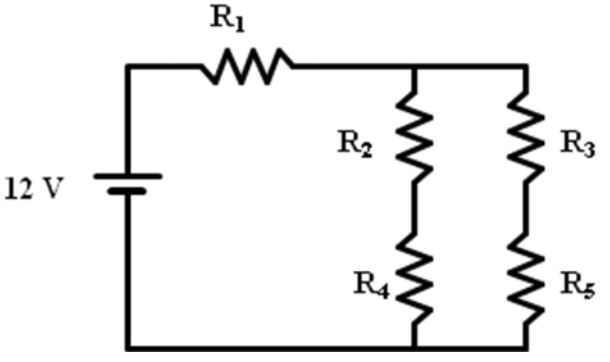


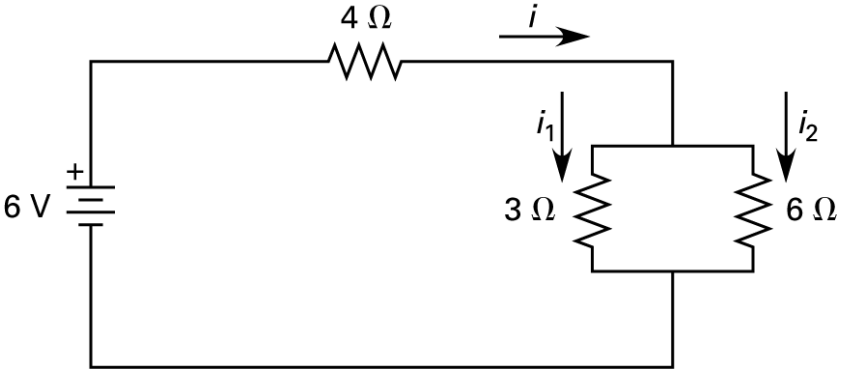
Figure 2.12: Electrical and electronic devices on a printed circuit board- IV

Self-assessment Problems & Questions – Segment 2

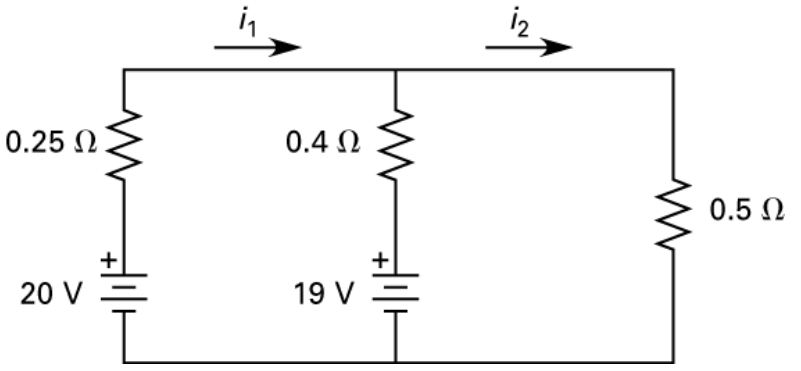
1. Determine the following for the DC circuit shown below if $R_1 = 5\Omega$, $R_2 = R_3 = 10\Omega$, and $R_4 = R_5 = 20\Omega$:
a) Current flowing through resistor R_1
b) Voltage across resistor R_5



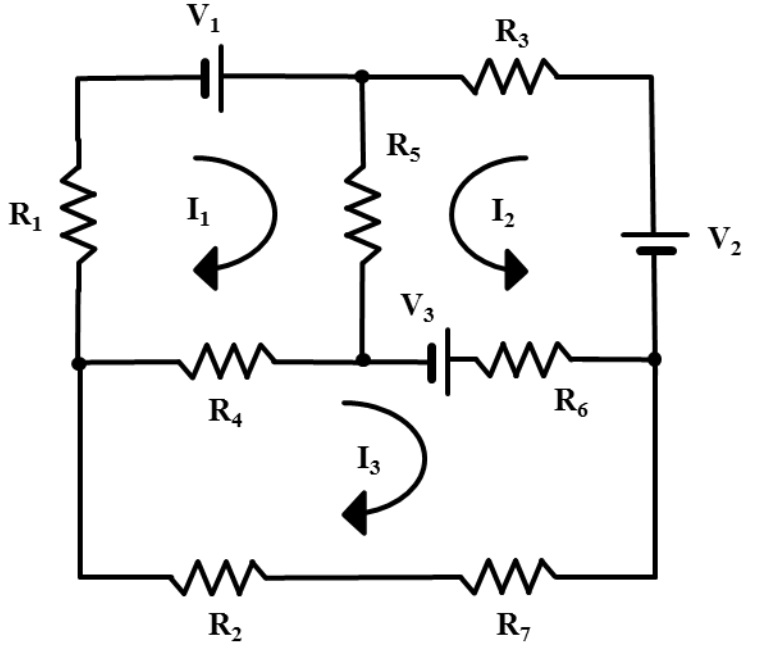
2. What is the current through the 6 Ω resistor in the circuit shown below?



3. Find the current through the 0.5 Ω resistor in the circuit shown below.



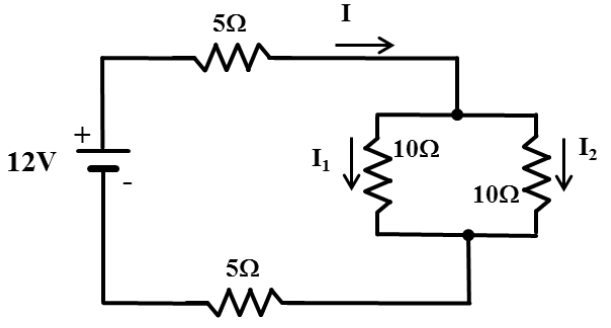
4. Determine the value of currents I_1, I_2 and I_3 in the circuit shown below if the voltage source V_3 fails in short circuit mode. The specifications of all components are listed in the table below:



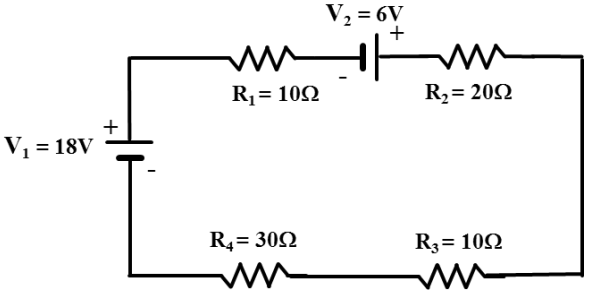
Resistor Specifications:

$\mathbf{R_1}$	10Ω
$\mathbf{R_2}$	2Ω
$\mathbf{R_3}$	3Ω
$\mathbf{R_4}$	4Ω
$\mathbf{R_5}$	7Ω
$\mathbf{R_6}$	3Ω
$\mathbf{R_7}$	5Ω
$\mathbf{V_1}$	20
$\mathbf{V_2}$	5
$\mathbf{V_3}$	12

5. Use current division to determine the value of current I_1 in the circuit below:

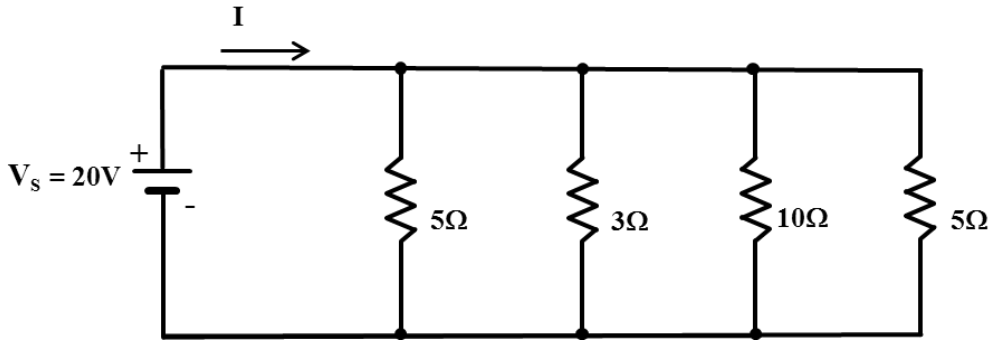


6. Using Kirchhoff's voltage law, calculate the current circulating in the series resistor network below:



7. Determine the value of voltage source ***current*** in the parallel circuit below using KCL, Kirchhoff's Current Law.

Ancillary question: If one of the 5Ω resistors is removed (or replaced with an open circuit) and the other one is replaced with a short circuit, what would be the source current?



APPENDICES

Appendix A
Solutions for Self-Assessment Problems

This appendix includes the solutions and answers to end of segment self-assessment problems and questions.

MADE AVAILABLE UPON PURCHASE OF COURSE

Appendix B
Common Units and Unit Conversion Factors

MADE AVAILABLE UPON PURCHASE OF COURSE

Appendix C
Greek Symbols Commonly Used in Electrical Engineering

MADE AVAILABLE UPON PURCHASE OF COURSE