5
Gas/Liquid Oxygen Transfer and Stripping

5.1
Transport by Diffusion

Oxygen is needed for all aerobic biological processes. Usually, it consumption rate depends on oxygen mass transfer[1]. If we wish to increase the rate of these processes, we must increase the gas/liquid interfacial area and the rate of the specific mass transfer, which consists of diffusion and convection. The specific mass transport rate of diffusion without convection is given by Fick's law:

$$J_D = -D \frac{dc'}{dx} \text{ mol m}^{-2}\text{ h}^{-1} \tag{5.1}$$

D is the diffusion coefficient. If c′ is the dissolved oxygen concentration in water, then $D = D_{O_2/H_2O}$ is the diffusion coefficient of oxygen in water. It can be calculated by using the Einstein equation (Daniels and Alberty 1955) or the Nernst-Einstein equation (Bird et al. 1962)

$$D = KT \frac{w_{O_2}}{F_{O_2}} \text{ m}^2\text{ h}^{-1} \tag{5.2}$$

where $K = 1.380\ 10^{-23}$ J K^{-1} (Boltzmann's constant), T is the temperature (K), w_{O_2} is the rate of diffusion of oxygen molecules (mol m h^{-1}) and F_{O_2} is the power of resistance of oxygen molecules (J mol m^{-1}).

F_{O_2} is obtained from Stokes' law for the motion influenced by high friction forces ($Re \ll 1$):

$$F_{O_2} = 6\,\pi\,\eta\,w_{O_2}\,R_{O_2} \text{ J mol m}^{-1} \tag{5.3}$$

where η is the dynamic viscosity of water (g m^{-1} h^{-1}) and R_{O_2} is the molecular radius of O_2 (m).

Introducing Eq. (5.3) in Eq. (5.2), this follows:

$$D = \frac{KT}{6\,\pi\,R_{O_2}\,\eta} \text{ m}^2\text{ h}^{-1} \tag{5.4}$$

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[1] Only for very low reaction rates is it not dependent on oxygen mass transfer.

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Wilke and Chang (Reid et al. 1987) calculated for T = 20 °C:

$$D_{20} = 6.48 \cdot 10^{-6}\ \mathrm{m^2\ h^{-1}}$$

and recommended that the influence of temperature should be taken into account using:

$$D_T = D_{20}\ \theta^{T-20} \tag{5.5}$$

with $\theta = 1.029$.

At the gas/liquid boundary, the specific transfer rate of oxygen in water calculated by using Eq. (5.1) is low if only diffusion is involved. This follows not only from the low diffusion coefficient, but also from the low solubility of oxygen in water:

$$c = H\ c^* \tag{5.6}$$

Equation (5.6) is called the Henry absorption equilibrium (Henry's law, see Eq. 4.46) and is valid up to oxygen pressures of nearly p = 10 bar ($c = p/RT$). H decreases with increasing temperature and concentration of chlorides (Table 5.1).

For 20 °C and deionized water H = 32.6 which means that, for an O_2 concentration in air of 298.7 mg L^{-1}, only 9.17 mg O_2 L^{-1} are dissolved in water. Therefore, oxygen transfer into water by diffusion is a slow process, due to the low diffusion coefficient and the low solubility (Fig. 5.1).

If we want to use oxygen for a chemical or biological process in water, we must: (a) use convection in addition to diffusion and (b) increase the gas/liquid interfacial area.

Furthermore, it may be necessary to use pure oxygen instead of air in some cases, resulting in a higher dissolved oxygen concentration at the boundary c^* by a factor of 4.8. But in our further discussion we will only consider the use of air.

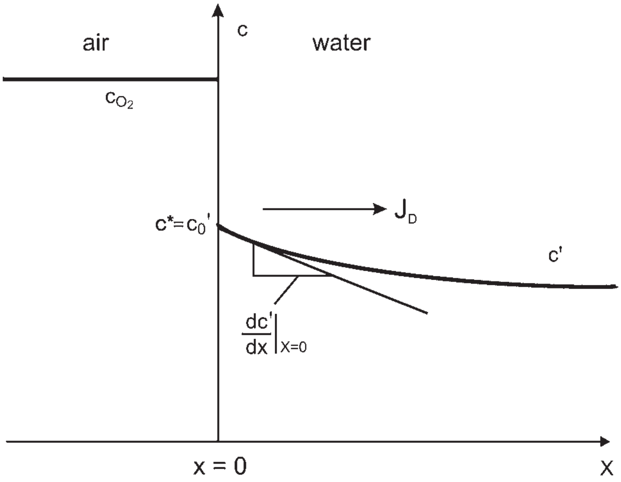

Fig. 5.1 Transport of oxygen from air into water only by diffusion.

Table 5.1 Dissolved oxygen concentration in water c′ for different temperatures and chloride concentrations; c = 20.9% O_2 in air; p_{air} = 760 bar (Chow et al. 1972).

Temperature (°C)	Chloride concentration (mg L^{-1})				
	0	5000	10 000	15 000	20 000
0	14.62	13.79	12.97	12.14	11.32
1	14.23	13.41	12.61	11.82	11.03
2	13.84	13.05	12.28	11.52	10.76
3	13.48	12.72	11.98	11.24	10.50
4	13.13	12.41	11.69	10.97	10.25
5	12.80	12.09	11.39	10.70	10.01
6	12.48	11.79	11.12	10.45	9.78
7	12.17	11.51	10.85	10.21	9.57
8	11.87	11.24	10.61	9.98	9.36
9	11.59	10.97	10.36	9.76	9.17
10	11.33	10.73	10.13	9.55	8.98
11	11.08	10.49	9.92	9.35	8.80
12	10.83	10.28	9.72	9.17	8.62
13	10.60	10.05	9.52	8.98	8.46
14	10.37	9.85	9.32	8.80	8.30
15	10.15	9.65	9.14	8.63	8.14
16	9.95	9.46	8.96	8.47	7.99
17	9.74	9.26	8.78	8.30	7.84
18	9.54	9.07	8.62	8.15	7.70
19	9.35	8.89	8.45	8.00	7.56
20	9.17	8.73	8.30	7.86	7.42
21	8.99	8.57	8.14	7.71	7.28
22	8.83	8.42	7.99	7.57	7.14
23	8.68	8.27	7.85	7.43	7.00
24	8.53	8.12	7.71	7.30	6.87
25	8.38	7.96	7.56	7.15	6.74
26	8.22	7.81	7.42	7.02	6.61
27	8.07	7.67	7.28	6.88	6.49
28	7.92	7.53	7.14	6.75	6.37
29	7.77	7.39	7.00	6.62	6.25
30	7.63	7.25	6.86	6.49	6.13

There are two possibilities to realize the conditions (a) and (b):

- We can produce small air bubbles, which rise to the surface of the water, causing an oxygen transfer into the water and to the suspended microorganisms.
- We can produce trickling films, which flow downwards at the surface of solids covered by biofilms.

In this chapter we will concentrate only on the transfer of oxygen from bubbles to water by diffusion and convection.

5.2
Mass Transfer Coefficients

5.2.1
Definition of Specific Mass Transfer Coefficients

In contrast to Fig. 5.1, where mass is only transferred by molecular motion (diffusion), we now want to augment it with convective motion (Fig. 5.2).

As a result of the high diffusion coefficient of oxygen in air, the concentration of oxygen in air remains locally constant as before. But in the water, the situation has changed completely: in addition to the molecular motion, total liquid mass is now moved by hydrodynamic forces, i.e. pressure, inertia and friction forces in the direct vicinity of a rising air bubble. This convective transport increases the local slope of the concentration at the boundary $dc'/dx|_{x=0}$ and the specific rate of mass transfer:

$$J_{D+C} = -D\frac{dc'}{dx|_{x=0}} \text{ mol } (m^2\, h)^{-1} \tag{5.7}$$

But the gradient is not measurable and can be calculated only for low liquid flow rates. In contrast, the concentration difference $c^* - c'$ can easily be measured (Fig. 5.2):

$$J_{D+C} = k_L (c^* - c') \text{ mol } (m^2\, h)^{-1} \tag{5.8}$$

where $k_L = \frac{D}{\delta}$ and δ is the thickness of the boundary (see Fig. 5.3).

The proportionality factor k_L in Eq. (5.8) depends on the diffusion coefficient D as well as the bubble velocity $\overline{w}$, the kinematic viscosity v and the surface tension σ,

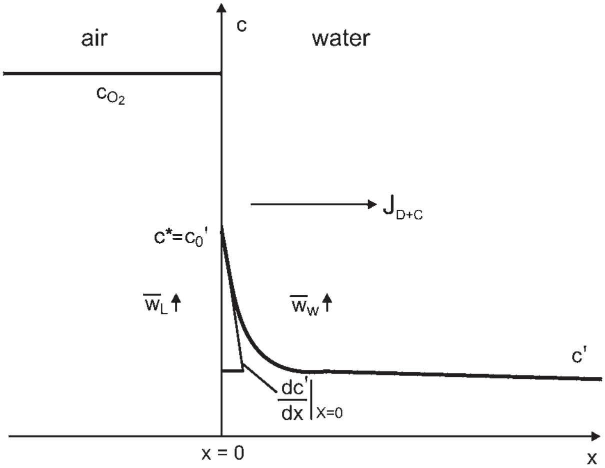

Fig. 5.2 Transport of oxygen from air into water by diffusion and convection.

for example. We cannot assume that we are able to measure the total boundary surface area A gas/liquid of all the bubbles:

$$J_{D+C}\,A = k_L\,A\,(c^* - c')\ \text{mol h}^{-1} \tag{5.9}$$

Applying Eq. (5.9) to the volume of the system V, the specific oxygenation capacity follows:

$$OC = J_{D+C}\,\frac{A}{V} = k_L a\ (c^* - c')\ \text{mol m}^{-3}\ \text{h}^{-1} \tag{5.10}$$

with k_La as the specific mass transfer coefficient.

k_La and c' are dependent on the temperature. For a temperature of 20 °C and $c' = 0$, we obtain the maximum specific standardized oxygenation capacity:

$$OC_{20} = (k_La)_{20} \cdot c^*_{20} \tag{5.11}$$

$(k_La)_{20}$ and c^*_{20} are dependent on the concentration of dissolved inorganic and organic substances. Applying the ratio of values effective for wastewater and clean water, we write:

$$\alpha_w = \frac{(k_La)_{20,w}}{(k_La)_{20,0}} \tag{5.12}$$

$$\beta_w = \frac{c'_{20,w}}{c'_{20,0}} \tag{5.13}$$

$$\gamma_w = \frac{(k_La)_{20,w}\,c'_{20,w}}{(k_La)_{20,0} \cdot c'_{20,0}} \tag{5.14}$$

reflecting the mass transfer coefficients for T = 20 °C gives:

$(k_La)_{20,w}$ for wastewater
$(k_La)_{20,0}$ for clean water

and concentrations of dissolved oxygen at the interface:

$c'_{20,w}$ for wastewater
$c'_{20,0}$ for clean water

Especially in industrial wastewater, different components may be dissolved which may desorb during the absorption of oxygen. Because of their low Henry coefficient H, a transport resistance occurs in both phases, in water *and* air. This case will be discussed in the next section.

5.2.2
Two Film Theory

Three models have been developed to improve the understanding of gas/liquid mass transfer: the surface renewal model of Higbie (1935) and Danckwerts (1951, 1970); the ‘still surface’ model of King (1964) and the ‘two film’ model of Whitman

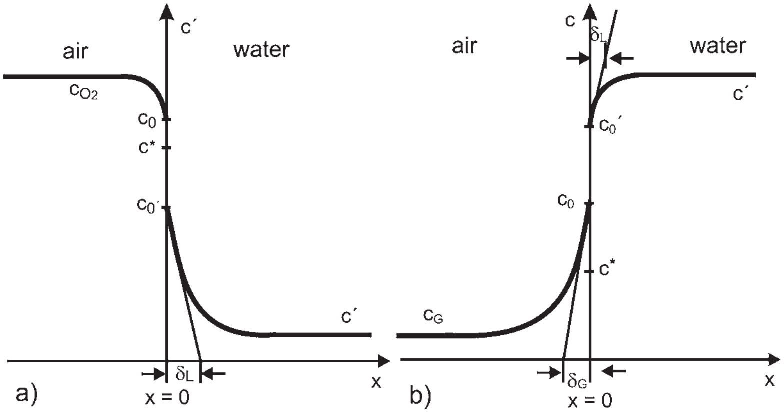

Fig. 5.3 Absorption and desorption of compounds dissolved in water with a low Henry coefficient, $H = c/c^* = c_0/c_0'$; (a) absorption, (b) desorption.

(1923). We will limit our discussion to the two film theory. A review of all theories is given by Danckwerts (1970).

Let us look at the mass transfer at the boundary water/air for desorption processes and for the case that two films are formed. The desorption of oxygen into an oxygen-free gas film, e.g. nitrogen, is a simple case which follows nearly directly from Fig. 5.2. But such equilibria, which are characterized by a low Henry coefficient, lead to concentration profiles in both water and air for absorption (Fig. 5.3a) and desorption (Fig. 5.3b).

The rate of desorption must now be calculated using the two film theory. Table 5.2 compiles some compounds with low H.

Note that in Fig. 5.3b the concentration c in the air bubble is much lower than c^*, the theoretical equilibrium concentration.

An overall mass transfer rate is given using K_La. If we define the specific overall mass transfer coefficient K_La by writing:

$$K_La\,(c' - c^*)$$

for the specific overall mass transfer rate, we can conclude that this rate is the same as in the liquid film:

$$K_La\,(c' - c^*) = k_La\,(c' - c_0') \tag{5.15}$$

which also agrees with the rate in the gas film:

$$k_La\,(c' - c_0') = k_Ga\,(c_0 - c_G) \tag{5.16}$$

with:

$$c_0' = \frac{1}{H}\,c_0 \tag{5.17}$$

Table 5.2 Solubility and Henry coefficient of different compounds (Nyer 1992, supplemented), Henry's law:

$$c^* = \frac{c}{H} \text{ and } c^* = \frac{P\,c_w}{K_P};\; H = \frac{K_P}{RT\,c_w}$$

Compound	Maximal solubility (c^*_{max}, mg L^{-1})	Henry coefficient (K_P, bar)	Henry coefficient (H, –)
1 Acetone	$1 \cdot 10^6$	0	0
2 Benzene	$1.75 \cdot 10^3$	230	0.1690
3 Bromodichloromethane	$4.4 \cdot 10^3$	127	0.0934
4 Bromoform	$3.01 \cdot 10^3$	35	0.0257
5 Carbon tetrachloride	$7.57 \cdot 10^2$	1282	0.943
6 Chlorobenzene	$4.66 \cdot 10^2$	145	0.107
7 Chloroform	$8.2 \cdot 10^3$	171	0.126
8 2-Chlorophenol	$2.9 \cdot 10^4$	0.93	$0.684 \cdot 10^{-3}$
9 p-Dichlorobenzene (1,4)	$7.9 \cdot 10^1$	104	0.0765
10 1,1-Dichloroethane	$5.5 \cdot 10^3$	240	0.176
11 1,2-Dichloroethane	$8.52 \cdot 10^3$	51	0.0375
12 1,1-Dichloroethylene	$2.25 \cdot 10^3$	1841	1.353
13 cis-1,2-Dichloroethylene	$3.5 \cdot 10^3$	160	0.117
14 trans-1,2-Dichloroethylene	$6.3 \cdot 10^3$	429	0.315
15 Ethylbenzene	$1.52 \cdot 10^2$	359	0.264
16 Hexachlorobenzene	$6 \cdot 10^{-3}$	37.8	0.0278
17 Methylene chloride	$2 \cdot 10^4$	89	0.0654
18 Methylethylketone	$2.68 \cdot 10^5$	1.16	$0.853 \cdot 10^{-3}$
19 Methyl naphthalene	$2.54 \cdot 10^1$	3.2	$0.235 \cdot 10^{-3}$
20 Methyl tert-butyl-ether	4.8	196	0.144
21 Naphthalene	$3.2 \cdot 10^1$	20	0.0147
22 Pentachlorophenol	$1.4 \cdot 10^1$	0.15	$0.1100 \cdot 10^{-3}$
23 Phenol	$9.3 \cdot 10^4$	0.017	$0.0125 \cdot 10^{-3}$
24 Tetrachloroethylene	$1.5 \cdot 10^2$	1035	0.761
25 Toluene	$5.35 \cdot 10^2$	217	0.160
26 1,1,1-Trichloroethane	$1.5 \cdot 10^3$	390	0.287
27 1,1,2-Trichloroethane	$4.5 \cdot 10^3$	41	0.030
28 Trichloroethylene	$1.1 \cdot 10^3$	544	0.400
29 Vinyl chloride	$2.67 \cdot 10^3$	355000	261.0
30 o-Xylene	$1.75 \cdot 10^2$	266	0.196

and:

$$c^* = \frac{1}{H}\,c_G \tag{5.18}$$

c'_0 follows from Eqs. (5.15) to (5.18). c_G is the concentration of the desorbed compound in the air and c^* is its equilibrium value at the interface:

$$c'_0 = \frac{c' + \dfrac{k_G a}{k_L a} H c^*}{1 + \dfrac{k_G a}{k_L a} H} \tag{5.19}$$

After introducing Eq. (5.19) into Eq. (5.15), we obtain:

$$\underset{\substack{\text{overall}\\\text{resistance}}}{\frac{1}{K_L a}} = \underset{\substack{\text{resistance of}\\\text{the air film}}}{\frac{1}{k_G a\, H}} + \underset{\substack{\text{resistance of}\\\text{the water film}}}{\frac{1}{k_L a}} \tag{5.20}$$

Note that:

- This simple equation is only applicable if we use $c' - c^*$ as the driving concentration difference to define the specific overall transfer coefficient $K_L a$.
- The resistance of the air film increases with decreasing Henry coefficient H, i.e. higher dissolution in water at lower concentration in air.
- The influence of H is increased at low $k_G a$ and high $k_L a$.

Looking at Table 5.2 and assuming similar values for $k_G a$ and $k_L a$, mass transfer resistance lies nearly completely on the air film side for compounds with low H (2-chlorophenol, methyl ethyl ketone, phenol and others) and nearly completely on the water film side for vinyl chloride with its very high H.

Although H = 32.6 (T = 20 °C) for oxygen/water (clean water) is relatively high (Section 5.1) and mass transfer is controlled for normal conditions by the resistance of the water film, we want to use the specific overall mass transfer coefficient $K_L a$ as well.

5.3
Measurement of Specific Overall Mass Transfer Coefficients $K_L a$

5.3.1
Absorption of Oxygen During Aeration

5.3.1.1 Steady State Method

The absorption of oxygen will first be discussed for a *c*ompletely *s*tirred *t*ank *r*eactor (CSTR) filled with clean water and equipped with a perforated annular tube below the stirrer (Fig. 5.4).

Sodium sulfite, Na_2SO_3, is dissolved in the clean water together with a cobalt catalyst. Sulfite ions oxidize immediately, consuming the dissolved oxygen:

$$SO_3^{2-} + \frac{1}{2} O_2 \xrightarrow{\text{catalyst}} SO_4^{2-} \tag{5.21}$$

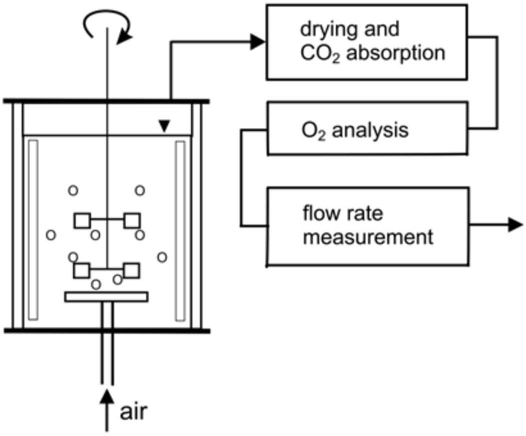

Fig. 5.4 Installation for measuring the mass transfer coefficient in clean water or wastewater without activated sludge.

Therefore, c′ is zero until the dissolved SO_3^{2-} is totally oxidized to SO_4^{2-}. If we want to measure in nearly salt-free water after aeration, only a small amount of Na_2SO_3 should be added – just enough for a few experiments. The exhaust air with a low O_2 concentration flows through absorption systems for drying and CO_2 removal before entering on-line oxygen gas analyzers and flow rate measurement devices.

The steady state oxygen balance in the completely mixed gas is:

$$0 = Q_G\, c_{in} - Q_G\, c_{out} - K_L a\, c^*_{out}\, V \tag{5.22}$$

where V is the water volume, Q_G is the gas flow rate and $c^*_{out} = \frac{c_{out}}{H}$ is the saturated dissolved oxygen concentration.

Solving Eq. (5.22) for $K_L a$ gives:

$$K_L a = \frac{Q_G\,(c_{in} - c_{out})\, H}{V\, c_{out}} \tag{5.23}$$

In larger aerated reactors, the O_2 gas concentration is often not completely mixed and better results can be obtained if an arithmetic mean value is used for:

$$\bar{c}^* = \frac{c^*_{in} + c^*_{out}}{2} \tag{5.24}$$

This relatively small simplification is only allowed for small decreases in the percent of oxygen volume inside the air bubbles, e.g. from 20.9 to 19.0%. For a greater decrease, a more complicated evaluation is necessary. Better results follow if a logarithmic mean value is used.

5.3.1.2 **Non-steady State Method**

Only a low amount of Na_2SO_3 is added to the water (clean water or wastewater free of activated sludge) using the dissolved O_2 for oxidation. After the Na_2SO_3 is completely oxidized, c′ increases. It is measured by one or several O_2 electrodes, which are combined with a computer for direct evaluation. For this purpose, the O_2 dissolved in the completely mixed water must be balanced:

$$\frac{dc'}{dt} = K_L a\,(c^* - c') \tag{5.25}$$

Considering the initial condition:

$$t = 0 \qquad c' = 0$$

the solution of Eq. (5.25) gives:

$$\ln \frac{c^* - c'}{c^*} = - K_L a\, t \tag{5.26}$$

A plot of ln $(c^* - c')/c^*$ versus t must give a straight line, if the assumption of a completely mixed system is to be met. Frequently, a better linear plot is obtained using an arithmetic mean value for c^*. Equation (5.26) should go over at least 1.5 decades of $\left(1 - \frac{c'}{c^*}\right)$.

5.3.1.3 Dynamic Method in Wastewater Mixed with Activated Sludge

The specific overall mass transfer coefficients K_La can be influenced not only by temperature and pressure (bubble diameter) but also by dissolved and suspended matter, such as minerals and activated sludge (Eqs. 5.12 to 5.14). Therefore, it is necessary to measure $(K_L a)_{20,w}$ in an activated sludge system which can be treated as a CSTR in continuous flow or in batch operation. We will describe this method for a batch system. After a period of continuous operation (Fig. 5.5, period I) and after an input of a given bacteria mass, the flow of wastewater and air is stopped, without reducing the stirrer speed to maintain the activated sludge in suspension. During period II, the concentration of dissolved oxygen c′ is decreased by respira-

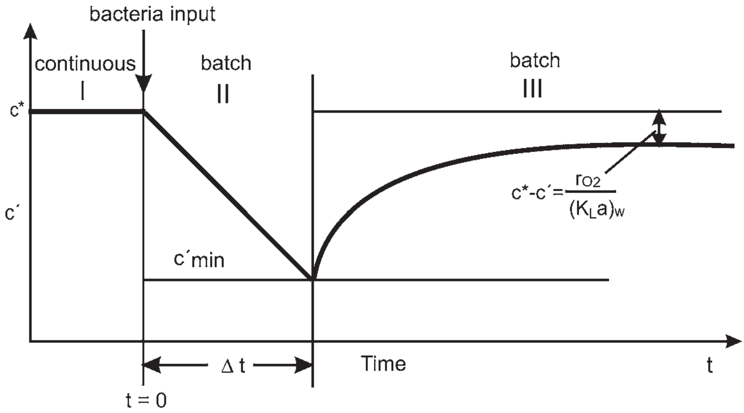

Fig. 5.5 Non-steady state method for measuring the mass transfer coefficients in wastewater with activated sludge.

tion. Period II is finished as soon as $c'_{min} \approx 2$ mg L^{-1} is reached. In the region of lower c' values, respiration rate may be limited by oxygen. Now aeration resumes and c' increases up to a constant value (period III).

The balance for dissolved oxygen in period II and the initial condition are:

$$\frac{dc'}{dt} = -r_{O_2} \tag{5.27}$$

$$\text{at} \quad t = 0 \quad c' = c^*$$

The respiration rate follows as:

$$r_{O_2} = \frac{c^* - c'_{min}}{\Delta t} \tag{5.28}$$

The balance for period III and the initial condition are:

$$\frac{dc'}{dt} = -r_{O_2} + (K_L a)_w (c^* - c') \tag{5.29}$$

$$\text{at} \quad t = 0 \quad c' = c'_{min}$$

Equation (5.29) can be solved by the separation of variables. After integration and consideration of the initial condition:

$$\ln\left[\frac{(K_L a)_w c^* - r_{O_2} - (K_L a)_w c'}{(K_L a)_w c^* - r_{O_2} - (K_L a)_w c'_{min}}\right] = -(K_L a)_w t$$

can be written, and finally:

$$c^* - c' = \left(c^* - c'_{min} - \frac{r_{O_2}}{(K_L a)_w}\right) \exp\left[-(K_L a)_w t\right] + \frac{r_{O_2}}{(K_L a)_w} \tag{5.30}$$

For higher values of time t:

$$c^* - c' \approx \frac{r_{O_2}}{(K_L a)_w}$$

and $(K_L a)_w$ can be calculated.

The experimental curve in period III is described in the best possible way obtaining a more exact value for $(K_L a)_w$. By comparing this value $(K_L a)_w$ to $K_L a$ measured in clean water, both at 20 °C, α_w can be determined (see Eq. 5.12).

5.3.2
Desorption of Volatile Components During Aeration

Dissolved volatile organic components (VOC) can be divided into two groups, according to their Henry coefficient H (see Table 5.2).

For $H \gg 1$ (e.g. oxygen/water), overall mass transfer resistance is given by liquid mass transfer:

$$\frac{1}{K_L a} = \frac{1}{K_G a\, H} + \frac{1}{k_L a} \tag{5.20}$$

$$K_L a \approx k_L a$$

and usually the desorption capacity for T = 20 °C follows from (see Fig. 5.3b):

$$(DC)_{20} = k_L a\,(c' - c'_0) \tag{5.31}$$

If the desorbing component has nearly the same diffusion coefficient as oxygen, the same $k_L a$ can be used. For molar masses of individual VOCs different from oxygen, it is most effective to use a corrected $k_L a$:

$$(DC)_{20} = k_L a \left(\frac{D_{VOC}}{D_{O_2}}\right)^n (c' - c'_0) \tag{5.32}$$

D_{VOC} can be measured or calculated using Eq. (5.4). The exponent n is not easily obtainable and depends on the mixing equipment.

For $H < 10$, the overall mass transfer resistance is normally given by liquid and gas mass transfer rates using the film model (Fig. 5.3 where $H < 1$). In this case, the desorption capacity at T = 20 °C is:

$$(DC)_{20} = K_L a\,(c' - c^*) \tag{5.33}$$

with:

$$K_L a = \frac{k_G a\, H \cdot k_L a}{k_G a\, H + k_L a} = \frac{k_L a}{1 + \dfrac{k_L a}{k_G a\, H}} \tag{5.34}$$

if the diffusion coefficient of the desorbing component is nearly the same as that of oxygen. Otherwise, Eqs. (5.33) and (5.34) must be corrected in a similar way to Eq. (5.32):

$$K_L a = \frac{k_L a \left(\dfrac{D_{VOC}}{D_{O_2}}\right)^n \cdot k_G a \left(\dfrac{D_{VOC}}{D_{O_2}}\right)^m H}{k_G a \left(\dfrac{D_{VOC}}{D_{O_2}}\right)^m H + k_L a \left(\dfrac{D_{VOC}}{D_{O_2}}\right)^n} \tag{5.35}$$

From Eq. (5.20), follows:

$$\frac{H}{K_L a} = \frac{1}{k_G a} + \frac{H}{k_L a} \tag{5.36}$$

For at least three different components with different Henry coefficients, $H/K_L a$ can be measured in the same mixing system at exactly the same conditions (stirrer speed, flow rate). A straight line must go through the date plotted as $H/K_L a$ versus H. The resulting value of $k_L a$ follows from the slope and $k_G a$ from the ordinate intercept.

Only the results of a few studies are available (Libra 1993) to date.

5.4
Oxygen Transfer Rate, Energy Consumption and Efficiency in Large-scale Plants

5.4.1
Surface Aeration

5.4.1.1 Oxygen Transfer Rate

Principally, the same methods are used for measuring overall specific mass transfer coefficients $K_L a$ as described before in Section 5.3, but often the following terms are used:

- OTR oxygenation transfer rate (ASCE 1997: Standard guidelines for in-process oxygen transfer testing).
- OC oxygenation capacity (ATV 1997: Guidelines according to ATV Arbeitsblatt M 209).

We select OTR, which is related to reactor volume V and is calculated from (see Fig. 5.3a):

$$\mathrm{OTR} = \frac{Q_G\,(c_{in} - c_{out})}{V} = K_L a\,(c^* - c')^{1)} \qquad (5.37)$$

However, the air flow rate Q_G as well as the oxygen concentration of the escaping air is normally not measurable. Therefore, the only way to measure $K_L a$ is by using data obtained from absorption measurements and the balances for oxygen absorption described in Section 5.3.1.2 (see Eq. (5.27) for period II and Eq. (5.29) for period III).

One problem must be noted: in Section 5.3 the reactors used for laboratory or pilot experiments have a small volume of several liters to 1–2 m^3; and, regardless of the aeration method (volume or surface aeration), a nearly completely mixed gas and liquid phase cannot be guaranteed. Large-scale basins of several hundreds to several thousands of cubic meters cannot be completely mixed by surface aerators, such as centrifugal aerators. Therefore $K_L a$ as well as c' are different at several points of a “mixed” tank with one or more aerators.

Nevertheless, the only way to obtain information about OTR in large-scale basins is to evaluate the measurements with Eq. (5.25) for clean water without sludge or Eqs. (5.27) and (5.29) for wastewater with sludge.

Three measures are necessary to standardize OTR:

- Use a fixed temperature (in Europe T = 20 °C).
- Limit the maximum value for the difference of concentration.
- Use clean water (index 0).

For the standardized oxygen transfer rate we write:

$$\mathrm{SOTR} = (K_L a)_{20,0}\, c^*_{20,0} \qquad (5.38)$$

---

1) Remember: Only for this definition of $K_L a$, it is not influenced by concentration; see Eq. (5.20).

For a measurement in wastewater with or without sludge, we obtain:

$$\mathrm{OTR}_{20,\mathrm{w}} = (K_L a)_{20}\, c^*_{20} \tag{5.39}$$

$$\alpha = \frac{(K_L a)_{20}}{(K_L a)_{20,0}} \tag{5.40}$$

Figure 5.6 shows a Simplex aerator. Different values of $K_L a$ can be selected via the rotation speed and depth of submersion. Frequently, a transmission makes three speeds possible.

For a Simplex aerator with the dimensions presented in Fig. 5.6, the standardized oxygen transfer rate (SOTR) is plotted versus power consumption P/V (see Fig. 5.7, in the next section).

We will now consider a surface aerator and the power required to achieve a desired oxygen transfer rate.

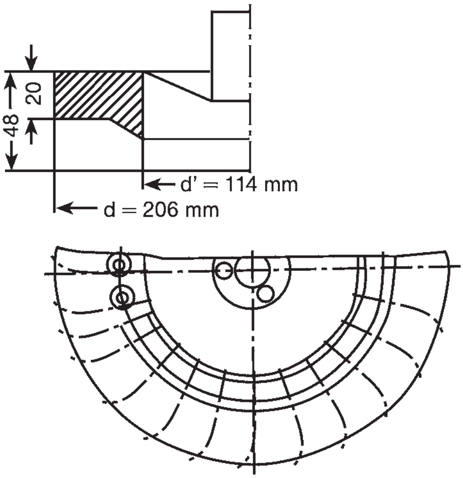

Fig. 5.6 Simplex aerator, number of blades = 24 (Zlokarnik 1979).

5.4.1.2 Power Consumption and Efficiency

The power consumption of a rotating turbine powered by an electric motor is usually given as the total power consumption P_t:

$$P_t = \frac{P}{\eta_m \cdot \eta_g} \tag{5.41}$$

where η_m is the efficiency of the electric motor and η_g is the efficiency of the transmission.

If we compare different kinds of aerators, we must use the effective power:

$$P = P_t\, \eta_m\, \eta_g \tag{5.42}$$

considering the measurable P_t:

$$P_t = \sqrt{3}\, U \cdot I \cos\varphi / (\eta_m \eta_g) \tag{5.43}$$

and the values for e.g. $\eta_m = 0.93$ and $\eta_g = 0.93$, or those obtained from the manufacturer (Zlokarnik 1979).

The efficiency of the aerator follows as:

$$E = \frac{\text{OTR}}{P/V} \tag{5.44}$$

The total efficiency, which cannot be used in comparisons because of different η_m and η_g, is (see Eq. 5.37):

$$E_t = \frac{\text{OTR}}{P_t/V} = \frac{K_L a\,(c^* - c')}{P_t/V} \tag{5.45}$$

E_t provides information about the total energy needed for the aeration of real wastewater with a given system at the effective conditions.

If we wish to compare this system with others with different temperatures and kinds of wastewater, we must use the standardized oxygen transfer rate OTR_{20} = SOTR (see Eq. 5.38) and the power P without considering the energy loss from the motor and the transmission:

$$E_n = \frac{\text{OTR}_{20,0}}{P/V} = \frac{\text{SOTR}}{P/V} \tag{5.46}$$

and we have to measure in clean water.

Some results for OTR, P/V and E are presented in Fig. 5.7 (Zlokarnik 1979).

In spite of the scattering of the measured data, it can be concluded from Fig. 5.7 that for specific powers $P/V = 15\ \text{kW m}^{-3}$ and a $\text{SOTR} = 45\ \text{kg O}_2\ (\text{m}^3\,\text{h})^{-1}$ an efficiency of $E \approx 3\ \text{kg O}_2\ (\text{kWh})^{-1}$ may be attained. However, the total electrical power

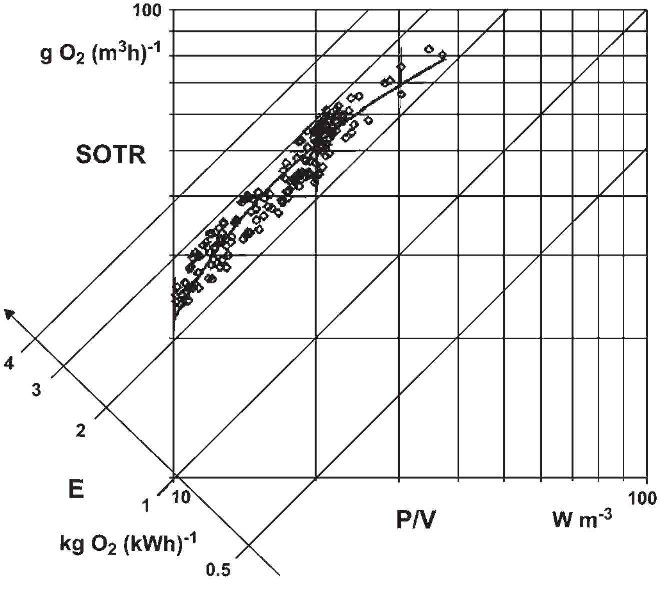

Fig. 5.7 Standardized oxygen transfer rate, power consumption and efficiency of a Simplex aerator (Schuchardt 2005, data from Zlokarnic 1979).

P_t which must be supplied is higher by the factor $1/\eta_m\ \eta_g = 1.16$, resulting in a lower efficiency of at most:

$$E_t = 2.6 \text{ kg } O_2 \text{ (kWh)}^{-1}$$

In reality $c' = 1–4$ mg L^{-1} and E_t is 10 to 40% lower for e.g. 15 °C water temperature (Eq. 5.45 and Table 5.1). We must not forget that these measurements were carried out in a very small test unit of only 100 L. It is not permissible to transfer these results to a larger tank with a larger aerator without further intermediate studies.

The question should be: can we use these results for a large-scale plant with the same type of aerator if we take the theory of similarity into account?

We will return to this question in Section 5.5.

5.4.2
Deep Tank Aeration

5.4.2.1 Preliminary Remarks

The aerators mostly used in deep tanks are tubes or plates made of porous ceramics or synthetic materials. We refer to these reactors as deep tanks if the water depth is greater than H = 6 m. Existing tanks frequently have a depth of 4–6 m, but already 6 years ago activated sludge reactors with H = 8–12 m were planned, constructed and commissioned (Pöpel et al. 1998). Reactors with L = 15–25 m have been built for the treatment of industrial effluents (Turmbiologie, Bio-Hochreaktor, deep-shaft reactor). Although most reactors are dug into the ground, some have been built above ground; we speak of “deep tanks” in both cases and use L to indicate the water depth (Turmbiologie, Bayer 1991; Bio-Hochreaktor, Leistner et al. 1979; deep-shaft reactor, Lock 1982).

Let us study mass transfer in deep tanks only in such systems which are approximately characterized by *completely mixed water* and *completely unmixed oxygen* in the rising air bubbles (plug flow). Reactors with L>12 m may be excluded, because nearly complete mixing of the water is not to be expected.

Mass transfer in bubble columns is a complicated process:

1. Oxygen concentration inside the rising bubbles decreases, resulting in a high $\Delta c'$ at the beginning and a lower $\Delta c'$ at the end before the bubbles reach the water surface.

2. The pressure caused by the water column decreases with decreasing depth, which has an effect on both:
   (a) the gas concentration of oxygen and inert components inside the bubble and
   (b) the bubble diameter as well as the velocity of the rising bubble, consequently the specific overall mass transfer coefficient K_La.

3. As the result of the mass transfer of oxygen into the water and the mass transfer of CO_2 into the bubbles, the molar fraction of O_2 changes in a complicated way and the concentration difference between dissolved and gaseous O_2 is influenced.

Let us discuss these influences on mass transfer separately.

5.4.2.2 **The Simple Plug Flow Model**

The situation is described in Fig. 5.8: all bubbles rise with the same velocity $\bar{w}$, which is not easy to measure. Therefore, the average void velocity is used:

$$\bar{w} = \frac{Q_G}{A_R} \tag{5.47}$$

with A_R as the cross-sectional area of the reactor (bubble column). We can balance the gaseous oxygen in the total volume of the bubble column; and it is not necessary to determine the total gas volume, which would be nearly impossible in large open tanks.

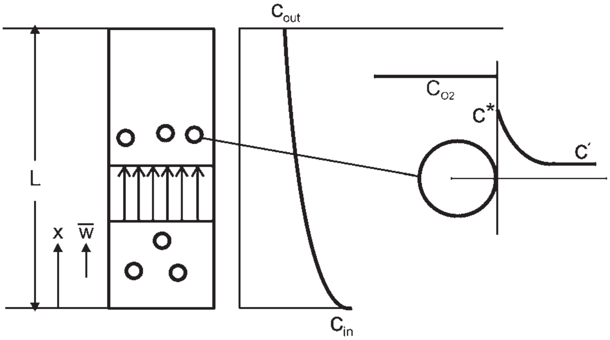

Fig. 5.8 Upward plug flow of water and flowing bubbles with corresponding diameters.

This steady-state experiment can be carried out:

- with clean water and Na_2SO_3, resulting in an oxygen-free liquid system,
- with real wastewater with or without activated sludge.

Let us discuss the first case. For the steady-state oxygen gas balance, we write:

$$0 = -\bar{w}\frac{dc}{dx} - K_L a\, c^* \tag{5.48}$$

and considering Henry's law:

$$c = H\, c^* \tag{5.6}$$

as well as the boundary condition:

$$x = 0 \quad c = c_{in}$$

the result for the concentration profile is:

$$\ln \frac{c}{c_{in}} = -\frac{K_L a}{H\,\bar{w}}\,x \tag{5.49}$$

Frequently, a mean O_2 concentration of the bubbles $\bar{c}$ is calculated using:

$$\bar{c} = \int_{c_{in}}^{c_{out}} \ln \frac{c}{c_{in}}\, dc = \frac{c_{in} - c_{out}}{\ln \frac{c_{in}}{c_{out}}} \tag{5.50}$$

which is introduced into the integral balance:

$$Q_G\,(c_{in} - c_{out}) = K_L a\, c^*\, V = K_L a\, \frac{\bar{c}}{H}\, V \tag{5.51}$$

Instead of Eq. (5.50), we need to write for the case of a finite dissolved oxygen concentration ($c' \neq 0$):

$$Q_G\,(c_{in} - c_{out}) = K_L a \cdot \left(\frac{\bar{c}}{H} - c'\right) V \tag{5.52}$$

$$\text{with} \quad \bar{c} = \frac{c_{in} - c_{out}}{\ln \frac{c_{in} - c}{c_{out} - c}} \tag{5.53}$$

Equations (5.51) and (5.52) are approximations for two reasons:

- A totally mixed liquid system cannot be passed through by gas bubbles flowing with the same velocity and exhibit a detectable decrease in the oxygen concentration.
- A plug-style flow of bubbles cannot be realized precisely when using a swarm of bubbles.

In wastewater technology, this plug flow model is used only seldom. Instead of an exponential decrease in oxygen concentration inside the bubbles, a linear decrease is considered with an arithmetic mean value:

$$\bar{c}_{arith} = \frac{c_{in} + c_{out}}{2}$$

which is a very rough approximation for larger, actual differences $(c_{in} - c_{out})/c_{in}$ (Zlokarnik 1979). Now, the oxygen transfer rate (OTR) can be calculated by using either:

$$\text{OTR} = K_L a\, \bar{c}/H \tag{5.54a}$$

or:

$$\text{OTE} = K_L a\, \bar{c}_{arith}/H \tag{5.54b}$$

A value of further interest is the oxygen transfer efficiency (OTE):

$$\mathrm{OTE} = \frac{c_{in} - c_{out}}{c_{in}} \tag{5.55}$$

Using Eq. (5.55), OTE can be obtained simply. We will now describe the mass transfer in more detail.

5.4.2.3 **Proposed Model of the American Society of Civil Engineers**

In this advanced kind of evaluation we should consider that the molar fraction of oxygen and the air flow rate Q_G will change as a result of both the consumption of O_2 and the uptake of CO_2 as well as the change of inert gases. The OTE can be written, while introducing the air flow rate Q_G into Eq. (5.55):

$$\mathrm{OTE} = \frac{Q_{G,in}\, c_{in} - Q_{G,out}\, c_{out}}{Q_{G,in}\, c_{in}} \tag{5.56}$$

with:

$$c_{in} = c_{O_2,in} = y_{O_2,in} \cdot \rho_{O_2} \tag{5.57}$$

and

$$c_{out} = c_{O_2,out} = y_{O_2,out}\, \rho_{O_2} \tag{5.58}$$

the O_2 concentration in the air flowing in and out of the reactor, Eq. (5.56) gives:

$$\mathrm{OTE} = 1 - \frac{Q_{G,out}\, \rho_{O_2}\, y_{O_2,out}}{Q_{G,in}\, \rho_{O_2}\, y_{O_2,in}} \tag{5.59}$$

with the molar fraction:

$$y_{O_2,in} = \frac{N_{O_2,in}}{N_{O_2,in} + N_{CO_2,in} + \sum_{i=1}^{n} N_{inert,in}} \tag{5.60}$$

where N_{O_2} is the number of moles of O_2, N_{CO2} is the number of moles of CO_2 and N_{inert} is the number of moles of inert components i.

A first approximation is obtained for neglecting the change in the flow rate Q_G ($Q_{G,in} = Q_{G,out}$) respectively:

$$\mathrm{OTE} = \frac{y_{O_2,in} - y_{O_2,out}}{y_{O_2,in}} \tag{5.61}$$

If we want to achieve greater precision, we must consider that the air flow rate Q_G changes during the rise of the bubbles. This change is caused by:

- the decrease in oxygen,
- the increase in carbon dioxide,
- the increase in water vapor and all other inert components inside the air bubble.

In the following model, we assume that the water vapor and the concentration of all other inert components do not change. But there is a further problem: it is seldom possible to measure the flow rate of the influent air. Therefore, the air flow rate is replaced by the nitrogen mass flow rate $Q_{N2,in}\,\rho_{N2}$, which must be corrected with Eq. (5.62) (Redmon et al. 1983):

$$Q_{G,in}\,\rho_{O_2}\,y_{O_2,in} = Q_{N_2,in}\,\rho_{N_2}\,\frac{M_{O_2}}{M_{N_2}}\,\frac{N_{O_2}}{N_{N_2}} \tag{5.62}$$

where $Q_{N_2,in}\,\rho_{N_2}$ is the mass flow rate of influent nitrogen (g N_2 h^{-1}), $\frac{M_{O_2}}{M_{N_2}}$ is the ratio of the molar masses oxygen/nitrogen in influent air $(\frac{gO_2\ mol\ N_2}{gN_2\ mol\ O_2})$ and $\frac{N_{O_2}}{N_{N_2}}$ is the ratio of the moles oxygen/nitrogen in the influent air $(\frac{mol\ O_2}{mol\ N_2})$.

Introducing Eq. (5.62), reflecting whether “in” or “out” into Eq. (5.59) gives:

$$OTE = 1 - \frac{Q_{N_2,out}\,\rho_{N_2}\,\frac{M_{O_2}}{M_{N_2}}\left(\frac{N_{O_2}}{N_{N_2}}\right)_{out}}{Q_{N_2,in}\,\rho_{N_2}\,\frac{M_{O_2}}{M_{N_2}}\left(\frac{N_{O_2}}{N_{N_2}}\right)_{in}} \tag{5.63}$$

or with $Q_{N_2,out} = Q_{N_2,in}$ gives:

$$OTE = 1 - \frac{\left(\frac{N_{O_2}}{N_{N_2}}\right)_{out}}{\left(\frac{N_{O_2}}{N_{N_2}}\right)_{in}} \tag{5.64}$$

with:

$$\left(\frac{N_{O_2}}{N_{N_2}}\right)_{in} = \frac{(N_{O_2}/(N_{O_2}+N_{N_2}+N_{CO_2}))_{in}}{(N_{N_2}/(N_{O_2}+N_{N_2}+N_{CO_2}))_{in}} = \frac{y_{O_2,in}}{y_{N_2,in}} \tag{5.65}$$

$$\left(\frac{N_{O_2}}{N_{N_2}}\right)_{out} = \frac{(N_{O_2}/(N_{O_2}+N_{N_2}+N_{CO_2}))_{out}}{(N_{N_2}/(N_{O_2}+N_{N_2}+N_{CO_2}))_{out}} = \frac{y_{O_2,out}}{y_{N_2,out}} \tag{5.66}$$

as well as:

$$y_{O_2} + y_{N_2} + y_{CO_2} = 1$$

or:

$$y_{N_2,in} = 1 - y_{O_2,in} - y_{CO_2,in}$$

and:

$$y_{N_2,out} = 1 - y_{O_2,out} - y_{CO_2,out}$$

Equation (5.64) can be transformed into:

$$\text{OTE} = 1 - \frac{y_{O_2,out}\left(1 - y_{O_2,in} - y_{CO_2,in}\right)}{y_{O_2,in}\left(1 - y_{O_2,out} - y_{CO_2,out}\right)} \tag{5.67}$$

If CO_2 is removed before the O_2 concentration is measured (e.g. in NaOH), from Eq. (5.67) we obtain:

$$\text{OTE} = 1 - \frac{y_{O_2,out}\left(1 - y_{O_2,in}\right)}{y_{O_2,in}\left(1 - y_{O_2,out}\right)} \tag{5.68}$$

Finally, the oxygen transfer rate can be calculated based on Eq. (5.37) by considering the changing gas flow rate and by using:

$$\text{OTR} = \frac{Q_{N_2,in}\,\rho_{N_2}\,\dfrac{M_{O_2}}{M_{N_2}}\left(\dfrac{N_{O_2}}{N_{N_2}}\right)_{in} - Q_{N_2,out}\,\rho_{N_2}\,\dfrac{M_{O_2}}{M_{N_2}}\left(\dfrac{N_{O_2}}{N_{N_2,out}}\right)_{out}}{V} \tag{5.69}$$

and with Eqs. (5.65) and (5.66):

$$\text{OTR} = \frac{Q_{N_2,in}\,\rho_{N_2}\,M_{O_2}/M_{N_2}}{V}\left[\frac{y_{O_2,in}}{y_{N_2,in}} - \frac{y_{O_2,out}}{y_{N_2,out}}\right] \tag{5.70}$$

$$\text{OTR} = \frac{Q_{N_2,in}\,\rho_{N_2}\,M_{O_2}/M_{N_2}}{V}\left[\frac{y_{O_2,in}}{1 - y_{O_2,in} - y_{CO_2,in}} - \frac{y_{O_2,out}}{1 - y_{O_2,out} - y_{CO_2,in}}\right] \tag{5.71}$$

If CO_2 is removed, we obtain:

$$\text{OTR} = \frac{Q_{N_2,in}\,\rho_{N_2}\,M_{O_2}/M_{N_2}}{V}\left[\frac{y_{O_2,in}\left(1 - y_{O_2,out}\right) - y_{O_2,out}\left(1 - y_{O_2,in}\right)}{\left(1 - y_{O_2,in}\right)\left(1 - y_{O_2,out}\right)}\right] \tag{5.72}$$

With Eq. (5.54a) or Eq. (5.54b), the overall mass transfer coefficient $K_L a$ can be calculated from OTR.

5.4.2.4 **Further Models**

Zlokarnik (1980) discussed how the increase in the hydrostatic pressure raises the oxygen concentration inside the air bubbles and the equilibrium concentration at the air/water interface, as well as the effects of temperature, which may change in tall bubble columns.

An estimate of the influences of pressure on OTE and OTR as well as the change in bubble surface, mass transfer coefficient and saturation concentration was presented by Pöpel et al. (1998). These considerations were complemented by measurements in a pilot plant with a water depth of up to 12.5 m.

5.4.2.5 **Oxygen Transfer Rate**

Until now, all systems discussed here have been either completely mixed (Sections 5.3.1.1 to 5.3.1.3, 5.3.2 and 5.4.2.3) or plug flow systems (Section 5.3.1.4). Therefore, models could be used and $k_L a$ or $K_L a$ could be applied. However, in large-scale tanks, the description of oxygen transfer using the concept of Eq. (5.8) is frequently not allowed because of the different locally structured air/water flows. This is demonstrated by Fig. 5.9 (Mueller et al. 2002).

Porous ceramic tubes are installed in such a way as to promote a secondary water circulation which sucks down a portion of the bubbles rising in the vicinity. But even for a system with a locally constant configuration of diffusers (Fig. 5.10), the oxygen transfer rate may change considerably over time because of the change in load over the course of 24 h and also with location, especially in longitudinal tanks (Schuchardt et al. 2002). We will come back to this point in Section 5.4.2.7.

$K_L a$ can be calculated from Eq. (5.54b):

$$\mathrm{OTR} = K_L a \, \frac{c_{in} + c_{out}}{2\,H} \tag{5.73}$$

and, after introducing $c = Hc^*$, we obtain:

$$\mathrm{OTR} = K_L a \, \frac{c^*_{in} + c^*_{out}}{2} \tag{5.74}$$

OTE was developed in Section 5.4.2.3, resulting in Eq. (5.68); and OTR can be obtained by using Eq. (5.72).

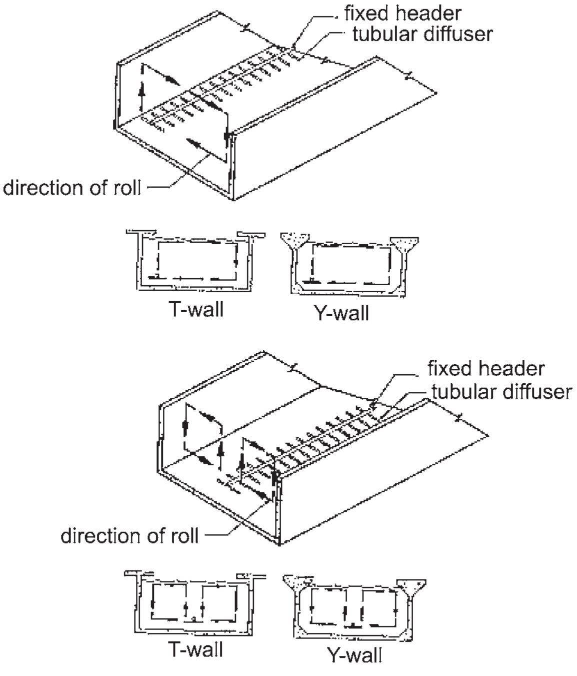

Fig. 5.9 Configuration of deep tank aeration system (Mueller et al. 2002).

Fig. 5.10 Porous ceramic tubes in deep tank aeration, second zone of a longitudinal tank at WWTP Waßmannsdorf, near Berlin (Schuchardt et al. 2002).

Considering the influence of temperature on c* at point x by introducing:

$$c^* = c^*_{20,x}\,\theta^{(20-T)} \tag{5.75}$$

with $\theta = 1.024$ (Stenstrom and Gilbert 1981) and the influence of the hydrostatic pressure on c^* at point $x = h$ (h = height of the water column):

$$c^*_{20,x} = c^*_{20,x=L} = \left(1 + \frac{1}{2}\;\frac{x}{10.32}\right) \tag{5.76}$$

we write the standardized oxygen transfer rate for deep tanks:[2]

$$\mathrm{SOTR} = K_L a\; c^*_{20,x=L}\;\theta^{(20-T)}\left(1 + \frac{1}{2}\;\frac{x}{10.32}\right) \tag{5.77}$$

With Eq. (5.77), the influence of aeration, height of the water column and temperature can be considered.

This SOTR value is valid for measurements in clean water. In wastewater, especially with activated sludge, a lower SOTR value is to be expected, resulting in (Stenstrom and Redman 1996):

$$\alpha\,(\mathrm{SOTR}) = \frac{\mathrm{SOTR}_w}{\mathrm{SOTR}} < 1 \tag{5.78}$$

---

[2] 10.32 m is the height of a water column causing a pressure of 1 bar. $c^*_{20,x} = p/H_P$ means that c^*_{20} increases linearly with x.

α-Values are dependent upon diffuser configuration, air flow rate, wastewater composition, local position x and temperature T.

We must keep the following simplifications in mind:

- Instead of using the logarithmic concentration difference (5.54a), which followed from the plug flow model, the approximation $\Delta c' = c_T^* - c'$ was applied.
- $K_L a$ was assumed to be constant, however there are several influences on both a and K_L (lower pressure, diffusing CO_2 into the bubble, coalescence of bubbles and increasing velocity).

5.4.2.6 Power Consumption and Efficiency

We have to distinguish two levels of power consumption:

- The power consumption of the blower P producing compressed air.
- The total power demand $P_T = P/(\eta_b \cdot \eta_m \cdot \eta_g)$, with $\eta_b = 0.8$ for the blower (newer equipment), $\eta_m = 0.95$ for the motor and $\eta_g = 0.95$ for the transmission (Mueller et al. 2002).

The power demanded for compression is:

$$P = \frac{Q_G \rho RT}{k} \left[\left(\frac{p_{in}}{p_{out}} \right)^k - 1 \right] \text{J h}^{-1} \tag{5.79}$$

where $k = 0.2857$ for dry air, p_{in} is the pressure before compression, p_{out} is the pressure after compression, T is the temperature (K), $R = 8.314$ J (mol K)$^{-1}$ is the gas constant and $Q_G\rho$ is the mass flow rate of air (kg h^{-1}).

Considering:

$$1 \frac{\text{J}}{\text{h}} = \frac{1}{3600 \text{ s}} \frac{\text{J}}{\text{s}} = \frac{1}{3600} \text{W} = \frac{10^{-6}}{3.6} \text{kW} = 0.278 \cdot 10^{-6} \text{ kW}$$

we obtain:

$$P = 0.278 \cdot 10^{-6} \frac{Q_G \rho RT}{k} \left[\left(\frac{p_{in}}{p_{out}} \right)^k - 1 \right] \text{kW} \tag{5.80}$$

Now we calculate the efficiencies using Eqs. (5.77) and (5.80):

$$E = \frac{\text{SOTR}}{P/V} \text{ kg O}_2 \text{ (kWh)}^{-1} \tag{5.81}$$

and for the total efficiency:

$$E_t = \frac{\text{SOTR} \cdot \eta_b \eta_m \eta_g}{P/V} \text{ kg O}_2 \text{ (kWh)}^{-1} \tag{5.82}$$

5.4.2.7 Monitoring of Deep Tanks

If we want to measure OTE, SOTR and E for a deep tank in operation, we must use the off-gas analysis method (Fig. 5.11).

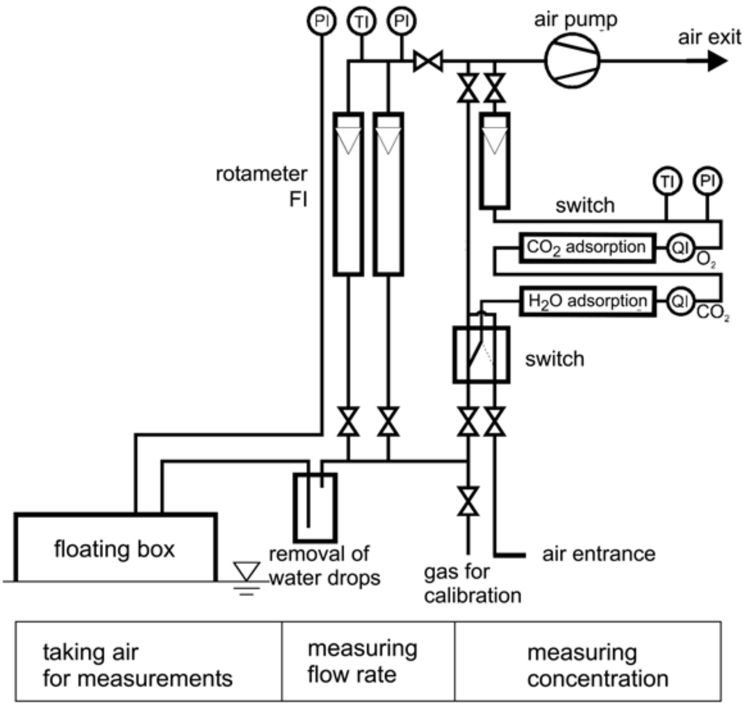

Fig. 5.11 Instrument for off-gas analysis, apparatus for manual operation (Schuchardt et al. 2002).

An off-gas apparatus consists of an off-gas hood with instruments for the measurement of flow rates and oxygen concentration. The floating hood may have a cross-sectional area of 2–4 m^2 (Schuchardt 2005). Air is drawn off by a fan passing first through a rotameter for flow rate measurement of the off-gas. A side-stream is passed through H_2O and CO_2 adsorbers and is then analyzed to measure O_2 consumption.

The results were obtained from measurements at the aerobic activated sludge basin of the WWTP Waßmannsdorf, near Berlin, equipped with ceramic tubes (see Fig. 5.12). For low and high flow rates of compressed air and specific power demands (P/V), the efficiency E decreases only slightly. Maximum efficiencies ($E = 2.5$–2.7 kg O_2 $(kWh)^{-1}$) are obtained only in a narrow band of $11 < P/V < 20$ ($W\ m^{-3}$) with values of $31 < SOTR < 52$ (g O_2 $(m^3\ h)^{-1}$) (Schuchardt et al. 2002; Schuchardt 2005). For lower and higher values of P/V, the efficiency decreases slightly, down to $E = 2.1$–2.2 kg O_2 $(kWh)^{-1}$. With optimized aeration equipment and optimized automatic process control, great amounts of energy can be saved.

If we compare these results with those presented in Fig. 5.7 for a small Simcar aerator ($d = 40$ cm) we establish a quite good correspondence. We should not misinterpret these relative small derivatives: We need more studies for the optimization of aeration systems of large WWTP's in a time of arrising costs for energy!

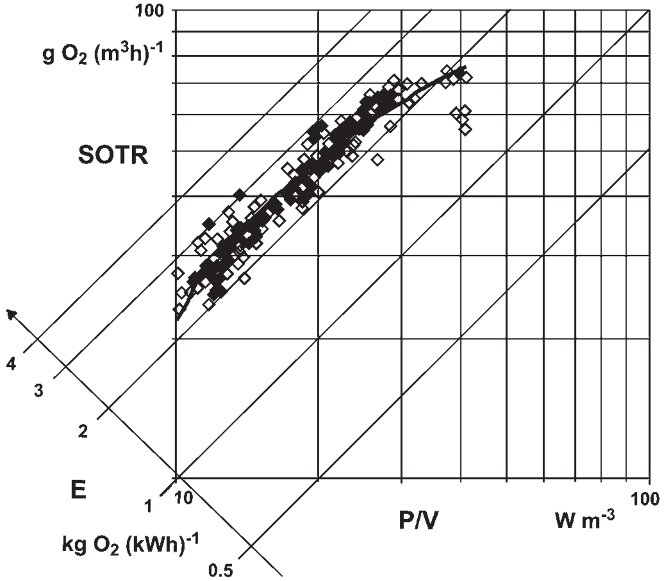

Fig. 5.12 Standardized oxygen transfer rate, power consumption, and efficiency of a deep tank aeration system equiped with porous aeration tubes of WWTP Waßmannsdorf (see Fig. 5.10) (Schuchardt 2005).

5.5
Dimensional Analysis and Transfer of Models

5.5.1
Introduction

There are some important questions to answer if we want to study an aeration process in a laboratory-scale or pilot-scale plant and apply the results to large-scale aeration and activated sludge systems. Let us discuss this problem for the example of a surface aerator constructed and processed as a turbine with a vertical shaft. We assume that it is possible to measure the oxygen transfer rate and the power consumption for clean water or wastewater with activated sludge. The volume, the type of tank and the mean residence time (aeration time) are given. Some important questions are:

- The diameter of the aerator in the laboratory-scale plant was 0.25 m, for example. What may be the best diameter for the large-scale plant?
- The range of speeds in the laboratory-scale experiments was 10–100 min^{-1}. What may be the suitable range of speeds for the large-scale aerator?
- In the laboratory-scale experiments, an efficiency of nearly E = 3 kg O_2 $(\text{kWh})^{-1}$ was obtained. Under what conditions can we expect a relatively high efficiency; and is it possible to find operating conditions yielding E = 3 kg O_2 $(\text{kWh})^{-1}$?

- Is it more economical to install two or four aerators instead of one? What is the optimal diameter, speed and depth of submersion?

These questions are not easy to answer. We need two methods, the method of dimensional analysis and the method to transfer models. Both will be explained for a simple case: the stirred non-aerated tank and the stirred aerated tank without suspended solids.

5.5.2
Power Consumption of a Stirred, Non-aerated Tank – A Simple Example

The first step is to find all parameters which are relevant to the question (Fig. 5.13). Will this parameter influence the power consumption in the stirred tank if changed?

Looking at Fig. 5.13, we will first mention all the geometric parameters:

D = diameter of the tank
d = diameter of the stirrer
H = height of the water
h = distance of the stirrer from the bottom

The stirrer may be equipped with three plates with a given width and distance from the wall. Usually, the stirrer diameter d is selected as the reference and we can define three dimensionless geometric numbers:

$$\frac{D}{d},\ \frac{H}{d},\ \frac{h}{d}$$

Tanks of different sizes are considered to be geometrically similar if these values are constant for all tanks.

Furthermore, we will consider a group of tanks with different stirrers of diameter d. Let us now follow the method of Zlokarnik (1999).

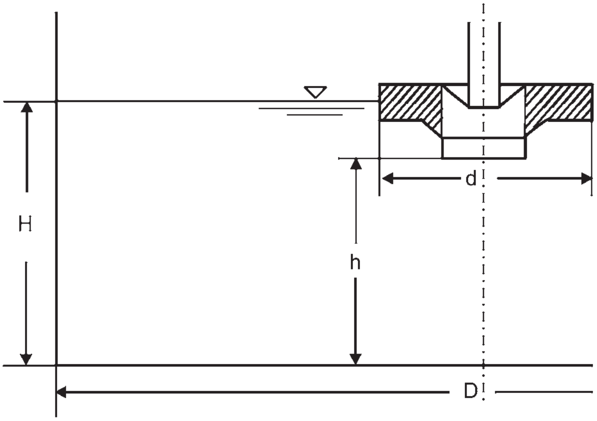

Fig. 5.13 Cylindrical tank with Simplex rotor, Type A, showing main dimensions.

The next step is to look for parameters which will influence the power input P. These parameters can be divided into:

- Geometric parameters: diameter of the stirrer d.
- Material parameters of the water: density ρ, kinematic viscosity ν.
- Process parameters: stirrer speed n.

Therefore, the list of relevant variables is: {P; d; ρ; ν; n}.

The next step is to formulate the dimensional matrix. Before doing this, we have to decide which parameters should occur only in one dimensionless characteristic number. It is most effective to select P and ν. After making this decision, the dimensional matrix can be formed (Table 5.3), with each number in the matrix representing the power of the unit dimension (length, time, mass) occurring in the variable.

This matrix must be transformed to obtain a unit matrix in the first 3×3 matrix, that is, with a diagonal of ones. To obtain a 0 instead of –3 in the first column (ρ) and a 1 instead of –1 in the third column (n), the Gaussian method is applied by performing linear combinations of the rows. In this manner –3 times the elements of the first row are added to the second row. The exponents listed in the last row need only be multiplied by –1 to obtain a 1 in the proper location (Table 5.4).

The numbers in the remaining matrix to the right of the unit matrix below P and ν give us the exponents of the first three variables which yield dimensionless num-

Table 5.3 Dimensional analysis of a stirrer, matrix of unit dimensions for relevant parameters with respect to selected P and ν (Zlokarnik 1999).

	ρ	d	n	P[1)]	ν
Mass *M*	1	0	0	1	0
Length *L*	–3	1	0	2	2
Time *T*	0	0	–1	–3	–1
	Core matrix			Rest matrix	

1) Note: $P = \frac{A}{t} = \frac{Kl}{t}$; K [$MLT^{-2}$]; P [$ML^2 T^{-3}$]
A = work, K = force.

Table 5.4 Transformation of the core matrix into a unit matrix, to obtain two dimensionless numbers with respect to selected P and ν (Zlokarnik 1999).

	ρ	d	n	P[1)]	ν
M	1	0	0	1	0
3 M + L	0	1	0	5	2
–T	0	0	1	3	1
	Unit matrix			Rest matrix	

bers. The first dimensionless number π_1 formed with P is known as the Newton number. The second, formed with v, is the Reynold's number:

$$\pi_1 = \frac{P}{\rho^1 n^3 d^5} \equiv \text{Ne (Newton number)} \tag{5.83}$$

and:

$$\pi_2 = \frac{v}{\rho^0 n^1 d^2} = \frac{v}{nd^2} = \text{Re}^{-1} \text{ (Reynolds number)} \tag{5.84}$$

There must be a function of the five parameters for which:

$$f\,(\text{Ne, Re}) = 0 \tag{5.85}$$

Another way to find Eq. (5.85) starting from the list of the five variables

$$f\,(P, \rho, n, d, v) = 0$$

is described below.

Looking for a function, with a reduced number of dimensionless parameter, we have at first to write:

$$\pi_i\,(P^\alpha, \rho^\beta, n^\gamma, d^\delta, v^\varepsilon)$$

and working in the MLT-system (mass, length, time) for the needed dimensions, we introduce P $[ML^2\,T^{-3}]$, ρ $[M\,L^{-3}]$, n $[T^{-1}]$, d $[L]$, v $[L^2\,T^{-1}]$:

$$\pi_i\,((ML^2\,T^{-3})^\alpha\,(ML^{-3})^\beta\,(T^{-1})^\gamma\,(L)^\delta\,(L^2\,T^{-1})^\varepsilon)$$

As can be demonstrated now, this problem can be described by using only two dimensionless numbers resulting from the fact, that we use five parameter with dimensions and three units ($5-3=2$). If we want to obtein two dimensionless numbers each with P and v, the exponents of ρ, n and d have to be replaced by the exponents of P and v. Writing

$$\pi_i\,((ML^2\,T^{-3})^\alpha\,(ML^{-3})^\beta\,(T^{-1})^\gamma\,(L)^\delta\,(L^2\,T^{-1})^\varepsilon)$$

we obtain three equations:

$$\begin{aligned} &M: \ \alpha + \beta = 0 \\ &L: \ 2\alpha - 3\beta + \delta + 2\varepsilon = 0 \\ &T: \ 3\alpha - \gamma - \varepsilon = 0 \end{aligned}$$

and the solutions

$$\begin{aligned} &\beta = -\alpha \\ &\gamma = -3\alpha - \varepsilon \\ &\delta = -5\alpha - 2\varepsilon \end{aligned}$$

which have to be introduced into $\pi_i\,(P^\alpha, \rho^\beta, n^\gamma, d^\delta, v^\varepsilon)$.

π_i is only dimensionless, if

$$\left(\frac{P}{\rho\, n^3\, d^5}\right)^\alpha = \text{const} \quad \text{and} \quad \left(\frac{v}{n\, d^2}\right)^\varepsilon = \text{const}$$

giving the numbers Ne and Re (see Eqs. 5.83 and 5.84).

5.5.3
Description of Oxygen Transfer, Power Consumption and Efficiency by Surface Aerators Using Dimensionless Numbers

With the results from above, we return to the experimental results during surface aeration with an aeration turbine (see Fig. 5.6). Oxygen transfer rate OTR, power consumption P/V and efficiency E were measured in clean water and plotted in Fig. 5.7. Now we will describe these results for a series of aerators (Simplex, Type A), which are characterized by geometric similarity.

However, the problems to solve are more difficult than those in Section 5.5.2:

- The stirred tank is aerated and buoyancy forces must be considered. Dimensional analysis now yields a set of three dimensionless numbers:

  $$f\,(\mathrm{Ne},\ \mathrm{Re},\ \mathrm{Fr}) = 0 \tag{5.86}$$

  with Froude number:

  $$\mathrm{Fr} \equiv \frac{n^2 d}{g} \tag{5.87}$$

- A further number must be developed which gives information about the rate of oxygen transfer. The relevance list is:

  $$K_L a\, V = f\,(d,\ \rho,\ \nu,\ \sigma,\ D,\ n,\ g) \tag{5.88}$$

  Using the method described above, we obtain:

  $$f\,(Y,\ \mathrm{Fr},\ \mathrm{Re},\ \mathrm{Sc},\ \sigma^*) = 0 \tag{5.89}$$

  with:

  $$\frac{K_L a\, V}{d^3}\left(\frac{\nu}{g^2}\right)^{1/3} \equiv Y \text{ (oxygen transfer number)} \tag{5.90}$$

  $$\frac{\nu}{D} \equiv \mathrm{Sc} \text{ (Schmidt number)} \tag{5.91}$$

  $$\frac{\sigma}{\rho\,(\nu^4\, g)^{1/3}} \equiv \sigma^* \text{ (dimensionless surface tension)} \tag{5.92}$$

Let us look at the system air/water and we can conclude that all material coefficients are constant, resulting in:

$\mathrm{Sc} = \mathrm{const}$
$\sigma^* = \mathrm{const}$

Therefore, the set of dimensionless numbers for air/water is:

$$f\,(Y,\ \mathrm{Fr},\ \mathrm{Re}) = 0 \tag{5.93}$$

If we do not consider aerators with small diameter d and low rotation speed n, $\mathrm{Re} \geq 10^4$ is valid (turbulent region of the flow) and Re does not influence Ne and Y remarkably. We can conclude that it must be possible to describe the results using (Zlokarnik 1979, 1980):

$$\mathrm{Ne} = f\,(\mathrm{Fr}) \tag{5.94}$$

and:

$$Y = f\,(\mathrm{Fr}) \tag{5.95}$$

Both results obtained by experiments can be represented as straight lines if we use double-logarithmic plots (Figs. 5.14 and 5.15). But the author chose to use a larger region in Fr numbers ($0.02 < \mathrm{Fr} < 0.8$) for power measurement (Fig. 5.13) compared with oxygen transfer measurements ($0.08 < \mathrm{Fr} < 0.35$; Fig. 5.14).

The straight lines in both figures are given by:

$$\mathrm{Ne} = 0.28\ \mathrm{Fr}^{-0.78} \tag{5.96}$$

and:

$$Y = 1.25 \cdot 10^{-3}\ \mathrm{Fr}^{0.89} \tag{5.97}$$

From these equations a dimensionless efficiency can be defined as:

$$E^{*} = \frac{Y}{\mathrm{Ne}} \tag{5.98}$$

and with Eqs. (5.96) and (5.97):

$$E^{*} = 4.46 \cdot 10^{-3}\ \mathrm{Fr}^{1.67} \tag{5.99}$$

can be used together with Eq. (5.96) and Eq. (5.97) for scale-up calculations, if the dimensionless geometric numbers are (nearly) constant.

5.5.4
Application of Dimensionless Numbers for Surface Aeration

From a fundamental point of view, it is not permissible to exceed the range of experimentally changed parameters considerably, if the results are to be reliable. If we wish to extract information about K_La from Fig. 5.14 for Simplex aerators (Fig. 5.6) of another size, all dimensionless geometric numbers must be constant and we must remain within a range of Fr numbers between $0.08 < \mathrm{Fr} < 0.35$.

Using $\mathrm{Fr}_{\mathrm{max}} = 0.35$ and assuming $n = 1\ \mathrm{s}^{-1}$, we can calculate:

$$\mathrm{Fr}_{\mathrm{max}} = \frac{n^2 d}{g}$$

$$d = \frac{\mathrm{Fr}_{\mathrm{max}}\, g}{n^2} = \frac{0.35 \cdot 9.81}{1^2} = 3.43\ \mathrm{m}$$

Although only a small range of Fr numbers was investigated, a relative high range of diameters d is included. However, is it possible to realize the required K_La? We will return to this question in Problem 5.

For practical applications, it is necessary to change geometric numbers and to study their influences. Frequently, the influence of relative small variations (e.g. $d/D = 4.0 \rightarrow d/D = 3.5$) is low and the freedom to move is given.

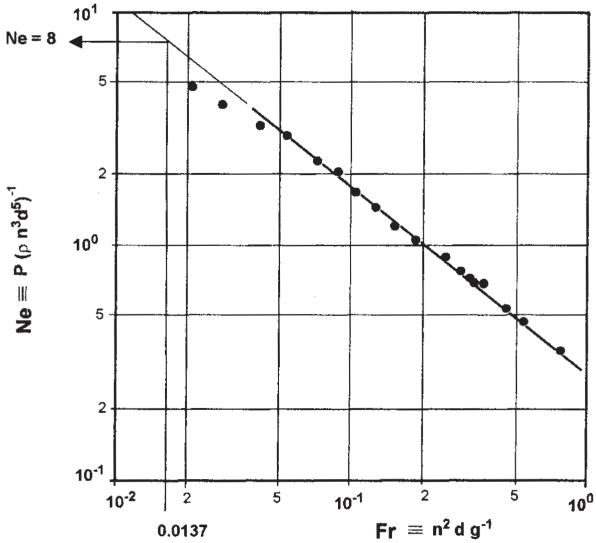

Fig. 5.14 Power correlation Ne = f (Fr) for surface aerator Symplex Type A, Pushing, H/d = 1.0, D/d = 4.37 (Zlokarnik 1975).

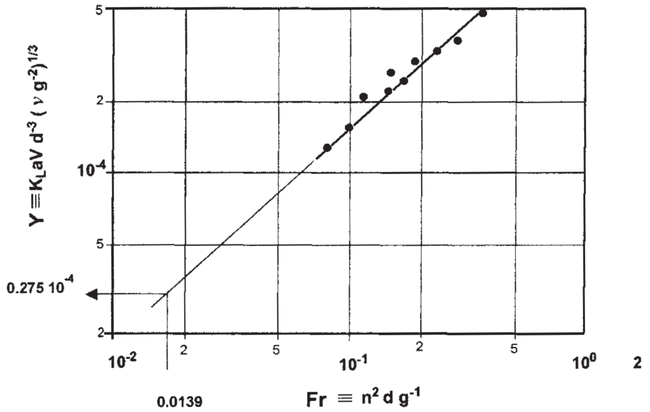

Fig. 5.15 Sorption correlation Y = f (Fr) for surface Aerator Simplex, Type A, Pushing, H/d = 1.0, D/d = 4.37 (Zlokarnik 1979).

It is very important to prove these methods experimentally with two or three aerators of the same type but different diameters, which differ by a factor of two or three. The same water should be used and all geometric numbers must be the same in all these experiments if the influence of relative small changes has not been studied before. Unfortunately, the expenditure for such experiments is high and such studies are very seldom.

PROBLEM

The oxygen transfer rate OTR and power demand P/V of a Simplex aerator (-Type A) with a diameter of d = 0.206 m (Fig. 5.6) was measured for different speeds. The efficiency E was calculated and the results plotted in Fig. 5.7. Following this, the dimensionless numbers Ne, Fr and Y were calculated using Eqs. (5.83), (5.87) and (5.90). First, Ne was plotted versus Fr in Fig. 5.14. In addition, Fig. 5.15 shows the plot of Y as a function of Fr.
A large aerobic tank with a volume of $V = 250\ \mathrm{m}^3$ and with only one Simplex aerator is to be constructed with an oxygen transfer rate of $\mathrm{OTR} = 100\ \mathrm{g\ O_2\ (m^3\ h)^{-1}}$. An aerator with a diameter of 2.0 m is available (Zlokarnik 1979).

1. What is the speed n needed for the demanded oxygen supply?
2. What is the efficiency E of the aeration system?

Constant parameters:

$\nu = 10^{-6}\ \mathrm{m^2\ s^{-1}}$, $g = 9.81\ \mathrm{m\ s^{-1}}$, $c^* = 8\ \mathrm{mg\ L^{-1}}$, $c' = 1\ \mathrm{mg\ L^{-1}}$

Solution

1. Calculation of the required speed n:

(a) $K_L a = ?$
$\mathrm{OTR} = K_L a\,(c^* - c')$ from Eq. (5.37):

$$K_L a = \frac{\mathrm{OTR}}{(c^* - c')} = \frac{100}{7}\ \frac{\mathrm{g\ O_2\ m^3}}{\mathrm{m^3\ h\ g\ O_2}} = 14.3\ \mathrm{h^{-1}} = 3.97 \cdot 10^{-3}\ \mathrm{s^{-1}}$$

(b) $Y = ?$, $\mathrm{Fr} = ?$

$$Y = \frac{K_L a\, V}{d^3}\left(\frac{\nu}{g^2}\right)^{1/3} \quad \text{from Eq. (5.90)}$$

$$Y = \frac{1}{d^3}\left[\frac{3.97 \cdot 10^{-3}\ 250\ 10^{-6/3}}{9.81^{2/3}}\right] \frac{\mathrm{m^3\ m^{2/3}\ s^{4/3}}}{\mathrm{m^3\ s\ s^{1/3}\ m^{2/3}}} \qquad Y = \frac{1}{d^3} \cdot 0.22 \cdot 10^{-3}$$

$$d = 2.0\ \mathrm{m};\ \ Y = \frac{0.22 \cdot 10^{-3}}{8.0} = 0.0275 \cdot 10^{-3}$$

From Eq. (5.97) it follows that:

$$\mathrm{Fr} = \left[\frac{Y}{1.25 \cdot 10^{-3}}\right]^{\frac{1}{0.89}} = 0.0137$$

(c) The speed n follows from:

$$\mathrm{Fr} = \frac{d\,n^2}{g}$$

$$n = \left(\frac{\mathrm{Fr}\,g}{d}\right)^{1/2} = \left(\frac{0.0137 \cdot 9.81}{2.0}\right)^{1/2} \left(\frac{\mathrm{m}}{\mathrm{s}^2\,\mathrm{m}}\right)^{1/2} = 0.260\ \mathrm{s}^{-1}$$

2. Calculation of the efficiency E:

(a) $P/V = ?$

$\mathrm{Fr} = 0.0139$ gives $\mathrm{Ne} = 8$

From $$\mathrm{Ne} = \frac{P}{\rho\,d^5\,n^3} \tag{5.83}$$

we obtain the specific power consumption:

$$\frac{P}{V} = \frac{\mathrm{Ne}\,\rho\,d^5\,n^3}{V}$$

$$\frac{P}{V} = \frac{8 \cdot 1000 \cdot 2^5 \cdot 0.26^3}{250}\ \frac{\mathrm{kg\,m^5}}{\mathrm{m^3\,m^2\,s^3}} = 18\ \frac{\mathrm{kg}}{\mathrm{m\,s^3}} = 18\ \frac{\mathrm{W}}{\mathrm{m^3}}$$

(b) With the given $\mathrm{OTR_v} = 100\ \mathrm{g\ (m^3\ h)^{-1}\ O_2}$ the efficiency follows:

$$E = \frac{\mathrm{OTR}}{P/V} = \frac{100}{18}\ \frac{\mathrm{g\,O_2\,m^3}}{\mathrm{h\,W\,m^3}} = 5.5\ \frac{\mathrm{kg\,O_2}}{\mathrm{k\,W\,h}}$$

The experimental results in Figs. 5.13 and 5.14 are valid for:

$H/d = 1$ and $D/d = 4.37$

For $d = 2$ m, we obtain $H = 2$ m and $D = 8.74$ m for a cylindrical tank with:

$$V = \frac{\pi D^2}{4} \cdot H = 120\ \mathrm{m^3}$$

In a tank with $V = 250\ \mathrm{m^3}$, H and/or D must differ from those values.

For $D = 12$ m, H follows:

$H = 4\,V/\pi D^2 = 2.21$ m

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