The Islamic University of Gaza

Faculty of Engineering

Civil Engineering Department

Hydraulics - ECIV 3322

Chapter 4

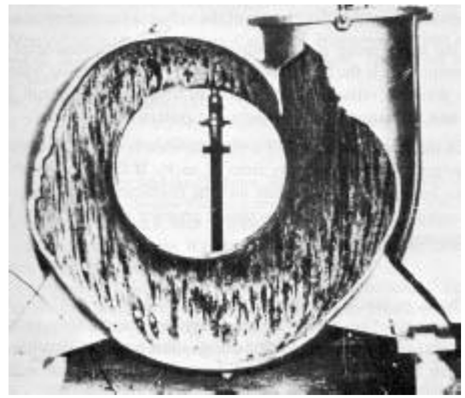
Unsteady Flow in Pipes

Water Hammer Phenomenon in pipelines

- A sudden change of flow rate in a large pipeline (due to valve closure, pump turnoff, etc.) may involve a great mass of water moving inside the pipe.
- The force resulting from changing the speed of the water mass may <u>cause a pressure rise</u> in the pipe with a magnitude several times greater than the normal static pressure in the pipe.
- The excessive pressure <u>may fracture the pipe</u> walls or cause other damage to the pipeline system.
- This phenomenon is commonly known as the water hammer phenomenon

Some typical damages

Burst pipe in power
sation Big Creek #3, USA



Pump damage in Azambuja
Portugal

Pipe damage in
power station Okigawa

The sudden change of pressure due to a valve closure may be viewed as the result of the force developed in the pipe necessary to stop the flowing water column. The column has a total mass M and is changing its velocity at the rate of dV/dt. According to Newton's second law of motion,

$$\boxed{F = m\frac{dV}{dt}}$$

If the velocity of the entire water column could be reduced to zero instantly

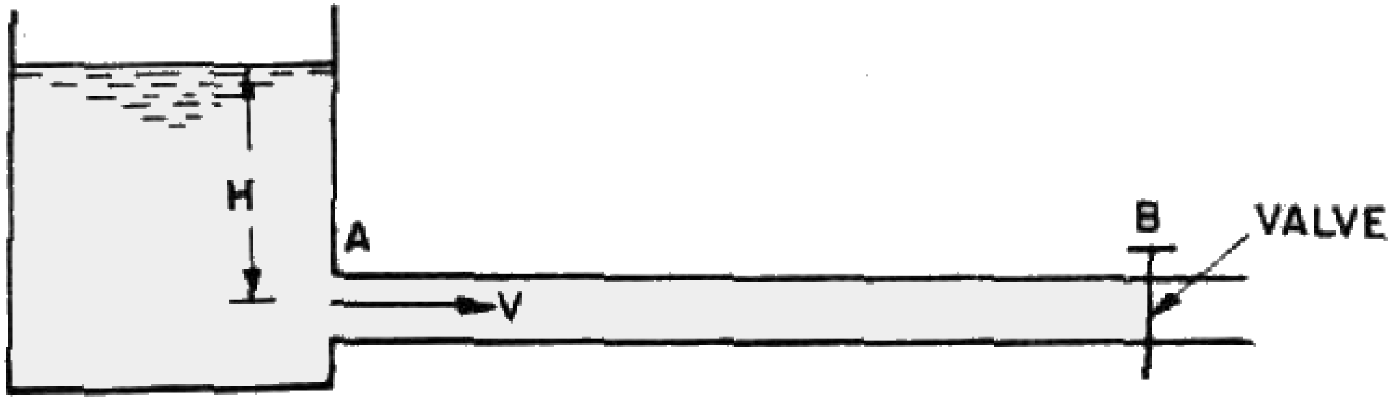
$$F = \frac{m(V_0 - 0)}{0} = \frac{mV_0}{0} = \infty$$

The resulting force (hence, pressure) would be infinite. Fortunately, such an instantaneous change is almost impossible because a mechanical valve requires a certain amount of time to complete a closure operation. In addition, neither the pipe walls nor the water column involved are perfectly rigid under large pressure. The elasticities of both the pipe walls and the water column play very important roles in the water hammer phenomenon.

Water Hammer

Consider a long pipe *AB*:

- Connected at one end to a reservoir containing water at a height *H* from the center of the pipe.
- At the other end of the pipe, a valve to regulate the flow of water is provided.



- If the valve is suddenly closed, the flowing water will be obstructed and momentum will be destroyed and consequently **a wave of high pressure will be created** which travels back and forth starting at the valve, traveling to the reservoir, and returning back to the valve and so on.

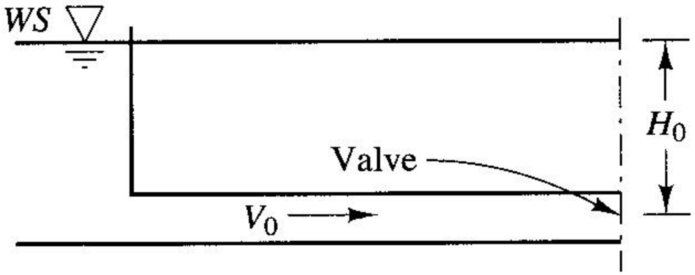
This wave of high pressure:

1. Has a very high speed **(called celerity, C)** which may reach the speed of sound wave and may create noise called ***knocking***,
2. Has the effect of hammering action on the walls of the pipe and hence is commonly known as the ***water hammer phenomenon***.

- The kinetic energy of the water moving through the pipe is converted into potential energy stored in the water and the walls of the pipe through the elastic deformation of both.
- The water is compressed and the pipe material is stretched.
- The following figure illustrates the formation and transition of the pressure wave due to the sudden closure of the valve

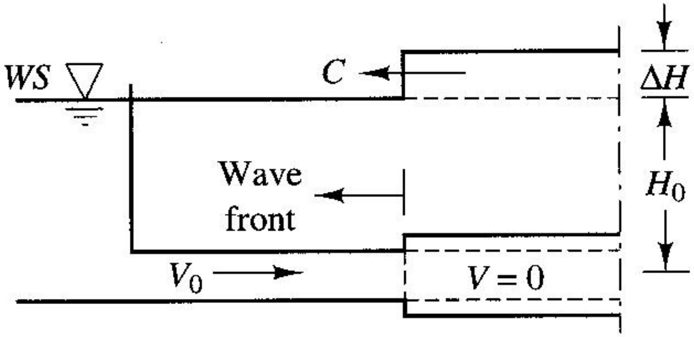
Propagation of water hammer pressure wave

Consider a pipe length L with inside diameter D, wall thickness e, and the modulus of elasticity E_p.



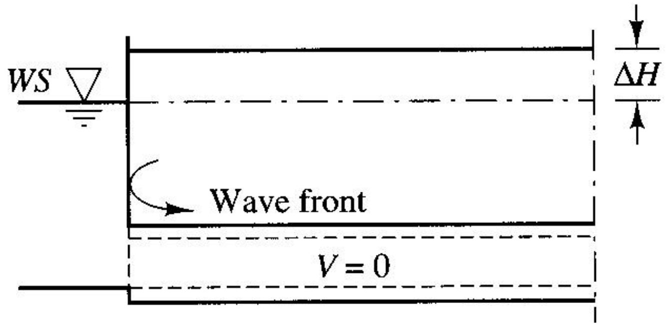
(a)

Steady state condition



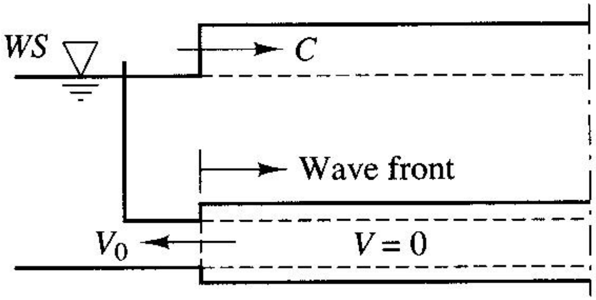
(b)

Transient condition $t < L/C$



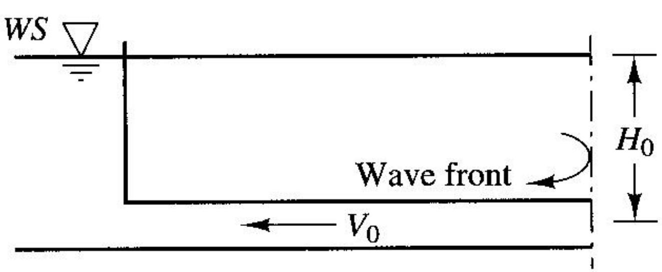
(c)

Transient condition $t = L/C$



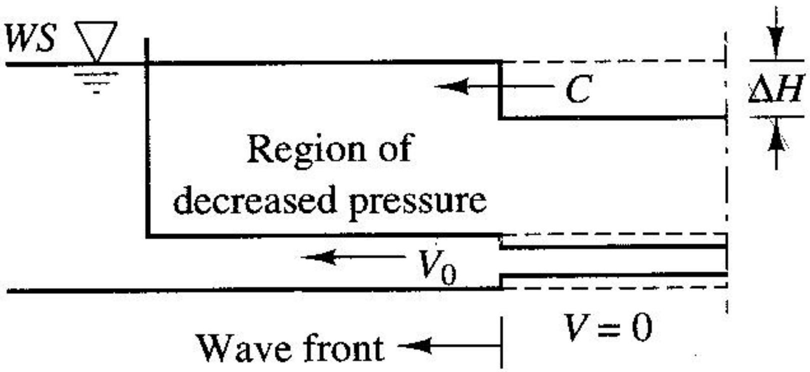
(d)

Transient condition $L/C > t > 2L/C$



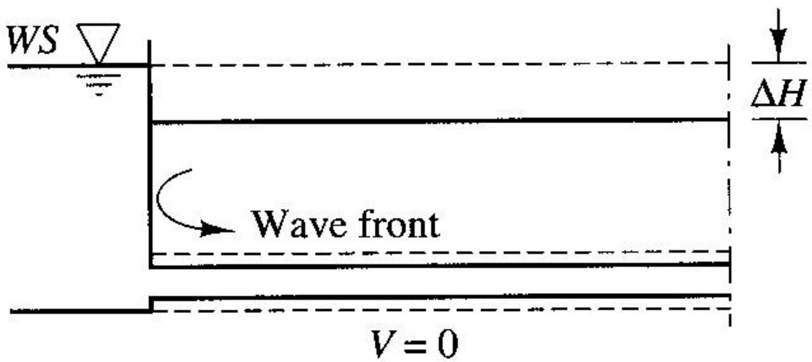
(e)

Transient condition $t = 2L/C$



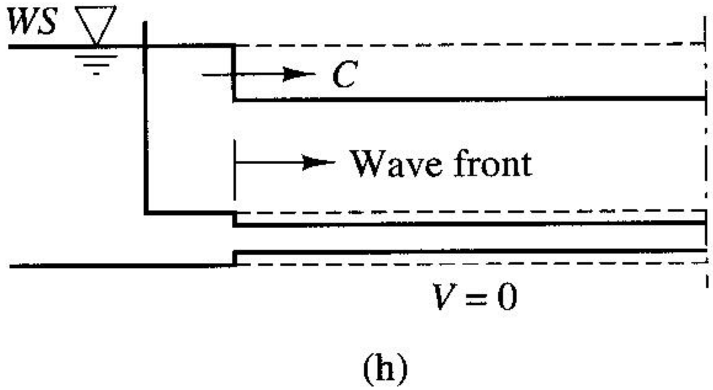
(f)

Transient condition 2L/C >t >3L/C

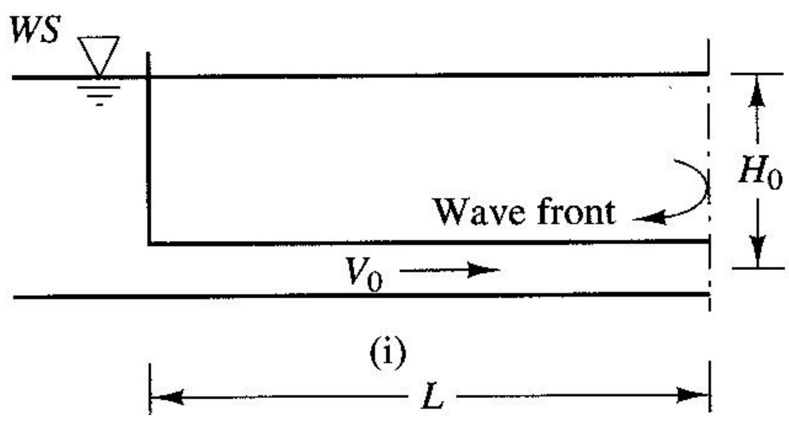


(g)

Transient condition t = 3L/C



Transient condition 3L/C >t >4L/C



Transient condition t = 4L/C

Analysis of Water Hammer Phenomenon

The pressure rise due to water hammer depends upon:

(a) The velocity of the flow of water in pipe,

(b) The length of pipe,

(c) Time taken to close the valve,

(d) Elastic properties of the material of the pipe.

The following cases of water hammer will be considered:

- Gradual closure of valve,
- Sudden closure of valve and pipe is rigid, and
- Sudden closure of valve and pipe is elastic.

- The time required for the pressure wave to travel from the valve to the reservoir and back to the valve is:

$$t = \frac{2L}{C}$$

Where:

L = length of the pipe (m)

C = speed of pressure wave, celerity (m/sec)

- If the valve time of closure is t_c , then
  - If $\boxed{t_c \succ \frac{2L}{C}}$ the closure is considered gradual
  - If $\boxed{t_c \leq \frac{2L}{C}}$ the closure is considered sudden

The speed of pressure wave *"C"* depends on :

- the pipe wall material.
- the properties of the fluid.
- the anchorage method of the pipe.

- $C=\sqrt{\dfrac{E_b}{\rho}}$ if the pipe is rigid
- $C=\sqrt{\dfrac{E_c}{\rho}}$ if the pipe is elastic

and $\dfrac{1}{E_c}=\dfrac{1}{E_b}+\dfrac{DK}{E_p e}$

Where:

- C = velocity (celerity) of pressure wave due to water hammer.
- ρ = water density (*1000 kg/m3*).
- E_b = bulk modulus of water ($2.1 \times 10^9\ N/m^2$).
- E_c = effective bulk modulus of water in elastic pipe.
- E_p = Modulus of elasticity of the pipe material.
- e = thickness of pipe wall.
- D = diameter of pipe.
- K = factor depends on the anchorage method:

$= \left(\frac{5}{4} - \varepsilon\right)$ for pipes free to move longitudinally,

$= (1 - \varepsilon^2)$ for pipes anchored at both ends against longitudinal movement

$= (1 - 0.5\varepsilon)$ for pipes with expansion joints.

- where ε = poisson’s ratio of the pipe material (0.25 - 0.35). It may take the value ε = 0.25 for common pipe materials.

If the longitudinal stress in a pipe can be neglected, $k = 1.0$, and Equation can be simplified

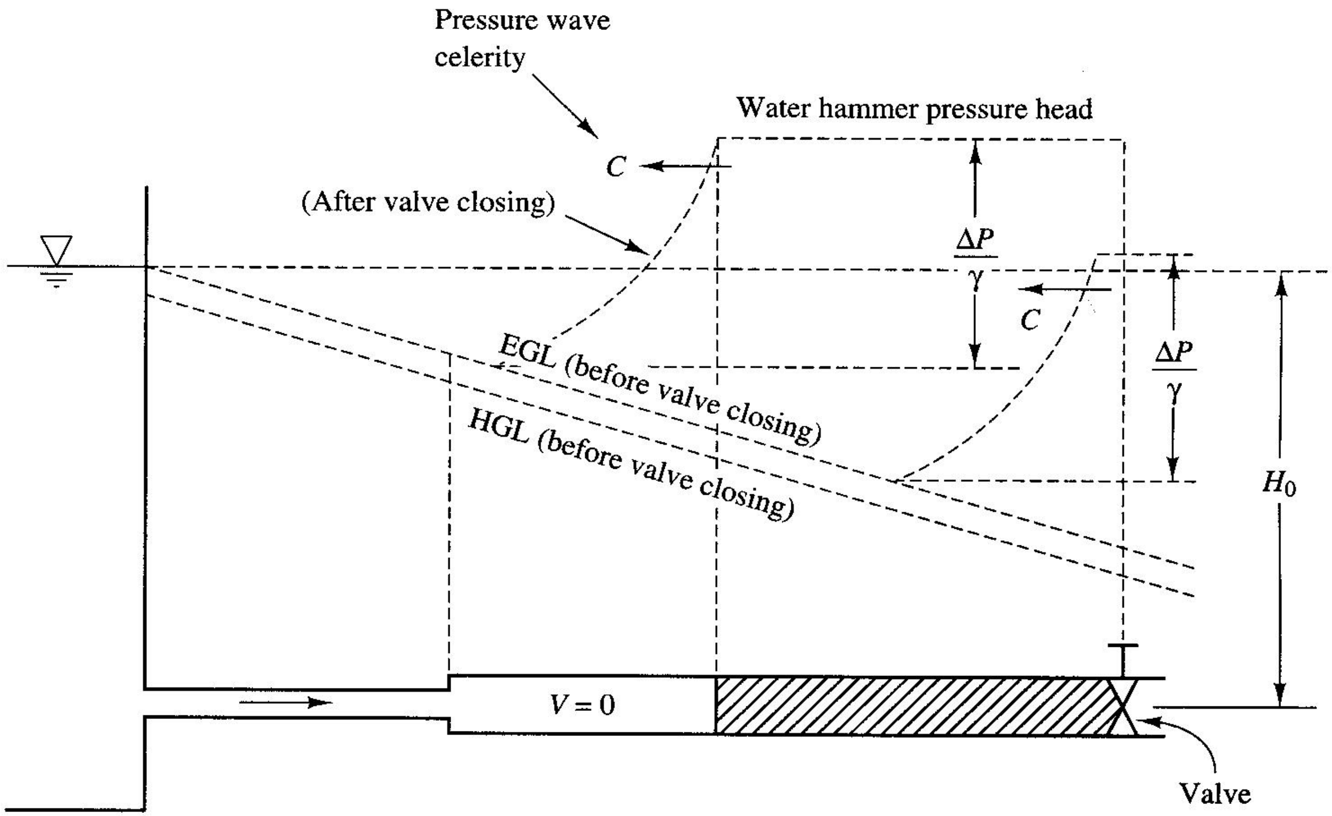
$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{D}{E_p e}$$

Pipe Material	$E_p (N/m^2)$	E_p (psi)
Aluminum	$7.0 \cdot 10^{10}$	$10 \cdot 10^6$
Brass, Bronze	$9.0 \cdot 10^{10}$	$13 \cdot 10^6$
Cast-iron, gray	$1.1 \cdot 10^{11}$	$16 \cdot 10^6$
Cast-iron, malleable	$1.6 \cdot 10^{11}$	$23 \cdot 10^6$
Concrete, reinforced	$1.6 \cdot 10^{11}$	$25 \cdot 10^6$
Glass	$7.0 \cdot 10^{10}$	$10 \cdot 10^6$
Lead	$3.1 \cdot 10^8$	$4.5 \cdot 10^4$
Lucite	$2.8 \cdot 10^8$	$4 \cdot 10^4$
Copper	$9.7 \cdot 10^{10}$	$14 \cdot 10^6$
Rubber, vulcanized	$1.4 \cdot 10^{10}$	$2 \cdot 10^6$
Steel	$1.9 \cdot 10^{11}$	$28 \cdot 10^6$

The Maximum pressure created by the water hammer

The total pressure experienced by the pipe is

$$P = \Delta P + P_0$$



Case 1: Gradual Closure of Valve

- If the time of closure $\boxed{t_c \succ \frac{2L}{C}}$, then the closure is said to be gradual and the increased pressure is

$$\boxed{\Delta P = \frac{\rho L V_0}{t}}$$

where,

- V_0 = initial velocity of water flowing in the pipe before pipe closure
- t = time of closure.
- L = length of pipe.
- ρ = water density.

- The pressure head caused by the water hammer is

$$\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V_0}{\rho g t} = \frac{L V_0}{g t}$$

Another method **for closure time (t > 2 L/C)**

* L. Allievi, *The Theory of Water Hammer* (translated by E. E. Halmos), *Trans. ASME* (1929). The maximum water hammer pressure calculated by the Allievi formula is

$$\boxed{\Delta P = P_o\left(\frac{N}{2} + \sqrt{\frac{N^2}{4} + N}\right)}$$

where P_0 is the static-state pressure in the pipe, and

$$\boxed{N = \left(\frac{\rho L V_o}{P_o t}\right)}$$

Case 2: Sudden Closure of Valve and Pipe is Rigid

- If the time of closure $\boxed{t_c \leq \frac{2L}{C}}$, then the closure is said to be Sudden.
- The pressure head due caused by the water hammer is

$$\Delta P = \rho C V_0$$

$$\boxed{\Delta H = \frac{C V_0}{g}}$$

- But for rigid pipe $C = \sqrt{\frac{E_b}{\rho}}$ so:

$$\Delta H = \frac{V_0}{g}\sqrt{\frac{E_b}{\rho}}$$

$$\Delta P = V_0 \sqrt{E_b \rho}$$

Case 3: Sudden Closure of Valve and Pipe is Elastic

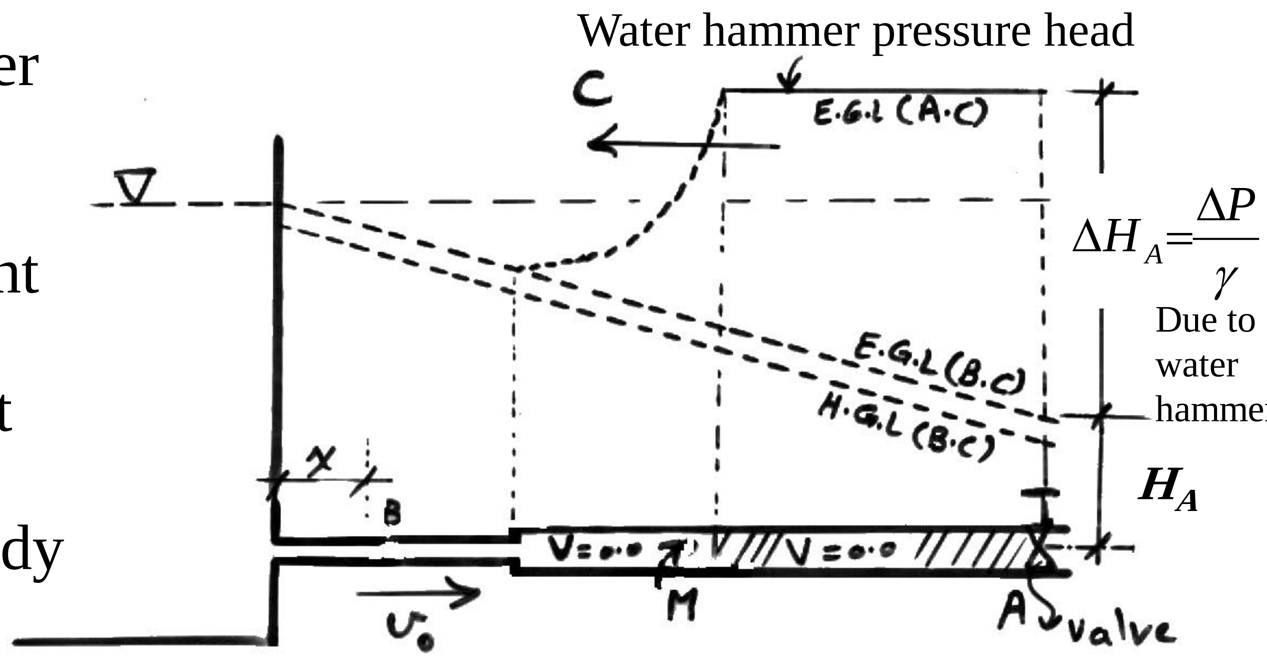
- If the time of closure $\boxed{t_c \leq \frac{2L}{C}}$, then the closure is said to be Sudden.
- The pressure head caused by the water hammer is

$$\Delta P = \rho C V_0 \qquad\qquad \boxed{\Delta H = \frac{C V_0}{g}}$$

- But for elastic pipe $C = \sqrt{\frac{E_c}{\rho}}$ so: $\Delta H = \frac{V_0}{g}\sqrt{\frac{1}{\rho(\frac{1}{E_b} + \frac{DK}{E_p e})}}$

$$\Delta P = V_0 \sqrt{\frac{\rho}{(\frac{1}{E_b} + \frac{DK}{E_p e})}}$$

- Applying the water hammer formulas we can determine the energy gradient line and the hydraulic gradient line for the pipe system under steady flow condition.



Water Hammer Pressure in a Pipeline

So the total pressure at any point M after closure (water hammer) *is*

$$\boxed{P_M = P_{M,before\,closure} + \Delta P}$$

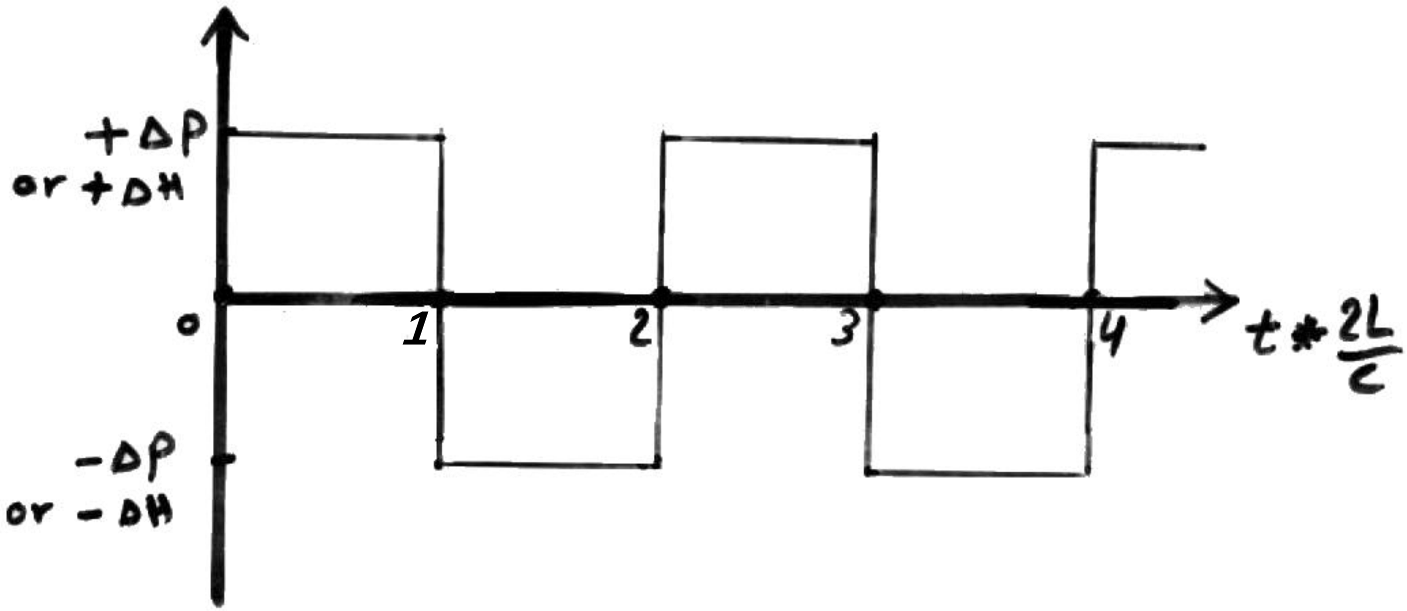
or

$$\boxed{H_M = H_{M,before\,closure} + \Delta H}$$

Time History of Pressure Wave (Water Hammer)

- The time history of the pressure wave for a specific point on the pipe is a graph that simply shows the relation between the pressure increase (ΔP) and time during the propagation of the water hammer pressure waves.

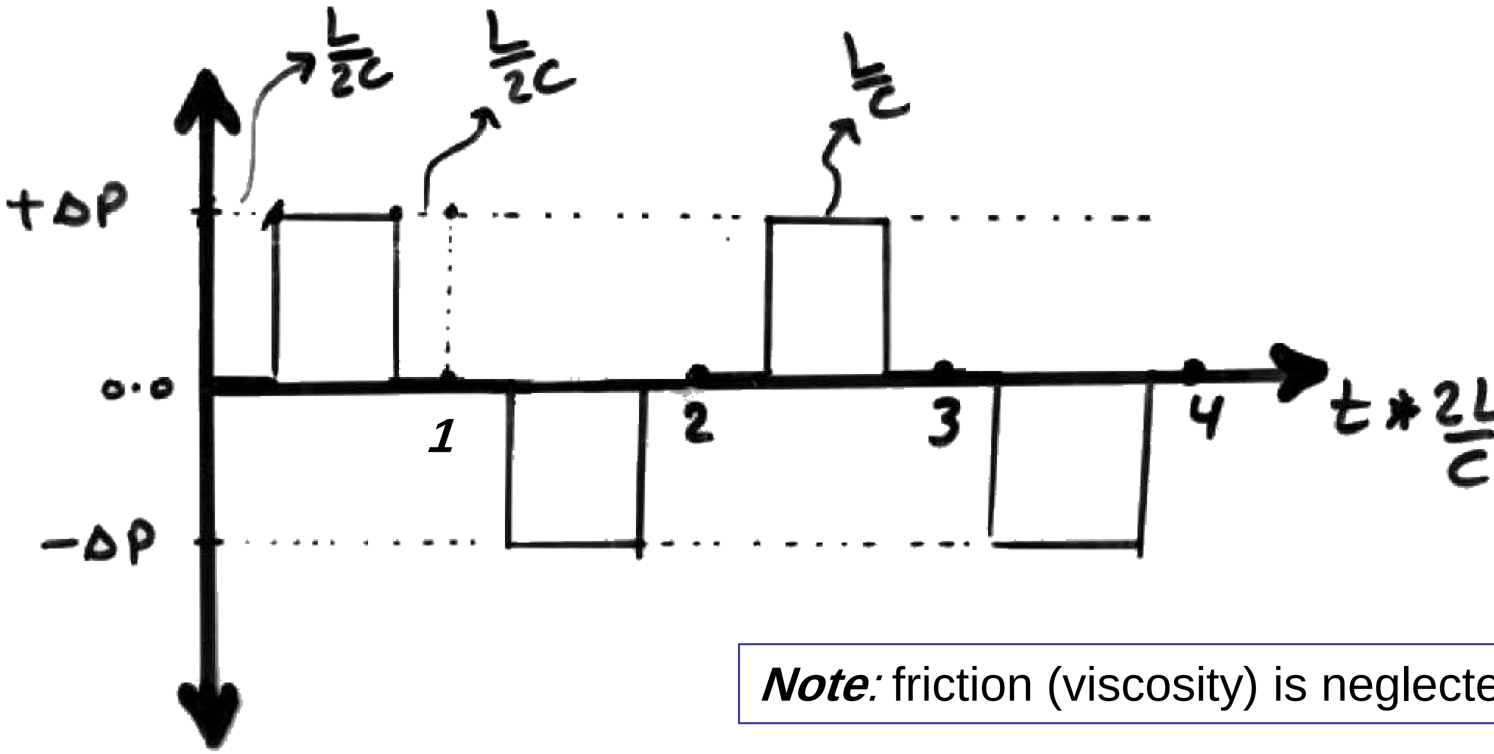
- For example, considering point "A" just to the left of the valve.



Time history for pressure at point "A" (after valve closure)

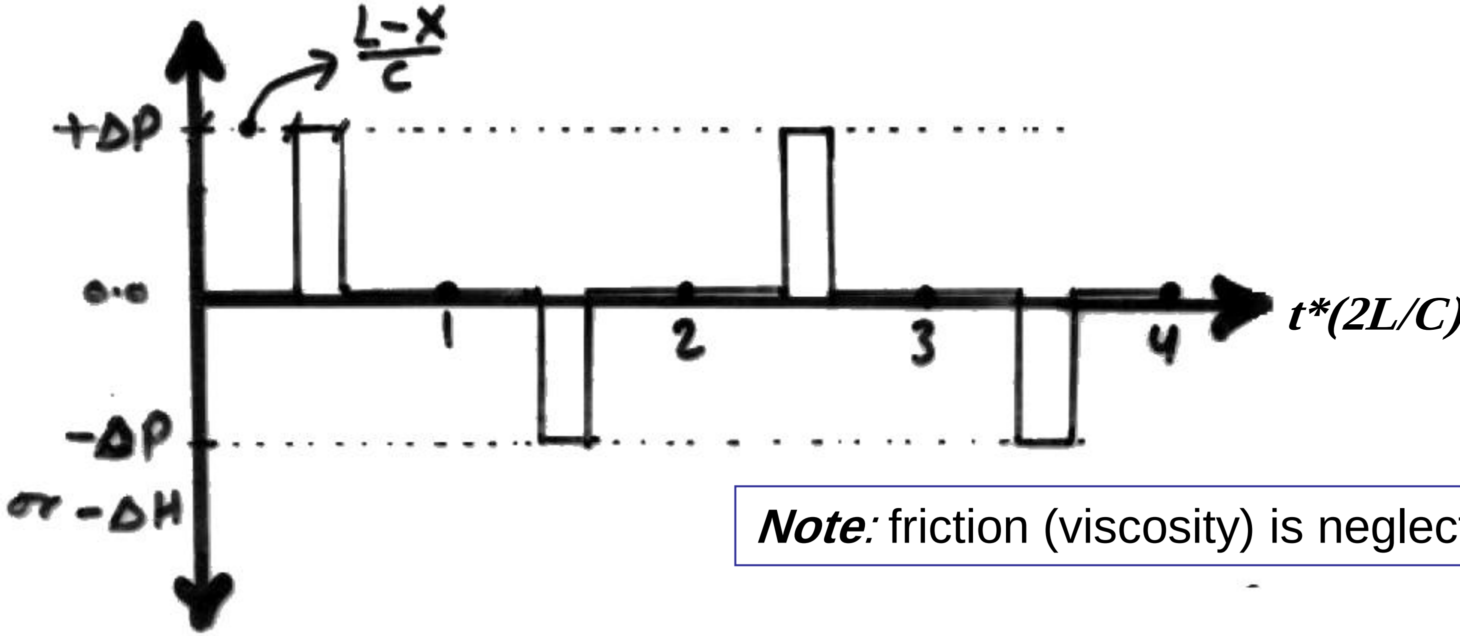
- ***Note:*** friction (viscosity) is neglected.

The time history for point "*M*" (at midpoint of the pipe)



Note: friction (viscosity) is neglected.

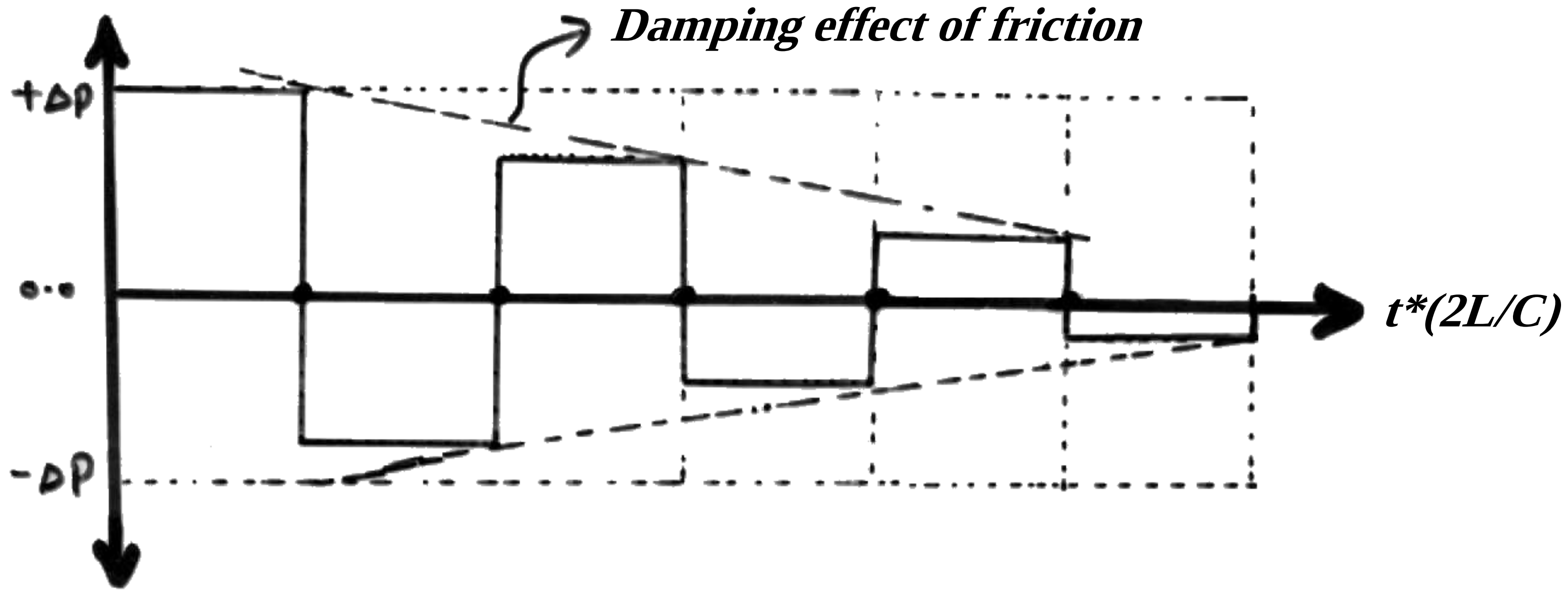
The time history for point *B* (at a distance x from the reservoir)



Note: friction (viscosity) is neglected.

This is a general graph where we can substitute any value for x (within the pipe length) to obtain the time history for that point.

In real practice friction effects are considered and hence a damping effect occurs and the pressure wave dies out, i.e.; energy is dissipated.



the time history for pressure at point "A" when friction (viscosity) is included

Stresses in the pipe wall

- After calculating the pressure increase due to the water hammer, we can find the stresses in the pipe wall:
- Circumferential (hoop) stress "f_c": $$f_c = \frac{P\,D}{2\,t_p}$$
- Longitudinal stress "f_L": $$f_L = \frac{P\,D}{4\,t_p}$$

where:

D = pipe inside diameter

t_p = pipe wall thickness

$P = P_0 + \Delta P =$ total pressure

= initial pressure (before valve closure) + pressure increase due water hammer.

Example 1

A steel pipe 5000 ft long laid on a uniform slope has an 18-in. diameter and a 2-in. wall thickness. The pipe carries water from a reservoir and discharges it into the air at an elevation 150 ft below the reservoir free surface. A valve installed at the downstream end of the pipe allows a flow rate of 25 cfs. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Neglect longitudinal stresses.

Solution

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{D}{E_p e}$$

where $E_b = 3.0 \cdot 10^5$ psi, and $E_p = 2.8 \cdot 10^7$ psi, The above equation may thus be written as

$$\frac{1}{E_c} = \frac{1}{3.0 \cdot 10^5} + \frac{18}{(2.8 \cdot 10^7) \cdot 2.0}$$

Hence,

$$E_c = 2.74 \cdot 10^5 \text{ psi}$$

$$C = \sqrt{\frac{E_c}{\rho}} = \sqrt{\frac{2.74 \cdot 10^5(144)}{1.94}} = 4510 \text{ ft/sec}$$

The time required for the wave to return to the valve is

$$t = \frac{2L}{C} = \frac{2 \cdot 5000}{4510} = 2.22 \text{ sec}$$

Because the water velocity in the pipe before valve closure is

$$V_0 = \frac{25}{\frac{\pi}{4} \cdot (1.5)^2} = 14.1 \text{ ft/sec}$$

the maximum water hammer pressure at the valve can be calculated.

$$\Delta P = \rho V_0 C = 1.94 \cdot 14.1 \cdot 4510 = 1.23 \cdot 10^5 \text{ lb/ft}^2 (854 \text{ psi})$$

To keep the water hammer pressure within manageable limits, valves are commonly design with closure times considerably greater than $2L/C$

Example

- A cast iron pipe with 20 cm diameter and 15 mm wall thickness is carrying water from a reservoir. At the end of the pipe a valve is installed to regulate the flow. The following data are available:
- *e = 0.15 mm (absolute roughness) ,*
- *L = 1500 m (length of pipe),*
- *Q = 40 l/sec (design flow) ,*
- *K =* 2.1×10^9 *N/m*^{2} *(bulk modulus of water),*
- *E =* 2.1×10^{11} *N/m*^{2} *(modulus of elasticity of cast iron),*
- $\varepsilon = 0.25$ *(poisson's ratio).*
- $\rho = 1000$ *kg/m*^{3}
- $T = 15^0$ *C.*

Find ΔP , ΔH , f_c , and f_L due to the water hammer produced for the following cases:

a) Assuming <u>rigid</u> pipe when $t_c = 10$ seconds, and $t_c = 1.5$ seconds.

b) Assuming <u>elastic</u> pipe when $t_c = 10$ seconds, and $t_c = 1.5$ seconds, if:

   1. the pipe is free to move longitudinally,
   2. the pipe is anchored at both ends and throughout its length,
   3. the pipe has expansion joints.

c) Draw the time history of the pressure wave for the case (b-3) at:

   1. a point just to the left of the valve, and
   2. a distance x = 0.35 L from the reservoir.

d) Find the total pressure for all the cases in (b-3).