Thermodynamics: An Engineering Approach, 6th Edition
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Chapter 17
COMPRESSIBLE FLOW

Objectives

- Develop the general relations for compressible flows encountered when gases flow at high speeds.
- Introduce the concepts of *stagnation state*, *speed of sound*, and *Mach number* for a compressible fluid.
- Develop the relationships between the static and stagnation fluid properties for isentropic flows of ideal gases.
- Derive the relationships between the static and stagnation fluid properties as functions of specific-heat ratios and Mach number.
- Derive the effects of area changes for one-dimensional isentropic subsonic and supersonic flows.
- Solve problems of isentropic flow through converging and converging–diverging nozzles.
- Discuss the shock wave and the variation of flow properties across the shock wave.
- Develop the concept of duct flow with heat transfer and negligible friction known as *Rayleigh flow*.
- Examine the operation of steam nozzles commonly used in steam turbines.

STAGNATION PROPERTIES

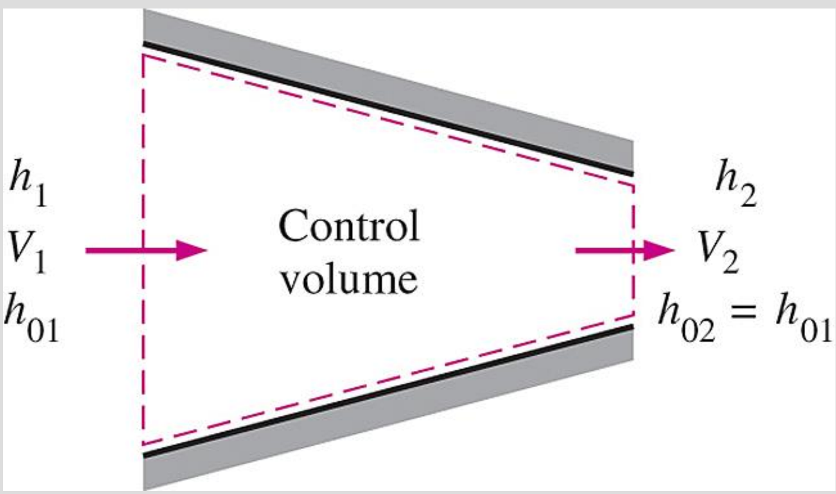
Stagnation (or **total**) **enthalpy**

$$h_0 = h + \frac{V^2}{2} \qquad \text{(kJ/kg)}$$

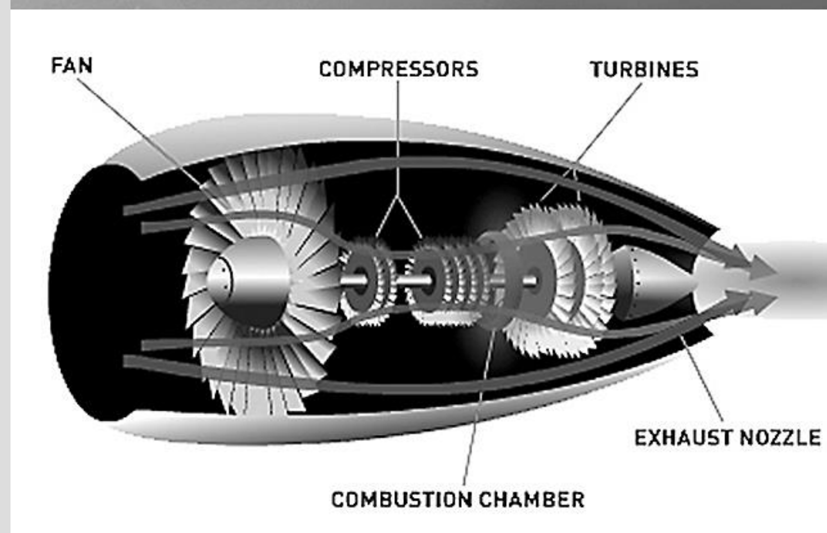
Static enthalpy: the ordinary enthalpy *h*

Energy balance (with no heat or work interaction, no change in potential energy)

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \longrightarrow \quad h_{01} = h_{02}$$



Steady flow of a fluid through an adiabatic duct.



Aircraft and jet engines involve high speeds, and thus the kinetic energy term should always be considered when analyzing them.

If the fluid were brought to a complete stop, the energy balance becomes

$$h_1 + \frac{V_1^2}{2} = h_2 = h_{02}$$

Stagnation enthalpy: The *enthalpy of a fluid when it is brought to rest adiabatically.*

During a stagnation process, the kinetic energy of a fluid is converted to enthalpy, which results in an increase in the fluid temperature and pressure.

The properties of a fluid at the stagnation state are called **stagnation properties** (stagnation temperature, stagnation pressure, stagnation density, etc.).

The stagnation state is indicated by the subscript 0.

Isentropic stagnation state: When the stagnation process is reversible as well as adiabatic (i.e., isentropic).

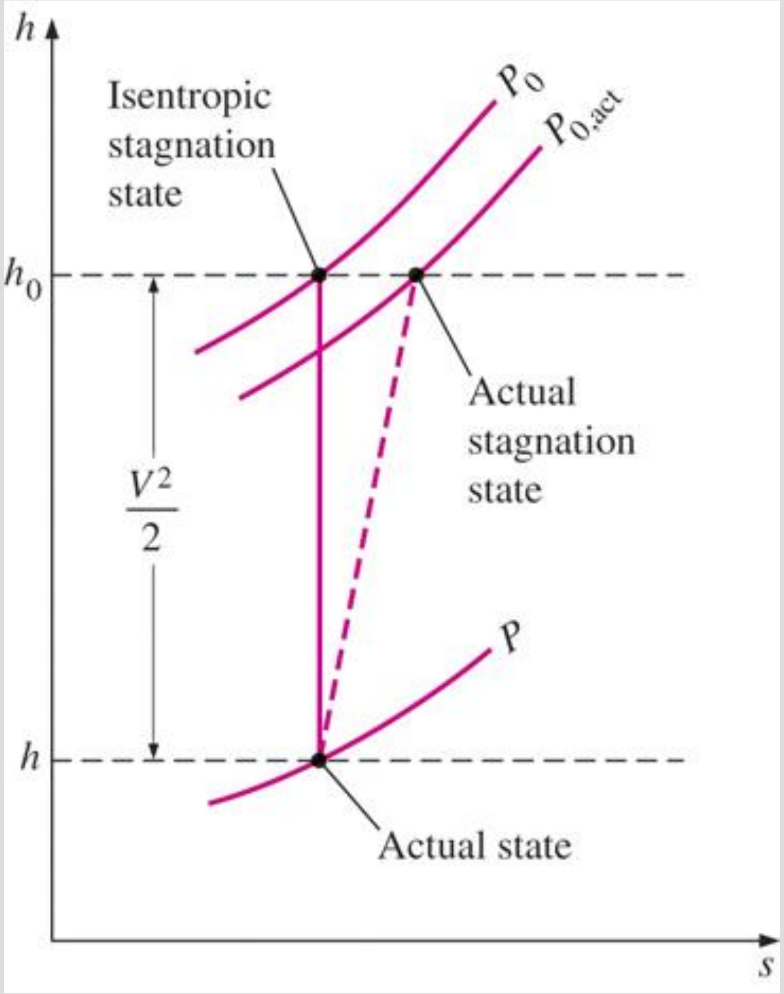
The stagnation processes are often approximated to be isentropic, and the isentropic stagnation properties are simply referred to as stagnation properties.

When the fluid is approximated as an *ideal gas* with constant specific heats

$$c_p T_0 = c_p T + \frac{V^2}{2} \rightarrow T_0 = T + \frac{V^2}{2c_p}$$

T_0 is called the **stagnation** (or **total**) **temperature**, and it represents *the temperature an ideal gas attains when it is brought to rest adiabatically.*

The term $V^2/2c_p$ corresponds to the temperature rise during such a process and is called the **dynamic temperature**.



The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an *h-s* diagram.

The pressure a fluid attains when brought to rest isentropically is called the **stagnation pressure** P_0.

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

Stagnation density ρ_0

$$\rho = 1/v$$

$$Pv^k = P_0 v_0^k$$

$$\rightarrow \quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

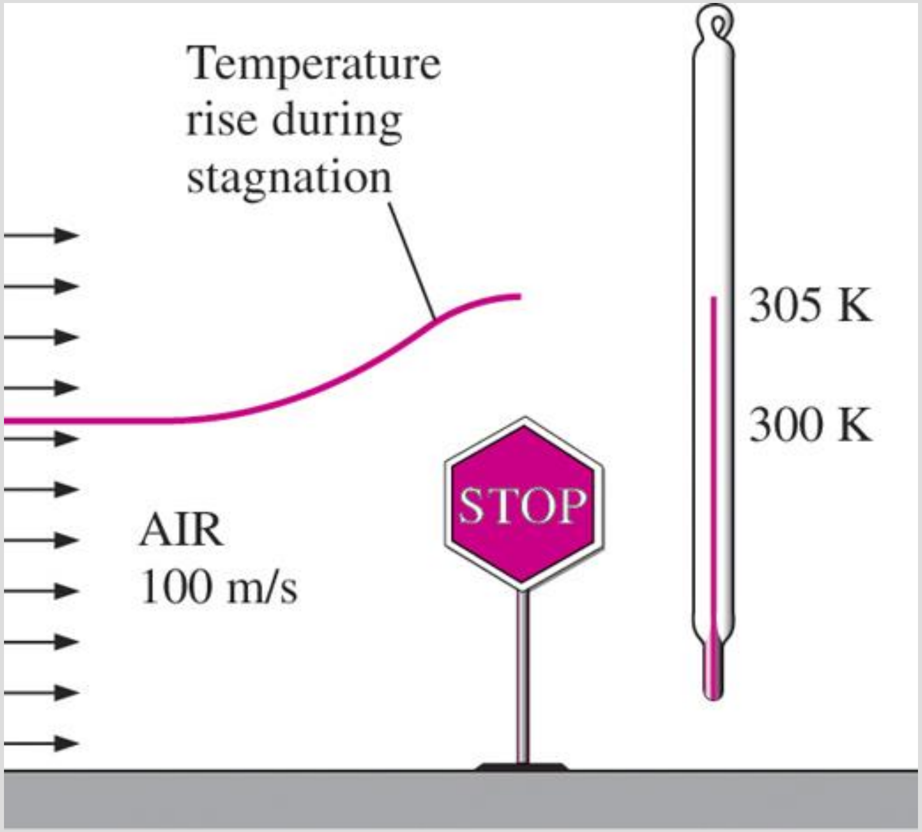
When stagnation enthalpies are used, the energy balance for a single-stream, steady-flow device

$$\dot{E}_{in} = \dot{E}_{out}$$

$$q_{in} + w_{in} + (h_{01} + gz_1) = q_{out} + w_{out} + (h_{02} + gz_2)$$

When the fluid is approximated as an *ideal gas* with constant specific heats

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = c_p(T_{02} - T_{01}) + g(z_2 - z_1)$$

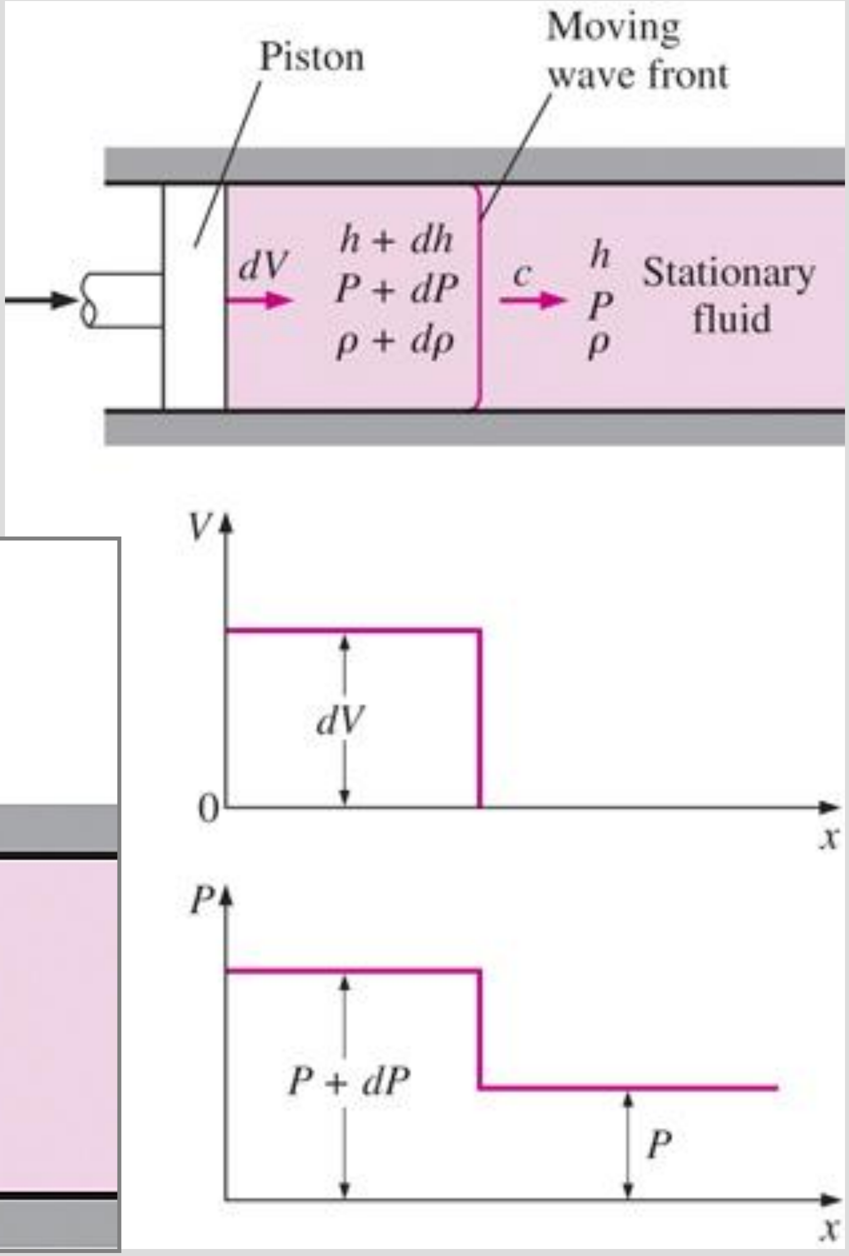


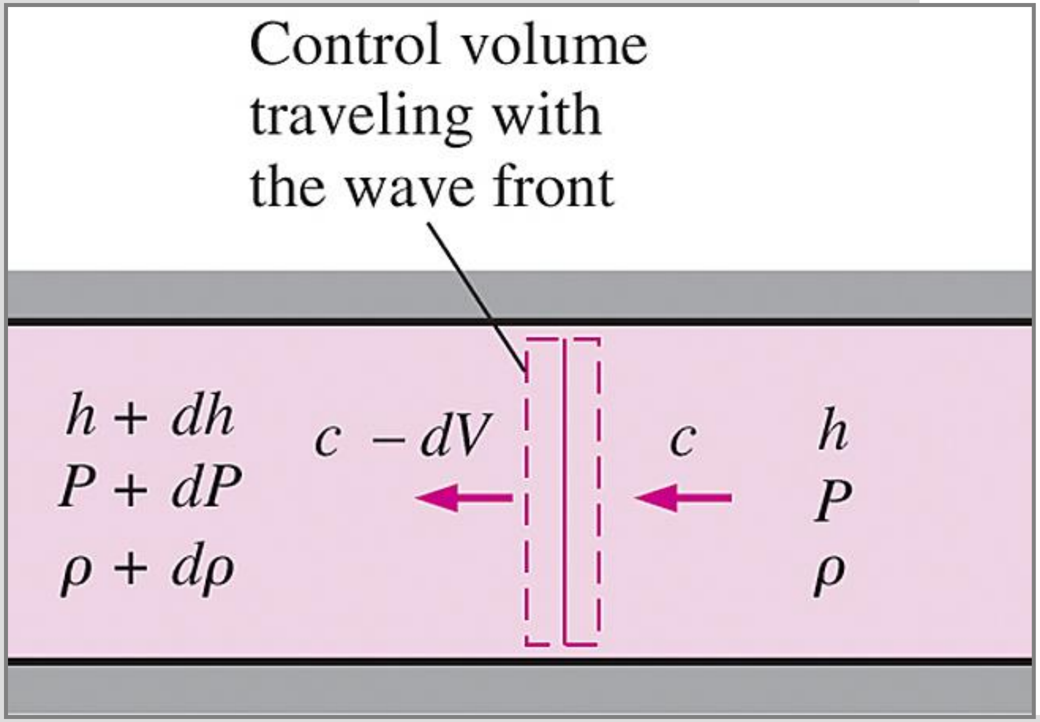
The temperature of an ideal gas flowing at a velocity V rises by $V^2/2c_p$ when it is brought to a complete stop.

SPEED OF SOUND AND MACH NUMBER

Speed of sound (or the **sonic speed**): The speed at which an infinitesimally small pressure wave travels through a medium.

To obtain a relation for the speed of sound in a medium, the systems in the figures are considered.





$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

Speed of sound

For an ideal gas $P = \rho RT$

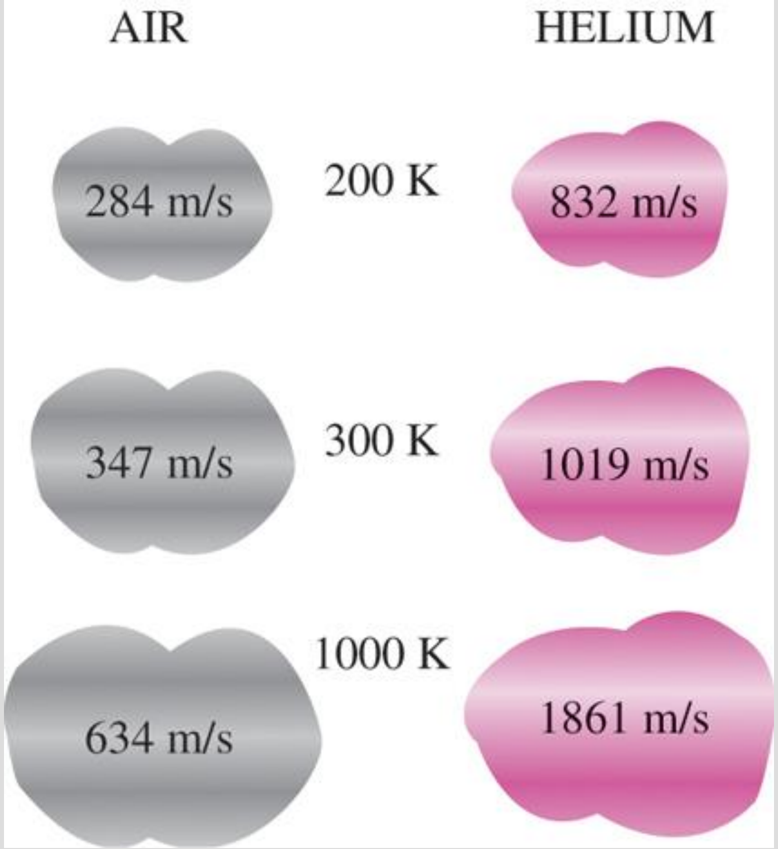
$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T = k\left[\frac{\partial(\rho RT)}{\partial \rho}\right]_T = kRT$$

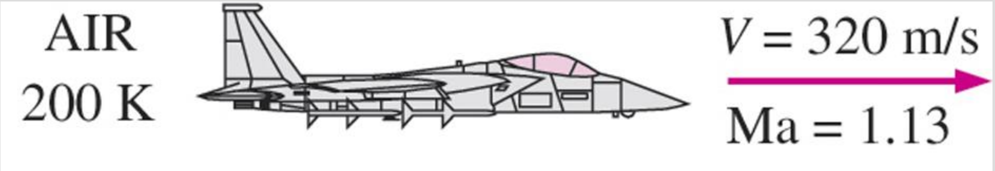
$$c = \sqrt{kRT}$$

$$\text{Ma} = \frac{V}{c}$$

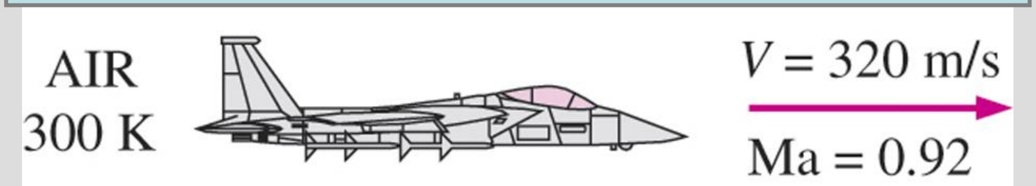
Mach number

The speed of sound changes with temperature and varies with the fluid.





The Mach number can be different at different temperatures even if the velocity is the same.

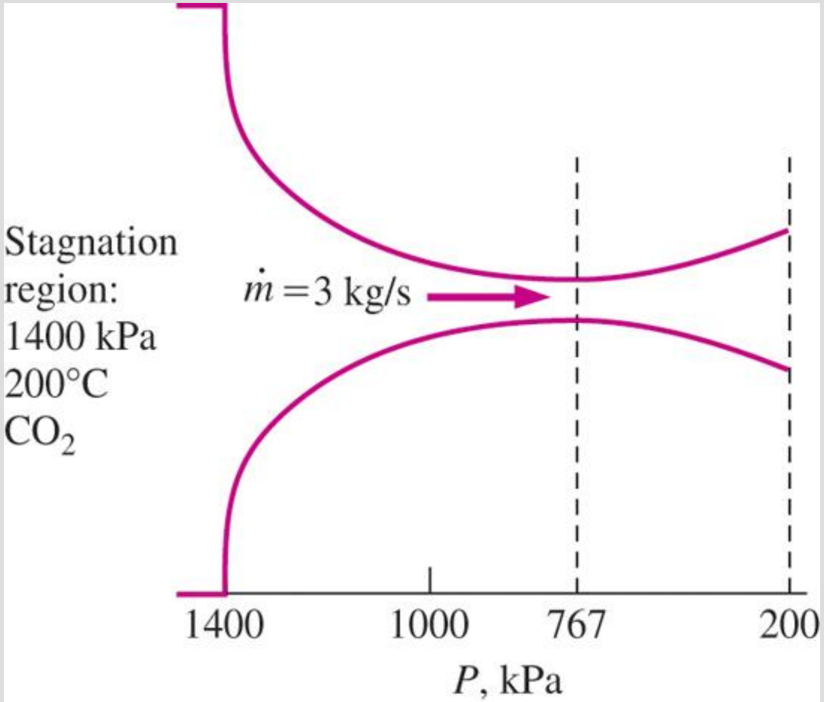


Ma = 1	Sonic flow
Ma < 1	Subsonic flow
Ma > 1	Supersonic flow
Ma >> 1	Hypersonic flow
Ma ≅ 1	Transonic flow

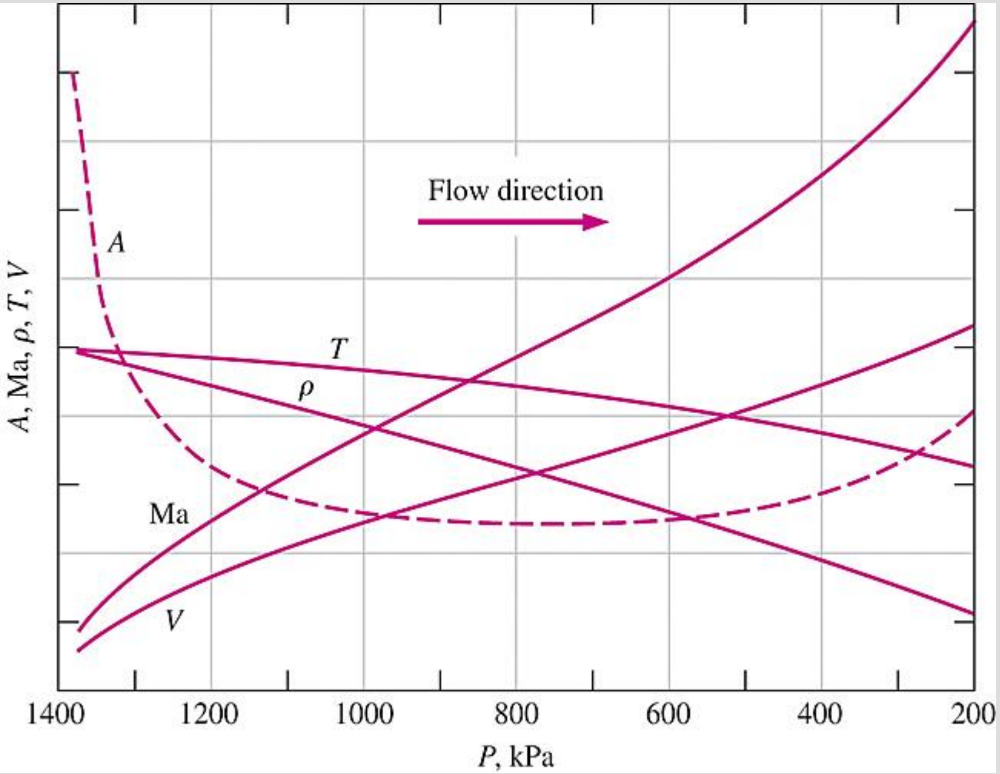
ONE-DIMENSIONAL ISENTROPIC FLOW

During fluid flow through many devices such as nozzles, diffusers, and turbine blade passages, flow quantities vary primarily in the flow direction only, and the flow can be approximated as one-dimensional isentropic flow with good accuracy.

EXAMPLE



A converging–diverging nozzle.



Variation of normalized fluid properties and cross-sectional area along a duct as the pressure drops from 1400 to 200 kPa.

We note from Example that the flow area decreases with decreasing pressure up to a critical-pressure value (Ma = 1), and then it begins to increase with further reductions in pressure.

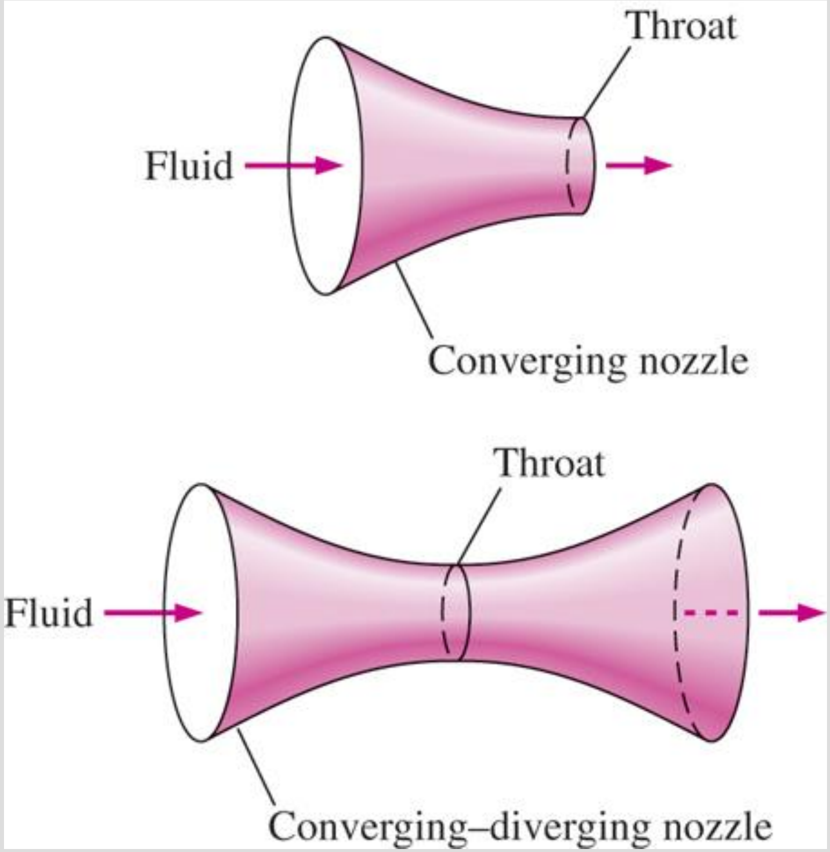
The Mach number is unity at the location of smallest flow area, called the **throat.**

The velocity of the fluid keeps increasing after passing the throat although the flow area increases rapidly in that region.

This increase in velocity past the throat is due to the rapid decrease in the fluid density.

The flow area of the duct considered in this example first decreases and then increases. Such ducts are called **converging–diverging nozzles**.

These nozzles are used to accelerate gases to supersonic speeds and should not be confused with *Venturi nozzles,* which are used strictly for incompressible flow.



The cross section of a nozzle at the smallest flow area is called the *throat*.

Variation of Fluid Velocity with Flow Area

In this section, the relations for the variation of static-to-stagnation property ratios with the Mach number for pressure, temperature, and density are provided.

$$\frac{dA}{A} = \frac{dP}{\rho V^2}\left(1 - \mathrm{Ma}^2\right)$$

This relation describes the variation of pressure with flow area.

At subsonic velocities, the pressure decreases in converging ducts (subsonic nozzles) and increases in diverging ducts (subsonic diffusers).

At supersonic velocities, the pressure decreases in diverging ducts (supersonic nozzles) and increases in converging ducts (supersonic diffusers).

CONSERVATION OF ENERGY
(steady flow, $w = 0$, $q = 0$, $\Delta \mathrm{pe} = 0$)

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or

$$h + \frac{V^2}{2} = \text{constant}$$

Differentiate,

$$dh + V\,dV = 0$$

Also,

$$\overset{0\ \text{(isentropic)}}{T\,ds} = dh - v\,dP$$

$$dh = v\,dP = \frac{1}{\rho}\,dP$$

Substitute,

$$\frac{dP}{\rho} + V\,dV = 0$$

Derivation of the differential form of the energy equation for steady isentropic flow.

$$\frac{dA}{A} = -\frac{dV}{V}(1 - \mathrm{Ma}^2)$$

This equation governs the shape of a nozzle or a diffuser in subsonic or supersonic isentropic flow.

For subsonic flow $(\mathrm{Ma} < 1)$, $\quad \dfrac{dA}{dV} < 0$

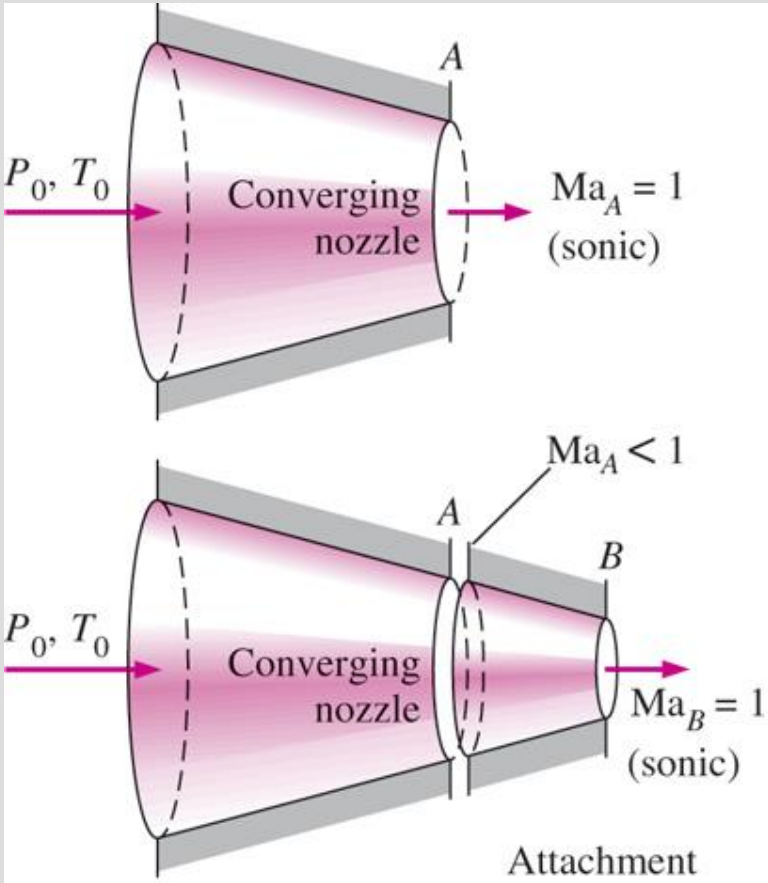
For supersonic flow $(\mathrm{Ma} > 1)$, $\quad \dfrac{dA}{dV} > 0$

For sonic flow $(\mathrm{Ma} = 1)$, $\quad \dfrac{dA}{dV} = 0$

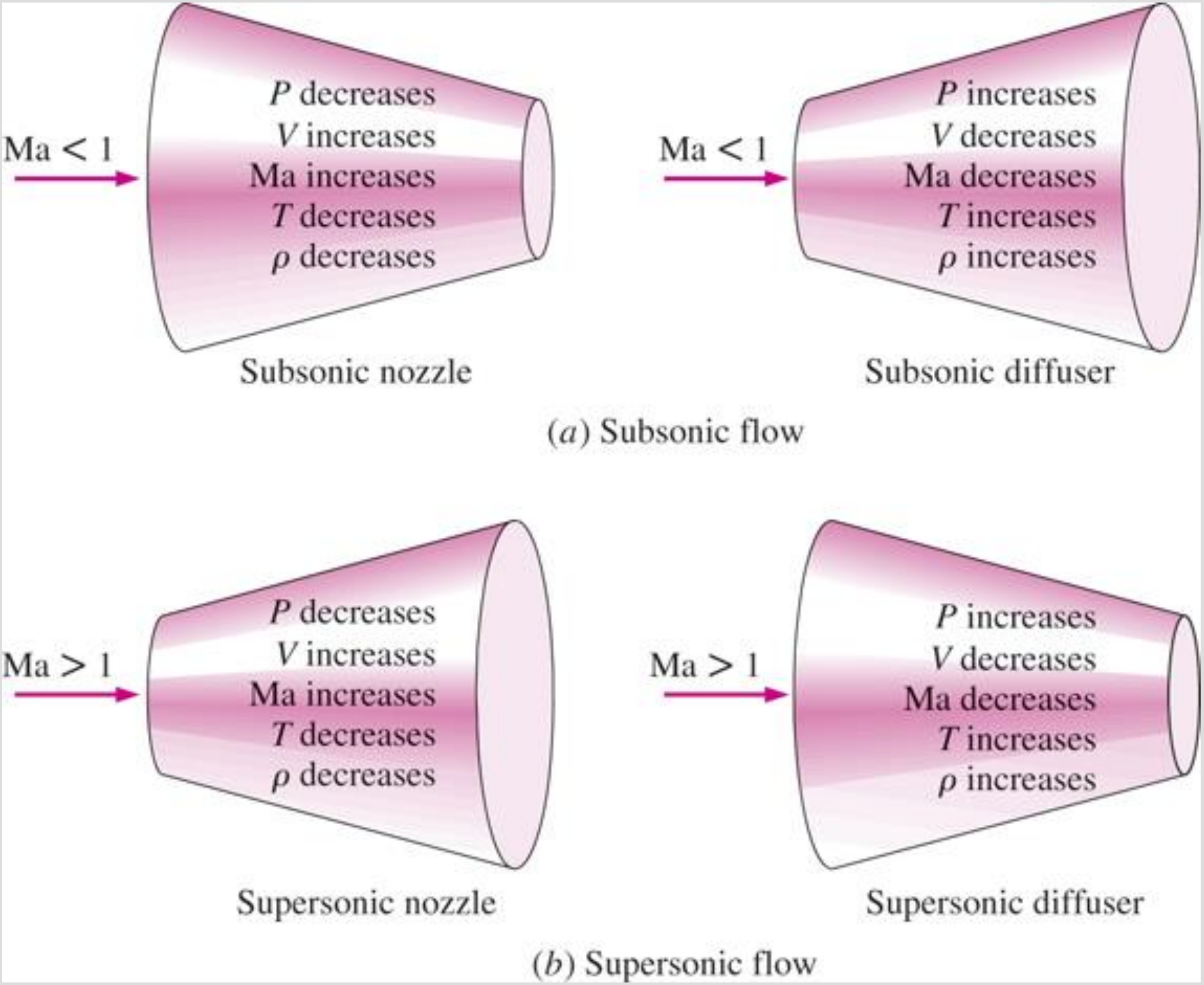
The proper shape of a nozzle depends on the highest velocity desired relative to the sonic velocity.

To accelerate a fluid, we must use a converging nozzle at subsonic velocities and a diverging nozzle at supersonic velocities.

To accelerate a fluid to supersonic velocities, we must use a converging–diverging nozzle.



We cannot obtain supersonic velocities by attaching a converging section to a converging nozzle. Doing so will only move the sonic cross section farther downstream and decrease the mass flow rate.



Variation of flow properties in subsonic and supersonic nozzles and diffusers.

Property Relations for Isentropic Flow of Ideal Gases

The relations between the static properties and stagnation properties of an ideal gas with constant specific heats

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{k/(k-1)}$$

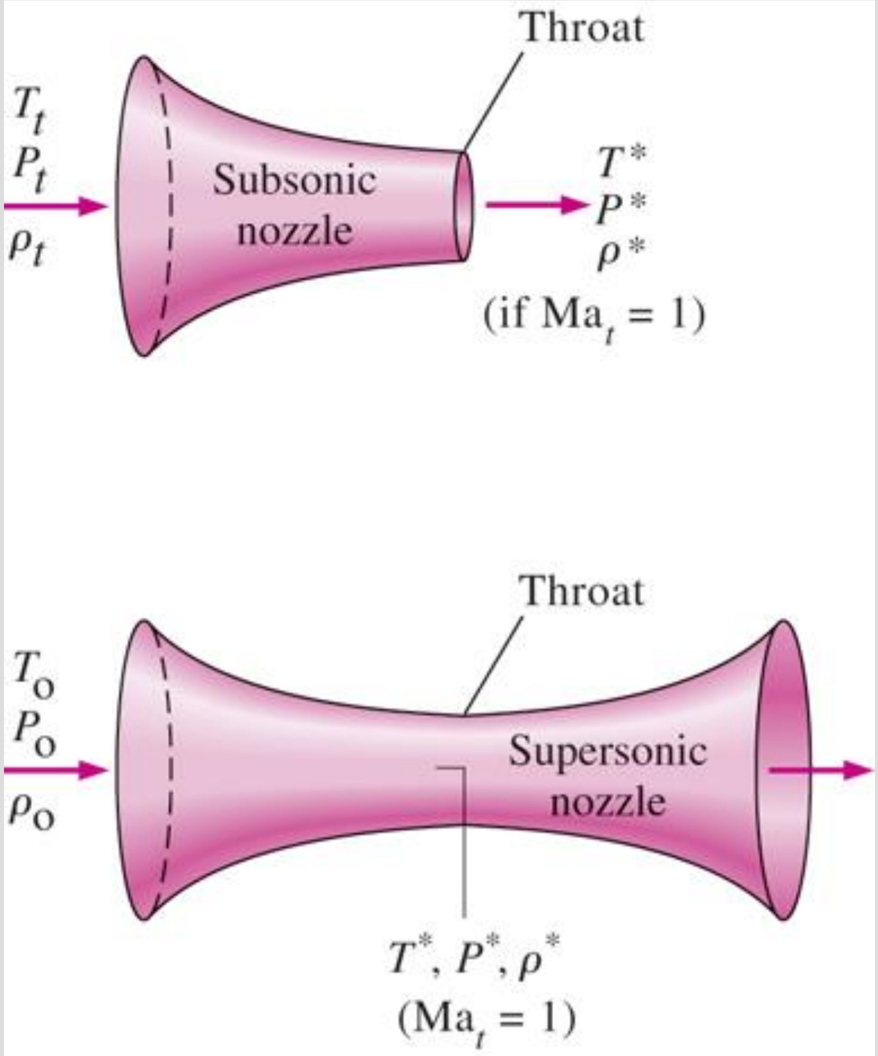
$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}^2\right]^{1/(k-1)}$$

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

Critical ratios (Ma=1)

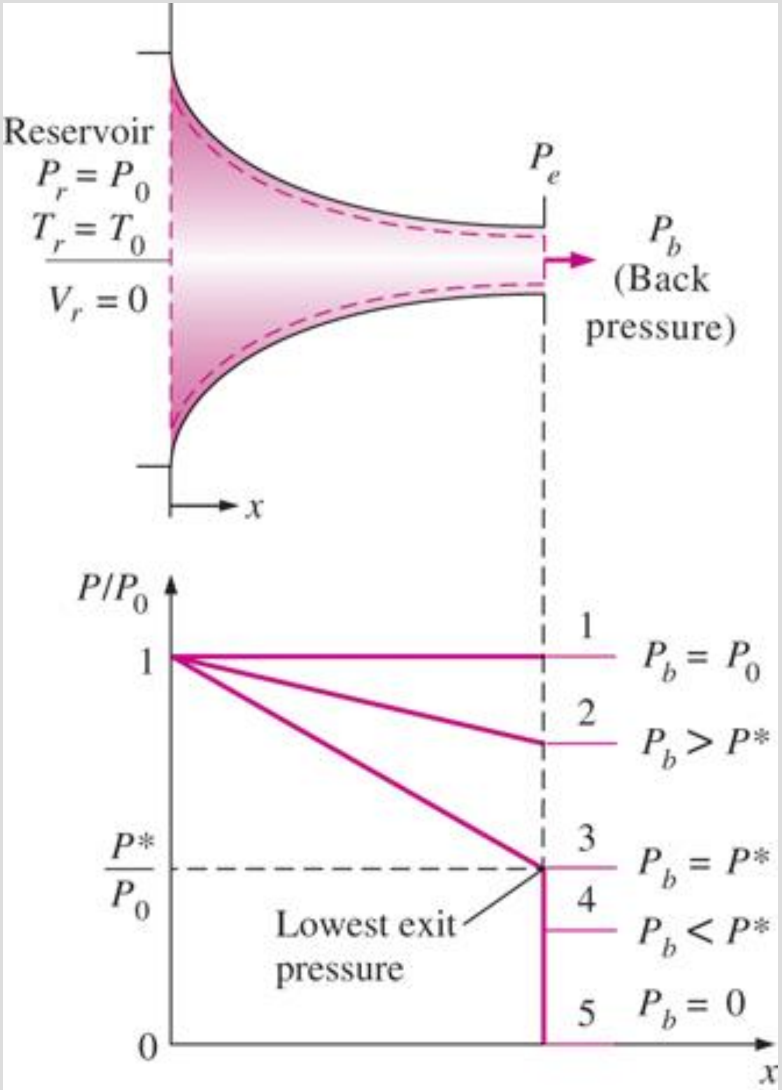


When $\mathrm{Ma}_t = 1$, the properties at the nozzle throat become the critical properties.

ISENTROPIC FLOW THROUGH NOZZLES

Converging or converging–diverging nozzles are found in steam and gas turbines and aircraft and spacecraft propulsion systems.

In this section we consider the effects of **back pressure** (i.e., the pressure applied at the nozzle discharge region) on the exit velocity, the mass flow rate, and the pressure distribution along the nozzle.



The effect of back pressure on the pressure distribution along a converging nozzle.

Converging Nozzles

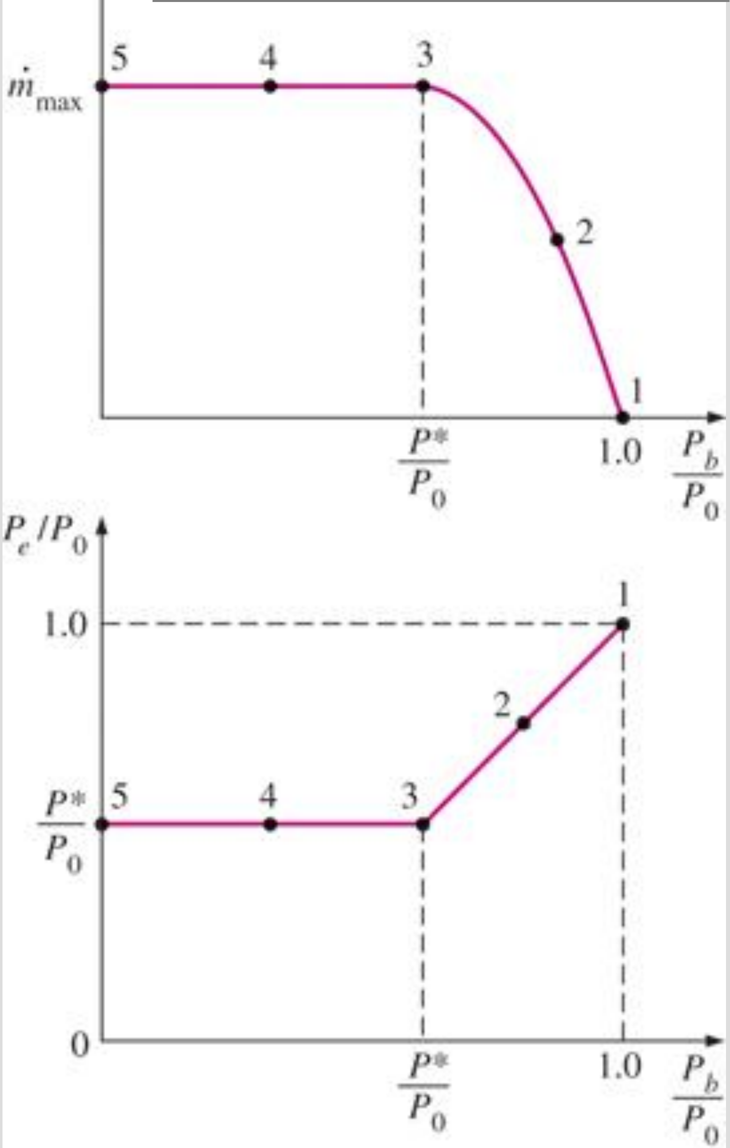
Mass flow rate through a nozzle

$$\dot{m} = \frac{A\mathrm{Ma}P_0\sqrt{k/(RT_0)}}{[1 + (k - 1)\mathrm{Ma}^2/2]^{(k+1)/[2(k-1)]}}$$

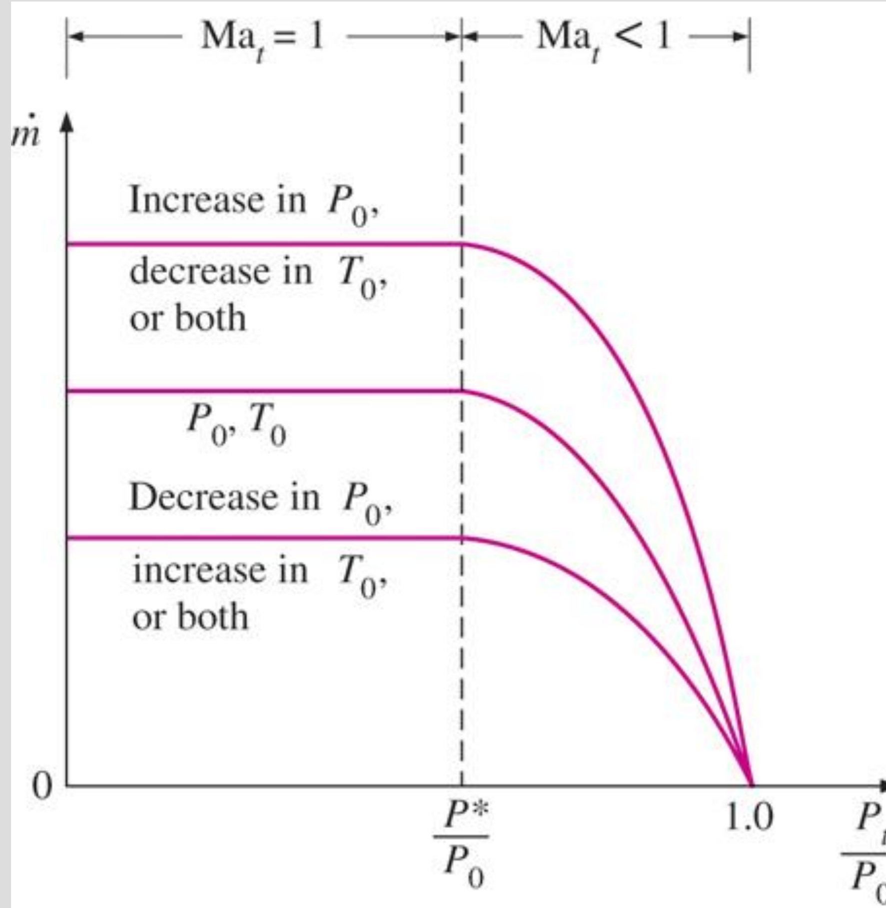
Maximum mass flow rate

$$\dot{m}_{\max} = A^*P_0\sqrt{\frac{k}{RT_0}\left(\frac{2}{k + 1}\right)^{(k+1)/[2(k-1)]}}$$

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$



The effect of back pressure P_b on the mass flow rate and the exit pressure P_e of a converging nozzle.



The variation of the mass flow rate through a nozzle with inlet stagnation properties.

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{(k+1)/[2(k-1)]}$$

$$\mathrm{Ma}^* = \frac{V}{c^*}$$

$$\mathrm{Ma}^* = \frac{V}{c}\frac{c}{c^*} = \frac{\mathrm{Ma}\,c}{c^*} = \frac{\mathrm{Ma}\sqrt{kRT}}{\sqrt{kRT^*}} = \mathrm{Ma}\sqrt{\frac{T}{T^*}}$$

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}}$$

Ma* is the local velocity nondimensionalized with respect to the sonic velocity at the *throat.*

Ma is the local velocity nondimensionalized with respect to the *local* sonic velocity.

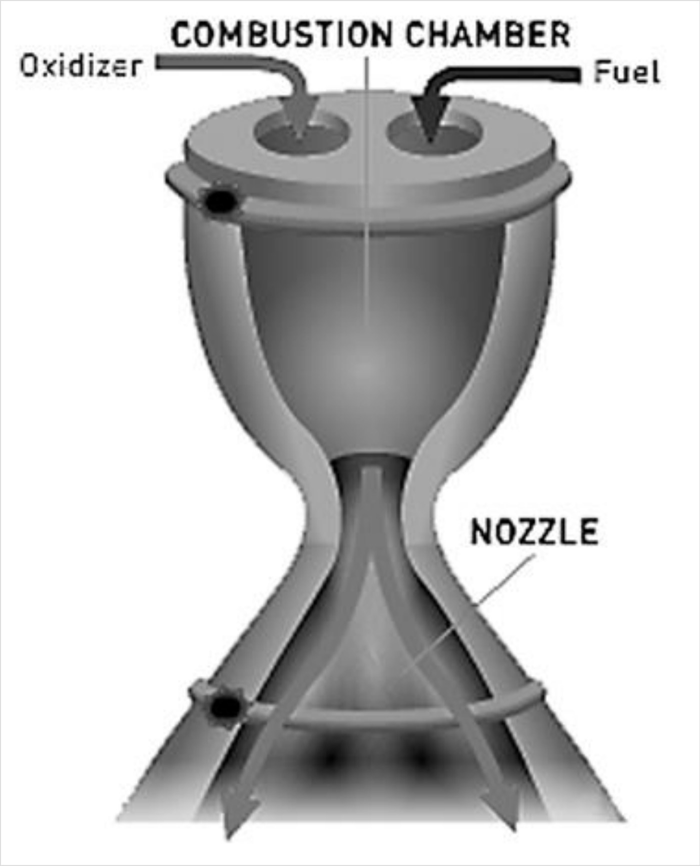
Ma	Ma*	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$
⋮	⋮	⋮	⋮	⋮	⋮
0.90	0.9146	1.0089	0.5913	⋮	⋮
1.00	1.0000	1.0000	0.5283	⋮	⋮
1.10	1.0812	1.0079	0.4684	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

Various property ratios for isentropic flow through nozzles and diffusers are listed in Table A–32 for $k = 1.4$ for convenience.

Converging–Diverging Nozzles

The highest velocity in a converging nozzle is limited to the sonic velocity (Ma = 1), which occurs at the exit plane (throat) of the nozzle.

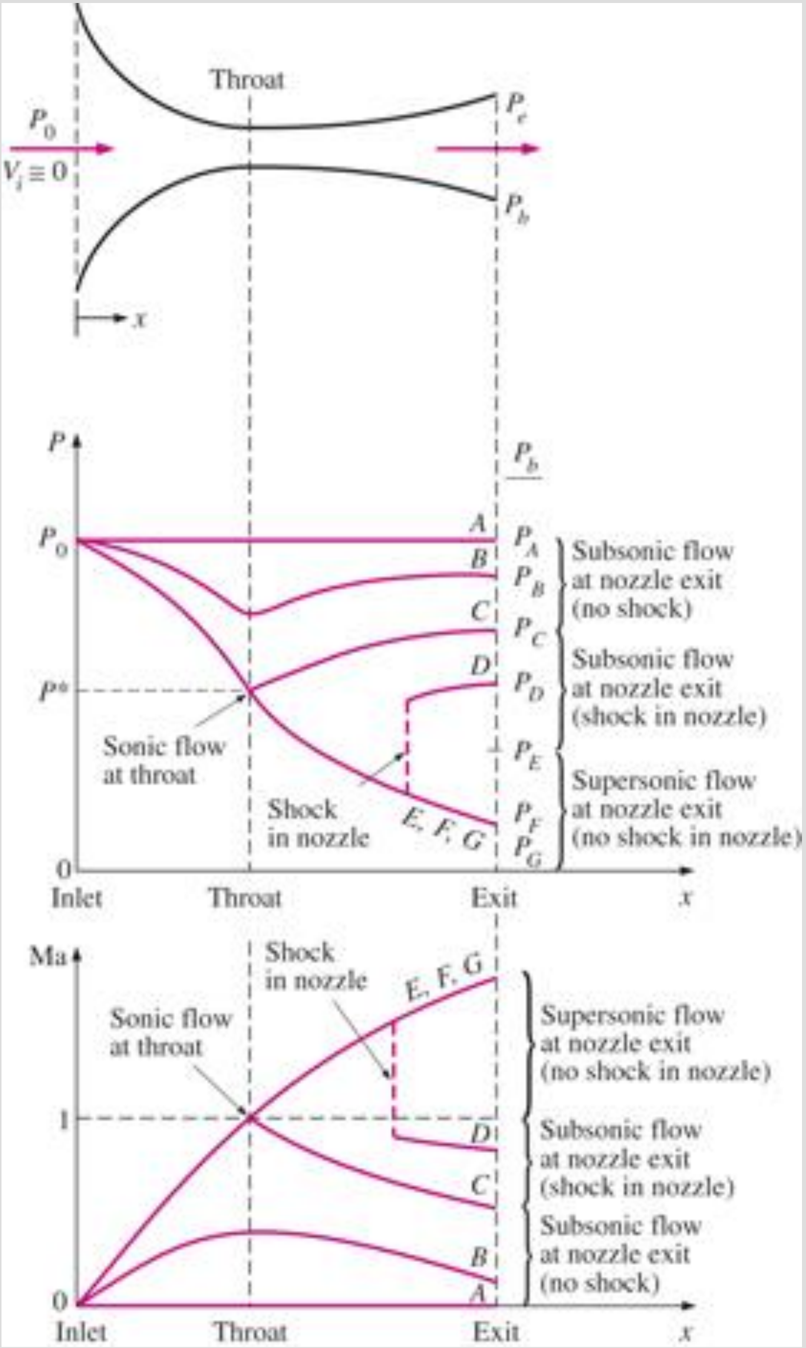
Accelerating a fluid to supersonic velocities (Ma > 1) can be accomplished only by attaching a diverging flow section to the subsonic nozzle at the throat (a converging–diverging nozzle), which is standard equipment in supersonic aircraft and rocket propulsion.



Converging–diverging nozzles are commonly used in rocket engines to provide high thrust.

When $P_b = P_0$ (case A), there will be no flow through the nozzle.

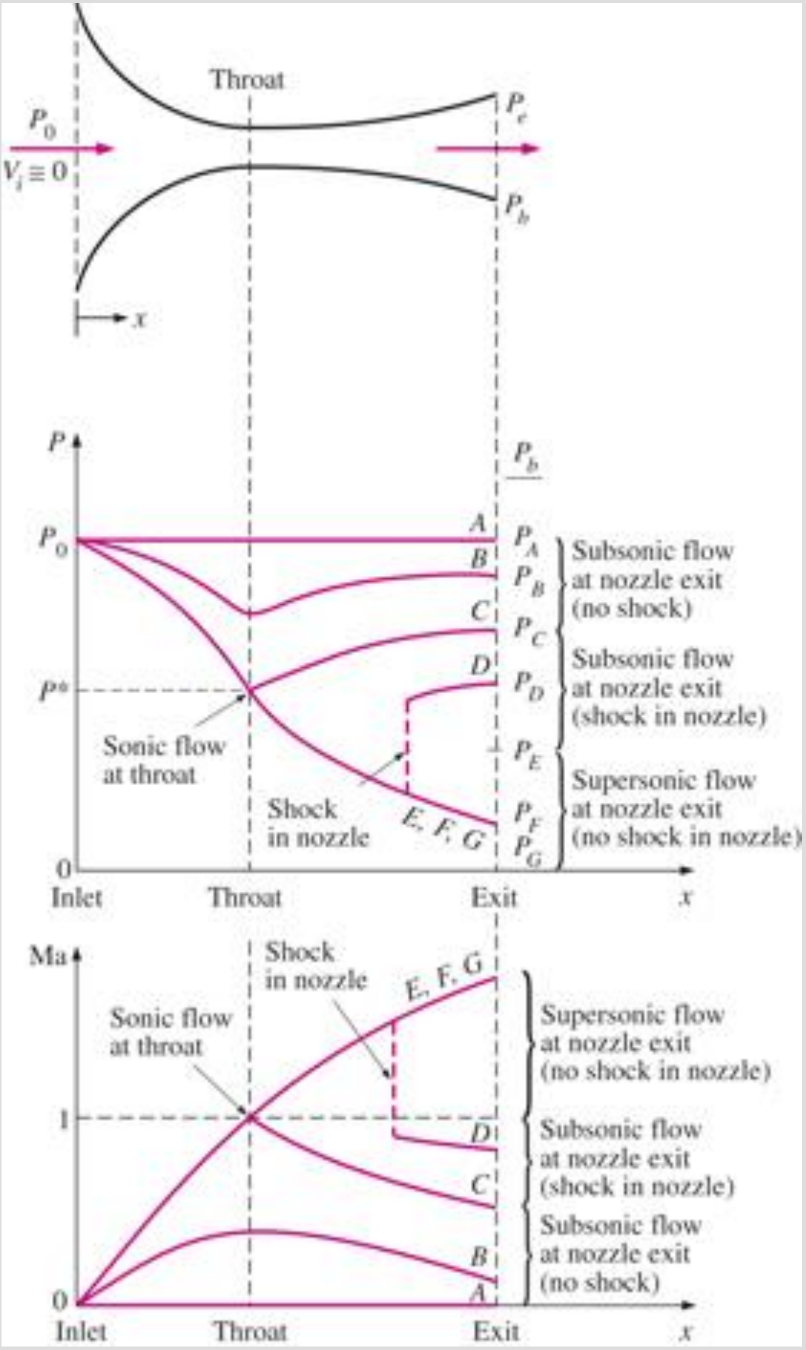
1. When $P_0 > P_b > P_C$, the flow remains subsonic throughout the nozzle, and the mass flow is less than that for choked flow. The fluid velocity increases in the first (converging) section and reaches a maximum at the throat (but Ma < 1). However, most of the gain in velocity is lost in the second (diverging) section of the nozzle, which acts as a diffuser. The pressure decreases in the converging section, reaches a minimum at the throat, and increases at the expense of velocity in the diverging section.



The effects of back pressure on the flow through a converging–diverging nozzle.

2. When $P_b = P_C$, the throat pressure becomes P^* and the fluid achieves sonic velocity at the throat. But the diverging section of the nozzle still acts as a diffuser, slowing the fluid to subsonic velocities. The mass flow rate that was increasing with decreasing P_b also reaches its maximum value.

3. When $P_C > P_b > P_E$, the fluid that achieved a sonic velocity at the throat continues accelerating to supersonic velocities in the diverging section as the pressure decreases. This acceleration comes to a sudden stop, however, as a **normal shock** develops at a section between the throat and the exit plane, which causes a sudden drop in velocity to subsonic levels and a sudden increase in pressure. The fluid then continues to decelerate further in the remaining part of the converging–diverging nozzle.

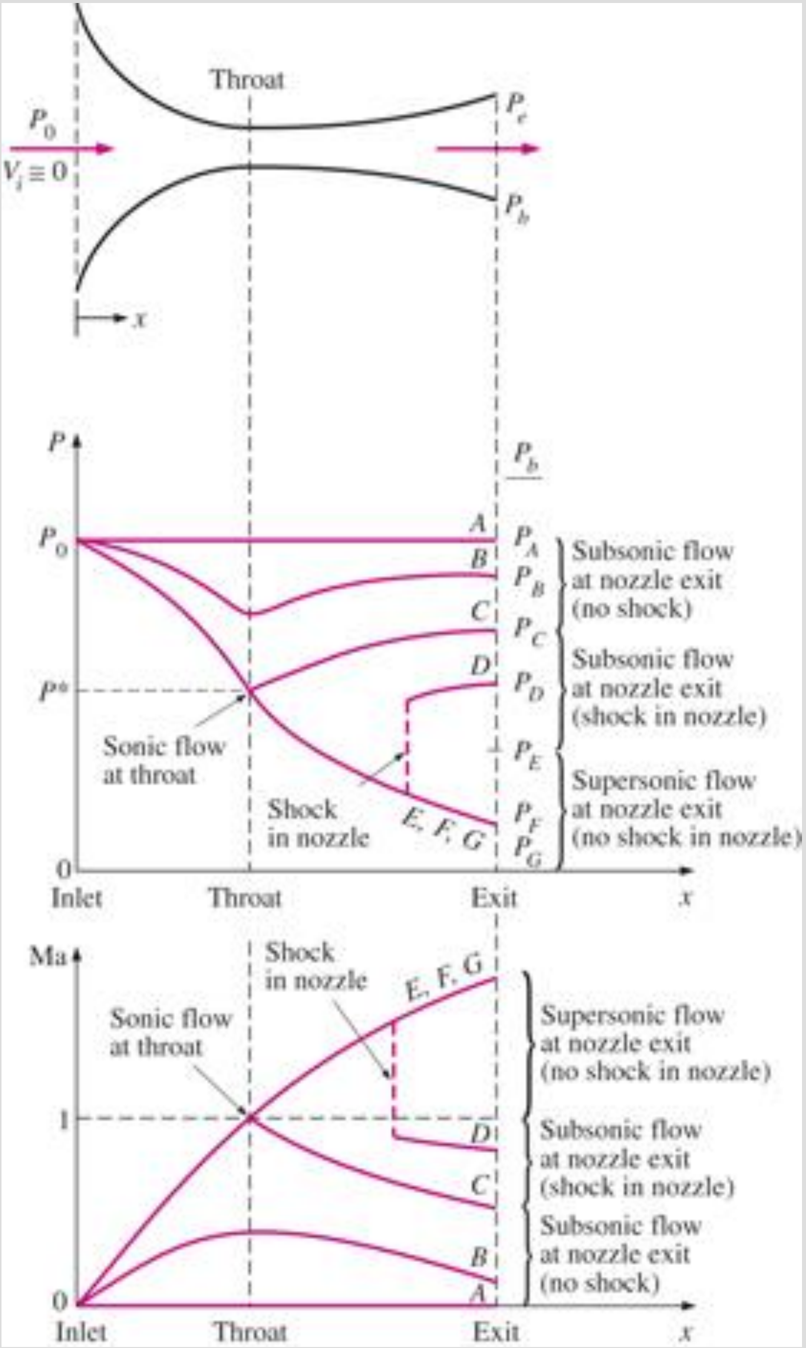


4. When $P_E > Pb > 0$, the flow in the diverging section is supersonic, and the fluid expands to P_F at the nozzle exit with no normal shock forming within the nozzle. Thus, the flow through the nozzle can be approximated as isentropic.

When $P_b = P_F$, no shocks occur within or outside the nozzle.

When $P_b < P_F$, irreversible mixing and expansion waves occur downstream of the exit plane of the nozzle.

When $P_b > P_F$, however, the pressure of the fluid increases from P_F to P_b irreversibly in the wake of the nozzle exit, creating what are called *oblique shocks.*



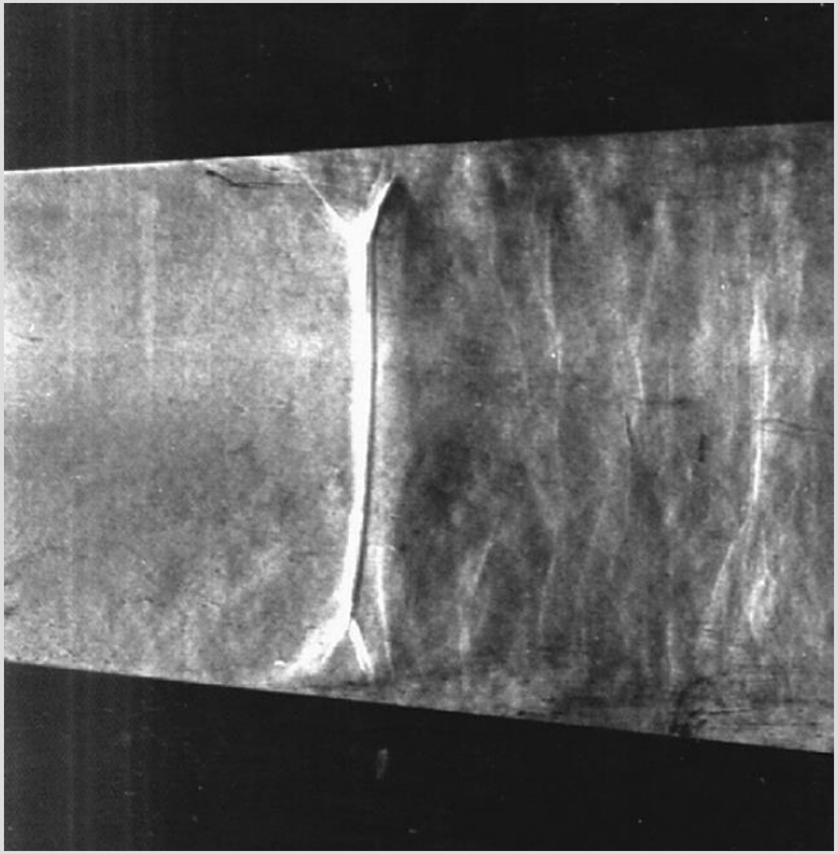
SHOCK WAVES AND EXPANSION WAVES

For some back pressure values, abrupt changes in fluid properties occur in a very thin section of a converging–diverging nozzle under supersonic flow conditions, creating a **shock wave**.

We study the conditions under which shock waves develop and how they affect the flow.

Normal Shocks

Normal shock waves: The shock waves that occur in a plane normal to the direction of flow. The flow process through the shock wave is highly irreversible.



Schlieren image of a normal shock in a Laval nozzle. The Mach number in the nozzle just upstream (to the left) of the shock wave is about 1.3. Boundary layers distort the shape of the normal shock near the walls and lead to flow separation beneath the shock.

Conservation of mass

$$\rho_1 A V_1 = \rho_2 A V_2 \rightarrow \rho_1 V_1 = \rho_2 V_2$$

Conservation of energy

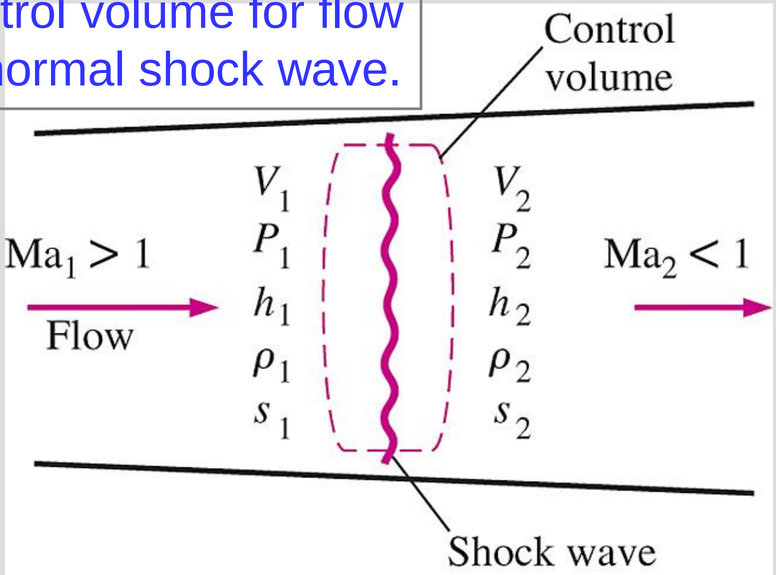
$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \rightarrow h_{01} = h_{02}$$

Conservation of momentum

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$

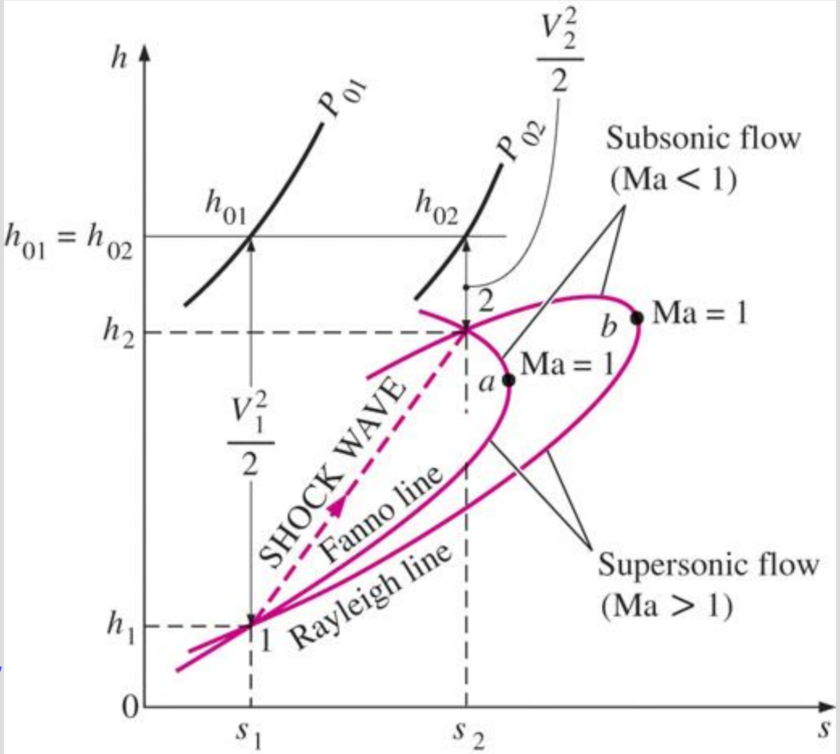
$s_2 - s_1 \geq 0$ Increase of entropy

Control volume for flow across a normal shock wave.



Fanno line: Combining the conservation of mass and energy relations into a single equation and plotting it on an *h-s* diagram yield a curve. It is the locus of states that have the same value of stagnation enthalpy and mass flux.

Rayleigh line: Combining the conservation of mass and momentum equations into a single equation and plotting it on the *h-s* diagram yield a curve.



The *h-s* diagram for flow across a normal shock.

The relations between various properties before and after the shock for an ideal gas with constant specific heats.

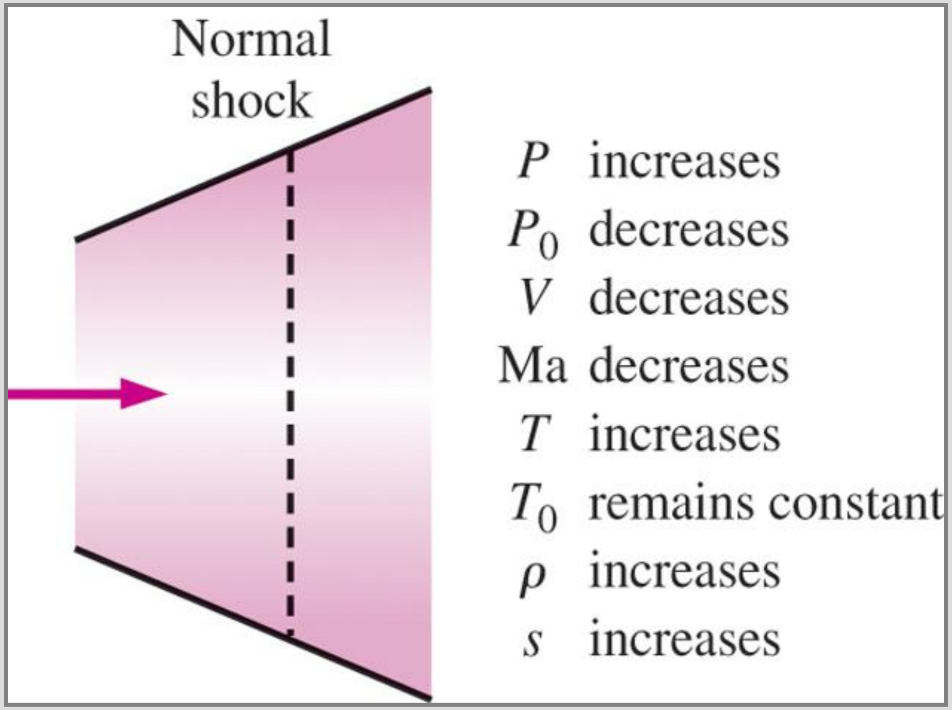
$$\frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{P_2\mathrm{Ma}_2c_2}{P_1\mathrm{Ma}_1c_1} = \frac{P_2\mathrm{Ma}_2\sqrt{T_2}}{P_1\mathrm{Ma}_1\sqrt{T_1}} = \left(\frac{P_2}{P_1}\right)^2\left(\frac{\mathrm{Ma}_2}{\mathrm{Ma}_1}\right)^2$$

$$\frac{P_2}{P_1} = \frac{\mathrm{Ma}_1\sqrt{1 + \mathrm{Ma}_1^2(k-1)/2}}{\mathrm{Ma}_2\sqrt{1 + \mathrm{Ma}_2^2(k-1)/2}}$$

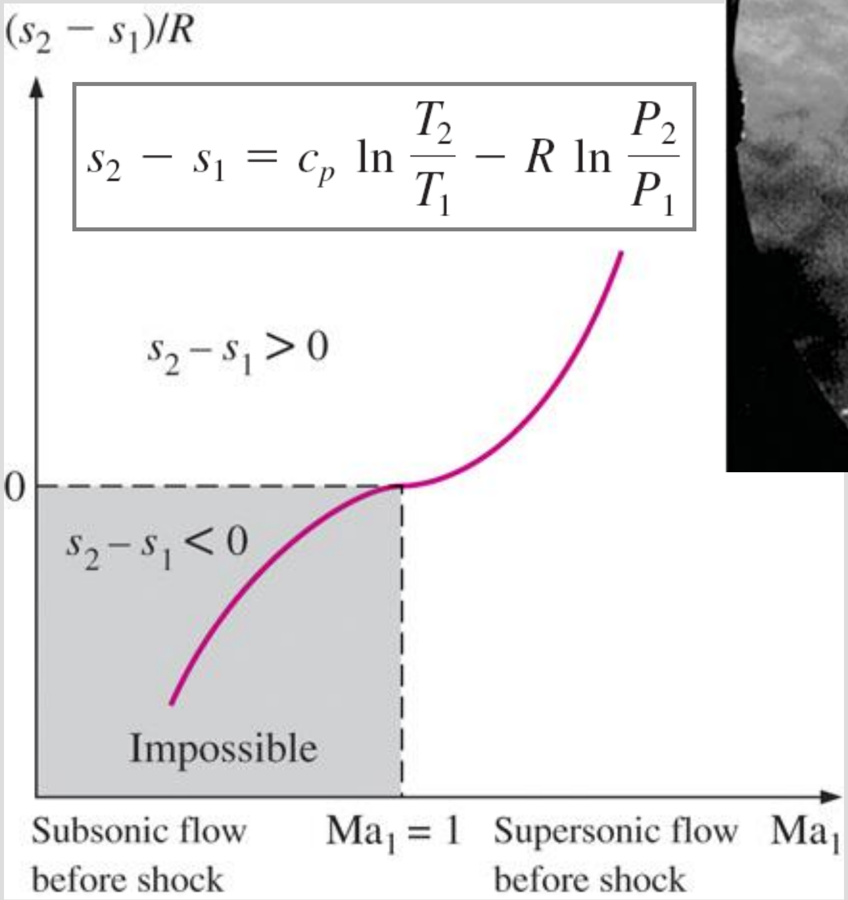
Various flow property ratios across the shock are listed in Table A–33.

$$\mathrm{Ma}_2^2 = \frac{\mathrm{Ma}_1^2 + 2/(k-1)}{2\mathrm{Ma}_1^2k/(k-1) - 1}$$

This represents the intersections of the Fanno and Rayleigh lines.



Variation of flow properties across a normal shock.



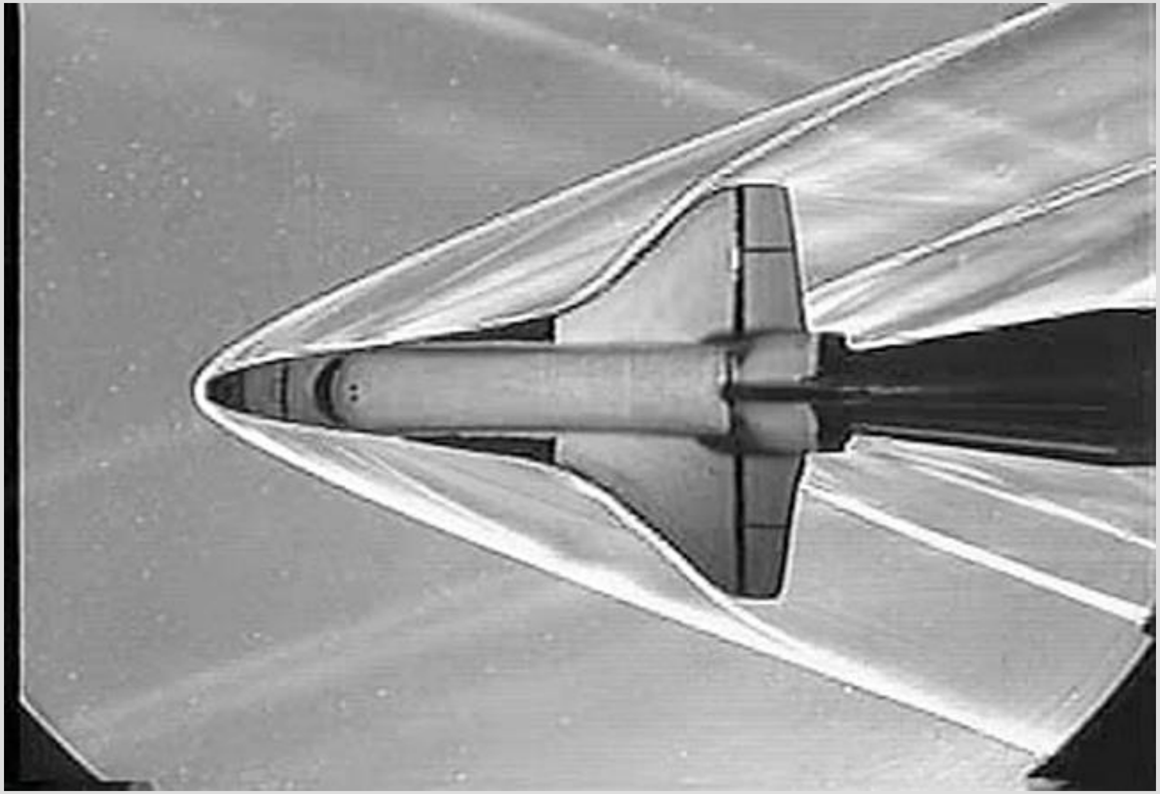
Entropy change across the normal shock.

Schlieren image of the blast wave (expanding spherical normal shock) produced by the explosion of a firecracker detonated inside a metal can that sat on a stool. The shock expanded radially outward in all directions at a supersonic speed that decreased with radius from the center of the explosion. The microphone at the lower right sensed the sudden change in pressure of the passing shock wave and triggered the microsecond flashlamp that exposed the photograph.

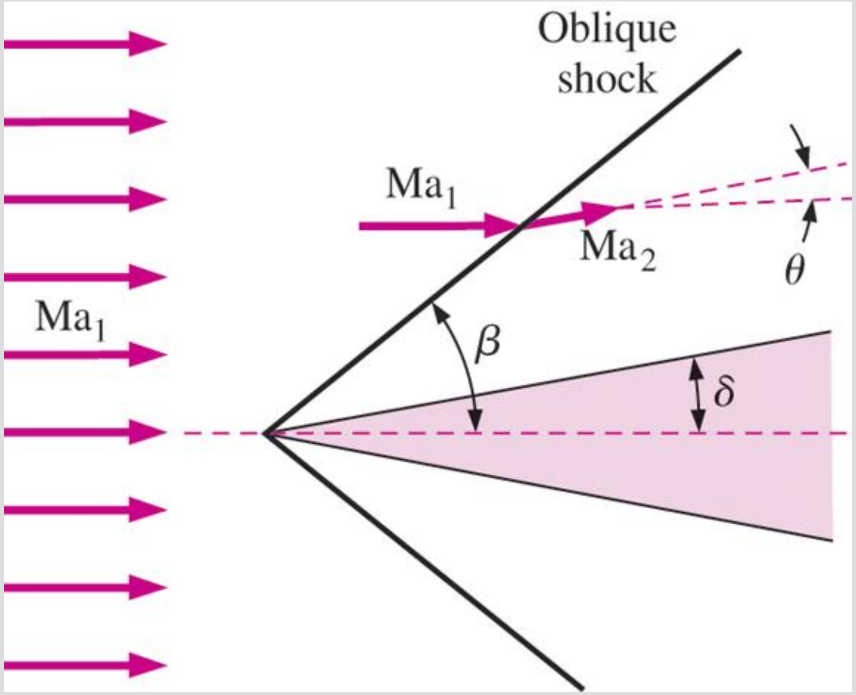
Oblique Shocks

When the space shuttle travels at supersonic speeds through the atmosphere, it produces a complicated shock pattern consisting of inclined shock waves called **oblique shocks**.

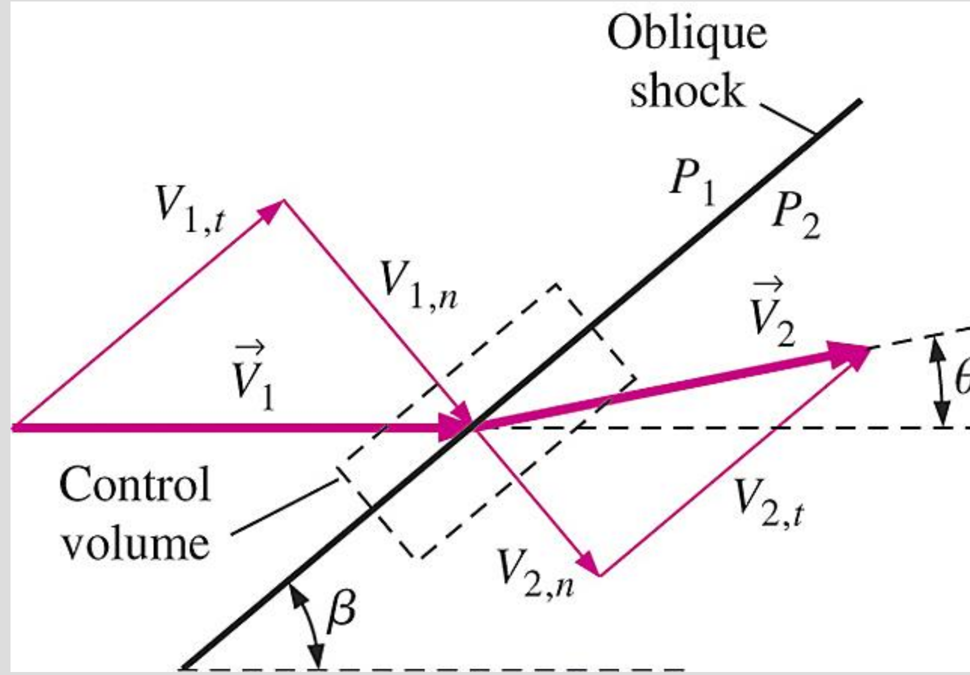
Some portions of an oblique shock are curved, while other portions are straight.



Schlieren image of a small model of the space shuttle *Orbiter* being tested at Mach 3 in the supersonic wind tunnel of the Penn State Gas Dynamics Lab. Several *oblique shocks* are seen in the air surrounding the spacecraft.

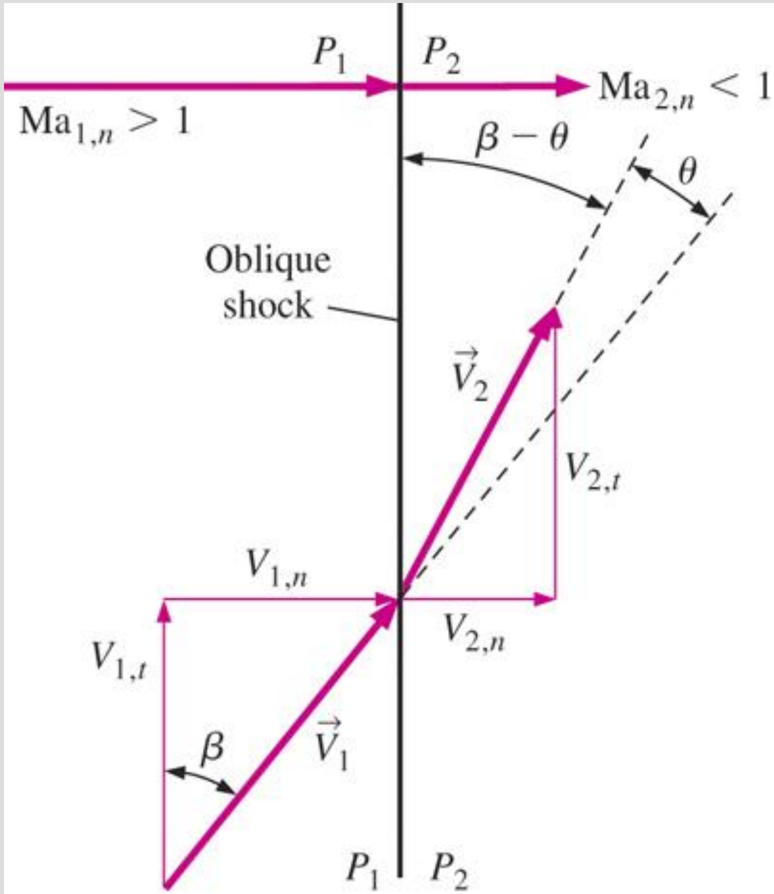


An oblique shock of ***shock angle (wave angle)*** β formed by a slender, two-dimensional wedge of half-angle δ. The flow is turned by ***deflection angle (turning angle)*** θ downstream of the shock, and the Mach number decreases.



Velocity vectors through an oblique shock of shock angle β and deflection angle θ.

Unlike normal shocks, in which the downstream Mach number is always subsonic, Ma_2 downstream of an oblique shock can be subsonic, sonic, or supersonic, depending on the upstream Mach number Ma_1 and the turning angle.



The same velocity vectors of Fig. 17–38, but rotated by angle $\pi/2-\beta$, so that the oblique shock is vertical. Normal Mach numbers $\mathrm{Ma}_{1,n}$ and $\mathrm{Ma}_{2,n}$ are also defined.

$$h_{01} = h_{02} \quad \rightarrow \quad T_{01} = T_{02}$$

$$\mathrm{Ma}_{2,n} = \sqrt{\frac{(k-1)\mathrm{Ma}_{1,n}^2 + 2}{2k\mathrm{Ma}_{1,n}^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{2k\,\mathrm{Ma}_{1,n}^2 - k + 1}{k+1}$$

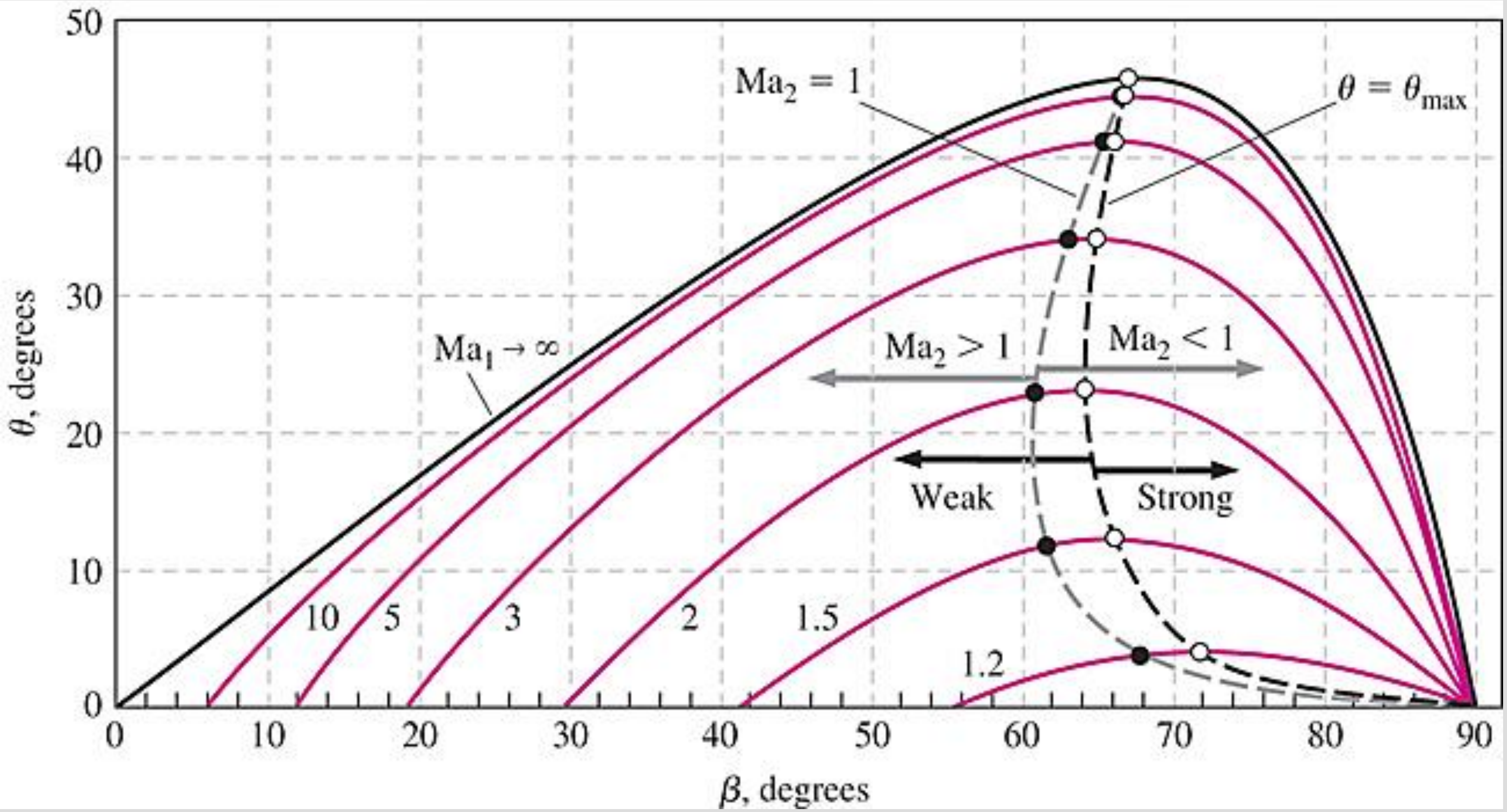
$$\frac{\rho_2}{\rho_1} = \frac{V_{1,n}}{V_{2,n}} = \frac{(k+1)\mathrm{Ma}_{1,n}^2}{2 + (k-1)\mathrm{Ma}_{1,n}^2}$$

$$\frac{T_2}{T_1} = [2 + (k-1)\mathrm{Ma}_{1,n}^2]\frac{2k\,\mathrm{Ma}_{1,n}^2 - k + 1}{(k+1)^2\mathrm{Ma}_{1,n}^2}$$

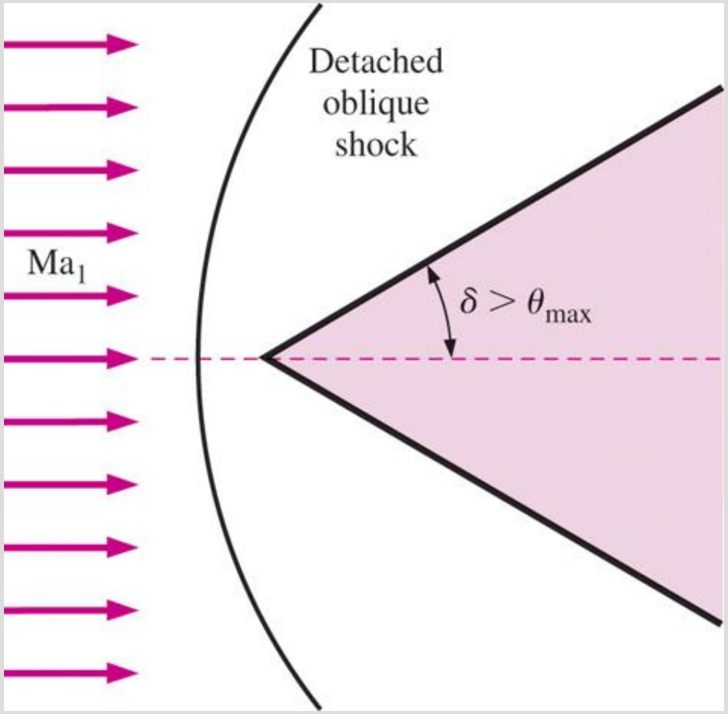
$$\frac{P_{02}}{P_{01}} = \left[\frac{(k+1)\mathrm{Ma}_{1,n}^2}{2 + (k-1)\mathrm{Ma}_{1,n}^2}\right]^{k/(k-1)}\left[\frac{(k+1)}{2k\,\mathrm{Ma}_{1,n}^2 - k + 1}\right]^{1/(k-1)}$$

Relationships across an oblique shock for an ideal gas in terms of the normal component of upstream Mach number $\mathrm{Ma}_{1,n}$.

All the equations, shock tables, etc., for normal shocks apply to oblique shocks as well, provided that we use only the **normal** components of the Mach number.



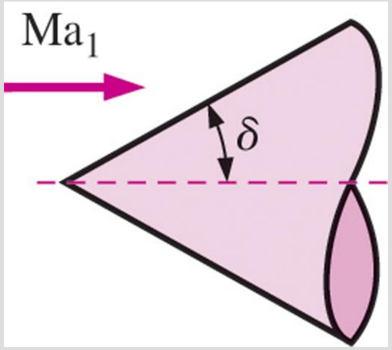
The dependence of straight oblique shock deflection angle θ on shock angle β for several values of upstream Mach number Ma_1. Calculations are for an ideal gas with $k = 1.4$. The dashed black line connects points of maximum deflection angle ($\theta = \theta_{max}$). *Weak oblique shocks* are to the left of this line, while *strong oblique shocks* are to the right of this line. The dashed gray line connects points where the downstream Mach number is *sonic* ($Ma_2 = 1$). *Supersonic downstream flow* ($Ma_2 > 1$) is to the left of this line, while *subsonic downstream flow* ($Ma_2 < 1$) is to the right of this line.

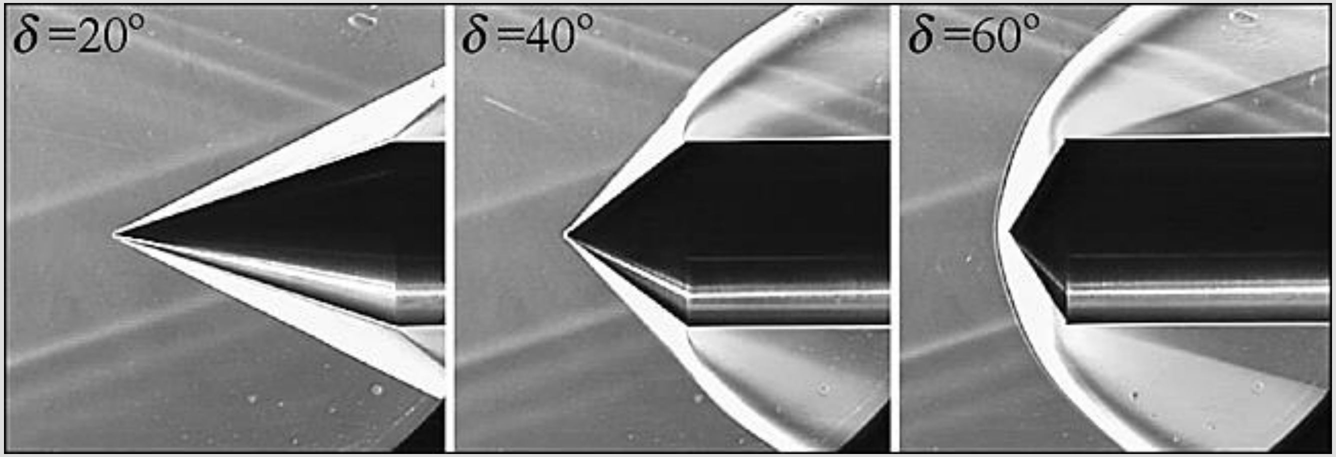


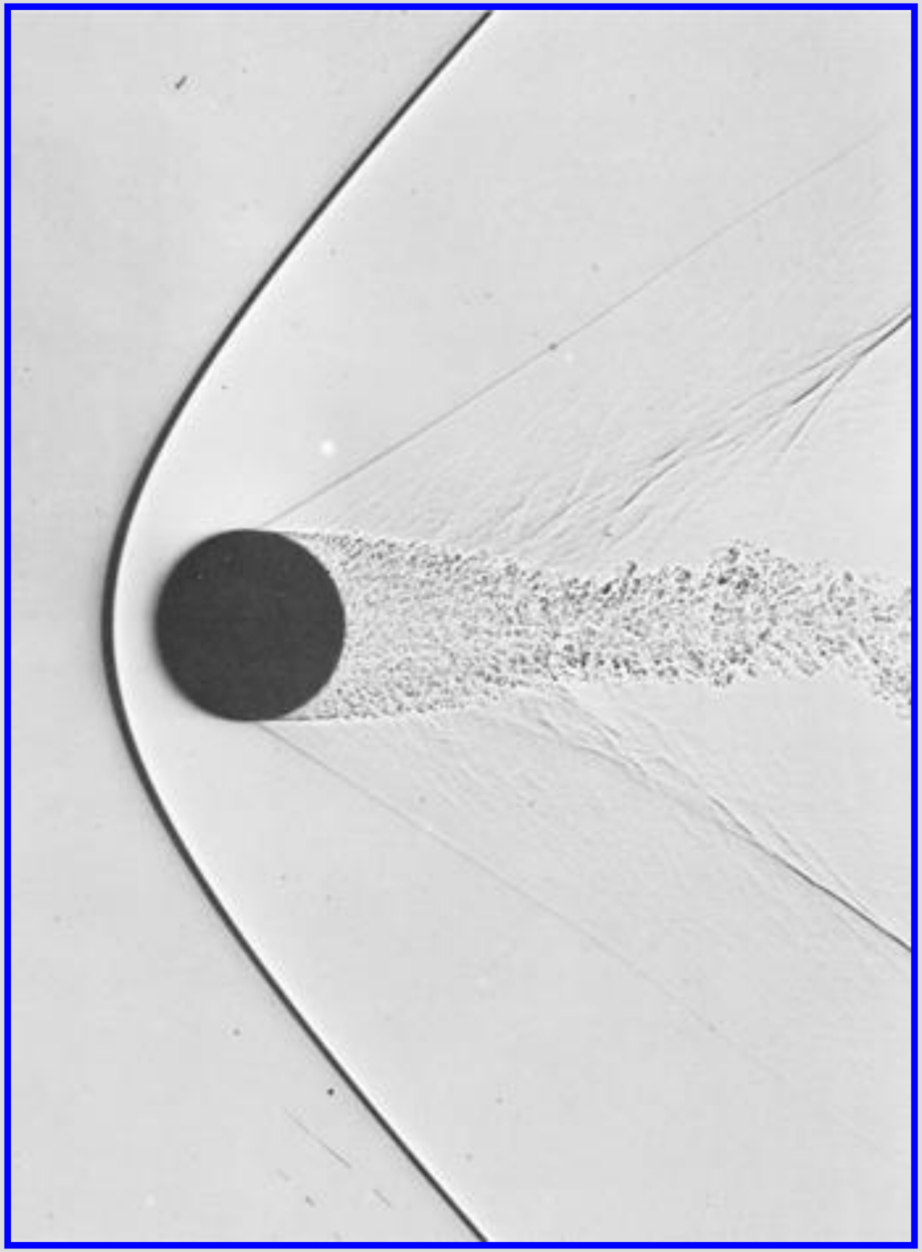
A *detached oblique shock* occurs upstream of a two-dimensional wedge of half-angle δ when δ is greater than the maximum possible deflection angle θ. A shock of this kind is called a *bow wave* because of its resemblance to the water wave that forms at the bow of a ship.

Still frames from schlieren videography illustrating the detachment of an oblique shock from a cone with increasing cone half-angle δ in air at Mach 3. At (*a*) δ=20° and (*b*) δ=40°, the oblique shock remains attached, but by (*c*) δ=60°, the oblique shock has detached, forming a bow wave.

Mach angle $\mu = \sin^{-1}(1/\mathrm{Ma}_1)$







Shadowgram of a one-half-in diameter sphere in free flight through air at Ma = 1.53. The flow is subsonic behind the part of the bow wave that is ahead of the sphere and over its surface back to about 45°. At about 90° the laminar boundary layer separates through an oblique shock wave and quickly becomes turbulent. The fluctuating wake generates a system of weak disturbances that merge into the second “recompression” shock wave.

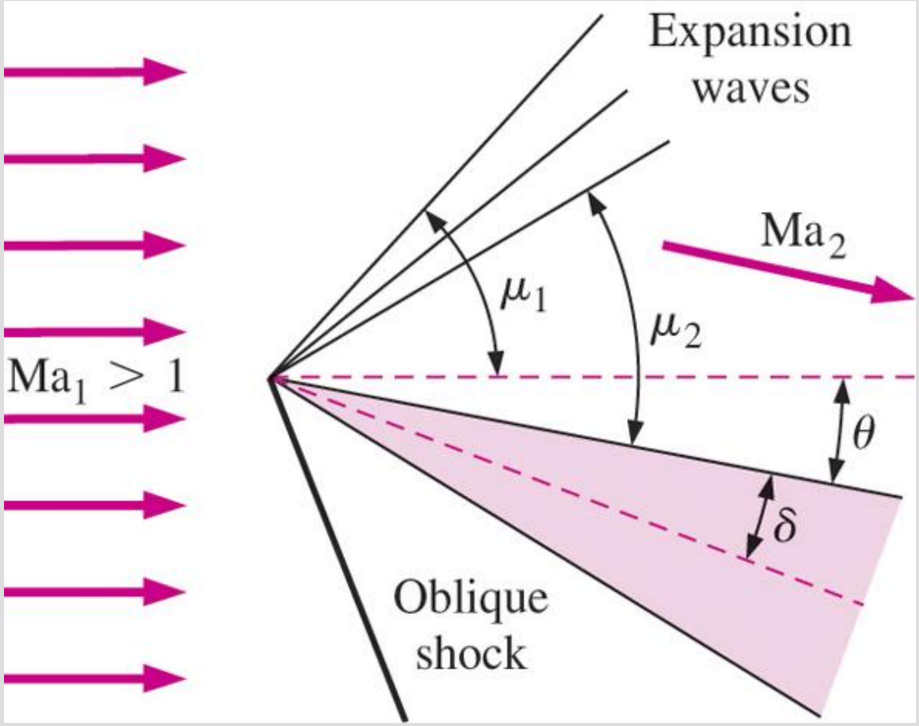
Prandtl–Meyer Expansion Waves

We now address situations where supersonic flow is turned in the *opposite* direction, such as in the upper portion of a two-dimensional wedge at an angle of attack greater than its half-angle δ.

We refer to this type of flow as an **expanding flow**, whereas a flow that produces an oblique shock may be called a **compressing flow**.

As previously, the flow changes direction to conserve mass. However, unlike a compressing flow, an expanding flow does *not* result in a shock wave.

Rather, a continuous expanding region called an **expansion fan** appears, composed of an infinite number of Mach waves called **Prandtl–Meyer expansion waves**.

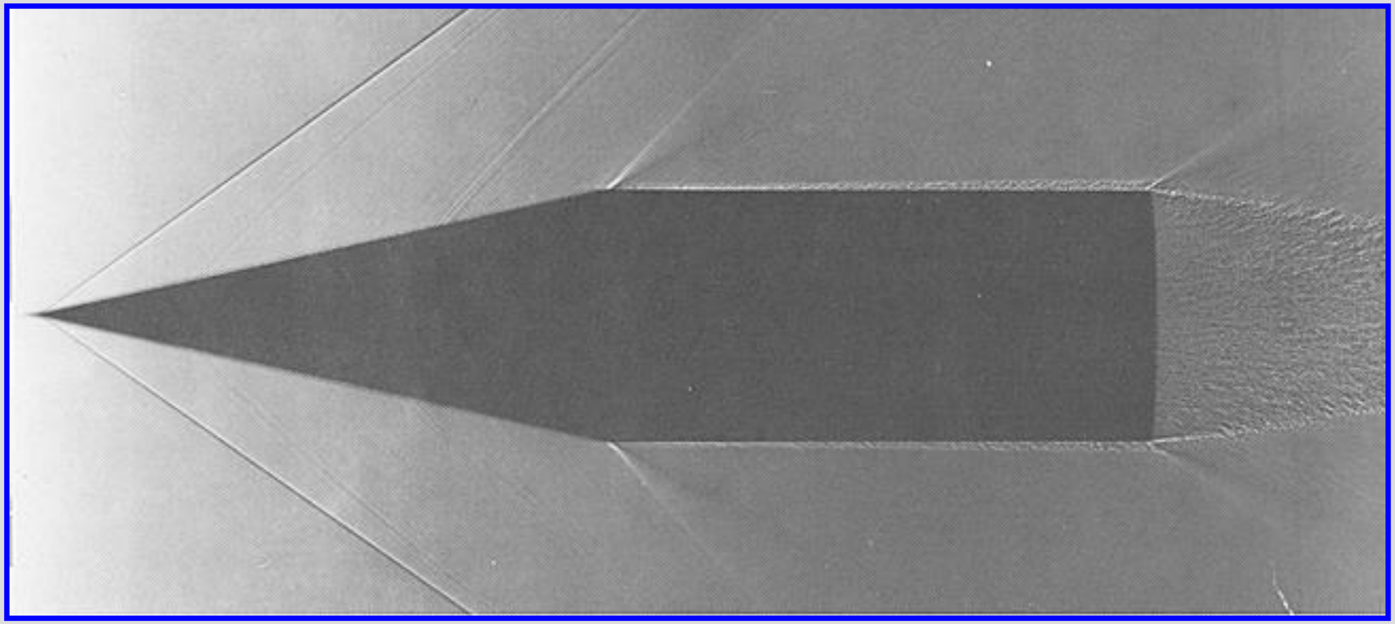


An expansion fan in the upper portion of the flow formed by a two-dimensional wedge at the angle of attack in a supersonic flow. The flow is turned by angle u, and the Mach number increases across the expansion fan. Mach angles upstream and downstream of the expansion fan are indicated. Only three expansion waves are shown for simplicity, but in fact, there are an infinite number of them. (An oblique shock is present in the bottom portion of this flow.)

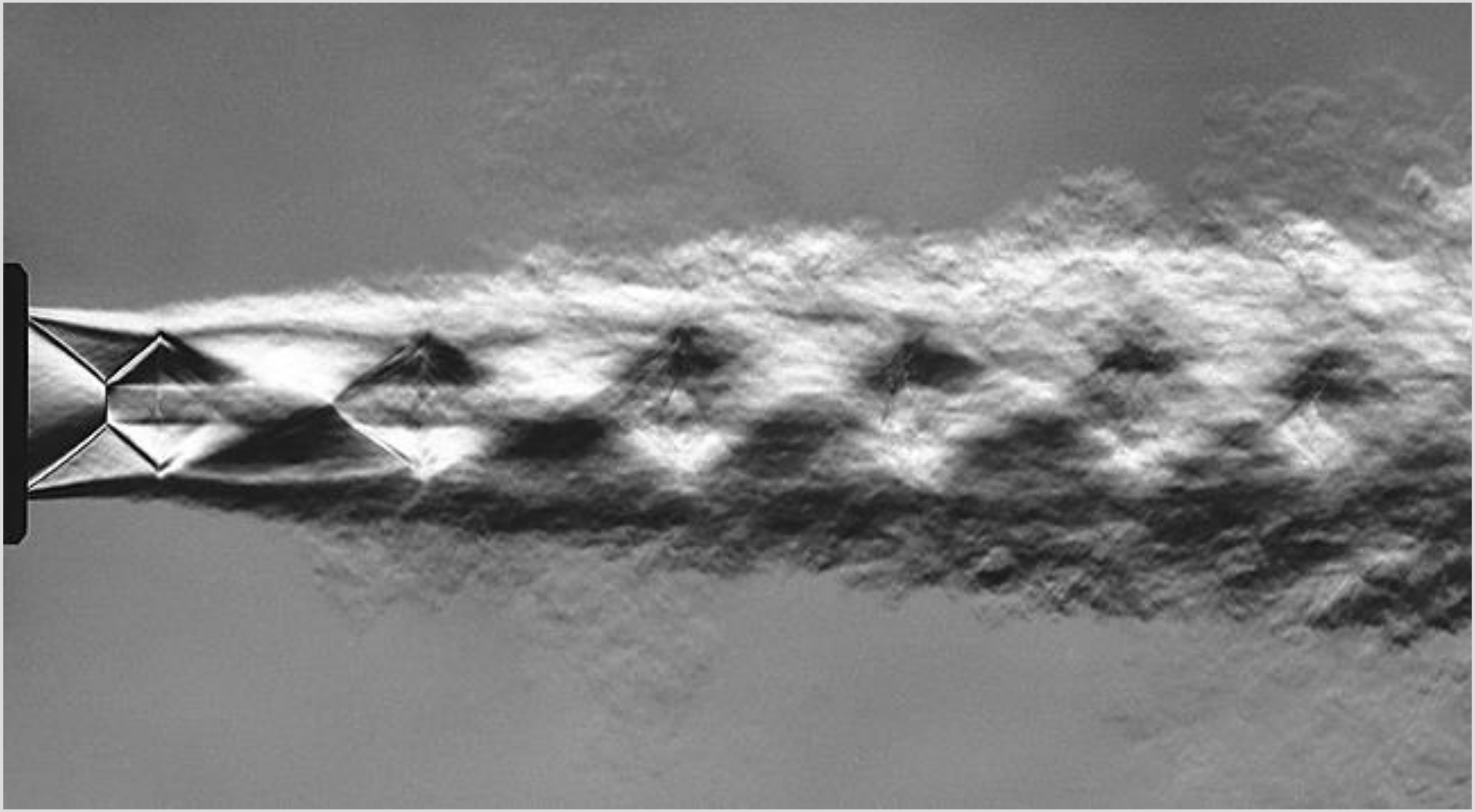
Turning angle across an expansion fan: $\theta = \nu(\mathrm{Ma}_2) - \nu(\mathrm{Ma}_1)$

Prandtl–Meyer function

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left[\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2-1)}\right] - \tan^{-1}\left(\sqrt{\mathrm{Ma}^2-1}\right)$$



A cone-cylinder of 12.5° half-angle in a Mach number 1.84 flow. The boundary layer becomes turbulent shortly downstream of the nose, generating Mach waves that are visible in this shadowgraph. Expansion waves are seen at the corners and at the trailing edge of the cone.



The complex interactions between shock waves and expansion waves in an “overexpanded” supersonic jet. The flow is visualized by a schlierenlike differential interferogram.

DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

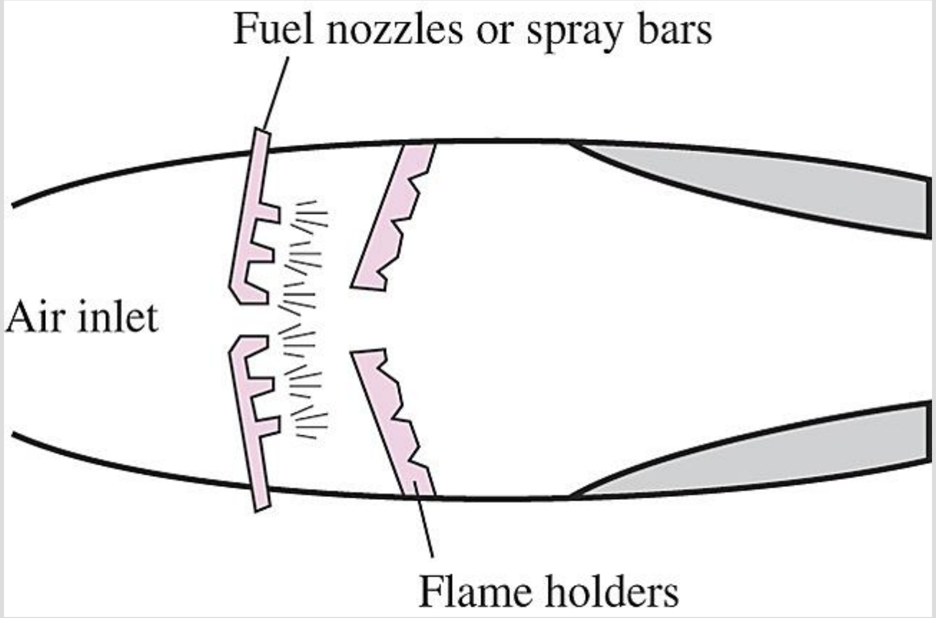
So far we have limited our consideration mostly to *isentropic flow* (no heat transfer and no irreversibilities such as friction).

Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall.

Such problems are difficult to analyze exactly since they may involve significant changes in chemical composition during flow, and the conversion of latent, chemical, and nuclear energies to thermal energy.

A simplified model is Rayleigh flow.

Rayleigh flows: Steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer, but with negligible friction.



Many practical compressible flow problems involve combustion, which may be modeled as heat gain through the duct wall.

$$\rho_1 V_1 = \rho_2 V_2$$ Mass equation

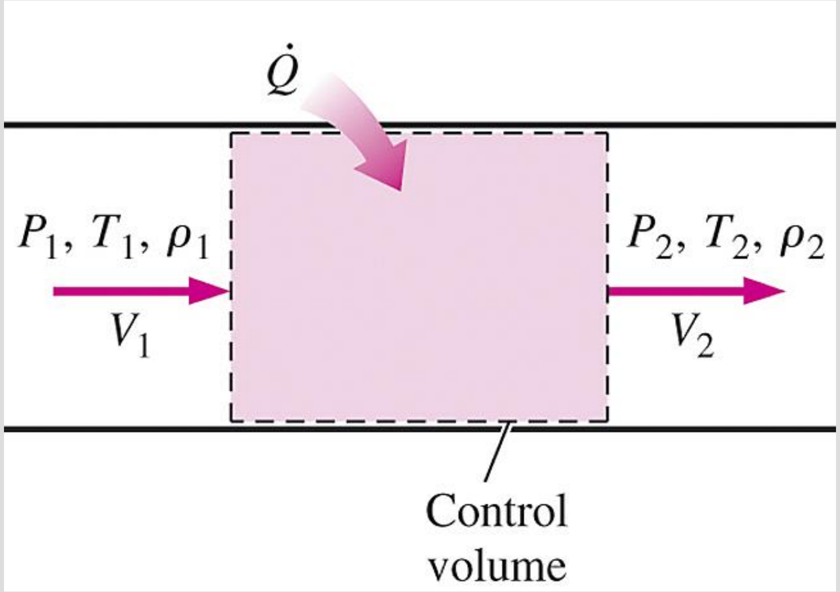
$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$ *x*-Momentum equation

$$q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$ Energy equation

$$q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$ Entropy change

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$ Equation of state

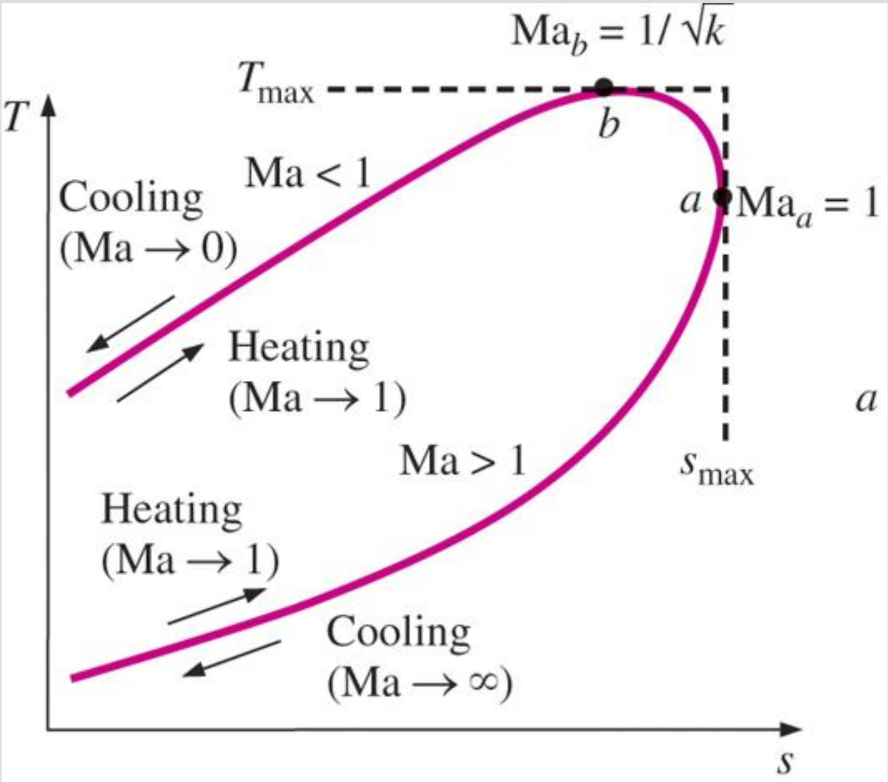


Control volume for flow in a constant-area duct with heat transfer and negligible friction.

Consider a gas with known properties R, k, and c_p. For a specified inlet state 1, the inlet properties P_1, T_1, ρ_1, V_1, and s_1 are known. The five exit properties P_2, T_2, ρ_2, V_2, and s_2 can be determined from the above equations for any specified value of heat transfer q.

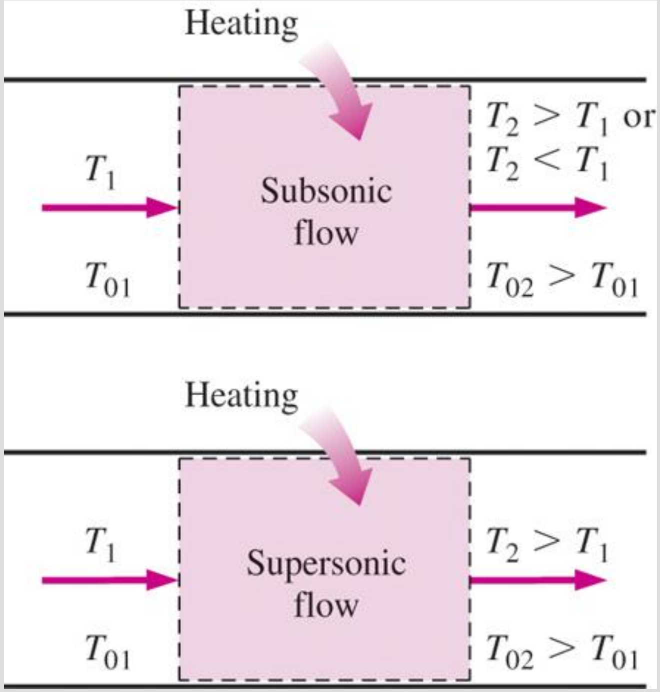
From the Rayleigh line and the equations

1. All the states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations are on the Rayleigh line.
2. Entropy increases with heat gain, and thus we proceed to the right on the Rayleigh line as heat is transferred to the fluid.
3. Heating increases the Mach number for subsonic flow, but decreases it for supersonic flow.
4. Heating increases the stagnation temperature T_0 for both subsonic and supersonic flows, and cooling decreases it.
5. Velocity and static pressure have opposite trends.
6. Density and velocity are inversely proportional.



T-s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow). This line is called **Rayleigh line**.

7. The entropy change corresponding to a specified temperature change (and thus a given amount of heat transfer) is larger in supersonic flow.



During heating, fluid temperature always increases if the Rayleigh flow is supersonic, but the temperature may actually drop if the flow is subsonic.

Heating or cooling has opposite effects on most properties. Also, the stagnation pressure decreases during heating and increases during cooling regardless of whether the flow is subsonic or supersonic.

TABLE 17–3

The effects of heating and cooling on the properties of Rayleigh flow

	Heating		*Cooling*	
Property	Subsonic	Supersonic	Subsonic	Supersonic
Velocity, V	Increase	Decrease	Decrease	Increase
Mach number, Ma	Increase	Decrease	Decrease	Increase
Stagnation temperature, T_0	Increase	Increase	Decrease	Decrease
Temperature, T	Increase for $\mathrm{Ma} < 1/k^{1/2}$ Decrease for $\mathrm{Ma} > 1/k^{1/2}$	Increase	Decrease for $\mathrm{Ma} < 1/k^{1/2}$ Increase for $\mathrm{Ma} > 1/k^{1/2}$	Decrease
Density, ρ	Decrease	Increase	Increase	Decrease
Stagnation pressure, P_0	Decrease	Decrease	Increase	Increase
Pressure, P	Decrease	Increase	Increase	Decrease
Entropy, s	Increase	Increase	Decrease	Decrease

Property Relations for Rayleigh Flow

$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2} \qquad \frac{T_2}{T_1} = \left[\frac{\mathrm{Ma}_2(1 + k\mathrm{Ma}_1^2)}{\mathrm{Ma}_1(1 + k\mathrm{Ma}_2^2)}\right]^2 \qquad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{\mathrm{Ma}_1^2(1 + k\mathrm{Ma}_2^2)}{\mathrm{Ma}_2^2(1 + k\mathrm{Ma}_1^2)}$$

$$\frac{P}{P^*} = \frac{1 + k}{1 + k\mathrm{Ma}^2} \qquad \frac{T}{T^*} = \left[\frac{\mathrm{Ma}(1 + k)}{1 + k\mathrm{Ma}^2}\right]^2 \quad \text{and} \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1 + k)\mathrm{Ma}^2}{1 + k\mathrm{Ma}^2}$$

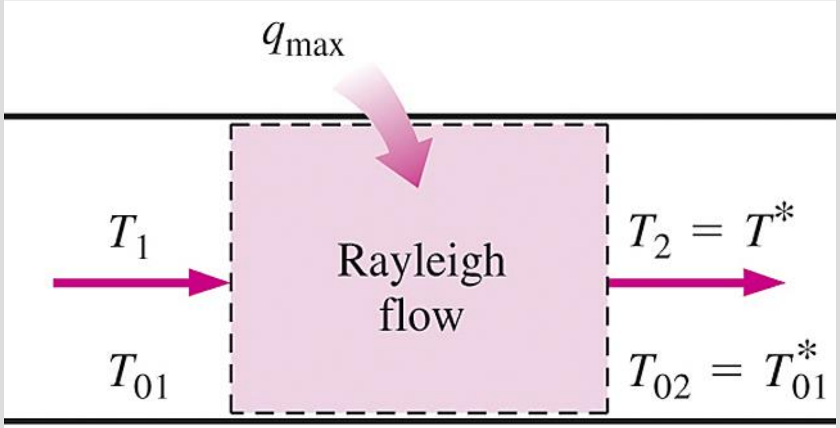
$$\frac{T_0}{T_0^*} = \frac{(k + 1)\mathrm{Ma}^2[2 + (k - 1)\mathrm{Ma}^2]}{(1 + k\mathrm{Ma}^2)^2} \qquad \frac{P_0}{P_0^*} = \frac{k + 1}{1 + k\mathrm{Ma}^2}\left[\frac{2 + (k - 1)\mathrm{Ma}^2}{k + 1}\right]^{k/(k-1)}$$

Representative results are given in Table A–34.

Choked Rayleigh Flow

The fluid at the critical state of Ma =1 cannot be accelerated to supersonic velocities by heating. Therefore, the flow is ***choked***. For a given inlet state, the corresponding critical state fixes the maximum possible heat transfer for steady flow:

$$q_{\max} = h_0^* - h_{01} = c_p(T_0^* - T_{01})$$



Choked flow

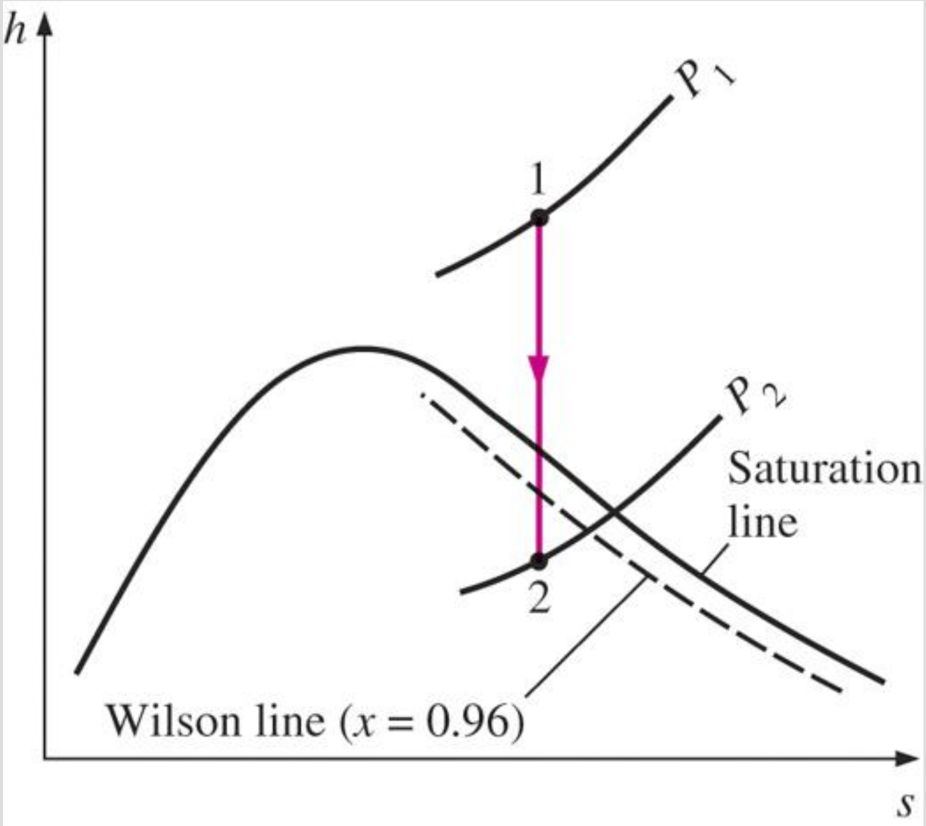
For a given inlet state, the maximum possible heat transfer occurs when sonic conditions are reached at the exit state.

STEAM NOZZLES

Water vapor at moderate or high pressures deviates considerably from ideal-gas behavior, and thus most of the relations developed in this chapter are not applicable to the flow of steam through the nozzles or blade passages encountered in steam turbines.

Supersaturated steam: The steam that exists in the wet region without containing any liquid. Supersaturation states are nonequilibrium (or metastable) states.

Wilson line: The locus of points where condensation takes place regardless of the initial temperatureand pressure at the nozzle entrance.



The *h-s* diagram for the isentropic expansion of steam in a nozzle.

When steam is assumed ideal gas with $k = 1.3$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.546$$

Summary

- Stagnation properties
- Speed of sound and Mach number
- One-dimensional isentropic flow
  - ✓ Variation of fluid velocity with flow area
  - ✓ Property relations for isentropic flow of ideal gases
- Isentropic flow through nozzles
  - ✓ Converging nozzles
  - ✓ Converging–diverging nozzles
- Shock waves and expansion waves
  - ✓ Normal shocks
  - ✓ Oblique shocks
  - ✓ Prandtl–Meyer expansion waves
- Duct flow with heat transfer and negligible friction (Rayleigh flow)
  - ✓ Property relations for Rayleigh flow
  - ✓ Choked Rayleigh flow
- Steam nozzles