Cranfield UNIVERSITY

PRESSURE SURGE

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Process Systems Engineering PSE05

Pressure Surge Lecture – Additional Notes

1. Brief History of Surge Analysis

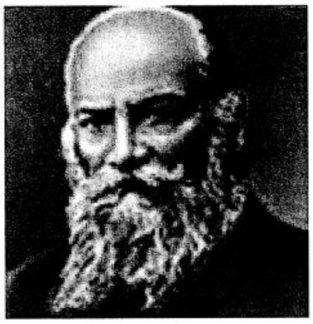
1.1 The Beginning

The story of surge analysis is a truly international one.

Korteweg

Dutch mathematician Diederik **Korteweg** derived the wavespeed equation for fluid in cylindrical pipes in 1878.

Nicolai Yegorovich Zhukovsky (generally romanised to **Joukowsky** or Joukovsky) in Moscow in 1897 carried out field tests for the Imperial Water Works which showed that the pressure rise caused by sudden valve closure was identical to acoustic phenomena. He published a paper with the 'Joukowsky equation' in 1898. He also studied surge protection devices such as surge vessels and safety valves. He derived the equation for the maximum (instantaneous) pressure rise due to a sudden valve closure.



Joukowsky

Independently, J P **Frizell** in the USA also published a paper containing the 'Joukowsky' equation, but his contribution was criticised and not accepted by his American contemporaries.

Lorenzo **Allievi** in Italy derived and was the first to solve the basic waterhammer equations in the early years of the 20th century.

However, there were earlier researchers who discovered the existence and many features of pressure surge, although they did not use this term.

William **Rankine** (Scotland) in 1870 had also derived the Joukowsky equation, but in a more general context.

Allievi

Thomas **Young** (England) in 1808 wrote a paper on linear wave propagation that derived the wavespeed in an incompressible fluid and implicity derived (but for elastic solids rather than fluids) an equivalent of the Joukowsky equation.

Johannes **von Kries** (German) carried out experiments on blood flow in arteries, some using rubber hoses, and demonstrated pressure transients. He published a paper in 1883 as well as a book. He was also aware of the phenomena of line packing and fluid-structure interaction. In addition, he looked at frequency-dependent damping and phase velocity, the first steps in the investigation of unsteady friction which is one of today's 'hot topics'. His work was limited by the instrumentation available at that time.

JOHANNES VON KRIES

L-F **Ménabréa** (Italy, 1858, 1862) knew of the existence of pressure surge, introduced the technique of energy analysis and came close to deriving the Joukowsky equation.

Jules **Michaud** (France, 1878) was familiar with the concept of pressure surge and also looked at surge suppression measures.

So the existence of and/or the equations for the propagation of pressure waves in pipes have been known for up to two centuries!

Solution methods have also been available. Analytical solutions are available for very simple cases. Numerical solution methods were also available, but incredibly time-consuming to carry out by hand. What was lacking, until comparatively recently, was a practical method of solving the equations for more complex systems.

1.2 Approximate Solutions

Graphical techniques produced by O Schnyder (?Switzerland, from 1929) and L Bergeron (France, from 1931) allowed practical analysis of pressure waves along pipes for a range of boundary conditions.

However, this could not deal with non-linear (i.e. friction) terms so pipes had zero friction. This was overcome by placing orifices with a suitable loss at intervals along the pipelines.

Although only very simple networks could be analysed using graphical methods, they were popular with many users because the method gave an insight into the physical processes.

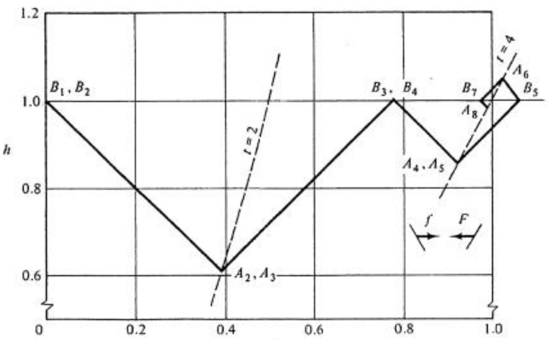


Figure 4-18 Valve opening.

80 FLUID TRANSIENTS

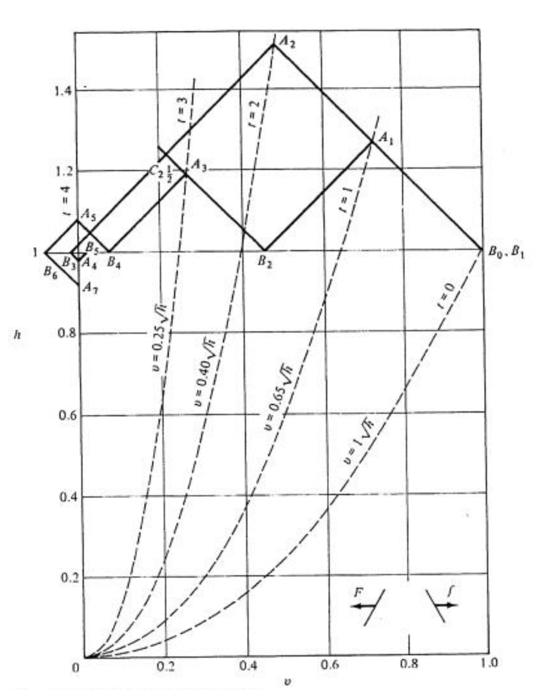


Figure 4-17 Gradual valve-closure solution.

1.3 The Arrival of Computers

With the arrival of computers in the 1960s, the first attempts at programming were focussed on automating the graphical methods, although this was abandoned fairly quickly.

From the 1970s, advances in computer software allowed the analysis of increasingly complex systems with branches and loops, using smaller timesteps and more complex boundary conditions.

In the 1980s, the availability of commercial software with graphical interfaces brought pressure surge from the academic world to the consulting engineer.

The continued increase in computing power allows modelling of larger networks and more complex control.

1.4 Current research areas

Until recently, if the software correctly predicted the timing and the size of the first pressure peak (high or low), analysts were satisfied. Usually, the magnitude and/or timing of subsequent peaks could not be used for design with confidence.

For many pump trip or valve closure events, it is the primary peak that is of most interest and the basis of the design of surge suppression equipment. However, for multiple events, or in complex piping systems, the timing and the magnitude of the subsequent peaks can be very important.

With the increase in computing power, it is now possible to start including 'second order' effects to improve the solution accuracy.

Research is actively taking place at present in the following main areas:

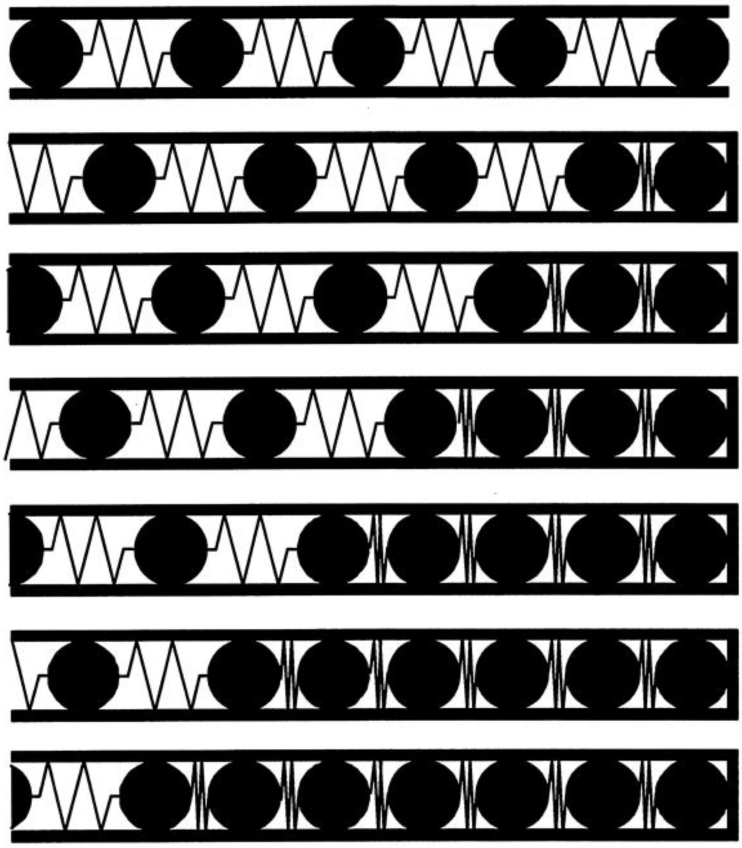
Unsteady friction	the changing velocity profiles during transients change the friction and hence the damping
Visco-elastic pipes	plastic pipes are visco-elastic, which means that there is noticeable damping within the material as the pipe wall expands and the strain (i.e. increase/decrease in pipe diameter) depends on the time since the change in pressure – this results in a changing wavespeed with time and additional damping
FSI	fluid-structure interaction is a challenging field that links transient fluid software (CFD as well as pressure surge) to stress analysis (FEA) codes to look at both the effects of the forces caused by fluid flow on the structure (e.g. due to pressure transients) and the effect of movement of the structure on fluid flow (e.g. due to earthquakes, external loadings)
Air	work is being carried out to enable predictions of air movement in pipelines which, hopefully, will feed through into the software to give more accurate predictions of surge effects when air does not remain at the point of admission
Leakage	generating (small!) pressure transients in networks to locate leaks or blockages – often a frequency domain analysis is used
Multiphase flow	work is starting to be carried out on transients in multiphase (gas/liquid) flow – this is particularly challenging as steady state multiphase flow is still not well understood for may pipe geometries (however, due to the gas content, the flow is closer to 'unsteady' than 'transient' in this context)

2. Concepts and Definitions

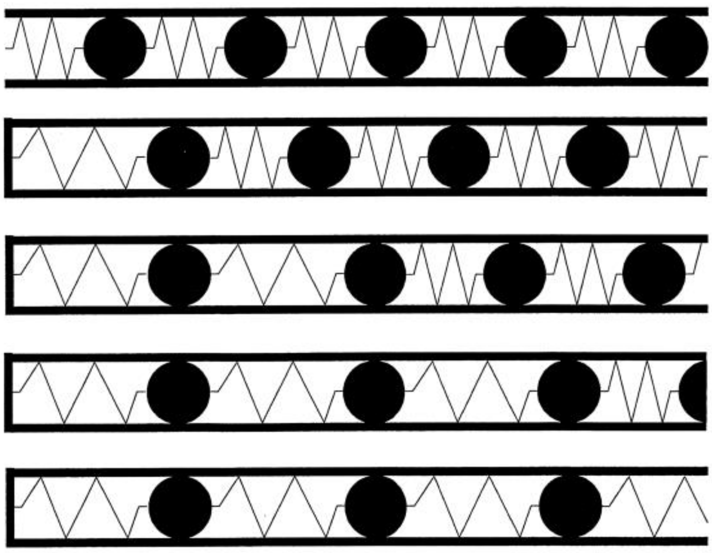
Pressure surge	'fast' changes in pressure (also known as pressure transients, fluid transients,waterhammer)
Unsteady flow	'slow' changes in flow
Wavespeed, a (or c)	the speed at which pressure waves travel through the fluid
Pipeline period	time for a pressure wave to travel the length, L, of the pipe and come back (= 2L/a)
Fast or Rapid	describes an event which occurs in a time less than the pipeline period
Slow	describes an event which occurs in a time greater than the pipeline period

Pressure surge with a positive head rise is often described using the analogy of a train with a number of carriages, or masses connected by springs to reflect the compressibility of liquids. The locomotive, or first mass, is brought suddenly to a halt and the carriages (or following masses) progressively crash into the back of each other, propagating the pressure wave upstream. The compression of the springs reflects the rise in pressure.

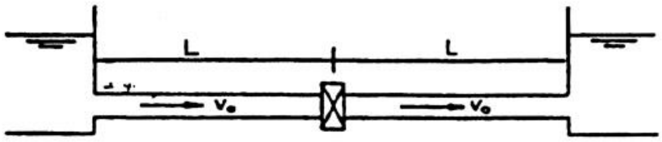
Pressure rise upstream of a rapidly closed valve (flow left to right):

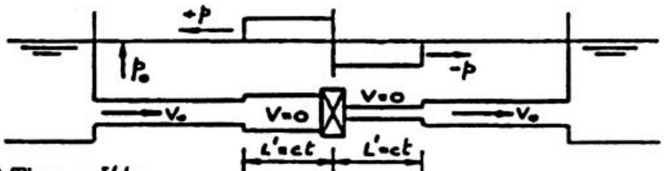


Pressure drop downstream of a rapidly closed valve (flow left to right):

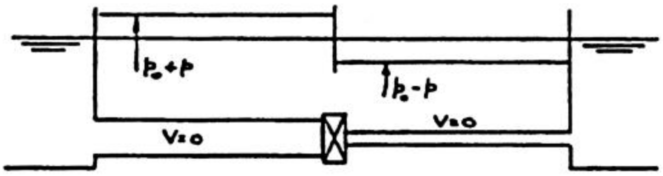


Pressure variations following valve closure in the middle of a pipeline:

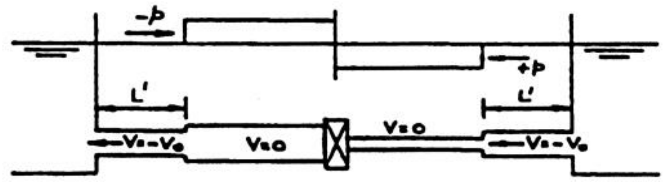




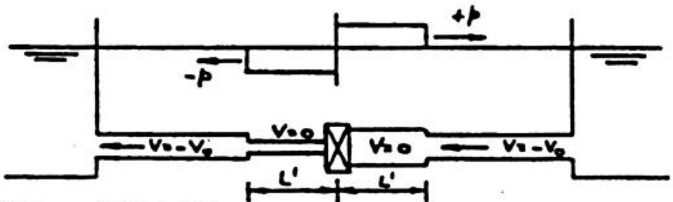
(a) Time = L'/c



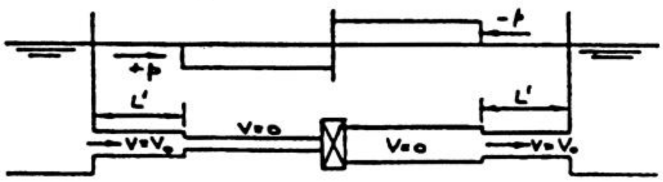
(b) Time = L/c



(c) Time = $L/c + L'/c$



(d) Time = $2L/c + L'/c$



(e) Time = $3L/c + L'/c$

<u>Joukowsky head</u>

The Joukowsky head is the head change (in metres of fluid) that would occur if the fluid was brought to rest instantaneously

Joukowsky head, $\Delta h = - \dfrac{a\,\Delta V}{g}$

where	a	wavespeed
	ΔV	change in fluid velocity
	g	acceleration due to gravity

Useful indicator of the likely scale of the problem

(Some people refer to the Joukowsky equation as the Joukowsky-Frizell or Allievi equation, presumably depending on which country they are from!)

IMPORTANT NOTE

The Joukowsky head is NOT the highest pressure rise that can ever be seen in a system.

Higher pressures can be generated by interactions between pressure waves, especially in more complex systems, due to pipeline network configuration and/or scheduling/control of pump/valve operations and/or line packing.

Line packing

In long pipelines, or systems with high initial steady flow frictional losses, the neglect of frictional effects can result in a gross underestimate of the peak pressure rise following valve closure. In these cases, the initial pressure at the valve is considerably less than the upstream reservoir pressure level. As the flow is retarded by valve closure, frictional pressure loss along the pipe is reduced as flow velocity falls and the pressure at the valve rises towards the reservoir level. As each layer of fluid between the valve and the reservoir is brought to rest by the passage of the +p pressure transient, so a series of secondary pressure waves, each of a magnitude equal to the friction pressure loss recovered within that cross sectional layer is transmitted towards the closed valve, resulting in the full frictional recovery effect being felt at the closed valve at the end of the first pipe period, 2L/c As the flow reverses in the pipe during the second pipe period, 2L/c to 4L/c, the opposite effect is recorded at the valve, due to the need to re-establish the high frictional loss. In long distance oil or water pipelines, these effects may contribute a large part of the pressure change on valve closure.

3. Wavespeed

3.1 Liquid only

Wavespeed depends on fluid AND pipe material.

Please note that the Young's modulus of GRP pipe, and other composite pipes of similar construction, depends both on the materials used and the method of construction (wound, chopped strand). Individual pipe manufacturers should be able to provide guidance on suitable values to use. In the absence of this data, the Young's modulus can be estimated from:

$$E = V_f E_f + (1-V_f)E_b \qquad \text{(Thorley)}$$

where subscripts f and b refer to the fibre and bonding material and V_f is the volume fraction of fibre

For **thin-walled pipes**, wavespeed, $a = \sqrt{\dfrac{1}{\rho\left[\dfrac{1}{K}+\dfrac{D}{Ee}\phi\right]}}$

(of circular cross-section)

where
- D pipe internal diameter
- E Young's Modulus of pipe material
- K bulk modulus of fluid
- e pipe wall thickness
- ρ fluid density
- φ restraint factor (usually taken as 1)

Thorley and Wylie & Streeter suggest the use of the following values of φ for different end conditions (where ν is Poisson's ratio):

Pipeline anchored at upstream end only

$$\varphi = 1 - \nu/2$$

Pipeline anchored against longitudinal movement throughout its length

$$\varphi = 1 - \nu^2$$

Pipeline has expansion joints throughout its length

$$\varphi = 1$$

In practice, none of these idealised conditions will be fully realised and taking φ as 1 will not introduce significant error.

For **thick-walled pipes** of circular cross-section (D/e ratios of about 10 or less), the wavespeed equation is the same, but the values of φ are given by:

Pipeline anchored at upstream end only

$$\varphi = \frac{2e}{D}(1+\nu) + \frac{D}{D+e}\left(1-\frac{\nu}{2}\right)$$

Pipeline anchored against longitudinal movement throughout its length

$$\varphi = \frac{2e}{D}(1+\nu) + \frac{D}{D+e}\left(1-\nu^2\right)$$

Pipeline has expansion joints throughout its length

$$\varphi = \frac{2e}{D}(1+\nu) + \frac{D}{D+e}$$

As can be seen, the type of constraint has little effect on the wavespeed. As e becomes small, the values approach those of the thin-walled pipe.

For **circular tunnels**, allowing the values for e to become larger and larger, the equation becomes:

$$\text{Wavespeed, } a = \sqrt{\frac{1}{\rho\left[\frac{1}{K} + \frac{2}{E_R}(1+\nu)\right]}}$$

where E_R modulus of rigidity of tunnel material
ν Poisson's ratio of tunnel material

For **lined circular tunnels**, e.g. steel lined rock tunnels, neglecting Poisson's ratio effects in both steel and rock, the equation becomes:

$$\text{Wavespeed, } a = \sqrt{\frac{1}{\rho\left[\frac{1}{K} + \frac{2D}{E_R D + 2Ee}\right]}}$$

The equations for rectangular and other non-circular cross-sections are considerably more complex. Refer in the first instance to Wylie & Streeter.

3.2 Gas with Liquid

Assuming homogeneous flow and a 'small' proportion of gas

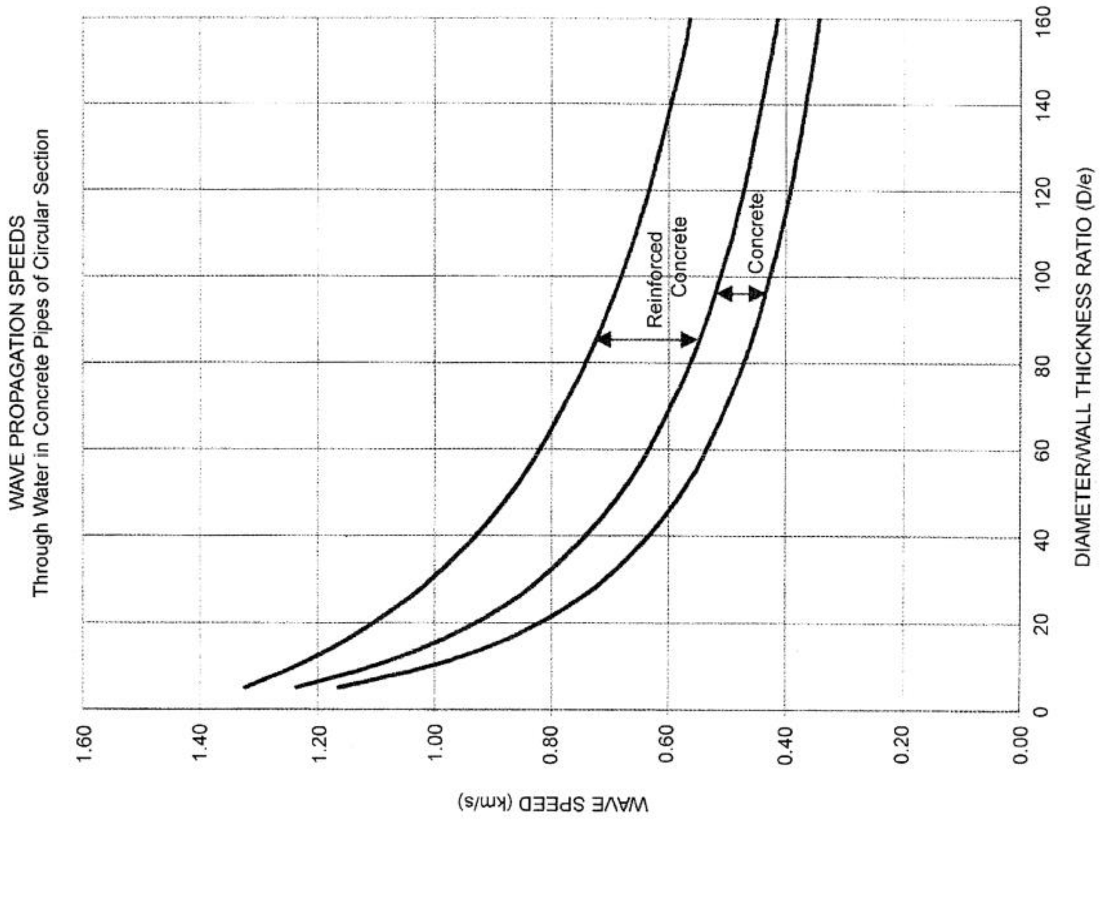
Wavespeed, a $\approx \sqrt{\dfrac{\rho_g a_g^2}{\rho_l \alpha(1-\alpha)}}$

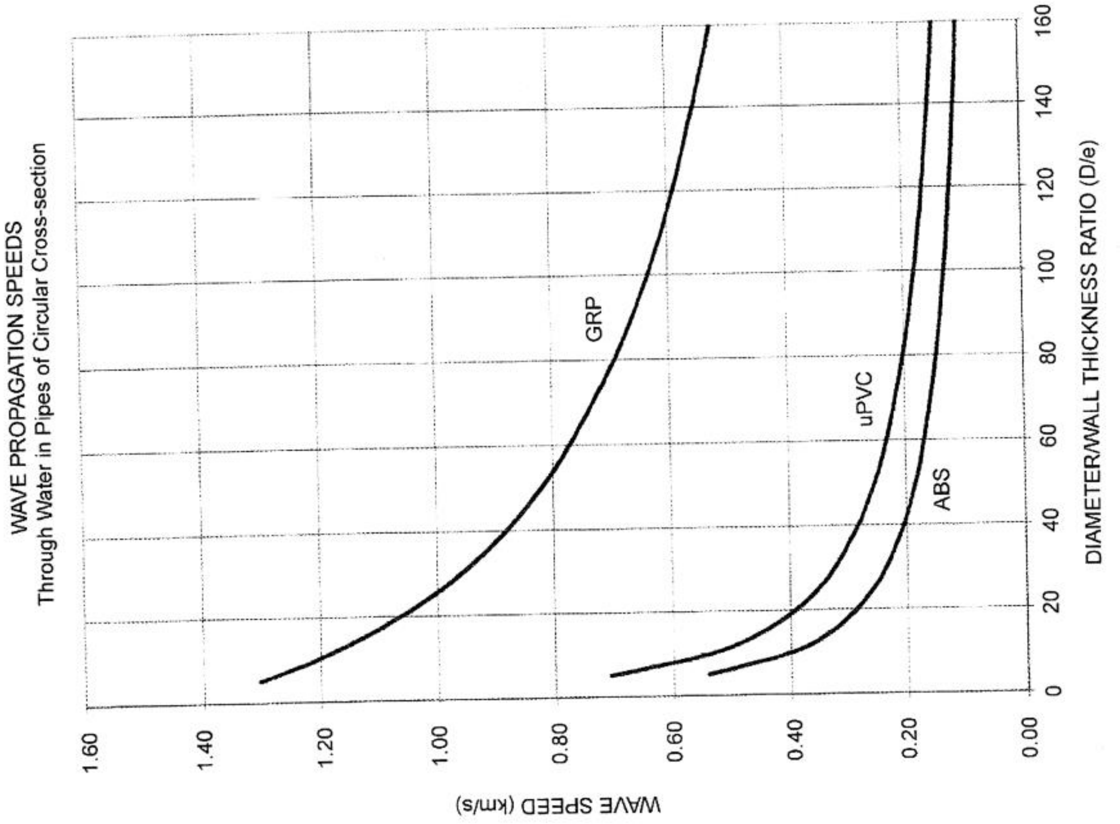
where	a_g	wavespeed in gas
	ρ_g	gas density
	ρ_l	liquid density
	α	ratio of gas to liquid by volume

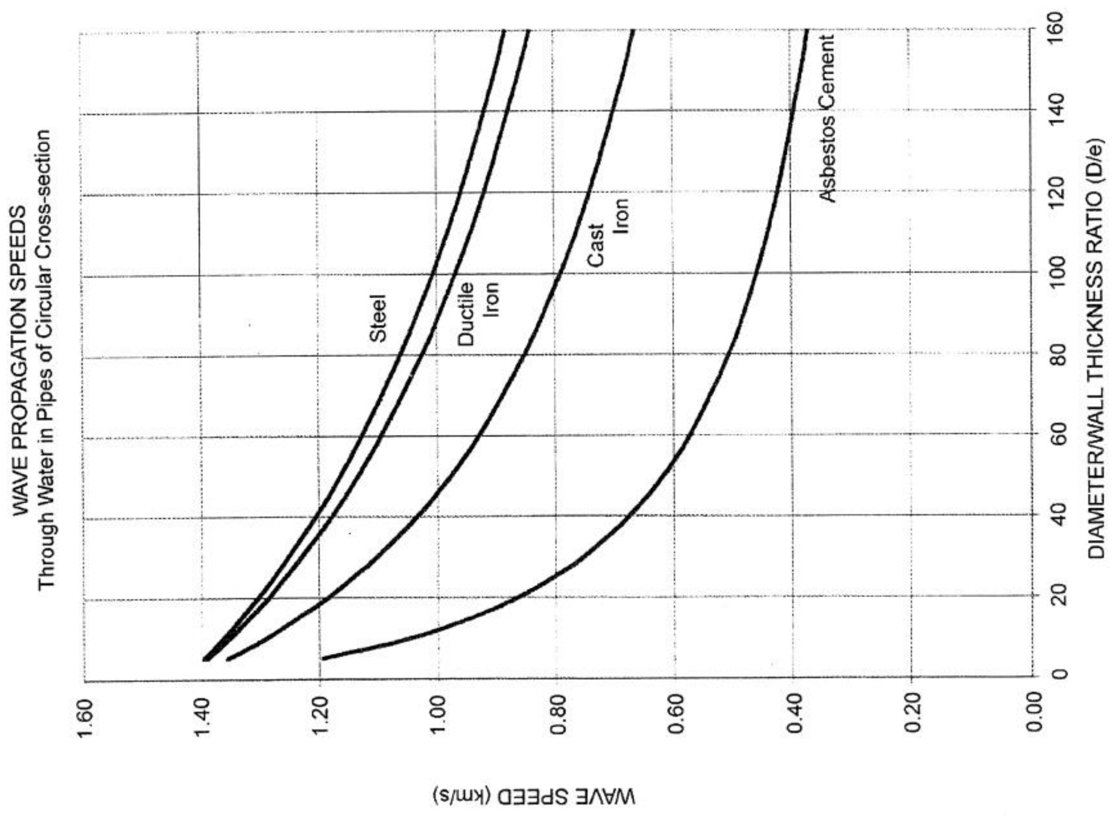
1% of gas by volume would reduce a wavespeed of 1400 m/s without gas to 120 m/s

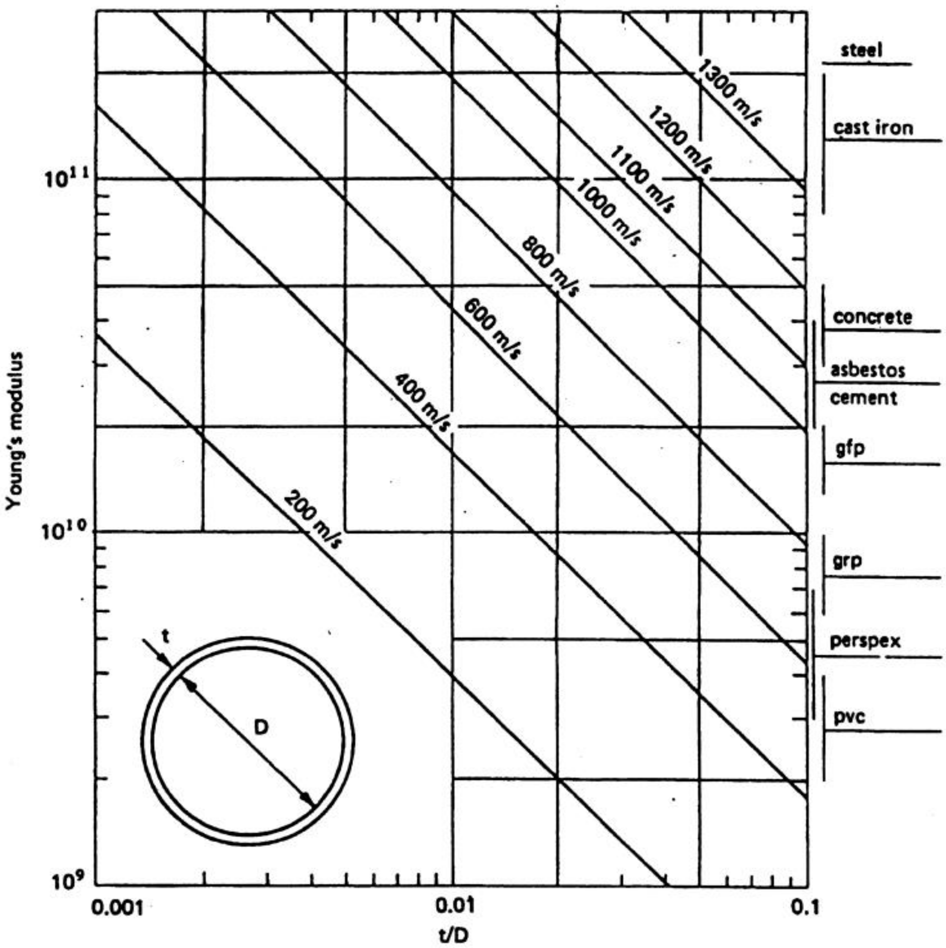
3.3 Ranges of wavespeeds

Ranges of wavespeeds (with water as the fluid) for common materials:

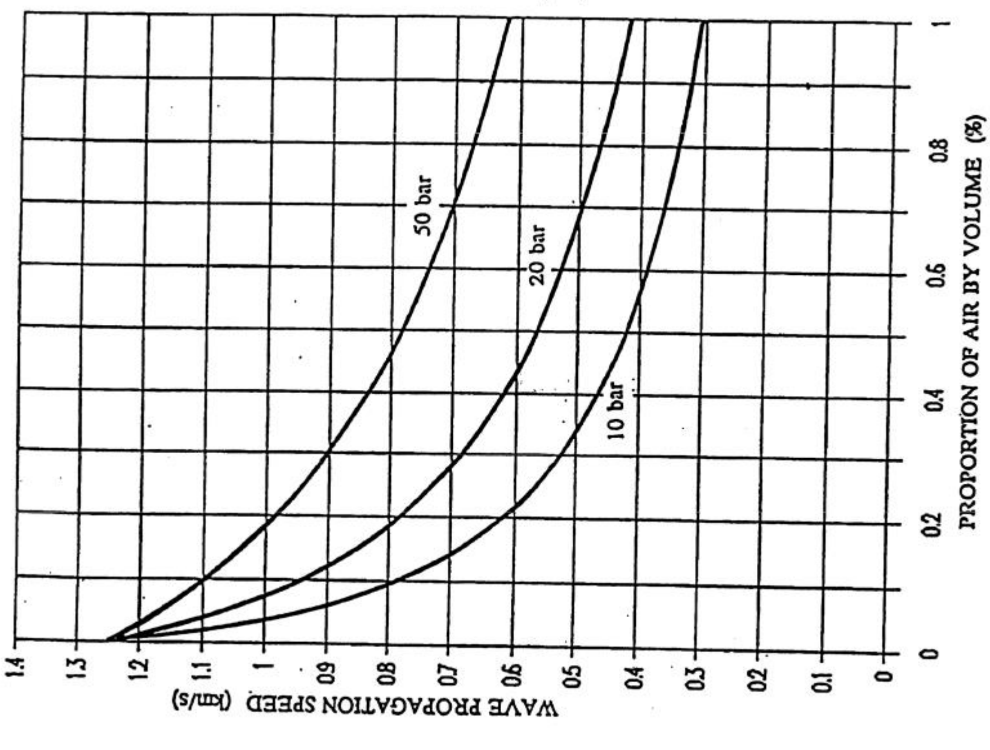



WAVE PROPAGATION SPEEDS
Through Water in Pipes of Circular Cross-section
WAVE SPEED (km/s)
DIAMETER/WALL THICKNESS RATIO (D/e)
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
0
20
40
60
80
100
120
140
160
GRP
uPVC
ABS


WAVE PROPAGATION SPEEDS
Through Water in Pipes of Circular Cross-section
WAVE SPEED (km/s)
DIAMETER/WALL THICKNESS RATIO (D/e)
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
0
20
40
60
80
100
120
140
160
Steel
Ductile Iron
Cast Iron
Asbestos Cement



Wavespeeds with varying proportions of air (for water in steel pipe with D/t ratio of 20)



3.3 Fluid and material properties

Physical properties of some common fluids at atmospheric pressure

Liquid	Temperature (°C)	Bulk Modulus K (GN/m^2)	Density ρ (kg/m^3)
Benzene	20	1.1	879
Ethyl Alcohol	20	1.32	789
Kerosene	20	1.32	805
Methanol	20	1.0	791
Mineral Oils	25	1.5 - 1.9	860-888
Phosphate Esters	25	0.9 - 0.95	1265 - 1315
Sea Water	15	2.27	1025
Sulphuric Acid	30	2.7	1330
Water	20	2.19	998

Wavespeeds of various liquids (in infinite volume of liquid, i.e. the maximum possible value)

Liquid	Temperature (°C)	Wavespeed a or c (m/s)
Ammonia	-77	2000
Ammonia	-33	1715
Crude oil	20	1400
Cyclohexane	20	1280
Ethylene glycol	20	1670
Ethanol	20	1170
Methane	-160	1320
Toluene	20	1330
Water	20	1480

Properties of common pipe materials

Material	Young's Modulus E (GN/m^2)	Shear Modulus G (GN/m^2)	Poisson's ratio ν
Aluminium,	69	27.6	0.33
Aluminium Alloys	69-75	26-28	0.33
Asbestos Cement	24		
Cast Iron	90-160	40-50	0.25
Concrete	20.3-	8.6-13	0.15
Ductile Iron	172	83	0.3
Glass	7-8	3	0.24
GRP (but see note above)	50	3.9	0.35
Mild Steel	200-210	80-84	0.28
Phosphor Bronze	120	43	0.38
ABS	1.7	0.65	0.33
Nylon	1.4-2.8	0.4-0.90	0.5
Perspex	6.2	2.3	0.33
UPVC (20°C)	3.3	1.1	0.5
Reinforced Concrete	30.60		
Stainless Steel	200-215	80.85	0.28
Titanium	103	45	0.34

4. Reading List

4.1 In Print

Fluid Transients in Pipeline Systems, A R D **Thorley**, 2nd Ed, Prof. Eng. Pub., 2004.

If you are only going to read/buy one book, it should be this one! Excellent coverage of methods of surge suppression and a very good chapter on the benefits and dangers of air valves. One of the few that assumes you will have access to commercial software and don't want to write your own! Very readable. There is a copy in the Cranfield University Library.

Pressure Transients in Water Engineering: A Guide to Analysis and Interpretation of Behaviour, J. **Ellis**, Thomas Telford, 2008.

I haven't actually seen a copy of this book as yet. Ellis was at Strathclyde University for many years lecturing on pressure transients before he retired and wrote this book. It seems to cover much the same ground as Thorley and the others.

Fluid Transients in Pipe Networks, A **Betâmio de Almeida** & E **Koelle**, Comp. Mech. Pub. / Elsevier Applied Science, 1992.

Written by two of the most well known people in the industry (from the Universities of Lisbon and São Paolo), this is another basic textbook covering similar ground to Wylie & Streeter, Thorley, etc. Contains sections on hydro-electric power stations and open channel transients. The book has an extensive bibliography at the end referencing books and papers. May not be in print but new copies were available on the internet in 2008 (from Computational Mechanics Publications). A little difficult to read due to the authors' style – neither are native English speakers.

Internal Flow Systems, D S **Miller**, 2nd Ed, BHR Group, 1990.

A key reference to pressure losses in components - free with Flowmaster New User training courses.

Proceedings of **Pressure Surges conference series** organised by BHR Group: 7th in 1996, 8th in 2000, 9th in 2004, 10th in 2008.

Case studies and discussions on surge protection and general surge issues plus academic 'state of the art' papers.

4.2 Out of Print

Fluid Transients, E B **Wylie** & V L **Streeter**, FEB Press, 1983.

The classic text for those who want a deeper understanding of the theory and the Method of Characteristics.

Transient Flow in Pipes, Open Channels and Sewers, J A **Fox**, Ellis Horwood Ltd, 1989.

Good general readable book including open channel surges.

Applied Hydraulic Transients, M H **Chaudry**, 2nd Ed, Van Nostrand Reinhold, 1987.

Good general text covering several industries as well as surges in open channels.

Fluid Transients, C **Jaeger**, Blackie & Son Ltd, 1977.

Concentrates on hydro-electric schemes.

5. Acknowledgements & Credits

The illustrations in the slides were mostly taken from Thorley or Wylie & Streeter. Pictures of valves, pumps and surge suppression equipment were taken from manufacturer's websites.

6. Theory and Method of Characteristics

These are extracts from Wylie & Streeter "Fluid Transients", Corrected Edition 1983, FEB Press. (This book is now out of print.) The nomenclature is at the end.

Resonance in a piping system is an oscillatory phenomenon in which the amplitude of the unsteady oscillations build up with time until failure occurs or until a steady-oscillatory flow of unusually large magnitude is reached. Resonance usually occurs at or near one of the natural periods, either the fundamental or a harmonic, of a system.

The term *valve stroking* is restricted in its meaning to the design of boundary conditions such that a transient occurs in a prescribed manner. *Liquid column separation* refers to the situation in a pipeline in which gas and (or) vapor collects at some section.

1-2 Arithmetic Derivation of Transient Flow Equations

The unsteady momentum equation is first applied to a control volume containing a section of pipe. The continuity equation is then developed for the fluid in the pipe; Poisson's ratio effects are introduced which requires definition of the means of supporting the pipe. Since more general derivations are made in Chapter 2, restricted conditions are imposed which aid in visualization of the transients.

The case of instantaneous stoppage of flow at a downstream valve is first described, then the continuity and momentum equations are applied to an incremental change in valve setting. In Fig. 1-1(*a*) friction and minor losses are neglected. The instant the valve is closed, the fluid immediately adjacent to it is brought from V_0 to rest by the impulse of the higher pressure developed at the face of the valve. As soon as the first layer is brought to rest, the same action is applied to the next layer of fluid bringing it to rest. In this manner, a pulse wave of high pressure is visualized as traveling upstream at some sonic wavespeed a and at a sufficient pressure to apply just the impulse to the fluid to bring it to rest.

The momentum equation is applied to a control volume, Fig. 1-1(*b*), within which the wave front is moving to the left with an absolute speed of $a - V_0$ due to a small change in valve setting. The head change ΔH at the valve is accompanied by a velocity change ΔV. The momentum equation for the x direction, in words, states that the resultant x component of force on the control volume is just equal to the time rate of increase of x momentum within the control volume plus the net efflux of x momentum from the control volume, or

$$-\gamma \Delta H A = \rho A(a - V_0)\Delta V + \rho A(V_0 + \Delta V)^2 - \rho A V_0^2$$

where

γ = specific weight of fluid
ρ = mass density of fluid (γ/g)
g = acceleration of gravity
A = cross-sectional area of pipe
V_0 = initial velocity
ΔV = increment of flow velocity
a = unknown wavespeed
ΔH = increment of head change

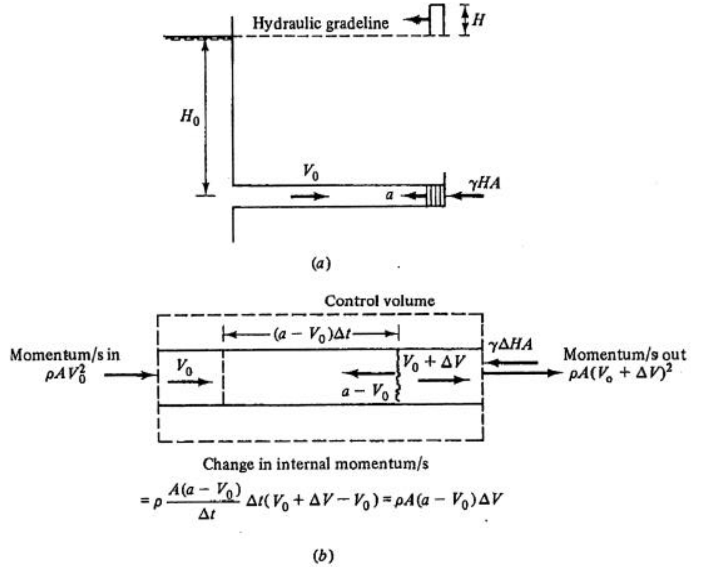


Figure 1-1 (*a*) Instantaneous stoppage of frictionless liquid in horizontal pipe; (*b*) Momentum equation applied to control volume.

The mass of fluid $\rho A(a - V_0)$ is having its velocity changed by ΔV in one second. By neglecting the small quantity containing ΔV^2, the equation reduces to

$$\Delta H = -\frac{a\Delta V}{g}\left(1 + \frac{V_0}{a}\right) \approx -\frac{a\Delta V}{g} \qquad (1\text{-}1)$$

V_0/a is generally very small compared with 1 for liquids in metal pipes. If the flow is completely stopped $\Delta V = -V_0$, and $\Delta H = aV_0/g$. Equation (1-1) also shows that for an increase in velocity at the gate the head there must be reduced. If the valve is on the downstream end of a long pipe, and is closed by increments, the equation becomes

$$\Sigma \Delta H = -\frac{a}{g}\Sigma \Delta V \qquad (1\text{-}2)$$

which holds for any movements of the valve, so long as the pressure pulse wave has not reached the upstream end of the pipe and returned as a reflected wave, i.e., so long as the time is less than $2L/a$, with L the pipe length.

For adjustments in an upstream gate, a similar derivation shows that $\Delta H = a\Delta V/g$, so

$$\Sigma \Delta H = \pm \frac{a}{g} \Sigma \Delta V \tag{1-3}$$

describes the change in flow related to a change in head. The minus sign must be used for waves traveling upstream and the plus sign for waves traveling downstream. It is the *basic equation of waterhammer* and always holds in the absence of reflections.

The magnitude of the wavespeed a has not been determined. Application of the continuity equation yields enough information so that, together with Eq. (1-2) the numerical value of a can be calculated. With reference to Fig. 1-2, if the gate at the downstream end of the pipe is suddenly closed the pipe may stretch in length Δs, depending on how it is supported. We may assume that it moves this distance in L/a seconds, or has the velocity $\Delta s\, a/L$. Hence the velocity of fluid at the gate has been changed by $\Delta V = \Delta s\, a/L - V_0$. During L/a seconds after gate closure the mass entering the pipe is $\rho A V_0 L/a$, which is accommodated within the pipe by increasing its cross-sectional area, by filling the extra volume due to pipe extension Δs, and by compressing the liquid due to its higher pressure, or

$$\rho A V_0 \frac{L}{a} = \rho L \Delta A + \rho A \Delta s + L A \Delta \rho \tag{1-4}$$

By use of $\Delta V = \Delta s\, a/L - V_0$ in eliminating V_0, Eq. (1-4) simplifies to

$$-\frac{\Delta V}{a} = \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

Now by use of Eq. (1-2) to eliminate ΔV

$$a^2 = \frac{g\,\Delta H}{\Delta A/A + \Delta\rho/\rho} \tag{1-5}$$

If the pipe is supported so that it cannot extend, then $\Delta s = 0$ and the same Eq. (1-5) is obtained, with or without expansion joints. By bringing in

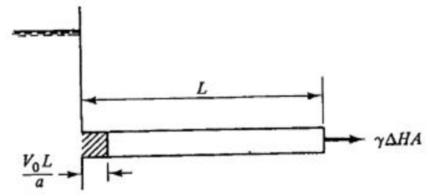


Figure 1-2 Continuity relations in pipe.

the bulk modulus of elasticity K of the fluid, defined by

$$K = \frac{\Delta p}{\Delta\rho/\rho} = -\frac{\Delta p}{\Delta \forall/\forall} \tag{1-6}$$

with $\Delta\forall/\forall$ the fractional volume change, Eq. (1-5) may be rearranged to yield

$$a^2 = \frac{K/\rho}{1 + (K/A)(\Delta A/\Delta p)} \tag{1-7}$$

For very thick-walled pipe $\Delta A/\Delta p$ is very small, and $a \approx \sqrt{K/\rho}$ the acoustic speed of a small disturbance in an infinite fluid. For very flexible pipe walls, the 1 in the denominator is small and becomes unimportant so that

$$a \approx \sqrt{\frac{A}{\rho}\frac{\Delta p}{\Delta A}} \tag{1-8}$$

Three support situations for a thin-walled pipeline are examined and the wavespeed formulas developed:

(*a*) pipe anchored at its upstream end only
(*b*) pipe anchored throughout against axial movement
(*c*) pipe anchored with expansion joints throughout

The term $\Delta A/(A\,\Delta p)$ must be evaluated for the 3 cases. First, Poisson's ratio, μ, is defined by

$$\mu = -\frac{\text{lateral unit strain}}{\text{axial unit strain}} = -\frac{\xi}{\xi_1} \tag{1-9}$$

The change in area is the result of a total change in lateral or circumferential strain, ξ_T,

$$\Delta A = \Delta\xi_T \frac{D}{2}\pi D \quad \text{or} \quad \frac{\Delta A}{A} = 2\,\Delta\xi_T$$

in which

$$\xi_T = \xi_2 + \xi$$
$$= \xi_2 - \mu\xi_1$$

Stress and strain are related by Young's modulus of elasticity E, thus

$$\xi_2 = \frac{\sigma_2}{E} \qquad \xi_1 = \frac{\sigma_1}{E} \tag{1-10}$$

in which

σ_1 = axial unit stress
σ_2 = lateral unit stress

All three cases have the same relationship for tensile stress σ_2 in the pipe wall, Fig. 1-3.

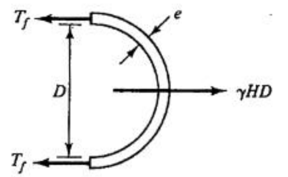


Figure 1-3 Forces on semicylinder of pipe due to waterhammer.

$$\sigma_2 = \frac{T_f}{e} = \frac{\gamma HD}{2e} \quad \text{or} \quad \Delta\sigma_2 = \frac{\gamma D\,\Delta H}{2e} = \frac{D\,\Delta p}{2e} \tag{1-11}$$

with e the pipe wall thickness and T_f the circumferential tensile force per unit length of pipe.

Case a The axial tensile stress is the force on the closed valve γHA divided by the area πDe, or

$$\sigma_1 = \frac{\gamma HA}{\pi De} \quad \text{or} \quad \Delta\sigma_1 = \frac{D\,\Delta p}{4e} \tag{1-12}$$

Then

$$\frac{\Delta A}{A\,\Delta p} = \frac{2\Delta\xi_T}{\Delta p} = \frac{2}{\Delta p}(\Delta\xi_2 - \mu\,\Delta\xi_1) = \frac{2}{\Delta pE}(\Delta\sigma_2 - \mu\,\Delta\sigma_1) = \frac{D}{Ee}\left(1 - \frac{\mu}{2}\right) \tag{1-13}$$

Case b For a pipe anchored throughout $\xi_1 = 0$, and $\sigma_1 = \mu\sigma_2$, so

$$\frac{\Delta A}{A\,\Delta p} = \frac{2}{\Delta pE}(\Delta\sigma_2 - \mu^2\,\Delta\sigma_2) = \frac{D}{Ee}(1 - \mu^2) \tag{1-14}$$

Case c For expansion joints throughout $\sigma_1 = 0$, and

$$\frac{\Delta A}{A\,\Delta p} = \frac{2\Delta\sigma_2}{\Delta pE} = \frac{D}{Ee} \tag{1-15}$$

By writing the wavespeed as

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + [(K/E)(D/e)]c_1}} \tag{1-16}$$

c_1 takes on the values:

(*a*) $c_1 = 1 - \dfrac{\mu}{2}$

(*b*) $c_1 = 1 - \mu^2$

(*c*) $c_1 = 1$

In these equations three pipe wall relations have been used. In Chapter 2 more general cases are developed.

Small amounts of entrained gas in the liquid, or gas which has come out of solution, greatly modify the acoustic speed in a pipe. After the pressure has been reduced in a pipeline, say by a pump slowdown or stoppage upstream, some gas usually comes out of solution which reduces the acoustic speed during the transient. The wavespeed does not fully recover its former high value rapidly under higher pressure conditions. The effect of free air or other gases is discussed in the next section.

For water at ordinary temperatures $\sqrt{K/\rho} = 1{,}440$ m/s. Wavespeeds for large steel pipelines may be as low as 1,000 m/s, whereas speeds in high-pressure small pipes may be 1,200 to 1,400 m/s.

With the wavespeed determined by Eq. (1-16), Eq. (1-3) yields a known relation between change in velocity and change in head. This equation is of great value in visualizing unsteady flow. For example with a/g of 100 s, a reduction of 1 m/s velocity creates an immediate head rise of 100 m.

The complete cycle, or period, that results from an instantaneous valve closure in a frictionless case is next described.

At the instant of valve closure ($t = 0$) the fluid nearest the valve is compressed, brought to rest, and the pipe wall stretched (Fig. 1-4). As soon as the first layer is compressed, the process is repeated for the next layer. The fluid upstream from the valve continues to move downstream with undiminished speed until successive layers have been compressed back to the source. The high pressure moves upstream as a wave, bringing the fluid to rest as it passes, compressing it, and expanding the pipe. When the wave reaches the upstream end of the pipe ($t = L/a$ s), all the fluid is under the extra head H, all the momentum has been lost, and all the kinetic energy has been converted into elastic energy.

There is an unbalanced condition at the upstream (reservoir) end at the instant of arrival of the pressure wave, since the reservoir pressure is unchanged. The fluid starts to flow backward (Fig. 1-4*b*), beginning at the upstream end. This flow returns the pressure to the value which was normal before closure, the pipe wall returns to normal, and the fluid has a velocity V_0 in the backward sense. This process of conversion travels downstream towards the valve at the speed of sound a in the pipe. At the instant $2L/a$, the wave arrives at the valve, pressures are back to normal along the pipe, and velocity is everywhere V_0 in the backward direction.

Since the valve is closed, no fluid is available to maintain the flow at the valve and a low pressure $-H$ develops such that the fluid is brought to rest. This low-pressure wave travels upstream at speed a and everywhere brings the fluid to rest, causes it to expand because of the lower pressure, and allows the pipe walls to contract. (If the static pressure in the pipe is not sufficiently high to sustain head $-H$ above vapor pressure, the liquid vaporizes in part and continues to move backward over a longer period of time.)

At the instant the negative-pressure wave arrives at the upstream end of the pipe, $3L/a$ sec after closure, the fluid is at rest, but uniformly at head $-H$ less than before closure. This leaves an unbalanced condition at the reservoir, and fluid flows into the pipe, acquiring a velocity V_0 forward and returning the pipe and

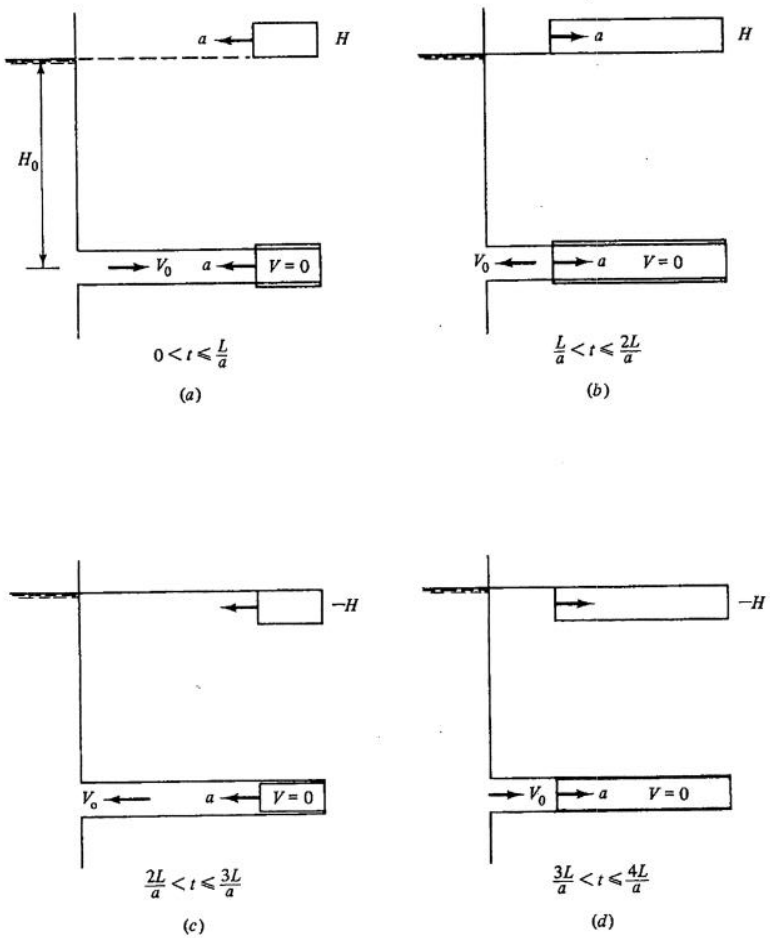


Figure 1-4 Sequence of events for one period after sudden closure of a valve.

fluid to normal conditions as the wave progresses downstream at speed a. At the instant this wave reaches the valve, conditions are exactly the same as at the instant of closure, $4L/a$ s earlier.

This process is then repeated every $4L/a$ s. The action of fluid friction and imperfect elasticity of fluid and pipe wall, neglected heretofore, damps out the vibration and eventually causes the fluid to come permanently to rest.

1-3 Effects of Air Entrainment

The propagation velocity of a pressure wave in a pipeline containing a liquid can be greatly reduced if gas bubbles are dispersed throughout the liquid. A straightforward calculation after Kobori, et al.,[56] and Pearsall[73] demonstrates that this is indeed true, even with a very small amount of air present in the form of bubbles in the liquid. These same authors have laboratory measurements that are in general agreement with their theory.

It is assumed that a pipeline contains a fluid which consists of a liquid with gas bubbles uniformly distributed throughout. Consider a section of the pipeline as a control volume. The total volume $\mathcal{V}$ of the fluid can be expressed as the sum of the liquid volume $\mathcal{V}_{liq}$ and the gas volume $\mathcal{V}_g$. A pressure change brings about a volume change which can be expressed

$$\Delta\mathcal{V} = \Delta\mathcal{V}_{liq} + \Delta\mathcal{V}_g$$

The gross bulk modulus of elasticity is defined by Eq. (1-6), and the bulk moduli of the individual components can be expressed by

$$K_{liq} = -\frac{\Delta p}{\Delta\mathcal{V}_{liq}/\mathcal{V}_{liq}}$$

and

$$K_g = -\frac{\Delta p}{\Delta\mathcal{V}_g/\mathcal{V}_g}$$

Combining these expressions with Eq. (1-6) yields

$$K = \frac{K_{liq}}{1 + (\mathcal{V}_g/\mathcal{V})(K_{liq}/K_g - 1)} \tag{1-17}$$

By expressing the mixture density in terms of the liquid and gas densities,

$$\rho = \rho_g \frac{\mathcal{V}_g}{\mathcal{V}} + \rho_{liq} \frac{\mathcal{V}_{liq}}{\mathcal{V}} \tag{1-18}$$

and, substituting into Eq. (1-16), one can obtain an expression for the wavespeed. If a small amount of gas is present, the effect of the pipe-wall elasticity becomes insignificant and that term is dropped:

$$a = \sqrt{\frac{K}{\rho}} \tag{1-19a}$$

where K is defined by Eq. (1-17) and ρ is defined by Eq. (1-18). Figure 1-5[56] illustrates the effect of air bubbles in a pipeline containing water. The good agreement between experimental results and theory is apparent. It can be noted that a small air content produces a wavespeed less than the speed of sound in air.

It is natural that the gas content in liquids tends to reduce the speed of the pressure pulse. For example, air bubbles in water could be visualized as springs

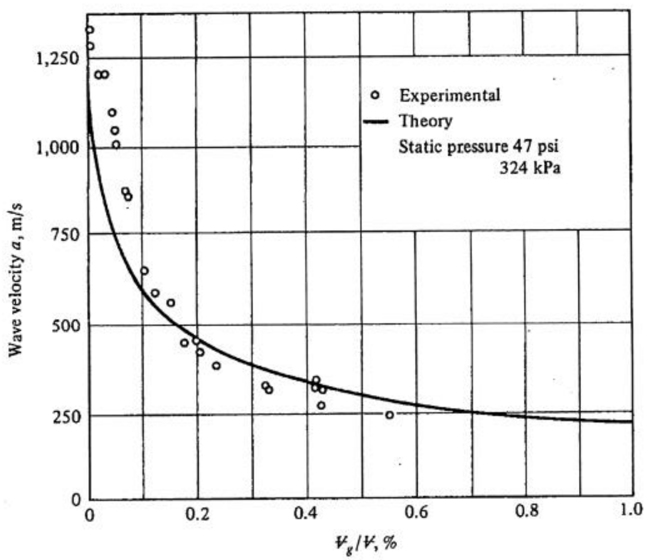


Figure 1-5 Propagation velocity a of a pressure wave in pipeline for varying air content (theoretical and experimental results).[56]

loaded with the water. A pressure pulse compresses the spring, which accelerates the water mass, which, in turn, compresses another spring. Thus the wave would travel through the fluid at a lower velocity than in a homogeneous liquid, in which the wave is transmitted directly from one particle to the next.

Example 1-1 Calculate the wavespeed in a pipeline containing water at atmospheric pressure with 1 percent air content. $K_g = 3000$ lb/ft², $K_{liq} = 4.23(10)^7$ lb/ft², $\rho_g = 0.00238$ slug/ft³, and $\rho_{liq} = 1.94$ slug/ft³. From Eq. (1-17), $K = 2.98(10)^5$ lb/ft², and from Eq. (1-18), $\rho = 1.92$. Therefore,

$$a = \sqrt{\frac{K}{\rho}} = 396 \text{ ft/s}$$

When free air is present under high pressure conditions, its volume is greatly reduced and the pipe elasticity becomes important. The equation for wavespeed is then

$$a = \sqrt{\frac{K/\rho}{1 + (K/E)(D/e)}} \tag{1-19b}$$

with K and ρ defined as in Eq. (1-19*a*). By considering a mass m of free air present per unit of volume, and that it compresses isothermally, $K_g = p$ the absolute pressure. Then

$$K = \frac{K_{liq}}{1 + (mRT/p)[(K_{liq}/p) - 1]}$$

with R the gas constant and T the absolute temperature, or

$$a = \sqrt{\frac{K_{liq}/\rho}{1 + K_{liq}D/Ee + (mRT/p)[(K_{liq}/p) - 1]}} \tag{1-19c}$$

Wavespeed for solid–liquid mixture When fine solid particles are in suspension in a liquid, the bulk modulus of elasticity for the mix is derived in the same manner as Eq. (1-17) yielding

$$K = \frac{K_{liq}}{1 + (\forall_s/\forall)(K_{liq}/K_s - 1)} \tag{1-19d}$$

in which K_s and $\forall_s$ are bulk modulus of elasticity of the solid particles and volume of solid particles, respectively. The mixture density is

$$\rho = \rho_s \frac{\forall_s}{\forall} + \rho_{liq} \frac{\forall_{liq}}{\forall} \tag{1-19e}$$

When a waterhammer pressure wave travels through a suspension, there is a motion of the solid particles relative to the liquid for a short time, until the viscous resistance forces between them damps out the motion. The acoustic wavespeed is given by Eq. (1-19*a*) using Eqs. (1-19*d*) and (1-19*e*). The relation between pressure change and velocity change due to a waterhammer wave is

$$\Delta p = -\rho a \Delta V \tag{1-19f}$$

neglecting relative motion of solid particles to liquid.

1-4 The Cause of Transients. Role of Valving

A change from steady-state flow in a piping system occurs because of a change in boundary conditions (except for pipe rupture). There are many kinds of boundary conditions that may introduce transients. Common ones that frequently require analyses are

1. Changes in valve settings, accidental or planned
2. Starting or stopping of pumps
3. Changes in power demand in turbines
4. Action of reciprocating pumps
5. Changing elevation of a reservoir
6. Waves on a reservoir
7. Turbine governor hunting
8. Vibration of impellers or guide vanes in pumps, fans, or turbines

1-7 Scope and Range of Problems in Unsteady Flow

Steady-oscillatory problems may be solved by transient-flow equations, but the converse is not practical. Unsteady-flow problems arising in the analysis and design of fluid systems may appear quite unrelated (e.g., flow in a large hydro system as compared with the waterhammer effects apparent in an oil-hydraulic lift for a farm tractor or flow in a diesel fuel injection system). The same methods of analysis apply, however, and it is the purpose of this treatment to present those methods that are of the most value in dealing with a wide variety of applications. Straight analysis of a system, correction of transients in a system, and design to avoid bad transients are all considered. The digital-computer methods are examined because of their many advantages. Where feasible, experimental evidence of the accuracy of the methods is included.

Problems

1-1 Derive Eq. (1-1) for complete stoppage of velocity V_0 by use of the control volume approach.

1-2 A valve at the downstream end of a pipe is opened suddenly so that the flow increases from 2 to 2.2 m/s. For $a = 1{,}100$ m/s what is the head change upstream?

1-3 For Problem 1-2 what would be the percent difference in head if the more exact formulation were used?

1-4 What thickness of a 1-in ID steel pipe is needed to withstand 100 psi pressure? Allowable maximum tensile stress is 10,000 psi. $c_1 = 1$.

1-5 A penstock near the power plant has a head of 350 ft and is 16 ft in diameter. For an overload of 100 percent what thickness of steel pipe wall is required? Maximum allowable stress = 10,000 psi. $c_1 = 1$.

1-6 A 2-m diameter pipe, $e = 20$ mm, has flow of water at 1 m/s under a head of 100 m. For steel pipe, ($E = 207$ GPa) for sudden valve closure, determine:

(a) The wave speed ($K = 2{,}070$ MPa) for the three cases of constraint ($\mu = 0.3$).
(b) the pipe circumferential stress before and after closure.
(c) the additional pipe area and its percentage change.

1-7 A steel pipe 4,000 ft long and 6 ft in diameter, $e = \frac{3}{4}$ in, $E = 3(10)^7$ psi, $K = 3(10)^5$ psi. It has water flowing at 3 ft/s. For case a restraint and sudden valve closure, how much flow enters the pipe after closure? How is this volume distributed between $\Delta\forall_L$, $\Delta\forall_{liq}$, and $\Delta\forall_r$?

1-8 Develop Eq. (1-19c) by considering a slice of liquid next to a slice of gas in a pipe segment by use of the momentum equation and the continuity equation. Neglect expansion of the pipe and Poisson ratio effects.

1-9 Find the wavespeed in a pipeline containing water with 2 percent air content. Assume standard atmospheric conditions at sea level.

1-10 At a pressure of 1 MPa in a pipeline, what air content in water would produce a wavespeed equal to the speed of sound in air alone?

CHAPTER
TWO
BASIC DIFFERENTIAL EQUATIONS FOR TRANSIENT FLOW

In this chapter the differential equations of motion and continuity are developed for use in later chapters. These two equations, in general, are simpler than the momentum and continuity equations for developing algebraic finite difference equations for solving transient problems. One special case of a highly deformable continuity equation is also worked out. The general wave equation solutions are given, as well as several special formulas for wavespeeds under various wall conditions. The subscripts x and t denote partial differentiation (i.e., $p_x = \partial p/\partial x$), and a dot over a dependent variable indicates the total derivative with respect to time.

2-1 Equation of Motion

The equation of motion is derived for liquid flow through a conical tube as well as for a cylindrical tube. The equation is in terms of centerline pressure $p(x, t)$ and average velocity $V(x, t)$. It is then converted to a form using the hydraulic grade line $H(x, t)$ sometimes called the *piezometric* head, or in short, the *head*. In most of this treatment H and the discharge $Q(x, t)$ are the preferred dependent variables. x and t are the independent variables.

Figure 2-1 shows a free body of fluid of cross sectional area A and thickness δx. The area A is, in general, a function of x, which is the coordinate distance along the axis of the tube from an arbitrary origin. The tube is inclined with the horizontal at an angle α, positive when the elevation increases in the $+x$ direction. The forces on the free body in the x direction are the surface contact normal pressures on the transverse faces, and shear and pressure components on the periphery. In addition gravity, the body force, has an x component. The shear

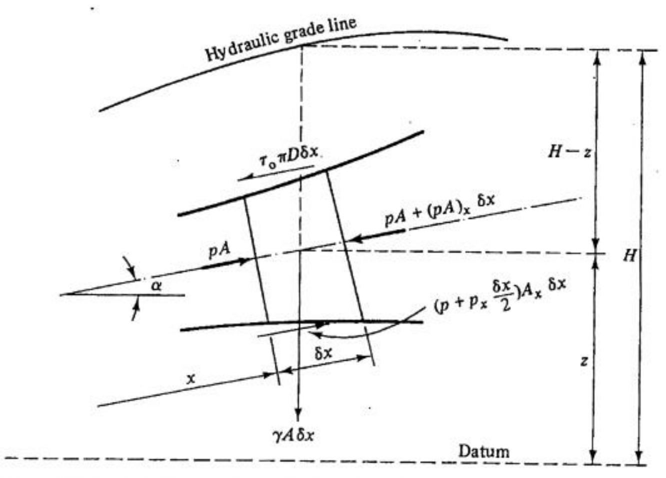


Figure 2-1 Freebody diagram for application of equation of motion.

force τ_0 is considered to act in the $-x$ direction. With reference to the figure the summation of forces on the slice of fluid is equated to its mass times its acceleration:

$$pA - [pA + (pA)_x \delta x] + \left(p + p_x \frac{\delta x}{2}\right) \dot{A}_x \delta x - \tau_0 \pi D\, \delta x - \gamma A\, \delta x \sin \alpha = \rho A\, \delta x\, \dot{V}$$

By dropping out the small quantity containing $(\delta x)^2$ and simplifying

$$p_x A + \tau_0 \pi D + \rho g A \sin \alpha + \rho A \dot{V} = 0 \tag{2-1}$$

In transient flow calculations the shear stress τ_0 is considered to be the same as if the velocity were steady, so in terms of the Darcy–Weisbach friction factor f[105]

$$\tau_0 = \frac{\rho f V|V|}{8} \tag{2-2}$$

This equation is developed from the Darcy–Weisbach equation

$$\Delta p = \frac{\rho f L}{D} \frac{V^2}{2} \tag{2-3}$$

with L the length of horizontal pipe, and from a force balance on the pipe in steady flow

$$\Delta p \frac{\pi D^2}{4} = \tau_0 \pi D L \tag{2-4}$$

by eliminating Δp. The absolute value sign on the velocity term in Eq. (2-2) ensures that the shear stress always opposes the direction of the velocity.

The acceleration term $\dot{V}$ in Eq. (2-1) is for a particle of fluid (the slice) having velocity V, hence

$$\dot{V} = V V_x + V_t \tag{2-5}$$

By use of Eqs. (2-2) and (2-5), Eq. (2-1) takes the form

$$\frac{p_x}{\rho} + V V_x + V_t + g \sin \alpha + \frac{f V|V|}{2D} = 0 \tag{2-6}$$

which is valid for converging or diverging pipe flow also. The piezometric head H (or elevation of hydraulic grade line above an arbitrary datum) may replace p. From Fig. 2-1

$$p = \rho g (H - z)$$

where z is the elevation of centerline of pipe at x. Then

$$p_x = \rho g (H_x - z_x) = \rho g (H_x - \sin \alpha) \tag{2-7}$$

This partial differentiation considered ρ to be substantially constant, as compared with H or z. Equation (2-6) is valid for gases, but Eq. (2-7) is restricted to liquids. Substitution into Eq. (2-6) yields

$$g H_x + V V_x + V_t + \frac{f V|V|}{2D} = 0 \tag{2-8}$$

also restricted to liquid flow. The hydraulic grade line form of the equation is somewhat simpler, as the slope of the pipeline drops out. Although V^2 friction was used in deriving the equations, an exponential law may be substituted, e.g., if $n = 1.85$ in a power law, then $fV|V|/2D$ may be replaced by

$$\lambda V |V^{n-1}| / D^m$$

λ, n, and m are determined to fit the formula desired.

Since Eq. (2-8) must hold for steady flow, a special case of unsteady flow, by setting $V_x = 0$ and $V_t = 0$, it becomes

$$\Delta H = -\frac{f \Delta x V|V|}{2gD}$$

which is the Darcy–Weisbach equation.

2-2 Continuity Equation

In this section a derivation of the continuity equation developed by T. P. Propson (private communication) is presented. It is quite general and has the advantage of portraying the various total derivatives, i.e., derivatives with respect to the motion. Two come directly into the continuity equation; (1) differentiation with respect to the axial motion of the pipe, and (2) differentiation with respect to a particle of fluid mass. The third total derivative is differentiation with respect to the acoustic wave motion which arises from the characteristics method developed in Chapter 3.

With reference to Fig. 2-2 a moving control volume of length δx at time t may be considered to be fixed relative to the pipe—it moves and stretches only as the inside surface of the pipe moves and stretches. The conservation of mass law may be stated that the rate of mass inflow into this control volume is just equal to the time rate of increase of mass within the control volume, or

$$-[\rho A(V-u)]_x\,\delta x = \frac{D'}{Dt}(\rho A\,\delta x) \tag{2-9}$$

Let the upstream face be at x, and u is the velocity of the pipe wall at x. The total derivative with respect to the axial motion of the pipe is given by

$$\frac{D'}{Dt} = u\frac{\partial}{\partial x} + \frac{\partial}{\partial t} \tag{2-10}$$

and the time rate of increase of length δx of the control volume is given by

$$\frac{D'}{Dt}\delta x = u_x\,\delta x \tag{2-11}$$

By partial expansion of Eq. (2-9) with use of Eq. (2-11)

$$(\rho AV)_x - (\rho Au)_x + \frac{D'}{Dt}(\rho A) + \rho A u_x = 0 \tag{2-12}$$

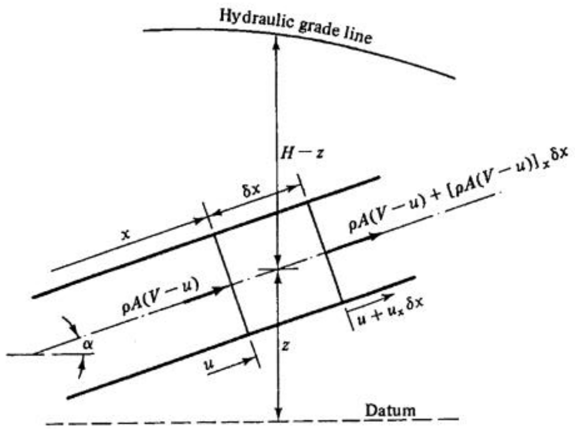


Figure 2-2 Control volume for continuity equation.

Additional expansion of Eq. (2-12), using Eq. (2-10) yields

$$(\rho AV)_x - (\rho A)_x u - \rho A u_x + u(\rho A)_x + (\rho A)_t + \rho A u_x = 0$$

or by simplifying

$$(\rho AV)_x + (\rho A)_t = 0 \tag{2-13}$$

which may now be written as

$$\rho A V_x + V(\rho A)_x + (\rho A)_t = 0$$

The last two terms represent the derivative of ρA with respect to motion of a mass particle, or

$$\frac{1}{\rho A}\frac{D}{Dt}(\rho A) + V_x = 0 \tag{2-14}$$

in which

$$\frac{D}{Dt} = V\frac{\partial}{\partial x} + \frac{\partial}{\partial t} \tag{2-15}$$

This total derivative is also indicated by a dot over the dependent variable, so

$$\frac{1}{\rho A}(\rho\dot{A} + \dot{\rho}A) + V_x = 0$$

or

$$\frac{\dot{A}}{A} + \frac{\dot{\rho}}{\rho} + V_x = 0 \tag{2-16}$$

This equation holds for converging or diverging tubes as well as cylindrical pipes. It is also valid for very flexible tubes, or for gas flow, as no simplifying assumptions have been required. Chapter 17, Sec. 1 deals with non-prismatic conduits. The rest of this chapter deals with prismatic conduits.

It is informative to again introduce the effect of Poisson's ratio on wave-speeds for the three cases handled in Chapter 1. With reference to Eq. (2-16)

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{p}}{K} \tag{2-17}$$

from the definition of bulk modulus of elasticity of a fluid (Eq. 1-6). The pipe wall expansion per unit of area per unit time $\dot{A}/A$ is

$$\frac{\dot{A}}{A} = 2\dot{\xi}_T \tag{2-18}$$

with terms defined as in Chapter 1. From Poisson's ratio relations

$$\dot{\xi}_T = \frac{1}{E}(\dot{\sigma}_2 - \mu\dot{\sigma}_1) \tag{2-19}$$

After use of Eqs. (2-17), (2-18), and (2-19) in Eq. (2-16)

$$\frac{2}{E}(\dot{\sigma}_2 - \mu\dot{\sigma}_1) + \frac{\dot{p}}{K} + V_x = 0 \qquad (2\text{-}20)$$

The transverse, or circumferential, tension is related to pressure by

$$\sigma_2 = pD/(2e)$$

or

$$\dot{\sigma}_2 = \dot{p}D/(2e) \qquad (2\text{-}21)$$

which holds for all three support conditions of Chapter 1. $\dot{D}$ changes so little with time, as compared to p in transient flow that it is considered constant for this differentiation.

The axial rate of change of tensile stress $\dot{\sigma}_1$ is given for the three cases as

$$(a)\quad \dot{\sigma}_1 = \frac{\dot{p}A}{\pi De} = \frac{\dot{p}D}{4e}$$

$$(b)\quad \dot{\sigma}_1 = \mu\dot{\sigma}_2 \qquad (2\text{-}22)$$

$$(c)\quad \dot{\sigma}_1 = 0$$

Equation (2-20) through substitution of Eqs. (2-21) and (2-22) may be written

$$\frac{\dot{p}}{\rho} + a^2 V_x = 0 \qquad (2\text{-}23)$$

in which

$$a^2 = \frac{K/\rho}{1 + [(K/E)(D/e)]c_1} \qquad (2\text{-}24)$$

with c_1 defined for each case

$$(a)\quad c_1 = 1 - \frac{\mu}{2}$$

$$(b)\quad c_1 = 1 - \mu^2 \qquad (2\text{-}25)$$

$$(c)\quad c_1 = 1$$

In Eq. (2-23) a^2 is considered to be a constant that is a collection of properties of the fluid, the pipe, and its means of support, and so far has been given no meaning relating it to acoustic speed.

The bulk modulus of elasticity of a fluid, Eq. (1-6), may also be defined in terms of piezometric head

$$K = \frac{\rho g \,\Delta H}{\Delta \rho / \rho} \qquad (2\text{-}26)$$

The continuity equation, Eq. (2-16), may then be written

or

$$VH_x + H_t + \frac{a^2}{g}V_x = 0 \qquad (2\text{-}28)$$

This is a convenient form of the continuity equation with V and H as dependent variables, and with x and t the independent variables. Through a^2 the fluid and wall properties are included.

2-3 Wavespeeds in Special Conduits

In addition to wavespeeds given for the previously discussed thin-walled pipe, we shall present here wavespeeds for a few special conduits.

Thick-walled elastic pipeline For pipes in which the walls are relatively thick in comparison with the diameter, the stress in the walls is not uniformly distributed throughout the walls. In this condition, as when the ratio D/e is less than approximately 25, the following coefficients[37] should be used.

Case a The pipeline is anchored at upstream end only, and

$$c_1 = \frac{2e}{D}(1+\mu) + \frac{D}{D+e}\left(1 - \frac{\mu}{2}\right) \qquad (2\text{-}29)$$

Case b The pipeline is anchored against longitudinal movement, and

$$c_1 = \frac{2e}{D}(1+\mu) + \frac{D(1-\mu^2)}{D+e} \qquad (2\text{-}30)$$

Case c The pipeline has expansion joints throughout its length, and

$$c_1 = \frac{2e}{D}(1+\mu) + \frac{D}{D+e} \qquad (2\text{-}31)$$

In the thick-walled pipeline the type of constraint has little effect on the wavespeed. It can be noted that as the thickness e becomes small, each coefficient approaches the corresponding c_1 for the thin-walled pipeline.

Circular tunnels By allowing the thickness e in the equations for thick-walled pipes to become larger and larger, c_1 approaches the value $(2e/D)(1+\mu)$. Substituting this value of c_1 into Eq. (1-16) yields

$$a = \sqrt{\frac{K/\rho}{1 + (2K/E_R)(1+\mu)}} \qquad (2\text{-}32)$$

This equation enables the wavespeed in a conduit through solid rock or concrete to be calculated. E_R and μ represent the modulus of rigidity and Poisson's ratio of the tunnel material, respectively.

Lined circular tunnels A steel liner in contact with the tunnel material increases the wavespeed to more than that which would exist in the tunnel alone. If Poisson's ratio effects are neglected in both the steel and tunnel material, a simple expression[37] can be presented for the coefficient c_1 in Eq. (1-16):

$$c_1 = \frac{2Ee}{E_R D + 2Ee} \tag{2-33}$$

Reinforced concrete pipe The pressure-pulse velocity in reinforced concrete pipe can be estimated by replacing the actual pipe with an "equivalent" steel pipe whose wall thickness is based upon the concrete thickness and the reinforcing bars in the pipe. The ratio of the moduli of concrete to steel multiplied by the concrete thickness yields an equivalent steel-pipe thickness. An allowance can be made for the probable cracking of the concrete pipe.

Equations including Poisson's ratio effect can be developed for lined tunnels;[37] in most cases, however, additional accuracy for this one factor is not warranted, since other uncertainties are likely to be equally important. Items which may be of some importance but which have not been considered include the nonlinear nature of the bulk modulus of the fluid, a nonperfect circular section, the nonlinear nature of some pipe materials, and frictional, viscoelastic, and hysteretic losses.

Plastic pipes The formulas as developed for metal pipes are satisfactory for calculating plastic tubing wavespeeds if the appropriate bulk moduli and Poisson's ratio are used.[117]

Rectangular and other noncircular cross sections For cross sections other than circular, theoretical wavespeeds may be calculated from Eq. (1-7) if the term $\Delta A/(A\,\Delta p)$ can be evaluated. Jenkner[45] has calculated values for both the square and rectangular cases.

For the square conduit of sides B and thickness of material e

$$\frac{\Delta A}{A\,\Delta p} = \frac{B}{eE} + \frac{B^3}{15e^3E} \tag{2-34}$$

The first term on the right side is due to the tension elongation of the sides and the second term is due to bending of the sides. In general the first term may be neglected.

For the rectangular cross section of width B and depth D, neglecting area increase due to tension,

$$\frac{\Delta A}{A\,\Delta p} = \frac{B^4R}{15e^3ED} \tag{2-35}$$

in which R is a "rectangular factor" given by

$$R = \frac{6-5\alpha}{2} + \frac{1}{2}\left(\frac{D}{B}\right)^5\left[6 - 5\alpha\left(\frac{B}{D}\right)^2\right]$$

with

$$\alpha = \frac{1 + (D/B)^3}{1 + D/B}$$

Thorley and Guymer[112] have studied thick-walled rectangular conduits including shear deformation as well as tension and bending. The shear and tension terms comprise 10–12 percent of the area change for a width to thickness ratio of 15. The terms become negligible for large ratios, say 100.

Example 2-1 A 750-mm-diameter pipe is filled with water. $K = 2.2$ GPa and $\rho = 998.2$ kg/m³ for 20°C.

(*a*) If the pipeline were considered completely rigid, the wavespeed would be

$$a = \sqrt{K/\rho} = \sqrt{2.2 \times 10^9/998.2} = 1484.6 \text{ m/s}$$

(*b*) Consider the three conditions of restraint on a steel pipeline ($e = 6.35$ mm, $\mu = 0.3$, $E = 207$ GPa, and $KD/Ee = 1.255$).

Case a

$$c_1 = 1 - \frac{\mu}{2} = 0.85 \qquad a = \frac{1484.6}{\sqrt{1 + 1.255 \times 0.85}} = 1032.7 \text{ m/s}$$

Case b

$$c_1 = 1 - \mu^2 = 0.91 \qquad a = \frac{1484.6}{\sqrt{1 + 1.255 \times 0.91}} = 1014.4 \text{ m/s}$$

Case c

$$c_1 = 1 \qquad a = \frac{1484.6}{\sqrt{1 + 1.255}} = 988.6 \text{ m/s}$$

(*c*) Consider the three conditions of restraint on a steel thick-walled pipeline of the same internal diameter. $e = 50$ mm, $KD/Ee = 0.1594$.

Case a

$$c_1 = \frac{2 \times 0.05}{0.75}(1 + 0.3) + \frac{0.75}{0.75 + 0.05}\left(1 - \frac{0.3}{2}\right) = 0.970$$

$$a = \frac{1484.6}{\sqrt{1 + 0.1594 \times 0.970}} = 1381.6 \text{ m/s}$$

Case b

$$c_1 = 0.173 + \frac{0.75}{0.80} \times 0.91 = 1.026 \qquad a = \frac{1484.6}{\sqrt{1 + 0.1594 \times 1.026}} = 1376.3 \text{ m/s}$$

Case c

$$c_1 = 0.173 + \frac{0.75}{0.80} = 1.11 \qquad a = \frac{1484.6}{\sqrt{1 + 0.1594 \times 1.11}} = 1368.4 \text{ m/s}$$

(*d*) Consider a 750-mm tunnel through a concrete dam. $E_R = 20.7$ GPa and $\mu = 0.3$

$$a = \frac{1484.6}{\sqrt{1 + 2 \times (2.2/20.7)(1 + 0.3)}} = 1314.1 \text{ m/s}$$

2-6 Show graphically the effect of the modulus of elasticity of the pipe material upon the wavespeed. Assume the fluid to be water and the ratio $D/e = 70$. Include points for the following:

Material	Modulus of elasticity $E \times 10^6$ psi	GPa
Steel	30.0	207.0
Aluminum	10.3	70.0
Copper	17.6	120.0
Cast iron	16.0	110.0
Reinforced concrete	4.3-8.7	30.0-60.0
Fiberglass reinf. plastic	1.4-4.0	10.0-27.3
Transite	3.4	23.0
PVC	0.4-0.6	3.0-4.4
Plastic	0.2	1.4
Rubber	0.01	0.07

2-7 Calculate the wavespeeds in Example 2-1c, using the equation for thin-walled tubing, and compare the results with those given for the thick-walled condition.

2-8 Compare the wavespeed in a water-filled 300-mm-diameter rubber pipeline, 6 mm thick, with a thick-walled pipeline, 70 mm thick. $E = 0.1$ GPa; $\mu = 0.45$. Assume the pipe to be anchored at one end only.

2-9 Calculate what the wavespeed would be in the thin-walled pipeline in Problem 2-8 if it were filled with oil. Specific gravity = 0.80; $K = 1.586$ GPa.

2-10 Find the wave velocity in a 5-m-diameter tunnel through solid rock. $E_R = 11$ GPa; $\mu = 0.3$.

2-11 Calculate the wavespeed in a water-filled copper tube installed without longitudinal restraint. ID = 25 mm; $e = 0.7$ mm; $E = 110$ GPa; $\mu = 0.3$.

CHAPTER
THREE
SOLUTION BY CHARACTERISTICS METHOD

A numerical solution of the equations that govern unsteady-fluid flow in pipelines is developed in this chapter. A general solution to the partial differential equations is not available, however, the partial differential equations may be transformed by the method of characteristics into particular total differential equations. These latter equations may then be integrated to yield finite difference equations which are conveniently handled numerically.

The equations in their simplest form, not including the smaller terms, are first organized for numerical computations. Various boundary conditions are presented, and simple examples are solved. The equations are presented in forms for direct substitution into a computer compiler language. Basic programs for the solution of unsteady-fluid flow problems are presented in FORTRAN. Examples of experimental confirmation of the calculations are included.

The method of characteristics is also applied to the nonsimplified equations. Complex systems are handled, and some of the distinctive features of this particular method of analysis are covered. This includes the handling of high friction systems, the inclusion of minor losses, and the alternative characteristics grid approach.

3-1 Characteristics Equations

The continuity and momentum Eqs. (2-8) and (2-28) form a pair of quasi-linear hyperbolic partial differential equations in terms of two dependent variables, velocity and hydraulic-grade-line elevation, and two independent variables, distance along the pipe and time. The equations are transformed into four ordinary differential equations by the characteristics method.[32,33,36,59,104] In this section, the terms of lesser importance are omitted from the equations in order to provide the simplest possible introduction to the theory in a single pipeline. Sec. 3-7 deals with the solution of the complete equations.

The simplified equations of motion and continuity are identified as L_1 and L_2 (from Eqs. 2-8 and 2-28)

$$L_1 = gH_x + V_t + \frac{f}{2D} V|V| = 0 \tag{3-1}$$

$$L_2 = H_t + \frac{a^2}{g} V_x = 0 \tag{3-2}$$

These equations are combined linearly using an unknown multiplier λ.

$$L = L_1 + \lambda L_2 = \lambda\left[H_x \frac{g}{\lambda} + H_t\right] + \left[V_x \lambda \frac{a^2}{g} + V_t\right] + \frac{fV|V|}{2D} = 0 \tag{3-3}$$

Any two real, distinct values of λ will again yield two equations in terms of the two dependent variables H and V that are in every way the equivalent of Eqs. (3-1) and (3-2). Appropriate selection of two particular values of λ leads to simplification of Eq. (3-3). In general both variables V and H are functions of x and t. If the independent variable x is permitted to be a function of t, then from calculus

$$\frac{dH}{dt} = H_x \frac{dx}{dt} + H_t \qquad \frac{dV}{dt} = V_x \frac{dx}{dt} + V_t \tag{3-4}$$

Now, by examination of Eq. (3-3) with Eqs. (3-4) in mind, it can be noted that if

$$\frac{dx}{dt} = \frac{g}{\lambda} = \frac{\lambda a^2}{g} \tag{3-5}$$

Eq. (3-3) becomes the ordinary differential equation

$$\lambda \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \tag{3-6}$$

The solution of Eq. (3-5) yields the two particular values of λ:

$$\lambda = \pm \frac{g}{a} \tag{3-7}$$

By substituting these values of λ back into Eq. (3-5), the particular manner in which x and t are related is given,

$$\frac{dx}{dt} = \pm a \tag{3-8}$$

This shows the change in position of a wave related to the change in time by the wave propagation velocity a. When the positive value of λ is used in Eq. (3-5), the positive value of λ *must* be used in Eq. (3-6). A similar parallelism exists

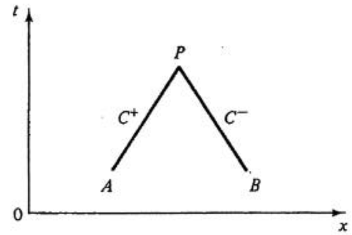


Figure 3-1 Characteristic lines in the xt plane.

for the negative λ. The substitution of these values of λ into Eq. (3-6) leads to two pairs of equations which are grouped and identified as C^+ and C^- equations.

$$\left.\begin{aligned} &\frac{g}{a}\frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 && (3\text{-}9)\\ &\frac{dx}{dt} = +a && (3\text{-}10)\end{aligned}\right\} C^+$$

$$\left.\begin{aligned} &-\frac{g}{a}\frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 && (3\text{-}11)\\ &\frac{dx}{dt} = -a && (3\text{-}12)\end{aligned}\right\} C^-$$

Thus the two real values of λ have been used to convert the original two partial differential equations to two total differential equations, Eqs. (3-9) and (3-11), each with the restriction that it is valid only when the respective Eqs. (3-10) and (3-12) are valid.

It is convenient to visualize the solution as it develops on the independent variable plane, i.e., the xt plane. Inasmuch as a is generally constant for a given pipe, Eq. (3-10) plots as a straight line on the xt plane; and similarly Eq. (3-12) plots as a different straight line (Fig. 3-1). These lines on the xt plane are the "characteristic" lines along which Eqs. (3-9) and (3-11) are valid. The latter equations are referred to as compatibility equations, each one being valid only on the appropriate characteristic line.

No mathematical approximations have been made in this transformation of the original partial differential equations. Thus, every solution of this set will be a solution of the original system given by Eqs. (3-1) and (3-2).

3-2 Finite-difference Equations

A pipeline is divided into N equal reaches, each Δx in length as shown in Fig. 3-2. A time-step size is computed, $\Delta t = \Delta x/a$, and Eq. (3-10) is satisfied by a positively sloped diagonal of the grid, shown by the line AP. If the dependent variables V and H are known at A, then Eq. (3-9), which is valid along the C^+

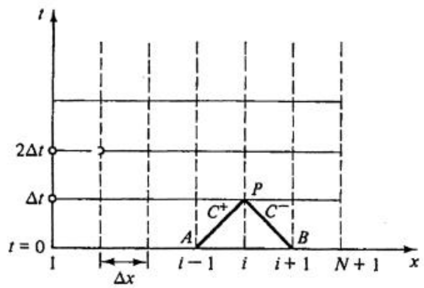


Figure 3-2 *xt* grid for solving single-pipe problems.

line, can be integrated between the limits A and P and thereby be written in terms of unknown variables V and H at point P. Equation (3-11) is satisfied by a negatively sloped diagonal of the grid, shown by BP. Integration of the C^- compatibility equation along the line BP, with conditions known at B and unknown at P, leads to a second equation in terms of the same two unknown variables at P. A simultaneous solution yields conditions at the particular time and position in the xt plane designated by point P.

By multiplying Eq. (3-9) by $a\,dt/g = dx/g$, and by introducing the pipeline area to write the equation in terms of discharge in place of velocity, the equation may be placed in a form suitable for integration along the C^+ characteristic, Fig. 3-2.

$$\int_{H_A}^{H_P} dH + \frac{a}{gA}\int_{Q_A}^{Q_P} dQ + \frac{f}{2gDA^2}\int_{x_A}^{x_P} Q|Q|\,dx = 0 \qquad (3\text{-}13)$$

The variation of Q with x under the integral in the last term is unknown *a priori*, so an approximation is introduced in this evaluation. A first-order approximation is satisfactory for most problems (in fact for all except the class of problems in which the friction term dominates, as discussed in Sec. 3-6). The integration of Eq. (3-13), and a similar integration along the C^- characteristic between B and P, yields

$$H_P - H_A + \frac{a}{gA}(Q_P - Q_A) + \frac{f\Delta x}{2gDA^2}Q_A|Q_A| = 0 \qquad (3\text{-}14)$$

$$H_P - H_B - \frac{a}{gA}(Q_P - Q_B) - \frac{f\Delta x}{2gDA^2}Q_B|Q_B| = 0 \qquad (3\text{-}15)$$

These two compatibility equations are basic algebraic relations that describe the transient propagation of pressure head and flow in a pipeline. By solving for H_P, these equations may be written

$$C^+: H_P = H_A - B(Q_P - Q_A) - RQ_A|Q_A| \qquad (3\text{-}16)$$

$$C^-: H_P = H_B + B(Q_P - Q_B) + RQ_B|Q_B| \qquad (3\text{-}17)$$

in which $B = a/gA$ and $R = f\Delta x/(2gDA^2)$.

These equations must hold for steady flow, which is a special case of unsteady flow. For steady conditions the flows are equal, $Q_A = Q_P = Q_B$, and $RQ_A|Q_A|$ is the steady-state friction head loss over the reach Δx. If an exponential friction formula is preferred the last term of Eq. (3-16), for example, would become $R'Q_A|Q_A|^{n-1}$ with n the exponent in the friction loss equation and R' the coefficient.

The solution to a problem in liquid transients usually begins with steady-state conditions at time zero, so that H and Q are known initial values at each computing section, Fig. 3-2, for $t = 0$. The solution consists of finding H and Q for each grid point along $t = \Delta t$, then proceeding to $t = 2\Delta t$, etc., until the desired time duration has been covered. At any interior grid intersection point, section i, the two compatibility equations are solved simultaneously for the unknowns Q_{P_i} and H_{P_i}. Equations (3-16) and (3-17) may be written in a simple form, namely

$$C^+: H_{P_i} = C_P - BQ_{P_i} \qquad (3\text{-}18)$$

$$C^-: H_{P_i} = C_M + BQ_{P_i} \qquad (3\text{-}19)$$

in which C_P and C_M are always known constants when the equations are applied:

$$C_P = H_{i-1} + BQ_{i-1} - RQ_{i-1}|Q_{i-1}| \qquad (3\text{-}20)$$

$$C_M = H_{i+1} - BQ_{i+1} + RQ_{i+1}|Q_{i+1}| \qquad (3\text{-}21)$$

By first eliminating Q_{P_i} in Eqs. (3-18) and (3-19)

$$H_{P_i} = (C_P + C_M)/2 \qquad (3\text{-}22)$$

Then Q_{P_i} may be found directly from either Eq. (3-18) or (3-19). The subscript notation used in the above equations, which is convenient for computer calculations, is shown in Fig. 3-2. It may be noted that section i refers to any grid intersection point in the x direction. Subscripted values of H and Q at each section are always available for the preceding time step, either as given initial conditions or as the results of a previous stage of the calculations. The new heads and flows at the current time during the transient have the letter P appended to the variables.

Examination of the grid in Fig. 3-2 shows that the end points of the system begin influencing the interior points after the first time step. Therefore, in order to complete the solution to any desired time, it is necessary to introduce the appropriate boundary conditions.

3-3 Basic Boundary Conditions

At either end of a single pipe only one of the compatibility equations is available in the two variables. For the upstream end (Fig. 3-3*a*), Eq. (3-19) holds along the C^- characteristic, and for the downstream boundary (Fig. 3-3*b*), Eq. (3-18) is valid along the C^+ characteristic. These are linear equations in Q_P and H_P; each conveys to their respective boundaries the complete behavior and response of the fluid in the pipeline during the transient. An auxiliary equation is needed in each

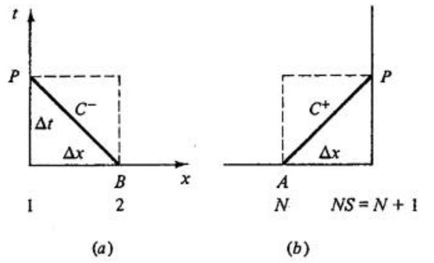


Figure 3-3 Characteristics at boundaries.

case that specifies Q_P, H_P, or some relation between them. That is, the auxiliary equation must convey information on the behavior of the boundary to the pipeline. Each boundary condition is solved independently of the other boundary, and independently of the interior point calculations. A few simple boundary conditions are now considered.

Reservoir at upstream end with elevation specified At a large upstream reservoir the elevation of the hydraulic grade line normally can be assumed constant during a short duration transient. This boundary condition is described, $H_{P_1} = H_R$, in which H_R is the elevation of the reservoir surface above the reference datum. If the reservoir level changes in a known manner, say as a sine wave, the boundary condition is

$$H_{P_1} = H_R + \Delta H \sin \omega t \tag{3-23}$$

in which ω is the circular frequency and ΔH is the amplitude of the wave. At each time step in either of the above cases H_{P_1} is known, and Q_{P_1} is determined by a direct solution of Eq. (3-19)

$$Q_{P_1} = (H_{P_1} - C_M)/B \tag{3-24}$$

The subscript 1 refers to the upstream section, Fig. 3-3*a*; C_M is a variable in the computational procedure but is dependent only upon known values from the previous time step, in this case from section 2.

Discharge as a specified function of time at upstream end The flow delivered from a positive displacement pump may be expressed as an explicit function of time, for example,

$$Q_{P_1} = Q_0 + \Delta Q |\sin \omega t| \tag{3-25}$$

With Q_{P_1} known at any instant, Eq. (3-19) is applied directly to find H_{P_1} at each time step.

Centrifugal pump at the upstream end with the head-discharge curve known The response of a centrifugal pump operating at constant speed may be included in an

analysis by defining the pump characteristic curve. In a computer program this may be accomplished by storing tabular data to describe the curve, or by using an equation to relate the variables. If the pump is supplying flow from a suction reservoir, whose surface elevation is used as the reference datum for the hydraulic grade line, an equation of the following form may be used

$$H_{P_1} = H_S + Q_{P_1}(a_1 + a_2 Q_{P_1}) \tag{3-26}$$

in which H_S is the shutoff head, and a_1 and a_2 are constants to describe the characteristic curve. Equation (3-26) provides an analytical relationship between the two variables which must be solved simultaneously with Eq. (3-19). The solution is

$$Q_{P_1} = \frac{1}{2a_2}\left[B - a_1 - \sqrt{(B - a_1)^2 + 4a_2(C_M - H_S)}\right] \tag{3-27}$$

With Q_{P_1} known, H_{P_1} can be obtained from either Eq. (3-19) or (3-26).

Dead end at downstream end of pipe The downstream end of a pipeline that is divided into N reaches is at section $NS = N + 1$, Fig. 3-3*b*. If the pipeline contains a closed end, then $Q_{P_{NS}} = 0$, and $H_{P_{NS}}$ is obtained directly from either Eq. (3-16) or (3-18).

Valve at downstream end of pipe If the datum for elevation of hydraulic grade line is taken at the valve, the orifice equation for steady-state flow through the valve is

$$Q_0 = (C_d A_G)_0 \sqrt{2gH_0} \tag{3-28}$$

in which Q_0 is the steady-state flow, H_0 the steady-state head loss across the valve, and $(C_d A_G)_0$ the area of valve opening times the discharge coefficient. For another opening, in general,

$$Q_P = C_d A_G \sqrt{2g\,\Delta H} \tag{3-29}$$

in which ΔH is the instantaneous drop in hydraulic grade line across the valve. After defining the dimensionless valve opening as

$$\tau = \frac{C_d A_G}{(C_d A_G)_0} \tag{3-30}$$

and dividing Eq. (3-29) by Eq. (3-28)

$$Q_P = \frac{Q_0}{\sqrt{H_0}}\,\tau\sqrt{\Delta H} \tag{3-31}$$

For steady flow, $\tau = 1$, and for no flow with the valve in the closed position, $\tau = 0$. The value of τ may be larger than unity if the valve is opened from the steady-state position. When the subscript for the downstream section, NS, is appended to the variables Q_P and $H_P(\Delta H = H_{P_{NS}})$, and Eqs. (3-18) and (3-31) are solved simultaneously

$$Q_{P_{NS}} = -BC_v + \sqrt{(BC_v)^2 + 2C_vC_P} \qquad (3\text{-}32)$$

in which $C_v = (Q_0\tau)^2/2H_0$. The corresponding value of $H_{P_{NS}}$ can be determined from either Eq. (3-18) or (3-31).

The hydraulic characteristics of valves differ greatly depending primarily upon the configuration of the flow path through the valve opening. The steady-state loss coefficients, as a function of valve position, for a few different valve types, are provided in Appendix B.[8a,113a]

Orifice at downstream end of pipe The same equations are used for the fixed orifice as for the valve with the simplification that $\tau = 1$.

3-4 Single-pipeline Applications

The procedure to solve a transient fluid-flow problem numerically involves a number of repetitious calculations. A computer program to solve a problem involving a single pipe leading from a reservoir to a valve, Fig. 3-4, has the following elements:

1. Read in values of data that describe the system and the character of the particular transient.
2. Calculate constants and initial steady-state conditions, store initial values of Q_i and H_i for $t = 0$.
3. Print out values of Q_i and H_i at each section, plus print out time and valve opening.
4. Increment the time by Δt and calculate the interior points Q_{P_2} to Q_{P_N}, H_{P_2} to H_{P_N}, and then calculate the boundary values Q_{P_1}, H_{P_1}, $Q_{P_{NS}}$, $H_{P_{NS}}$.
5. Store all values of Q_{P_i}, H_{P_i} in Q_i, H_i, respectively.
6. Transfer back to the print statement (No. 3), or to increment time (No. 4) and check to see if T_{max} has been exceeded. If not, continue with the calculations.

Example 3-1 The valve closure relationship for the pipeline shown in Fig. 3-4 is given by the equation

$$\tau = \left(1 - \frac{t}{t_c}\right)^{E_m}$$

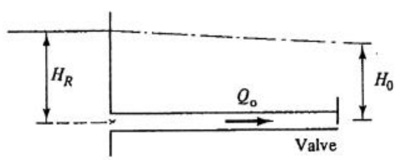


Figure 3-4 Single pipeline.

in which t_c is the time of closure. The initial valve opening ($\tau = 1$) is specified by a value of $(C_dA_G)_0$ in Eq. (3-28). The input data for the problem are: $L = 600$ m, $a = 1{,}200$ m/s, $D = 0.5$ m, $f = 0.018$, $H_R = 150$ m, $t_c = 2.1$ s, $T_{max} = 4.3$ s, $E_m = 1.5$, $(C_dA_G)_0 = 0.009$, $g = 9.806$ m/s², $N = 5$.

Figure 3-5 presents a computer program, written in the FORTRAN compiler language, to solve for the pressure head and flow response as a result of the specified valve closure. The input data are listed at the end of the program. One additional piece of input data is shown as well as all of the data listed above. This is a printout control parameter, IPR, which controls the number of time increments between each printout of calculated results. Steady-state discharge is calculated in the program to balance the friction and valve losses with the energy available in the reservoir.

The computer output is shown in Fig. 3-6, and a graphical display of head at the valve, discharge at the reservoir, and valve position as a function of time are shown in Fig. 3-7.

```
/COMPILE
C BASIC WATERHAMMER PROGRAM. RESERVOIR AT UPSTREAM END OF PIPE AND
C VALVE AT DOWNSTREAM END. HGL DATUM AT VALVE. DARCY WEISBACH FRICTION.
C HR, & INITIAL VALUE OF VALVE CDA=SQRT(QO**2/(2*G*HO)) GIVEN IN DATA.
      DIMENSION HP(11),QP(11),H(11),Q(11)
      NAMELIST/DIN/A,XL,D,F,G,HR,CDA,TC,EM,TMAX,N,IPR
   10 READ (5,DIN,END=99)
      NS=N+1
      R=F*XL/(2.*G*D**5*.7854**2*FLOAT(N))
      B=A/(G*.7854*D*D)
      DT=XL/(A*FLOAT(N))
C FIND STEADY STATE FLOW AND STORE INITIAL VARIABLES
      QO=SQRT(2.*G*CDA*CDA*HR/(R*FLOAT(N)*2.*G*CDA*CDA+1.))
      DO 20 I=1,NS
      H(I)=HR-(I-1)*R*QO*QO
   20 Q(I)=QO
      CVP=.5*QO*QO/H(NS)
      T=0.
      K=0
      TAU=1.
      WRITE (6,25) A,XL,D,F,HR,H(NS),QO,CDA,TC,EM,G,TMAX,DT,B,N,IPR
   25 FORMAT(' A, XL, D, F=',2F8.1,2F8.4/' HR, HO, QO =',2F8.2,F8.3/
     2' CDA, TC, EM=',3F8.3/' G,TMAX,DT,B=',F8.3,F8.1,2F8.3/' N, IPR='
     32I4//'   HEADS AND DISCHARGES ALONG THE PIPE'//'     TIME X/L=
     40.      .2      .4      .6      .8      1.     TAU')
   30 WRITE (6,35)T,(H(I),I=1,NS),TAU,(Q(I),I=1,NS)
   35 FORMAT (1HO,F7.3,5H   H=,6F8.2,F7.3/10X,3H Q=,6F8.3)
   40 T=T+DT
      IF (T.GT.TMAX) GO TO 10
      K=K+1
C COMPUTATION OF INTERIOR POINTS
      DO 50 I=2,N
      CP=H(I-1)+Q(I-1)*(B-R*ABS(Q(I-1)))
      CM=H(I+1)-Q(I+1)*(B-R*ABS(Q(I+1)))
      HP(I)=.5*(CP+CM)
   50 QP(I)=(HP(I)-CM)/B
C BOUNDARY CONDITIONS
      HP(1)=HR
      QP(1)=Q(2)+(HP(1)-H(2)-R*Q(2)*ABS(Q(2)))/B
      IF (T-TC) 55,60,60
   55 TAU=(1.-T/TC)**EM
      CV=TAU*TAU*CVP
      GO TO 70
   60 TAU=0.
      CV=0.
   70 CP=H(N)+Q(N)*(B-R*ABS(Q(N)))
      QP(NS)=-CV*B+SQRT(CV*CV*B*B+CV*CP*2.)
      HP(NS)=CP-B*QP(NS)
      DO 80 I=1,NS
      H(I)=HP(I)
   80 Q(I)=QP(I)
      IF(K/IPR*IPR-K) 40,30,40
   99 STOP
      END
/DATA
 &DIN A=1200.,XL=600.,D=.5,F=.018,G=9.806,HR=150.,CDA=0.009,
 TC=2.1,EM=1.5,TMAX=4.3,N=5,IPR=1, &END
```

Figure 3-5 Basic liquid transient program in FORTRAN.

The authors wish to recognize the contributions of the many fellow researchers in this field and to acknowledge the contributions of the continuous stream of graduate students who have passed through the program in fluid transients at The University of Michigan. Although a few may be singled out as having made particularly outstanding contributions, it is the continuing effort over an extended period of time that has brought about a relevant impact in this field. The work began with Chintu Lai in 1960 under the guidance of Professor Streeter, and has continued with D. N. Contractor, R. A. Baltzer, D. C. Wiggert, W. Zielke, T. P. Propson, M. A. Stoner, J. L. Caves, M. E. Weyler, W. Yow, B. Petry, M. F. El-Erian, C. N. Papadakis, and C. P. Liou. Their contributions are evident throughout this treatment.

Foreword for the Corrected Edition

The opportunity to produce a new printing of this publication was welcomed as it offered a chance to correct errors which had not been removed in earlier printings. Our thanks are extended to course participants and other readers for calling attention to errors in the previous edition. An extensive revision, which might have been appropriate at this time, was not undertaken in the interest of keeping the book in circulation. However, reference material from the period 1977 to 1982 has been appended to the Bibliography in this corrected edition.

We would like to extend our thanks to a number of individuals whose enthusiasm motivated us to proceed with the publication of this edition. Professor Chia-Shun Yih, our colleague at The University of Michigan, and Dr. Michael A. Stoner undoubtedly offered the most influential encouragement.

E. B. W.
V. L. S., 1982

NOMENCLATURE

A	area of pipe
A_e	equivalent area
A_G	area of opening of a valve
A_m	amplitude of the mth harmonic
A_n	nozzle area
A_P	area of piston in dashpot assembly, and in reciprocating pump
A_s	area of surge tank
A_T	penstock area; total area
$\mathrm{A_{TH}}$	minimum surge tank area
a	speed of pressure pulse
a'	wavespeed with free gas
a_n	dimensionless nozzle area
a_1	product of area of orifice and discharge coefficient for flow into surge tank
a_2	product of area of orifice and discharge coefficient for flow out of surge tank
B	width of rectangular section; width of open channel bottom
B	isothermal wave speed
B	Allievi constant or pipeline constant, $aV_0/(gH_0)$, or a/gA
B'	dimensionless Allievi constant
b	aquifer thickness
b_P	permanent speed droop
b_t	temporary speed droop
C	pipeline capacitance $= gA/a^2$
C_D	orifice discharge coefficient
CDG	combined discharge coefficient

CG	characteristics grid
C_g	constant relating flyball speed to turbine speed
C_K	proportionality factor
C_M, C_P	known constants in characteristics equations
C_M	constant in Manning formula
C_v	number of piston displacement volumes; specific heat at constant volume
C^+, C^-	name for characteristic equations
c	wavespeed in open channel; isentropic wavespeed
c	loss coefficient for simple surge tank
c_1	dimensionless parameter that describes the effect of pipe constraint condition on the wavespeed
D	subscript for downstream end of pipe; depth of rectangular section
D	pipe diameter; characteristic dimension of turbomachine
E	energy; modulus of elasticity; collection of variables in Energy equation
E_m	exponent of τ-equation
E_R	modulus of rigidity of rock or concrete
e	pipe wall thickness
$\mathbf{F}$	transfer matrix
F	pressure wave traveling in the $-x$ direction in the pipe; linearized resistance factor
f	pressure wave traveling in the $+x$ direction in the pipe
f	Darcy–Weisbach friction factor; frequency in cycles/sec
G	shear modulus of elasticity
g	gravitational acceleration
gpm	gallons per minute
H	instantaneous piezometric head
H_a	absolute pressure head
$\bar{H}$	average or mean pressure head; barometric head
H^+	oscillatory pressure head wave traveling in the $+x$ direction
H^-	oscillatory pressure head wave traveling in the $-x$ direction
HB	barometric head
H_0	steady-state or mean pressure head; head drop at a valve; standard pressure head absolute
H_P	piezometric head at unknown computational point in xt plane of characteristics grid
H_R	rated pressure head of turbomachine; reservoir head
H_{sub}	complex number notations for instantaneous head at a point. The subscript designates the location
$H(x)$	non-time-varying complex number for pressure head
HP	horsepower
HV	vapor pressure head
h	dimensionless pressure head, H/H_0 or H/H_R; dimensionless distance
h'	instantaneous oscillatory pressure head

h_a	atmospheric pressure in length units of fluid flowing (abs.)
h_m	dimensionless maximum pressure head, H_{max}/H_0
h_v	vapor pressure in length units of fluid flowing (abs.)
I	polar moment of inertia of rotating parts, WR^2/g; index number
IND	name of indexing array
i	denotes section number along a pipe; $\sqrt{-1}$
J	subscript denoting a particular pipe in a complex system
J	subscript in algebraic waterhammer to refer to time, $t = J\Delta t$
K	minor loss coefficient; bulk modulus of elasticity; hydraulic conductivity
K'	combined modulus of elasticity of fluid and container
K_e	equivalent bulk modulus
K_s	spring constant
k	entrance loss coefficient
L	pipe length; pipeline inertance $= 1/gA$
L_1, L_2	identification of continuity and momentum equation in characteristics method
L_{cav}	length of cavity
l	pipe length
M	number of harmonics used in an harmonic analysis to describe a periodic motion; number of pumps in parallel pump system; mass
M	slope of head-discharge curve; mass rate of flow of gas
m	mass; integer identifying a particular harmonic in a Fourier series; thickness of confining soil layer; exponent of diameter in friction term; isothermal Mach number
$\dot{m}$	rate of mass of gas released
N	rotational speed; number of reaches a pipeline is subdivided into for computations
N_R	rated speed of turbomachine
N_w	number of wells
NPSH	net positive suction head
N_S	specific speed
n	number of free vibration mode
n	exponent of the velocity in the turbulent friction term
n	Manning's roughness coefficient; porosity of soil; polytropic exponent
o	as a subscript refers to steady-state condition
P	integer in algebraic waterhammer, $P\Delta t = L/a$
P	solution point in xt plane
P	power produced by a turbine; wetted perimeter
$\mathbf{P}$	point transfer matrix
P_G	power absorbed by the generator
P_R	rated power output of turbine
P_1, P_2	pressures in the cylinder of the dashpot assembly
PR	pressure ratio or amplification factor
p	pressure

p^*	choke pressure
Q	instantaneous discharge at a section
Q_H	heat added to control volume/unit time
$\bar{Q}$	average discharge at a section
Q_R	rated discharge of turbomachine
Q_{sub}	complex number for instantaneous discharge at a point. The subscript designates the location
$Q(x)$	non-time-varying complex number for discharge
Q_1	unit discharge of turbomachine, $Q/(D^2 H^{1/2})$
q	heat added per unit mass/unit time
q	distributed outflow per unit length
q'	instantaneous oscillatory discharge; fluctuations from the average
R	hydraulic radius A/P; parameter in nonlinear soil equation; gas constant
R	resistance coefficient; radius of gyration
R	radial distance in aquifer; crank radius on reciprocating pump
R	nondimensionalizing constant for turbine governor, $C_4 L_2/(L_1 + L_2)$
Re	real part of complex number
R_S	real part of the complex impedance at point S
r	pipe radius; radial distance
r_0	pipe radius corresponding to initial conditions
S	connecting rod length; coefficient of storage; slope of hydraulic grade line
S	solubility coefficient
SI	international metric system
S_K	spring constant
S_s	specific storage $= S/b$
s	complex-valued frequency, $s = \sigma + i\omega$; constant in gas flow for a reach
T	theoretical period of pipe, $4L/a$; tensile force in tube wall; period of surge tank system
T	particular solution (harmonic oscillation) of second-order differential equation
T	instantaneous torque on turbine or pump; absolute temperature
T	top width of prismatic section; soil transmissivity $= K \cdot b$
T_b	beat period
T_D	dashpot time constant in turbine governor
T_F	period corresponding to the forcing frequency
T_g	time of valve closure corresponding to maximum servo velocity
T_m	mechanical starting time of turbine
T_m	period of the mth harmonic, $T_m = 2\pi/m\omega$
T_n	net torque applied to the turbine
T_R	rated torque of turbomachine
T_w	hydraulic inertia time
T_α	promptitude time constant, ratio of change in speed deviation to change in relative servo velocity

t	time; as a subscript denotes partial differentiation
tdh	total dynamic head
t'	dimensionless time, $t/(2L/a)$
t_c	time of closure of a valve
t_1	a specified time
U	subscript for upstream end of pipe; displacement of spool valve from closed position
$\mathbf{U}$	overall transfer matrix
u	dimensionless displacement of spool valve, $U/(R\omega_0)$
u	soil particle displacement in x direction
u	peripheral speed of the centerline of the turbine buckets
V	instantaneous velocity
V_e	velocity at point of lowest pressure in pump suction system
V_0	initial fluid velocity; steady state or mean velocity
V_P	velocity at unknown computational point in xt plane of characteristics solution
V_T	velocity entering the penstock at surge tank
$\forall$	volume
$\forall_g'$	volume of gas reduced to standard conditions
v	dimensionless velocity, V/V_0 or Q/Q_R
v_s	phase velocity in soil dynamics
v_{0_1}	initial steady-state dimensionless velocity
v_{0_2}	final steady-state dimensionless velocity
W	aquifer leakage parameter; width of element in 2-D aquifer
WB, WH	dimensionless turbomachine characteristics
WR^2/g	polar moment of inertia of rotating parts
X	solution of second-order differential equation, function of x only
x	distance along pipe from left end
x	as a subscript denotes partial differentiation
x	displacement from fully extended position of piston
x'	displacement from fully extended position of piston on discharge stroke
x_1	distance along pipe from right end, $x_1 = L - x$
Y	displacement of main servomotor in turbine governor
y	difference in elevation between reservoir and surge tank surface
y, y_1	depth of flow in open channel
Z_C	characteristic impedance
Z_{sub}	hydraulic impedance at a point denoted by the subscript
$Z(x)$	hydraulic impedance, complex ratio of head fluctuation to discharge fluctuation
Z	impedance
z	elevation of pipe above datum; depth of soil in vertical direction
z	compressibility of gas
α	angular position of connecting rod in reciprocating pump
α	pipe or channel slope; parameter in nonlinear soil equation
α	dimensionless speed ratio; ω/ω_0 or N/N_R

α	inertia multiplier in groundwater flow and in natural gas flow
β	change of tube diameter per unit length
β	dimensionless torque ratio, T/T_0 or T/T_R
β	slope of characteristic line on hv plane, $\tan^{-1}(aV_0/gH_0)$
Γ	reflection coefficient, complex ratio of the reflected pressure head to the incident pressure head
γ	complex number called the propagation constant
γ	unit weight of fluid; shearing strain, $\partial u/\partial z$
γ	dimensionless pressure error; weighing factor in implicit method
γ_m	factor indicating portion of x momentum leaving laterally; permissible variation in wave speed
δ	amplitude of oscillatory motion of valve
δ_a	displacement of restoring spring housing
δ_C	displacement of dashpot cylinder
δ_D	nondimensionalizing length for turbine governor, $c_4\omega_0 L_2/L_1$
δ	variation of the manual speed control setting from steady-state position
δ_p	displacement of dashpot piston from steady-state position
ε	complex number representing small change in s; $\Delta\varepsilon = \Delta\sigma + i\Delta\omega$
ε	unit strain in tube wall; fractional value to limit error in aquifer flow
ζ	vane angle of the turbine buckets, interpolation constant; inertial factor
η	turbine or pump efficiency; piston displacement in dashpot cylinder
θ	characteristics grid mesh ratio, $\Delta t/\Delta x$
θ	angular position of crank on suction stroke
θ_m	phase angle of the head fluctuation of the mth harmonic
θ_0	angular displacement when check valve opens in reciprocating pump
λ	multiplier in characteristics method; wave length
μ	Poisson's ratio; absolute or dynamic viscosity; coefficient of viscosity
ν	kinematic viscosity
ξ	displacement of restoring spring in dashpot assembly, $\delta_p - \delta_a$; unit lateral strain
ξ_1	unit longitudinal strain or axial unit strain
ξ_2	unit lateral strain
ξ_T	total lateral or circumferential unit strain
ρ	mass density
σ	real part of complex valued frequency; dimensionless friction term
σ	unit stress in tube wall
σ'	unit stress in tube wall
σ_1	axial unit stress
σ_2	lateral unit stress
τ	shearing stress; τ_m = maximum shearing strength
τ	dimensionless number describing the discharge coefficient times area of opening at a valve, $(C_DA)/(C_DA)_0$
τ_0	dimensionless number describing the valve position corresponding to mean flow conditions
τ_0	wall shear stress

ϕ	angular position of crank on discharge stroke; phase angle
ϕ	speed factor of turbomachine, $DN/(1838H^{1/2})$
ϕ_0	angular position of crank at which pumping starts
ϕ_q	phase angle of the complex impedance
ω	angular frequency; angular velocity
ω'	angular velocity of crank shaft
ω_{cf}	angular frequency
ω_F	angular frequency at which a system is being forced
ω_n	natural frequency of resonator
ω^*	dimensionless frequency