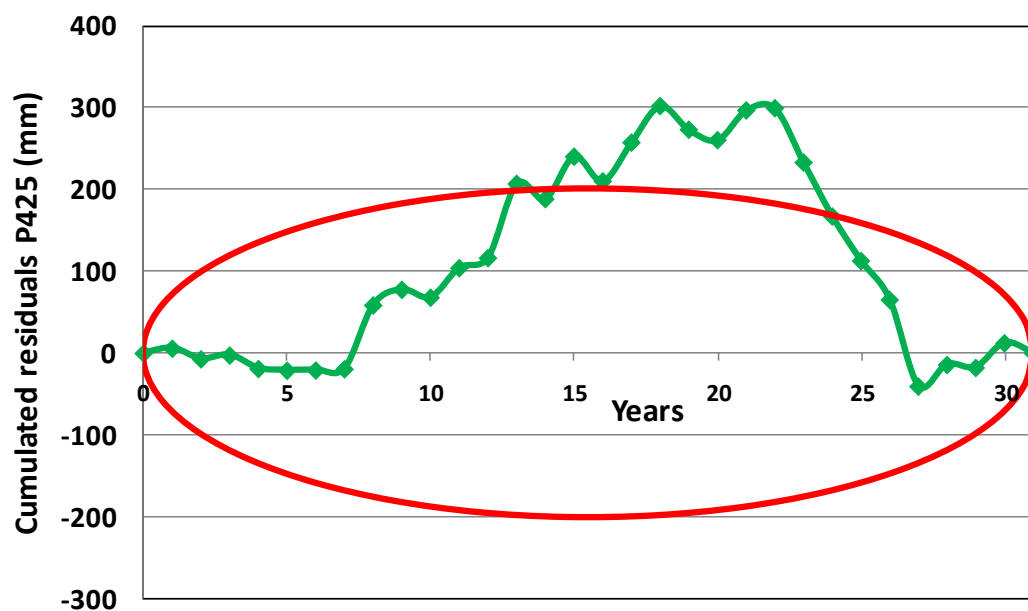


# Workshop on Hydrology

Homogeneity test for P425 at 80 % probability level





# **Workshop on Hydrology**



## **PREFACE**

This workshop includes rainfall data processing exercises, assignments on the computation of evaporation, composition of rating curves, flood routing, aspects of rainfall-runoff modelling, reservoir operation, computation of unsaturated flow and actual evapotranspiration. All applications use Microsoft Excel spreadsheet software.

The use of spreadsheet software for data screening and analyses has an advantage over specialized hydrological software because 1) spreadsheet software is widely available and 2) the participants have to write the algorithms themselves and go through the underlying theories and practises of data processing and analyses while designing their own calculation spreadsheets and graphs.

Data of hydrologic phenomena form the basis of most hydrological work. Through data, theories are verified and phenomena quantified. It should be realized that hydrological and meteorological data, even from renowned institutes, may contain large errors. Therefore, data from observations need cautious treatment. Data screening is a first step in the process of applying data. A simple procedure, such as plotting the time series is a powerful tool to spot suspicious data in a glance. Some well-known screening techniques are exercised in these notes.

Data analysis techniques are often used for data screening, but the purpose of data analysis is much wider: to describe or present a hydrological phenomenon in other terms than the basic observations. Statistical analyses are in this sense important. The analyses described in these notes are spatial homogeneity, K-day rainfall, analysis of extremes, double mass, cumulated residuals, frequency distributions and homogeneity tests. In the application of the techniques a distinction is made between daily, monthly or annual rainfall data. Special attention is given to techniques for data completion.

Participants are supposed to have some background of statistics for successful understanding of the exercises. In data screening and analysis use is made of statistic parameters (means, moments) and theories for fitting distributions (Normal, Lognormal, Gumbel) on observed data. For data completion the principles of regression and correlation are applied. The statistical background for some special topics related to homogeneity tests is dealt with in the appendices. Each assignment is accompanied by an explanatory text on the principles of application of the technique.

In most water balances evaporation plays a dominant role. Calculating reference evaporation from meteorological data using the Equation of Penman-Monteith or the Radiation Method is often the most reliable basis for estimating potential and actual evapotranspiration. These computations are easily carried out with spreadsheets. In the workshop attention is given to a comparison of different methods.

Rating curves are an important link in processing hydrological data. Most discharge data are obtained via the rating curve. The water level-discharge relation is, however, by no means time-invariant. Re-establishment of the rating curve is to be carried out regularly. This activity is easily done with the help of a spreadsheet.

There are several methods for hydrological flood routing in a river of which the Muskingum method is probably the best known. The required parameters can be optimised from a repeated regression analysis.

In the field of rainfall-runoff modelling, concepts such as hydrograph separation, unit hydrograph derivation and convolution are often used. One assignment involves a more complicated analysis of the rainfall-runoff relation of the Umbeluzi River which runs from Swaziland into Mozambique. The mentioned concepts as well as flood routing are applied to compute the effective precipitation and the surface water unit hydrograph.

Data from the Jatiluhur reservoir on Java, Indonesia are used for various assignments on reservoir operation. First the reservoir capacity will be determined using the cumulative mass curve or Rippl diagram and the Sequent Peak Algorithm. The storage in the reservoir will be simulated using monthly data and the effect of rainfall and evaporation on the reservoir is taken into account. The use of rule curves will also be exercised. Finally, the size of the spillway has to be determined by routing a design flood through the reservoir.

Estimation of actual evapotranspiration is important in, for example water balance studies and irrigation water management. A simple approach for estimating actual evapotranspiration from the potential values is based on soil moisture accounting. In these notes use will be made of soil physical data of an international soil series to estimate the soil parameters. The same data are used for an assignment of the derivation of the Van Genuchten model parameters and the derivation of soil pressure and soil moisture profiles using the equation of Darcy.

The theoretical background of the exercises in this workshop is not, or only briefly, discussed in these notes. This workshop is linked to the lecture notes on Hydrology (De Laat & Savenije, 2008) and Soil-Water-Atmosphere (De Laat, 2009). For a more elaborate discussion on the principles of hydrology the reader is referred to text books such as Shaw et al. (2010).

Pieter de Laat

# CONTENTS

|           |                                                 |           |
|-----------|-------------------------------------------------|-----------|
| <b>1</b>  | <b>DAILY RAINFALL DATA .....</b>                | <b>1</b>  |
| 1.1       | TABULAR COMPARISON .....                        | 1         |
| 1.2       | GRAPHICAL COMPARISON .....                      | 3         |
| 1.3       | SPATIAL HOMOGENEITY .....                       | 5         |
| 1.4       | DURATION CURVES FOR K-DAY VALUES .....          | 7         |
| 1.5       | ANALYSES OF EXTREMES AND EXCEEDANCES .....      | 9         |
| <b>2</b>  | <b>MONTHLY RAINFALL DATA .....</b>              | <b>15</b> |
| 2.1       | TABULAR COMPARISON .....                        | 15        |
| 2.2       | DATA COMPLETION THROUGH LINEAR REGRESSION ..... | 16        |
| 2.3       | DOUBLE MASS ANALYSIS .....                      | 18        |
| 2.4       | METHOD OF CUMULATIVE RESIDUALS .....            | 21        |
| 2.5       | FREQUENCY DISTRIBUTION .....                    | 23        |
| <b>3</b>  | <b>YEARLY RAINFALL DATA .....</b>               | <b>27</b> |
| 3.1       | SPEARMAN'S RANK CORRELATION METHOD .....        | 27        |
| 3.2       | F-TEST FOR THE STABILITY OF THE VARIANCE .....  | 29        |
| 3.3       | STUDENTS-T TEST FOR STABILITY OF THE MEAN ..... | 29        |
| <b>4</b>  | <b>EVAPORATION .....</b>                        | <b>31</b> |
| 4.1       | PENMAN-MONTEITH .....                           | 31        |
| 4.2       | RADIATION METHOD .....                          | 35        |
| <b>5</b>  | <b>COMPOSITION OF A RATING CURVE.....</b>       | <b>37</b> |
| <b>6.</b> | <b>FLOOD ROUTING .....</b>                      | <b>41</b> |
| 6.1       | INTRODUCTION .....                              | 41        |
| 6.2       | MUSKINGUM METHOD .....                          | 42        |
| <b>7</b>  | <b>RAINFALL RUNOFF MODELLING.....</b>           | <b>45</b> |
| 7.1       | INTRODUCTION .....                              | 45        |
| 7.2       | FLOOD ROUTING .....                             | 46        |
| 7.3       | BASE FLOW SEPARATION .....                      | 46        |
| 7.4       | ESTIMATING EFFECTIVE PRECIPITATION .....        | 48        |
| 7.5       | DERIVATION OF THE UNIT HYDROGRAPH .....         | 49        |
| <b>8</b>  | <b>RESERVOIR OPERATION .....</b>                | <b>55</b> |
| 8.1       | INTRODUCTION .....                              | 55        |
| 8.2       | DETERMINATION OF THE RESERVOIR CAPACITY .....   | 55        |
| 8.3       | RESERVOIR SIMULATION.....                       | 57        |
| 8.4       | RULE CURVES .....                               | 61        |
| 8.5       | RESERVOIR ROUTING .....                         | 64        |

|                         |                                                                    |           |
|-------------------------|--------------------------------------------------------------------|-----------|
| <b>9</b>                | <b>SOIL MOISTURE .....</b>                                         | <b>67</b> |
| 9.1                     | ACTUAL EVAPOTRANSPIRATION THROUGH SOIL MOISTURE ACCOUNTING .....   | 67        |
| 9.2                     | OPTIMISATION OF THE VAN GENUCHTEN PARAMETERS.....                  | 70        |
| 9.3                     | COMPUTATION OF MOISTURE PROFILES USING THE EQUATION OF DARCY ..... | 72        |
| <b>REFERENCES .....</b> |                                                                    | <b>73</b> |
| <b>APPENDICES .....</b> |                                                                    | <b>75</b> |
| A                       | STATISTICAL BACKGROUNDS.....                                       | 75        |
| B                       | STUDENT T-DISTRIBUTION .....                                       | 89        |
| C                       | PERCENTILE POINTS OF THE FISCHER-F DISTRIBUTION .....              | 91        |
| D                       | TABLE OF STANDARD NORMAL DISTRIBUTION .....                        | 93        |
| E                       | PARAMETERS FOR GUMBEL TYPE I DISTRIBUTION.....                     | 95        |
| F                       | SOIL PHYSICAL DATA INTERNATIONAL SOIL SERIES.....                  | 97        |



# 1 DAILY RAINFALL DATA

This chapter deals with screening of daily data. For this workshop rainfall data are used, but the same procedures are also applicable to other types of daily data. The screening of these data is restricted to identifying suspicious values. No attempt will be made correcting the data, because additional information on how these values were obtained and processed is lacking. The homogeneity of time series will be dealt with in the next chapter. Some analyses do not require daily rainfall data, but k-day values, where k usually varies from 2 to 30. Ranking k-day rainfall data over at least one year may result in Depth-Duration curves, which give the percentage of time a certain rainfall depth is exceeded. If a long time series, say at least 20 years of daily rainfall data are available, an extreme value analysis may be carried out, giving the probability that an extreme daily rainfall event will be exceeded. In these notes the extreme value distribution of Gumbel (1941) will be used.

In this workshop use will be made of daily rainfall data of four stations in the Umbeluzi catchment area in Mozambique. The length of the records varies, but the period 1958/59 - 1981/82 is always included.

| Station code | Period of records | File name                       |
|--------------|-------------------|---------------------------------|
| P5           | 1945/46 - 1981/82 | i:\groupwork\hydata\p5_dr.txt   |
| P6           | 1951/52 - 1988/89 | i:\groupwork\hydata\p6_dr.txt   |
| P119         | 1913/14 - 1986/87 | i:\groupwork\hydata\p119_dr.txt |
| P425         | 1958/59 - 1984/85 | i:\groupwork\hydata\p425_dr.txt |

A map of the catchment and location of the stations is provided in figure 1.1. The computations will be carried out with the "Microsoft Excel Spreadsheet Program".

## 1.1 Tabular comparison

In the first assignment rainfall data of four stations in the Umbeluzi catchment of the same year will be screened through tabular comparison. For this purpose the data have to be entered into an empty worksheet. The original (rough) data are available in text files. The files may be retrieved from the IHE network using the names given above. Missing data are indicated by the value -1.0.

### Assignment

For each of the four rainfall stations the data for the year assigned to you is to be copied to your own spreadsheet file. The procedure is as follows.

- Start the Microsoft Excel software and open the first file *P5\_dr.txt* in the directory *groupwork/hydata* on the *i*-drive. Make sure that you have selected in the menu **Files of type:** *All files (\*.\*)* otherwise the directory looks empty. Select: *next* and *finish*.
- Select the range of cells containing the year assigned to you (including the two lines on top with the station number and year) and copy to clipboard.
- Paste contents clipboard into blank workbook and close file *P5\_dr.txt*
- Open file *P6\_dr.txt*, copy required data and paste in cell Q1
- Open file *P119\_dr.txt*, copy required data and paste in cell AG1
- Open file *P425\_dr.txt*, copy required data and paste in cell AW1
- Save this spreadsheet in your own directory on the H-drive under the name **Anum**, where *num* is your (three digit) locker number (e.g. A063.xlsx if your locker number is 63).

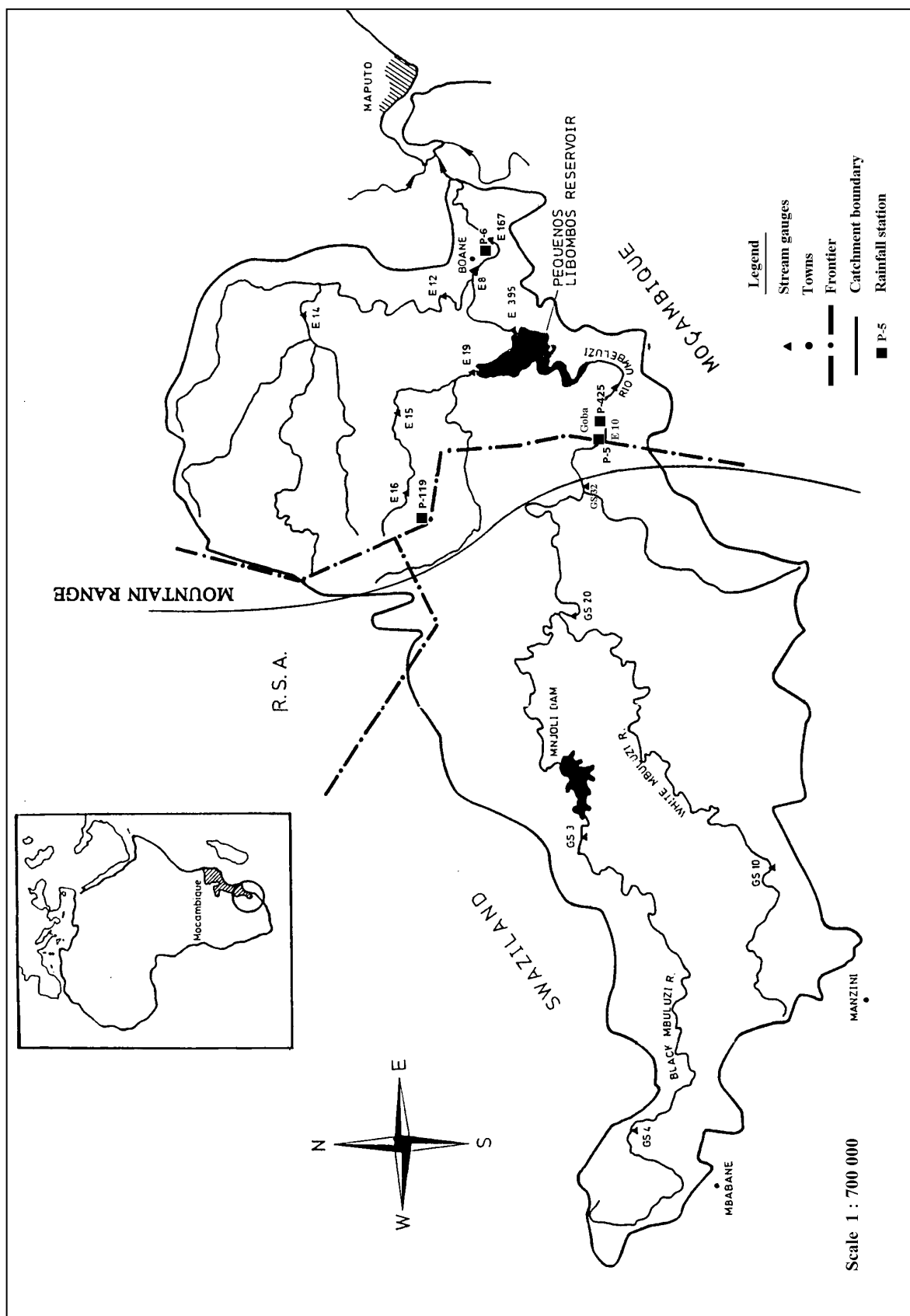


Fig. 1.1 Map of catchment and location of stations

The spreadsheet now contains 4 tables, each of which represents one year of daily rainfall data. The format of each table is as follows:

Rows are day numbers from 1 to 31.

Columns are months.

The first data column is for the month of October (month X), which corresponds to the start of the hydrological year in Mozambique.

- Insert a row (Click with right mouse button in cell A3, and select: *Insert, Entire row*).
- Write in this row above each column the name of the month (starting with October). Right align and change letter type to bold. Copy this heading to the other three tables.
- Freeze titles (Click in cell A4 and select: *View, Freeze panes, Freeze Panes*).
- Delete at the end of the each column data for non-existing days (mind leap years during which 29 February does exist). Repeat this for each of the four stations.
- Save the spreadsheet under the same name. (It is advisable to make a copy on your memory stick).
- Have a close look at the daily values in the table. Check for strange values, long sequences of rainfall, long sequences of 0.0, extreme high values. At the same time compare the data of one station with the other. Do not change any of the values when they are correctly copied from the original files, just be alert for suspicious values.
- Make rows at the bottom of your table for the monthly minimum and maximum rainfall value (Mmin and Mmax). Use the functions **MIN**(range) and **MAX**(range). Also make a row for the monthly total (Mtot), using the function **SUM**(range).
- Finally fill a row by hand (Missing) indicating the number of missing days. An example of the result is given in table 1.1.
- Compute the year total as the sum of all monthly values. Copy all 4 lines at the bottom of the table to each of the other tables. Compute missing values by hand in the other tables. Note: exclude months with missing values in your calculations. A monthly or yearly total in this case is left blank.
- By comparing the monthly totals, minima and maxima of the four stations, suspicious data are easily detected.

## 1.2 Graphical comparison

Comparison of data between the 4 stations is facilitated by the graphical features of Excel. We could create stacked-column graphs for daily rainfall data. It is expected that the daily rainfall data are closely correlated so any suspicious data are easily spotted.

In this exercise we will not compare daily values but monthly totals of the 4 stations using a stacked-column graph. As an example of the graph to create, see figure 1.2.

### Assignment

Make sure you have opened your spreadsheet **Anum** with the daily data of four stations of one and the same year.

- Select: *View, Freeze Panes, Unfreeze Panes* and click on an empty cell (not close to an array of values) where you want the chart to appear.
- Select: *Insert, Column* and choose the middle icon of the first row of 2-D column charts.



- Select: *Select data, Add*, Series Name: type *P5*, Series values: highlight the monthly totals of station *P5*, repeat this for the other three stations. Click *edit* in Horizontal (Category) Axis labels and highlight the names of the months of one of the stations. Click *OK OK*
- Select an appropriate chart layout and fill in the title and legends.
- The properties of the chart are easily changed by double clicking on the appropriate item.
- Save the spreadsheet.

### 1.3 Spatial homogeneity

In spatial homogeneity tests data of a base station are related to data of surrounding stations. In principle good correlations are expected with stations nearby. This can be expressed by a negative exponential function (see lecture notes Hydrology)

$$\rho_r = \rho_0 e^{\left(\frac{-r}{r_0}\right)} \quad (1.1)$$

where

- $\rho_r$  correlation at distance  $r$
- $\rho_0$  correlation at distance  $0$
- $r$  distance between stations
- $r_0$  coefficient

The correlation of monthly data will generally be better than daily data. Apart from the time period and distance, a third parameter affecting the correlation is the type of rainfall that can be expected (convective, orographic, or depression type of rainfall).

For the Umbeluzi catchment with mixed convective, orographic and depression type of rainfall, values for the coefficients are taken as follows:  $\rho_0 = 0.94$  and  $r_0 = 300$  km. A maximum distance  $r_{max}$  between the base station and neighbouring stations is defined as the limit where correlation becomes insignificant ( $\rho < 0.75$ ).

#### Assignment

- Find the distances between P5 and the other stations from figure 1.1 and compute the correlation coefficients for these three distances.
- Compute the maximum correlation distance  $r_{max}$ , assuming a minimum value of the correlation coefficient of  $\rho = 0.75$ .
- Are there distances between stations that fall beyond this limit?

To investigate the reliability of point observations, the measurements,  $P_{meas}(t)$ , of one station are compared with estimated values,  $P_{est}(t)$ , based on a weighted calculation using the rainfall at neighbouring stations. Only stations with a correlation distance smaller than  $r_{max}$  are taken into consideration. The weights are inversely proportional to some power of the distance between the base station and the neighbouring stations.

The estimated daily rainfall is calculated with

$$P_{est}(t) = \frac{\sum \left( \frac{P_i(t)}{r_i^b} \right)}{\sum \left( \frac{1}{r_i^b} \right)} \quad (1.2)$$

where

- $P_{est}(t)$  estimated rainfall of base station at time  $t$
- $P_i(t)$  measured rainfall of station  $i$  at time  $t$
- $r_i$  distance to station  $i$
- $b$  power of distance (usually  $b = 2$ )

The difference between the observed value  $P_{meas}(t)$  and the estimated value  $P_{est}(t)$  is considered to be insignificant if the following conditions are met

- 1 Absolute criterium

$$|P_{meas}(t) - P_{est}(t)| \leq X_{abs} \quad (1.3)$$

- 2 Relative criterium

$$|P_{meas}(t) - P_{est}(t)| \leq X_{rel} \cdot S_{P_{meas}(t)} \quad (1.4)$$

where

- $X_{abs}$  admissible absolute difference
- $X_{rel}$  multiplier of standard deviation
- $S_{P_{meas}(t)}$  standard deviation of values of neighbouring stations at time  $t$  within  $r_{max}$

The calculation of  $S_{P_{meas}(t)}$  does not yield a realistic value if only a few stations are involved.

For this situation an alternative relative criterium can be used

$$F_1 \leq \frac{P_{est}(t)}{P_{meas}(t)} \leq F_2 \quad (1.5)$$

where  $F_1$  is the admissible relative difference coefficient.

### Assignment

Make sure you are in spreadsheet **Anum** with the tables of daily rainfall data for the four stations. Spatial correlation will be performed between the utmost left table (base station P5) and the others as neighbouring stations. Assume that the parameter  $b$  equals 2.

- Make a small table with in the first column the distances  $r_i$  and in the next column the correlation coefficient according to equation 1.1. In the last column appear values for  $1/r_i^2$  and the sum of these values at the bottom.
- Create a new table below the table with daily values for station P5 with values for  $P_{est}$ . Write in the first cell of this table the equation using absolute cell references for  $1/r_i^2$  and the sum, and relative cell references for the values  $P_i$  of the neighbouring stations. Copy this equation to create a table with 12 x 31  $P_{est}(t)$  values.
- Create another table below this one with absolute differences between the measured rainfall of station P5 and the estimated values, thus  $|P_{meas}(t) - P_{est}(t)|$ .
- Finally create a table with the ratio of estimated over observed rainfall values for P5, thus  $P_{est}(t)/P_{meas}(t)$ . You may use an *IF*-function to avoid dividing by zero, e.g. *IF(b4=0,"",b54/b4)*.

From the last two tables it is possible to inspect in a glance the suspected values, i.e. those values that are out of the admissible range, say greater than  $X_{abs} = 50$  mm or values for which the ratio is not between  $F_1 = 0.1$  and  $F_2 = 5$ . (You may use the formatting function of Excel to give these values a different colour. Select: *Home, Conditional formatting*, etc.).

**Question:** Please write in the top left corner of your spreadsheet a brief assessment of the quality of your data set.

Save the sheet **Anum**.

## 1.4 Duration curves for k-day values

*Duration curves*, which give the percentage of time a certain average discharge or rainfall amount is exceeded, are frequently used in hydrology. In this assignment the construction of a rainfall duration curve for different rainfall durations (k-day rainfall, where  $k = 1, 3, 10$  and  $30$  days) will be practised. In this context k-day rainfall values refer to the sum of rainfall over the *previous*  $k$  days, including the day under consideration. If the duration curve is based on one year of data, it gives information on that particular year. It is obvious that if the data include many years, say more than 20 or 30 years, the curve obtains the character of a probability curve that has a more general use. In this assignment we will derive a duration curve based on ten years.

### Assignment

Each participant will be assigned one hydrological year for station P119 to carry out the analysis of k-day rainfall data. Retrieve this specific year from the data file and prepare the tables as follows.

- Open file i:\groupwork\hydata\p119\_dr.txt and select the year assigned to you. Copy the data of this year to the clipboard.
- Open a new workbook, click cell G1 and paste clipboard contents.
- Delete non-existing days.
- Fill A5 to A370 with values from 1 to 365 or 366 (Put in A5 the value 1 and in A6 the value 2. Select both cells. Put the cursor in the small black square and drag downwards with the left or right mouse button pressed until cell A370. (If you drag over the range with the right mouse button pressed a small menu will appear from which you may choose an option). Delete cell A370 if this is not a leap year.
- Move the daily rainfall data into column B. (It is convenient to use the keyboard keys END and the arrow keys  $\uparrow\downarrow$  to move to the beginning and end of a column. The combination Ctr-Home moves the cursor to A1). Put above this column the title: **1k**.
- For safety reasons it is better to already save the worksheet now as **Bnum**.
- Click in P119\_dr.txt in the taskbar at the bottom of your screen and copy from this spreadsheet the data for the remaining nine years into cell G1.
- Move the daily rainfall data into column B as done earlier for the months of the year assigned to you.
- Extend the values in column A to  $3650 + 3$  or  $4$  (depending on the number of leap years).
- Compute in column C, D, and E respectively, the 3, 10 and 30 k-day rainfall data, make use of the function **SUM(range)**. Save again for safety reasons your worksheet.

| Cumulative number of occurrences greater than or equal to the class bottom |      |      |      |      | Percentage of time the k-day rainfall depth is exceeded |       |       |       |       |
|----------------------------------------------------------------------------|------|------|------|------|---------------------------------------------------------|-------|-------|-------|-------|
|                                                                            | 1K   | 3K   | 10K  | 30K  |                                                         | 1K    | 3K    | 10K   | 30K   |
| 0                                                                          | 3653 | 3651 | 3644 | 3624 | 0                                                       | 100.0 | 100.0 | 100.0 | 100.0 |
| 10                                                                         | 285  | 782  | 2007 | 3111 | 10                                                      | 7.8   | 21.4  | 55.1  | 85.8  |
| 20                                                                         | 152  | 480  | 1532 | 2779 | 20                                                      | 4.2   | 13.1  | 42.0  | 76.7  |
| 30                                                                         | 90   | 306  | 1165 | 2560 | 30                                                      | 2.5   | 8.4   | 32.0  | 70.6  |
| 40                                                                         | 48   | 197  | 819  | 2335 | 40                                                      | 1.3   | 5.4   | 22.5  | 64.4  |
| 50                                                                         | 23   | 137  | 628  | 2077 | 50                                                      | 0.6   | 3.8   | 17.2  | 57.3  |
| 60                                                                         | 15   | 102  | 527  | 1922 | 60                                                      | 0.4   | 2.8   | 14.5  | 53.0  |
| 70                                                                         | 11   | 68   | 409  | 1735 | 70                                                      | 0.3   | 1.9   | 11.2  | 47.9  |
| 80                                                                         | 8    | 43   | 314  | 1537 | 80                                                      | 0.2   | 1.2   | 8.6   | 42.4  |
| 90                                                                         | 5    | 34   | 236  | 1390 | 90                                                      | 0.1   | 0.9   | 6.5   | 38.4  |
| 100                                                                        | 3    | 25   | 180  | 1215 | 100                                                     | 0.1   | 0.7   | 4.9   | 33.5  |
| 110                                                                        | 2    | 18   | 148  | 1051 | 110                                                     | 0.1   | 0.5   | 4.1   | 29.0  |
| 120                                                                        | 2    | 14   | 119  | 871  | 120                                                     | 0.1   | 0.4   | 3.3   | 24.0  |
| 130                                                                        | 2    | 12   | 97   | 740  | 130                                                     | 0.1   | 0.3   | 2.7   | 20.4  |
| 140                                                                        | 2    | 9    | 87   | 628  | 140                                                     | 0.1   | 0.2   | 2.4   | 17.3  |
| 150                                                                        | 2    | 8    | 63   | 579  | 150                                                     | 0.1   | 0.2   | 1.7   | 16.0  |
| 160                                                                        | 0    | 6    | 55   | 514  | 160                                                     | 0.0   | 0.2   | 1.5   | 14.2  |
| 170                                                                        | 0    | 2    | 37   | 446  | 170                                                     | 0.0   | 0.1   | 1.0   | 12.3  |
| 180                                                                        | 0    | 1    | 29   | 371  | 180                                                     | 0.0   | 0.0   | 0.8   | 10.2  |
| 190                                                                        | 0    | 0    | 23   | 322  | 190                                                     | 0.0   | 0.0   | 0.6   | 8.9   |
| 200                                                                        | 0    | 0    | 22   | 246  | 200                                                     | 0.0   | 0.0   | 0.6   | 6.8   |
| 210                                                                        | 0    | 0    | 21   | 203  | 210                                                     | 0.0   | 0.0   | 0.6   | 5.6   |
| 220                                                                        | 0    | 0    | 19   | 181  | 220                                                     | 0.0   | 0.0   | 0.5   | 5.0   |
| 230                                                                        | 0    | 0    | 19   | 159  | 230                                                     | 0.0   | 0.0   | 0.5   | 4.4   |
| 240                                                                        | 0    | 0    | 16   | 135  | 240                                                     | 0.0   | 0.0   | 0.4   | 3.7   |
| 250                                                                        | 0    | 0    | 9    | 130  | 250                                                     | 0.0   | 0.0   | 0.2   | 3.6   |
| 260                                                                        | 0    | 0    | 7    | 124  | 260                                                     | 0.0   | 0.0   | 0.2   | 3.4   |
| 270                                                                        | 0    | 0    | 6    | 118  | 270                                                     | 0.0   | 0.0   | 0.2   | 3.3   |
| 280                                                                        | 0    | 0    | 1    | 111  | 280                                                     | 0.0   | 0.0   | 0.0   | 3.1   |
| 290                                                                        | 0    | 0    | 0    | 107  | 290                                                     | 0.0   | 0.0   | 0.0   | 3.0   |
| 300                                                                        | 0    | 0    | 0    | 74   | 300                                                     | 0.0   | 0.0   | 0.0   | 2.0   |
| 310                                                                        | 0    | 0    | 0    | 55   | 310                                                     | 0.0   | 0.0   | 0.0   | 1.5   |
| 320                                                                        | 0    | 0    | 0    | 44   | 320                                                     | 0.0   | 0.0   | 0.0   | 1.2   |
| 330                                                                        | 0    | 0    | 0    | 37   | 330                                                     | 0.0   | 0.0   | 0.0   | 1.0   |
| 340                                                                        | 0    | 0    | 0    | 36   | 340                                                     | 0.0   | 0.0   | 0.0   | 1.0   |
| 350                                                                        | 0    | 0    | 0    | 35   | 350                                                     | 0.0   | 0.0   | 0.0   | 1.0   |
| 360                                                                        | 0    | 0    | 0    | 35   | 360                                                     | 0.0   | 0.0   | 0.0   | 1.0   |
| 370                                                                        | 0    | 0    | 0    | 33   | 370                                                     | 0.0   | 0.0   | 0.0   | 0.9   |
| 380                                                                        | 0    | 0    | 0    | 28   | 380                                                     | 0.0   | 0.0   | 0.0   | 0.8   |
| 390                                                                        | 0    | 0    | 0    | 22   | 390                                                     | 0.0   | 0.0   | 0.0   | 0.6   |
| 400                                                                        | 0    | 0    | 0    | 15   | 400                                                     | 0.0   | 0.0   | 0.0   | 0.4   |

Table 1.2 Example computation k-day duration curves for 1951-1961

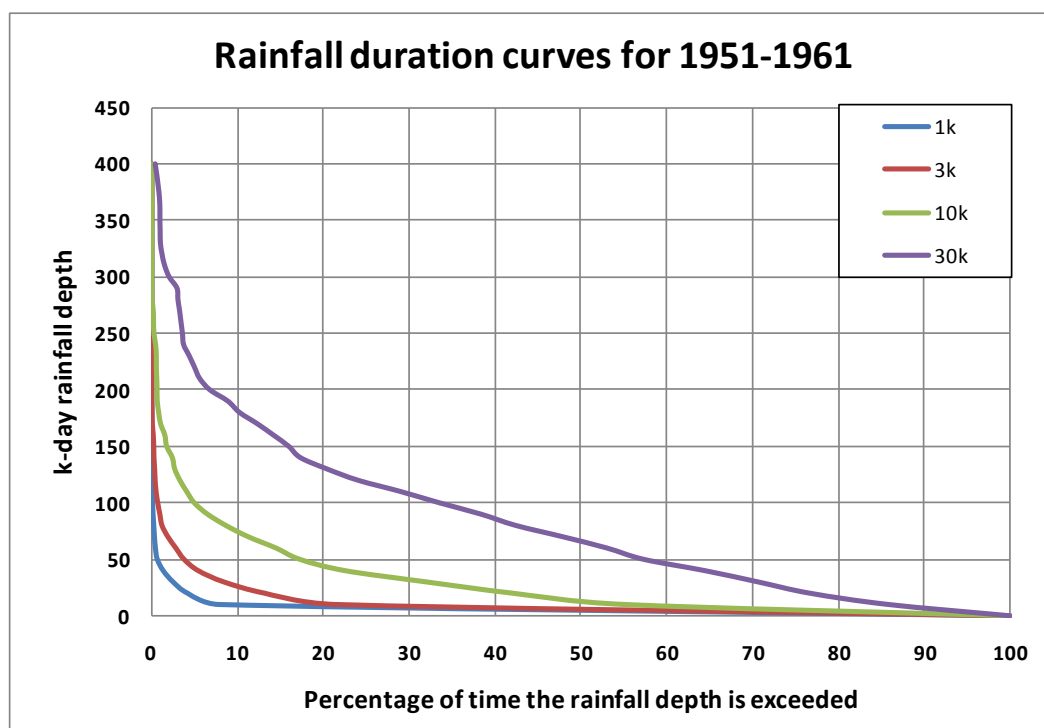


Fig. 1.3 Rainfall duration curves for 1951-1961



- Create a table similar to table 1.2, with in the first column the class bottom (use class intervals of 10 mm, or if your data range is very large use 20 mm) and in the subsequent columns the number of intervals greater or equal to the class bottom. To count the number of intervals greater than or equal to the class bottom us the function **COUNTIF**(range;"condition").
- Compute in the next table (see also table 1.2, right hand side) the percentage of time the rainfall depth is exceeded.
- Make a chart similar to figure 1.3, with on the horizontal axis the percentage of time the rainfall depth is exceeded and on the vertical axis the rainfall depth. (Click in a cell some distance away from the table, Select: *Insert, Scatter, Select data, Add*, etc.).

**Please write in the left top corner of your spreadsheet answers to the following two questions:**

- 1. How many percent of the time is 20 mm or more rainfall in a 30 day period exceeded?**
- 2. Is this chart useful to estimate the rainfall depth that will be exceeded once in 100 years?**

Save the worksheet **Bnum**.

## 1.5 Analyses of extremes and exceedances

The following two methods will be used for analysing extreme daily rainfall events

### 1 Analysis of extremes (annual series)

From each (hydrological) year the maximum value is registered. The extremes for a large number of years are fitted into a distribution, for example Gumbel type I.

### 2 Analysis of exceedances (partial duration series)

All the values exceeding a certain level (threshold) of daily rainfall are registered, regardless the number of times in a year they occur. This method, also known as 'Peaks Over Threshold' (POT), is more correct as all values above a certain threshold are included, but it involves more work. Moreover, there is a greater risk that the extreme values are not independent of each other. The method is most easily compared with the annual series if the threshold is selected in such a way that the number of values above the threshold equals the number of years of data.

Langbein (see Chow, 1964) has shown that the following relation exists between the two methods

$$\frac{1}{T} = 1 - \exp\left(\frac{-1}{T_p}\right) \quad (1.6)$$

where  $T$  is the return period of annual extremes and  $T_p$  the return period for the partial duration series (exceedances). The relation, which is summarized in table 1.3, shows that for rare events, say  $T > 10$  years, the difference between the two methods is very small.

In this workshop both methods will be applied and the results will be plotted in a chart. About 30 years of data from file P119\_dr.txt will be used for this analysis.

| Return period | $X_{exc}/X_{ext}$ |
|---------------|-------------------|
| 2 years       | 1.090             |
| 5 years       | 1.023             |
| 10 years      | 1.009             |

where  
 $X_{ext}$  = rainfall depth referring to annual extremes  
 $X_{exc}$  = rainfall depth referring to exceedances

**Table 1.3** Relation between annual extremes and exceedances

### ***Analysis of extremes (annual series)***

In this workshop the well-known Extreme Value Distribution developed by Gumbel (1941) will be applied. This distribution has been used with success to describe the populations of many hydrological events. When applied to extreme values, the fundamental theorem can be stated as follows.

If  $X_1, X_2, X_3, \dots, X_N$  are independent extreme values observed in  $N$  samples of equal size  $n$  (e.g. years), and if  $X$  is an unlimited exponentially-distributed variable, then as  $n$  and  $N$  approach infinity, the cumulative probability  $q$  that any of the extremes will be less than a given value  $X_i$  is given by

$$q = \exp(-\exp(-y)) \quad (1.7)$$

where  $q$  is the probability of non-exceedance,  $y$  is the reduced variate. If the probability that  $X$  will be exceeded is defined as  $p = 1 - q$ , then (1.7) yields

$$y = -\ln(-\ln(1 - p)) = -\ln(-\ln(1 - 1/T)) \quad (1.8)$$

where  $T$  is the return period measured in sample sizes  $N$  (e.g. years).

According to Gumbel, there is a linear relation between  $X$  and  $y$

$$y = a (X - b) \quad (1.9)$$

where  $a$  is the dispersion factor and  $b$  is the mode.

If the samples are finite, which they always are (already a series of 30 years ( $N = 30$ ) is large), the coefficients  $a$  and  $b$  are computed according to the following equations

$$b = \bar{X}_{ext} - s_{ext} \frac{y_N}{\sigma_N} \quad (1.10)$$

$$a = \frac{\sigma_N}{s_{ext}} \quad (1.11)$$

where  $\bar{X}_{ext}$  is the mean of  $X$  and  $s_{ext}$  the standard deviation of the sample. Values for  $y_N$  (the mean of the reduced variate) and  $\sigma_N$  (the standard deviation of the reduced variate) are tabulated as a function of  $N$  in appendix E. With  $X = X_{Gum}$  equation (1.9) modifies to

$$X_{Gum} = \bar{X}_{ext} + \frac{s_{ext}}{\sigma_N} (y - y_N) \quad (1.12)$$

On probability paper where the horizontal axis is linear in  $y$ , (1.12) plots a straight line. To plot the data points on the horizontal axis a, so-called, plotting position, or estimator of the probability of non-exceedance  $q$  is required. The following plotting position is used

$$q = 1 - p = 1 - \frac{m - \alpha}{N + 1 - 2\alpha} \quad (1.13)$$

where  $m$  is the rank number of the maximum occurrences in decreasing order,  $N$  is the total number of years of observations and  $\alpha = 0.44$  as proposed by Gringerton for Gumbel distributions. For  $\alpha = 0$  equation (1.13) yields the well-known Weibull formula.

### Assignment

- Create a table similar to table 1.4 as follows.
- Open file P119\_dr.txt and copy years assigned to you into a new spreadsheet. Close file P119\_dr.txt.
- Compute for each year the maximum and minimum daily rainfall in columns  $N$  and  $O$ , respectively (use the functions **MAX** and **MIN**).
- Delete years with missing data (delete all rows of years with minimum rainfall equal to -1).
- Save the file in your own directory as **Cnum**.
- Copy the annual extremes (only the *values*) in column  $N$  to column  $S$  (Copy to Clipboard, Paste, Paste Special, Values, OK).
- Sort data in column  $S$  in descending order (Select with mouse data range, Select: Data, Sort, Largest to smallest, OK)
- Rank the values in column  $R$  similar to table 1.4 (Put the values 1 and 2 in the first and second cell, select both cells and drag the square dot to end of the column with the left mouse button pressed). There should be about 30 values.
- In the next column the plotting position  $p$  (the probability of exceedance), is calculated for each value of  $X_{ext}$  using the formula of Gringorten (see Cunnane, 1978)

$$p = \frac{m - 0.44}{N + 0.12} \quad (1.14)$$

- The log of the return period  $T = 1/p$  is computed in the next column. Sometimes a near linear relation may be found between  $\log T$  and the annual extreme rainfall depth.
- Create a graph of the XY(scatter)-type, similar to figure 1.4, showing this relation. (Select: Insert, Scatter, etc.). Add a straight regression line as follows: Click right mouse button on one of the dots in the chart of the series, Select: Add Trendline, Linear, Close.
- Create a column next to the column of logarithmic values for the computation of the reduced variate  $y$  using equation 1.8.

- Calculate the average  $\bar{X}_{ext}$  with function **AVERAGE** and standard deviation  $s_{ext}$  with function **STDEV** of the sample.
- Given the number of years  $N$ , values for  $y_N$  and  $\sigma_N$  may be found in appendix *E* and copied to cells somewhere in the spreadsheet for later use.
- For each value of  $y$  calculate the corresponding rainfall value  $X_{Gum}$  according to Gumbel (equation 1.12).
- Make another chart similar to figure 1.5 with on the horizontal axis the  $y$  values and on the vertical axis the series  $X_{ext}$  and  $X_{Gum}$ .

| Rank<br>(m) | Xext  | p=(m-0.44)<br>/(N+0.12) | LogT=<br>Log (1/p) | Y=-Ln<br>(-Ln(1-p)) | Xgum  | Lower<br>Conf level | Upper<br>Conf level | Xexc  |
|-------------|-------|-------------------------|--------------------|---------------------|-------|---------------------|---------------------|-------|
| 1           | 221.7 | 0.018                   | 1.745              | 4.01                | 229.6 | 165.4               | 293.7               | 221.7 |
| 2           | 183.2 | 0.050                   | 1.300              | 2.97                | 187.6 | 139.1               | 236.2               | 183.2 |
| 3           | 182.9 | 0.082                   | 1.085              | 2.46                | 167.0 | 126.0               | 208.0               | 182.9 |
| 4           | 142.9 | 0.114                   | 0.942              | 2.11                | 153.0 | 117.0               | 189.0               | 142.9 |
| 5           | 138.1 | 0.147                   | 0.834              | 1.84                | 142.3 | 110.1               | 174.6               | 138.1 |
| 6           | 130.3 | 0.179                   | 0.748              | 1.63                | 133.6 | 104.3               | 162.8               | 135.2 |
| 7           | 110.0 | 0.211                   | 0.676              | 1.44                | 126.1 | 99.4                | 152.9               | 130.3 |
| 8           | 107.5 | 0.243                   | 0.615              | 1.28                | 119.6 | 95.0                | 144.3               | 110.0 |
| 9           | 104.0 | 0.275                   | 0.561              | 1.13                | 113.8 | 91.0                | 136.6               | 107.5 |
| 10          | 100.9 | 0.307                   | 0.513              | 1.00                | 108.5 | 87.2                | 129.7               | 104.0 |
| 11          | 100.0 | 0.339                   | 0.469              | 0.88                | 103.6 | 83.7                | 123.5               | 101.5 |
| 12          | 95.0  | 0.371                   | 0.430              | 0.77                | 99.0  | 80.4                | 117.7               | 100.9 |
| 13          | 78.1  | 0.404                   | 0.394              | 0.66                | 94.7  | 77.1                | 112.3               | 100.0 |
| 14          | 76.3  | 0.436                   | 0.361              | 0.56                | 90.6  | 74.0                | 107.2               | 100.0 |
| 15          | 75.0  | 0.468                   | 0.330              | 0.46                | 86.7  | 70.8                | 102.5               | 99.3  |
| 16          | 74.6  | 0.500                   | 0.301              | 0.37                | 82.9  | 67.7                | 98.1                | 95.0  |
| 17          | 73.5  | 0.532                   | 0.274              | 0.27                | 79.2  | 64.5                | 93.9                | 92.4  |
| 18          | 73.4  | 0.564                   | 0.249              | 0.19                | 75.6  | 61.3                | 89.8                | 92.4  |
| 19          | 69.0  | 0.596                   | 0.224              | 0.10                | 72.0  | 58.1                | 86.0                | 88.6  |
| 20          | 65.4  | 0.629                   | 0.202              | 0.01                | 68.5  | 54.7                | 82.4                | 81.6  |
| 21          | 62.5  | 0.661                   | 0.180              | -0.08               | 65.0  | 51.2                | 78.8                | 78.1  |
| 22          | 60.2  | 0.693                   | 0.159              | -0.17               | 61.4  | 47.5                | 75.4                | 76.5  |
| 23          | 58.3  | 0.725                   | 0.140              | -0.26               | 57.8  | 43.6                | 72.1                | 76.3  |
| 24          | 55.9  | 0.757                   | 0.121              | -0.35               | 54.1  | 39.5                | 68.8                | 75.0  |
| 25          | 54.9  | 0.789                   | 0.103              | -0.44               | 50.3  | 35.1                | 65.4                | 74.8  |
| 26          | 54.0  | 0.821                   | 0.085              | -0.54               | 46.2  | 30.4                | 62.1                | 74.6  |
| 27          | 50.5  | 0.853                   | 0.069              | -0.65               | 41.8  | 25.1                | 58.6                | 73.5  |
| 28          | 48.4  | 0.886                   | 0.053              | -0.77               | 37.0  | 19.1                | 54.8                | 73.4  |
| 29          | 47.8  | 0.918                   | 0.037              | -0.92               | 31.3  | 11.9                | 50.6                | 69.0  |
| 30          | 46.5  | 0.950                   | 0.022              | -1.10               | 24.0  | 2.6                 | 45.4                | 67.5  |
| 31          | 41.5  | 0.982                   | 0.008              | -1.39               | 12.1  | -12.9               | 37.2                | 67.5  |
| Xavg=       |       | 89.8                    |                    |                     |       |                     |                     |       |
| Std=        |       | 44.9                    |                    |                     |       |                     |                     |       |

Table 1.4 Example computation of Gumbel distribution

### Adding confidence limits

The longer the time series the more confidence we have in the derived extreme value distribution. In particular for short series it may be wise to indicate the level of confidence for the data we have used. The procedure is the following.

First the standard error of estimate  $SE_X$  is computed in terms of the reduced variate  $y$ . The equation varies with the type of probability distribution used, and may be written for Gumbel as

$$SE_X = \frac{s_{ext}}{\sqrt{N}} \left[ 1 + \frac{1.14}{\sigma_N} (y - y_N) + \frac{1.10}{\sigma_N^2} (y - y_N)^2 \right]^{0.5} \quad (1.15)$$

From the Student's t-distribution the critical values for a 95 % confidence interval may be found given the degree of freedom  $\nu$ , which equals the sample size minus one, or

$$\nu = N - 1 \quad (1.16)$$

The confidence limits  $t_c$  are read from appendix *B*. For the example data set in table 1.4 the degree of freedom is 30 and the critical t-values are  $t_c = \pm 2.04$ .

Assuming that the errors of the estimated extremes  $X_{Gum}$  are normally distributed, the upper and lower limit of the confidence interval  $X_c$  is as follows related to the standardized values  $t_c$

$$\pm t_c = \frac{X_c - X_{Gum}}{SE_X} \quad (1.17)$$

or

$$X_c = X_{Gum} \pm t_c SE_X \quad (1.18)$$

### Assignment

- Find the critical t-values from Appendix B for the degree of freedom  $\nu$ .
- Add two more columns for the lower and upper confidence limits (see table 1.4) and use equation (1.17) for the computation.
- Add the confidence limits to the chart (see figure 1.5).

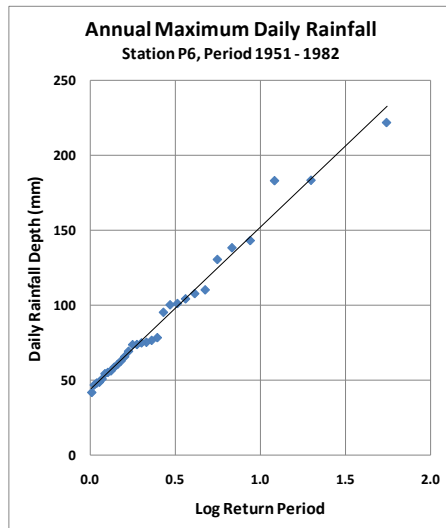


Fig. 1.4 Logarithmic distribution of annual maximum daily rainfall

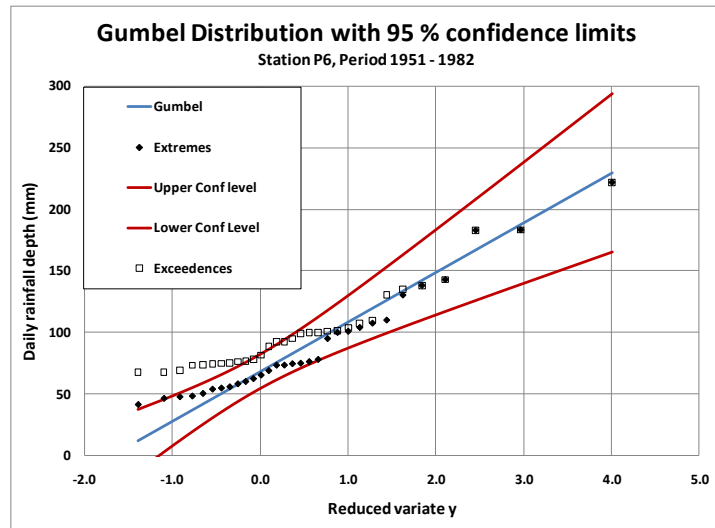


Fig. 1.5 Analyses of extremes and exceedances

### Analysis of exceedances (partial duration series or POT)

#### Assignment

Compute for the same series as used above the  $N$  largest daily rainfall values ( $X_{exc}$ ). This is most easily done by sorting the columns with  $N$  years of original data in descending order. Copy all values larger than a set limit to a column in the same table that you used for analysis of extremes. The lower limit should be such that the number of values larger than this limit is larger than  $N$ . Sort these  $X_{exc}$  values in descending order. See table 1.4.

According to the Langbein theory we only analyze the number of highest values equal to the number of years of the data series. You can erase now all lower values. Indicate  $X_{exc}$  in the Gumbel graph by filling an additional plot range with the column  $X_{exc}$ . See figure 1.5.

Compare the results and notice that in particular for low return periods  $X_{exc}/X_{ext} > 1$ .

**Remark**

It should be noted that sometimes a procedure is used for the computation of the extreme value distribution according to Gumbel that does not take into account the sample size. This applies for example for the Time Series Analysis software (TSA) developed by Guzman and Chu (2003). The difference between the two methods can be significant for small sample sizes.

**Question:** Please write in the top left corner of your spreadsheet a brief assessment of how well your data fit the Gumbel distribution.

**Use both distributions (Gumbel and Logarithmic) to compute the 50 year annual extreme rainfall.**

Save the file as **Cnum**.

## 2 MONTHLY RAINFALL DATA

In a previous exercise monthly data have been generated through aggregation of daily data. Each participant performed this for each of the four stations over a particular year. Through combining all the years, tables can be generated with monthly data over the years of observation per station. As an example see Table 2.1. A worksheet file containing the monthly rainfall values of all four stations is named **i:\groupwork\hydata\month\_31.txt**. The period of observations is 1951/52 until 1981/82.

### 2.1 Tabular comparison

#### Assignment

Retrieve the file with tables of monthly data over the period 1951/52 - 1981/82 of the 4 stations into the spreadsheet program. Each participant will be assigned a particular month for which a statistical analysis is to be carried out.

In order to retrieve the data file and to establish the table with monthly data of the 4 stations the following procedure could be applied.

- Open the file with monthly data of the 4 stations: **i:\groupwork\hydata\month\_31.txt**
- Prepare a table (starting in cell A45) with columns for the year numbers and monthly values of the stations P425, P119, P5 and P6 (in this order). Fill the columns with values over the years 51/52 - 81/82 for the month assigned to you.

| Station_code: P119 |       |       |       |       |       |       |       |       |      |       |      |       | Yearly  |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|------|-------|---------|
| Month<br>Year      | X     | XI    | XII   | I     | II    | III   | IV    | V     | VI   | VII   | VIII | IX    | Total   |
| 51/52              | 127.6 | 8.8   | 91.4  | 108.8 | 109.6 | 50.9  | 22.2  | 19.1  | 40.9 | 82.9  | 9.5  | 1.6   | 673.3   |
| 52/53              | 43.2  | 194.2 | 119.6 | 305.4 | 188.1 | 78.8  | 23.2  | 11.7  | 0.0  | 14.1  | 2.4  | 50.4  | 1,031.1 |
| 53/54              | 76.3  | 144.5 | 28.1  | 84.2  | 95.4  | 115.1 | 85.7  | 30.5  | 12.3 | 1.5   | 30.1 | 198.5 | 902.2   |
| 54/55              | 155.9 | 187.4 | 94.1  | 293.8 | 292.6 | 183.1 | 76.1  | 73.5  | 24.7 | 0.0   | 0.0  | 2.0   | 1,383.2 |
| 55/56              | 193.9 | 297.5 | 114.3 | 123.0 | 375.7 | 172.1 | 6.6   | 56.4  | 19.0 | 7.6   | 2.0  | 125.5 | 1,493.6 |
| 56/57              | 18.0  | 78.5  | 160.7 | 87.4  | 95.1  | 132.0 | 151.5 | 15.7  | 33.5 | 99.3  | 68.3 | 104.9 | 1,044.9 |
| 57/58              | 132.4 | 30.9  | 152.7 | 253.1 | 133.9 | 82.7  | 60.0  | 0.0   | 25.1 | 4.0   | 5.3  | 45.3  | 925.4   |
| 58/59              | 34.8  | 68.9  | 136.8 | 123.7 | 92.6  | 60.9  | 8.5   | 133.3 | 14.0 | 10.4  | 5.5  | 70.3  | 759.7   |
| 59/60              | 85.2  | 41.5  | 102.0 | 63.5  | 85.8  | 70.8  | 179.8 | 7.2   | 21.0 | 10.0  | 23.3 | 63.0  | 753.1   |
| 60/61              | 49.4  | 144.3 | 296.1 | 49.1  | 108.0 | 155.5 | 47.4  | 5.9   | 48.5 | 10.6  | 35.5 | 55.3  | 1,005.6 |
| 61/62              | 140.3 | 62.2  | 49.2  | 118.1 | 26.3  | 57.8  | 81.6  | 1.6   | 22.7 | 3.0   | 54.7 | 7.5   | 625.0   |
| 62/63              | 67.4  | 127.8 | 97.8  | 115.8 | 204.6 | 95.6  | 31.3  | 32.1  | 69.0 | 152.4 | 15.0 | 6.2   | 1,015.0 |
| 63/64              | 51.6  | 72.4  | 52.0  | 173.2 | 42.2  | 7.5   | 96.9  | 5.0   | 2.9  | 3.1   | 1.8  | 3.4   | 512.0   |
| 64/65              | 107.4 | 63.0  | 163.8 | 56.2  | 69.0  | 139.1 | 33.6  | 4.1   | 13.5 | 4.7   | 31.5 | 59.7  | 745.6   |
| 65/66              | 48.7  | 131.5 | 62.7  | 672.2 | 124.9 | 9.0   | 15.3  | 51.1  | 41.4 | 9.9   | 31.3 | 22.2  | 1,220.2 |
| 66/67              | 75.8  | 36.3  | 99.2  | 190.1 | 353.6 | 174.0 | 114.4 | 2.1   | 12.3 | 8.3   | 2.5  | 2.0   | 1,070.6 |
| 67/68              | 89.3  | 113.4 | 52.0  | 113.7 | 227.9 | 112.7 | 32.6  | 14.1  | 25.5 | 58.4  | 37.1 | 19.6  | 896.3   |
| 68/69              | 33.1  | 114.6 | 57.7  | 188.7 | 58.2  | 154.3 | 110.0 | 56.2  | 0.0  | 22.0  | -1   | 31.1  |         |
| 69/70              | 236.4 | 43.1  | 117.5 | 13.0  | 47.2  | 52.9  | 9.5   | 37.1  | 9.5  | 4.0   | 7.5  | 17.7  | 595.4   |
| 70/71              | 89.3  | 96.0  | 84.7  | 98.6  | 63.4  | 147.4 | 95.9  | 53.7  | 13.8 | 0.6   | 4.6  | 20.5  | 768.5   |
| 71/72              | 96.3  | 104.3 | 236.5 | 320.6 | 327.1 | 176.6 | 74.5  | 171.7 | 5.0  | 14.0  | 1.0  | 5.0   | 1,532.6 |
| 72/73              | 86.6  | 122.7 | 96.8  | 57.2  | 141.8 | 52.2  | 98.0  | 11.7  | 12.1 | 0.3   | 22.0 | 217.5 | 918.9   |
| 73/74              | 85.5  | 122.9 | 379.2 | 209.6 | 107.7 | 70.6  | 115.3 | 10.8  | 0.0  | 54.8  | 3.5  | 11.3  | 1,171.2 |
| 74/75              | 44.6  | 118.0 | 70.3  | 183.8 | 356.5 | 114.8 | 107.4 | 12.9  | 36.1 | 1.6   | 5.6  | 37.4  | 1,089.0 |
| 75/76              | 35.8  | 71.4  | 200.3 | 416.7 | 255.4 | 141.0 | 76.1  | 21.4  | 8.5  | 0.6   | 0.4  | 3.5   | 1,231.1 |
| 76/77              | 42.3  | 157.3 | 104.9 | 150.9 | 580.5 | 228.6 | 22.6  | 21.2  | 3.3  | 0.0   | 43.5 | 89.4  | 1,444.5 |
| 77/78              | 34.8  | 27.4  | 114.8 | 354.0 | 86.9  | 205.0 | 76.5  | 19.7  | 24.8 | 61.7  | 3.5  | 151.6 | 1,160.7 |
| 78/79              | 85.1  | 168.5 | 153.4 | 171.2 | 25.1  | 122.1 | 125.4 | 15.7  | 15.3 | 2.6   | 16.1 | 21.4  | 921.9   |
| 79/80              | 51.6  | 56.6  | 75.7  | 157.7 | 237.4 | 54.2  | 104.1 | 17.7  | 0.0  | 6.2   | 30.8 | 113.1 | 905.1   |
| 80/81              | 10.3  | 142.2 | 71.9  | 112.5 | 229.3 | 141.9 | 26.6  | 48.1  | 7.5  | 18.6  | 32.5 | 120.7 | 962.1   |
| 81/82              | 115.0 | 175.3 | 156.1 | 96.1  | 36.2  | 57.1  | 139.6 | 4.6   | 5.0  | 0.0   | 5.4  | 23.8  | 814.2   |
| MIS                | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0    | 0     | 1    | 0     | 1       |
| AVG                | 82.1  | 107.2 | 122.3 | 176.2 | 167.0 | 110.2 | 72.5  | 31.2  | 18.3 | 21.5  |      | 54.9  |         |
| STD                | 51.3  | 61.5  | 74.6  | 133.3 | 130.5 | 56.8  | 46.6  | 38.0  | 16.2 | 35.5  |      | 58.8  |         |
| MIN                | 10.3  | 8.8   | 28.1  | 13.0  | 25.1  | 7.5   | 6.6   | 0.0   | 0.0  | 0.0   |      | 1.6   |         |
| MAX                | 236.4 | 297.5 | 379.2 | 672.2 | 580.5 | 228.6 | 179.8 | 171.7 | 69.0 | 152.4 |      | 217.5 |         |
| P20                | 39.0  | 55.5  | 59.7  | 64.2  | 57.4  | 62.5  | 33.4  | -0.8  | 4.7  | -8.3  |      | 5.5   |         |
| P80                | 125.2 | 158.9 | 185.0 | 288.2 | 276.6 | 157.9 | 111.7 | 63.1  | 31.9 | 51.3  |      | 104.3 |         |

Table 2.1 Monthly rainfall data sheet for station P119

- Erase the original data of the four stations (erase the rows A1..A43). Your table is now at the left top of the spreadsheet.
- It is better to save the file now as **Dnum**.
- Copy the table to a place right of the original table.
- Delete in the original table all missing data (-1) in order to carry out the following calculations at the bottom of each column (see also table 2.1):
  - 1) Number of missing values
  - 2) Monthly average (use function **AVERAGE**)
  - 3) Standard deviation (use **STDEV**)
  - 4) Calculate the minimum value (**MIN**)
  - 5) Calculate the maximum value (**MAX**)
  - 6) Probability of non-exceedance with 20% by  
 $P_{20} = \text{AVERAGE} - 0.84 * \text{STDEV}$  (assuming a normal distribution)<sup>1</sup>
  - 7) Probability of non-exceedance with 80% by  
 $P_{80} = \text{AVERAGE} + 0.84 * \text{STDEV}$  (assuming a normal distribution)

If the data are normally distributed 60 % of the values are between  $P_{20}$  and  $P_{80}$ .

## 2.2 Data completion through linear regression

One way of data completion is through linear regression.

With linear regressions a mathematical relation is defined between data of a base station and other stations of the form:

$$Y = C + C_1X_1 + C_2X_2 + C_3X_3 + \text{etc.} \quad (2.1)$$

where

- $Y$  a series of values of the base station (dependent variable)
- $X_i$  a series of values of neighbouring station  $i$  (independent variable)
- $C$  the equation's constant
- $C_i$  the equation's coefficients

Multiple regression means that more than one neighbouring station (independent variable) is regarded. In case of a base station and one neighbouring station, the equation reduces to

$$Y = C + C_1X_1 \quad (2.2)$$

The method is based on fitting the 'best' straight line through observations. In general there always will be a difference between a calculated value,  $Y'$  according to this line and a measured value. In linear regression this difference ( $Y - Y'$ ) is minimized using the method of 'least squares'. The quality of the fit obtained can be investigated by calculating the coefficient of determination  $\rho^2$ . The coefficient of determination is a number between 0 and 1, where 1 represents perfect fit. A minimum value of  $\rho^2$  should be applied giving the limit of a realistic regression (minimum of  $\rho^2$  should be around 0.5). This is also a criterion for the selection of stations to be included in the regression analysis.

---

<sup>1</sup>If you were assigned data from the dry period, the mean value is very low and the standard deviation high. Consequently  $P_{20}$  may be calculated as a negative value, indicating that the normal distribution is not very suitable for this set of data.



The square root of the coefficient of determination is the correlation coefficient  $\rho$  which value varies between  $-1$  and  $+1$ . The correlation coefficient  $\rho$  is related to the differences between measured and calculated values of the dependent variable ( $Y - Y'$ ) and defined as (see also appendix A5)

$$\rho = \left( \frac{\sum (Y' - \bar{Y})^2}{\sum (Y - \bar{Y})^2} \right)^{0.5} \quad (2.3)$$

Among the statistical applications of the Excel spreadsheet program is the option of multiple linear regression. This provides directly the coefficient of determination  $\rho^2$  (in the spreadsheet known as *R Square*), as well as the coefficients and the *Intercept* of the regression equation.

### Assignment

Open the file **Dnum** with the monthly data of the four stations. The (multiple) regression analysis will be performed between P425 (dependent variable) and P119, P5 and P6 as neighbouring stations (independent variables). A worked out example as a reference for the exercise is provided in table 2.2. The correlation analysis will be carried out for only those years in which all 4 stations have data.

- Copy the table in the sheet to an area below the original table and delete all rows in which not all stations have data (see e.g. table 2.2, notice that some years are missing).

This table will be used to calculate the constant (intercept) and coefficients of the regression.

First the individual correlation coefficients between P6 (independent  $X$ ) station and each of the other (dependent  $Y$ ) stations will be investigated.

| January | P425  | P119  | P5    | P6    | <b>SUMMARY OUTPUT</b>        |           |
|---------|-------|-------|-------|-------|------------------------------|-----------|
| 58/59   | 175.3 | 123.7 | 162.9 | 79.5  | <b>P425 - P6</b>             |           |
| 59/60   | 79.5  | 63.5  | 76.4  | 84.3  | <i>Regression Statistics</i> |           |
| 60/61   | 56.0  | 49.1  | 110.0 | 84.5  | Multiple R                   | 0.8564737 |
| 61/62   | 142.4 | 118.1 | 93.1  | 188.6 | R Square                     | 0.7335473 |
| 62/63   | 95.7  | 115.8 | 111.8 | 84.7  | Adjusted R Square            | 0.7202246 |
| 63/64   | 249.0 | 173.2 | 210.3 | 215.5 | Standard Error               | 60.297187 |
| 64/65   | 12.3  | 56.2  | 14.3  | 40.4  | Observations                 | 22        |
| 65/66   | 546.8 | 672.2 | 625.1 | 587.6 | <i>Coefficients</i>          |           |
| 66/67   | 76.6  | 190.1 | 48.5  | 162.1 | Intercept                    | 10.380164 |
| 67/68   | 121.5 | 113.7 | 92.0  | 71.4  | X Variable 1                 | 0.7697746 |
| 68/69   | 157.7 | 188.7 | 125.5 | 111.4 | <b>SUMMARY OUTPUT</b>        |           |
| 69/70   | 4.7   | 13.0  | 98.4  | 9.6   | <b>P425 and P119, P5, P6</b> |           |
| 70/71   | 51.8  | 98.6  | 87.9  | 53.1  | <i>Regression Statistics</i> |           |
| 71/72   | 218.0 | 320.6 | 156.8 | 210.4 | Multiple R                   | 0.9044524 |
| 72/73   | 56.7  | 57.2  | 79.1  | 63.3  | R Square                     | 0.8180341 |
| 73/74   | 108.6 | 209.6 | 151.7 | 299.5 | Adjusted R Square            | 0.7877065 |
| 74/75   | 81.3  | 183.8 | 138.7 | 232.3 | Standard Error               | 52.524354 |
| 75/76   | 210.2 | 416.7 | 311.4 | 275.0 | Observations                 | 22        |
| 76/77   | 44.3  | 150.9 | 77.0  | 115.5 | <i>Coefficients</i>          |           |
| 77/78   | 122.0 | 354.0 | 305.6 | 202.9 | Intercept                    | 3.821143  |
| 78/79   | 122.5 | 171.2 | 60.5  | 129.0 | X Variable 1                 | 0.0918169 |
| 79/80   | 62.2  | 157.7 | 52.9  | 33.8  | X Variable 2                 | 0.4495039 |
|         |       |       |       |       | X Variable 3                 | 0.2729471 |

**Table 2.2 Example of (multiple) linear regression between 2 and 4 stations**

Perform linear regression between P6 and P425 as follows.

- Select: *Data, Data Analysis*<sup>2</sup>, *Regression*, *OK*, Input Y-Range, (use mouse to select P425 data), X-Range (use mouse to select P6 data), Output Range (click cell for output), *OK*.
- Repeat the analysis for P6 and P119 and also for P6 and P5
- The multiple regression is carried out in the same way except that a range covering three columns has to be specified for the independent (X) data. If the intercept appears to be negative, values estimated with this equation may also be negative. In that case the regression is to be repeated with the intercept forced to zero (Click: Constant is zero).

The results of the correlation analysis may now be used to complete missing data, provided the coefficient of determination (*R Square*) is larger than 0.5.

- Use the result of the individual regressions with station P6 to complete missing data (cells with the value -1) in the right hand side table at the top, of the spreadsheet.
- Use the results of the multiple regression analysis to complete data for station P425 in the years 1951/52 - 1957/58.
- Calculate from the completed time series the mean, standard deviation,  $P_{20}$  and  $P_{80}$  (assuming a normal distribution) and compare the result with the values you have been calculating for the uncompleted series. Look for suspicious values.
- Save sheet **Dnum**.

## 2.3 Double mass analysis

The principle of double mass analysis is to plot accumulated values of the station under investigation against accumulated values of another station, or accumulated values of the average of other stations, over the same period of time.

Through a double mass curve inhomogeneities in the time series (gradual changes and jumps) can be investigated. The series may be not homogeneous if, for example, there was a change in the type of instrument, the position of the instrument, the observer, or due to the growth of trees, etc. This is indicated in the curve of a double mass plot, showing an inflection point in the straight line.

The principle of double mass curve analysis will be exercised through plotting accumulated monthly rainfall of station P425 against the mean of the other three stations. A worked out example is provided in table 2.3 and figure 2.1.

### Assignment

- The completed data for the years 1951/52 - 1981/82, as obtained in the previous exercise, will be used for the double mass analysis. Make sure you are in spreadsheet **Dnum**.
- Create a table in the area to the right of the table with completed data (see e.g. table 2.3). The new table will contain 4 columns with accumulated values of the 4 stations.
- Compute in the next column the average of the stations P119, P5 and P6.
- Insert an XY-chart with the average values of the stations P119, P5 and P6 as X, against the accumulated values of P425 as Y (see e.g. figure 2.1). A straight line indicates that the data of station P425 may be considered homogeneous.

---

<sup>2</sup>If *Data Analysis* does not appear in the 'Data' task bar, this Add-In has to be installed. Click: *Office button, Excel Options, Add-Ins, Go, Select: Analysis toolpak, OK*. Installation may take some time.

| January | P425  | P119  | P5    | P6    | P425   | P119   | P5     | P6     | Average    | Average | Residual |
|---------|-------|-------|-------|-------|--------|--------|--------|--------|------------|---------|----------|
|         |       |       |       |       | sum    | sum    | sum    | sum    | P119,P5,P6 | Linear  | mass     |
|         |       |       |       |       | 0      | 0      | 0      | 0      | 0          | 0       | 0        |
| 51/52   | 84.5  | 108.8 | 114.2 | 70.8  | 84.5   | 108.8  | 114.2  | 70.8   | 97.9       | 78.1    | 6.4      |
| 52/53   | 162.6 | 305.4 | 186.2 | 172.3 | 247.1  | 414.2  | 300.4  | 243.1  | 319.2      | 254.4   | -7.4     |
| 53/54   | 62.9  | 84.2  | 87.4  | 44.3  | 310.0  | 498.4  | 387.8  | 287.4  | 391.2      | 311.8   | -1.8     |
| 54/55   | 164.2 | 293.8 | 154.1 | 235.0 | 474.2  | 792.2  | 541.9  | 522.4  | 618.8      | 493.2   | -19.0    |
| 55/56   | 68.6  | 123.0 | 85.6  | 54.9  | 542.8  | 915.2  | 627.5  | 577.3  | 706.7      | 563.2   | -20.4    |
| 56/57   | 57.9  | 87.4  | 66.2  | 59.8  | 600.7  | 1002.6 | 693.7  | 637.1  | 777.8      | 619.9   | -19.2    |
| 57/58   | 171.1 | 253.1 | 216.2 | 171.7 | 771.8  | 1255.7 | 909.9  | 808.8  | 991.5      | 790.2   | -18.4    |
| 58/59   | 175.3 | 123.7 | 162.9 | 79.5  | 947.1  | 1379.4 | 1072.8 | 888.3  | 1113.5     | 887.5   | 59.6     |
| 59/60   | 79.5  | 63.5  | 76.4  | 84.3  | 1026.6 | 1442.9 | 1149.2 | 972.6  | 1188.2     | 947.0   | 79.6     |
| 60/61   | 56.0  | 49.1  | 110.0 | 84.5  | 1082.6 | 1492.0 | 1259.2 | 1057.1 | 1269.4     | 1011.7  | 70.9     |
| 61/62   | 142.4 | 118.1 | 93.1  | 188.6 | 1225.0 | 1610.1 | 1352.3 | 1245.7 | 1402.7     | 1117.9  | 107.1    |
| 62/63   | 95.7  | 115.8 | 111.8 | 84.7  | 1320.7 | 1725.9 | 1464.1 | 1330.4 | 1506.8     | 1200.9  | 119.8    |
| 63/64   | 249.0 | 173.2 | 210.3 | 215.5 | 1569.7 | 1899.1 | 1674.4 | 1545.9 | 1706.5     | 1360.1  | 209.7    |
| 64/65   | 12.3  | 56.2  | 14.3  | 40.4  | 1582.0 | 1955.3 | 1688.7 | 1586.3 | 1743.4     | 1389.5  | 192.5    |
| 65/66   | 546.8 | 672.2 | 625.1 | 587.6 | 2128.8 | 2627.5 | 2313.8 | 2173.9 | 2371.7     | 1890.3  | 238.5    |
| 66/67   | 76.6  | 190.1 | 48.5  | 162.1 | 2205.4 | 2817.6 | 2362.3 | 2336.0 | 2505.3     | 1996.7  | 208.7    |
| 67/68   | 121.5 | 113.7 | 92.0  | 71.4  | 2326.9 | 2931.3 | 2454.3 | 2407.4 | 2597.7     | 2070.3  | 256.6    |
| 68/69   | 157.7 | 188.7 | 125.5 | 111.4 | 2484.6 | 3120.0 | 2579.8 | 2518.8 | 2739.5     | 2183.4  | 301.2    |
| 69/70   | 4.7   | 13.0  | 98.4  | 9.6   | 2489.3 | 3133.0 | 2678.2 | 2528.4 | 2779.9     | 2215.5  | 273.8    |
| 70/71   | 51.8  | 98.6  | 87.9  | 53.1  | 2541.1 | 3231.6 | 2766.1 | 2581.5 | 2859.7     | 2279.2  | 261.9    |
| 71/72   | 218.0 | 320.6 | 156.8 | 210.4 | 2759.1 | 3552.2 | 2922.9 | 2791.9 | 3089.0     | 2461.9  | 297.2    |
| 72/73   | 56.7  | 57.2  | 79.1  | 63.3  | 2815.8 | 3609.4 | 3002.0 | 2855.2 | 3155.5     | 2515.0  | 300.9    |
| 73/74   | 108.6 | 209.6 | 151.7 | 299.5 | 2924.4 | 3819.0 | 3153.7 | 3154.7 | 3375.8     | 2690.5  | 233.9    |
| 74/75   | 81.3  | 183.8 | 138.7 | 232.3 | 3005.7 | 4002.8 | 3292.4 | 3387.0 | 3560.7     | 2837.9  | 167.8    |
| 75/76   | 210.2 | 416.7 | 311.4 | 275.0 | 3215.9 | 4419.5 | 3603.8 | 3662.0 | 3895.1     | 3104.4  | 111.5    |
| 76/77   | 44.3  | 150.9 | 77.0  | 115.5 | 3260.2 | 4570.4 | 3680.8 | 3777.5 | 4009.6     | 3195.6  | 64.6     |
| 77/78   | 122.0 | 354.0 | 305.6 | 202.9 | 3382.2 | 4924.4 | 3986.4 | 3980.4 | 4297.1     | 3424.8  | -42.5    |
| 78/79   | 122.5 | 171.2 | 60.5  | 129.0 | 3504.7 | 5095.6 | 4046.9 | 4109.4 | 4417.3     | 3520.6  | -15.9    |
| 79/80   | 62.2  | 157.7 | 52.9  | 33.8  | 3566.9 | 5253.3 | 4099.8 | 4143.2 | 4498.8     | 3585.5  | -18.6    |
| 80/81   | 160.0 | 112.5 | 183.2 | 194.4 | 3726.9 | 5365.8 | 4283.0 | 4337.6 | 4662.1     | 3715.7  | 11.2     |
| 81/82   | 29.2  | 96.1  | 31.7  | 24.5  | 3756.2 | 5461.9 | 4314.7 | 4362.1 | 4712.9     | 3756.2  | 0.0      |

Table 2.3 Double mass analysis

- In order to assess the homogeneity, the relation is compared with a straight line, the average linear increase  $AL$ , which is computed in the next column as follows

$$AL_i = \frac{\sum_{j=1}^n Y_j}{\sum_{j=1}^n X_j} \sum_{j=1}^i X_j \quad (2.4)$$

where

$AL_i$  Average Linear increase at year  $i$

$X_j$  Monthly rainfall in year  $j$  of station  $X$

$Y_j$  Monthly rainfall in year  $j$  of station  $Y$

$i, j$  1,...,n where  $n$  is the total number of years considered

- Add the average linear increase to the XY-chart. Click in the chart, *Chart tools, Select data, Add*, select for the X-values the average of the 3 stations and for the Y-values the average linear increase  $AL$ . The line you have now plotted connects the total accumulated P425 values with the origin.
- Add one more column with the difference between the accumulated P425 values and the average linear increase  $AL$ . These differences are known as the *residuals*.
- Insert another XY-chart with on the horizontal axis the accumulated P425 values and the residuals on the Y-axis. As an example see figure 2.2. This chart is known as the *residual mass curve*.

The residual mass curve shows more clearly the deviations of station  $Y$  from the mean (indicated as inflection points in the double mass analysis). The curve can be interpreted as follows:

- an upward deviation from the linear relation indicates relative high values of station Y
- a parallel line indicates a constant relation between station X and Y
- a downward deviation from the linear relation indicates relative low values of station Y

**Question:** Please write in the top left corner of your spreadsheet the largest accumulated difference as a percentage and in which year it occurs.

Save **Dnum**.

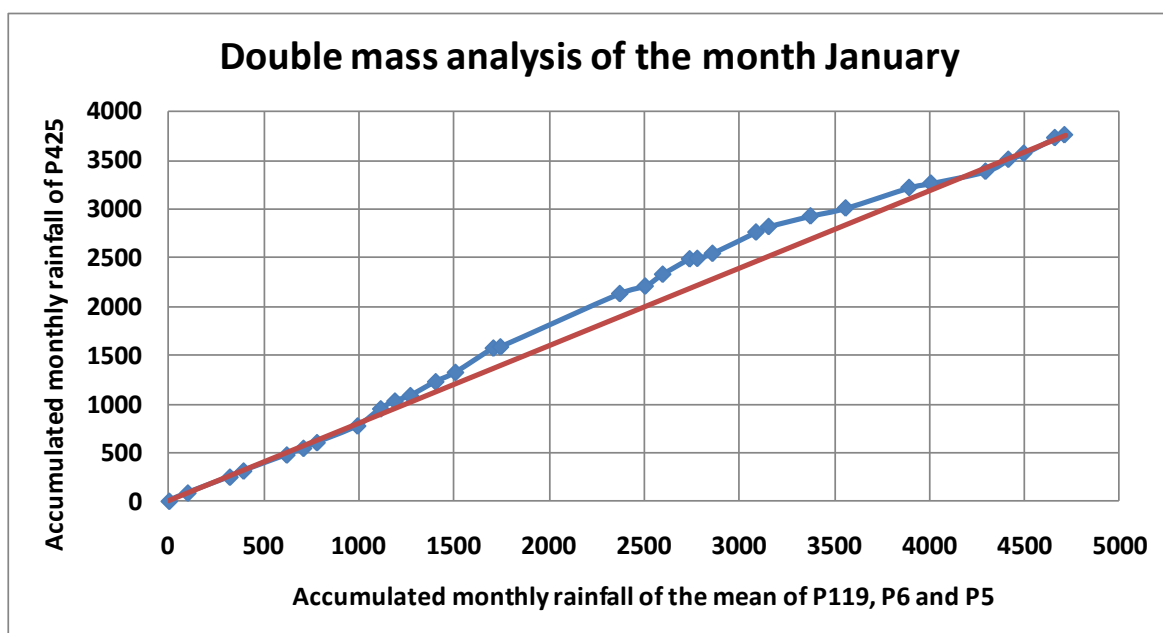


Fig. 2.1 Double mass analysis of monthly rainfall

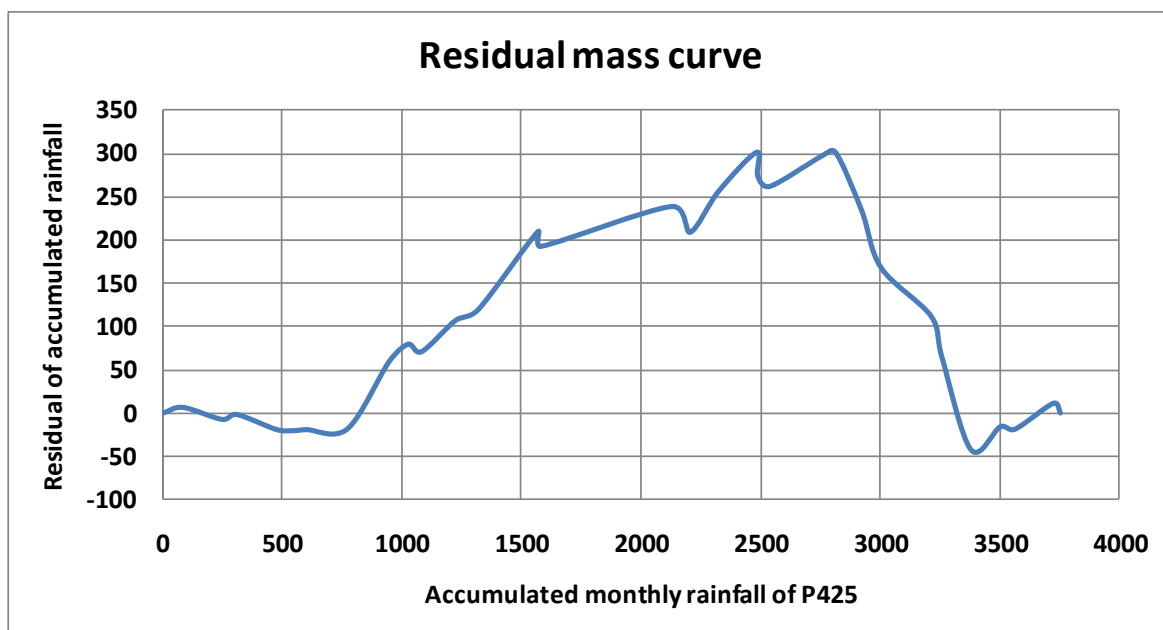


Fig. 2.2 Residual mass curve

## 2.4 Method of cumulative residuals

Application of the double mass technique provides an opportunity to visually inspect the homogeneity of a time series. The method does, however, not give a criterion for accepting or rejecting the hypothesis of homogeneity. For this purpose the method of *Cumulative Residuals* may be used (Allen et al., 1998). The method of cumulative residuals makes use of the original series (not the accumulated values). In this analysis we will test the homogeneity of the 31 values of station P425 against the average of the 3 other stations (P5, P6 and P119).

### Assignment

Retrieve the file **Dnum** with the double mass analysis of the completed monthly data of the four stations.

- Copy the column with 'filled-in' monthly data of station P425 to the right hand side of your sheet (e.g. to column AA). Use *Paste Special, Values!*
- Compute in the next column the average of the monthly values of the stations P119, P5 and P6. The series to be tested is P425 and these data are the *Y* values, while *X* refers to the average of the other three stations.
- Plot the monthly rainfall of P425 against the mean of the other stations (see e.g. figure 2.3) and show the regression line including the equation and the coefficient of determination,  $R^2$  (Click on a data point in the chart with the right mouse button, choose: *Add Trendline*, click: *Display equation on chart* and *Display R-squared value on chart*).
- Use the regression equation to compute in the next column the values estimated for *Y* (P425) from the other three stations. The estimated values are known as *Yest*.
- Compute in the next column the residuals of the observed *Y* values to the regression line, thus  $Y - Y_{est}$ .
- Compute in the next column the cumulated residuals  $E_i$ .
- Select a probability for accepting the hypothesis of homogeneity. The value of non-exceedance  $q = 0.8$  (or 80 %) is commonly utilized. The relation between the probability of exceedance  $p$  ( $p = 1 - q$ ) and the standardized or reduced variate  $t$  is given in appendix D and summarized here in a table 2.5.
- Compute the parameters  $\alpha$  and  $\beta$  as follows:

$$\alpha = \frac{n}{2}$$

$$\beta = \frac{n t s_{y,x}}{\sqrt{n-1}}$$

where  $n$  is the number of years and  $s_{y,x}$  is the standard deviation of the residuals.

- Add a column with the values 0 to 31 and another with values varying from 0 to  $2\pi$ .
- The equation of the ellipse is then

$$x = \alpha \cos(\theta)$$

$$y = \beta \sin(\theta)$$

with  $\theta$  varying from 0 to  $2\pi$ .

| q % | t    |
|-----|------|
| 60  | 0.25 |
| 70  | 0.52 |
| 80  | 0.84 |
| 85  | 1.04 |
| 90  | 1.28 |
| 95  | 1.64 |

**Table 2.5**  
The reduced variate  $t$  for the probability of non-exceedance  $q$

| P425            | Average<br>P119,P5,P6 | Yest    | Residual | Ei    | Year | Theta | x     | y      | x-shifted |
|-----------------|-----------------------|---------|----------|-------|------|-------|-------|--------|-----------|
| Y               | X                     |         |          | 0     | 0    | 0.00  | 15.5  | 0.0    | 31.0      |
| 84.5            | 97.9                  | 78.7    | 5.8      | 5.8   | 1    | 0.20  | 15.2  | 40.5   | 30.7      |
| 162.6           | 221.3                 | 175.6   | -13.0    | -7.2  | 2    | 0.41  | 14.2  | 79.4   | 29.7      |
| 62.9            | 72.0                  | 58.3    | 4.6      | -2.6  | 3    | 0.61  | 12.7  | 115.0  | 28.2      |
| 164.2           | 227.6                 | 180.5   | -16.3    | -18.9 | 4    | 0.81  | 10.7  | 145.9  | 26.2      |
| 68.6            | 87.8                  | 70.8    | -2.2     | -21.1 | 5    | 1.01  | 8.2   | 170.9  | 23.7      |
| 57.9            | 71.1                  | 57.7    | 0.3      | -20.8 | 6    | 1.22  | 5.4   | 188.8  | 20.9      |
| 171.1           | 213.7                 | 169.6   | 1.5      | -19.3 | 7    | 1.42  | 2.3   | 199.0  | 17.8      |
| 175.3           | 122.0                 | 97.6    | 77.7     | 58.4  | 8    | 1.62  | -0.8  | 201.1  | 14.7      |
| 79.5            | 74.7                  | 60.5    | 19.0     | 77.4  | 9    | 1.82  | -3.9  | 194.9  | 11.6      |
| 56.0            | 81.2                  | 65.6    | -9.6     | 67.9  | 10   | 2.03  | -6.8  | 180.7  | 8.7       |
| 142.4           | 133.3                 | 106.4   | 36.0     | 103.8 | 11   | 2.23  | -9.5  | 159.2  | 6.0       |
| 95.7            | 104.1                 | 83.5    | 12.2     | 116.0 | 12   | 2.43  | -11.8 | 131.1  | 3.7       |
| 249.0           | 199.7                 | 158.6   | 90.4     | 206.4 | 13   | 2.63  | -13.6 | 97.7   | 1.9       |
| 12.3            | 37.0                  | 30.8    | -18.5    | 187.9 | 14   | 2.84  | -14.8 | 60.3   | 0.7       |
| 546.8           | 628.3                 | 495.1   | 51.7     | 239.6 | 15   | 3.04  | -15.4 | 20.4   | 0.1       |
| 76.6            | 133.6                 | 106.7   | -30.1    | 209.5 | 16   | 3.24  | -15.4 | -20.4  | 0.1       |
| 121.5           | 92.4                  | 74.3    | 47.2     | 256.7 | 17   | 3.45  | -14.8 | -60.3  | 0.7       |
| 157.7           | 141.9                 | 113.2   | 44.5     | 301.2 | 18   | 3.65  | -13.6 | -97.7  | 1.9       |
| 4.7             | 40.3                  | 33.5    | -28.8    | 272.4 | 19   | 3.85  | -11.8 | -131.1 | 3.7       |
| 51.8            | 79.9                  | 64.5    | -12.7    | 259.7 | 20   | 4.05  | -9.5  | -159.2 | 6.0       |
| 218.0           | 229.3                 | 181.8   | 36.2     | 295.9 | 21   | 4.26  | -6.8  | -180.7 | 8.7       |
| 56.7            | 66.5                  | 54.0    | 2.7      | 298.5 | 22   | 4.46  | -3.9  | -194.9 | 11.6      |
| 108.6           | 220.3                 | 174.8   | -66.2    | 232.4 | 23   | 4.66  | -0.8  | -201.1 | 14.7      |
| 81.3            | 184.9                 | 147.0   | -65.7    | 166.7 | 24   | 4.86  | 2.3   | -199.0 | 17.8      |
| 210.2           | 334.4                 | 264.3   | -54.1    | 112.6 | 25   | 5.07  | 5.4   | -188.8 | 20.9      |
| 44.3            | 114.5                 | 91.7    | -47.4    | 65.2  | 26   | 5.27  | 8.2   | -170.9 | 23.7      |
| 122.0           | 287.5                 | 227.5   | -105.5   | -40.4 | 27   | 5.47  | 10.7  | -145.9 | 26.2      |
| 122.5           | 120.2                 | 96.2    | 26.3     | -14.1 | 28   | 5.68  | 12.7  | -115.0 | 28.2      |
| 62.2            | 81.5                  | 65.8    | -3.6     | -17.6 | 29   | 5.88  | 14.2  | -79.4  | 29.7      |
| 160.0           | 163.4                 | 130.1   | 30.0     | 12.3  | 30   | 6.08  | 15.2  | -40.5  | 30.7      |
| 29.2            | 50.8                  | 41.7    | -12.4    | -0.1  | 31   | 6.28  | 15.5  | 0.0    | 31.0      |
|                 |                       | Stdev = | 42.3     |       |      |       |       |        |           |
|                 |                       | n =     | 31       |       |      |       |       |        |           |
|                 |                       | Alpha = | 15.5     |       |      |       |       |        |           |
|                 |                       | t =     | 0.84     |       |      |       |       |        |           |
| Intercept =     | 1.7969                |         |          |       |      |       |       |        |           |
| X-coefficient = | 0.7852                | Beta =  | 201.3    |       |      |       |       |        |           |

Table 2.4 Method of cumulative residuals

- Add an extra column with values for  $x+\alpha$  which causes the ellipse to be shifted.
- Plot  $Ei$  against time (0 to 31) and add to this chart the ellipse (plot  $(x+\alpha)$  against  $y$ ).
- If the  $Ei$  values lie inside the ellipse, the hypothesis of homogeneity is accepted at the 80 % level of confidence or any other level that was selected.

**Question: Do you think that the results of the homogeneity test for P425 will be better if less data were missing? Why?**

Save worksheet **Dnum**.

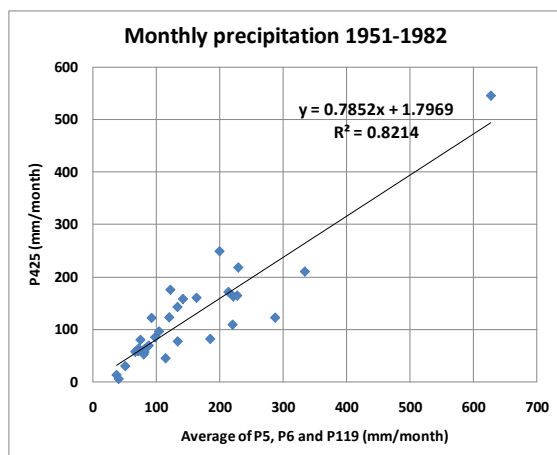


Fig. 2.3 Regression between P425 and average of other 3 stations

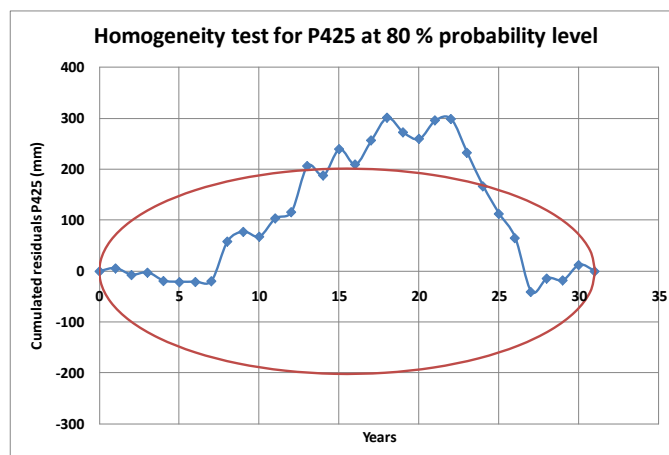


Fig. 2.4 Homogeneity test for P425 at 80 % probability level

## 2.5 Frequency distribution

The purpose of this exercise is to present data by a statistical distribution. The monthly rainfall totals of a particular month over a large number of years are used.

Two distributions will be considered:

- normal distribution
- log-normal distribution

The monthly rainfall data of station P119 are used for this exercise. Each participant will be assigned a particular month containing approximately 70 years of monthly values.

A normal distribution assumes a symmetrical distribution around the mean where also negative values are possible. Since negative rainfall values do not exist, a normal distribution does not seem very suitable. However, if the logarithm of the rainfall data are considered, the values may be approximately normally distributed. This is known as a log-normal distribution.

### Assignment

Create a table similar to the example table 2.5 using the month assigned to you and the monthly data of station P119.

- Open file i:groupwork\hydata\p119\_mr.txt
- Copy the values in the column of the month assigned to you to the clipboard.
- Close the data file and open a new workbook, click in cell B3 and paste the data.
- Rank the data in descending order (Select with the mouse the data to be sorted, Select: *Data, Sort, Select: Largest to Smallest, OK*)
- Delete the negative values at the bottom.
- Fill the first column with the rank values (Put the values 1 and 2 in the first two cells, select both cells with the mouse, double click square dot in bottom right corner).

| Rank<br>(m) | X     | LnX  | $p=(m-0.375)/(N+0.25)$ | t      | X<br>cal | LnX<br>cal |
|-------------|-------|------|------------------------|--------|----------|------------|
| 1           | 297.5 | 5.70 | 0.01                   | 2.370  | 199.7    | 5.80       |
| 2           | 236.4 | 5.47 | 0.02                   | 1.993  | 178.8    | 5.50       |
| 3           | 193.9 | 5.27 | 0.04                   | 1.782  | 167.1    | 5.33       |
| 4           | 188.1 | 5.24 | 0.05                   | 1.630  | 158.7    | 5.21       |
| 5           | 183.3 | 5.21 | 0.07                   | 1.508  | 151.9    | 5.12       |
| 6           | 155.9 | 5.05 | 0.08                   | 1.405  | 146.2    | 5.04       |
| -           | -     | -    | -                      | -      | -        | -          |
| -           | -     | -    | -                      | -      | -        | -          |
| 65          | 14.8  | 2.69 | 0.92                   | -1.405 | -9.3     | 2.84       |
| 66          | 13.6  | 2.61 | 0.93                   | -1.508 | -15.0    | 2.76       |
| 67          | 11.2  | 2.42 | 0.95                   | -1.630 | -21.8    | 2.66       |
| 68          | 11.1  | 2.41 | 0.96                   | -1.782 | -30.2    | 2.54       |
| 69          | 10.3  | 2.33 | 0.98                   | -1.993 | -41.9    | 2.38       |
| 70          | 10.0  | 2.30 | 0.99                   | -2.370 | -62.8    | 2.08       |
| AVG =       | 68.5  | 3.94 |                        |        |          |            |
| STD =       | 55.4  | 0.78 |                        |        |          |            |

Table 2.5 Application of Normal and Log-normal distribution

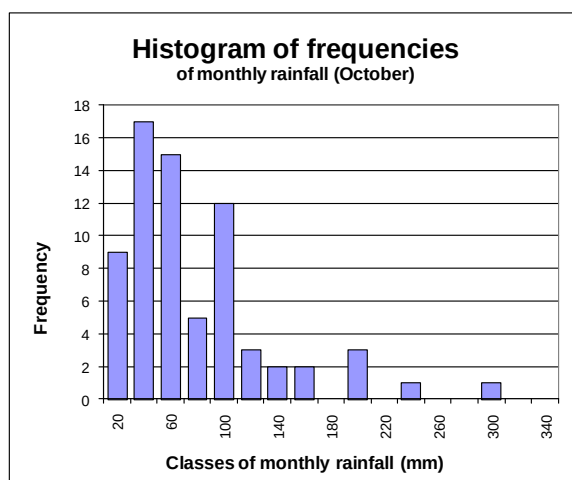
- Compute in the third column the natural logarithm of  $X$ ,  $\ln(X)$
- Save the sheet in your own directory as **Enum**
- Make another table (as an example see table 2.6) with columns for classes and the number of elements within each class interval. Fill this table with classes from 0 to an upper limit that can include your maximum value with class-width of 10 or 20. Count the number of values within each class.
- Make a histogram of the frequency distribution similar to figure 2.5 with on the horizontal axis the classes and vertical axis the number of elements. (*Insert, Column, choose first type, Select data, Add*, highlight column with number of values ( $n$ ) in each class interval in table 2.6, *OK*, in Horizontal axis (category) labels click: *Edit*, highlight class intervals, *OK, OK*).
- Extend the table with  $\ln(X)$  class values and plot the frequencies into another histogram.

As you have noticed now, the frequency distribution of monthly rainfall values is (right) skewed and the one with the logarithm of the monthly rainfall values is not. The frequency distribution of  $\ln(X)$  looks more like a normal distribution. Therefore, it is expected that a normal distribution applied on  $\ln(X)$  values gives more satisfactory results in representing the data.

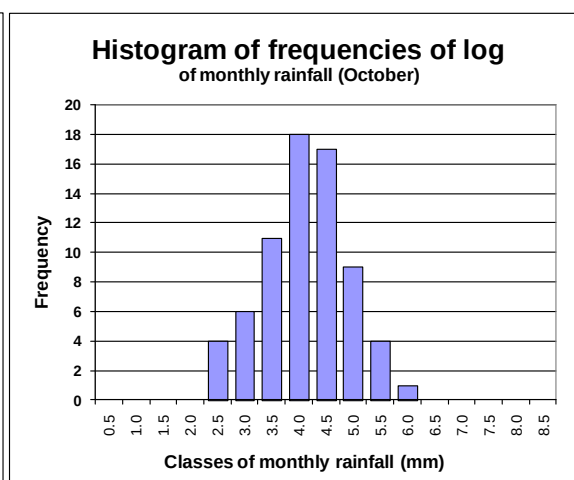
A characteristic of a normal distribution is that data represented by this distribution on probability paper plot a straight line. The horizontal axis of this paper expresses probability of exceedance  $p$ , or non-exceedance  $q$ , on a non-linear scale and the vertical axis the

| Monthly values |    | Monthly Ln-values |    |
|----------------|----|-------------------|----|
| class interval | n  | class interval    | n  |
| 0 - 20         | 9  | 0 - 0.5           |    |
| 20 - 40        | 17 | 0.5 - 1.0         |    |
| 40 - 60        | 15 | 1.0 - 1.5         |    |
| 60 - 80        | 5  | 1.5 - 2.0         |    |
| 80 - 100       | 12 | 2.0 - 2.5         | 4  |
| 100 - 120      | 3  | 2.5 - 3.0         | 6  |
| 120 - 140      | 2  | 3.0 - 3.5         | 11 |
| 140 - 160      | 2  | 3.5 - 4.0         | 18 |
| 160 - 180      | 0  | 4.0 - 4.5         | 17 |
| 180 - 200      | 3  | 4.5 - 5.0         | 9  |
| 200 - 220      | 0  | 5.0 - 5.5         | 4  |
| 220 - 240      | 1  | 5.5 - 6.0         | 1  |
| 240 - 260      | 0  | 6.0 - 6.5         |    |
| 260 - 280      | 0  | 6.5 - 7.0         |    |
| 280 - 300      | 1  | 7.0 - 7.5         |    |
| 300 - 320      | 0  | 7.5 - 8.0         |    |
| 320 - 340      | 0  | 8.0 - 8.5         |    |
|                | 70 |                   | 70 |

**Table 2.6** Distribution of the (log of) monthly values over the class intervals



**Fig. 2.5** Histogram of frequencies of monthly rainfall



**Fig. 2.6** Histogram of frequencies of the logarithm of monthly rainfall



values analyzed. In order to plot the distribution in a spreadsheet the scale of the horizontal axis has to be linear. A linear scale is obtained in terms of the normalized value  $t$ , derived from a normal distribution which is related to the probability of exceedance (or non-exceedance). The relation between  $t$  and  $p$  is read from a table of the normal distribution, see appendix D.

The relation between the standardized or reduced variate  $t$  and  $X$  in the normal distribution is given as

$$t = \frac{X - \bar{X}}{s_X} \quad (2.5)$$

where

$\bar{X}$  average value of series  $X$   
 $s_X$  standard deviation of series  $X$

Next the theoretical line of the normal distribution will be fitted through the monthly values of your series and subsequently through the  $\ln$ -values of the same series, after which the results will be compared.

### Assignment

- Create in your table of ranked values for  $X$  and  $\ln(X)$  a column for the probability of exceedance  $p$ , using the formula of Blom (see Cunnane, 1978)

$$p = \frac{m - 0.375}{N + 0.25} \quad (2.6)$$

where  $m$  is the rank and  $N$  the sample size.

- Use the table of the normal distribution to find for each value of  $p$  the related  $t$ -value or use the spreadsheet function NORMSINV, which returns the inverse of the standard normal distribution, i.e. the value for  $t$  for  $P(X < x)$ .
- Create columns for  $X_{cal}$  and  $\ln(X_{cal})$  and calculate for each  $t$ -value the theoretical corresponding  $X_{cal}$  and  $Y_{cal}$ , where  $Y_{cal} = \ln(X_{cal})$  using

$$X_{cal} = \bar{X} + t s_X \quad (2.7)$$

$$Y_{cal} = \bar{Y} + t s_Y \quad (2.8)$$

- Make a graph which plots  $t$  (X-axis) against  $X$  (Y-axis) and the line  $X_{cal}$ , similar to figure 2.7.
- Make another graph which plots  $t$  (X-axis) against  $\ln(X)$  (Y-axis) and the line  $\ln(X_{cal})$ , similar to figure 2.8.
- Do not forget to save sheet **Enum**.

**Question:** Please write in the top left corner of your spreadsheet which distribution (Normal or Lognormal) is most appropriate for a series of monthly minimum flows?

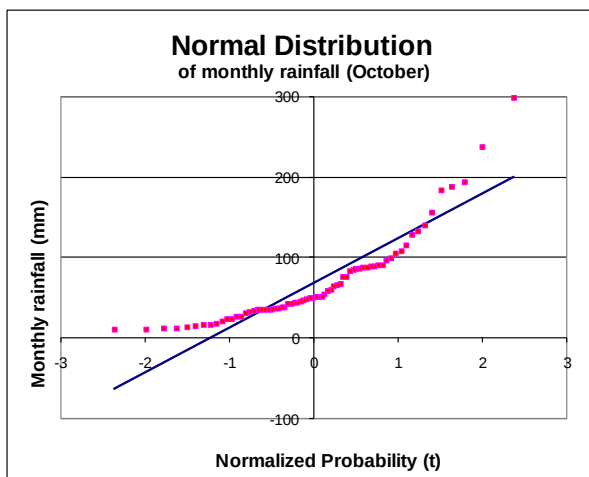


Fig. 2.7 Normal distribution of monthly rainfall

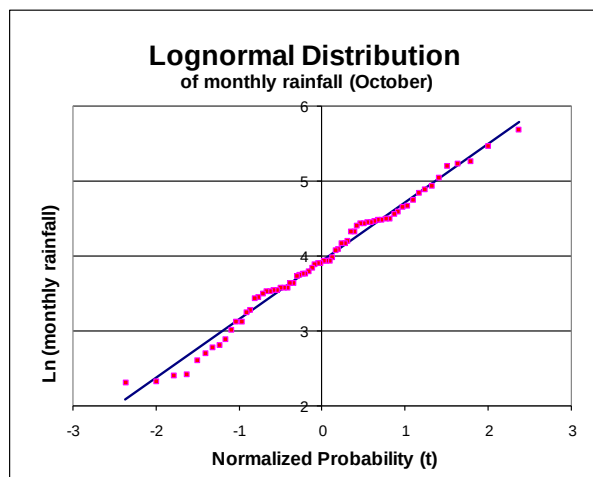


Fig. 2.8 Log-normal distribution of monthly rainfall

### 3 YEARLY RAINFALL DATA

This chapter is about homogeneity tests. Hydrological time series may exhibit trends referred to as inconsistencies or non-homogeneities. Inconsistencies result from changes in the amount of systematic errors associated with recording of data, such as those arising from changes in instrumentation or observational practises. Non-homogeneity is defined as a change in the statistics of the data set which are caused by natural or man-made changes (e.g. change in landuse (deforestation!), water use, climatic change, etc.). Split record tests on variances and means are applied to detect the presence of inconsistencies or non-homogeneities. These tests are referred to as the F-test for stability of the variances and t-test for stability of the mean. These two tests can be reinforced by a third test, Spearman's rank correlation test, for indicating absence of trends.

All three tests determine the presence or absence of 'absolute' consistency or homogeneity, as they are performed on an individual data series without comparison with other series. The theoretical background of these tests is explained in appendix A.

#### 3.1 Spearman's rank correlation method

##### Assignment

- Time series of 39 years of annual rainfall data of 4 stations are found in file i:\groupwork\hydata\yeartot.txt. Open this file and delete the columns with data except for the station assigned to you. Save the sheet as **Fnum**.
- Prepare a table with columns for:

|            |                                               |
|------------|-----------------------------------------------|
| $X_{obs}$  | yearly rainfall in order of observation       |
| $X_{rank}$ | yearly rainfall in ascending order            |
| $Kx_i$     | the rank of the data as observed              |
| $Ky_i$     | the rank of the same data in ascending order. |
| $D_i$      | $Kx_i - Ky_i$                                 |
| $D_i^2$    | square of $D_i$                               |

- Fill the columns  $Kx_i$  and  $Ky_i$  with the rank (the values 1 to 39).
- Copy the values in column  $X_{obs}$  into the column  $X_{ranked}$ .
- To rank the data in ascending order, select with the mouse data in the columns D and E, Select: *Data, Sort, Sort by column D, Smallest to Largest, OK*.
- Compute  $D_i = Kx_i - Ky_i$  and in the last column the square of  $D_i$ .
- Then calculate in the spreadsheet the Spearman's rank correlation coefficient  $R_{sp}$  and the test statistics  $t$ , as

$$R_{sp} = 1 - \frac{6 \sum D_i^2}{n(n^2 - 1)} \quad (3.1)$$

$$t = R_{sp} \left( \frac{n-2}{1 - R_{sp}^2} \right)^{0.5} \quad (3.2)$$

where  $n$  represents the number of observations. An example is presented in table 3.1.

| Year  | Xobs   | Kxi | Xranked | Kyi | Di         | Di^2   |
|-------|--------|-----|---------|-----|------------|--------|
| 46/47 | 521.2  | 1   | 473.3   | 18  | -17        | 289    |
| 47/48 | 524.7  | 2   | 475.4   | 25  | -23        | 529    |
| 48/49 | 770.8  | 3   | 502.4   | 19  | -16        | 256    |
| 49/50 | 801.8  | 4   | 502.7   | 37  | -33        | 1089   |
| 50/51 | 633.7  | 5   | 508.2   | 16  | -11        | 121    |
| 51/52 | 528.6  | 6   | 521.2   | 1   | 5          | 25     |
| 52/53 | 850.9  | 7   | 524.7   | 2   | 5          | 25     |
| 53/54 | 597.9  | 8   | 528.6   | 6   | 2          | 4      |
| 54/55 | 921.0  | 9   | 529.6   | 24  | -15        | 225    |
| 55/56 | 828.9  | 10  | 537.1   | 34  | -24        | 576    |
| 56/57 | 734.2  | 11  | 570.2   | 14  | -3         | 9      |
| 57/58 | 593.8  | 12  | 593.8   | 12  | 0          | 0      |
| 58/59 | 594.2  | 13  | 594.2   | 13  | 0          | 0      |
| 59/60 | 570.2  | 14  | 597.9   | 8   | 6          | 36     |
| 60/61 | 836.0  | 15  | 619.5   | 33  | -18        | 324    |
| 61/62 | 508.2  | 16  | 626.7   | 27  | -11        | 121    |
| 62/63 | 830.7  | 17  | 633.7   | 5   | 12         | 144    |
| 63/64 | 473.3  | 18  | 638.5   | 36  | -18        | 324    |
| 64/65 | 502.4  | 19  | 657.3   | 22  | -3         | 9      |
| 65/66 | 971.9  | 20  | 702.7   | 32  | -12        | 144    |
| 66/67 | 768.0  | 21  | 721.3   | 39  | -18        | 324    |
| 67/68 | 657.3  | 22  | 734.2   | 11  | 11         | 121    |
| 68/69 | 800.1  | 23  | 768.0   | 21  | 2          | 4      |
| 69/70 | 529.6  | 24  | 770.8   | 3   | 21         | 441    |
| 70/71 | 475.4  | 25  | 786.7   | 38  | -13        | 169    |
| 71/72 | 988.2  | 26  | 800.1   | 23  | 3          | 9      |
| 72/73 | 626.7  | 27  | 801.8   | 4   | 23         | 529    |
| 73/74 | 830.9  | 28  | 828.9   | 10  | 18         | 324    |
| 74/75 | 896.1  | 29  | 830.7   | 17  | 12         | 144    |
| 75/76 | 839.9  | 30  | 830.9   | 28  | 2          | 4      |
| 76/77 | 1179.8 | 31  | 836.0   | 15  | 16         | 256    |
| 77/78 | 702.7  | 32  | 839.9   | 30  | 2          | 4      |
| 78/79 | 619.5  | 33  | 850.9   | 7   | 26         | 676    |
| 79/80 | 537.1  | 34  | 896.1   | 29  | 5          | 25     |
| 80/81 | 954.8  | 35  | 921.0   | 9   | 26         | 676    |
| 81/82 | 638.5  | 36  | 954.8   | 35  | 1          | 1      |
| 82/83 | 502.7  | 37  | 971.9   | 20  | 17         | 289    |
| 83/84 | 786.7  | 38  | 988.2   | 26  | 12         | 144    |
| 84/85 | 721.3  | 39  | 1179.8  | 31  | 8          | 64     |
|       |        |     |         |     |            |        |
|       |        |     |         |     | Sum Di^2 = | 8454   |
|       |        |     |         |     | Rsp =      | 0.14   |
|       |        |     |         |     | t =        | 0.89   |
|       |        |     |         |     | tcr =      | +-2.02 |

Table 3.1 Spearman's rank correlation test

The test variable  $t$  has a Student's  $t$ -distribution with  $\nu = n - 2$  degrees of freedom. Use a table of the Student's- $t$  distribution (appendix B) to define with a level of significance of 5% the critical region (see Appendix A6.2)

$$\{-\infty, t(\nu, 2.5\%)\} \cup \{t(\nu, 97.5\%), +\infty\} \quad (3.3)$$

Thus the time series does not have a trend if

$$t(\nu, 2.5\%) < t < t(\nu, 97.5\%) \quad (3.4)$$

Based on your results specify the absence or presence of a trend.

### 3.2 F-test for the stability of the variance

#### Assignment

The same series that was investigated for the presence of a trend will be subjected to an F-test. Make sure you are in the right spreadsheet. Make a quick investigation on the spread of the variance by plotting the years against the yearly rainfall data. Is there any suspicious period?

An investigation will now be carried out to see whether there is prove for a significant difference between the variance of the first half of the series and the second half.

Create a table with two columns  $X_1$  and  $X_2$  with yearly rainfall data from the original series of approximately equal length, where

$X_1$  yearly rainfall first half of the series

$X_2$  yearly rainfall second half of the series

Calculate for each series the variances  $s_1^2$  and  $s_2^2$  and the test statistic

$$F_t = \frac{s_1^2}{s_2^2} \quad (3.5)$$

Use a table of the Fisher-F distribution (appendix C) to define with a level of significance of 5% the critical region (see Appendix A6.3)

$$\{ 0, F(v_1, v_2, 2.5\%) \} \cup \{ F(v_1, v_2, 97.5\%), \infty \} \quad (3.6)$$

where  $v_1$  and  $v_2$  are the respective numbers of degrees of freedom of the numerator and dominator.  $v_1 = n_1 - 1$  and  $v_2 = n_2 - 1$  where  $n_1$  and  $n_2$  are the number of observations in each sub-set.

Based on your results specify the stability or non-stability of the variance of the series.

### 3.3 Students-t test for stability of the mean

#### Assignment

The same series that was investigated for the presence of a trend and the stability of the variance will be subjected to a test for stability of the mean. Make sure you are in sheet **Fnum**.

You will investigate now whether there is prove for a significant difference between the mean of the first half of the series and the second half.

Use the same subsets  $X_1$  and  $X_2$  as used in the F-test.

Calculate for each subset the averages  $\bar{X}_1$  and  $\bar{X}_2$  and the test statistic  $t$ .

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\left( \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \left( \frac{1}{n_1} + \frac{1}{n_2} \right) \right)^{0.5}} \quad (3.7)$$

where

$n_i$  the number of data in subset  $i$

$\bar{X}_i$  the mean of subset  $i$

$s_i^2$  the variance of subset  $i$

Use a table of the Student's-t distribution (appendix B) to define with a level of significance of 5% the critical region (see Appendix A6.4)

$$\{ -\infty , t ( v , 2.5\% ) \} \cup \{ t ( v , 97.5\% ), +\infty \} \quad (3.8)$$

where  $v$  is the number of degrees of freedom,  $v = n_1 + n_2 - 2$

Based on your results indicate if the mean of the time series is stable.

Save the sheet as **Fnum**.

**Question:** Please write in the top left corner of your spreadsheet answers to the following questions.

- 1 Can you apply the students-t test to a series for which the variance is not stable?
- 2 Can you apply the split record test to more than 2 subsets (e.g. 3 or 4)?
- 3 If the series shows a linear trend, does that also mean that the mean is not stable?

## 4 EVAPORATION

Evapotranspiration rates will be calculated for the meteorological station Lelystad, The Netherlands. For a period of 20 years (1961 - 1980) meteorological data are available in file `i:\groupwork\hydata\MeteoData.txt`. The data are specified as an average value per decade. With decades the month is divided into three parts: 1 - 10, 11 - 20 and 21 - end of month. Thus the third decade in the month consists of 8, 9, 10 or 11 days.

The first computation concerns the evapotranspiration according to the Penman-Monteith equation. This quantity is often used as reference evaporation  $ET_{ref}$  for estimating the potential evapotranspiration of crops  $ET_{pot}$  as follows

$$ET_{pot} = k_c ET_{ref} \quad (4.1)$$

where  $k_c$  is the crop coefficient. The reference evapotranspiration according to Penman-Monteith, using the FAO defined constants, yields the evapotranspiration rate of grass with a length of 12 cm (see lecture notes on Hydrology).

An alternative for the equation of Penman-Monteith is the *Radiation Method*, a more simple approach proposed by Makkink (1957), which formula also computes the potential evapotranspiration of grass, using only global radiation and temperature data. In this assignment both methods are applied and compared.

### 4.1 Penman-Monteith

The Penman-Monteith formula is written as

$$ET_{P-M} = \frac{C}{L} \frac{sR_N + c_p \rho_a (e_a - e_d) / r_a}{s + \gamma(1 + r_c / r_a)} \quad (4.2)$$

where

- $ET_{P-M}$  potential evapotranspiration of grass in  $\text{mm.d}^{-1}$
- $C$  constant to convert units from  $\text{kg.m}^{-2}.\text{s}^{-1}$  to  $\text{mm.d}^{-1}$  ( $C = 86400$ )
- $R_N$  net radiation at the earth's surface in  $\text{W.m}^{-2}$
- $L$  latent heat of vaporization ( $L = 2.45 \cdot 10^6 \text{ J.kg}^{-1}$ )
- $s$  slope of the temperature-saturation vapour pressure curve ( $\text{kPa.K}^{-1}$ )
- $c_p$  specific heat of air at constant pressure ( $c_p = 1004.6 \text{ J.kg}^{-1}.\text{K}^{-1}$ )
- $\rho_a$  density of air ( $\rho_a = 1.2047 \text{ kg.m}^{-3}$  at sea level)
- $e_d$  actual vapour pressure of the air at 2 m height in kPa
- $e_a$  saturation vapour pressure for the air temperature at 2 m height in kPa
- $\gamma$  psychrometric constant ( $\gamma = 0.067 \text{ kPa.K}^{-1}$  at sea level)
- $r_a$  aerodynamic resistance in  $\text{s.m}^{-1}$
- $r_c$  crop resistance in  $\text{s.m}^{-1}$  (FAO takes for grass  $r_c = 70 \text{ s.m}^{-1}$ )

The aerodynamic resistance  $r_a$  is a function of the wind speed. The following expression for  $r_a$  ( $\text{s.m}^{-1}$ ) is used by the FAO for wind velocities,  $U_2$  ( $\text{m.s}^{-1}$ ), observed at a height of 2 m over grass

$$r_a = \frac{208}{U_2} \quad (4.3)$$

Values for  $e_a$  (kPa) and  $s$  (kPa.K<sup>-1</sup>) may be obtained from

$$e_a = 0.6108 e^{\frac{17.27 T_a}{237.3 + T_a}} \quad (4.4)$$

$$s = \frac{4098 e_a}{(237.3 + T_a)^2} \quad (4.5)$$

where  $T_a$  is the 24 hour mean temperature of the air in °C.

The actual or dewpoint vapour pressure  $e_d$  (kPa) is calculated from measurements of the relative humidity  $RH$ , thus

$$e_d = e_a \frac{RH}{100} \quad (4.6)$$

The net outgoing long wave radiation  $R_{nL}$  (W.m<sup>-2</sup>) is estimated with the empirical equation

$$R_{nL} = 5.6745 \times 10^{-8} (273 + T_a)^4 (0.34 - 0.139 \sqrt{e_d}) (0.1 + 0.9 n/N) \quad (4.7)$$

and the short wave or global radiation  $R_S$  (W.m<sup>-2</sup>) with

$$R_S = (0.20 + 0.60 n/N) R_A \quad (4.8)$$

where  $N$  is the day length (hours) and  $n$  the actual number of hours of sunshine during the day.

The net radiation  $R_N$  is calculated as the incoming short wave radiation at the earth's surface (or global radiation)  $R_S$  minus the fraction that is reflected and minus the net outgoing long wave radiation  $R_{nL}$ , hence

$$R_N = (1 - r) R_S - R_{nL} \quad (4.9)$$

where  $r$  is the reflection coefficient (FAO uses  $r = 0.23$  for grass)

Values for radiation received at the outer limits of the atmosphere  $R_A$  are read for a given date and latitude from table 4.1 and values for the day length  $N$  from table 4.2.

### Assignment

- Retrieve file *i:\groupwork\hydata\Meteo.txt*. Delete all data except for the year assigned to you and save the sheet as **Gnum**. The meteorological data are observed at a height of 2 m and refer to 24-hour totals or means, which are averaged over a decade.
- Extend the columns in the spreadsheet similar to table 4.3 and compute the Penman-Monteith evapotranspiration  $ET_{P-M}$ . Read values for  $N$  and  $R_A$  from tables 4.1 and 4.2



respectively, for a northern latitude of 52 degrees which applies to The Netherlands. A more accurate approach (see table 4.5) is the computation of  $N$  and  $R_A$  for given latitude and Julian day number, using the algorithms specified in the text boxes in this chapter.

It is interesting to compare the global radiation flux and the evapotranspiration rate. To do this  $R_S$  ( $\text{W.m}^{-2} = \text{J.s}^{-1}.\text{m}^{-2}$ ) has to be converted into the unit  $\text{mm/d}$ , which is accomplished through dividing  $R_S$  by the latent heat of vaporization  $L$  ( $\text{J.kg}^{-1}$ ), giving  $\text{kg.s}^{-1}.\text{m}^{-2}$ , which is approximately  $\text{mm.s}^{-1}$ . Multiplying by  $C = 86400$  s yields  $\text{mm.d}^{-1}$ , thus

$$R_S^* \approx \frac{C}{L} R_S \quad (4.10)$$

- Make a separate column to compute the global radiation  $R_S^*$  in  $\text{mm.d}^{-1}$ .
- Plot in one graph the global radiation flux and the evapotranspiration rate according to Penman-Monteith (see e.g. figure 4.1).

| North Lats.<br>South Lats. | Jan<br>July | Feb<br>Aug | Mar<br>Sept | Apr<br>Oct | May<br>Nov | June<br>Dec | July<br>Jan | Aug<br>Feb | Sept<br>Mar | Oct<br>Apr | Nov<br>May | Dec<br>June |
|----------------------------|-------------|------------|-------------|------------|------------|-------------|-------------|------------|-------------|------------|------------|-------------|
| 60                         | 6.7         | 9.0        | 11.7        | 14.5       | 17.1       | 18.6        | 17.9        | 15.5       | 12.9        | 10.1       | 7.5        | 5.9         |
| 58                         | 7.2         | 9.3        | 11.7        | 14.3       | 16.6       | 17.9        | 17.3        | 15.3       | 12.8        | 10.3       | 7.9        | 6.5         |
| 56                         | 7.6         | 9.5        | 11.7        | 14.1       | 16.2       | 17.4        | 16.9        | 15.0       | 12.7        | 10.4       | 8.3        | 7.0         |
| 54                         | 7.9         | 9.7        | 11.7        | 13.9       | 15.9       | 16.9        | 16.5        | 14.8       | 12.7        | 10.5       | 8.5        | 7.4         |
| 52                         | 8.3         | 9.9        | 11.8        | 13.8       | 15.6       | 16.5        | 16.1        | 14.6       | 12.7        | 10.6       | 8.8        | 7.8         |
| 50                         | 8.5         | 10.0       | 11.8        | 13.7       | 15.3       | 16.3        | 15.9        | 14.4       | 12.6        | 10.7       | 9.0        | 8.1         |
| 48                         | 8.8         | 10.2       | 11.8        | 13.6       | 15.2       | 16.0        | 15.6        | 14.3       | 12.6        | 10.9       | 9.3        | 8.3         |
| 46                         | 9.1         | 10.4       | 11.9        | 13.5       | 14.9       | 15.7        | 15.4        | 14.2       | 12.6        | 10.9       | 9.5        | 8.7         |
| 44                         | 9.3         | 10.5       | 11.9        | 13.4       | 14.7       | 15.4        | 15.2        | 14.0       | 12.6        | 11.0       | 9.7        | 8.9         |
| 42                         | 9.4         | 10.6       | 11.9        | 13.4       | 14.6       | 15.2        | 14.9        | 13.9       | 12.6        | 11.1       | 9.8        | 9.1         |
| 40                         | 9.6         | 10.7       | 11.9        | 13.3       | 14.4       | 15.0        | 14.7        | 13.7       | 12.4        | 11.2       | 10.0       | 9.3         |
| 35                         | 10.1        | 11.0       | 11.9        | 13.1       | 14.0       | 14.5        | 14.3        | 13.5       | 12.4        | 11.9       | 10.3       | 9.8         |
| 30                         | 10.4        | 11.1       | 12.0        | 12.9       | 13.6       | 14.0        | 13.9        | 13.2       | 12.4        | 12.0       | 10.6       | 10.8        |
| 25                         | 10.7        | 11.3       | 12.0        | 12.7       | 13.3       | 13.7        | 13.5        | 13.0       | 12.3        | 12.0       | 10.9       | 10.6        |
| 20                         | 11.0        | 11.5       | 12.0        | 12.6       | 13.1       | 13.3        | 13.2        | 12.8       | 12.3        | 12.0       | 11.2       | 10.9        |
| 15                         | 11.3        | 11.6       | 12.0        | 12.5       | 12.8       | 13.0        | 12.9        | 12.6       | 12.2        | 12.0       | 11.4       | 11.2        |
| 10                         | 11.6        | 11.8       | 12.0        | 12.3       | 12.6       | 12.7        | 12.6        | 12.4       | 12.1        | 12.0       | 11.6       | 11.5        |
| 5                          | 11.8        | 11.9       | 12.0        | 12.2       | 12.3       | 12.4        | 12.3        | 12.3       | 12.1        | 12.0       | 11.9       | 11.8        |
| Equator 0                  | 12.0        | 12.0       | 12.0        | 12.0       | 12.0       | 12.0        | 12.0        | 12.0       | 12.0        | 12.0       | 12.0       | 12.0        |

**Table 4.1 Mean daily duration of maximum possible sunshine hours (N)**

|                     | Lat | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| NORTHERN HEMISPHERE |     |     |     |     |     |     |     |     |     |     |     |     |     |
| Equator             | 60  | 40  | 103 | 200 | 317 | 417 | 469 | 446 | 360 | 243 | 134 | 57  | 26  |
|                     | 52  | 91  | 157 | 252 | 357 | 440 | 475 | 457 | 389 | 292 | 192 | 111 | 74  |
|                     | 50  | 106 | 172 | 263 | 363 | 443 | 475 | 460 | 392 | 297 | 203 | 126 | 89  |
|                     | 40  | 177 | 240 | 317 | 395 | 455 | 477 | 466 | 420 | 346 | 266 | 194 | 160 |
|                     | 30  | 232 | 300 | 366 | 420 | 460 | 472 | 463 | 435 | 386 | 320 | 260 | 226 |
|                     | 20  | 309 | 354 | 400 | 435 | 449 | 452 | 452 | 440 | 412 | 369 | 323 | 297 |
|                     | 10  | 366 | 397 | 423 | 435 | 429 | 423 | 426 | 429 | 423 | 406 | 375 | 357 |
|                     | 0   | 415 | 429 | 435 | 420 | 397 | 383 | 389 | 409 | 426 | 429 | 417 | 409 |
|                     | 10  | 455 | 449 | 432 | 397 | 357 | 334 | 343 | 375 | 412 | 440 | 449 | 452 |
|                     | 20  | 489 | 457 | 415 | 357 | 306 | 277 | 289 | 332 | 389 | 437 | 469 | 483 |
|                     | 30  | 492 | 452 | 386 | 312 | 246 | 214 | 226 | 277 | 352 | 423 | 477 | 500 |
|                     | 40  | 495 | 432 | 349 | 254 | 183 | 149 | 160 | 217 | 306 | 395 | 472 | 509 |
|                     | 50  | 483 | 403 | 297 | 192 | 117 | 83  | 97  | 154 | 249 | 357 | 457 | 503 |
|                     | 60  | 472 | 360 | 237 | 123 | 51  | 26  | 37  | 89  | 186 | 309 | 432 | 500 |
| SOUTHERN HEMISPHERE |     |     |     |     |     |     |     |     |     |     |     |     |     |

**Table 4.2 Short wave radiation  $R_A$  received at the outer limits of the atmosphere expressed in  $\text{W.m}^{-2}$**

| 1971 | U   | n     | RH | T <sub>a</sub> | N     | n/N  | e <sub>a</sub> | s     | e <sub>d</sub> | r <sub>a</sub> | R <sub>A</sub>   | R <sub>S</sub>   | R <sub>S</sub> | R <sub>nl</sub>  | R <sub>N</sub>   | E <sub>P-M</sub> | E <sub>Mak</sub> |
|------|-----|-------|----|----------------|-------|------|----------------|-------|----------------|----------------|------------------|------------------|----------------|------------------|------------------|------------------|------------------|
|      | m/s | Hours | %  | °C             | Hours | -    | kPa            | kPa/K | kPa            | s/m            | W/m <sup>2</sup> | W/m <sup>2</sup> | mm/d           | W/m <sup>2</sup> | W/m <sup>2</sup> | mm/d             | mm/d             |
| 1    | 2.5 | 1.8   | 91 | -1.8           | 8.3   | 0.22 | 0.54           | 0.04  | 0.49           | 83             | 91               | 30               | 1.06           | 22               | 1                | 0.16             | 0.26             |
| 2    | 3.5 | 2.3   | 90 | 2.0            | 8.3   | 0.28 | 0.71           | 0.05  | 0.64           | 59             | 91               | 33               | 1.18           | 26               | 0                | 0.26             | 0.33             |
| 3    | 5.5 | 0.5   | 90 | 5.2            | 8.3   | 0.06 | 0.88           | 0.06  | 0.80           | 38             | 91               | 21               | 0.76           | 11               | 5                | 0.44             | 0.24             |
| 4    | 3.0 | 0.7   | 89 | 4.0            | 9.9   | 0.07 | 0.81           | 0.06  | 0.72           | 69             | 157              | 38               | 1.34           | 12               | 17               | 0.47             | 0.40             |
| 5    | 4.0 | 2.9   | 88 | 3.6            | 9.9   | 0.29 | 0.79           | 0.06  | 0.70           | 52             | 157              | 59               | 2.08           | 27               | 18               | 0.54             | 0.61             |
| 6    | 3.5 | 3.4   | 83 | 3.4            | 9.9   | 0.34 | 0.78           | 0.06  | 0.65           | 59             | 157              | 64               | 2.25           | 31               | 18               | 0.65             | 0.66             |
| 7    | 3.0 | 3.7   | 85 | -2.4           | 11.8  | 0.31 | 0.51           | 0.04  | 0.44           | 69             | 252              | 98               | 3.45           | 29               | 46               | 0.63             | 0.81             |
| 8    | 4.0 | 3.0   | 85 | 5.3            | 11.8  | 0.25 | 0.89           | 0.06  | 0.76           | 52             | 252              | 89               | 3.13           | 25               | 44               | 0.94             | 0.98             |
| 9    | 3.5 | 3.7   | 81 | 5.0            | 11.8  | 0.31 | 0.87           | 0.06  | 0.71           | 59             | 252              | 98               | 3.45           | 29               | 46               | 1.06             | 1.07             |
| 10   | 3.5 | 4.1   | 86 | 6.8            | 13.8  | 0.30 | 0.99           | 0.07  | 0.85           | 59             | 357              | 135              | 4.76           | 27               | 77               | 1.33             | 1.56             |
| 11   | 3.5 | 4.9   | 83 | 7.4            | 13.8  | 0.36 | 1.03           | 0.07  | 0.85           | 59             | 357              | 147              | 5.20           | 31               | 82               | 1.53             | 1.73             |
| 12   | 3.5 | 7.6   | 70 | 8.5            | 13.8  | 0.55 | 1.11           | 0.08  | 0.78           | 59             | 357              | 189              | 6.68           | 46               | 100              | 2.28             | 2.30             |
| 13   | 3.0 | 9.6   | 70 | 11.4           | 15.6  | 0.62 | 1.35           | 0.09  | 0.94           | 69             | 440              | 250              | 8.83           | 50               | 143              | 3.12             | 3.28             |
| 14   | 3.0 | 7.9   | 82 | 14.0           | 15.6  | 0.51 | 1.60           | 0.10  | 1.31           | 69             | 440              | 222              | 7.82           | 39               | 132              | 2.77             | 3.09             |
| 15   | 2.5 | 5.8   | 80 | 12.9           | 15.6  | 0.37 | 1.49           | 0.10  | 1.19           | 83             | 440              | 186              | 6.56           | 31               | 112              | 2.44             | 2.53             |
| 16   | 3.5 | 6.2   | 81 | 15.3           | 16.5  | 0.38 | 1.74           | 0.11  | 1.41           | 59             | 475              | 202              | 7.13           | 30               | 126              | 2.84             | 2.90             |
| 17   | 3.5 | 3.6   | 82 | 12.3           | 16.5  | 0.22 | 1.43           | 0.09  | 1.17           | 59             | 475              | 157              | 5.54           | 21               | 100              | 2.15             | 2.10             |
| 18   | 3.0 | 7.0   | 79 | 14.0           | 16.5  | 0.42 | 1.60           | 0.10  | 1.26           | 69             | 475              | 216              | 7.61           | 34               | 132              | 2.90             | 3.01             |
| 19   | 2.5 | 12.0  | 79 | 18.1           | 16.1  | 0.75 | 2.08           | 0.13  | 1.64           | 83             | 457              | 296              | 10.43          | 51               | 177              | 4.09             | 4.48             |
| 20   | 3.0 | 7.5   | 71 | 16.1           | 16.1  | 0.47 | 1.83           | 0.12  | 1.30           | 69             | 457              | 219              | 7.73           | 37               | 131              | 3.45             | 3.19             |
| 21   | 3.5 | 4.3   | 84 | 17.6           | 16.1  | 0.27 | 2.01           | 0.13  | 1.69           | 59             | 457              | 165              | 5.81           | 22               | 105              | 2.57             | 2.47             |
| 22   | 3.0 | 5.5   | 80 | 17.5           | 14.6  | 0.38 | 2.00           | 0.13  | 1.60           | 69             | 389              | 166              | 5.84           | 29               | 98               | 2.62             | 2.48             |
| 23   | 2.5 | 4.7   | 78 | 17.2           | 14.6  | 0.32 | 1.96           | 0.12  | 1.53           | 83             | 389              | 153              | 5.39           | 26               | 91               | 2.51             | 2.28             |
| 24   | 3.0 | 5.0   | 79 | 16.3           | 14.6  | 0.34 | 1.85           | 0.12  | 1.46           | 69             | 389              | 158              | 5.56           | 28               | 94               | 2.49             | 2.31             |
| 25   | 2.5 | 8.1   | 75 | 15.5           | 12.7  | 0.64 | 1.76           | 0.11  | 1.32           | 83             | 292              | 170              | 6.00           | 48               | 83               | 2.36             | 2.45             |
| 26   | 1.5 | 5.6   | 78 | 13.0           | 12.7  | 0.44 | 1.50           | 0.10  | 1.17           | 139            | 292              | 136              | 4.78           | 36               | 69               | 1.70             | 1.85             |
| 27   | 1.5 | 3.2   | 85 | 12.8           | 12.7  | 0.25 | 1.48           | 0.10  | 1.26           | 139            | 292              | 103              | 3.62           | 23               | 56               | 1.32             | 1.39             |
| 28   | 2.0 | 6.9   | 85 | 11.8           | 10.6  | 0.65 | 1.38           | 0.09  | 1.18           | 104            | 192              | 113              | 4.00           | 48               | 39               | 1.03             | 1.50             |
| 29   | 4.5 | 2.6   | 80 | 10.2           | 10.6  | 0.25 | 1.24           | 0.08  | 1.00           | 46             | 192              | 67               | 2.35           | 24               | 28               | 1.24             | 0.85             |
| 30   | 3.0 | 4.6   | 84 | 9.0            | 10.6  | 0.43 | 1.15           | 0.08  | 0.96           | 69             | 192              | 88               | 3.12           | 36               | 32               | 0.95             | 1.09             |
| 31   | 4.0 | 3.5   | 84 | 9.0            | 8.8   | 0.40 | 1.15           | 0.08  | 0.96           | 52             | 111              | 49               | 1.72           | 33               | 4                | 0.69             | 0.60             |
| 32   | 3.0 | 1.4   | 84 | 5.2            | 8.8   | 0.16 | 0.88           | 0.06  | 0.74           | 69             | 111              | 33               | 1.16           | 18               | 7                | 0.52             | 0.36             |
| 33   | 4.0 | 0.5   | 91 | 4.5            | 8.8   | 0.06 | 0.84           | 0.06  | 0.77           | 52             | 111              | 26               | 0.92           | 11               | 9                | 0.37             | 0.28             |
| 34   | 2.5 | 0.6   | 92 | 4.9            | 7.8   | 0.08 | 0.87           | 0.06  | 0.80           | 83             | 74               | 18               | 0.64           | 12               | 2                | 0.21             | 0.20             |
| 35   | 4.5 | 0.8   | 90 | 6.3            | 7.8   | 0.10 | 0.95           | 0.07  | 0.86           | 46             | 74               | 19               | 0.68           | 14               | 1                | 0.38             | 0.22             |
| 36   | 4.0 | 1.7   | 88 | 4.8            | 7.8   | 0.22 | 0.86           | 0.06  | 0.76           | 52             | 74               | 24               | 0.86           | 22               | -3               | 0.36             | 0.27             |

Table 4.3 Computation of reference evapotranspiration for Lelystad, 1971, using decades

**Algorithm for the computation of the day length  $N$  (hours)**  
**given the *Latitude* in degrees and the number of the day in the year  $DAYN$**

```

A = LATITUDE*PI/180
IF(A<0) B = 0.4014*SIN(2*PI*(DAYN-259)/365)
IF(A>=0) B = 0.4014*SIN(2*PI*(DAYN-77)/365)
X1 = 1-(-SIN(A)/COS(A)*B)^2
X2 = -SIN(A)/COS(A)*SIN(B)/COS(B)
Y1 = ASIN(SQRT(X1/(X1+X2*X2)))
IF(B>0) Y2 = PI-Y1
IF(B<=0) Y2 = Y1
N = 24*Y2/PI

```

**Algorithm for the computation of the extraterrestrial radiation  $R_A$  ( $W.m^{-2}$ )**  
**given the *Latitude* in degrees and the number of the day in the year  $DAYN$**

```

A = LATITUDE*PI/180
X1 = 0.0172*(DAYN-2.5)
X2 = X1 - 1.35512 + 0.0335*SIN(X1) + 0.00035*SIN(2*X1)
X3 = ASIN(0.397949*SIN(X2))
X4 = -TAN(A)*TAN(X3)
IF(ABS(X4)<1.0) X5 = ACOS(X4)
IF(X4>=1.0) X5 = 0.0
IF(X4<=-1.0) X5 = PI
X6 = 1.00028 + 0.0167547*COS(X2+1.35512)
RA=430.673*X6*X6*(X5*SIN(X3)*SIN(A)+COS(A)*COS(X3)*SIN(X5))

```

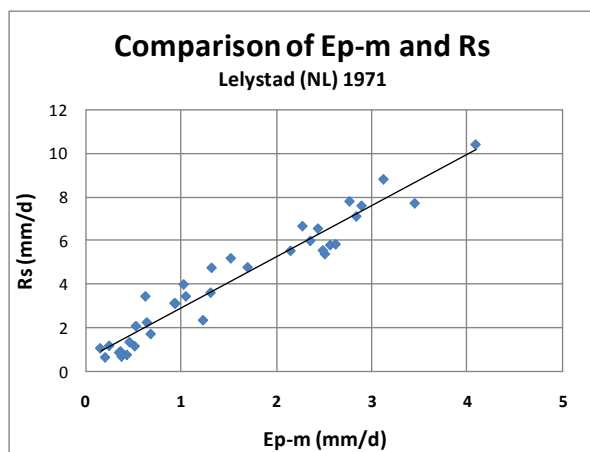


Fig. 4.1 Comparison of evapotranspiration and global radiation

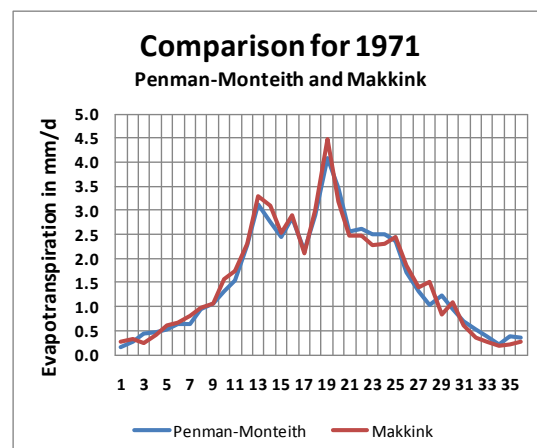


Fig. 4.2 Comparison of potential evapotranspirations

| Decade | EP-M   | EMak   |
|--------|--------|--------|
| days   | mm/dec | mm/dec |
| 10     | 1.6    | 2.6    |
| 10     | 2.6    | 3.3    |
| 11     | 4.8    | 2.6    |
| 10     | 4.7    | 4.0    |
| 10     | 5.4    | 6.1    |
| 8      | 5.2    | 5.3    |
| 10     | 6.3    | 8.1    |
| 10     | 9.4    | 9.8    |
| 11     | 11.6   | 11.7   |
| 10     | 13.3   | 15.6   |
| 10     | 15.3   | 17.3   |
| 10     | 22.8   | 23.0   |
| 10     | 31.2   | 32.8   |
| 10     | 27.7   | 30.9   |
| 11     | 26.8   | 27.8   |
| 10     | 28.4   | 29.0   |
| 10     | 21.5   | 21.0   |
| 10     | 29.0   | 30.1   |
| 10     | 40.9   | 44.8   |
| 10     | 34.5   | 31.9   |
| 11     | 28.2   | 27.2   |
| 10     | 26.2   | 24.8   |
| 10     | 25.1   | 22.8   |
| 11     | 27.4   | 25.4   |
| 10     | 23.6   | 24.5   |
| 10     | 17.0   | 18.5   |
| 10     | 13.2   | 13.9   |
| 10     | 10.3   | 15.0   |
| 10     | 12.4   | 8.5    |
| 11     | 10.4   | 12.0   |
| 10     | 6.9    | 6.0    |
| 10     | 5.2    | 3.6    |
| 10     | 3.7    | 2.8    |
| 10     | 2.1    | 2.0    |
| 10     | 3.8    | 2.2    |
| 11     | 4.0    | 2.9    |
| 365    | 563    | 570    |

Table 4.4 Comparison of  $E_{P-M}$  and  $E_{Makkink}$

## 4.2 Radiation Method

It may be seen from figure 4.1 that the global radiation and the evapotranspiration are closely related. This was reason for Makkink (1957) to propose a simplified equation for the computation of evaporation, based on radiation and temperature data alone. The method is known as the Radiation Method.

The Makkink formula is written as

$$ET_{Makkink} = C C_M \frac{s}{s + \gamma} \frac{R_s}{L} \quad (4.11)$$

where for grass in The Netherlands  $C_M = 0.65$ . Equation (4.11) with  $C_M = 0.80$  may be used to estimate the evaporation of open water.

### Assignment

- Compute the potential evapotranspiration of grass according to the Makkink formula.
- Compare in one figure (see e.g. figure 4.2) the results of the Radiation method with Penman-Monteith.
- Compute for both methods the annual total, similar to the example given in table 4.4. The difference between the totals should not be more than 10 %.
- Carry out a sensitivity analysis by changing the meteorological data by 10 %.
- Save the sheet **Gnum**

**Question:** Show which of the meteorological parameters (data) is most sensitive to the computation of  $E_{P-M}$  and  $E_{Makkink}$ .

| Latitude =<br>1971 | 52  | U     | n  | PI = | T <sub>a</sub> | D <sub>WV</sub> | B   | X1   | X2  | Y1  | Y2  | N     | N     | A =     | n/N  | e <sub>a</sub> | s     | e <sub>d</sub> | f <sub>a</sub> | X1   | X2   | X3   | X4  | X5  | X6    | R <sub>A</sub>   | R <sub>a</sub>   | R <sub>g</sub> | R <sub>h</sub>   | R <sub>h</sub>   | E <sub>pot</sub> | E <sub>max</sub> |
|--------------------|-----|-------|----|------|----------------|-----------------|-----|------|-----|-----|-----|-------|-------|---------|------|----------------|-------|----------------|----------------|------|------|------|-----|-----|-------|------------------|------------------|----------------|------------------|------------------|------------------|------------------|
|                    | m/s | Hours | %  | °C   | DAWN           |                 |     |      |     |     |     | Hours | Hours | 0.90757 |      | kPa            | kPa/K | kPa            | s/m            |      |      |      |     |     |       | W/m <sup>2</sup> | W/m <sup>2</sup> | mm/d           | W/m <sup>2</sup> | W/m <sup>2</sup> | mm/d             | mm/d             |
| 1                  | 2.5 | 1.8   | 91 | -1.8 | 5              | -0.4            | 0.8 | 0.5  | 1.0 | 1.0 | 1.0 | 8.0   | 8.0   | 0.23    | 0.54 | 0.04           | 0.49  | 83             | 0.043          | -1.3 | -0.4 | 0.5  | 1.0 | 1.0 | 78.0  | 78               | 26               | 0.92           | 23               | -2               | 0.13             | 0.22             |
| 2                  | 2.3 | 2.3   | 90 | -2.0 | 15             | -0.4            | 0.8 | 0.5  | 1.1 | 1.1 | 1.1 | 8.3   | 8.3   | 0.28    | 0.71 | 0.05           | 0.64  | 59             | 0.215          | -1.1 | -0.4 | 0.5  | 1.1 | 1.0 | 89.0  | 89               | 33               | 1.15           | 26               | -1               | 0.25             | 0.32             |
| 3                  | 5.5 | 0.5   | 90 | 5.2  | 25             | -0.3            | 0.8 | 0.4  | 1.1 | 1.1 | 1.1 | 8.8   | 8.8   | 0.06    | 0.88 | 0.06           | 0.80  | 38             | 0.387          | -1.0 | -0.3 | 0.4  | 1.1 | 1.0 | 105.5 | 105              | 25               | 0.87           | 11               | 8                | 0.46             | 0.27             |
| 4                  | 3.0 | 0.7   | 89 | 4.0  | 36             | -0.3            | 0.9 | 0.3  | 1.2 | 1.2 | 1.2 | 9.3   | 9.3   | 0.07    | 0.81 | 0.06           | 0.72  | 69             | 0.576          | -0.8 | -0.3 | 0.4  | 1.2 | 1.0 | 129.6 | 130              | 32               | 1.12           | 12               | 12               | 0.41             | 0.34             |
| 5                  | 4.0 | 2.9   | 88 | 3.6  | 46             | -0.2            | 0.9 | 0.3  | 1.3 | 1.3 | 1.3 | 10.0  | 10.0  | 0.29    | 0.79 | 0.06           | 0.70  | 52             | 0.748          | -0.6 | -0.2 | 0.3  | 1.3 | 1.0 | 156.3 | 156              | 59               | 2.07           | 27               | 18               | 0.53             | 0.61             |
| 6                  | 3.5 | 3.4   | 83 | 3.4  | 56             | -0.1            | 1.0 | 0.2  | 1.4 | 1.4 | 1.4 | 10.6  | 10.6  | 0.32    | 0.78 | 0.06           | 0.65  | 59             | 0.920          | -0.4 | -0.2 | 0.2  | 1.4 | 1.0 | 186.9 | 187              | 73               | 2.59           | 29               | 27               | 0.74             | 0.76             |
| 7                  | 3.0 | 3.7   | 85 | -2.4 | 64             | -0.1            | 1.0 | 0.1  | 1.5 | 1.5 | 1.5 | 11.1  | 11.1  | 0.33    | 0.51 | 0.04           | 0.44  | 69             | 1.058          | -0.3 | -0.1 | 0.1  | 1.4 | 1.0 | 213.4 | 213              | 85               | 3.01           | 30               | 35               | 0.55             | 0.71             |
| 8                  | 4.0 | 3.0   | 85 | 5.3  | 74             | 0.0             | 1.0 | 0.0  | 1.5 | 1.5 | 1.5 | 11.8  | 11.8  | 0.25    | 0.89 | 0.06           | 0.76  | 52             | 1.220          | -0.1 | 0.0  | 0.0  | 1.5 | 1.0 | 248.1 | 248              | 87               | 3.08           | 25               | 43               | 0.93             | 0.96             |
| 9                  | 3.5 | 3.7   | 81 | 5.0  | 84             | 0.0             | 1.0 | 0.1  | 1.5 | 1.6 | 1.6 | 12.5  | 12.5  | 0.30    | 0.87 | 0.06           | 0.71  | 59             | 1.402          | 0.1  | 0.0  | 0.0  | 1.6 | 1.0 | 283.6 | 284              | 107              | 3.78           | 28               | 55               | 1.14             | 1.17             |
| 10                 | 3.5 | 4.1   | 86 | 6.8  | 95             | 0.1             | 1.0 | -0.2 | 1.4 | 1.7 | 1.7 | 13.2  | 13.2  | 0.31    | 0.99 | 0.07           | 0.85  | 59             | 1.591          | 0.3  | 0.1  | -0.1 | 1.7 | 1.0 | 322.2 | 322              | 124              | 4.39           | 28               | 68               | 1.23             | 1.44             |
| 11                 | 3.5 | 4.9   | 83 | 7.4  | 105            | 0.2             | 0.9 | -0.2 | 1.3 | 1.8 | 1.8 | 13.9  | 13.9  | 0.35    | 1.03 | 0.07           | 0.85  | 59             | 1.763          | 0.4  | 0.2  | -0.2 | 1.8 | 1.0 | 355.7 | 356              | 147              | 5.17           | 31               | 82               | 1.52             | 1.72             |
| 12                 | 3.5 | 7.6   | 70 | 8.5  | 115            | 0.2             | 0.9 | -0.3 | 1.2 | 1.9 | 1.9 | 14.5  | 14.5  | 0.53    | 1.11 | 0.08           | 0.78  | 59             | 1.935          | 0.6  | 0.2  | -0.3 | 1.9 | 1.0 | 386.7 | 387              | 199              | 7.02           | 44               | 109              | 2.39             | 2.42             |
| 13                 | 3.0 | 9.6   | 70 | 11.4 | 125            | 0.3             | 0.9 | -0.4 | 1.2 | 2.0 | 2.0 | 15.0  | 15.0  | 0.64    | 1.35 | 0.09           | 0.94  | 69             | 2.107          | 0.9  | 0.3  | -0.4 | 2.0 | 1.0 | 414.4 | 414              | 242              | 8.52           | 51               | 135              | 3.01             | 3.16             |
| 14                 | 3.0 | 7.9   | 82 | 14.0 | 135            | 0.3             | 0.8 | -0.4 | 1.1 | 2.0 | 2.0 | 15.5  | 15.5  | 0.51    | 1.60 | 0.10           | 1.31  | 69             | 2.279          | 0.9  | 0.3  | -0.4 | 2.0 | 1.0 | 438.0 | 438              | 221              | 7.80           | 39               | 132              | 2.76             | 3.08             |
| 15                 | 2.5 | 5.8   | 80 | 12.9 | 145            | 0.4             | 0.8 | -0.5 | 1.1 | 2.1 | 2.1 | 15.9  | 15.9  | 0.36    | 1.49 | 0.10           | 1.19  | 83             | 2.451          | 1.1  | 0.4  | -0.5 | 2.1 | 1.0 | 456.7 | 457              | 191              | 6.74           | 31               | 117              | 2.51             | 2.60             |
| 16                 | 3.5 | 6.2   | 81 | 15.3 | 156            | 0.4             | 0.7 | -0.5 | 1.0 | 2.1 | 2.1 | 16.2  | 16.2  | 0.38    | 1.74 | 0.11           | 1.41  | 59             | 2.640          | 1.3  | 0.4  | -0.5 | 2.1 | 1.0 | 470.8 | 471              | 202              | 7.13           | 30               | 125              | 2.84             | 2.90             |
| 17                 | 3.5 | 3.6   | 82 | 12.3 | 166            | 0.4             | 0.7 | -0.5 | 1.0 | 2.1 | 2.1 | 16.3  | 16.3  | 0.22    | 1.43 | 0.09           | 1.17  | 59             | 2.812          | 1.5  | 0.4  | -0.6 | 2.2 | 1.0 | 477.3 | 477              | 159              | 5.60           | 21               | 101              | 2.17             | 2.12             |
| 18                 | 3.0 | 7.0   | 79 | 14.0 | 176            | 0.4             | 0.7 | -0.5 | 1.0 | 2.1 | 2.1 | 16.3  | 16.3  | 0.43    | 1.60 | 0.10           | 1.26  | 69             | 2.984          | 1.6  | 0.4  | -0.6 | 2.2 | 1.0 | 477.5 | 478              | 219              | 7.72           | 34               | 134              | 2.92             | 3.05             |
| 19                 | 2.5 | 12.0  | 79 | 18.1 | 186            | 0.4             | 0.8 | -0.5 | 1.0 | 2.1 | 2.1 | 16.1  | 16.1  | 0.75    | 2.08 | 0.13           | 1.64  | 83             | 3.156          | 1.8  | 0.4  | -0.5 | 2.1 | 1.0 | 471.4 | 471              | 305              | 10.77          | 51               | 184              | 4.22             | 4.62             |
| 20                 | 3.0 | 7.5   | 71 | 16.1 | 196            | 0.4             | 0.8 | -0.5 | 1.1 | 2.1 | 2.1 | 15.8  | 15.8  | 0.48    | 1.83 | 0.12           | 1.30  | 69             | 3.328          | 2.0  | 0.4  | -0.5 | 2.1 | 1.0 | 459.3 | 459              | 223              | 7.87           | 38               | 134              | 3.49             | 3.25             |
| 21                 | 3.5 | 4.3   | 84 | 17.6 | 206            | 0.3             | 0.8 | -0.4 | 1.1 | 2.0 | 2.0 | 15.3  | 15.3  | 0.28    | 2.01 | 0.13           | 1.69  | 59             | 3.500          | 2.1  | 0.3  | -0.5 | 2.0 | 1.0 | 441.7 | 442              | 163              | 5.74           | 23               | 103              | 2.53             | 2.44             |
| 22                 | 3.0 | 5.5   | 80 | 17.5 | 217            | 0.3             | 0.9 | -0.4 | 1.2 | 1.9 | 1.9 | 14.7  | 14.7  | 0.37    | 2.00 | 0.13           | 1.60  | 69             | 3.689          | 2.3  | 0.3  | -0.4 | 2.0 | 1.0 | 416.9 | 417              | 177              | 6.23           | 29               | 107              | 2.77             | 2.65             |
| 23                 | 2.5 | 4.7   | 78 | 17.2 | 227            | 0.2             | 0.9 | -0.3 | 1.3 | 1.9 | 1.9 | 14.1  | 14.1  | 0.33    | 1.96 | 0.12           | 1.53  | 83             | 3.861          | 2.5  | 0.2  | -0.3 | 1.9 | 1.0 | 390.1 | 390              | 156              | 5.90           | 27               | 93               | 2.54             | 2.32             |
| 24                 | 3.0 | 5.0   | 79 | 16.3 | 237            | 0.2             | 1.0 | -0.2 | 1.4 | 1.8 | 1.8 | 13.5  | 13.5  | 0.37    | 1.85 | 0.12           | 1.46  | 69             | 4.033          | 2.7  | 0.2  | -0.2 | 1.8 | 1.0 | 360.2 | 360              | 132              | 5.36           | 30               | 87               | 2.39             | 2.22             |
| 25                 | 2.5 | 8.1   | 75 | 15.5 | 248            | 0.1             | 1.0 | -0.1 | 1.5 | 1.7 | 1.7 | 12.8  | 12.8  | 0.63    | 1.76 | 0.11           | 1.32  | 83             | 4.223          | 2.8  | 0.1  | -0.2 | 1.7 | 1.0 | 324.5 | 324              | 188              | 6.64           | 48               | 97               | 2.60             | 2.71             |
| 26                 | 1.5 | 5.6   | 78 | 13.0 | 258            | 0.0             | 1.0 | 0.0  | 1.6 | 1.6 | 1.6 | 12.1  | 12.1  | 0.46    | 1.50 | 0.10           | 1.17  | 139            | 4.395          | 3.0  | 0.1  | -0.1 | 1.6 | 1.0 | 290.5 | 290              | 139              | 4.89           | 37               | 70               | 1.72             | 1.89             |
| 27                 | 1.5 | 3.2   | 85 | 12.8 | 268            | -0.1            | 1.0 | 0.1  | 1.5 | 1.5 | 1.5 | 11.4  | 11.4  | 0.28    | 1.48 | 0.10           | 1.26  | 139            | 4.567          | 3.2  | 0.0  | 0.0  | 1.6 | 1.0 | 256.0 | 256              | 94               | 3.32           | 25               | 48               | 1.17             | 1.28             |
| 28                 | 2.0 | 6.9   | 85 | 11.8 | 278            | -0.1            | 1.0 | 0.2  | 1.4 | 1.4 | 1.4 | 10.8  | 10.8  | 0.64    | 1.38 | 0.09           | 1.18  | 104            | 4.739          | 3.3  | -0.1 | 0.1  | 1.5 | 1.0 | 221.9 | 222              | 130              | 4.58           | 48               | 52               | 1.24             | 1.72             |
| 29                 | 4.5 | 2.6   | 80 | 10.2 | 288            | -0.2            | 0.9 | 0.2  | 1.3 | 1.2 | 1.2 | 9.5   | 10.1  | 0.26    | 1.24 | 0.08           | 1.00  | 46             | 4.911          | 3.5  | -0.1 | 0.2  | 1.4 | 1.0 | 189.3 | 189              | 67               | 2.37           | 24               | 27               | 1.23             | 0.85             |
| 30                 | 3.0 | 4.6   | 84 | 9.0  | 298            | -0.2            | 0.9 | 0.3  | 1.2 | 1.2 | 1.2 | 9.5   | 9.5   | 0.48    | 1.15 | 0.08           | 0.96  | 69             | 5.083          | 3.7  | -0.2 | 0.3  | 1.3 | 1.0 | 159.2 | 159              | 78               | 2.76           | 39               | 21               | 0.80             | 0.96             |
| 31                 | 4.0 | 3.5   | 84 | 9.0  | 309            | -0.3            | 0.9 | 0.4  | 1.2 | 1.2 | 1.2 | 8.9   | 8.9   | 0.39    | 1.15 | 0.08           | 0.96  | 52             | 5.272          | 3.9  | -0.3 | 0.4  | 1.2 | 1.0 | 130.1 | 130              | 57               | 2.00           | 33               | 11               | 0.76             | 0.70             |
| 32                 | 3.0 | 1.4   | 84 | 5.2  | 319            | -0.3            | 0.8 | 0.5  | 1.1 | 1.1 | 1.1 | 8.4   | 8.4   | 0.17    | 0.88 | 0.06           | 0.74  | 69             | 5.444          | 4.1  | -0.3 | 0.4  | 1.1 | 1.0 | 108.2 | 108              | 32               | 1.14           | 19               | 6                | 0.51             | 0.36             |
| 33                 | 4.0 | 0.5   | 91 | 4.5  | 329            | -0.4            | 0.8 | 0.5  | 1.1 | 1.1 | 1.1 | 8.0   | 8.0   | 0.06    | 0.84 | 0.06           | 0.77  | 52             | 5.616          | 4.2  | -0.4 | 0.5  | 1.1 | 1.0 | 91.2  | 91               | 22               | 0.76           | 11               | 5                | 0.34             | 0.23             |
| 34                 | 2.5 | 0.6   | 92 | 4.9  | 339            | -0.4            | 0.7 | 0.5  | 1.0 | 1.0 | 1.0 | 7.8   | 7.8   | 0.08    | 0.87 | 0.06           | 0.80  | 83             | 5.788          | 4.4  | -0.4 | 0.5  | 1.0 | 1.0 | 79.5  | 79               | 20               | 0.69           | 12               | 3                | 0.22             | 0.21             |
| 35                 | 4.5 | 0.8   | 90 | 6.3  | 349            | -0.4            | 0.7 | 0.5  | 1.0 | 1.0 | 1.0 | 7.7   | 7.7   | 0.10    | 0.95 | 0.07           | 0.86  | 46             | 5.960          | 4.6  | -0.4 | 0.6  | 1.0 | 1.0 | 73.2  | 73               | 19               | 0.68           | 14               | 1                | 0.38             | 0.22             |
| 36                 | 4.0 | 1.7   | 88 | 4.8  | 359            | -0.4            | 0.7 | 0.5  | 1.0 | 1.0 | 1.0 | 7.7   | 7.7   | 0.22    | 0.86 | 0.06           | 0.76  | 52             | 6.132          | 4.8  | -0.4 | 0.6  | 1.0 | 1.0 | 72.3  | 72               | 24               | 0.85           | 22               | -4               | 0.36             | 0.26             |

Table 4.5 Same computation as table 4.3 but using algorithms for computing daylength and extraterrestrial radiation

## 5 COMPOSITION OF A RATING CURVE

A rating curve gives the relation between the discharge and the gauge reading (also referred to as stage or water level reading) in a certain cross section at a fixed geographical location (gauging station). The water level-discharge relation is often approximated with the following formula

$$Q = a (H - H_0)^b \quad (5.1)$$

where

$Q$  discharge in m<sup>3</sup>/s  
 $H$  gauge reading in m  
 $H_0$  gauge reading for zero discharge  
 $a, b$  coefficients

This equation is compatible with the Chézy formula where the cross sectional area  $A$  and the hydraulic radius  $R$  are functions of  $(H - H_0)$ . Assuming a wide river, where approximately  $A = B(H - H_0)$  and  $R = H - H_0$  it can be shown that

$$Q = C B (H - H_0)^{1.5} \sqrt{S} \quad (5.2)$$

where

$C$  Chézy roughness coefficient  
 $S$  bed slope

The coefficient  $b$  has a value of 1.67 in a rectangular channel for which  $B \gg H$ , and a value of 2.67 in a triangular channel. When the coefficients  $a$  and  $b$  are fixed, plotting  $Q$  against  $H - H_0$  produces a straight line on double logarithmic paper as can be seen from the transformed equation of the rating curve

$$\text{Log}(Q) = \text{Log}(a) + b \text{Log}(H - H_0) \quad (5.3)$$

In reality a cross section of a riverbed is a composite of sections. Consequently a rating curve on double logarithmic paper also can be a composite of several straight lines, each with its own values for  $a$  and  $b$ . Often one distinguishes between conditions under normal and bankfull flow.

Once a rating curve is established, regular measurements of water level can be converted to discharges. It should be realized that a rating curve is only valid over the range of discharges or water levels that were used for its establishment. Moreover, a rating curve has to be updated regularly, since sedimentation and scour changes the cross sectional profile and slope of the river.

In this workshop data will be used from the gauging station 'Boane' in the lower part of the Umbeluzi catchment (see figure 1.1). Discharge measurements are available from the period 1951 until 1989, as well as water level readings over the same period. Each participant will establish a rating curve for the year assigned to him/her.

The average bed slope of the river Boane is 0.0005. The cross sectional profile is depicted in figure 5.1.

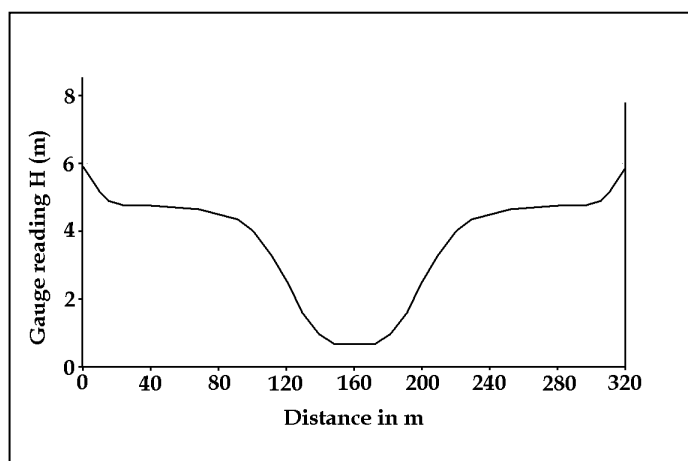
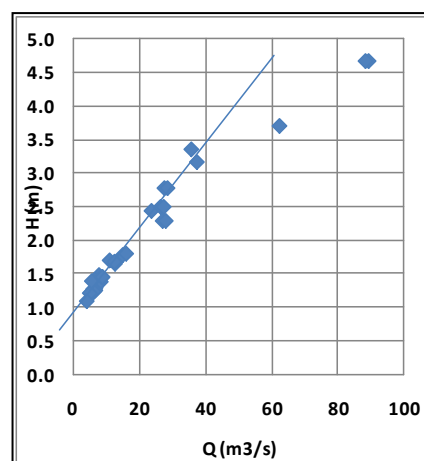


Fig. 5.1 Cross section of the Umbeluzi at Boane

Fig. 5.2 First estimate of  $H_0$ 

| Date     | H    | Q      | LOG(H-H <sub>0</sub> ) | LOG(Q) | LOG(Qcal) | Ho = 0.5 |                         |
|----------|------|--------|------------------------|--------|-----------|----------|-------------------------|
| 10-01-81 | 1.47 | 7.652  | -0.013                 | 0.884  | 0.946     |          |                         |
| 10-02-81 | 4.67 | 89.284 | 0.620                  | 1.951  | 1.917     | Ho       | R Square                |
| 11-02-81 | 4.67 | 88.339 | 0.620                  | 1.946  | 1.917     | 0.9      | 0.9672                  |
| 18-02-81 | 3.35 | 35.599 | 0.455                  | 1.551  | 1.663     | 0.8      | 0.9747                  |
| 17-03-81 | 2.77 | 27.450 | 0.356                  | 1.439  | 1.512     | 0.7      | 0.9778                  |
| 18-03-81 | 2.77 | 28.395 | 0.356                  | 1.453  | 1.512     | 0.6      | 0.9789                  |
| 21-03-81 | 2.43 | 23.536 | 0.286                  | 1.372  | 1.404     | 0.5      | 0.9791                  |
| 27-03-81 | 2.28 | 26.916 | 0.250                  | 1.430  | 1.350     | 0.4      | 0.9787                  |
| 28-03-81 | 2.28 | 27.861 | 0.250                  | 1.445  | 1.350     | 0.3      | 0.9780                  |
| 31-03-81 | 3.16 | 37.279 | 0.425                  | 1.571  | 1.618     | 0.2      | 0.9772                  |
| 03-04-81 | 2.49 | 26.302 | 0.299                  | 1.420  | 1.424     | 0.1      | 0.9762                  |
| 04-04-81 | 2.49 | 27.247 | 0.299                  | 1.435  | 1.424     | 0.0      | 0.9752                  |
| 15-04-81 | 1.69 | 10.865 | 0.076                  | 1.036  | 1.082     |          |                         |
| 18-04-81 | 1.64 | 12.505 | 0.057                  | 1.097  | 1.053     |          |                         |
| 29-04-81 | 1.44 | 8.700  | -0.027                 | 0.940  | 0.925     |          |                         |
| 30-04-81 | 1.43 | 8.383  | -0.032                 | 0.923  | 0.918     |          |                         |
| 05-05-81 | 1.67 | 12.338 | 0.068                  | 1.091  | 1.070     |          | SUMMARY OUTPUT          |
| 15-05-81 | 1.37 | 8.057  | -0.060                 | 0.906  | 0.873     |          |                         |
| 16-05-81 | 1.37 | 8.175  | -0.060                 | 0.912  | 0.873     |          | Regression Statistics   |
| 20-05-81 | 1.77 | 14.857 | 0.104                  | 1.172  | 1.125     |          | Multiple F 0.989478     |
| 21-05-81 | 1.79 | 15.879 | 0.111                  | 1.201  | 1.135     |          | R Square 0.979066       |
| 10-06-81 | 1.40 | 6.539  | -0.046                 | 0.816  | 0.896     |          | Adjusted R 0.978432     |
| 15-06-81 | 1.26 | 6.381  | -0.119                 | 0.805  | 0.783     |          | Standard Error 0.054187 |
| 23-06-81 | 1.28 | 6.617  | -0.108                 | 0.821  | 0.800     |          | Observations 35         |
| 06-07-81 | 1.33 | 7.231  | -0.081                 | 0.859  | 0.842     |          |                         |
| 22-07-81 | 1.26 | 5.881  | -0.119                 | 0.769  | 0.783     |          | ANOVA                   |
| 24-08-81 | 1.22 | 5.344  | -0.143                 | 0.728  | 0.747     |          | df                      |
| 28-08-81 | 1.24 | 6.529  | -0.131                 | 0.815  | 0.765     |          | Regression 1            |
| 07-09-81 | 1.22 | 6.075  | -0.143                 | 0.784  | 0.747     |          | Residual 33             |
| 29-10-81 | 1.20 | 4.853  | -0.155                 | 0.686  | 0.728     |          | Total 34                |
| 11-11-81 | 1.08 | 3.931  | -0.237                 | 0.595  | 0.603     |          |                         |
| 23-11-81 | 1.38 | 5.479  | -0.056                 | 0.739  | 0.881     |          | Coefficients            |
| 27-11-81 | 1.44 | 8.545  | -0.027                 | 0.932  | 0.925     |          | Intercept 0.965892      |
| 03-12-81 | 3.70 | 62.297 | 0.505                  | 1.794  | 1.741     |          | X Variable 1.533624     |
| 15-12-81 | 1.44 | 8.693  | -0.027                 | 0.939  | 0.925     |          |                         |

Table 5.1 Discharge measurements and regression analysis for 1981

### Assignment

Create a table similar to table 5.1 with level and discharge data for the year that is assigned to you.

- Open file *i:\groupwork\hydata\umb\_e8.txt*
- Select cells in the columns A-C for the year assigned to you and copy to clipboard.
- Open New Blank Workbook, Click in cell A4 and paste.
- Save file as **Hnum**.
- Complete table similar to table 5.1.
- Insert a Scatter Chart showing the data of  $Q$  (horizontal axis) in relation to  $H$ .
- Sketch a line through the lower data points and estimate  $H_0$  similar to figure 5.2 (*Insert, Shapes*, click on the *line* icon and draw line). Write the estimate of  $H_0$  in an empty cell.
- Make two columns with values for  $\text{Log}(H-H_0)$  and  $\text{Log}(Q)$ .
- Perform linear regression between  $\text{Log}(H-H_0)$  as the independent X-variable and  $\text{Log}(Q)$  as the dependent Y-variable (Select: *Data, Data Analysis, Regression*, etc.)
- Follow the procedure below to optimize  $H_0$ .

Make a note of the value of the regression coefficient  $R^2$  and repeat the regression for a different value of  $H_0$ . Make a table in the spreadsheet with  $H_0$ -values and the corresponding values for  $R^2$  that are obtained from these regressions. Determine the  $H_0$ -value for which the correlation coefficient is maximum. This is the best estimate of  $H_0$ . Repeat the regression analysis for the best estimate of  $H_0$  and take the regression constant (*Intercept*) as  $\text{Log}(a)$  and the *X-Variable* as parameter  $b$  in the rating curve equation 5.3.

In general there are plenty of low flow measurements available and only few flood events. Since the rating curve is often used for extreme floods or even for extrapolation to events for which no observations exist, as many flood events as possible should be considered in the establishment of the rating curve. The flood events in the period 1951 to 1989 in the Umbeluzi catchment are separately listed in the same file *umb\_e8.txt* (see also table 5.2).

### Assignment

- Copy the list with flood events in file *i:\groupwork\hydata\umb\_e8.txt* to the clipboard and paste it to the bottom of the columns with the data of the year assigned to you in your sheet **Hnum**.
- Extend also the columns with  $\text{Log}(H-H_0)$  and  $\text{Log}(Q)$  values.
- Compute in the next column the  $\text{Log}(Q_{cal})$  according to equation 5.3, where  $\text{Log}(a)$  is the *Intercept* and  $b$  the *X-Variable* as computed with the regression analysis after optimization of  $H_0$ .
- Make a chart similar to figure 5.3.

This graph clearly shows that extreme events do not fit well the flood relation that was derived for the low flows. This does not come as a surprise in view of the shape of the cross section of the river Boane as presented in figure 5.1. The cross section

| DATE       | H    | Q     |
|------------|------|-------|
| 24-2-1972  | 8.29 | 656.3 |
| 11-2-1985  | 7.48 | 633.8 |
| 6-10-1989  | 7.07 | 620.7 |
| 18-2-1975  | 7.57 | 513.4 |
| 11-2-1977  | 7.13 | 486.0 |
| 12-2-1985  | 7.20 | 446.6 |
| 23-3-1972  | 6.48 | 355.9 |
| 22-12-1973 | 6.13 | 328.4 |
| 20-2-1975  | 5.99 | 321.7 |
| 1-2-1979   | 5.75 | 245.7 |
| 1-2-1974   | 5.75 | 244.7 |
| 12-2-1977  | 5.77 | 208.9 |
| 23-12-1973 | 5.76 | 208.7 |
| 7-12-1989  | 5.94 | 208.7 |
| 22-2-1967  | 5.14 | 207.9 |
| 14-12-1989 | 5.60 | 207.9 |
| 22-3-1972  | 5.66 | 207.6 |
| 5-10-1989  | 5.54 | 202.0 |
| 19-2-1975  | 5.38 | 201.8 |
| 4-1-1978   | 5.45 | 174.2 |
| 2-2-1974   | 5.50 | 174.2 |
| 28-2-1967  | 5.48 | 170.8 |
| 1-3-1967   | 5.48 | 169.8 |
| 18-2-1955  | 5.38 | 168.2 |
| 6-2-1955   | 5.48 | 168.1 |
| 7-2-1955   | 5.55 | 162.3 |

**Table 5.2** Recorded flood flows

shows a dramatic change for a value of  $H$  approximately equal to 4.5 m. The rating curve could, therefore best be presented by two straight lines intersecting for  $H = \pm 4.5$  m. Consequently, linear regression is to be performed on two sets of data, with stage values below and above 4.5 m. The procedure is explained below.

### Assignment

- Copy the table with data to another place in the spreadsheet.
- Sort the table for  $H$  in ascending order: highlight with the mouse all columns of copied data, Select: *Data, Sort, Sort by (select column with  $H$ -data), Smallest to largest, OK.*
- Some of these flood events may be from the same date in the year that is assigned to you. Check for double dates and delete rows with flood events that are already listed in the same column.
- Perform a regression analysis for the  $\text{Log}(H - H_0)$  as the independent X-variable and  $\text{Log}(Q)$  as the dependent Y-variable for all values for which  $H < 4.5$ .
- Do the same for the lower part of the table for which  $H \geq 4.5$ .
- The regression analysis results in two sets of parameters for equation 5.3. Compute (by hand) the point of intersection  $\text{Log}(H - H_0)$  of the two linear approximations of the rating curve and add this point to your table so that it can be plotted.
- Extend the table with another column for the computation of  $\text{Log}(Q_{cal})$  using equation 5.3.
- Apply one set of parameter values for the  $H$ -values less than the point of intersection and the other set for the levels larger than the point of intersection.
- Make a chart similar to figure 5.4.
- Save the sheet **Hnum**.

**Question:** Please write in the top left corner of your spreadsheet for which range of discharges your rating curve applies.

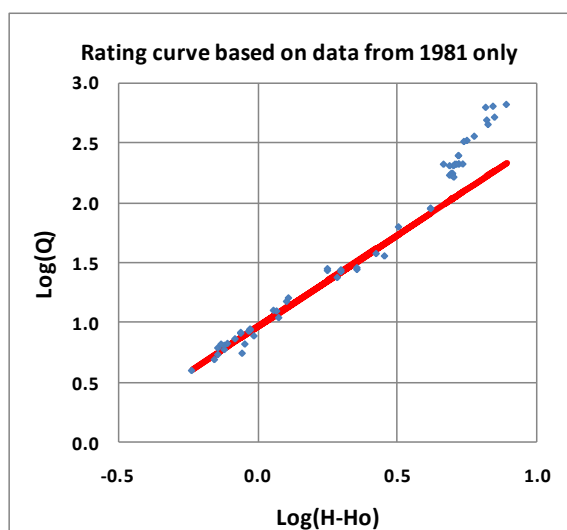


Fig. 5.3 Rating curve represented by a single straight line

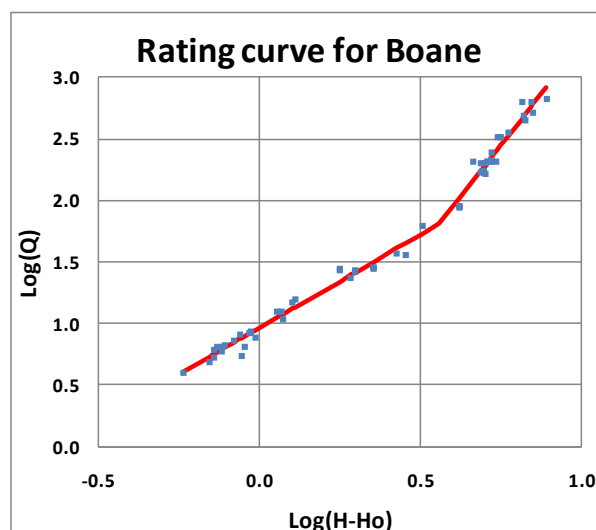


Fig. 5.4 Rating curve represented by two straight lines



## 6. FLOOD ROUTING

### 6.1 Introduction

The shape of a hydrograph changes when it travels along the river from point A to point B (see figure 6.1). If lateral in- and outflows between A and B are negligible, the peak of the hydrograph arriving in B will be lower (*attenuation*) and the time of arrival later (*translation*).

Lateral or tributary flow, however is seldom negligible. The map of the Umbeluzi catchment (figure 1.1) indicates that a recorded flood hydrograph at Goba (E10) when propagating to Boane (E8) is increased by hydrographs resulting from tributaries draining the Mozambican part of the catchment. The hydrographs of tributaries are superimposed on the flood wave coming from Goba resulting in a composite hydrograph in Boane. Consequently peak flow and flood volume in Boane can exceed peak flow and flood volume in Goba. As an example, a flood hydrograph as recorded in January 1971 at Boane (E8) is shown in figure 6.2. On the same time axis the hydrograph as registered at Goba (E10) at the Mozambican border with Swaziland is presented.

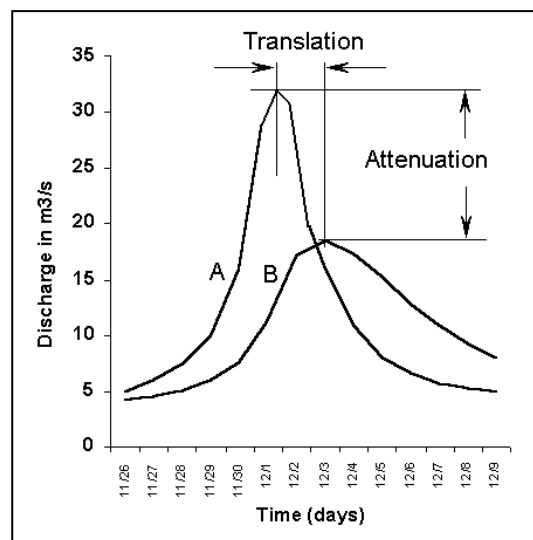


Fig. 6.1 Hydrograph translation and attenuation

In this chapter the movement of a flood wave through a channel reach without lateral in- and outflows will be analyzed, using the Muskingum method for flood routing.

In the next chapter the contribution of the intermediate catchment between Goba and Boane (completely within the Mozambican part of the catchment) is studied with the aid of the unit hydrograph method. Finally the combination of these methods enables us to predict quantitatively flood waves at the location of the river at Boane on the basis of river flow at Goba taking into account the rainfall in the catchment in between these two locations.

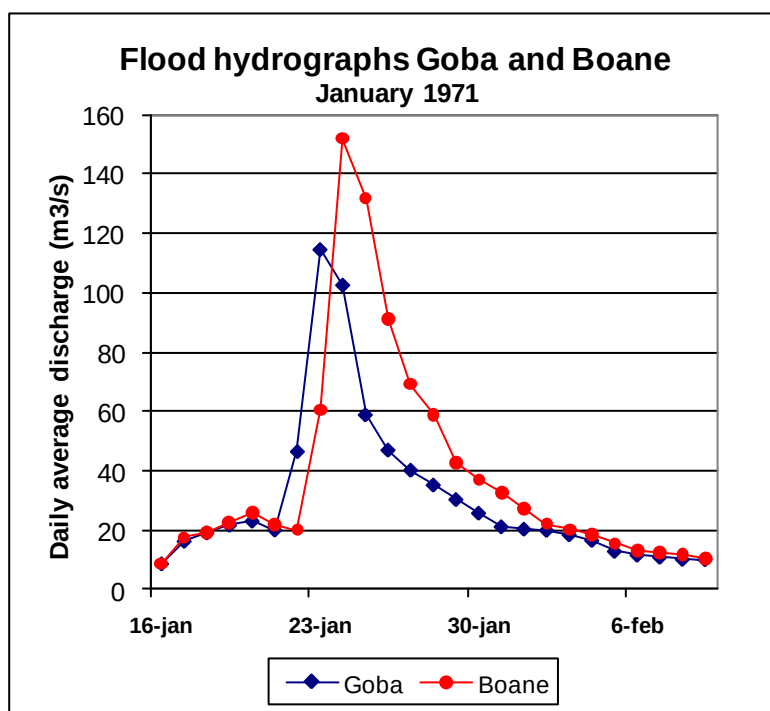


Fig. 6.2 Flood hydrographs at Goba and Boane

## 6.2 Muskingum method

A procedure for the routing of floods which is based on the continuity equation and the assumption of wedge storage in the river reach, is known as the Muskingum method. The storage  $S$  is as follows defined as a function of the inflow  $I$  and the outflow  $Q$  in the considered reach of the river

$$S = K [xI + (1-x)Q] \quad (6.1)$$

In the Muskingum formula the parameter  $x$  is a dimensionless weighing factor indicating the relative importance of  $I$  and  $Q$  in determining the storage in the reach. The value of  $x$  is limited between 0 and 0.5. The parameter  $K$  has the dimension of time. Both  $K$  and  $x$  are constants for a certain river reach. Neglecting lateral inflow,  $K$  and  $x$  can be determined if the input and output hydrographs of the river reach are known.

One method of obtaining values for  $K$  and  $x$  is by plotting values of  $S$  against  $[xI + (1-x)Q]$  for different trial values of  $x$ . Theoretically for a correct value of  $x$  the result will plot a straight line with  $K$  as the tangent as demonstrated in figure 6.3.

Values for the storage  $S$  are derived from continuity. Using the finite difference notation the continuity equation over a time interval  $\Delta t$  from  $t = i\Delta t$  to  $t = (i+1)\Delta t$  is written as

$$\frac{S_{i+1} - S_i}{\Delta t} = \frac{I_{i+1} + I_i}{2} - \frac{Q_{i+1} + Q_i}{2} \quad (6.2)$$

and

$$S_t = S_0 + \sum_{i=1}^t (S_{i+1} - S_i) \quad (6.3)$$

It should be noted that for the solution of  $K$  and  $x$  the initial storage  $S_0$  is irrelevant.

For the purpose of predicting outflow at time  $t = (i+1)\Delta t$ , rearrangement and substitution of the value of  $S$  according to equation 6.2 and the Muskingum equation 6.1 into the continuity equation 6.3 yields the following routing equation

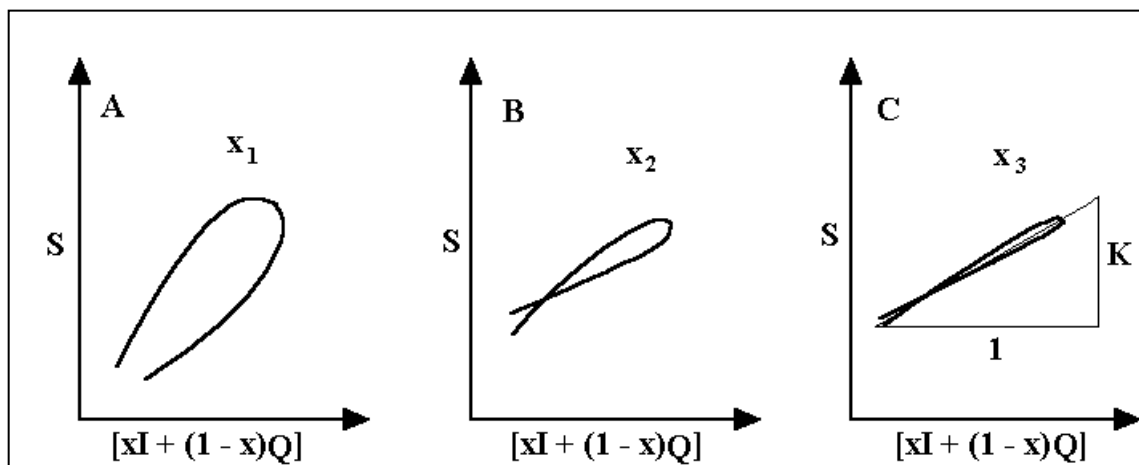


Fig. 6.3 Trial plots for obtaining the Muskingum parameters  $K$  and  $x$

$$Q_{i+1} = c_1 I_i + c_2 I_{i+1} + c_3 Q_i \quad (6.4)$$

where

$$c_1 = \frac{\Delta t + 2Kx}{\Delta t + 2K - 2Kx} \quad (6.5a)$$

$$c_2 = \frac{\Delta t - 2Kx}{\Delta t + 2K - 2Kx} \quad (6.5b)$$

$$c_3 = \frac{-\Delta t + 2K - 2Kx}{\Delta t + 2K - 2Kx} \quad (6.5c)$$

It should be noted that the sum of the parameters  $c_1$ ,  $c_2$  and  $c_3$  equals 1.

Table 6.1 gives the data of a flood recorded in Goba (E10) and Boane (E8) in November/December 1970. The flood occurred in a period that it was very dry in Mozambique. Consequently, the inflow in the river reach between Goba and Boane was negligible. In file *i:\groupwork\hydata\flood.txt* floods for similar situations are found. The flood in table 6.1 is plotted in figure 6.4 and used in this text as an example.

### Assignment

- Open the text file *i:\groupwork\hydata\flood.txt* with MS Excel software, copy the year assigned to you and paste it into an empty worksheet. Save the file with the name **Inum**.
- You may now plot the two hydrographs, select: *Insert, Line, Select Data, Add*, etc. The flood originated from rainfall in Swaziland as no rainfall was recorded in the part of the catchment situated in Mozambique. The data are 24 hour mean values of the discharge specified in m<sup>3</sup>/s.
- Create columns for the computation of
  - $I_{av} = (I_i + I_{i+1})/2$ , the average input over two consecutive days
  - $Q_{av} = (Q_i + Q_{i+1})/2$ , the average output over two consecutive days
  - $\Delta S = I_{av} - Q_{av}$ , the change in storage over two consecutive days
  - $S$ , accumulated storage as from the start of the flood wave at E10. Add a zero value at the top of the column
  - $[xI + (1-x)Q]$ , where the reference for  $x$  is an absolute cell address (set  $x$  equal to 0.2)
- Make a graph similar to figure 6.3 with  $[xI + (1-x)Q]$  on the X-axis and  $S$  on the Y-axis.
- Carry out a regression analysis (*Data, Data Analysis*, etc.) on the last two columns with  $[xI + (1-x)Q]$  as the independent X-value and  $S$  as the dependent Y-value.
- Optimize the parameter  $x$  by repeating the regression analysis for values of  $x$  ranging from 0.2 to 0.5.
- The parameter  $x$  is selected from the regression analysis that has the lowest  $R^2$  value. The parameter  $K$  is found from the same regression analysis as the *X-Variable*.
- Use equations 6.5 to compute the parameters  $c_1$ ,  $c_2$  and  $c_3$ . Take the time step  $\Delta t = 1$  day. Check if the three parameters add up to exactly 1.

| Date      | E10   | E8    |
|-----------|-------|-------|
| 26-Nov-70 | 4.96  | 4.22  |
| 27-Nov-70 | 6.35  | 5.43  |
| 28-Nov-70 | 7.91  | 5.43  |
| 29-Nov-70 | 12.40 | 7.51  |
| 30-Nov-70 | 27.90 | 11.10 |
| 1-Dec-70  | 36.12 | 22.20 |
| 2-Dec-70  | 29.87 | 28.08 |
| 3-Dec-70  | 16.40 | 28.98 |
| 4-Dec-70  | 10.90 | 24.32 |
| 5-Dec-70  | 7.98  | 15.87 |
| 6-Dec-70  | 6.62  | 11.34 |
| 7-Dec-70  | 5.72  | 8.23  |
| 8-Dec-70  | 5.32  | 6.98  |
| 9-Dec-70  | 4.91  | 4.75  |

**Table 6.1 Recorded flood hydrographs**

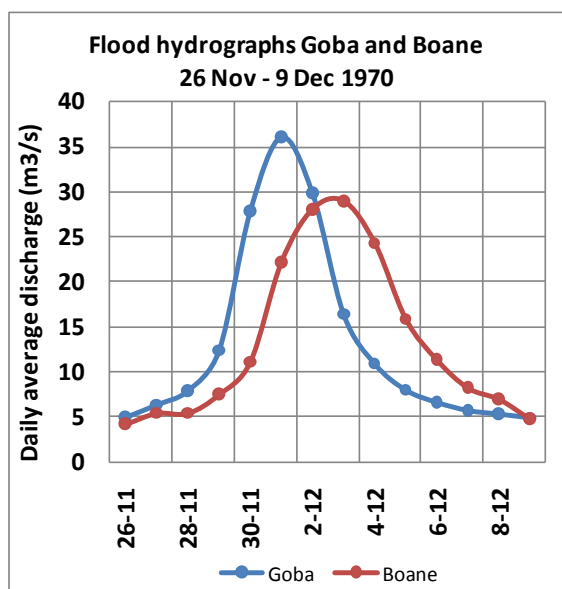


Fig. 6.4 Observed flood hydrographs in Goba and Boane

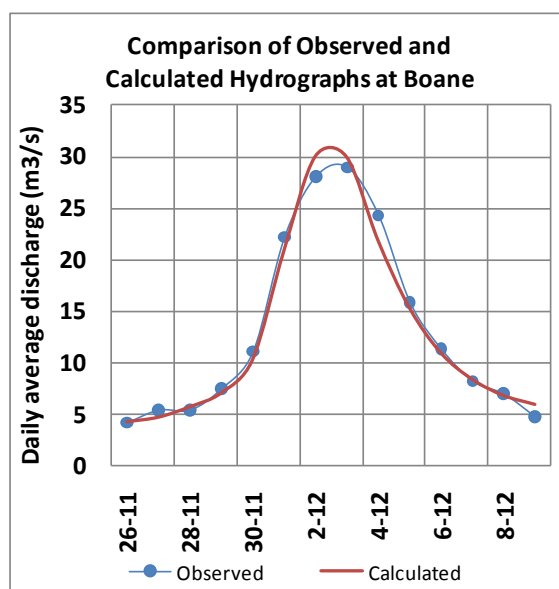


Fig. 6.5 Comparison of observed and calculated flood hydrographs

- Copy the first two columns of the spreadsheet to another location and add one column, using equation 6.4 to calculate the discharge in Boane from observed discharge in Goba. Take the first value equal to the observed discharge.
- Compare in a graph the calculated and observed discharges in Boane similar to figure 6.5 (Use as graph type *Line*).
- Save sheet **Inum**.

**Question:** Please write in the top left corner of your spreadsheet the last value of  $S$  in the unit  $m^3$  and explain why it is not equal to zero.

## 7 RAINFALL RUNOFF MODELLING

### 7.1 Introduction

In this chapter a unit hydrograph will be derived which applies to the part of the Umbeluzi catchment that is situated in Mozambique. A map of the catchment in the Mozambican part is shown in figure 7.1. There are two gauging stations. One station is at the boarder with Swaziland in Goba (station E10), where the river enters Mozambique. In Boane is another gauging station. The procedure for the derivation of the unit hydrograph is the following.

- The Muskingum method will be used to route the discharge at Goba to Boane.
- The hydrograph representing the runoff from the Mozambican part of the catchment is the difference between the measured discharge in Boane and the contribution from Goba as routed with the Muskingum method to Boane.
- The Mozambican hydrograph is subject the further analysis. Base flow and surface runoff will be separated.
- The surface hydrograph will then be used to estimate the effective rainfall.
- Precipitation data from 4 stations are used to determine the areal rainfall.

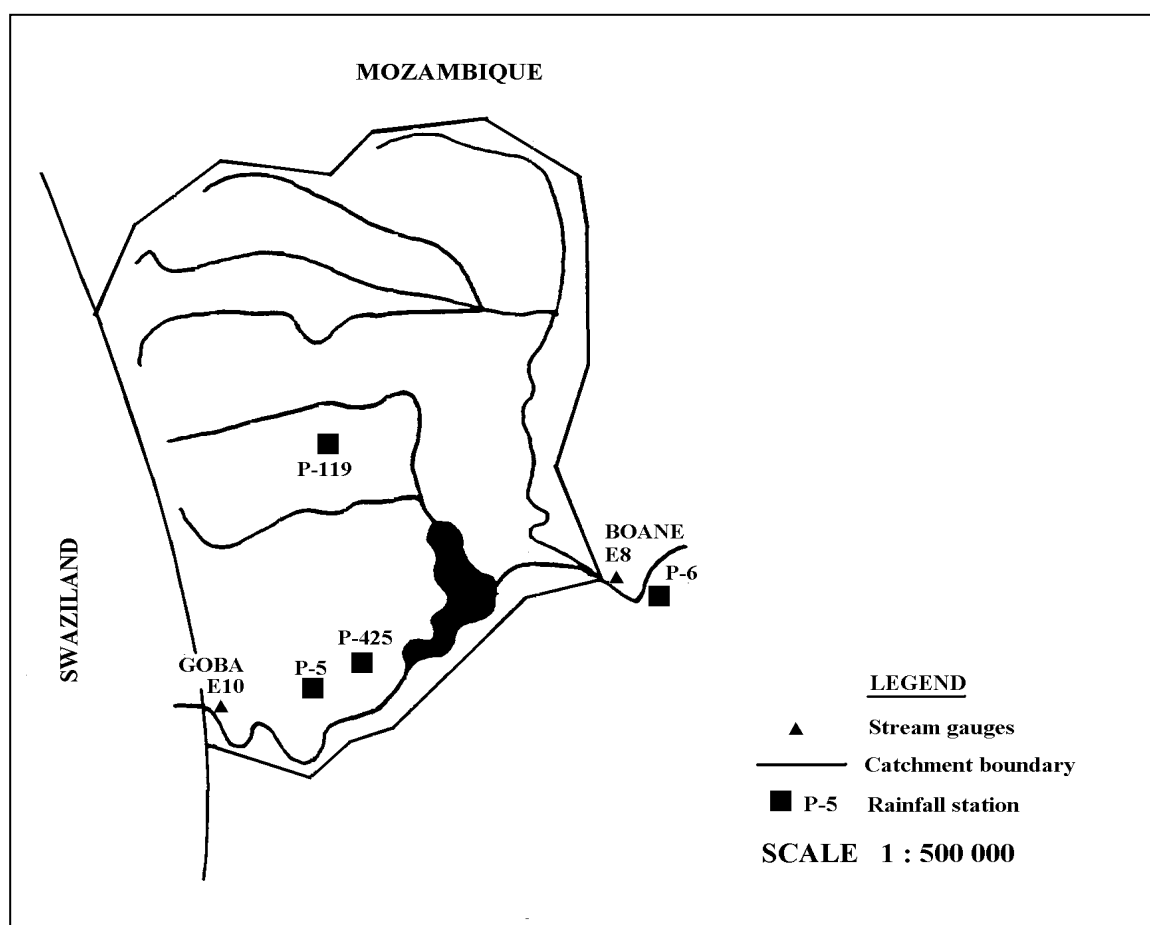


Fig. 7.1 Mozambican part of the Umbeluzi catchment

- The difference between the areal rainfall and the effective rainfall yields the losses from which the  $\Phi$ -index can be determined.
- From a multiple regression analysis the unit hydrograph is finally derived.
- The method will be validated for a storm occurring in a different period.

## 7.2 Flood routing

The observed hydrograph in Boane will be decomposed into two components. Part of the hydrograph consists of discharge entering Mozambique in Goba and the other part originates from the rainfall excess in the Mozambican part of the catchment. The hydrograph observed in Goba (E10) is first routed to Boane (E8) and then subtracted from the hydrograph as observed in E8. Table 7.1 shows the data of the observed hydrographs in Goba and Boane as well as  $Q_{mus}$ , the hydrograph routed with the Muskingum method from Goba to Boane, using  $K = 1.24$  and  $x = 0.40$ . The three hydrographs are plotted in figure 7.2.

Subtracting  $Q_{mus}$  from the observed hydrograph in Boane yields the hydrograph  $Q_{Moz}$ , which represents the runoff from the tributaries draining the Mozambican part of the catchment (see table 7.1 and figure 7.4).

## 7.3 Base flow separation

In order to separate the base flow from the hydrograph  $Q_{Moz}$  it is assumed that the depletion curve may be represented by the following equation

$$Q_t = Q_0 e^{\frac{-(t-t_0)}{K}} \quad (7.1)$$

The parameter  $K$  can be found from a logarithmic plot of the depletion curve (Log  $Q$  against time). The best approach is to find the *average K-value* from the *master depletion curve* which may be derived from a plot of many depletion curves. Since there is only one such curve available for this problem, the  $K$ -value is found from plotting  $Q_{Moz}$  in a semi-logarithmic chart as shown in figure 7.3. Our interest is not so much the  $K$ -value, but the date at which the depletion curve actually starts. Depletion starts after a dry period when the contribution of the fast runoff component has ceased. The depletion curve ends when the next rainstorm causes surface runoff. In this example the rain stops on 26 December

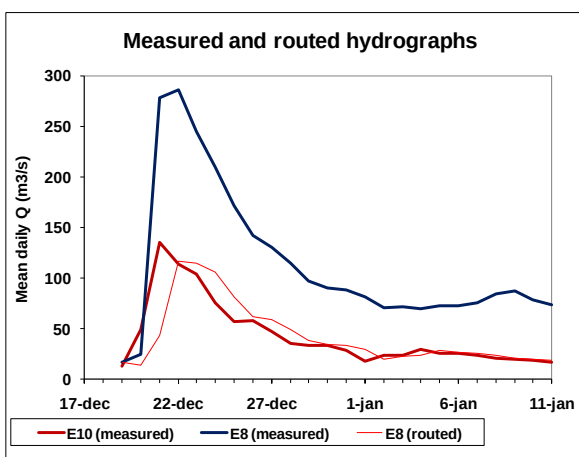


Fig. 7.2 Hydrographs at Goba and Boane

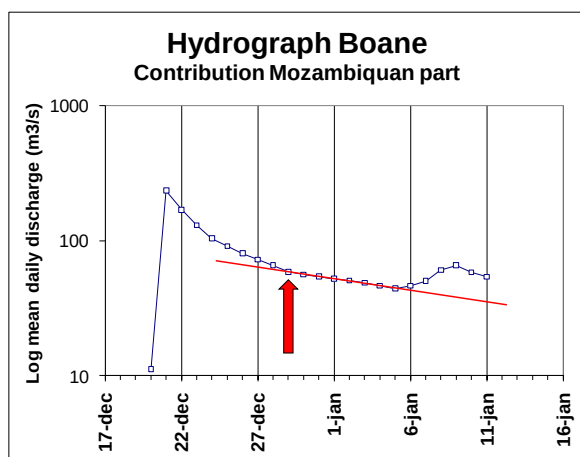


Fig. 7.3 Log plot of the discharge from the Mozambican part and determination of the start of the depletion curve

|           | E10   | E8    | E8    | E8     |    |                                       |         |          |           |
|-----------|-------|-------|-------|--------|----|---------------------------------------|---------|----------|-----------|
|           | GOBA  | BOANE |       |        |    |                                       | Surface | Areal    | Effective |
|           | Qm    | Qm    | Qmusk | Qmoz   |    | Baseflow                              | flow    | rainfall | rainfall  |
|           | m3/s  | m3/s  | m3/s  | m3/s   |    | m3/s                                  | m3/s    | mm       | mm        |
| 19-dec-73 | 12.7  | 17.1  | 17.1  | 0.00   |    | 0.00                                  | 0.00    | 40       | 0         |
| 20-dec-73 | 49.1  | 25.0  | 13.8  | 11.20  | 1  | 5.89                                  | 5.31    | 103      | 56        |
| 21-dec-73 | 135.0 | 278.0 | 42.9  | 235.10 | 2  | 11.78                                 | 223.32  | 62       | 15        |
| 22-dec-73 | 114.0 | 286.5 | 117.2 | 169.30 | 3  | 17.67                                 | 151.63  | 15       | 0         |
| 23-dec-73 | 104.0 | 244.7 | 114.5 | 130.20 | 4  | 23.56                                 | 106.64  | 0        | 0         |
| 24-dec-73 | 75.3  | 209.9 | 105.8 | 104.10 | 5  | 29.45                                 | 74.65   | 5        | 0         |
| 25-dec-73 | 57.5  | 172.0 | 81.1  | 90.90  | 6  | 35.34                                 | 55.56   | 11       | 0         |
| 26-dec-73 | 57.8  | 142.4 | 62.0  | 80.40  | 7  | 41.23                                 | 39.17   | 0        | 0         |
| 27-dec-73 | 47.1  | 130.8 | 58.5  | 72.30  | 8  | 47.12                                 | 25.18   | 0        | 0         |
| 28-dec-73 | 36.0  | 114.8 | 49.2  | 65.60  | 9  | 53.01                                 | 12.59   | 0        | 0         |
| 29-dec-73 | 33.5  | 97.4  | 38.5  | 58.90  | 10 | 58.90                                 | 0.00    | 0        | 0         |
| 30-dec-73 | 33.5  | 90.7  | 34.5  | 56.20  |    |                                       |         |          |           |
| 31-dec-73 | 28.2  | 88.0  | 33.7  | 54.30  |    |                                       |         | 236      | 71        |
| 1-jan-74  | 18.1  | 81.3  | 29.2  | 52.10  |    |                                       |         |          |           |
| 2-jan-74  | 23.5  | 71.0  | 20.3  | 50.70  |    |                                       |         |          |           |
| 3-jan-74  | 23.4  | 71.7  | 22.9  | 48.80  |    | Total =                               | 6.0E+07 | m3       |           |
| 4-jan-74  | 30.1  | 69.6  | 23.3  | 46.30  |    | Total =                               | 71      | mm       |           |
| 5-jan-74  | 25.9  | 73.0  | 28.8  | 44.20  |    |                                       |         |          |           |
| 6-jan-74  | 25.4  | 72.8  | 26.4  | 46.40  |    |                                       |         |          |           |
| 7-jan-74  | 23.4  | 75.8  | 25.6  | 50.20  |    | Constant loss rate ( $\Phi$ -index) = |         |          |           |
| 8-jan-74  | 20.7  | 84.5  | 23.8  | 60.70  |    |                                       |         |          | 47        |
| 9-jan-74  | 19.8  | 87.1  | 21.3  | 65.80  |    |                                       |         |          |           |
| 10-jan-74 | 19.1  | 78.3  | 20.1  | 58.20  |    |                                       |         |          |           |
| 11-jan-74 | 17.3  | 73.3  | 19.3  | 54.00  |    |                                       |         |          |           |

Table 7.1 Derivation effective rainfall (see text)

and starts again on 1 January in Swaziland and 5 January in Mozambique. So the dry period is rather short. The discharge observations in this dry period are used to determine the date,  $t_0$  at which the depletion curve is starting. From the graph (figure 7.3) this appears to be 29 December

1973, since on 28 December and earlier the runoff is larger than is found from the extrapolated linear semi-log plot of the depletion curve. The base flow is now assumed to increase linearly from the start of the storm on 19 December ( $Q_{Moz} = 0 \text{ m}^3/\text{s}$ ) to 29 December when  $Q_{Moz} = 58.90 \text{ m}^3/\text{s}$ . The computation is shown in table 7.1 and the separation line

between base flow and surface runoff is plotted in figure 7.4.

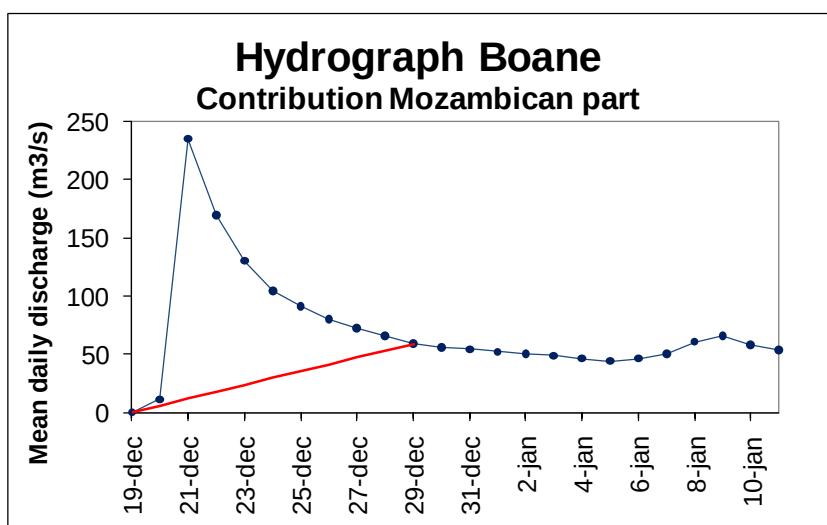


Fig. 7.4 Discharge from the Mozambican part and the base flow separation

## 7.4 Estimating effective precipitation

Subtracting the base flow hydrograph from  $Q_{Moz}$  yields the surface hydrograph of the Mozambican part of the catchment (see table 7.1). The surface runoff in  $m^3$  divided by the catchment area ( $850 km^2$ ) gives the effective precipitation ( $P_e$ ). In this example  $P_e = 71 mm$ .

Part of the areal rainfall  $P$  that does not belong to the fast runoff component (the surface hydrograph) is known as the losses, thus

$$Losses = P - P_e \quad (7.2)$$

where  $P$  is the areal rainfall.

The precipitation that caused the surface hydrograph fell in the period 19 - 25 December 1973. The daily rainfall data in the four gauging stations in the area are presented in table 7.2. For each day an isohyetal map was drawn, taking into consideration the orographic effect related to the mountain range of the Libombos and Pequeno Libombos (see for example figure 7.5). The areal rainfall expressed in millimetres over the total catchment of  $850 km^2$  is then calculated for each day (see table 7.2). The areal rainfall over the entire

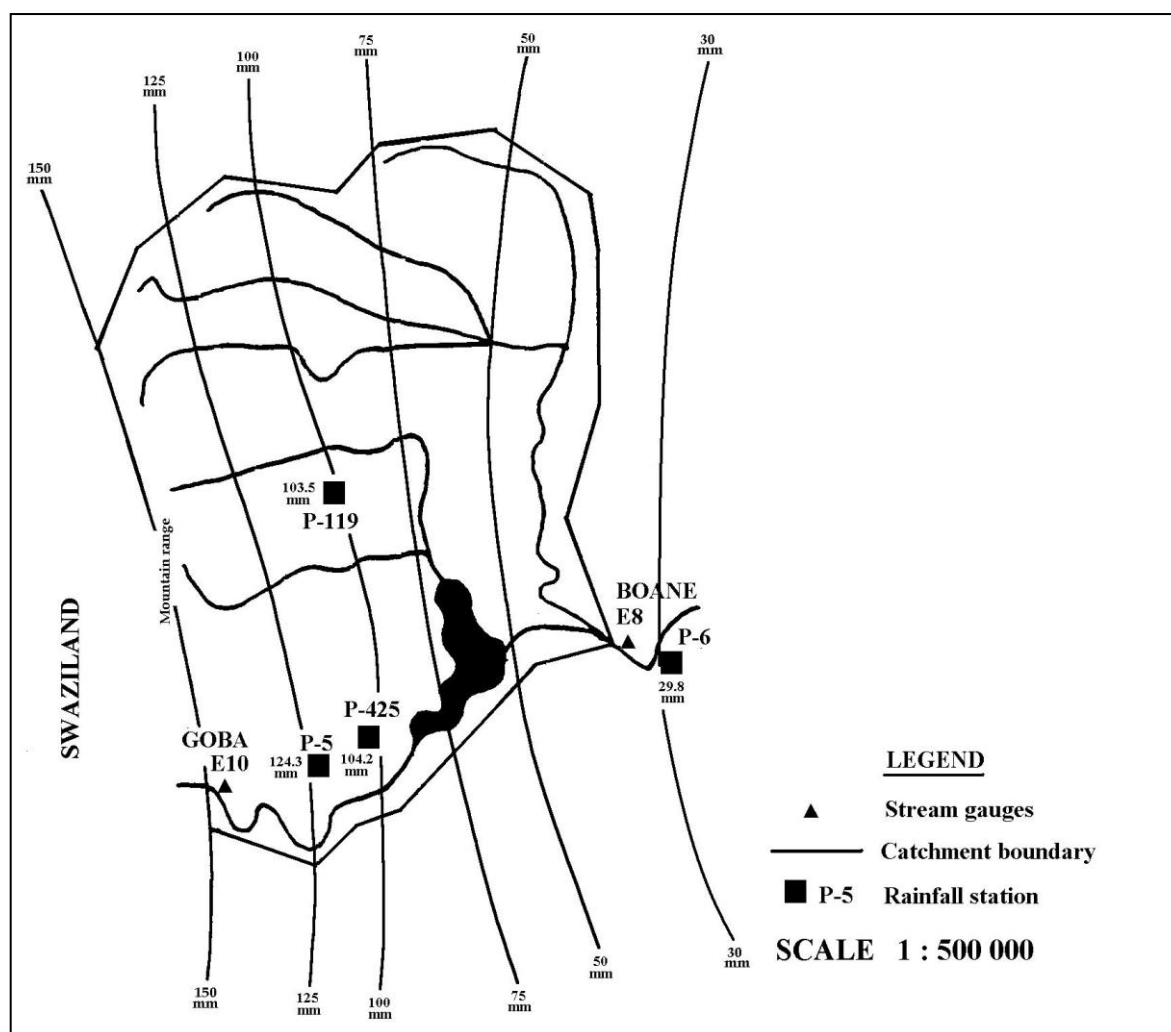


Fig. 7.5 Isohyets Mozambican part of the Umbeluzi catchment for 20 December 1973



period of 7 days is  $P = 236$  mm. The total losses are thus  $236 - 71 = 165$  mm. In this workshop use will be made of the  $\Phi$ -index, which assumes a constant loss rate during the duration of the rainstorm. A first estimate of the  $\Phi$ -index of 45 mm/d brings the total losses to  $2 \times 45 + 40 + 15 + 5 + 11 = 161$  mm. Increasing the  $\Phi$ -index to 47 mm/d yields a loss of exactly 165 mm over the entire storm period. Subtracting the loss from the areal rainfall gives the effective rainfall  $P_e$  as shown in table 7.1.

| Date     | P119  | P5    | P425  | P6   | Areal Rainfall |
|----------|-------|-------|-------|------|----------------|
| 19-12-73 | 52.8  | 40.0  | 0.0   | 48.7 | 40             |
| 20-12-73 | 103.5 | 124.3 | 104.2 | 29.8 | 103            |
| 21-12-73 | 87.6  | 54.0  | 40.6  | 13.5 | 62             |
| 22-12-73 | 22.0  | 0.0   | 6.0   | 3.0  | 15             |
| 23-12-73 | 0.0   | 0.0   | 0.0   | 0.0  | 0              |
| 24-12-73 | 4.5   | 12.0  | 8.1   | 84.0 | 5              |
| 25-12-73 | 17.9  | 9.0   | 5.2   | 11.1 | 11             |
|          |       |       |       |      | 236            |

Table 7.2 Computation areal rainfall Mozambican part of the Umbeluzi catchment

## 7.5 Derivation of the unit hydrograph

The theory of the unit hydrograph was introduced by Sherman in 1932. The method is based on the assumption that the physical characteristics within a river basin (such as slope, size, drainage network, etc.) do not change significantly, and consequently there should be a great similarity in the shape of the hydrographs resulting from similar high intensity rainfalls. The unit hydrograph is defined as the runoff of a catchment to a unit depth of effective rainfall (e.g. 1 mm) falling uniformly in space and time during a period  $T$  (minute, hour, day). It should be noted that the intensity of the rainfall during this period  $T$  is equal to  $1/T$  in order to obtain unit depth. The requirement of an effective precipitation falling uniformly in space limits the application of the unit hydrograph theory to catchments smaller than 500 - 1000 km<sup>2</sup>, since for larger basins the assumption of a uniform distribution of the rainfall is hardly ever valid.

The specific period of time for the excess rainfall  $T$  is known as the 'unit storm period'. For small to medium sized drainage basins there is a certain unit storm period for which the shape of the hydrograph is not significantly affected by changes in the time distribution of the excess rainfall over this unit storm period. This means that equal depths of excess rainfall with different time-intensity patterns produce hydrographs of direct runoff which are the same when the duration of this excess rainfall is equal to or shorter than the unit storm period.

An example of a unit hydrograph is given in figure 7.6, where the effective rainfall,  $P_e$  and the unit hydrograph, DUH (Distribution Unit Hydrograph) are expressed in the same units: mm/d. The unit hydrograph has a length of 4 days. The memory of the rainfall-runoff system is 3 days, since 3 days after the rain has stopped, the last rainfall excess comes to runoff. If the ordinates of the DUH are expressed in the same unit as the rainfall excess, they should sum to one. This will not be so if the ordinates convert unites from e.g. mm/d to m<sup>3</sup>/s.

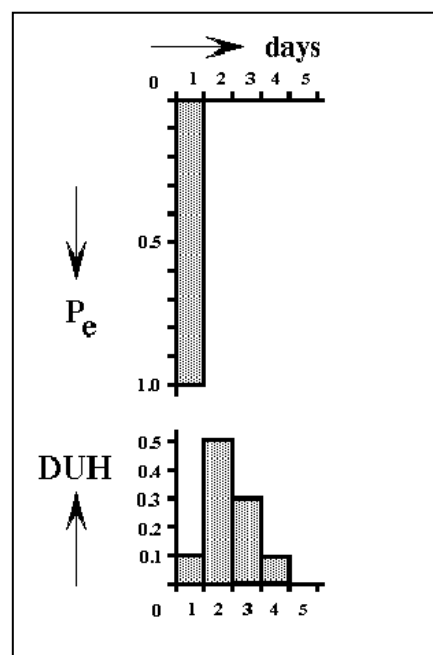


Fig. 7.6 Example of unit hydrograph

The unit hydrograph theory is based on the following assumptions:

- **The rainfall-runoff system is linear.** This means that the duration of the surface runoff is constant for a given unit storm period, and the runoff is proportional to the effective rainfall depth. Thus, for a rainfall intensity twice the unit depth, the ordinates of the unit hydrograph have to be multiplied by two in order to obtain the corresponding surface runoff.
- **The principle of superposition applies.** This is demonstrated with an example in figure 7.7 for a rainstorm that lasts 3 days. The effective rainfall on these three days is 1, 3 and 2 mm, respectively. The rainfall of the first day produces a runoff  $Q_1$  equal to the unit hydrograph (DUH in figure 7.6). The rain of 3 mm on the second day produces a runoff  $Q_2$  starting on the second day and with ordinates three times the unit hydrograph (principle of linearity above). Finally the 2 mm rain on the third day results in a hydrograph  $Q_3$  with ordinates twice as large as the DUH and starting on day 3. The principle of superposition means that the rainfall-runoff relation of each day is independent of events on other days, so that the combined effect of the three day rainstorm may be found by adding the runoff ( $Q_1 + Q_2 + Q_3$ ) produced by each single day as shown on the bottom of figure 7.7. This process of computing the runoff for each time step and the subsequent shifting and adding is known as *convolution*. The process of convolution shown in figure 7.7 is numerically worked out in table 7.3.
- **Time-invariance.** This means that the unit hydrograph does not change with time. So, in summer and winter, dry or wet season, the same direct runoff response to rainfall excess applies.

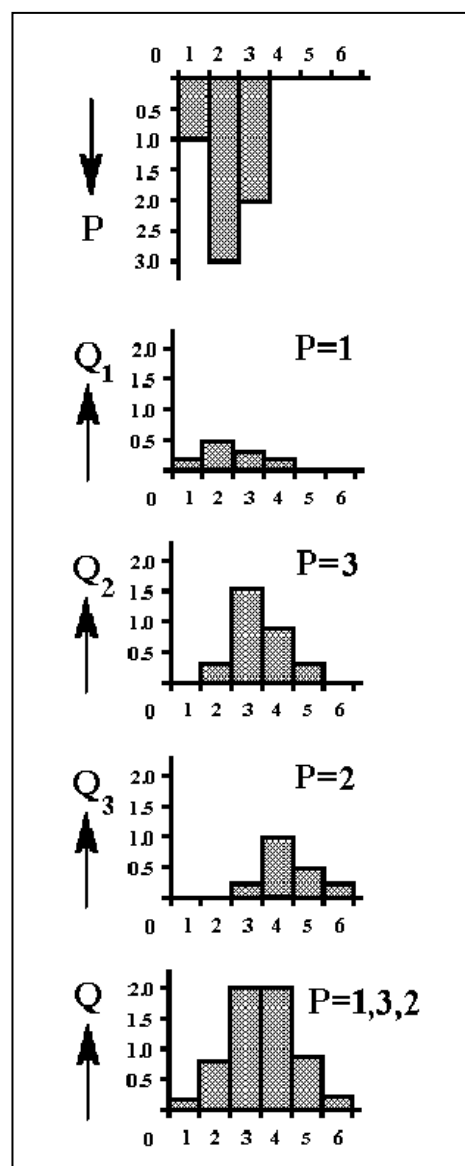


Fig. 7.7 The process of convolution

| Time  | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| DUH   | 0.1 | 0.5 | 0.3 | 0.1 |     |     |     |
| P     | 1   | 3   | 2   |     |     |     |     |
| $Q_1$ | 0.1 | 0.5 | 0.3 | 0.1 |     |     |     |
| $Q_2$ |     | 0.3 | 1.5 | 0.9 | 0.3 |     |     |
| $Q_3$ |     |     | 0.2 | 1.0 | 0.6 | 0.2 |     |
| Q     | 0.1 | 0.8 | 2.0 | 2.0 | 0.9 | 0.2 | 0.0 |

Table 7.3 Numerical example of the convolution procedure

Thus if the rainfall-runoff system may be assumed *linear* and *time-invariant*, the unit hydrograph may be convoluted with the effective rainfall to yield the surface hydrograph, as demonstrated with an example in table 7.3. The convolution procedure is mathematically described below.

Consider a rain storm lasting three 3 time steps (say days) for which the effective rainfall is given by  $P_1, P_2$  and  $P_3$ . The unit hydrograph consists of 4 ordinates,  $U_1, U_2, U_3$  and  $U_4$ . The convolution procedure as explained in table 7.3 may be written as follows.

$$\begin{aligned} Q_1 &= P_1 U_1 \\ Q_2 &= P_2 U_1 + P_1 U_2 \\ Q_3 &= P_3 U_1 + P_2 U_2 + P_1 U_3 \\ Q_4 &= 0 + P_3 U_2 + P_2 U_3 + P_1 U_4 \\ Q_5 &= 0 + 0 + P_3 U_3 + P_2 U_4 \\ Q_6 &= 0 + 0 + 0 + P_3 U_4 \end{aligned}$$

Since  $\sum U_i = 1$  it follows that  $\sum Q_i = \sum P_i$ , thus the total of effective rainfall equals the surface runoff. (The convolution procedure does not account for losses). The set of equations shows that if  $M$  is the total number of rainfall ordinates and  $J$  the length of the unit hydrograph, the total number of runoff ordinates  $N$  is found from

$$N = M + J - 1 \quad (7.3)$$

The general expression for the set of equations may be written as

$$Q_n = \sum_{i=1}^n U_i P_{n-(i-1)} \quad (7.4)$$

where  $U_i = 0$  for  $i > J$  and  $P_i = 0$  for  $i > M$ .

The set of equations may also be written in matrix form

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{pmatrix} = \begin{pmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_1 & 0 & 0 \\ P_3 & P_2 & P_1 & 0 \\ 0 & P_3 & P_2 & P_1 \\ 0 & 0 & P_3 & P_2 \\ 0 & 0 & 0 & P_3 \end{pmatrix} \times \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} \quad (7.5)$$

or

$$Q = P U \quad (7.6)$$

from which  $U$  could be solved as  $Q P^{-1}$ . However, the inverse of matrix  $P$  can only be obtained if  $P$  is a square matrix. Multiplying both sides of equation 7.6 by the transpose  $P^T$  yields a square matrix ( $P^T P$ ) for which the inverse exists. Hence

$$P^T Q = P^T P U \quad (7.7)$$

and the unknown vector  $U$  is found from

$$U = (P^T P)^{-1} P^T Q \quad (7.8)$$

The *matrix inversion method* is one of the methods to solve the unit hydrograph from a set of rainfall-runoff data. Other solutions to this problem are discussed in various text books. In the spreadsheet the matrix inversion method is available in the form of a multiple linear regression. The above problem is considered to consist of 6 linear equations of the type

$$Y = c_1 X_1 + c_2 X_2 + c_3 X_3 + c_4 X_4 \quad (7.9)$$

where the dependent  $Y$ -variable is the discharge, the  $X$ -coefficients the ordinates of the unit hydrograph and the independent  $X$ -variables the precipitation values. The procedure is worked out for the storm derived in table 7.1. The surface hydrograph is reproduced in table 7.4. The units ( $m^3/s$ ) are first changed to  $mm/d$ , so they are compatible to the precipitation units. The precipitation matrix is set up according to the procedure as explained above. The effective rainstorm for this example consists of two rainfall ordinates. The result of the multi-linear regression analysis is also shown in table 7.4. The unit hydrograph (the  $X$ -variables) is plotted in figure 7.6. The shape of the unit hydrograph should show a continuous rising limb and after the peak a continuous decreasing recession curve. It may be seen from figure 7.6 that the unit hydrograph obtained from the analysis has a reasonable shape. Moreover, table 7.4 shows that  $\sum U$  is very close to one ( $\sum U = 0.996$ ). If sufficient data are available it is advisable to repeat the procedure for a number of flood events in order to obtain an 'average' unit hydrograph.

Figure 7.7 finally shows the effective rainfall and the surface runoff from the Mozambican part of the Umbeluzi catchment.

| Qobs<br>m3/s                 | Qobs<br>mm/d | 56 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|------------------------------|--------------|----|----|----|----|----|----|----|----|----|----|
| 223.32                       | 22.70        | 15 | 56 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 151.63                       | 15.41        | 0  | 15 | 56 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 106.64                       | 10.84        | 0  | 0  | 15 | 56 | 0  | 0  | 0  | 0  | 0  | 0  |
| 74.65                        | 7.59         | 0  | 0  | 0  | 15 | 56 | 0  | 0  | 0  | 0  | 0  |
| 55.56                        | 5.65         | 0  | 0  | 0  | 0  | 15 | 56 | 0  | 0  | 0  | 0  |
| 39.17                        | 3.98         | 0  | 0  | 0  | 0  | 0  | 15 | 56 | 0  | 0  | 0  |
| 25.18                        | 2.56         | 0  | 0  | 0  | 0  | 0  | 0  | 15 | 56 | 0  | 0  |
| 12.59                        | 1.28         | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 15 | 56 | 0  |
| 0.00                         | 0.00         | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 15 | 56 |
| <b>SUMMARY OUTPUT</b>        |              |    |    |    |    |    |    |    |    |    |    |
| <b>Regression Statistics</b> |              |    |    |    |    |    |    |    |    |    |    |
| Multiple R                   | 0.9999784    |    |    |    |    |    |    |    |    |    |    |
| R Square                     | 0.9999568    |    |    |    |    |    |    |    |    |    |    |
| Adjusted R Square            | -0.000389    |    |    |    |    |    |    |    |    |    |    |
| Standard Error               | 0.2061633    |    |    |    |    |    |    |    |    |    |    |
| Observations                 | 10           |    |    |    |    |    |    |    |    |    |    |
| <b>Coefficients</b>          |              |    |    |    |    |    |    |    |    |    |    |
| Intercept                    | 0.000        |    |    |    |    |    |    |    |    |    |    |
| X Variable 1                 | 0.010        |    |    |    |    |    |    |    |    |    |    |
| X Variable 2                 | 0.403        |    |    |    |    |    |    |    |    |    |    |
| X Variable 3                 | 0.167        |    |    |    |    |    |    |    |    |    |    |
| X Variable 4                 | 0.149        |    |    |    |    |    |    |    |    |    |    |
| X Variable 5                 | 0.096        |    |    |    |    |    |    |    |    |    |    |
| X Variable 6                 | 0.075        |    |    |    |    |    |    |    |    |    |    |
| X Variable 7                 | 0.051        |    |    |    |    |    |    |    |    |    |    |
| X Variable 8                 | 0.032        |    |    |    |    |    |    |    |    |    |    |
| X Variable 9                 | 0.013        |    |    |    |    |    |    |    |    |    |    |
| Total                        | 0.996        |    |    |    |    |    |    |    |    |    |    |

Table 7.4 Derivation of the unit hydrograph

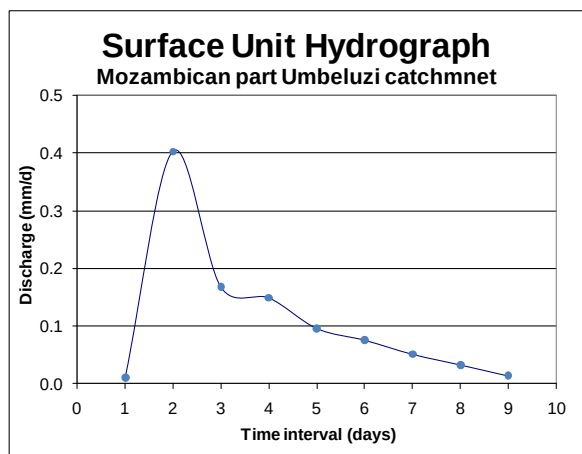


Fig. 7.6 Derived unit hydrograph

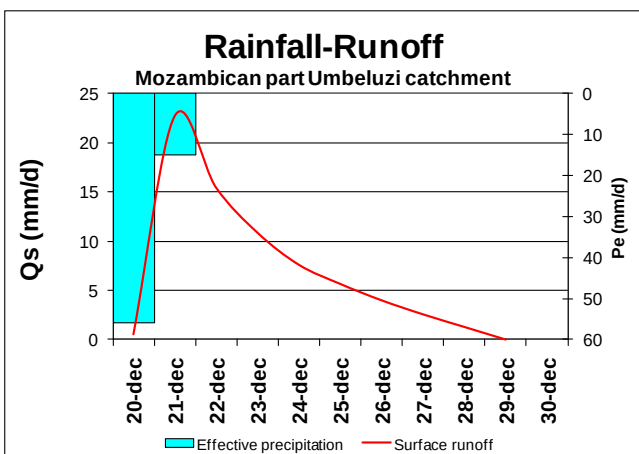


Fig. 7.7 Effective rainfall and surface runoff of the Mozambican part of the Umbeluzi catchment

### Assignment

Derive the surface unit hydrograph for the Mozambican part of the Umbeluzi catchment using data shown in table 7.5. The data may be retrieved from file *i:groupwork\hydata\uh.txt*. Follow the procedure as described above, consisting of the following steps.

- Open file *i:groupwork\hydata\uh.txt* and save it in your own directory as **Jnum**.
- Use the Muskingum method to route the discharge from Goba (E10) to Boane (E8). Use the following parameters:  $K = 1.24$  and  $x = 0.40$ .

| Date      | E10<br>GOBA<br>Qm<br>m3/s | E8<br>BOANE<br>Qm<br>m3/s | Rainfall     |            |              |            |
|-----------|---------------------------|---------------------------|--------------|------------|--------------|------------|
|           |                           |                           | P119<br>(mm) | P5<br>(mm) | P425<br>(mm) | P6<br>(mm) |
| 26-Jan-75 | 20.7                      | 20.60                     | 0.0          | 0.0        | 0.0          | 0.0        |
| 27-Jan-75 | 24.2                      | 20.71                     | 2.2          | 3.8        | 0.0          | 0.0        |
| 28-Jan-75 | 36.0                      | 23.53                     | 3.1          | 10.7       | 4.3          | 0.0        |
| 29-Jan-75 | 256.0                     | 34.27                     | 73.4         | 79.0       | 75.7         | 39.2       |
| 30-Jan-75 | 315.0                     | 239.17                    | 104.5        | 129.3      | 98.1         | 63.0       |
| 31-Jan-75 | 362.0                     | 476.65                    | 53.0         | 62.2       | 43.3         | 20.6       |
| 1-Feb-75  | 477.0                     | 761.36                    | 8.3          | 19.7       | 1.8          | 0.0        |
| 2-Feb-75  | 275.0                     | 742.04                    | 0.0          | 7.2        | 0.0          | 1.0        |
| 3-Feb-75  | 168.0                     | 494.48                    | 0.0          | 0.0        | 3.0          | 2.0        |
| 4-Feb-75  | 394.0                     | 322.91                    | 0.0          | 0.0        | 0.0          | 0.0        |
| 5-Feb-75  | 145.0                     | 446.56                    | 0.0          | 0.0        | 0.0          | 0.0        |
| 6-Feb-75  | 81.4                      | 255.53                    | 0.0          | 0.0        | 0.0          | 0.0        |
| 7-Feb-75  | 61.5                      | 152.89                    | 0.0          | 0.0        | 0.0          | 0.0        |
| 8-Feb-75  | 55.3                      | 115.59                    | 0.0          | 0.0        | 0.0          | 0.0        |
| 9-Feb-75  | 49.8                      | 99.85                     | 0.0          | 0.0        | 0.0          | 0.0        |
| 10-Feb-75 | 48.0                      | 89.24                     | 0.0          | 0.0        | 0.0          | 0.0        |
| 11-Feb-75 | 46.8                      | 82.9                      | 0.0          | 0.0        | 0.0          | 0.0        |
| 12-Feb-75 | 45.9                      | 78.1                      | 0.0          | 0.0        | 0.0          | 0.0        |
| 13-Feb-75 | 44.3                      | 74.2                      | 0.0          | 0.0        | 0.0          | 0.0        |
| 14-Feb-75 | 43.7                      | 70.0                      | 0.0          | 0.0        | 0.0          | 0.0        |
| 15-Feb-75 | 49.3                      | 72.3                      | 8.7          | 9.3        | 7.6          | 2.1        |
| 16-Feb-75 | 56.7                      | 83.3                      | 16.9         | 18.1       | 12.9         | 6.4        |
| 17-Feb-75 | 55.4                      | 87.7                      | 11.1         | 12.6       | 7.3          | 1.9        |

Table 7.5 Rainfall and discharge data Umbeluzi catchment for January/February 1975

- Plot the observed hydrographs in Goba and Boane as well as the routed hydrograph in one figure.
- Subtract the routed hydrograph from the observed hydrograph in Boane.
- Plot the resulting hydrograph  $Q_{Moz}$  in a chart with the vertical axis in a log-scale and determine the start of the depletion curve.
- Compute the base flow as a linear increase of the discharge from the start of  $Q_{Moz}$  to the start of the depletion curve and plot the base flow separation with  $Q_{Moz}$  in a chart (see e.g. figure 7.4).
- Subtract the base flow from  $Q_{Moz}$  to obtain the surface hydrograph from the Mozambican part of the Umbeluzi catchment.
- Draw isohyets on the maps (which are found in the back of these notes) and compute for each day the areal rainfall, using the data in table 7.5.
- Compute the total surface runoff from the Mozambican part of the Umbeluzi catchment in  $m^3$  as well as in mm.
- Make a chart showing the effective precipitation and the surface runoff in mm/d. Compute the  $\Phi$ -index and determine the effective rainfall in the Mozambican part of the Umbeluzi catchment.

Derivation of the unit hydrograph:

- Construct a matrix with effective precipitation values similar to table 7.4.
- Find the unit hydrograph from a multiple-linear regression on the surface runoff (values in mm) as the Y-dependent variable and the effective precipitation as the X-independent variable. Put the Y-intercept to zero.
- Check if the sum of the ordinates of the unit hydrograph sum to approximately one.
- Make a chart showing the unit hydrograph.

Validation procedure:

- Use the validation data, which apply for the period 14-25 Dec 1975.
- Use Muskingum to route the discharge from Goba to Boane.
- Convolute effective areal rainfall with the derived unit hydrograph (equation 7.5).
- Add the computed surface runoff from Mozambique to the discharge from Swaziland.
- Compare the observed and computed hydrograph in a chart. See e.g. figure 7.8.
- Save the sheet as **Jnum**.

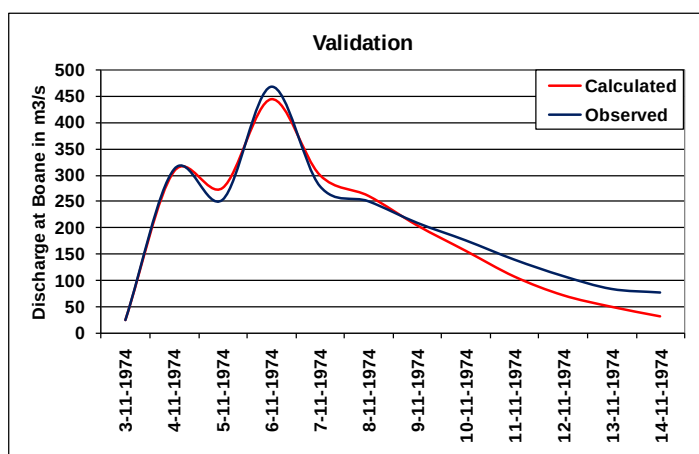


Fig. 7.8 Example validation

**Question:** Please write in the top left corner of your spreadsheet your opinion on the “goodness of fit”, in particular with regard to the recession curve.

## 8 RESERVOIR OPERATION

### 8.1 Introduction

The spreadsheet exercises presented below are developed to illustrate the role of the mass balance calculations in the design and operation of a reservoir. They aim to give insight in the determination of the reservoir capacity, the dimension of the spillway and the use of rule curves. It should be realized that reservoir design and operation is a rather complex matter which cannot be dealt with in a few exercises. This applies in particular if the reservoir is to serve more than one purpose and if the socio-economical aspects are taken into consideration.

The exercises make use of a long time series (1920 - 1980) of monthly rainfall and runoff data. The data refer to the Jatiluhur reservoir in the Citarum river basin on Java (Indonesia). Part of the data (in particular the rainfall data) is synthetic. The size of the catchment upstream the dam site is 4550 km<sup>2</sup>. The rainfall in the catchment varies from 1500 to 3500 mm/a, with a monthly average at the reservoir site of 187 mm. The average monthly runoff in Jatiluhur varies from 6 to 610 m<sup>3</sup>/s with a long-term average of 183 m<sup>3</sup>/s.

### 8.2 Determination of the reservoir capacity

There are two basic methods to determine the capacity of a reservoir: the graphical approach (Rippl method) and the numerical approach (Sequent Peak Algorithm). Both methods make use of the critical period, which is the period when the outflow (demand, yield or draft) from the reservoir is larger than the inflow. The difference in the accumulated draft and the accumulated inflow during the critical period is the storage that is required to supply the requested draft in the critical period (or to assure safe yield). If the considered time includes more than one critical period, the largest reservoir storage is taken as the reservoir capacity. It is obvious that if the computation of the reservoir capacity is based on one year of data, it may not be representative, since the considered year may be more dry or more wet than normal. A long time series, say 20 years or more is, therefore, recommended as the design period. In this assignment only one year is considered in order to reduce the computational effort.

#### ***The Graphical Approach (Rippl method)***

The graphical approach can only be applied to a constant draft from the reservoir. The method requires that the inflow is accumulated and plotted, which is usually done on a monthly basis. The required storage is then obtained by drawing tangents forward in time from the start of the critical period and from the end of the critical period. The critical period includes those months during which the inflow is less than the draft. Table 8.1 gives an example for the Jatiluhur reservoir for the year 1965. It may be noticed that the computations are carried out for 24 months. The series for 1965 is repeated, because the critical period may not be terminated before the end of the year. The incoming flow is first converted from m<sup>3</sup>/s to MCM (10<sup>6</sup> m<sup>3</sup>) per month, where a month is taken to have a length of 30.4 days. The draft is taken constant and equal to the average inflow, which is known as the *ideal reservoir* case. The example shows that from May onwards the draft is larger than the inflow. From that time the reservoir is depleting to reach a minimum level in November after which the inflow is larger than the draft. The difference between the cumulative inflow in November and the tangent to the cumulative inflow in April is the required storage to sustain the constant draft. The computed mass curves and tangents are plotted in figure 8.1.

| Draft D = Qav "Ideal Reservoir case" |            |           |               |                |                |                |
|--------------------------------------|------------|-----------|---------------|----------------|----------------|----------------|
| Month                                | Qin (m3/s) | Qin (MCM) | cum Qin (MCM) | cum Qout (MCM) | cum Qout (MCM) | Capacity (MCM) |
| 1965                                 |            |           | 0             |                |                |                |
| Jan                                  | 457        | 1200      | 1200          |                |                |                |
| Feb                                  | 466        | 1224      | 2424          |                |                |                |
| Mar                                  | 186        | 489       | 2913          |                |                |                |
| Apr                                  | 171        | 449       | 3362          | 3362           |                |                |
| May                                  | 121        | 318       | 3680          | 3779           |                |                |
| Jun                                  | 55         | 144       | 3824          | 4196           |                |                |
| Jul                                  | 38         | 100       | 3924          | 4614           |                |                |
| Aug                                  | 15         | 39        | 3963          | 5031           |                |                |
| Sep                                  | 9          | 24        | 3987          | 5448           |                |                |
| Oct                                  | 15         | 39        | 4027          | 5865           |                |                |
| Nov                                  | 75         | 197       | 4224          | 6282           | 4224           | 2059           |
| Dec                                  | 298        | 783       | 5006          | 6699           | 4641           | 2059           |
| Jan                                  | 457        | 1200      | 6207          |                | 5058           |                |
| Feb                                  | 466        | 1224      | 7431          |                | 5475           |                |
| Mar                                  | 186        | 489       | 7919          |                | 5892           |                |
| Apr                                  | 171        | 449       | 8368          | 8368           |                |                |
| May                                  | 121        | 318       | 8686          | 8785           |                |                |
| Jun                                  | 55         | 144       | 8830          | 9203           |                |                |
| Jul                                  | 38         | 100       | 8930          | 9620           |                |                |
| Aug                                  | 15         | 39        | 8970          | 10037          |                |                |
| Sep                                  | 9          | 24        | 8993          | 10454          |                |                |
| Oct                                  | 15         | 39        | 9033          | 10871          |                |                |
| Nov                                  | 75         | 197       | 9230          | 11289          | 9230           | 2059           |
| Dec                                  | 298        | 783       | 10012         | 11706          | 9647           | 2059           |
| Qav=                                 | 159        | 417       |               |                |                |                |

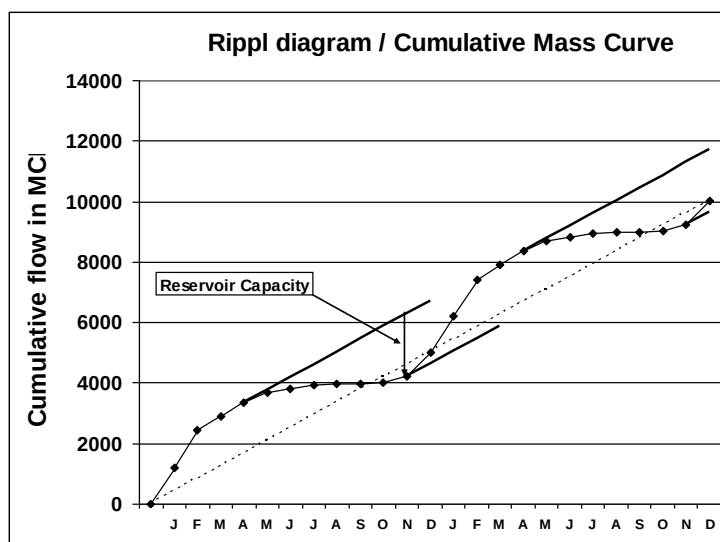


Table 8.1 The graphical approach (Rippl diagram) Ideal Reservoir Case

Fig. 8.1 The Ideal Reservoir Case: Example of the graphical approach (Rippl diagram)

The draft is generally taken smaller than the average inflow in order to reduce the height and thus the cost of the dam. If the draft is taken equal to 2/3 of the average inflow the required reservoir capacity reduces to approximately half of the value that was computed for the *ideal reservoir case* (see table 8.2 and figure 8.2). As a consequence 1/3 of the inflow leaves the reservoir through the spillway and is lost for e.g. energy production.

### The Numerical Approach (Sequent Peak Algorithm)

The numerical approach is particularly suitable for a draft which is not constant in time. The procedure computes for each month  $t$  the storage deficit  $K$  in the reservoir as follows

$$K_t = K_{t-1} + D_t - Q_t \quad K_t > 0$$

$$K_t = 0 \quad \text{otherwise} \quad (8.1)$$

where  $D$  is the draft and  $Q$  is the inflow. To illustrate the flexibility of the method, the previous set of data is used with a variable demand as follows:

| Constant Draft D = 2/3*Qav |            |           |               |                |                |                          |
|----------------------------|------------|-----------|---------------|----------------|----------------|--------------------------|
| Month                      | Qin (m3/s) | Qin (MCM) | cum Qin (MCM) | cum Qout (MCM) | cum Qout (MCM) | Reservoir Capacity (MCM) |
| 1965                       |            |           | 0             |                |                |                          |
| Jan                        | 457        | 1200      | 1200          |                |                |                          |
| Feb                        | 466        | 1224      | 2424          |                |                |                          |
| Mar                        | 186        | 489       | 2913          |                |                |                          |
| Apr                        | 171        | 449       | 3362          |                |                |                          |
| May                        | 121        | 318       | 3680          | 3680           |                |                          |
| Jun                        | 55         | 144       | 3824          | 3958           |                |                          |
| Jul                        | 38         | 100       | 3924          | 4236           |                |                          |
| Aug                        | 15         | 39        | 3963          | 4514           |                |                          |
| Sep                        | 9          | 24        | 3987          | 4792           |                |                          |
| Oct                        | 15         | 39        | 4027          | 5070           |                |                          |
| Nov                        | 75         | 197       | 4224          | 5349           | 4224           | 1125                     |
| Dec                        | 298        | 783       | 5006          | 5627           | 4502           |                          |
| Jan                        | 457        | 1200      | 6207          | 5905           | 4780           |                          |
| Feb                        | 466        | 1224      | 7431          | 6183           | 5058           |                          |
| Mar                        | 186        | 489       | 7919          |                | 5336           |                          |
| Apr                        | 171        | 449       | 8368          |                | 5614           |                          |
| May                        | 121        | 318       | 8686          | 8686           |                |                          |
| Jun                        | 55         | 144       | 8830          | 8964           |                |                          |
| Jul                        | 38         | 100       | 8930          | 9242           |                |                          |
| Aug                        | 15         | 39        | 8970          | 9520           |                |                          |
| Sep                        | 9          | 24        | 8993          | 9799           |                |                          |
| Oct                        | 15         | 39        | 9033          | 10077          |                |                          |
| Nov                        | 75         | 197       | 9230          | 10355          | 9230           | 1125                     |
| Dec                        | 298        | 783       | 10012         | 10633          | 9508           |                          |
| Qav=                       | 159        | 417       |               |                |                |                          |
| 2/3*Qav                    |            | 278       |               |                |                |                          |

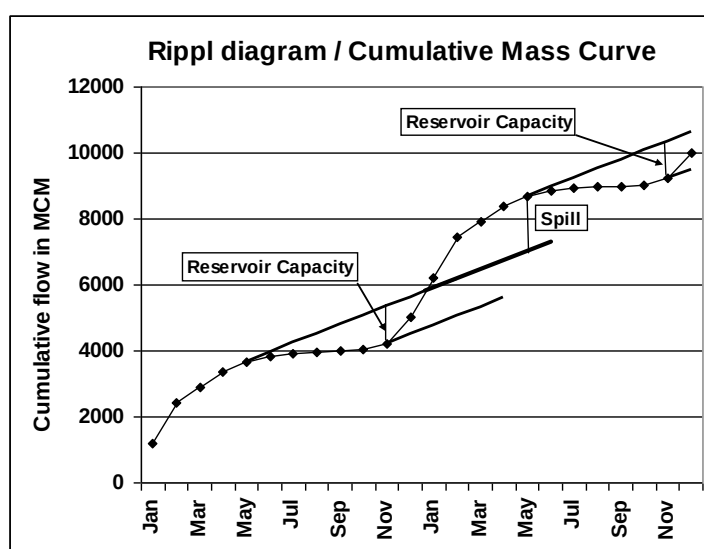


Table 8.2 Rippl method with D = 2/3

Fig. 8.2 Graphical Approach, Rippl method with D = 2/3



for the first eight months the draft equals 0.5  $Q_{av}$  and in the last four months  $D = 2 Q_{av}$ . It should be noted that the average annual draft equals  $2/3 Q_{av}$ , similar as in the previous example.

### Assignment

Use both methods (Graphical Approach/Rippl Diagram and the Sequent Peak Algorithm) to determine the minimum reservoir capacity for the year with inflow data that has been assigned to you. The Graphical Approach /Rippl Diagram will be used for a constant draft  $D$  equal to the average inflow  $Q_{av}$  and subsequently for a draft  $D = 2/3 Q_{av}$ . The Sequent Peak Algorithm will be applied for a variable draft, such that  $D_{av} = 2/3 Q_{av}$ . The inflow data are found in file *i:\groupwork\hydata\jatiluhur.txt*

- Copy the inflow data of only the first year assigned to you into a new spreadsheet.
- Copy and paste these data below, to yield a time series of 24 months.
- Complete a table similar to table 8.1.
- Make a chart with the cumulative mass curves and the tangents (see figure 8.1).
- Repeat the Cumulative Mass Approach for a draft equal to  $2/3$  of the average inflow (see table 8.2 and figure 8.2).
- Notice the large difference in reservoir capacity for a relatively small decrease in demand.
- Make a third table similar to table 8.3 for the Sequent Peak Algorithm.
- Use a variable demand as follows: the first 8 months  $D = 1/3 Q_{av}$  and the last four months  $D = 4/3 Q_{av}$ .
- Compute the reservoir capacity and compare with the Rippl method.
- Save the worksheet as **Knum**.

| Month      | $Q_{in}$<br>(m <sup>3</sup> /s) | $Q_{in}$<br>(MCM) | D<br>(MCM) | K<br>(MCM)          |
|------------|---------------------------------|-------------------|------------|---------------------|
| 1965       |                                 |                   |            | 0                   |
| Jan        | 457                             | 1200              | 139        | 0                   |
| Feb        | 466                             | 1224              | 139        | 0                   |
| Mar        | 186                             | 489               | 139        | 0                   |
| Apr        | 171                             | 449               | 139        | 0                   |
| May        | 121                             | 318               | 139        | 0                   |
| Jun        | 55                              | 144               | 139        | 0                   |
| Jul        | 38                              | 100               | 139        | 39                  |
| Aug        | 15                              | 39                | 139        | 139                 |
| Sep        | 9                               | 24                | 556        | 672                 |
| Oct        | 15                              | 39                | 556        | 1188                |
| Nov        | 75                              | 197               | 556        | <b>1548 Maximum</b> |
| Dec        | 298                             | 783               | 556        | 1321                |
| Jan        | 457                             | 1200              | 139        | 260                 |
| Feb        | 466                             | 1224              | 139        | 0                   |
| Mar        | 186                             | 489               | 139        | 0                   |
| Apr        | 171                             | 449               | 139        | 0                   |
| May        | 121                             | 318               | 139        | 0                   |
| Jun        | 55                              | 144               | 139        | 0                   |
| Jul        | 38                              | 100               | 139        | 39                  |
| Aug        | 15                              | 39                | 139        | 139                 |
| Sep        | 9                               | 24                | 556        | 672                 |
| Oct        | 15                              | 39                | 556        | 1188                |
| Nov        | 75                              | 197               | 556        | <b>1548 Maximum</b> |
| Dec        | 298                             | 783               | 556        | 1321                |
| $Q_{av} =$ | 159                             | 417               | 278        |                     |

**Table 8.3 Example Sequent Peak Algorithm**

## 8.3 Reservoir simulation

When simulating the water balance of a reservoir, the most important components are the inflow and the draft, as used in the previous exercise. If the storage capacity of the reservoir is fixed, the reservoir may run dry or become full and start to spill. Taking these aspects into account the water balance may be written as follows:

$$S_t = S_{t-1} + (Q - Q_{leak} - D - Sp + Sh) \Delta t \quad (8.2)$$

|            |                                                                               |
|------------|-------------------------------------------------------------------------------|
| $S_{t-1}$  | Storage in the reservoir at the beginning of the time step (m <sup>3</sup> )  |
| $S_t$      | Storage in the reservoir at the end of the time step (m <sup>3</sup> )        |
| $Q$        | Inflow into the reservoir during the time step (m <sup>3</sup> /time)         |
| $Q_{leak}$ | Leakage underneath the dam site (m <sup>3</sup> /time)                        |
| $D$        | Target release from the reservoir during the time step (m <sup>3</sup> /time) |
| $Sp$       | Spill from reservoir during the time step (m <sup>3</sup> /time)              |
| $Sh$       | Shortage during the time step (m <sup>3</sup> /time)                          |
| $\Delta t$ | Length of time step (time)                                                    |

Equation 8.2 is first applied with  $D$  taken equal to the target release. The actual release is found after comparing  $S_t$  with the maximum and minimum storage in the reservoir.

The length of the time interval may vary from 1 hour to 1 month, depending on the size of the reservoir and the purpose of the simulation. For a study of flood control problems a small time step (1 hour or 1 day) is required, but for the simulation of reservoir operation steps of one week or one month may be more appropriate. In general, the larger the reservoir, the larger the time step to be used.

### Assignment

Simulate the water balance of a reservoir for the two years assigned to you, using a time step of one month. The following data are applicable:

The reservoir capacity equals the storage determined in the previous exercise with the Rippl diagram.

Minimum required storage in the reservoir (dead storage) = 20% of the reservoir capacity.

Target release: during January - April: 1.5  $Q_{av}$

during May - December:  $2/3 Q_{av}$

There is a constant leakage under the dam site  $Q_{leak} = 0.5 \text{ m}^3/\text{s}$

The storage at the start of the simulation is two third of its maximum value.

The procedure includes the following steps:

- Copy the inflow data of the two years assigned to you to an empty spreadsheet.
- Complete the next two columns, similar to table 8.4.
- Compute in the next column the volume in the reservoir as follows:  
 $S_t = S_{t-1} + Q - Q_{leak} - D$  and limit this value with IF-statements to the maximum and minimum reservoir storage.
- Spilling occurs if  $S_t$  is larger than the reservoir capacity.
- Shortage occurs if  $S_t$  is smaller than the dead storage.
- The actual release equals the target release minus the shortage and plus the spilling.
- Make a chart similar to figure 8.3 with the target and the actual release.
- Save the worksheet as **Lnum**.

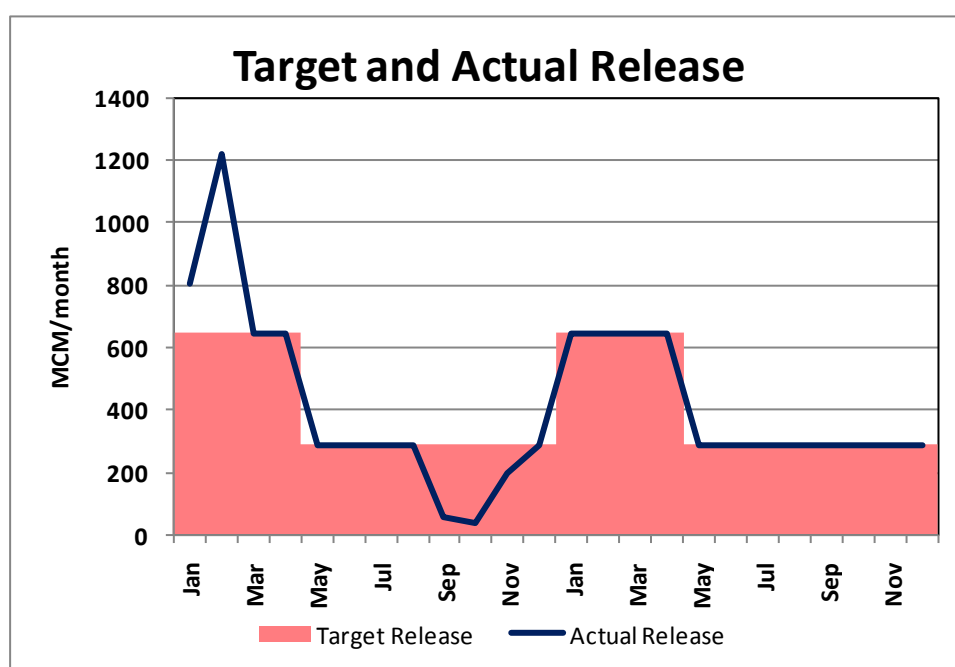


Fig. 8.3 Target and actual release

| Month<br>1965/66                    | Qin<br>(m3/s) | Qin<br>(MCM) | Target<br>Release<br>(MCM) | Stored<br>Volume<br>(MCM) | Spilling<br>(MCM) | Shortage<br>(MCM) | Actual<br>Release<br>(MCM) |
|-------------------------------------|---------------|--------------|----------------------------|---------------------------|-------------------|-------------------|----------------------------|
|                                     |               |              |                            | 787                       |                   |                   |                            |
| Jan                                 | 457           | 1200         | 647                        | 1181                      | 159               | 0                 | 805                        |
| Feb                                 | 466           | 1224         | 647                        | 1181                      | 576               | 0                 | 1223                       |
| Mar                                 | 186           | 489          | 647                        | 1021                      | 0                 | 0                 | 647                        |
| Apr                                 | 171           | 449          | 647                        | 822                       | 0                 | 0                 | 647                        |
| May                                 | 121           | 318          | 287                        | 852                       | 0                 | 0                 | 287                        |
| Jun                                 | 55            | 144          | 287                        | 707                       | 0                 | 0                 | 287                        |
| Jul                                 | 38            | 100          | 287                        | 518                       | 0                 | 0                 | 287                        |
| Aug                                 | 15            | 39           | 287                        | 269                       | 0                 | 0                 | 287                        |
| Sep                                 | 9             | 24           | 287                        | 236                       | 0                 | 232               | 55                         |
| Oct                                 | 15            | 39           | 287                        | 236                       | 0                 | 249               | 38                         |
| Nov                                 | 75            | 197          | 287                        | 236                       | 0                 | 92                | 196                        |
| Dec                                 | 298           | 783          | 287                        | 730                       | 0                 | 0                 | 287                        |
| Jan                                 | 261           | 686          | 647                        | 768                       | 0                 | 0                 | 647                        |
| Feb                                 | 235           | 617          | 647                        | 737                       | 0                 | 0                 | 647                        |
| Mar                                 | 346           | 909          | 647                        | 997                       | 0                 | 0                 | 647                        |
| Apr                                 | 178           | 468          | 647                        | 817                       | 0                 | 0                 | 647                        |
| May                                 | 152           | 399          | 287                        | 927                       | 0                 | 0                 | 287                        |
| Jun                                 | 96            | 252          | 287                        | 891                       | 0                 | 0                 | 287                        |
| Jul                                 | 33            | 87           | 287                        | 689                       | 0                 | 0                 | 287                        |
| Aug                                 | 22            | 58           | 287                        | 458                       | 0                 | 0                 | 287                        |
| Sep                                 | 27            | 71           | 287                        | 240                       | 0                 | 0                 | 287                        |
| Oct                                 | 167           | 439          | 287                        | 390                       | 0                 | 0                 | 287                        |
| Nov                                 | 202           | 531          | 287                        | 631                       | 0                 | 0                 | 287                        |
| Dec                                 | 315           | 827          | 287                        | 1170                      | 0                 | 0                 | 287                        |
| <b>Average</b>                      | <b>164.2</b>  | <b>431.2</b> | <b>407.2</b>               |                           |                   |                   | <b>413.9</b>               |
| <b>Reservoir capacity =</b>         |               |              | <b>1181 MCM</b>            |                           |                   |                   |                            |
| <b>Dead Storage =</b>               |               |              | <b>236 MCM</b>             |                           |                   |                   |                            |
| <b>Draft Jan - April: 3/2*Qav =</b> |               |              | <b>647 MCM</b>             |                           |                   |                   |                            |
| <b>Draft May - Dec: 2/3*Qav =</b>   |               |              | <b>287 MCM</b>             |                           |                   |                   |                            |
| <b>Qleak</b>                        |               |              | <b>1.3 MCM</b>             |                           |                   |                   |                            |
| <b>0.50 m3/s</b>                    |               |              |                            |                           |                   |                   |                            |

Table 8.4 Example of reservoir simulation

So far the effect of evaporation from and precipitation on the reservoir has not been taken into consideration. These components are generally neglected in case of flood routing, using small time steps. However, when simulating large time periods these contributions may be significant, in particular if the surface area of the reservoir is large in relation to the storage.

If the flow series used for simulation were obtained at the dam site, before the dam was built, the discharge values include the precipitation on the area occupied by the

reservoir. With regard to the evaporation losses, only the increase due to changing the land use from a vegetated area with actual evapotranspiration  $Ea$  to open water with an evaporation rate  $Eo$  has to be considered. Hence the water balance equation may be written as

$$S_t = S_{t-1} + [Q - Q_{leak} - D' - (Eo - Ea)A] \Delta t \quad (8.3a)$$

where

$D'$  Actual release ( $m^3/\text{time}$ )

$Eo$  Open water evaporation ( $m/\text{time}$ )

$Ea$  Actual evapotranspiration before the dam was built ( $m/\text{time}$ )

$A$  Surface area of the reservoir ( $m^2$ )

In this workshop the flow series is considered as inflow into the reservoir after the construction of the dam, hence equation 8.3 takes the form

$$S_t = S_{t-1} + [Q - Q_{leak} - D' - (P - Ea)A] \Delta t \quad (8.3b)$$

where  $P$  is the precipitation measured at the dam site in  $m/\text{time}$ .

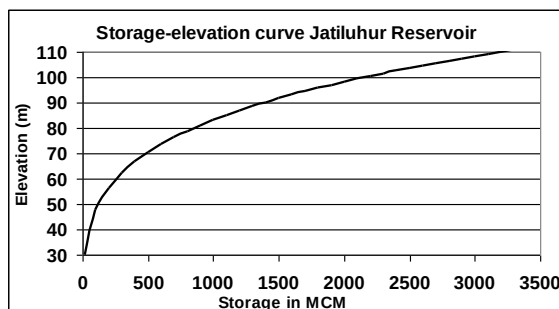


Fig. 8.4 Storage-Elevation curve Jatiluhur Reservoir

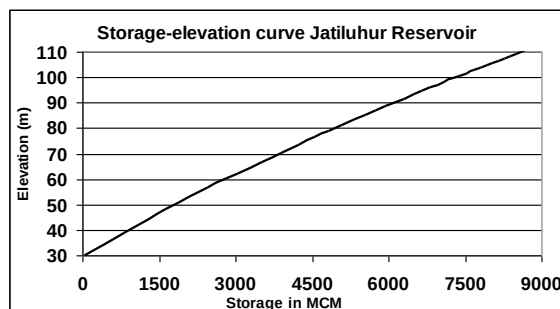


Fig. 8.5 Area-Elevation curve Jatiluhur Reservoir

The surface area of the reservoir changes with the amount of water stored. The solution of equation 8.3, therefore, requires a relation between  $A$  and  $S$ . The average value of  $A$  for time step  $\Delta t$  may then be found as the mean of  $A$  for the storages  $S_{t-1}$  and  $S_t$ . Since  $S_t$  is unknown, equation 8.3 has to be solved by iteration.

The following analytical equations approximate the relations between storage, area and water level for the Jatiluhur reservoir reasonably well.

$$A = 0.3 H^2 + 65 H - 2165 \quad (8.4)$$

$$H = 10^{0.24 \log S + 1.2} \quad (8.5a)$$

$$S = 10^{4.1667 \log H - 5} \quad (8.5b)$$

where

- $A$  area of the reservoir in ha
- $H$  water level in the reservoir in m
- $S$  storage in the reservoir in MCM ( $= 10^6 \text{ m}^3$ )

Equation 8.5b is the inverse of equation 8.5a. This equation is required in some assignments. The relationships are also depicted in figures 8.4 and 8.5.

### Assignment

Extend the table in worksheet **Lnum** to include in the water balance computation the precipitation on and the open water evaporation from the reservoir. Precipitation and evaporation data may be found in file *i:\groupwork\hydata\jatiluhur.txt*. The computations include the following steps:

- Make a first estimate of  $S_t$  neglecting precipitation and evaporation (Use the water balance equation with  $S_{t-1}$  equal to the third (last) estimate of the previous time step).
- Compute in the next columns estimates for  $H_t$  and  $A_t$  using equations 8.5 and 8.4.
- The water balance component  $(P - Eo)$  is estimated as  $(P - Eo \cdot 30.4) \cdot (A_t + A_{t-1}) / 2 \cdot 10^{-5}$  MCM, where  $P$  is in mm/month,  $Eo$  in mm/d and the area  $A$  in ha.
- Apply equation 8.2b to compute a second estimate of  $S_t$  and limit this value with IF-statements to the maximum and minimum reservoir storage.
- Table 8.5 shows that a third estimate is not required (the values of the third and second estimate are exactly the same).

| With rainfall and evaporation |     |      |          |                       |     |      |          |                      |                |                |                      |         |            |
|-------------------------------|-----|------|----------|-----------------------|-----|------|----------|----------------------|----------------|----------------|----------------------|---------|------------|
| First estimate (MCM)          | H m | A ha | P-Eo MCM | Second estimate (MCM) | H m | A ha | P-Eo MCM | Third estimate (MCM) | Spilling (MCM) | Shortage (MCM) | Actual Release (MCM) | Eo mm/d | P mm/month |
| 787                           | 79  | 4790 |          | 787                   | 79  | 4790 |          | 787                  |                |                |                      |         |            |
| 1181                          | 87  | 5710 | 15.6     | 1181                  | 87  | 5710 | 15.6     | 1181                 | 174            | 0              | 821                  | 3.5     | 404        |
| 1181                          | 87  | 5710 | 18.3     | 1181                  | 87  | 5710 | 18.3     | 1181                 | 594            | 0              | 1241                 | 3       | 411        |
| 1021                          | 84  | 5366 | 3.5      | 1025                  | 84  | 5374 | 3.5      | 1025                 | 0              | 0              | 647                  | 4       | 185        |
| 826                           | 79  | 4893 | 1.9      | 828                   | 79  | 4897 | 1.9      | 828                  | 0              | 0              | 647                  | 4.5     | 173        |
| 857                           | 80  | 4972 | -0.1     | 857                   | 80  | 4972 | -0.1     | 857                  | 0              | 0              | 287                  | 4.5     | 135        |
| 712                           | 77  | 4583 | -2.5     | 710                   | 77  | 4576 | -2.5     | 710                  | 0              | 0              | 287                  | 4.5     | 84         |
| 521                           | 71  | 3976 | -2.8     | 518                   | 71  | 3966 | -2.8     | 518                  | 0              | 0              | 287                  | 4.5     | 71         |
| 269                           | 61  | 2884 | -3.4     | 265                   | 60  | 2865 | -3.4     | 265                  | 0              | 0              | 287                  | 5       | 53         |
| 236                           | 59  | 2697 | -3.3     | 236                   | 59  | 2697 | -3.3     | 236                  | 0              | 239            | 48                   | 5.5     | 48         |
| 236                           | 59  | 2697 | -3.2     | 236                   | 59  | 2697 | -3.2     | 236                  | 0              | 253            | 35                   | 6       | 64         |
| 236                           | 59  | 2697 | -1.1     | 236                   | 59  | 2697 | -1.1     | 236                  | 0              | 93             | 195                  | 5       | 110        |
| 730                           | 77  | 4633 | 5.9      | 736                   | 77  | 4650 | 5.9      | 736                  | 0              | 0              | 287                  | 4       | 282        |
| 773                           | 78  | 4753 | 6.4      | 780                   | 78  | 4770 | 6.4      | 780                  | 0              | 0              | 647                  | 3.5     | 242        |
| 749                           | 78  | 4686 | 6.2      | 755                   | 78  | 4703 | 6.2      | 755                  | 0              | 0              | 647                  | 3       | 222        |
| 1016                          | 83  | 5353 | 9.9      | 1026                  | 84  | 5376 | 9.9      | 1026                 | 0              | 0              | 647                  | 4       | 319        |
| 845                           | 80  | 4942 | 2.2      | 847                   | 80  | 4948 | 2.2      | 847                  | 0              | 0              | 647                  | 4.5     | 179        |
| 958                           | 82  | 5219 | 1.1      | 959                   | 82  | 5222 | 1.1      | 959                  | 0              | 0              | 287                  | 4.5     | 159        |
| 922                           | 82  | 5134 | -1.1     | 921                   | 82  | 5131 | -1.1     | 921                  | 0              | 0              | 287                  | 4.5     | 115        |
| 719                           | 77  | 4602 | -3.4     | 716                   | 77  | 4592 | -3.4     | 716                  | 0              | 0              | 287                  | 4.5     | 67         |
| 485                           | 70  | 3845 | -4.0     | 481                   | 70  | 3830 | -4.0     | 481                  | 0              | 0              | 287                  | 5       | 58         |
| 263                           | 60  | 2852 | -3.2     | 260                   | 60  | 2834 | -3.1     | 260                  | 0              | 0              | 287                  | 5.5     | 73         |
| 410                           | 67  | 3551 | 0.0      | 410                   | 67  | 3551 | 0.0      | 410                  | 0              | 0              | 287                  | 6       | 181        |
| 651                           | 75  | 4403 | 2.2      | 654                   | 75  | 4409 | 2.2      | 654                  | 0              | 0              | 287                  | 5       | 208        |
| 1181                          | 87  | 5710 | 8.8      | 1181                  | 87  | 5710 | 8.8      | 1181                 | 20             | 0              | 307                  | 4       | 295        |
| Average P - Eo =              |     |      |          |                       |     |      | 2.2 MCM  | Average release =    |                | 415.7 MCM      |                      |         |            |

Table 8.5 Example reservoir simulation including rainfall and evaporation

- Compute in the next columns the Spilling, Shortage and Actual release as done previously.
- Save sheet **Lnum**.

## 8.4 Rule curves

In the reservoir simulation for yield analysis the routing of the inflow followed the Standard Operating Rule (SOR), which may be summarized as follows:

- 1 the target demand is not released if the storage at the end of the time period will be less than the dead storage;
- 2 if at the end of the time interval the reservoir is full, the actual release equals the target value plus the amount that is spilling.

In the analysis of reservoir yield, the time series of inflow, precipitation and evaporation are known values. This allows the computation of the unknown parameters, such as the variation of the storage over time and the actual release or outflow of the reservoir. From the yield analysis the target release may be approximated as a function of time. After the construction of the reservoir the SOR is not likely to be fully applicable, because the reservoir may serve more than one objective, such as water conservation, power generation, flood retention and recreation. For multi-purpose reservoirs, the operators have to control the releases, taking all objectives into consideration and knowing that future inflows are subject to hydrological uncertainty. For this purpose series of reservoir water levels (rule curves) are established that vary throughout the year. The rule curves are used

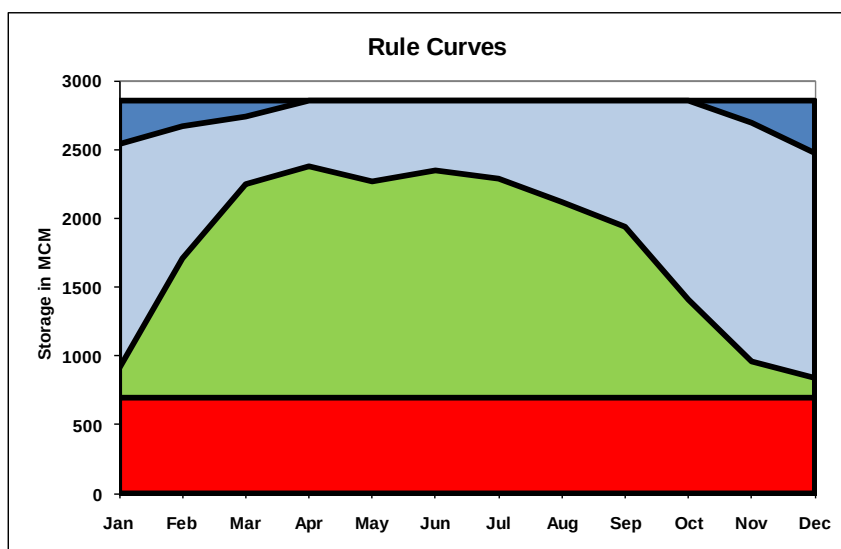


Fig. 8.6 Example rule curves

by the operators as guidelines for the actions to be taken. In the example in this workshop the Jatiluhur reservoir serves two purposes: water conservation (for irrigation) and flood retention (the rule curves are given in figure 8.6).

The following operation rules generally apply for this situation:

- 1 The Flood Rule Curve (*FRC*) represents the maximum storage levels necessary to store large floods. It is usually considered a hard boundary, meaning that the level may not be crossed. The operator may not always succeed in obeying this rule, in particular on a day-to-day basis. The success depends on the means of the operator to release water. If the reservoir e.g. uses a bottom gate with a maximum capacity  $Q_b$ , the operation rule is as follows:

If  $S > FRC$  and  $S - FRC < Q_b$  then  $Q = D + (S - FRC)$  and  $S = FRC$

If  $S > FRC$  and  $S - FRC > Q_b$  then  $Q = D + Q_b$  and  $S = S - Q_b$  (in this case the storage  $S$  exceeds the *FRC*, where  $D$  is the target draft and  $Q$  is the actual release).

- 2 The Conservation Rule Curve (*CRC*) is a soft boundary (it may be crossed). In this example the *CRC* refers to the conservation of water for irrigation. If the storages crosses the *CRC* the release from the reservoir is reduced by a certain rationing percentage  $r$  as follows:

If  $S < CRC$ , then  $Q = r \cdot D$  and  $S$  is recomputed with  $Q = r \cdot D$

In this way the effect of shortages on the water users is minimized. The application of water rationing is also referred to as *hedging*. Instead of reducing the release with a certain percentage, special rules curves (hedging rule curves) which apply in case of water shortages, may be used.

- 3 The Dead Storage Curve (*DSC*) is a hard boundary. The storage may never drop below this boundary as a result of releases. The dead storage requirement is often for environmental or ecological reasons. If  $S$  drops below *DSC*, the release is reduced as follows:

If  $S < DSC$ , then  $Q = S + D - DSC$  and  $S = DCS$

( $Q$  needs to be corrected if it appears to be negative due to evaporation)

| Month<br>1965/6                 | Qin<br>6 | Qin<br>m3/s | Qin<br>MCM | Target<br>Release<br>MCM | Full<br>Supply<br>Level<br>MCM | Flood<br>Rule<br>Curve<br>MCM | Conser-<br>vation<br>Rule<br>Curve<br>MCM | Dead<br>Storage<br>Curve<br>MCM | P - Eo<br>MCM | Adjust<br>for flood<br>control<br>MCM | Adjust<br>for<br>Conser-<br>vation<br>MCM | Shortage<br>MCM | Spilling<br>MCM | Stored<br>Volume<br>MCM | H<br>m | A<br>ha | Eo<br>mm/d | P<br>mm/month | Actual<br>Release<br>MCM |
|---------------------------------|----------|-------------|------------|--------------------------|--------------------------------|-------------------------------|-------------------------------------------|---------------------------------|---------------|---------------------------------------|-------------------------------------------|-----------------|-----------------|-------------------------|--------|---------|------------|---------------|--------------------------|
| Jan                             | 457      | 1200        | 647        | 2856                     | 2540                           | 910                           | 700                                       | 20.7                            | 0             | 0                                     | 0                                         | 0               | 0               | 1904                    | 97     | 6972    | 3.5        | 404           | 647                      |
| Feb                             | 466      | 1224        | 647        | 2856                     | 2670                           | 1710                          | 700                                       | 24.8                            | 263           | 0                                     | 0                                         | 0               | 0               | 2477                    | 103    | 7764    | 3          | 411           | 910                      |
| Mar                             | 186      | 489         | 647        | 2856                     | 2740                           | 2250                          | 700                                       | 5.2                             | 0             | 0                                     | 0                                         | 0               | 0               | 2660                    | 105    | 7991    | 4          | 185           | 647                      |
| Apr                             | 171      | 449         | 647        | 2856                     | 2856                           | 2380                          | 700                                       | 2.9                             | 0             | 0                                     | 0                                         | 0               | 0               | 2463                    | 103    | 7747    | 4.5        | 173           | 647                      |
| May                             | 121      | 318         | 287        | 2856                     | 2856                           | 2270                          | 700                                       | -0.1                            | 0             | 0                                     | 0                                         | 0               | 0               | 2493                    | 104    | 7784    | 4.5        | 135           | 287                      |
| Jun                             | 55       | 144         | 287        | 2856                     | 2856                           | 2350                          | 700                                       | -4.1                            | 0             | 72                                    | 0                                         | 0               | 0               | 2417                    | 103    | 7687    | 4.5        | 84            | 215                      |
| Jul                             | 38       | 100         | 287        | 2856                     | 2856                           | 2290                          | 700                                       | -5.1                            | 0             | 72                                    | 0                                         | 0               | 0               | 2295                    | 102    | 7526    | 4.5        | 71            | 215                      |
| Aug                             | 15       | 39          | 287        | 2856                     | 2856                           | 2120                          | 700                                       | -7.5                            | 0             | 72                                    | 0                                         | 0               | 0               | 2110                    | 100    | 7273    | 5          | 53            | 215                      |
| Sep                             | 9        | 24          | 287        | 2856                     | 2856                           | 1940                          | 700                                       | -8.7                            | 0             | 72                                    | 0                                         | 0               | 0               | 1909                    | 97     | 6979    | 5.5        | 48            | 215                      |
| Oct                             | 15       | 39          | 287        | 2856                     | 2856                           | 1410                          | 700                                       | -8.3                            | 0             | 0                                     | 0                                         | 0               | 0               | 1651                    | 94     | 6574    | 6          | 64            | 287                      |
| Nov                             | 75       | 197         | 287        | 2856                     | 2695                           | 960                           | 700                                       | -2.8                            | 0             | 0                                     | 0                                         | 0               | 0               | 1557                    | 93     | 6415    | 5          | 110           | 287                      |
| Dec                             | 298      | 783         | 287        | 2856                     | 2475                           | 840                           | 700                                       | 10.3                            | 0             | 0                                     | 0                                         | 0               | 0               | 2062                    | 99     | 7204    | 4          | 282           | 287                      |
| Jan                             | 261      | 686         | 647        | 2856                     | 2540                           | 910                           | 700                                       | 9.8                             | 0             | 0                                     | 0                                         | 0               | 0               | 2109                    | 99     | 7271    | 3.5        | 242           | 647                      |
| Feb                             | 235      | 617         | 647        | 2856                     | 2670                           | 1710                          | 700                                       | 9.5                             | 0             | 0                                     | 0                                         | 0               | 0               | 2087                    | 99     | 7241    | 3          | 222           | 647                      |
| Mar                             | 346      | 909         | 647        | 2856                     | 2740                           | 2250                          | 700                                       | 14.3                            | 0             | 0                                     | 0                                         | 0               | 0               | 2362                    | 102    | 7616    | 4          | 319           | 647                      |
| Apr                             | 178      | 468         | 647        | 2856                     | 2856                           | 2380                          | 700                                       | 3.2                             | 0             | 162                                   | 0                                         | 0               | 0               | 2346                    | 102    | 7595    | 4.5        | 179           | 485                      |
| May                             | 152      | 399         | 287        | 2856                     | 2856                           | 2270                          | 700                                       | 1.7                             | 0             | 0                                     | 0                                         | 0               | 0               | 2459                    | 103    | 7741    | 4.5        | 159           | 287                      |
| Jun                             | 96       | 252         | 287        | 2856                     | 2856                           | 2350                          | 700                                       | -1.7                            | 0             | 0                                     | 0                                         | 0               | 0               | 2421                    | 103    | 7692    | 4.5        | 115           | 287                      |
| Jul                             | 33       | 87          | 287        | 2856                     | 2856                           | 2290                          | 700                                       | -5.4                            | 0             | 72                                    | 0                                         | 0               | 0               | 2286                    | 101    | 7514    | 4.5        | 67            | 215                      |
| Aug                             | 22       | 58          | 287        | 2856                     | 2856                           | 2120                          | 700                                       | -7.1                            | 0             | 72                                    | 0                                         | 0               | 0               | 2120                    | 100    | 7287    | 5          | 58            | 215                      |
| Sep                             | 27       | 71          | 287        | 2856                     | 2856                           | 1940                          | 700                                       | -6.9                            | 0             | 72                                    | 0                                         | 0               | 0               | 1967                    | 98     | 7067    | 5.5        | 73            | 215                      |
| Oct                             | 167      | 439         | 287        | 2856                     | 2856                           | 1410                          | 700                                       | -0.1                            | 0             | 0                                     | 0                                         | 0               | 0               | 2118                    | 100    | 7284    | 6          | 181           | 287                      |
| Nov                             | 202      | 531         | 287        | 2856                     | 2695                           | 960                           | 700                                       | 4.1                             | 0             | 0                                     | 0                                         | 0               | 0               | 2364                    | 102    | 7618    | 5          | 208           | 287                      |
| Dec                             | 315      | 827         | 287        | 2856                     | 2475                           | 840                           | 700                                       | 13.2                            | 263           | 0                                     | 0                                         | 0               | 0               | 2653                    | 105    | 7984    | 4          | 295           | 550                      |
| Average                         |          | 431.2       |            |                          |                                |                               |                                           |                                 | 2.6           |                                       |                                           |                 |                 |                         |        |         |            |               | 401.3                    |
| Reservoir capacity =            |          |             |            |                          | 2856 MCM                       |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Dead Storage =                  |          |             |            |                          | 700 MCM                        |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Draft Jan - April: 3/2*Qav =    |          |             |            |                          | 647 MCM                        |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Draft May - Dec: 2/3*Qav =      |          |             |            |                          | 287 MCM                        |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Qleak 0.5 m3/s =                |          |             |            |                          | 1.3 MCM                        |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Rationing percentage =          |          |             |            |                          | 75 %                           |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |
| Capacity bottom gate 100 m3/s = |          |             |            |                          | 263 MCM                        |                               |                                           |                                 |               |                                       |                                           |                 |                 |                         |        |         |            |               |                          |

Table 8.6 Example of reservoir operation rule curves

**Assignment**

Simulate the reservoir storage for the two years assigned to you using the following rule curves:

- The Full Supply Level (*FSL*) is 2856 MCM (water level of 107 m).
- The Dead Storage Curve (*DSC*) is horizontal and equal to a storage of 700 MCM.
- The Flood Rule Curve (*FRC*) and the Conservation Rule Curve (*CRC*) are given in the table below in MCM.

|            | Jan  | Feb  | Mar  | Apr  | May  | Jun  | Jul  | Aug  | Sep  | Oct  | Nov  | Dec  |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|
| <b>FRC</b> | 2540 | 2670 | 2740 | 2856 | 2856 | 2856 | 2856 | 2856 | 2856 | 2856 | 2695 | 2475 |
| <b>CRC</b> | 910  | 1710 | 2250 | 2380 | 2270 | 2350 | 2290 | 2120 | 1940 | 1410 | 960  | 840  |

The following data apply:

- The leakage underneath the dam is  $0.5 \text{ m}^3/\text{s}$ .
- The rationing in case of water shortage is 75 %.
- The capacity of the bottom gate is  $100 \text{ m}^3/\text{s}$ .
- The initial storage is 2/3 of the reservoir capacity.
- Other data (target release, precipitation, evaporation, storage - area relationships) are the same as in the previous assignment.

Take the following steps:

- Copy the two years with inflow data to a fresh worksheet.
- Add columns with target release data, rule curves and the contribution from precipitation and evaporation (using the surface area of the previous time step), similar to table 8.6.

- Add a column to compute the release for flood control.
- Compute in the next column the reduction  $(1 - r)*D$  for the situation that the storage is less than the *CRC*.
- Shortage and spilling are computed similar to the previous assignment.
- Compute the stored volume taking into account the corrections made.

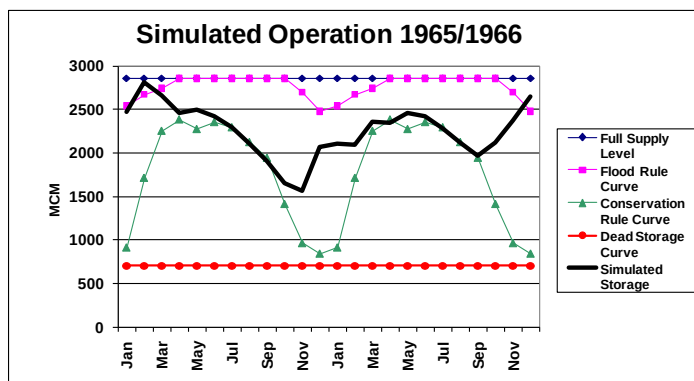


Fig. 8.7 Example simulation of reservoir operation

- The actual release is then found as the target release plus the adjustment for flood control, minus the correction for conservation, minus the shortage and plus the spilling.
- Make a chart similar to figure 8.7 showing the rule curves and the simulated storage.
- Check your spreadsheet as follows:  
If there is spilling the storage must equal 2856 MCM  
If there is shortage the storage must equal 700 MCM  
Overall water balance: compute averages of  $Q_{in}$ ,  $(P-E_o)$  and actual release  $Q_{act}$ .  
Storage in reservoir at the end of period =  $1904 + 24[Q_{in} + (P-E_o) - Q_{act} - 1.3]$ .
- Save the sheet as **Mnum**.

## 8.5 Reservoir routing

A reservoir must contain a spillway designed such that the largest flood which is expected to occur can pass without overtopping the dam and thus endangering the structure with potential disastrous consequences downstream. The design flood is selected as the maximum probable flood which could occur upstream of the reservoir. Instead of determining the probable maximum flood, also the flood with a return period of  $X$  years is used, where the value of  $X$  may vary from 10 to 10,000 years depending on the possible damage and the risk of the loss of lives.

The design flood is routed through the reservoir for a given width of the spillway. The simulation yields the maximum water level above the crest of the spillway during the passage of the flood. To obtain the ultimate height of the dam a safety margin should be added to the maximum water level during the passage of the flood to take into account wave run-up and wind set-up. The routing procedure may be repeated for a different width of the spillway resulting in a different height of the dam. The optimal dimensions are then determined by minimizing the construction costs.

The same water balance equation as discussed earlier may be applied for reservoir routing, but the time step is usually much smaller than for the yield analysis. With time steps in the order of hours, the precipitation and evaporation components in the water balance equation may be neglected, resulting in the following procedure:

1. Assume a full reservoir at the start of the design storm, or  $H_o = H_c$ , where  $H_o$  is the water level in the reservoir at the start of the simulation and  $H_c$  is the level of the crest of the spillway.
2. Estimate the flow across the spillway,  $Q$ , with the following formula



$$Q = K B (H - H_c)^c \quad (8.7)$$

where  $H$  is the water level in the reservoir ( $H = H_o$ ),  $K$  and  $c$  are parameters depending on the type of spillway and  $B$  is the width of the spillway crest.

3. Assuming that the outflow over the crest of the spill way is constant during the time step:  $Q = Q(H)$ , a first approximation of the reservoir storage  $S_t^*$  at the end of the time step is found from the water balance equation (neglecting precipitation and evaporation) as

$$S_t^* = S_{t-1} + (I - Q) \Delta t \quad (8.8a)$$

where  $I$  is the inflow during time step  $t$ . The equation is also called the *predictor*.

4. This first estimate of the storage  $S_t^*$  may be used in equation 8.5a to find the corresponding water level  $H_t^*$ .
5. A better estimate of the flow across the spillway  $Q$  is then found with equation 8.7 where  $H = (H_{t-1} + H_t^*)/2$ .
6. Equation 8.8a is subsequently computed again with  $Q = Q^*$

$$S_t = S_{t-1} + (I - Q^*) \Delta t \quad (8.8b)$$

This equation is called the *corrector*.

7. The corresponding water level  $H_t$  is then found from equation 8.5a with  $S = S_t$ .
8. If necessary steps 5 to 7 are repeated with  $H_t$  instead of  $H_t^*$  until no more significant change in  $S_t$  occurs.

An example of the above procedure, given in table 8.7, shows that after two iterations the simulated value for the outflow  $Q$  is almost constant ( $Q^* \approx Q^{**}$ ). Figure 8.8 is a plot of the inflow  $I$  and the (extended series of simulated) outflow  $Q$ . It shows that the maximum outflow occurs at the point of intersection with the inflow hydrograph. The reason is that the maximum water level in the reservoir (and thus the outflow  $Q$ ) occurs for  $I = Q$ .

### Assignment

Route a design flood through the Jatiluhur reservoir. The time step to be used is 3 hours (0.125 days). The design flood has a triangular shape with a duration of 3 days. Each participant is given the peak flow rate ( $\text{m}^3/\text{s}$ ) and a maximum reservoir level. Apply the above procedure with two iterations. Simulate the maximum water level in the reservoir for a width of the spillway of 10 m. Repeat the simulation for different widths to find the size of the spillway that keeps the water level in the reservoir below the given maximum. The crest of the spillway is at a level of 107 m and at the start of the simulation the reservoir is full. The procedure is the following (see table 8.7):

Generate in the first column 160 time steps of 3 hours (a period of 20 days).

- Compute the triangular inflow hydrograph for the given peak flow rate in the next column.
- Set up the headings of the remaining columns and set for  $t = 0$  the initial values for  $S = 2856 \text{ MCM}$  and  $H = 107 \text{ m}$ .
- Use equation 8.7 with  $K = 1.5$  and  $c = 1.5$  and  $B = 10$  in the third column to estimate  $Q$  for  $H$  in the previous time step.
- Apply the predictor equation 8.8a to compute  $S_t^*$  in the next column.

| Level spillway crest = |                     | 107 m                   |           |         |            |            |          |             |          |             |        |
|------------------------|---------------------|-------------------------|-----------|---------|------------|------------|----------|-------------|----------|-------------|--------|
| L =                    |                     | 45 m                    |           |         |            |            |          |             |          |             |        |
|                        |                     |                         |           |         |            |            |          |             |          | Max Level = | 110.01 |
| Time<br>Days           | Inflow<br>I<br>m3/s | Outflow<br>Q(H)<br>m3/s | S*<br>MCM | H*<br>m | Q*<br>m3/s | S**<br>MCM | H**<br>m | Q**<br>m3/s | S<br>MCM | H<br>m      |        |
| 0.000                  | 0                   |                         |           |         |            |            |          |             | 2856     | 107.0       |        |
| 0.125                  | 250                 | 0.0                     | 2859      | 107.0   | 0.1        | 2859       | 107.0    | 0.1         | 2859     | 107.0       |        |
| 0.250                  | 500                 | 0.2                     | 2864      | 107.1   | 0.7        | 2864       | 107.1    | 0.7         | 2864     | 107.1       |        |
| 0.375                  | 750                 | 1.3                     | 2872      | 107.1   | 2.4        | 2872       | 107.1    | 2.4         | 2872     | 107.1       |        |
| 0.500                  | 1000                | 3.7                     | 2883      | 107.2   | 5.7        | 2883       | 107.2    | 5.7         | 2883     | 107.2       |        |
| 0.625                  | 1250                | 8.0                     | 2896      | 107.4   | 11.1       | 2896       | 107.4    | 11.1        | 2896     | 107.4       |        |
| 0.750                  | 1500                | 14.6                    | 2912      | 107.5   | 19.1       | 2912       | 107.5    | 19.1        | 2912     | 107.5       |        |
| 0.875                  | 1750                | 24.0                    | 2931      | 107.7   | 30.1       | 2931       | 107.7    | 30.1        | 2931     | 107.7       |        |
| 1.000                  | 2000                | 36.7                    | 2952      | 107.9   | 44.7       | 2952       | 107.9    | 44.6        | 2952     | 107.9       |        |
| 1.125                  | 2250                | 53.1                    | 2976      | 108.1   | 63.0       | 2976       | 108.1    | 63.0        | 2976     | 108.1       |        |
| 1.250                  | 2500                | 73.5                    | 3002      | 108.3   | 85.6       | 3002       | 108.3    | 85.6        | 3002     | 108.3       |        |
| 1.375                  | 2750                | 98.3                    | 3030      | 108.5   | 112.8      | 3030       | 108.5    | 112.7       | 3030     | 108.5       |        |
| 1.500                  | 3000                | 127.8                   | 3061      | 108.8   | 144.8      | 3061       | 108.8    | 144.7       | 3061     | 108.8       |        |
| 1.625                  | 2750                | 162.3                   | 3089      | 109.0   | 178.6      | 3089       | 109.0    | 178.5       | 3089     | 109.0       |        |
| 1.750                  | 2500                | 195.3                   | 3114      | 109.2   | 210.7      | 3113       | 109.2    | 210.6       | 3113     | 109.2       |        |
| 1.875                  | 2250                | 226.2                   | 3135      | 109.4   | 240.2      | 3135       | 109.4    | 240.1       | 3135     | 109.4       |        |
| 2.000                  | 2000                | 254.4                   | 3154      | 109.6   | 266.9      | 3154       | 109.6    | 266.8       | 3154     | 109.6       |        |
| 2.125                  | 1750                | 279.4                   | 3170      | 109.7   | 290.2      | 3170       | 109.7    | 290.2       | 3170     | 109.7       |        |
| 2.250                  | 1500                | 301.0                   | 3183      | 109.8   | 310.0      | 3183       | 109.8    | 309.9       | 3183     | 109.8       |        |
| 2.375                  | 1250                | 319.0                   | 3193      | 109.9   | 326.1      | 3193       | 109.9    | 326.0       | 3193     | 109.9       |        |
| 2.500                  | 1000                | 333.1                   | 3200      | 110.0   | 338.2      | 3200       | 110.0    | 338.2       | 3200     | 110.0       |        |
| 2.625                  | 750                 | 343.3                   | 3204      | 110.0   | 346.5      | 3204       | 110.0    | 346.4       | 3204     | 110.0       |        |
| 2.750                  | 500                 | 349.6                   | 3206      | 110.0   | 350.8      | 3206       | 110.0    | 350.7       | 3206     | 110.0       |        |
| 2.875                  | 250                 | 351.9                   | 3205      | 110.0   | 351.1      | 3205       | 110.0    | 351.1       | 3205     | 110.0       |        |
| 3.000                  | 0                   | 350.3                   | 3201      | 110.0   | 347.6      | 3201       | 110.0    | 347.6       | 3201     | 110.0       |        |

Table 8.7 Example reservoir routing

- In the next column  $H_t^*$  is computed with equation 8.5a.
- The value of  $Q_t^*$  is estimated from equation 8.7 for using the average value of  $H_t^*$  (this time step) and  $H$  (previous time step).
- Apply in the next column the corrector equation 8.8b to find  $S_t^{**}$ .
- Repeat in the next columns the computations of  $H$ ,  $Q$ ,  $S$  and  $H$  in a second iteration.
- Copy the first row to the other time steps.
- Repeat the computations for different widths of the spillway, and determine the minimum width (with an accuracy of 5 m) that keeps the peak water level below the given maximum.
- Make a chart with the inflow and outflow hydrograph.
- Show in a chart the change of the water level in the reservoir with time.
- Save the sheet as **Nnum**.

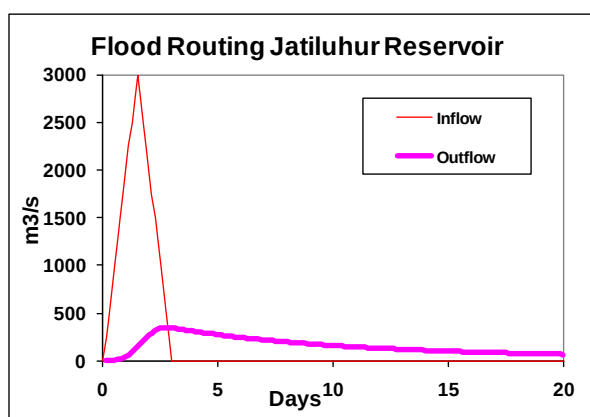


Fig. 8.8 Example flood routing

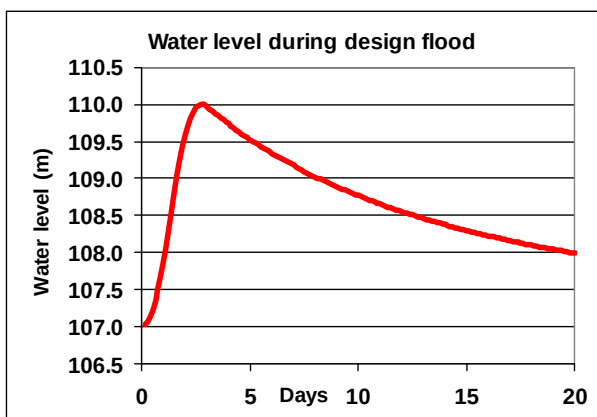


Fig. 8.9 Simulated water level

## 9 SOIL MOISTURE

### 9.1 Actual evapotranspiration through soil moisture accounting

In case of water stress in the root zone the actual evapotranspiration may become less than its potential value. A simple model as shown in figure 9.1 may be used to estimate relative evapotranspiration in relation to the available moisture. This requires that the amount of water in the root zone is continuously monitored: soil moisture accounting.

For this workshop daily precipitation and potential evapotranspiration values of grass for the meteorological station De Bilt (The Netherlands) are available for the period 1911 - 2002 in file *i:\groupwork\hydata\P&E DeBilt.txt*. Soil physical data of the international soil series are found in file *i:\groupwork\hydata\ISS.txt*.

The soil moisture accounting model keeps track of the actual amount of available moisture  $M_t$  stored at time  $t$  in the root zone. The (maximum) Available Moisture  $AM$  is defined as

$$AM = D_r (\theta_{FC} - \theta_{WP}) \quad (9.1)$$

where  $D_r$  is the depth of the root zone, and  $\theta_{FC}$  and  $\theta_{WP}$  the moisture content in the root zone at Field Capacity ( $h = -100$  cm) and Wilting Point ( $h = -16000$  cm), respectively. A fraction  $p$  of  $AM$  is readily available moisture ( $RAM = pAM$ ), which means that during the consumption of this water by the plant the actual evapotranspiration equals the potential evapotranspiration ( $E_{act} = E_{pot}$ ). Figure 9.1 shows that water in excess of the moisture content at field capacity and less than wilting point is not available for the crop.

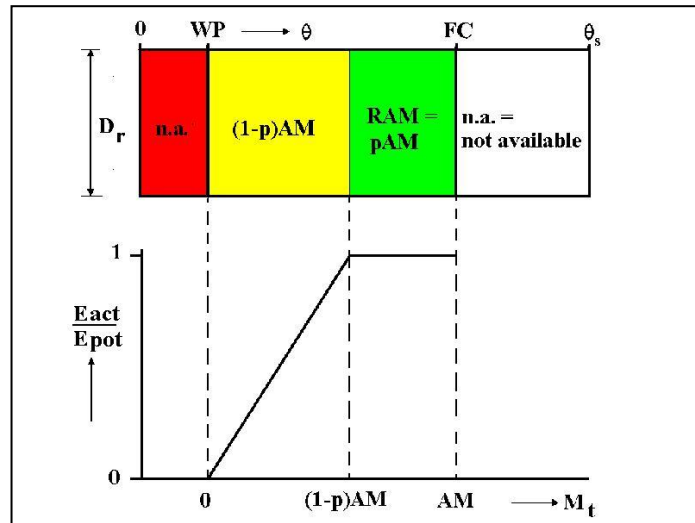


Fig. 9.1 Relative evapotranspiration in relation to moisture availability in the root zone

The value of  $p$  depends on the type of crop and the evaporative demand  $E_{pot}$  and ranges from 0.2 to 0.8, but an average value of 0.5 is often used.

For the situation that at time  $t$  the actual available moisture content in the root zone  $M_t$  reaches a value less than  $(1-p)AM$ , it is assumed that the relative evapotranspiration then reduces linearly from one to ultimately zero when all available moisture has been used. Thus

$$\text{for } M_t \geq (1-p)AM \quad \frac{E_{act}}{E_{pot}} = 1 \quad (9.2a)$$

$$\text{and for } M_t < (1-p)AM \quad \frac{E_{act}}{E_{pot}} = \frac{M_t}{(1-p)AM} \quad (9.2b)$$

The water balance of the root zone may be written as

$$M_t = M_{t-1} + P - E_{act} - D \quad (9.3)$$

where  $P$  is precipitation and  $D$  is drainage from the bottom of the root zone into the subsoil. It should be noted that for this water balance equation capillary rise is neglected (assume a deep water table) and that all precipitation enters into the soil (no interception, no surface runoff). The drainage from the root zone into the subsoil is simply computed as all water in excess of field capacity, hence

$$\text{for } M_t \geq AM \quad D = M_t - AM \quad (9.4a)$$

$$\text{and for } M_t < AM \quad D = 0 \quad (9.4b)$$

### Assignment

The computation of the actual evapotranspiration through soil moisture accounting is to be carried out for a calendar year. At the start of the computations the moisture content in the root zone is assumed at field capacity. The depth of the root zone  $D_r = 30$  cm.

- Open file *i:\groupwork\hydata\P&E DeBilt.txt* and copy the date, precipitation and potential evapotranspiration of the year assigned to you into an empty spreadsheet.
- Enter at the top of the spreadsheet the parameters related to the chosen soil:  $\theta_{FC}$  and  $\theta_{WP}$ ,  $D_r$ ,  $p$ ,  $AM$  and  $(1-p)AM$ . Take  $p = 0.4$ . The soil physical data of the soil assigned to you are found in Appendix F.
- At the start of the simulation the root zone is assumed at field capacity ( $h = -100$  cm).
- Take for each new time step the initial value for  $M_{t-1}$  equal to  $M_t$  of the previous time step.
- Make a first estimate of  $M_t^* = M_{t-1} + P - E_{pot}$ . Make sure the value is not negative.
- Compute in the next column the drainage with equation 9.4.
- Compute in the next column the actual evapotranspiration with equation 9.2.
- Compute in the next column  $M_t$  with equation 9.3.
- Make a chart showing the change of the actual and potential evapotranspiration over the year assigned to you (see for example figure 9.2).
- Compute the relative evapotranspiration ( $E_{act}/E_{pot}$ ) for the soil chosen by you as well for Coarse Sand (soil 1). Plot both series in a chart (see for example figure 9.3).
- Save the sheet as **Vnum**.

### Questions:

- 1- For which minimum depth of the root zone is the relative evapotranspiration always equal to one?
- 2- Do you think that such a depth of the root zone is realistic?
- 3- Why do farmers in general not prefer Coarse Sand for agricultural production?

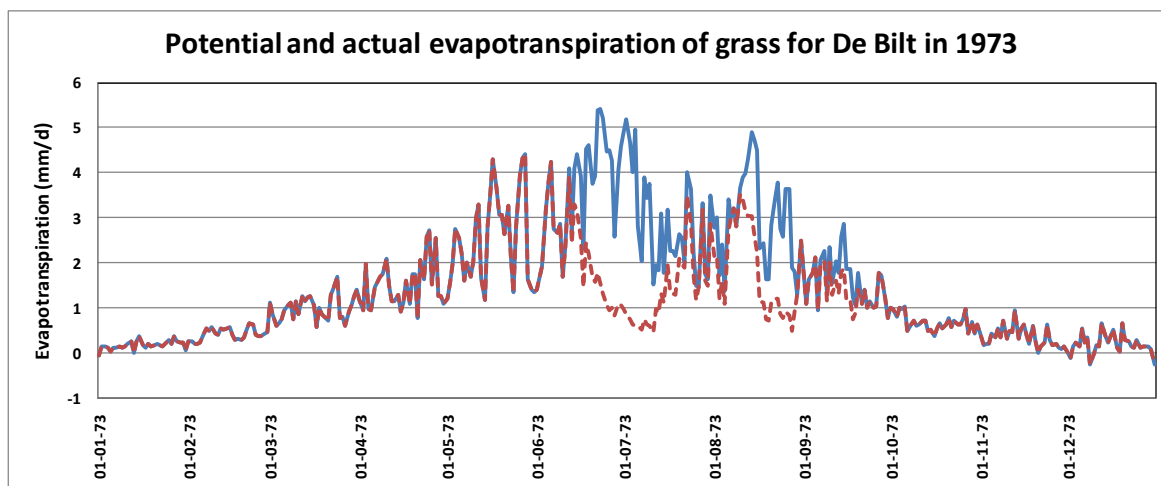


Fig. 9.2 Example simulated actual and potential evapotranspiration

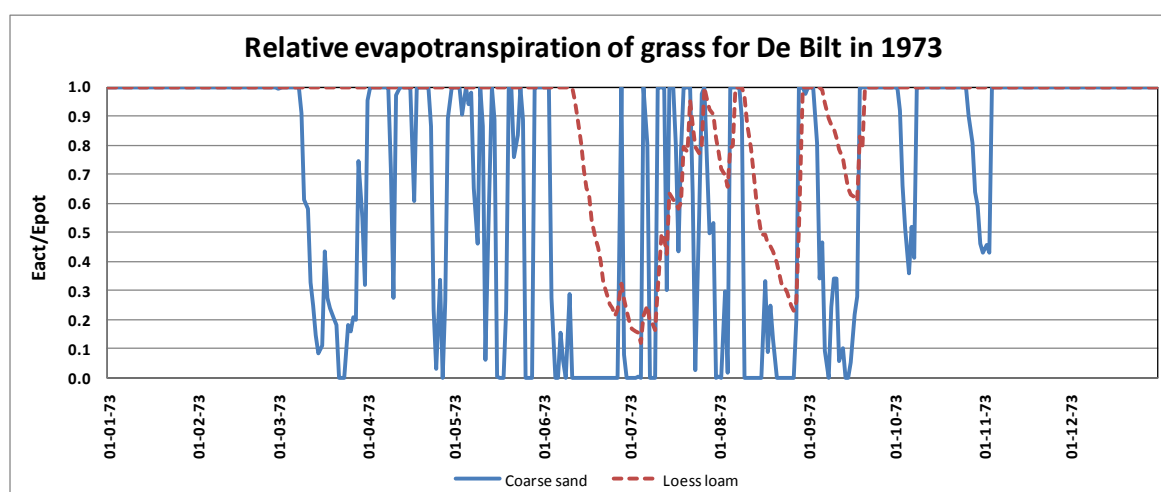


Fig. 9.3 Example comparison relative evapotranspiration Loess loam and Coarse sand

| Loess Loam |           |              |              |             |                  |              |            |           |      |
|------------|-----------|--------------|--------------|-------------|------------------|--------------|------------|-----------|------|
| FC=        | 0.34      |              |              |             |                  |              |            |           |      |
| WP=        | 0.11      |              |              |             |                  |              |            |           |      |
| Dr=        | 300.0 mm  |              |              |             |                  |              |            |           |      |
| p=         | 0.4       |              |              |             |                  |              |            |           |      |
| AM=        | 69.0 mm   |              |              |             |                  |              |            |           |      |
| (1-p)AM=   | 41.4 mm   |              |              |             |                  |              |            |           |      |
| Date       | P<br>mm/d | Ep-m<br>mm/d | M(t-1)<br>mm | M*(t)<br>mm | Drainage<br>mm/d | Eact<br>mm/d | M(t)<br>mm | Eact/Epot |      |
|            |           |              |              |             |                  |              | 69.0       |           |      |
| 1-1-1973   | 0.0       | -0.06        | 69.0         | 69.1        | 0.1              | -0.06        | 69.0       | 1.00      | 1.00 |
| 2-1-1973   | 0.0       | 0.15         | 69.0         | 68.9        | 0.0              | 0.15         | 68.9       | 1.00      | 1.00 |
| 3-1-1973   | 0.0       | 0.15         | 68.9         | 68.7        | 0.0              | 0.15         | 68.7       | 1.00      | 1.00 |
| 4-1-1973   | 0.0       | 0.11         | 68.7         | 68.6        | 0.0              | 0.11         | 68.6       | 1.00      | 1.00 |

Table 9.1 Example computation soil moisture accounting

## 9.2 Optimisation of the Van Genuchten parameters

In appendix *F* the soil physical data of twenty soils are tabulated. The data are based on a worldwide survey. When modelling unsaturated flow, the use of soil moisture characteristics  $\theta(h)$  and hydraulic conductivity relations  $K(h)$  in tabular form is rather inconvenient. Several authors have suggested analytical expressions for these relations, e.g. Brooks & Corey (1964), Campbell (1974) and Van Genuchten (1980). The equations proposed by Van Genuchten are most frequently used and specified below.

The empirical Van Genuchten equation for the soil moisture characteristic reads

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + |\alpha h|^n\right]^m} \quad (9.5)$$

where

- $\theta_r$  residual soil water content ( $L^3/L^3$ ), water content for  $h \rightarrow -\infty$  or non-capillary-bound water
- $\theta_s$  saturated soil water content or porosity ( $L^3/L^3$ )
- $h$  matric pressure in cm (L)
- $\alpha$  parameter corresponding roughly with reciprocal of the air entry value in  $cm^{-1}$  (1/L)
- $n$  dimensionless empirical shape parameter
- $m = 1 - 1/n$

Van Genuchten combined equation (9.5) with the statistical pore size distribution model of Mualem (1976) resulting in the following expression for the hydraulic conductivity relation

$$K(h) = K_s \frac{\left[1 - |\alpha h|^{n-1} \left(1 + |\alpha h|^n\right)^{-m}\right]^2}{\left[1 + |\alpha h|^n\right]^{m\lambda}} \quad (9.6)$$

where

- $K$  hydraulic conductivity in cm/d
- $K_s$  saturated hydraulic conductivity in cm/d
- $\lambda$  dimensionless parameter influencing the slope  $dK/dh$

The parameters of the Van Genuchten relations are constrained as follows:

- $\alpha > 0$
- $n > 1$
- $-10 < \lambda < 10$

### Assignment

- Open file *i:\groupwork\hydata\ISS.txt*, copy the data of the soil assigned to you, and prepare a table similar to table 9.2 in a blank workbook which is then saved as **Wnum**.
- Add one column  $\theta_{calc}$  with the computation of  $\theta$  using equation (9.5).

| Silty Clay |             |          | Van Genuchten relations |                 |          |       |          |                     |
|------------|-------------|----------|-------------------------|-----------------|----------|-------|----------|---------------------|
|            | $\alpha =$  | 0.01 cm  |                         |                 |          |       |          |                     |
|            | $n =$       | 1.35     |                         |                 |          |       |          |                     |
|            | $\lambda =$ | -3.23    |                         |                 |          |       |          |                     |
|            | $m =$       | 0.26     |                         |                 |          |       |          |                     |
| pF         | h           | $\theta$ | $\theta_{calc}$         | Diff $\theta^2$ | K        | LogK  | LogKcalc | Diff K <sup>2</sup> |
| 0          | 0           | 0.507    | 0.507                   |                 | 1.3      | 0.11  | 0.11     | 0.0000              |
| 1.0        | -10         | 0.492    | 0.504                   | 0.0001          | 0.8      | -0.10 | -0.38    | 0.0802              |
| 1.3        | -20         | 0.485    | 0.500                   | 0.0002          | 0.498    | -0.30 | -0.55    | 0.0619              |
| 1.5        | -31         | 0.482    | 0.495                   | 0.0002          | 0.294    | -0.53 | -0.69    | 0.0255              |
| 1.7        | -50         | 0.474    | 0.487                   | 0.0002          | 0.118    | -0.93 | -0.88    | 0.0027              |
| 2.0        | -100        | 0.463    | 0.466                   | 0.0000          | 0.045    | -1.35 | -1.21    | 0.0195              |
| 2.4        | -250        | 0.440    | 0.427                   | 0.0002          | 0.012    | -1.92 | -1.74    | 0.0332              |
| 2.7        | -500        | 0.422    | 0.396                   | 0.0007          | 0.0047   | -2.33 | -2.18    | 0.0209              |
| 3.0        | -1000       | 0.391    | 0.368                   | 0.0005          | 0.0018   | -2.74 | -2.64    | 0.0100              |
| 3.4        | -2500       | 0.352    | 0.338                   | 0.0002          | 0.00049  | -3.31 | -3.27    | 0.0020              |
| 3.7        | -5000       | 0.317    | 0.321                   | 0.0000          | 0.00019  | -3.72 | -3.74    | 0.0003              |
| 4.0        | -10000      | 0.280    | 0.307                   | 0.0008          | 0.000071 | -4.15 | -4.21    | 0.0038              |
| 4.2        | -16000      | 0.257    | 0.300                   | 0.0018          | 0.000037 | -4.43 | -4.53    | 0.0100              |
|            |             |          |                         | 0.0049          |          |       |          | 0.2700              |

Table 9.2 Example optimization Van Genuchten parameters for Silty Clay

- Compute in the next column the square of the difference between the given and computed soil moisture content  $\theta$ .
- Use *Solver* to optimise the parameters  $\alpha$  and  $n$ .
- Copy the values of the hydraulic conductivity from appendix F into your spreadsheet and estimate these values with equation (9.6). It is most convenient to use the logarithm of  $K$  in the optimization procedure.
- Use the sum of the squares of the differences to optimize the parameter  $\lambda$  with *Solver*.
- Compare the tabulated soil physical data and the optimized relations in a chart (see for example the figures 9.4 and 9.5 which apply for Silty clay).
- Save **Wnum**.

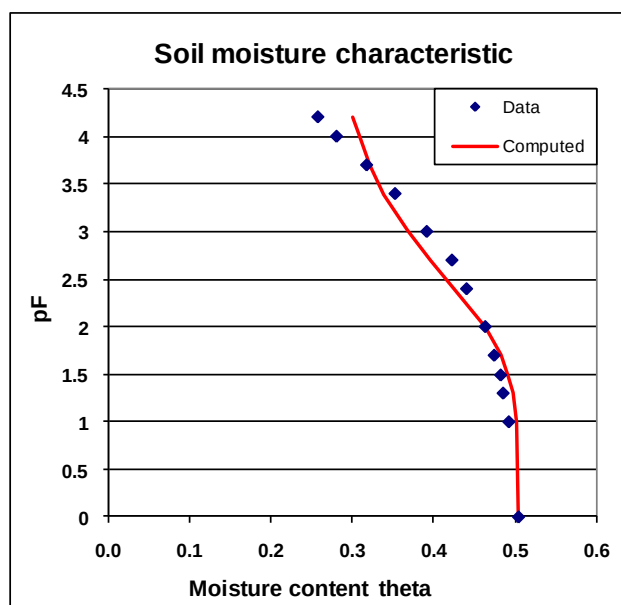


Fig. 9.4 Optimized pF curve compared with the data

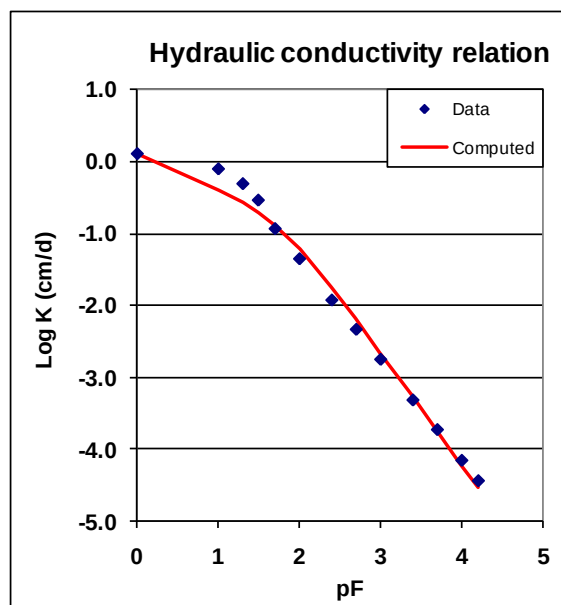


Fig. 9.5 Optimized K – h relation compared with the data

### 9.3 Computation of moisture profiles using the equation of Darcy

Steady vertical flow in unsaturated media  $q$  (cm/d) is described by Darcy's law, which may be written as

$$dz = \frac{-dh}{1 + \frac{q}{K(h)}} \quad (9.7)$$

Writing the equation of Darcy in this form allows the computation of a relation between the height above the water table  $z$  and the matric pressure at that height  $h$  for a steady flow  $q$ . The *pressure profile* is easily changed into a *moisture profile* given the soil moisture characteristic. Because the hydraulic conductivity  $K$  is a function of  $h$ , it is not possible to compute  $h$  for a given value of  $z$ . However,  $dz$  can be computed for a change in the matric pressure  $dh$ . Because the relation between  $K$  and  $h$  is very nonlinear, the change in  $dh$  has to be small for values of  $h$  close to zero, and may become larger for more negative values of  $h$ .

#### Assignment

- Set up a computational scheme as depicted in table (9.3) in sheet **Wnum** and use the soil for which the Van Genuchten parameters have been determined.
- Use small changes in  $h$  for wet conditions and larger steps (say -1000 cm) for dry conditions. Extend the table for values of  $h$  up to -16000 cm.
- After a change in  $h$  the moisture content  $\theta$  is computed with equation (9.5) for the average  $h$  value.
- Equation (9.6) may then be used to compute the average hydraulic conductivity  $K$  for the average  $h$ .
- Application of the equation of Darcy yields  $dz$ , and hence the new  $z$  value corresponding to the  $h$  value.
- Plot the moisture content up to a height of maximum 3 meters above the water table for  $q = -0.1$  cm/d.
- Save **Wnum**

#### Question:

Compute the maximum height of capillary rise for the same eight steady flow situations as in your lecture notes *Soil-Water-Atmosphere* (De Laat, 2009) and compare these values with those in the lecture notes. Explain the difference.

| Steady flow             |           | q =   | -0.1 |     |        |        |
|-------------------------|-----------|-------|------|-----|--------|--------|
| Application Darcy's law |           |       |      |     |        |        |
| h                       | average h | θ     | K    | dh  | dz     | z      |
| 0                       |           | 0.507 |      |     |        | 0      |
| -5                      | -2.5      | 0.507 | 0.68 | -5  | 5.9    | 5.9    |
| -10                     | -7.5      | 0.505 | 0.47 | -5  | 6.3    | 12.2   |
| -15                     | -12.5     | 0.503 | 0.37 | -5  | 6.8    | 19.0   |
| -20                     | -17.5     | 0.501 | 0.31 | -5  | 7.4    | 26.5   |
| -25                     | -22.5     | 0.499 | 0.26 | -5  | 8.1    | 34.6   |
| -30                     | -27.5     | 0.497 | 0.22 | -5  | 9.0    | 43.6   |
| -40                     | -35       | 0.493 | 0.18 | -10 | 21.9   | 65.5   |
| -50                     | -45       | 0.489 | 0.15 | -10 | 31.3   | 96.8   |
| -60                     | -55       | 0.484 | 0.12 | -10 | 57.9   | 154.7  |
| -80                     | -70       | 0.478 | 0.09 | -20 | 1000.0 | 1154.7 |
| -100                    | -90       | 0.470 | 0.07 | -20 | 1000.0 | 2154.7 |

Table 9.3 Example computation moisture profile

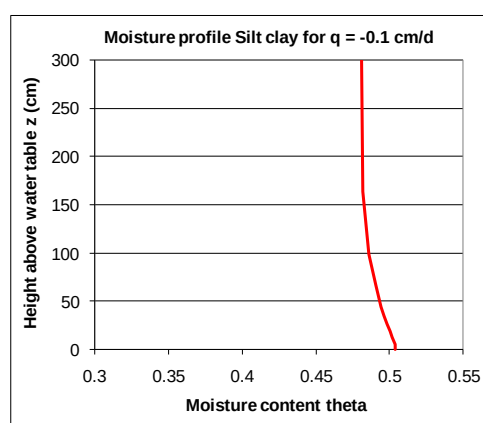


Fig. 9.6 Moisture profile for  $q = -0.1$  cm/d



## REFERENCES

- Allen, R.G., L.S. Pereira, D. Raes and M. Smith, 1998. Crop evapotranspiration guidelines for computing crop water requirements, Annex 4: Statistical analysis of weather data sets, FAO Irrigation and Drainage Paper no. 56, FAO, Rome: 229-243
- Brooks, R.H. and A.T. Corey, 1964. Hydraulic properties of porous media. Fort Collins, CO, Colorado State University Hydrology Paper 3
- Campbell, G.S., 1974. A simple method for determining unsaturated conductivity from moisture retention data. *Soil Science* 117: 311-314
- Chow, V.T. (Ed.), 1964. *Handbook of Applied Hydrology*. McGraw-Hill
- Cunnane, C., 1978. Unbiased plotting positions - a review. *J. of Hydrol.* 37:205-222
- Dahmen, E.R. and M.J. Hall, 1990. Screening of Hydrological Data: Tests for Stationarity and Relative Consistency. ILRI Pub. 49, ILRI, Wageningen.
- De Laat, P.J.M., 2009. Soil-Water-Atmosphere. Lecture note LN0413/09/1, UNESCO-IHE, Delft
- De Laat, P.J.M. & H.H.G. Savenije, 2008. Hydrology. Lecture note LN0262/08/1, UNESCO-IHE, Delft
- Gumbel, E.J., 1941. The Return Period of Flood Flows. *Ann. Math. Statist.* 12(2):163-190
- Guzman, J.A. and Chu, M.L., 2003. SPELL-Stat statistical analysis program. Universidad Industrial de Santander, Colombia.
- Mualem, Y., 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resour. Res.* 12: 513-522
- Rijtema, P.E., 1969. Soil moisture forecasting. Nota 513, ICW, Wageningen
- Shaw, E.M., K.J. Beven, N.A. Chappell & R. Lamb, 2010. *Hydrology in Practice*. Taylor & Francis Ltd, 560 p.
- Van Genuchten, M.T., 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal* 44: 892-898



## APPENDICES

### A Statistical backgrounds

#### A1 Terminology

Quantitative scientific data may be classified as either experimental or historical. The experimental data are measured through experiments and can usually be obtained repeatedly by experiments. The historical data, however, are collected from natural phenomena that can be observed only once and then will not occur again. Most hydrological data are historical data and can be treated as statistical variables. In their simplest form, statistical data will consist of a set of values of a variable, say the maximum seasonal floods observed during 30 years.

The following basic statistical terms may be noted:

- a *population* is the whole collection of values under consideration. It may be finite or infinite.
- a *sample* is a set of observed values, more or less representative of the population from which it is drawn.
- a *variable* (X) is the characteristic of a sample, for example the depth of rainfall.
- a *variate* (x) is an individual observation or the value of any variable.
- a *discrete variable* can contain only a finite number of values (or as many values as there are whole numbers), for example the number of rainy days.
- a *continuous variable* can contain all values within a certain range, for example the depth of rainfall.

A 30-year continuous record of flow at a hydrological station is only a sample of the stream flow history at that point. The population will be the set of all possible records at the station, under a fixed set of conditions. In order to draw conclusions about the population, the data in the sample must be random, independent and homogeneous.

A *random* sample is a sample where every value in the population has an equal chance of being included. In hydrology this is difficult to obtain since we have very little control over the selection of the sample. We will have to use the set of data being sampled over a period of time.

The assumption of having *independent* data in our sample may also be difficult to obtain in hydrology. The degree of independence varies with the nature of the data. Successive daily discharges are clearly not independent. Monthly flow data are much more independent of each other. A set of data representing the maximum seasonal floods for a 30-year period may be safely considered independent.

*Homogeneous* data belong to the same population; they have originated in the same way. Construction of a reservoir upstream of a station will cause a break in a homogeneous series of flow data from the station. A sample consisting of maximum floods where some are caused by snowmelt and some by heavy rain is another example of non-homogeneous data.

#### A2 Frequency

For discrete random variables, the number of occurrences of a variate is generally called *frequency*. When the number of occurrences of a variate, or the frequency, is plotted

against the variate as the abscissa, a pattern of distribution is obtained. This pattern is called the frequency *distribution*. It may be practicable to divide the range of values of the variate into equal class-intervals and then count the frequency in each interval. The frequency in any class-interval can be expressed as a fraction of the total frequency, and this fraction is called the relative *frequency* of the values in the interval. In the following example we have 75 observations of a variable with values between 2 and 12:

| Variable | Frequency<br>(f) | Relative<br>Frequency<br>% | Cumulative<br>frequency(X)<br>% |
|----------|------------------|----------------------------|---------------------------------|
| 2        | 1                | 1.3                        | 1.3                             |
| 3        | 3                | 4.0                        | 5.3                             |
| 4        | 7                | 9.3                        | 14.6                            |
| 5        | 15               | 20.1                       | 34.7                            |
| 6        | 20               | 26.7                       | 61.4                            |
| 7        | 13               | 17.4                       | 78.8                            |
| 8        | 8                | 10.6                       | 89.4                            |
| 9        | 4                | 5.3                        | 94.7                            |
| 10       | 2                | 2.7                        | 97.4                            |
| 11       | 1                | 1.3                        | 98.7                            |
| 12       | 1                | 1.3                        | 100.0                           |
| Total    | 75               | 100.0                      |                                 |

The first column gives the value of  $X$ , the variable. The next column gives the value of  $f$ , the frequency, showing how many times each particular value of  $X$  occurs. The third column gives the relative frequency in percent, and the last column shows the cumulative frequency.

A frequency table may be represented graphically by a *histogram*, which is a "bar-graph". With each class-interval as a base, a rectangle is constructed whose area represents the frequency in the interval. If the class-intervals are equal, as is usually the case, the heights

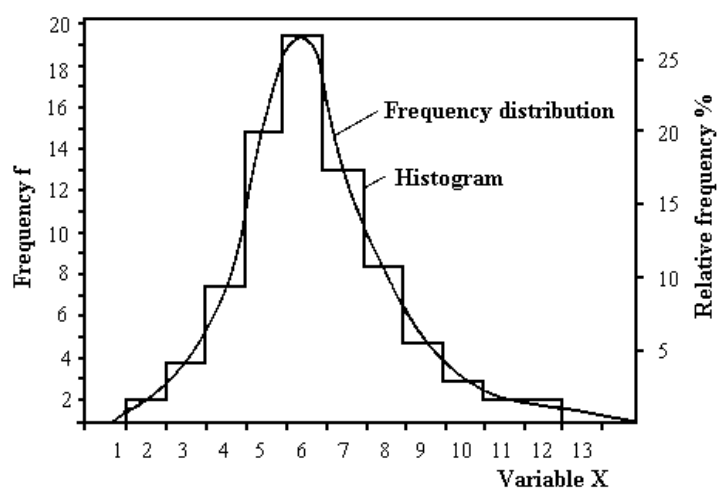


Fig. A2.1 Histogram and frequency distribution

of the rectangles will be proportional to the frequencies represented. The total frequency is represented by the total area. The pattern of distribution produced when the frequency is plotted against the variable is called the *frequency distribution*. The histogram for the values above is plotted in figure A2.1

Another way of showing a distribution in this example is by a cumulative frequency curve, see figure A2.2. The curve is drawn from the lowest value,  $x_1$

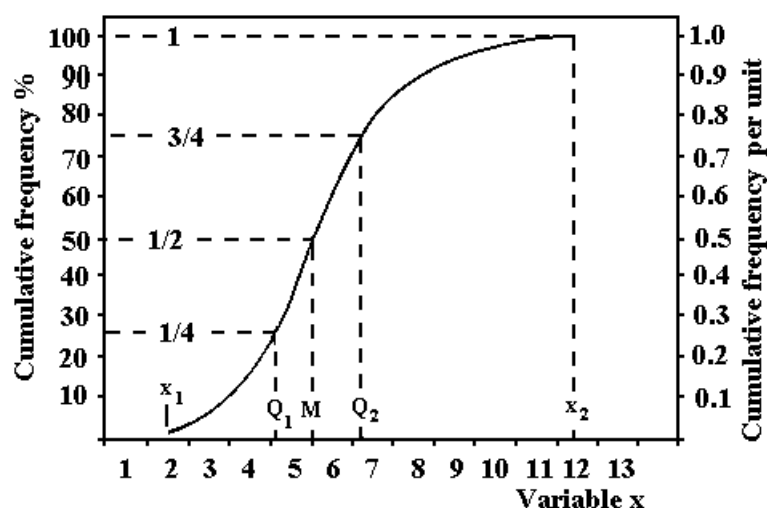


Fig. A2.2 Cumulative frequency curve

to the highest value  $x_2$  and illustrates how the total frequency is built up. It is useful in estimating the proportion of the distribution between set limits. The graded values of the variable are divided into two groups by the *median*  $M$ . We may further bisect these groups of values of the variate called the *quartiles*  $Q_1$ , and  $Q_2$ . Thus half of the total frequency will occur between the quartiles.

### A3 Statistical parameters

A great number of parameters are used to describe the characteristics of a statistical distribution. Some of the most important will be defined in what follows.

#### Measures of location

The **arithmetic mean** is the average most frequently used. It is obtained by adding together all the variates,  $\Sigma x$  and dividing by the total number of variates,  $N$ . This is expressed by

$$\bar{x} = \frac{\sum x}{N} \quad (\text{A.1})$$

The **median** is the middle value of the variate which divides the frequencies in a distribution into two equal portions. This is illustrated in figure A2.2. The arithmetic mean is more commonly used than the median. For skew distributions, however, the mean may be misleading. In such cases, the median will provide a better indication, because all variates greater or less than the median always occur half the time.

The **mode** is the variate which occurs most frequently. It corresponds to the peak of the frequency curve. For a grouped distribution the modal class can be defined as the class with the greatest frequency. The weighted mean of a set of numbers  $x_1, x_2, x_3, \dots, x_n$  whose relative importance is expressed numerically by some numbers  $w_1, w_2, w_3, \dots, w_n$  called the weight, is defined as

$$\bar{x}_w = \frac{\sum w x}{\sum w} \quad (\text{A.2})$$

This formula can be used, for example, in calculating the mean areal rainfall (Thiessen method) where the rain-gauge stations represent different portions of the total area.

#### Measures of variability

The **mean deviation** is defined as the mean of the absolute deviations of values from their mean, or

$$m.d. = \frac{\sum |x - \bar{x}|}{N} \quad (A.3)$$

The **standard deviation** is the measure of variability, or spread, which is most adaptable to statistical analysis. It is the square root of the mean-squared deviation of individual observations from their mean, or

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}} \quad (A.4)$$

which represents the standard deviation of the population. An estimate of this parameter from the sample is denoted by  $s$  and computed by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{N - 1}} = \sqrt{\frac{N}{N - 1} (\bar{x}^2 - \bar{x})^2} \quad (A.5)$$

where  $\bar{x}^2 = (\sum x^2) / N$

The **variance** is the square of the standard deviation which is denoted by  $\sigma^2$  or the population. The unbiased estimate of the sample variance is  $s^2$ .

The **coefficient of variation** is a measure of spread of the sample in relative terms:

$$C_v = \frac{s}{\bar{x}} \quad (A.6)$$

**Quartiles** and **percentiles** may be considered as measures of spread about the median (the 50-percentile value). With the variates arranged in ascending order of magnitude, the lower quartile is the value at the first quarter of the data series (25 percentile). The upper quartile marks the beginning of the top quarter of the data (the 75-percentile).

#### A4 Normal Distribution

The normal or Gaussian frequency distribution is the most important in statistical theory. It is a bell-shaped symmetrical distribution of a variate which may range from  $-\infty$  to  $+\infty$  (see figure A4.1). An important property of the normal curve is that it is completely determined if we are given its mean ( $\mu$ ) and standard deviation ( $\sigma$ ), the only two statistical parameters included in the mathematical equation for the curve:

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (A.7)$$

where  $y$  or  $f(x)$  is the frequency of occurrence of  $x$ . Most hydrological data are not normally distributed, but they can sometimes be normalized by various methods, such as using the logarithms or cube root of the random variates of the sample. For purposes of comparison, it is convenient to take the value of the mean ( $\mu$ ) as the origin of coordinates

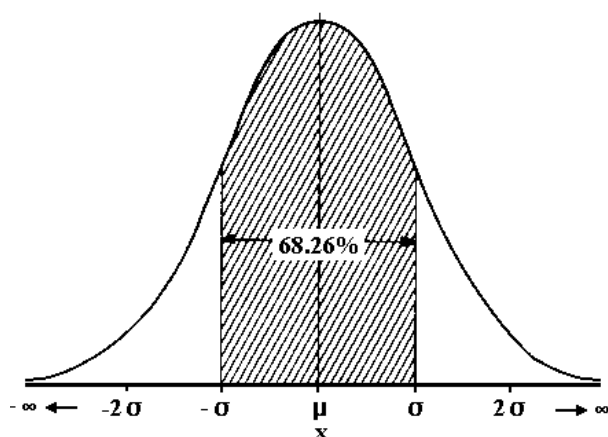
and measure along the horizontal axis in intervals of the standard deviation ( $\sigma$ ) as shown in figure A4.1. In this distribution the mean, median and mode are the same. It can be shown that:

50 % of the observations lie within  $\pm 0.67\sigma$

68 % of the observations lie within  $\pm 1\sigma$

95 % of the observations lie within  $\pm 2\sigma$

99 % of the observations lie within  $\pm 3\sigma$



This means that if a distribution is closely approximated by a normal curve, roughly 68 % of the cases will fall within one standard deviation from the mean, and 95 % of the cases will fall within two standard deviations. It is an aid to drawing the normal curve to know that it has inflexions at  $x = \pm\sigma$ , the mean being taken as origin, and that the tangents at these inflexions meet the x-axis at  $\pm 2\sigma$ . The curve can hardly be plotted over a wider range than  $\pm 3\sigma$  from the mean.

Fig. A4.1 Normal frequency distribution

#### *Probability and the standard normal distribution*

What do we mean by expressions such as ‘nine times out of ten’, or ‘odds of four to one’? How does an insurance firm calculate its premiums? How can a builder estimate the chance of wet weather delaying certain stages of a construction project? All these are aspects of probability, i.e. the branch of mathematics which enables us to calculate the likelihood of any particular outcome.

In any problem or experiment, each separate result is called an outcome. The particular happening we are looking for will be called the event. If  $n$  is the number of observations and the event we are looking for is the outcome on  $m$  occasions, we define the *probability* of the event

$$p = \frac{m}{n} = \text{the probability of success} \quad (\text{A.8})$$

and

$$q = \frac{n-m}{n} = 1 - p = \text{the probability of failure} \quad (\text{A.9})$$

thus  $p + q = 1$ .

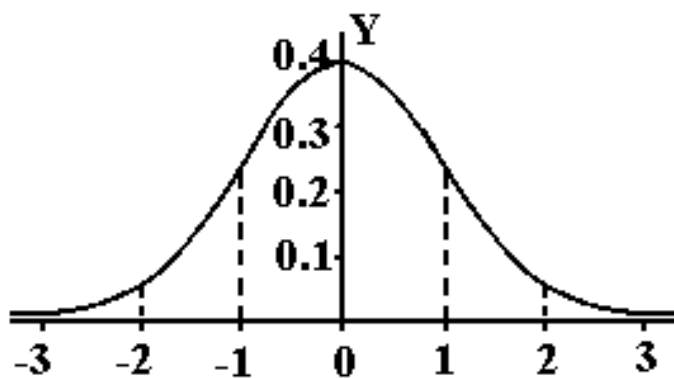


Fig. A4.2 Standard normal distribution curve

The extreme values of  $p$  are 0 and 1, corresponding to certain failure and certain success (if  $n$  is finite). When a probability is expressed in the form of odds, the ratio used is of favourable to unfavourable occurrences or vice versa. To say that the odds against an event are 7 to 2 means that the probability of its occurrence is 2/9.

The area between the normal frequency distribution curve (see figure A4.1) and the x-axis is equal to the total number of observations. For probability estimations a standardized form is used for the normal probability distribution curve in which the total area bounded by the curve and the x-axis is equal to unity.

Values of the variable are standardized to give a series of values ( $t$ ):

$$t = \frac{x - \mu}{\sigma} \quad (\text{A.10})$$

and the curve is defined by the equation

$$y = f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \quad (\text{A.11})$$

In such case we say that  $t$  is normally distributed with the mean,  $\mu_t = 0$  and a variance and standard deviation,  $\sigma_t^2$  and  $\sigma_t$  both equal to one (see figure A4.2).

Since the total area is one, the area under the curve between two ordinates  $a$  and  $b$  represents the probability that the variable lies between  $a$  and  $b$ . The table in appendix D gives the areas under this curve bounded by any positive value of  $t$  and  $t \rightarrow \infty$ . From this table the area between any two ordinates can be found by using the symmetry of the curve about  $t = 0$ . Example, the table gives an area of 0.242 for  $t \geq 0.70$ . This figure represents the probability that  $t$  is greater than 0.70. The probability that  $t$  lies between 0.0 and 0.70 is then the probability that  $t \geq 0$  minus the probability that  $t \geq 0.7$ , thus  $0.5 - 0.242 = 0.258$ .

#### Linearization

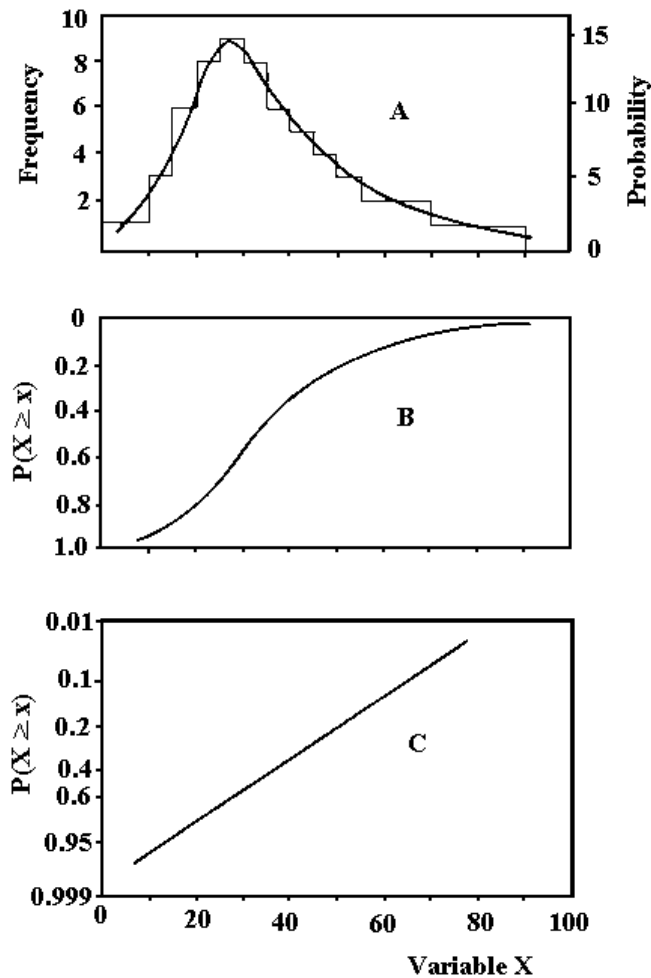
The probability distribution, also known as the *probability density function* (PDF), may be expressed as

$$p(a \leq X \leq b) = \int_a^b f(x) dx \quad ; \quad \int_{-\infty}^{\infty} f(x) dx = 1 \quad (\text{A.12})$$



where  $X$  is a specified value of the variable  $x$ ,  $f(x)$  is the frequency of occurrence of  $x$ , and  $p()$  is the probability that  $X$  lies between the values  $a$  and  $b$ . The same information can also be written in the form of the *cumulative distribution function* (CDF)

$$p(X \leq x) = F(x) = \int_{-\infty}^x f(x) dx \quad (\text{A.13})$$



**Fig. A4.3 Linearization of probability distribution.**  
**A - Frequency or probability density function (PDF),**  
**B - Cumulative distribution function (CDF),**  
**C - Linearized CDF on probability paper**

where  $p()$  is the probability of non-exceedance. The principle of linearization of a probability distribution is shown in figure A4.3. A variety of different distributions is used in hydrology and some of these are considered in these notes. The cumulative probability of a distribution may be represented graphically on probability paper which is specially designed for the distribution (e.g. Gumbel paper, normal- or lognormal distribution paper). The ordinate and the abscissa are so designed that the distribution plots as a straight line and the data to be fitted appear close to the straight line. The object of using the probability paper is to linearize the distribution so that the plotted data can be easily analysed for purposes of extrapolation or comparison. If we want to plot the cumulative distribution function using a spreadsheet, we cannot make use of the special designed plotting paper. Instead we plot the variable  $x$  against the reduced variate  $t$ , according to the equation

$$t = a(x - b) \quad (\text{A14})$$

where  $a$  and  $b$  are constants depending on the type of distribution (see for the Gumbel distribution page 10 and for the normal distribution equation 2.5). We then plot in the same graph the observed data and compare these with the linear relation. The disadvantage of this approach is that the graph is more difficult to read than the graph on special paper, since the reduced variate is to be 'translated' in the probability through an equation (e.g. equation 1.7 for Gumbel) or a table (e.g. appendix D for the normal distribution).

### A5 Correlations

Through correlations the interrelationship between two or more variables is measured, therefore it is an example of a statistical association method. A correlation coefficient,  $\rho$  explains the degree of the association as a linear dependence. Correlation coefficients measure only the degree of linear association and for example the correlation of a parabola is zero, because it has no linear term in it.

There are several types of correlation coefficients used in statistics. For hydrologic purposes the most commonly used correlation coefficient is the Pearson Product-Moment correlation coefficient. This coefficient of linear correlation ( $-1 < \rho < +1$ ), between two variables  $X$  and  $Y$  is defined as

$$\rho = \frac{\frac{1}{n} \sum ((x_i - \bar{x}) * (y_i - \bar{y}))}{\left( \frac{1}{n} \sum (x_i - \bar{x})^2 * \frac{1}{n} \sum (y_i - \bar{y})^2 \right)^{0.5}} = \frac{\sum ((x_i - \bar{x}) * (y_i - \bar{y}))}{\left( \sum (x_i - \bar{x})^2 * \sum (y_i - \bar{y})^2 \right)^{0.5}} \quad (\text{A.15})$$

where  $n$  total number of observations  
 $i$  1, 2, 3, etc.  
 $x_i, y_i$   $i^{\text{th}}$  observation of series  $x$  and  $y$

In the first part of this equation, the numerator is called the covariance,  $s_{xy}$ , and the two terms in the denominator are the standard deviations of variables  $x$  and  $y$  respectively. Then the equation can be written as

$$\rho = \frac{s_{xy}}{s_x * s_y} \quad (\text{A.16})$$

### A6 Tests for stationarity and homogeneity

#### A6.1 Principles

In tests on *stationarity* and *homogeneity* of time series, basic statistical tests are performed, which are generally used for the comparison of samples. For the analysis it is often necessary that the original series is split into two or more parts.

A certain parameter of the series will be analysed. This can be the mean, variance or another parameter like the correlation coefficient. In order to carry out the analysis a parameter is defined, which is known as the test-statistic. This parameter can be the mean or variance itself, a derivative of the mean or variance or another defined parameter.

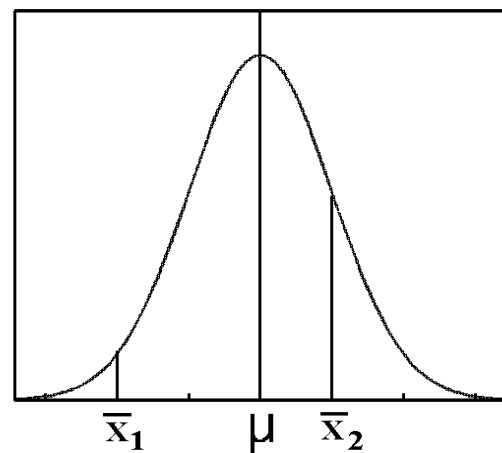


Fig. A6.1 Scatter of sample means around the true mean

When analysing the values of test-statistics, the problem arises to qualify a comparison. For instance for absence of trends in a time series we want to prove that the mean of the first 10 years of a series is not significantly different from the last 10 years. However, both values will never be equal. But what difference is accepted and what is not? Obviously, the larger the series, the closer will be the two values of the mean. Both values will also come closer to the true mean ( $\mu$ ) when the number of years increases.

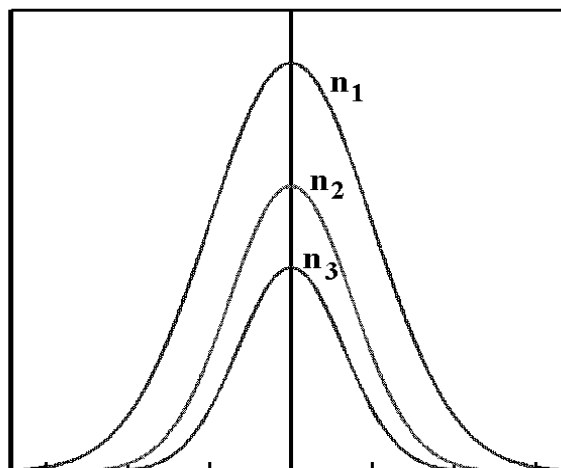


Fig. A6.2 Distributions as a function of the number of elements in a sample

In case one takes several samples (all with equal number of years) from the infinite time series (called the population), the values of the mean of these samples may differ in magnitude, thus  $\bar{x}_1 \neq \bar{x}_2 \neq \bar{x}_3 \neq \dots$  etc. A histogram of these values can be represented by a smooth curve, defining the scatter of the values around the true mean ( $\mu$ ), see figure A6.1.

The number of elements,  $n$  in a sample has influence on the shape of the curve, see figure A6.2. The curves are distributions, which are mathematically defined and often tabulated. Such a curve can be a normal distribution. Other distributions are Student's-t or Fisher-F distributions.

Often we have a situation where the real mean,  $\mu$  is not known. From the sample a mean,  $\bar{x}$  can be calculated. We may now assume a certain real mean,  $\mu_0$  and test on the basis of our sample whether there is ground to reject the assumption. The statistical procedure introduces a null hypothesis,  $H_0$  and an alternative hypothesis,  $H_1$ , thus

$$H_0 : \mu = \mu_0$$

$$H_1 : \mu < > \mu_0$$

On the basis of a sample it will never be possible to prove with 100% certainty that  $H_0$  nor  $H_1$  is correct. One should realize that even when accepting one of the two hypotheses there is a chance of making the error that it is still not the correct one. When  $H_0$  is always stated in the way that it is the hypothesis we wish to prove, the worst that can happen is to reject  $H_0$ , while it is true (like the judge who is convicting an innocent suspect). In statistics this is called a type I error.

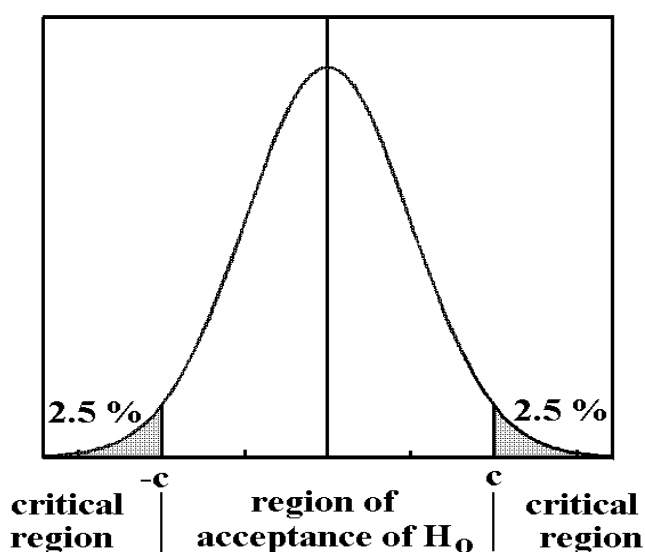


Fig. A6.3 Critical regions and critical values (confidence level 95%)

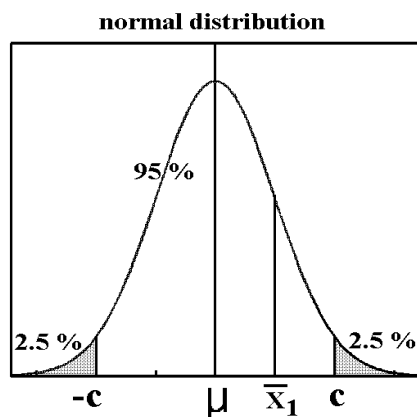


Fig. A6.4 Normal distribution

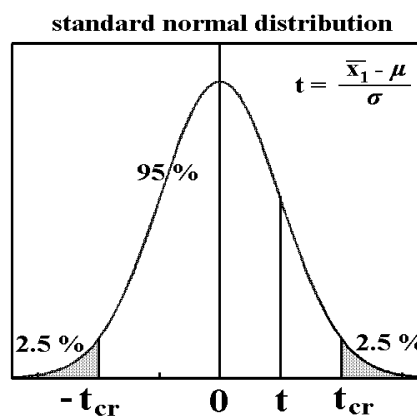


Fig. A6.5 Standard Normal distribution

The region representing the area of rejecting  $H_0$  must be minimized to an acceptable level. This area is called the critical region, bordered by critical values, or confidence limits ( $\pm c$ ), see figure A6.3.

According to the distribution of  $x$  (for instance Normal, Students-t, Fisher-F) the critical values mark the probability of a type I error. The probability of a type I error is called the significance level  $\alpha$ , while  $1 - \alpha$  is the confidence level. A confidence level of 95% is often applied ( $\alpha = 0.05$ ).

One can use the theory of a normal distribution, in case it is valid, and its relation with the standard normal distribution to calculate the critical values (confidence limits  $\pm c$  and  $-c$ ), see figures A6.4 and A6.5.

For a level of significance of 5%, the confidence limits,  $t_{cr}$ , for the standard normal distribution are:  $t_{cr} = \pm 1.96$ . The confidence limits  $\pm c$  for the real distribution are then calculated using its relation with the standard normal distribution as follows

$$t_{cr} = \frac{c - \mu}{\sigma} \quad (\text{A.17})$$

where  $\mu$  true mean of the population  
 $\sigma$  true standard deviation of the population

For  $\sigma$  is known that

$$\sigma = \frac{s}{\sqrt{n}} \quad (\text{A.18})$$

where  $s$  standard deviation of the sample means  
 $n$  number of elements in the sample

Now we conclude that, in case  $\mu$  is known, it can be verified whether a certain calculated mean is accepted as representing the population with a confidence level of 95%. In case  $\mu$  is unknown we can verify an assumption  $\mu = \mu_0$  with a certain confidence level.

What we did until now in fact was comparing one sample with its population, through the test statistic *mean*. However, in general the population parameters  $\mu$  and  $\sigma$  are not known. It is also possible to compare statistics of samples drawn from two different populations. In case both are normal distributions, the difference of the means  $d = \bar{x}_1 - \bar{x}_2$  is also a normal distribution with

$$\mu_d = \mu_1 - \mu_2 \quad (\text{A.19})$$

and

$$\sigma_d^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \quad (\text{A.20})$$

Assuming that  $\sigma_1 \approx \sigma_2$ , the pooled variance  $\sigma_d^2$  may as follows be estimated from the sample variances  $s_1^2$  and  $s_2^2$

$$\sigma_d^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} * \left( \frac{1}{n_1} + \frac{1}{n_2} \right) \quad (\text{A.21})$$

In case the two samples belong to the same population,  $\mu_1 = \mu_2$  and  $\mu_d = 0$ .

Again according to the principles of a standard normal distribution, transformation from the normal distribution is by (also see Figure A6.6)

$$t = \frac{d - \mu_d}{\sigma_d} \quad (\text{A.22})$$

To test now whether the two samples are from the same population is testing  $\mu_1 = \mu_2$  (the null hypothesis,  $H_0$ ) against  $\mu_1 < > \mu_2$  (the alternative hypothesis,  $H_1$ ).

As indicated above this is done by defining the confidence limits under assumption of  $H_0$ , while not committing a type I error.

Assuming  $H_0: \mu_d = 0$

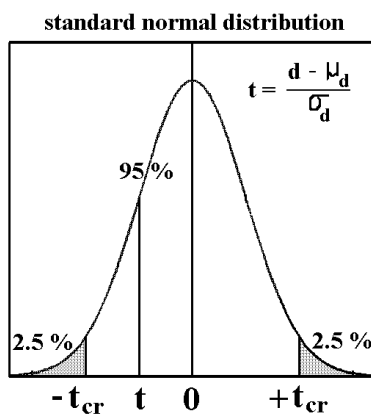


Fig. A6.6 Standard Normal distribution

$$t = \frac{d}{\sigma_d} \quad (\text{A.23})$$

For the confidence limits this means

$$t_{cr} = \pm \frac{c}{\sigma_d} \quad (\text{A.24})$$

For  $-t_{cr} < t < t_{cr}$  it is accepted with a confidence level according the confidence limits that the averages  $\bar{x}_1$  and  $\bar{x}_2$  originate from the same population and that there is no trend.

Note that the test statistic for the standard normal distribution is the variable

$$t_t = \frac{d}{\sigma_d} \quad (\text{A.25})$$

or

$$t_t = \frac{\bar{x}_1 - \bar{x}_2}{\left( \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} + \left[ \frac{1}{n_1} + \frac{1}{n_2} \right] \right)^{0.5}} \quad (\text{A.26})$$

With small samples ( $n < 30$ ) the standard normal distribution does not any longer give satisfactory results. Then one can better use the so-called Student's-t distribution. For the comparison of means, the test statistic remains as given above.

Also for other parameters acceptance within predefined levels of significance can be defined. In all cases a test statistic must be defined as well as its distribution. This distribution is not necessarily always a normal distribution or the Students-t distribution. For instance for significance of variance the Fisher distribution is applied. In a Spearman rank test the significance of a correlation coefficient is tested through a Students-t test.

#### A6.2 The spearman's rank test; test for absence of trend

The hypothesis is tested that there is no correlation between the order in which the data are observed and the increase (or decrease) in magnitude of those data. The test is usually performed on the whole data series but it is possible to select specific periods.

Two series are compared related to the rank of the data.  $Kx_i$  is the rank of the data as it was measured.  $Ky_i$  is the series of the rank of the same data in ascending or descending order.

The Spearman coefficient of rank correlation  $R_{sp}$  is then defined as

$$R_{sp} = 1 - \frac{6 * \sum D_i^2}{n * (n^2 - 1)} \quad (\text{A.27})$$

where

$$D_i = Kx_i - Ky_i \quad (\text{A.28})$$

When two or more observations have the same value, the average rank  $Ky_i$  is calculated.

A test-statistic  $t_t$  is used to test the null hypothesis  $H_0: R_{sp} = 0$  against the alternative hypothesis  $H_1: R_{sp} < > 0$ . The test statistic is defined as

$$t_t = R_{sp} * \left( \frac{n - 2}{1 - R_{sp}^2} \right)^{0.5} \quad (\text{A.29})$$

The test statistic  $t_t$  has Student's t-distribution with  $\nu = n - 2$  degrees of freedom, where  $n$  is the number of elements in a sample. Appendix B contains a table of the Student's-t distribution for a level of significance of 5% (two-tailed). The two sided critical region  $U$  of the test statistic  $t_t$  for a level of significance of 5% is bounded by

$$\{-\infty, t(v, 2.5\%)\} \cup \{t(v, 97.5\%), +\infty\} \quad (\text{A.30})$$

and the hypothesis  $H_0$  is accepted when the computed  $t_t$  is not contained in the critical region. In other words, one concludes that there is no trend when

$$t(v, 2.5\%) < t_t < t(v, 97.5\%) \quad (\text{A.31})$$

### A6.3 F-test for the stability of the variance

The appropriate test statistic is the ratio of the variances of two non-overlapping sub-sets of a series. The distribution of the variance ratio of samples from a normal distribution is known as the F-distribution or Fisher distribution. Even in the absence of the normal distribution it is generally accepted that the F-test provides a useful indication for the stability of the variance.

The number of data  $n$  in the test series should be equal to or greater than 10.

The test statistic is thus

$$F_t = \frac{\sigma_1^2}{\sigma_2^2} = \frac{s_1^2}{s_2^2} \quad (\text{A.32})$$

The null hypothesis for the F-test is the equality of variances,  $H_0: s_1^2 = s_2^2$  and the alternative hypothesis is  $H_1: s_1^2 \neq s_2^2$ . The rejection region is bounded by

$$\{0, F(v_1, v_2, 2.5\%)\} \cup \{F(v_1, v_2, 97.5\%), +\infty\} \quad (\text{A.33})$$

where  $v_1$  and  $v_2$  are the respective numbers of degrees of freedom of the numerator and dominator. It should be noted that  $v_1 = n_1 - 1$  and  $v_2 = n_2 - 1$  where  $n_1$  and  $n_2$  are the number of observations in each sub-set.

In other words, the variability of the data is considered to be stable and the standard deviation  $s$  can be used as an estimate for the population standard deviation, when

$$F(v_1, v_2, 2.5\%) < F_t < F(v_1, v_2, 97.5\%) \quad (\text{A.34})$$

The F-distribution is not symmetrical for the number of degrees of freedom of the numerator and dominator. Tables should therefore be applied properly with usually  $v_1$  horizontally and  $v_2$  vertically. See appendix C for a condensed table of the F-distribution with a confidence level of 5%.

The procedure to apply the F-test on data series is to subdivide the series in two or three (approximately) equal non-overlapping sub-sets. The standard deviation is computed for each subset. The limits of a sub-set can also be selected in such a way that the set will cover a suspect period. Such a period is then compared with a non-suspect period or periods.

#### A6.4 t-test for stability of the mean

The t-test is only valid if the variance of the time series is stable. Hence, the F-test for the stability of the variance has to be performed before this test.

The means of the same subsets can be compared to verify whether the mean is stable during the whole period of observations. A suitable test statistic for testing the null hypothesis  $H_0 : \bar{x}_1 = \bar{x}_2$  against the alternative hypothesis  $H_1 : \bar{x}_1 \neq \bar{x}_2$  is

$$t_t = \frac{\bar{x}_1 - \bar{x}_2}{\left( \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} + \left[ \frac{1}{n_1} + \frac{1}{n_2} \right] \right)^{0.5}} \quad (\text{A.35})$$

where

$n_i$  the number of data in subset  $i$

$\bar{x}_i$  the mean of the subset  $i$

$s_i^2$  the variance of the subset

The test statistic,  $t_t$  has Student's-t distribution for a sample which is normally distributed. The test may also be applied for non-normal distributions, best for approximately equal lengths of subsets.

The two sided critical region U for the test statistic is defined as

$$\{-\infty, t(v, 2.5\%)\} \cup \{t(v, 97.5\%), +\infty\} \quad (\text{A.36})$$

and the number of degrees of freedom is  $v = n_1 + n_2 - 2$

The null-hypothesis  $H_0$  is accepted when the computed  $t_t$  is not contained in the critical region. In other words, one concludes that  $\bar{x}_1 = \bar{x}_2$  when

$$t(v, 2.5\%) < t_t < t(v, 97.5\%) \quad (\text{A.37})$$



## B Student t-distribution

Percentile points of Student t-distribution for a 5% level of significance

| $p = P(t \leq t_p)$ : | 0.025 | 0.975 |
|-----------------------|-------|-------|
| $v$                   |       |       |
| 4                     | -2.78 | 2.78  |
| 5                     | -2.57 | 2.57  |
| 6                     | -2.54 | 2.54  |
| 7                     | -2.36 | 2.36  |
| 8                     | -2.31 | 2.31  |
| 9                     | -2.26 | 2.26  |
| 10                    | -2.23 | 2.23  |
| 11                    | -2.20 | 2.20  |
| 12                    | -2.18 | 2.18  |
| 14                    | -2.14 | 2.14  |
| 16                    | -2.12 | 2.12  |
| 18                    | -2.10 | 2.10  |
| 20                    | -2.09 | 2.09  |
| 24                    | -2.06 | 2.06  |
| 30                    | -2.04 | 2.04  |
| 40                    | -2.02 | 2.02  |
| 60                    | -2.00 | 2.00  |
| 100                   | -1.98 | 1.98  |
| 160                   | -1.97 | 1.97  |
|                       | -1.96 | 1.96  |

Remark: Take the next higher value for  $v$  if the required number of degrees of freedom is not listed.



### C Percentile points of the Fischer-F distribution for 5% level of significance

| p =      |       | v1: 4 | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 14   | 16   |
|----------|-------|-------|------|------|------|------|------|------|------|------|------|------|
| P(F<=Fp) |       |       |      |      |      |      |      |      |      |      |      |      |
| 0.025    | v2: 5 | .107  | .140 | .169 |      |      |      |      |      |      |      |      |
| 0.975    |       | 7.39  | 7.15 | 6.98 |      |      |      |      |      |      |      |      |
| 0.025    | 6     |       | .143 | .172 | .195 |      |      |      |      |      |      |      |
| 0.975    |       |       | 5.99 | 5.82 | 5.70 |      |      |      |      |      |      |      |
| 0.025    | 7     |       |      | .176 | .200 | .221 |      |      |      |      |      |      |
| 0.975    |       |       |      | 5.12 | 4.99 | 4.90 |      |      |      |      |      |      |
| 0.025    | 8     |       |      |      | .204 | .226 | .244 |      |      |      |      |      |
| 0.975    |       |       |      |      | 4.53 | 4.43 | 4.36 |      |      |      |      |      |
| 0.025    | 9     |       |      |      |      | .230 | .248 | .265 |      |      |      |      |
| 0.975    |       |       |      |      |      | 4.10 | 4.03 | 3.96 |      |      |      |      |
| 0.025    | 10    |       |      |      |      |      | .252 | .269 | .284 |      |      |      |
| 0.975    |       |       |      |      |      |      | 3.78 | 3.72 | 3.66 |      |      |      |
| 0.025    | 11    |       |      |      |      |      |      | .273 | .288 | .301 |      |      |
| 0.975    |       |       |      |      |      |      |      | 3.53 | 3.47 | 3.43 |      |      |
| 0.025    | 12    |       |      |      |      |      |      |      | .292 | .305 | .328 |      |
| 0.975    |       |       |      |      |      |      |      |      | 3.32 | 3.28 | 3.21 |      |
| 0.025    | 14    |       |      |      |      |      |      |      |      | .312 | .336 | .355 |
| 0.975    |       |       |      |      |      |      |      |      |      | 3.05 | 2.98 | 2.92 |

continued

[illegible]







## E Parameters for Gumbel Type I distribution

| N  | $\bar{y}_N$ | $\sigma_N$ | N  | $\bar{y}_N$ | $\sigma_N$ | N    | $\bar{y}_N$ | $\sigma_N$ |
|----|-------------|------------|----|-------------|------------|------|-------------|------------|
| 8  | 0.48430     | 0.90430    | 35 | 0.54034     | 1.12847    | 64   | 0.55330     | 1.17930    |
| 9  | 0.49020     | 0.92880    | 36 | 0.54100     | 1.13130    | 66   | 0.55380     | 1.18140    |
| 10 | 0.49520     | 0.94970    | 37 | 0.54180     | 1.13390    | 68   | 0.55430     | 1.18340    |
| 11 | 0.49960     | 0.96760    | 38 | 0.54240     | 1.13630    | 70   | 0.55477     | 1.18536    |
| 12 | 0.50350     | 0.98330    | 39 | 0.54300     | 1.13880    | 72   | 0.55520     | 1.18730    |
| 13 | 0.50700     | 0.99720    | 40 | 0.54362     | 1.14132    | 74   | 0.55570     | 1.18900    |
| 14 | 0.51000     | 1.00950    | 41 | 0.54420     | 1.14360    | 76   | 0.55610     | 1.19060    |
| 15 | 0.51280     | 1.02057    | 42 | 0.54480     | 1.14580    | 78   | 0.55650     | 1.19230    |
| 16 | 0.51570     | 1.03160    | 43 | 0.54530     | 1.14800    | 80   | 0.55688     | 1.19382    |
| 17 | 0.51810     | 1.04110    | 44 | 0.54580     | 1.14990    | 82   | 0.55720     | 1.19530    |
| 18 | 0.52020     | 1.04930    | 45 | 0.54630     | 1.15185    | 84   | 0.55760     | 1.19670    |
| 19 | 0.52200     | 1.05660    | 46 | 0.54680     | 1.15380    | 86   | 0.55800     | 1.19800    |
| 20 | 0.52355     | 1.06283    | 47 | 0.54730     | 1.15570    | 88   | 0.55830     | 1.19940    |
| 21 | 0.52520     | 1.06960    | 48 | 0.54770     | 1.15740    | 90   | 0.55860     | 1.20073    |
| 22 | 0.52680     | 1.07540    | 49 | 0.54810     | 1.15900    | 92   | 0.55890     | 1.20200    |
| 23 | 0.52830     | 1.08110    | 50 | 0.54854     | 1.16066    | 94   | 0.55920     | 1.20320    |
| 24 | 0.52960     | 1.08640    | 51 | 0.54890     | 1.16230    | 96   | 0.55950     | 1.20440    |
| 25 | 0.53086     | 1.09145    | 52 | 0.54930     | 1.16380    | 98   | 0.55980     | 1.20550    |
| 26 | 0.53200     | 1.09610    | 53 | 0.54970     | 1.16530    | 100  | 0.56002     | 1.20649    |
| 27 | 0.53320     | 1.10040    | 54 | 0.55010     | 1.16670    | 150  | 0.56461     | 1.22534    |
| 28 | 0.53430     | 1.10470    | 55 | 0.55040     | 1.16810    | 200  | 0.56715     | 1.23598    |
| 29 | 0.53530     | 1.10860    | 56 | 0.55080     | 1.16960    | 250  | 0.56878     | 1.23292    |
| 30 | 0.53622     | 1.11238    | 57 | 0.55110     | 1.17080    | 300  | 0.56993     | 1.24786    |
| 31 | 0.53710     | 1.11590    | 58 | 0.55150     | 1.17210    | 400  | 0.57144     | 1.25450    |
| 32 | 0.53800     | 1.11930    | 59 | 0.55180     | 1.17340    | 500  | 0.57240     | 1.25880    |
| 33 | 0.53880     | 1.12260    | 60 | 0.55208     | 1.17467    | 750  | 0.57377     | 1.26506    |
| 34 | 0.53960     | 1.12550    | 62 | 0.55270     | 1.17700    | 1000 | 0.57450     | 1.26851    |





## F Soil Physical Data International Soil Series

Reference: Rijtema(1969)

| Pressure<br>head(cm)               | 0       | -10     | -20    | -31   | -50    | -100    | -250     | -500     | -1000    | -2500     | -5000     | -10000    | -16000 |
|------------------------------------|---------|---------|--------|-------|--------|---------|----------|----------|----------|-----------|-----------|-----------|--------|
| 1. Coarse sand                     |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.395                              | 0.215   | 0.145   | 0.107  | 0.066 | 0.032  | 0.024   | 0.018    | 0.016    | 0.015    | 0.014     | 0.013     | 0.012     |        |
| 1120.0                             | 1100.00 | 119.000 | 10.100 | 0.144 | 0.0001 | 0.00004 | 0.000013 | 0.000005 | 0.000001 | 0.0000005 | 0.0000002 | 0.0000001 |        |
| 2. Medium coarse sand              |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.365                              | 0.331   | 0.300   | 0.274  | 0.160 | 0.095  | 0.062   | 0.052    | 0.038    | 0.031    | 0.025     | 0.020     | 0.017     |        |
| 300.0                              | 75.50   | 19.000  | 4.160  | 0.302 | 0.0010 | 0.00028 | 0.000100 | 0.000040 | 0.000011 | 0.0000042 | 0.0000016 | 0.0000008 |        |
| 3. Medium fine sand                |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.350                              | 0.325   | 0.316   | 0.305  | 0.260 | 0.155  | 0.077   | 0.061    | 0.050    | 0.043    | 0.032     | 0.025     | 0.023     |        |
| 110.0                              | 48.30   | 21.200  | 8.600  | 1.800 | 0.0300 | 0.00140 | 0.000550 | 0.000210 | 0.000058 | 0.0000220 | 0.0000083 | 0.0000043 |        |
| 4. Fine sand                       |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.365                              | 0.352   | 0.335   | 0.328  | 0.292 | 0.196  | 0.147   | 0.129    | 0.092    | 0.065    | 0.053     | 0.047     | 0.042     |        |
| 50.0                               | 30.30   | 18.400  | 10.600 | 4.100 | 0.3370 | 0.00480 | 0.001800 | 0.000680 | 0.000190 | 0.0000720 | 0.0000270 | 0.0000140 |        |
| 5. Humous loamy medium coarse sand |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.470                              | 0.460   | 0.448   | 0.440  | 0.424 | 0.405  | 0.336   | 0.293    | 0.233    | 0.174    | 0.140     | 0.117     | 0.105     |        |
| 1.0                                | 0.76    | 0.584   | 0.434  | 0.261 | 0.0680 | 0.00660 | 0.002500 | 0.000950 | 0.000260 | 0.0000990 | 0.0000380 | 0.0000190 |        |
| 6. Light loamy medium coarse sand  |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.394                              | 0.374   | 0.363   | 0.353  | 0.326 | 0.280  | 0.232   | 0.205    | 0.180    | 0.151    | 0.130     | 0.111     | 0.100     |        |
| 2.3                                | 1.31    | 0.747   | 0.403  | 0.138 | 0.0083 | 0.00230 | 0.000870 | 0.000330 | 0.000092 | 0.0000350 | 0.0000130 | 0.0000068 |        |
| 7. Loamy medium coarse sand        |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.301                              | 0.282   | 0.272   | 0.265  | 0.247 | 0.209  | 0.171   | 0.141    | 0.100    | 0.056    | 0.043     | 0.030     | 0.021     |        |
| 0.4                                | 0.27    | 0.188   | 0.124  | 0.060 | 0.0091 | 0.00092 | 0.000350 | 0.000130 | 0.000037 | 0.0000140 | 0.0000053 | 0.0000027 |        |
| 8. Loamy fine sand                 |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.439                              | 0.399   | 0.355   | 0.307  | 0.249 | 0.179  | 0.140   | 0.115    | 0.099    | 0.085    | 0.072     | 0.065     | 0.060     |        |
| 26.5                               | 17.80   | 12.000  | 7.720  | 3.620 | 0.4950 | 0.00720 | 0.002700 | 0.001000 | 0.000290 | 0.0001100 | 0.0000410 | 0.0000210 |        |
| 9. Sandy loam                      |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.465                              | 0.442   | 0.426   | 0.419  | 0.360 | 0.260  | 0.180   | 0.142    | 0.118    | 0.092    | 0.079     | 0.068     | 0.061     |        |
| 16.5                               | 7.90    | 3.780   | 1.680  | 0.414 | 0.0100 | 0.00011 | 0.000040 | 0.000015 | 0.000004 | 0.0000016 | 0.0000006 | 0.0000003 |        |
| 10. Loess loam                     |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.455                              | 0.436   | 0.410   | 0.385  | 0.373 | 0.340  | 0.269   | 0.232    | 0.203    | 0.170    | 0.143     | 0.122     | 0.110     |        |
| 14.5                               | 8.88    | 5.440   | 3.170  | 1.250 | 0.1080 | 0.00990 | 0.003800 | 0.001400 | 0.000390 | 0.0001500 | 0.0000570 | 0.0000290 |        |
| 11. Fine sandy loam                |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.504                              | 0.488   | 0.486   | 0.482  | 0.468 | 0.423  | 0.255   | 0.224    | 0.175    | 0.132    | 0.112     | 0.096     | 0.087     |        |
| 12.0                               | 11.90   | 9.360   | 7.130  | 4.450 | 1.2900 | 0.03100 | 0.004400 | 0.001700 | 0.000460 | 0.0001800 | 0.0000670 | 0.0000340 |        |
| 12. Silt loam                      |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.509                              | 0.497   | 0.487   | 0.484  | 0.474 | 0.461  | 0.400   | 0.279    | 0.205    | 0.150    | 0.125     | 0.103     | 0.092     |        |
| 6.5                                | 5.32    | 4.360   | 3.500  | 2.390 | 0.8800 | 0.04400 | 0.007900 | 0.003000 | 0.000830 | 0.0003100 | 0.0001200 | 0.0000620 |        |
| 13. Loam                           |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.503                              | 0.486   | 0.483   | 0.480  | 0.467 | 0.420  | 0.281   | 0.248    | 0.213    | 0.167    | 0.142     | 0.116     | 0.098     |        |
| 5.0                                | 3.97    | 3.150   | 2.440  | 1.580 | 0.4960 | 0.01600 | 0.002400 | 0.000910 | 0.000250 | 0.0000950 | 0.0000360 | 0.0000190 |        |
| 14. Sandy clay loam                |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.432                              | 0.407   | 0.387   | 0.376  | 0.359 | 0.338  | 0.309   | 0.288    | 0.263    | 0.240    | 0.215     | 0.194     | 0.180     |        |
| 23.5                               | 16.50   | 11.600  | 7.870  | 4.020 | 0.6890 | 0.01500 | 0.005600 | 0.002100 | 0.000590 | 0.0002200 | 0.0000840 | 0.0000440 |        |
| 15. Silty clay loam                |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.475                              | 0.438   | 0.421   | 0.410  | 0.394 | 0.372  | 0.335   | 0.305    | 0.279    | 0.250    | 0.222     | 0.195     | 0.185     |        |
| 1.5                                | 1.18    | 0.934   | 0.719  | 0.459 | 0.1400 | 0.01600 | 0.006000 | 0.002300 | 0.000630 | 0.0002400 | 0.0000900 | 0.0000470 |        |
| 16. Clay loam                      |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.445                              | 0.429   | 0.424   | 0.421  | 0.415 | 0.411  | 0.385   | 0.366    | 0.344    | 0.342    | 0.286     | 0.265     | 0.255     |        |
| 1.0                                | 0.78    | 0.609   | 0.464  | 0.289 | 0.0840 | 0.00200 | 0.000280 | 0.000110 | 0.000030 | 0.0000110 | 0.0000042 | 0.0000022 |        |
| 17. Light clay                     |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.453                              | 0.435   | 0.418   | 0.405  | 0.390 | 0.360  | 0.336   | 0.315    | 0.294    | 0.270    | 0.245     | 0.224     | 0.215     |        |
| 3.5                                | 2.94    | 2.470   | 2.040  | 1.470 | 0.6140 | 0.04500 | 0.009300 | 0.003500 | 0.000970 | 0.0003700 | 0.0001400 | 0.0000720 |        |
| 18. Silty clay                     |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.507                              | 0.492   | 0.485   | 0.482  | 0.474 | 0.463  | 0.440   | 0.422    | 0.391    | 0.352    | 0.317     | 0.280     | 0.257     |        |
| 1.3                                | 0.80    | 0.498   | 0.294  | 0.118 | 0.0450 | 0.01200 | 0.004700 | 0.001800 | 0.000490 | 0.0001900 | 0.0000710 | 0.0000370 |        |
| 19. Basin clay                     |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.540                              | 0.533   | 0.529   | 0.527  | 0.526 | 0.519  | 0.493   | 0.470    | 0.443    | 0.402    | 0.375     | 0.344     | 0.321     |        |
| 0.2                                | 0.14    | 0.094   | 0.062  | 0.030 | 0.0077 | 0.00210 | 0.000810 | 0.000310 | 0.000085 | 0.0000320 | 0.0000120 | 0.0000063 |        |
| 20. Peat                           |         |         |        |       |        |         |          |          |          |           |           |           |        |
| 0.863                              | 0.832   | 0.824   | 0.816  | 0.796 | 0.763  | 0.704   | 0.649    | 0.505    | 0.356    | 0.323     | 0.289     | 0.265     |        |
| 5.3                                | 1.86    | 0.656   | 0.208  | 0.029 | 0.0110 | 0.00300 | 0.001100 | 0.000430 | 0.000120 | 0.0000450 | 0.0000170 | 0.0000089 |        |



