Pipe Flow-Friction Factor Calculations with Excel, Course #505

Presented by:

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Several kinds of pipe flow calculations can be made with the Darcy-Weisbach equation and the Moody friction factor. These calculations can be conveniently carried out with an Excel spreadsheet. Many of the calculations require an iterative solution, so they are especially suitable for an Excel spreadsheet solution. This course includes discussion of the Darcy-Weisbach equation and the parameters in the equation along with the U.S. and S.I. units to be used. Example calculations and sample Excel spreadsheets for making the calculations are also included.

This course is intended primarily for civil engineers, mechanical engineers, chemical engineers, and environmental engineers. After completing this course you will be able to make calculations with the Darcy Weisbach equation and the Moody friction factor equations to calculate several different unknown parameters when sufficient input data is provided. You will also be prepared to use Excel spreadsheets to efficiently make the calculations

To receive credit for this course, each student must pass an online quiz consisting of twenty (20) questions. A passing score is 70% or better. Completion of this course and successfully passing the quiz will qualify the student for **three (3)** hours of continuing education credit.

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1. Introduction

Several kinds of pipe flow calculations can be made with the Darcy-Weisbach equation and the Moody friction factor. These calculations can be conveniently carried out with an Excel spreadsheet. Many of the calculations require an iterative solution, so they are especially suitable for an Excel spreadsheet solution.

This course includes discussion of the Darcy-Weisbach equation and the parameters in the equation along with the U.S. and S.I. units to be used. Example calculations and sample Excel spreadsheets for making the calculations are also included.



Image Credit: Wikimedia Commons

2. Learning Objectives

At the conclusion of this course, the student will

- Be able to calculate the Reynolds number for pipe flow with specified flow conditions
- Be able to determine whether a specified pipe flow is laminar or turbulent flow for specified flow conditions
- Be able to calculate the entrance length for pipe flow with specified flow conditions
- Be able to obtain a value for the friction factor using the Moody diagram for given Re and ε/D .
- Be able to calculate a value for the friction factor for specified Re and ϵ/D , using the appropriate equation for f.
- Be able to determine a value of the Moody friction factor from the Moody diagram, for given Re and ε/D.
- Be able to calculate a value of the Moody friction factor for given Re and ε/D, using the Moody friction factor equations.
- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the frictional head loss and frictional pressure drop for a given flow rate of a specified fluid through a pipe with known diameter, length and roughness.
- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the required diameter for a given flow rate of a specified fluid through a pipe with known length and roughness, with specified allowable head loss.
- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the fluid flow rate through a pipe with known diameter, length and roughness, with specified frictional head loss.

3. Topics Covered in this Course

- I. Pipe Flow Background
- II. Laminar and Turbulent Flow in Pipes
- III. The Entrance Length and Fully Developed Flow
- IV. The Darcy Weisbach Equation
- V. Obtaining a Value for the Friction Factor
- VI. Calculation of Frictional Head Loss/Pressure Drop Excel Spreadsheet
 - A. Straight Pipe Head Loss
 - B. Minor Losses
- VII. Calculation of Flow Rate Excel Spreadsheet
- VIII. Calculation of Required Pipe Diameter Excel Spreadsheet
- IX. Summary
- X. References and Websites

4. Pipe Flow Background

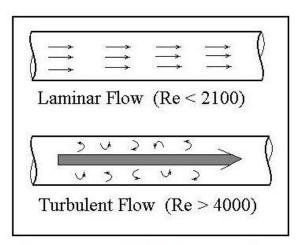
The term pipe flow in this course is being taken to mean flow under pressure in a pipe, piping system, or closed conduit with a non-circular cross-section. Calculations for gravity flow in a circular pipe, like a storm sewer, are done with open channel flow equations, and will not be discussed in this course.

The driving force for pressure pipe flow is typically pressure generated by a pump and/or flow from an elevated tank.

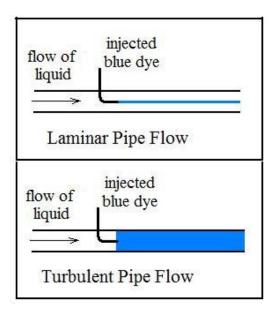
5. Laminar and Turbulent Flow in Pipes

It is often useful to be able to determine whether a given pipe flow is laminar or turbulent. This is necessary, because different methods of analysis or different equations are sometimes needed for the two different flow regimes.

Laminar flow takes place for flow situations with low fluid velocity and high fluid viscosity. In laminar flow, all of the fluid velocity vectors line up in the direction of flow. Turbulent flow, on the other hand, is characterized by turbulence and mixing in the flow. It has point velocity vectors in all directions, but the overall flow is in one direction. Turbulent flow takes place in flow situations with high fluid velocity and low fluid viscosity. The figures below illustrate differences between laminar and turbulent flow in a pipe.



Laminar and Turbulent Pipe Flow



The figure at the right above illustrates the classic experiments of Osborne Reynolds, in which he injected dye into fluids flowing under a variety of conditions and identified the group of parameters now known as the Reynolds Number for determining whether pipe flow will be laminar or turbulent. He observed that under laminar flow conditions, the dye flows in a streamline and doesn't mix into the rest of the flowing fluid. Under turbulent flow conditions, the turbulence mixes dye into all of the flowing fluid. Based on Reynold's experiments and subsequent measurements, the

criteria now in widespread use is that pipe flow will be laminar for a Reynolds Number (Re) less than 2100 and it will be turbulent for a Re greater than 4000. For 2100 < Re < 4000, called the transition region, the flow may be either laminar or turbulent, depending upon factors like the entrance conditions into the pipe and the roughness of the pipe surface.

The Reynolds Number for flow in pipes is defined as: $Re = DV\rho/\mu$, where

- D is the diameter of the pipe in ft (m for S.I.)
- V is the average fluid velocity in the pipe in ft/sec (m/s for S.I) (V = Q/A, where Q = volumetric flow rate and A = cross-sectional area of flow)
- ρ is the density of the fluid in slugs/ft³ (kg/m³ for S.I.)
- μ is the viscosity of the fluid in lb-sec/ft² (N-s/m² for S.I.)

Transport of water or air in a pipe is usually turbulent flow. Also pipe flow of other gases or liquids whose viscosity is similar to water will typically be turbulent flow. Laminar flow typically takes place with liquids of high viscosity, like lubricating oils.

Example #1: Water at 50°F is flowing at 0.6 cfs through a 4" diameter pipe. What is the Reynolds number of this flow? Is the flow laminar or turbulent?

Solution: From the table below the density and viscosity of water at 50° F are: $\rho = 1.94 \text{ slugs/ft}^3$ and $\mu = 2.73 \times 10^{-5} \text{ lb-sec/ft}^2$.

The water velocity, V, can be calculated from $V = Q/A = Q/(\pi D^2/4) = 0.6/[\pi(1/3)^2/4] = 6.9$ ft/sec.

Substituting values into Re = DV ρ/μ gives: Re = $(1/3)(6.9)(1.94)/(2.73 \times 10^{-5})$, or **Re** = **1.6** x **10**⁵.

The Reynolds Number is greater than 4000, so the flow is turbulent.

The density and viscosity of the flowing fluid are often needed for pipe flow calculations. Values of density and viscosity of many fluids can be found in handbooks, textbooks and websites. Table 1, below gives values of density and viscosity for water over a range of temperatures from 32 to 70°F.

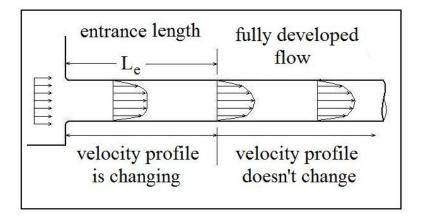
Dynamic Density, slugs/ft3 Viscosity, lb-s/ft2 Temperature, °F 3.732 x 10⁻⁵ 32 1.940 3.228 x 10-5 40 1.940 2.730 x 10⁻⁵ 50 1.940 2.334 x 10⁻⁵ 60 1.938 2.037 x 10-5 70 1.936

Table 1. Density and Viscosity of Water

6. The Entrance Length and Fully Developed Flow

The Darcy Weisbach equation, which will be discussed in the next section, applies only to the fully developed portion of the pipe flow. If the pipe in question is long in comparison with its entrance length, then the entrance length effect is often neglected and the total length of the pipe is used for calculations. If the entrance length is a significant part of the total pipe length, however, then it may be necessary to make separate calculations for the entrance region.

The entrance region refers to the initial portion of the pipe in which the velocity profile is not "fully developed," as shown in the diagram below. At the entrance to the pipe, the flow is often constant over the pipe cross-section. In the region near the entrance, the flow in the center of the pipe is unaffected by the friction between the pipe wall and the fluid. At the end of the entrance region, the pattern of velocity across the pipe (the velocity profile) has reached its final shape.



The entrance length, L_e, can be estimated if the Reynolds Number, Re, is known.

For turbulent flow (Re > 4000): $L_e/D = 4.4 \text{ Re}^{1/6}$.

For laminar flow (Re < 2100): $L_e/D = 0.06$ Re.

Example #2: Estimate the entrance length for the water flow described in example #1: (1.2 cfs of water at 50°F, flowing through a 4" diameter pipe).

Solution: From Example # 1, Re 1.6 x 10^5 . Since Re > 4000, so this is turbulent flow, and $L_e/D = L_e/(1/3) = 4.4(1.6 \text{ x } 10^5)^{1/6} = 32.4$ Thus $L_e = (1/3)(32.4) = 10.8 \text{ ft} = L_e$

7. The Darcy Weisbach Equation

The **Darcy Weisbach equation** is a widely used empirical relationship among several pipe flow variables. The equation is: $\mathbf{h}_{L} = \mathbf{f}(\mathbf{L}/\mathbf{D})(\mathbf{V}^{2}/2\mathbf{g})$, where:

- L = pipe length, ft (m for S.I. units)
- **D** = pipe diameter, ft (m for S.I. units)
- V = average velocity of fluid (= Q/A), ft/sec (m/s for S.I. units)
- **h**_L = frictional head loss due to flow at an ave. velocity, V, through a pipe of diameter, D, and length, L, ft (ft-lb/lb) (m or N-m/m for S.I. units).
- $\mathbf{g} = \text{acceleration due to gravity} = 32.2 \text{ ft/sec}^2 (9.81 \text{ m/s}^2)$
- $\mathbf{f} = \text{Moody friction factor (a dimensionless empirical factor that is a function of Reynolds Number and <math>\varepsilon/D$, where:
- ε = an empirical pipe roughness, ft (m or mm for S.I. units)

NOTE: Although the Darcy Weisbach equation is an empirical equation, it is also a dimensionally consistent equation. That is, there are no dimensional constants in the equation, so any consistent set of units that gives the same dimensions on both sides of the equation can be used. A typical set of U.S. units and a typical set of S.I. units are shown in the list above.

Table 2, below, shows some typical values of pipe roughness for common pipe materials, drawn from information on several websites.

Table 2. Pipe Roughness Values

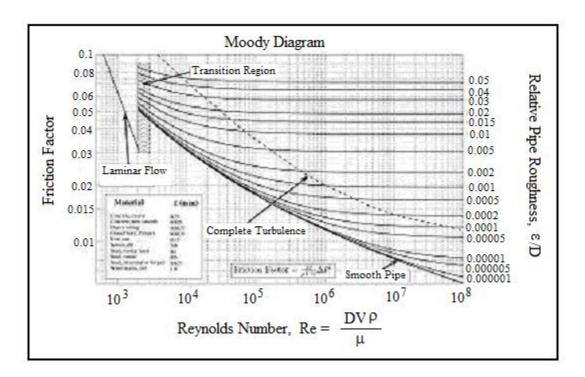
	Roughness, 8
Pipe Material	feet
drawn brass or copper	0.000005
PVC pipe	0.000005
commercial steel	0.000150
wrought iron	0.000150
asphalted cast iron	0.000400
galvanized iron	0.000500
cast iron	0.000850
concrete	0.001 - 0.01

Frictional pressure drop for pipe flow is related to the frictional head loss through the equation: $\Delta P_f = \rho g h_L = \gamma g h_L$, where:

- h_L = frictional head loss (ft or m) as defined above
- ρ = fluid density, slugs/ft³ (kg/m³ for S.I.)
- $g = acceleration due to gravity, ft/sec^2 (m/s^2 for S.I.)$
- γ = specific weight, lb/ft³ (N/m³ for S.I.)

8. Obtaining a Value for the Friction Factor

A value of the Moody friction factor, f, is needed for any calculations with the Darcy Weisbach equation other than empirical determination of the friction factor by measuring all of the other parameters in the equation. A common method of obtaining a value for f is graphically, from the Moody friction factor diagram, first presented by L. F. Moody in his classic 1944 paper in the *Transactions of the ASME*. (Ref. #1). The Moody friction factor diagram, shown in the diagram below, is now available in many handbooks and textbooks and on many websites.



Source: http://www.thefullwiki.org/Moody diagram

When using Excel spreadsheets for pipe flow calculations with the Darcy Weisbach equation, it is more convenient to use equations for the Moody friction factor, f, rather than a graph like the Moody diagram. There are indeed equations available that give the relationships between Moody friction factor and Re & ϵ/D for four different portions of the Moody diagram. The four portions of the Moody diagram are:

- i) **laminar flow** (Re < 2100 the straight line at the left side of the Moody diagram);
- ii) **smooth pipe turbulent flow** (the dark curve labeled "smooth pipe" in the Moody diagram f is a function of Re only in this region);
- iii) **completely turbulent region** (the portion of the diagram above and to the right of the dashed line labeled "complete turbulence" f is a function of ε/D only in this region);

iv) **transition region** (the portion of the diagram between the "smooth pipe" solid line and the "complete turbulence" dashed line – f is a function of both Re and ε/D in this region).

The equations for these four regions are shown in the box below:

Laminar Flow:
$$f = \frac{64}{Re}$$

0.316

Smooth Pipe Turbulent Flow:
$$f = \frac{0.316}{Re^{1/4}}$$

Completely Turbulent Flow:
$$f = \left[1.14 + 2 \log_{10} \left(\frac{D}{\epsilon}\right)\right]^{-2}$$

Transition Region:
$$f = \left\{-2 \log_{10} \left[\frac{(\epsilon/D)}{3.7} + \frac{2.51}{\text{Re}(f^{1/2})}\right]\right\}^{-2}$$

Example #3: Calculate the value of the Moody friction factor for pipe flow with Re = 10^7 and $\epsilon/D = 0.005$.

Solution: From the Moody diagram above, it is clear that the point, Re = 10^7 and $\epsilon/D = 0.005$, is in the "complete turbulence" region of the diagram. Thus:

$$f = [1.14 + 2 \log_{10}(D/\epsilon)]^{-2} = [1.14 + 2 \log_{10}(1/0.005)]^{-2} = 0.0303 = f$$

9. Calculation of Frictional Head Loss/Pressure Drop

A. Straight Pipe Head Loss

Calculation of the frictional head loss for a specified flow rate of a given fluid at a given temperature through a pipe of known diameter, length, and material can be done using the Darcy Weisbach equation. The step-by-step process for this calculation is as follows:

- a. Determine the density and viscosity of the flowing fluid at the specified temperature
- b. Calculate the fluid velocity from $V = Q/A = Q/(\pi D^2/4)$
- c. Calculate the Reynolds Number (Re = $DV\rho/\mu$)
- d. Find the pipe roughness value (ϵ) for the specified pipe material
- e. Calculate the pipe roughness ratio, (ε/D)
- f. Determine the Moody friction factor value using the Moody diagram and/or friction factor equation(s), with the calculated values of Re and ϵ/D
- g. Calculate the frictional head loss using the Darcy Weisbach equation and the specified or calculated values of f, L, D, and V.
- h. If desired calculate the frictional pressure drop

Example #4: Calculate the frictional head loss in ft and the frictional pressure drop in psi, for a flow rate of 0.60 cfs of water at 50°F, through 100 ft of 6 inch diameter wrought iron pipe. Use the Moody diagram to get the value for f.

Solution: Following the steps outlined above:

- a. At 50°F, the needed properties of water are: density = $\rho = 1.94 \text{ slugs/ft}^3$, viscosity = $\mu = 2.73 \times 10^{-5} \text{ lb-s/ft}^2$
- b. Water velocity = $V = Q/(\pi D^2/4) = 0.60/(\pi (6/12)^2/4) = 3.1$ ft/sec
- c. Reynolds number = $Re = DV\rho/\mu = (0.5)(3.1)(1.94)/(2.73 \times 10^{-5}) = 1.08 \times 10^{5}$
- d. From the pipe roughness table on page 8, for wrought Iron: $\epsilon = 0.0005$ ft
- e. Pipe roughness ratio = $\epsilon/D = 0.0005/0.5 = 0.001$
- f. From the Moody diagram, the point for Re = 1.08×10^5 and $\epsilon/D = 0.001$, is in the transition region and the friction factor, f, is approximately: f = 0.02
- g. Substituting the given values: D = 6 inches = 0.5 ft and L = 100 ft, along with the calculated values: V = 3.1 ft/sec and f = 0.02, and the acceleration due to gravity, g = 32.2 ft/sec², the Darcy Weisbach equation becomes: $h_L = f(L/D)(V^2/2g) = (0.02)(100/0.5)[3.1^2/(2*32.2)] = 0.58$ ft = h_L
- h. The frictional pressure drop is $\Delta P_f = \rho g h_L = 1.94*32.2*0.58$ = 36 lb/ft² = 36/144 psi = **0.25 psi** = ΔP_f

Example #5: Repeat the calculations in Example #4, for the frictional head loss in ft and the frictional pressure drop in psi, for a flow rate of 0.60 cfs of water at 50°F, through 100 ft of 6 inch diameter wrought iron pipe, using an Excel spreadsheet and the appropriate Moody friction factor equation(s).

NOTE: The Excel spreadsheet shown in the image on page 16 can be obtained from www.engineeringexceltemplates.com. It isn't necessary to download the spreadsheet to complete this course and take the quiz, but it can be obtained there.

Solution: The image on page 16 (referred to above) shows an Excel spreadsheet created to make these calculations. **Step 1** in the solution consists of entering a value for each of the following inputs in the column on the left at the upper part of the spreadsheet:

- the pipe diameter, D_{in} , in inches;
- the pipe roughness, ε , in ft;
- the pipe length, L, in ft;
- the pipe flow rate, **Q**, in cfs;
- the fluid density, ρ , in slugs/ft³; and
- the fluid viscosity, μ , in lb-sec/ft².

When all the required input values are entered, the spreadsheet will calculate the parameters shown in the "calculations" column at the right side of the spreadsheet:

- pipe diameter, **D**, in ft, which is needed to calculate:
- Moody friction factor using $f = [1.14 + 2\log_{10}(D/\epsilon)]^{-2}$ (That is, assuming the flow is in the "completely turbulent flow" region.)
- cross-sectional area of flow in the pipe, **A**, in ft², which is needed to calculate:
- average velocity of the fluid in the pipe, V, in ft/sec
- Reynolds number, Re

Step 2 in the spreadsheet is the iterative calculation of the Moody friction factor, f, using the transition region equation: $\mathbf{f} = \{-2\log_{10}[((\epsilon/\mathbf{D})/3.7) + (2.51/(\mathbf{Re}\ \mathbf{f}^{1/2}))]\}^{-2}$, as follows:

- a. Calculate f with the transition region equation, using the Re value just calculated and the f value calculated assuming completely turbulent flow.
- b. If this calculated value of f is different than the one calculated with the completely turbulent flow assumption, then the flow **is** in transition region and f should be calculated again with the transition region equation, using the last calculated value of f in the right hand side of the equation.
- c. This is repeated until two subsequent calculations give the same value of f.

Step 3 in the spreadsheet is calculation of h_L using the Darcy Weisbach equation and calculation of ΔP_f using $\Delta P_f = \rho g h_L$. The calculated frictional head loss and frictional pressure drop, as shown in the spreadsheet image are:

$$h_L = 0.64 \text{ ft}$$
 and $\Delta P_f = 0.28 \text{ psi}$

These values were calculated using the final value of f calculated in the spreadsheet: f = 0.0220.

For comparison the value of f = 0.020 estimated from the Moody diagram led to these values:

$$h_L = 0.58 \text{ ft}$$
 and $\Delta P_f = 0.25 \text{ psi}$

The values calculated by the Excel spreadsheet using the Moody friction factor equations are the best estimates for h_L and ΔP_f . The value of f from the Moody diagram did, however give values of h_L and ΔP_f close to those from the equations. A larger, better Moody diagram would allow a more precise estimate of f.

Calculation of He	ad Loss.	h _L , and	Fric	tional Pre	essure Dr	op, ΔP _f ,	
for given flow rate							
pipe roughness,			// -	77.77	,,		
pipe rougililess,	s, and nu	ia piopi	ei ue.	s , ρ α μ.			
1. Determine Friction F	actor, f, ass	uming co	mplete	ely turbulent	flow {f = [1.14 + 2 log	₁₀ (D/ε)]
Inputs				Calculati	ons		
Pipe Diameter, D in =	6	in		Pipe Diame	ter, D =	0.5000	ft
Pipe Roughness, ε =	0.0005	ft		Friction Fac	tor, f =	0.01962	
Pipe Length, L =	100	ft		Cross-Sect.	Area, A =	0.1963	ft²
Pipe Flow Rate, Q =	0.600	cfs		Ave. Velocit	y, V =	3.1	ft/sec
Fluid Density, p =	1.94	slugs/ft ³		Reynolds no	umber, Re =	108,575	
Fluid Viscosity, µ =	0.0000273	lb-sec/ft ²					
2. Check on whether th	e given flov	v is "comp					
	e given flov	v is "comp	and se			culated above	.)
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2. Check on whether the (Calculate f with the transition Region Repeat calc of f under the Repeat again 3. Calculate h _L and ΔP _L (h _L = f(L/D)(V ² /2g)) Frictional Head Loss, h _L	nsition region (3.7)+(2.51) Friction Fact sing new value in if necessary in using the first and ΔF	v is "comp n equation /(Re*(f ^{1/2} tor, f: ue of f:	f = f = f = f =	0.0221 0.0220	m the one ca	Iculated above	.)
2. Check on whether the (Calculate f with the transition Region Repeat calc of f under the Repeat again 3. Calculate h _L and ΔP _L (h _L = f(L/D)(V ² /2g)) Frictional Head Loss, h _L	nsition region (3.7)+(2.51) Friction Fact sing new value in if necessary in using the first and ΔF	v is "comp n equation /(Re*(f ^{1/2} tor, f: ue of f:	f = f = f = f =	0.0221 0.0220	m the one ca	Iculated above	.)
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B. Minor Losses

The term "minor losses" refers to head loss or pressure drop due to flow through pipe fittings (e.g. valves, elbows, or tees), entrances and exits, changes in pipe diameter, and any other causes of head loss besides the straight pipe portion of the flow.

The general equation for calculating the minor loss of a fitting in a pipe system is: $\mathbf{h}_L = \mathbf{K}(\mathbf{V}^2/2\mathbf{g})$, where K is the minor loss coefficient for the particular fitting. For several fittings in a pipe system, all of the K values can be summed to give: $\mathbf{h}_L = \Sigma \mathbf{K}(\mathbf{V}^2/2\mathbf{g})$.

Values of K for fittings, changes in cross-section, etc are available in many engineering handbooks and textbooks and on internet sites. Table 3, below gives typically used values of K for some common pipe fittings.

Table 3. Minor Loss Coefficients for Pipe Fittings

Fitting	Loss Coefficient, K
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Swing check valve, fully open	2.5
Gate valve, fully open	0.2
Short-radius elbow	0.9
Medium-radius elbow	0.8
Long-radius elbow	0.6
45 degree elbow	0.4
Close return bend	2.2
Standard tee, flow through run	0.6
Standard tee, flow through branc	h 1.8
Square entrance	0.5
Exit	1.0

Source: U.S. EPA, EPANET2 User Manual,

http://www.epa.gov/nrmrl/wswrd/dw/epanet/EN2manual.PDF

Example #6: What is the total head loss due to minor losses for the 0.60 cfs of water at 50°F flowing through 100 ft of 6 inch diameter pipe as in Examples #4 and #5, if the piping system contains one fully open check valve, three medium radius elbows, and one standard tee with flow through the branch.

Solution: The sum of the minor loss coefficients is:

$$\Sigma K = 2.5 + (3)(0.8) + 1.8 = 6.7$$

 $h_L = \Sigma K(V^2/2g) = (6.7)[3.1^2/(2*32.2)] = 1.0 \text{ ft } = h_L \text{ (minor losses)}$

10. Calculation of Required Pipe Diameter

Calculation of the required pipe diameter for a specified flow rate of a given fluid at a given temperature through a pipe of known length and material, with a specified allowable head loss can be done using the Darcy Weisbach equation. The step-by-step process for this calculation is as follows:

- a. Determine the density, ρ , and viscosity, μ , of the flowing fluid at the specified temperature.
- b. Obtain a value of pipe roughness, ε , for the specified pipe material.
- c. Identify the known values for frictional head loss, h_L, flow rate, Q, and pipe length, L.
- d. Select an assumed pipe diameter, D, to use as a starting point.
 - <u>NOTE</u>: Standard U.S. pipe sizes are given at the bottom of the spreadsheet image on page 21.
- e. Using the pipe roughness, ε, and the assumed pipe diameter, D, calculate the Moody friction factor, f, assuming completely turbulent flow.

- f. Use the assumed pipe diameter to calculate the pipe cross-sectional area, A, and use it together with the specified flow rate through the pipe, to calculate the average fluid velocity in the pipe, V.
- g. Calculate the Reynolds number: $Re = DV\rho/\mu$.
- h. Use the calculated values of Re and f to recalculate f using the transition region equation (which gives f as a function of Re and f).
- i. Use the following iterative process to calculate the Moody friction factor, f: If the value of f calculated in step h is different from that calculated in step e, repeat step h using the most recent value calculated for f. Repeat as many times as necessary until two subsequent calculations give the same value of f.
- j. Using known values for h_L and L, and calculated values for f and V, calculate pipe diameter D with the Darcy Weisbach equation.
- k. Use the following iterative process to calculate the pipe diameter, D:
 - If the value of D calculated in step j is larger than the assumed value, replace the assumed value of D with a larger standard pipe diameter and recalculate D (steps e through j).
 - If the value of D calculated in step j is smaller than the assumed value, replace the assumed value of D with a smaller standard pipe diameter and recalculate D (steps e through j).
 - Repeat as many times as necessary to find the smallest standard pipe diameter for which the next calculated value of D is smaller than the assumed value. This is the required value of D.

Example #7: Use an Excel spreadsheet to calculate the required pipe diameter to carry 0.60 cfs of water at 50°F, through 100 ft of galvanized pipe, with an allowable head loss of 20 ft.

Pipe Flow/Frict	ion Fac	ctor Cal	culations	II: (U.S.	units)	<u>.</u>
Calculation of pipe	diameter	, D, for giv	ven flow rate	, Q, pipe ler	ngth, L,	
pipe roughness, ϵ , l	head los	s, h _L , and	fluid propert	i es , ρ & μ.		
(NOTE: This is an itera	tive calcul	ation. An as	ssumed value o	of D will be use	d to start.)	
1. Determine Friction Fact	or, f, assum	ning complete	ely turbulent flov	v {f = [1.14 + 2	2 log ₁₀ (D/ε)] ⁻² }
Inputs			Calculatio	ns_		
Allowable Head Loss, h _L =	20	ft	Assumed Pip	e Diam, D _{in} * =	3.5	in
Pipe Roughness, ε =	0.0005	ft	Pipe Diamete	r, D =	0.2917	ft
Pipe Length, L =	100	ft	Friction Facto	or, f =	0.02247	
Pipe Flow Rate, Q =	0.600	cfs	Cross-Sect. A	Area, A =	0.0668	ft²
Fluid Density, ρ =	1.94	slugs/ft ³	Ave. Velocity	, V =	9.0	ft/sec
Fluid Viscosity, µ =	0.000027	lb-sec/ft ²	Reynolds nur	nber, Re =	188,197	
2. Check on whether the g	jiven flow i	s "completely	turbulent flow"		1	
(Calculate f with the transit		880 8		e one calculated	above.)	
[f = $\{-2*\log_{10}[((\epsilon/D)/3)]$.7)+(2.51/	(Re*(f ^{1/2}))]} ⁻⁴	1			
Transistion Region F	riction Facto	or, f. f	= 0.0236			
Repeat calc of fusing	g new value	of f: f	= 0.0235			
Repeat again if	necessary:	f	= 0.0235			
2. Check on whether the g (Calculate f with the transit [f = {-2*log ₁₀ [((ε/D)/3 Transistion Region F Repeat calc of f using	tion region e .7)+(2.51/(s "completely quation and se (Re*(f ^{1/2}))]}-2 or, f: f of f: f	turbulent flow" e if differs from the control of th			

$[D = f(L/h_L)(V^2/2g)]$						
Pipe Diameter, D =	0.1475	ft	=	1.8	in	
NOTE: This iterative proced	ure doesn't co	nverge smo	oothly to	a solution. If	the calculated	pipe diameter
in this step is larger than the	e assumed pip	e diameter	above, th	nen replace tl	ne assumed pi	pe diameter
value with the next larger sta	andard pipe siz	ze. Repeat	until you	find the sma	allest standard	pipe diameter
that gives a smaller calculat	ed required pir	oe diameter	r. That is	your solutio	n.	(20)
				83		
With the example values giv	en here, an as	sumed pip	e diamet	er of 3 inches	gives a calcul	ated pipe
diameter requirement of 3.4						
(3.5 inches) gives a calcula				Name and Administration of the Control of the Contr	Contractive of the Otton Contractive	
minimum standard pipe dian	3000		THORE OF	1.0 menes, s	0 u 0.0 men o	20 13 1110
minimum standard pipe diar	neter triat will	do the job.				
*Standard nominal U.S. pipe	sizes in inch	es:				
*Standard nominal U.S. pipe	sizes in inch	es:				
*Standard nominal U.S. pipe			2, 4, 5, 6	5, 1 0, 1 2, 1 4	1, 16, 18, 20, 2	4,
			/2, 4, 5, 6	5, 10, 12, 14	4, <mark>16, 18</mark> , 20, 2	4,

Solution: The Excel spreadsheet in the two images above shows the solution to this example. (The spreadsheet shown can be obtained from the same <u>link</u> mentioned earlier on page 13.) The spreadsheet calculations correlate with the steps a through k outlined above for this type of calculation.

- Steps a, b, & c consist of entering values for the required inputs, h_L , ϵ , L, Q, ρ , and μ in the column on the left in the upper part of the spreadsheet.
- Step d is assumption of a starting value for the pipe diameter.

- Steps e, f, & g are the calculations in the column on the right in the upper part of the spreadsheet.
- Steps h and i are the iterative calculation of the Moody friction factor using the transition region equation, shown as step 2 in the spreadsheet.
- Steps j and k are the iterative calculation of the required pipe diameter, shown as step 3 in the spreadsheet.

The spreadsheet image above only shows the final assumed and calculated values for D. In the original solution to this example, the first assumed value for D was 6 inches. Subsequent calculated and assumed values for D, leading to the final solution are as follows:

Assumed D, in	Calculated D, in.
6	0.2
4	1.0
3	3.4
3.5	1.8

This shows 3.5 inches to be the smallest standard pipe size that will be adequate. The next smaller size, 3 inch diameter, isn't large enough, because the calculated D is larger than 3 inches.

11. Calculation of Pipe Flow Rate

Calculation of the fluid flow rate for a given fluid temperature through a pipe of known diameter, length, and material, with a specified allowable head loss can be done using the Darcy Weisbach equation. This is also an iterative process, because a value for the unknown fluid flow rate is needed to calculate the Reynolds number, which is needed to get a value for the friction factor. The step-by-step process for this calculation is as follows:

- a. Determine the density, ρ , and viscosity, μ , of the flowing fluid at the specified temperature.
- b. Obtain a value of pipe roughness, ε , for the specified pipe material.
- c. Identify the known values for frictional head loss, h_L and pipe diameter, D, and length, L.
- d. Using the known values for D and ε , calculate the Moody friction factor, f, assuming completely turbulent flow.
- e. Assume a starting value for the fluid flow rate, Q.
- f. Calculate the fluid velocity, V.
- g. Calculate the Reynolds number, Re
- h. Use the calculated values of Re and f to recalculate f using the transition region equation (which gives f as a function of Re and f).
- i. Use the following iterative process to calculate the Moody friction factor, f: If the value of f calculated in step h is different from that calculated in step d, repeat step h using the most recent value calculated for f. Repeat as many times as necessary until two subsequent calculations give the same value of f.
- j. Using known values for h_L , D, and L, and the calculated value for f, calculate the average fluid velocity, V, with the Darcy Weisbach equation and calculate the fluid flow rate from $Q = V(\pi D^2/4)$.
- k. If this calculated value of Q is different than the assumed value, then replace the assumed Q value with the calculated value and carry out steps f through j to get another calculated value for Q. Repeat as many times as necessary to the calculated Q value to be the same as the assumed Q value. This iteration typically converges rapidly.

Example #8: Use an Excel spreadsheet to calculate the flow rate of water at 50°F, delivered through 100 ft of 6 inch diameter galvanized pipe, with a head loss of 1.2 ft.

Solution: The Excel spreadsheet in the image on the next page shows the solution to this example. (The spreadsheet shown can be downloaded from a nk in the same article mentioned earlier on page 13.) The spreadsheet calculations correlate with the steps a through k outlined above for this type of calculation.

- Steps a, b, & c consist of entering values for the required inputs, h_L, ε,
 L, Q, ρ, and μ in the column on the left in the upper part of the spreadsheet.
- Step d is calculation of the Moody friction factor, f, assuming completely turbulent flow.
- Step e is assumption of a starting value for the fluid flow rate, Q.
- Steps f and g are calculation of the average fluid velocity, V, and then calculation of the Reynolds number, Re.
- Steps h and i are the iterative calculation of the Moody friction factor using the transition region equation, shown as step 2 in the spreadsheet.
- Steps j and k are the iterative calculation of the fluid flow rate, shown as step 3 in the spreadsheet.

	Flow Rat	10.4		1 112			
for given frictional h	ead loss	, h _L , pip	e dia	am., D, pi	pe length, l	-1	
pipe roughness, ε, ε	and fluid	properti	ies,	ρ & μ.			
					1 2 2 2 2		2.
1. Determine Friction Fact	or, f, assum	ing compl	etely	turbulent flo	w {t = [1.14	+ 2 log ₁₀ (D	/ε)] ⁻
<u>Inputs</u>				Calculati	ons		
Pipe Diameter, D in =	6	in		Pipe Diame	ter, D =	0.5000	ft
Pipe Roughness, ε =	0.0005	ft		Friction Fac	tor f -	0.01962	
ripe Rougilliess, & -	0.0003	н		i netion i ac	tor, 1 –	0.01302	
Pipe Length, L =	100	ft		Cross-Sect.	Area, A =	0.1963	ft²
Allowable Head Loss, h _L =	1.2	ft		Assumed F	low Rate, Q =	0.83	cfs
Fluid Density, ρ =	1.94	slugs/ft ³		Ave. Velocit	y, V =	4.2	ft/sec
Fluid Viscosity, μ =	0.0000273	lb-sec/ft ²		Reynolds no	ımber, Re =	150,196	
5.61 1 1 1 1 1 1				1111111111	.,		
2. Check on whether the g	liven flow is	complete	eiy tu	rbulent flow	8		
						N.	
(Calculate f with the transit	ti <mark>o</mark> n region eq	uation and	see if		he one calculate	d above.)	
(Calculate f with the transit $f = \{-2*log_{10}[((\epsilon/D)/3.7)]$	×		_		he one cal <mark>c</mark> ulate	d above.)	
	×		-2		he one calculate	d above.)	
	7)+(2.51/(R	e*(f ^{1/2}))]}	_		he one calculate	d above.)	
$f = \{-2^{\circ}\log_{10}[((\epsilon/D)/3.7)]$	7)+(2.51/(R ction Factor,	e*(f^{1/2}))]} f:	-2	differs from t	he one calculate	d above.)	
f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Fri	7)+(2.51/(R ction Factor,	e*(f^{1/2}))]} f:	-2 f =	differs from t	he one calculate	d above.)	
f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Fri	7)+(2.51/(R ction Factor, g new value o	e*(f^{1/2}))]} f:	-2 f =	differs from t	he one calculate	d above.)	
f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Fri Repeat calc of f using	r)+(2.51/(R ction Factor, g new value on necessary:	e*(f ^{1/2}))]} f:	-2 f = f =	0.0215 0.0214			30.00
f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Fri Repeat calc of f using Repeat again if	r)+(2.51/(R ction Factor, g new value of necessary:	e*(f ^{1/2}))]} f: ff: alue for f	f = f = calcu	0.0215 0.0214			quatic
f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Fri Repeat calc of f using	r)+(2.51/(R ction Factor, g new value of necessary:	e*(f ^{1/2}))]} f: ff: alue for f	f = f = calcu	0.0215 0.0214			quatic
f = {-2*log ₁₀ [((ε/D)/3.7) Transition Region Fri Repeat calc of f using Repeat again if	r)+(2.51/(R ction Factor, g new value of necessary:	e*(f ^{1/2}))]} f: ff: alue for f	f = f = calcu	0.0215 0.0214			quatic
f = {-2*log ₁₀ [((ε/D)/3.7] Transition Region Fri Repeat calc of f using Repeat again if the second seco	r)+(2.51/(R ction Factor, g new value of necessary: g the final v	e*(f ^{1/2}))]} f. f. alue for f = V(πD ² /4	f = f = calcu	0.0215 0.0214			quatic

The spreadsheet image on the previous page only shows the final assumed and calculated values for Q. In the original solution to this example, the first assumed value for Q was 1.5 cfs. Subsequent calculated and assumed values for Q, leading to the final solution are as follows:

Assumed Q, cfs	Calculated Q, cfs
1.5	0.85
0.85	0.83
0.83	0.83

As shown in the table the iterative calculation converged rapidly to the solution: $\mathbf{Q} = \mathbf{0.83}$ cfs.

12. Calculations with S.I. Units

Use of S.I. units for pipe flow/friction factor/Darcy Weisbach calculations is rather straightforward. The Darcy Weisbach equation, the Moody diagram, and the Moody friction factor equations remain exactly the same.

The only change from calculations with U.S. units is use of mm instead of inches (or m instead of ft) for pipe diameter, D; m instead of ft for pipe length, L; m^3/s instead of cfs for fluid flow rate, Q; mm or m instead of ft for ϵ ; kg/m^3 instead of slugs/ft³ for density, ρ ; N-s/m² instead of lb-sec/ft² for μ ; and m instead of ft for frictional head loss, h_L . The following example illustrates S.I. calculations.

Example #9: Calculate the frictional head loss in m and the frictional pressure drop in kN/m², for a flow rate of 0.017 m³/s of water at 10°C, through a 30 m length of 150 mm diameter wrought iron pipe, using an Excel spreadsheet and the appropriate Moody friction factor equation(s).

Solution: The steps in the solution are the same as those given above for Example #5 (a similar problem in U.S. units). The spreadsheet image on the next page shows the solution.

Calculation of Head	Loss, h	, or Frid	ction	nal Pressur	e Drop, Z	ΔP _f ,	
for given flow rate, (Q, pipe d	liam., D	, pip	e length, L	<u>.</u>		
pipe roughness, ε, ε	Carlotte Et. II		No.	1000001110			
p.p g			,				
1. Determine Friction Fact	or, f, assum	ing comp	letely	turbulent flov	v {f = [1.1	4 + 2 log ₁₀ (D)/ε)] ⁻²
Inputs				Calculatio	<u>ns</u>		
Pipe Diameter, D =	150	mm		Pipe Diamete	r, D =	0.1500	m
Pipe Roughness, ε =	0.15	mm		Friction Facto	or, f =	0.01962	
Pipe Length, L =	30	m		Cross-Sect. A	Area, A =	0.0177	m ²
Pipe Flow Rate, Q =	0.017	m³/s		Ave. Velocity	V =	1.0	m/s
Fluid Density, ρ =	1000	kg/m ³		Reynolds nun	nber, Re =	111,000	
Fluid Viscosity, µ =	0.0013	N-s/m ²					
2. Check on whether the g	iven flow is	"complet	tely tu	urbulent flow"			
2. Check on whether the g (Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7	ion region e	quation and	d see		111 120 20	ated above.)	
NAME OF A PARTY OF A PARTY OF THE PARTY OF T	ion region e	quation and	d see		111 120 20	ated above.)	
(Calculate f with the transit	ion region e	quation and	d see		111 120 20	ated above.)	
(Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Frid	ion region er	quation and Re*(f ^{1/2}))]] f:	d see	if differs from th	111 120 20	ated above.)	
(Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7	ion region er	quation and Re*(f ^{1/2}))]] f:	f =	o.0221	111 120 20	ated above.)	
(Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7 Transition Region Frid)+(2.51/(F ction Factor,	quation and Re*(f ^{1/2}))]] f:	f =	o.0221	111 120 20	ated above.)	
(Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7) Transition Region Frid Repeat calc of f using	tion region en ()+(2.51/(F)) etion Factor, g new value of the control of the cont	quation and Re*(f ^{1/2}))]]] f:	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	
(Calculate f with the transit f = {-2*log ₁₀ [((ε/D)/3.7) Transition Region Frid Repeat calc of f using	tion region editor (C.51/(F.) ction Factor, grew value of the cecessary:	quation and Re*(f ^{1/2}))]] f: of f:	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	
(Calculate f with the transit $f = \{-2^*log_{10}[((\epsilon/D)/3.7)]$ Transition Region Frid Repeat calc of f using Repeat again if r	tion region editor (C.51/(F.) ction Factor, grew value of the cecessary:	quation and Re*(f ^{1/2}))]] f: of f:	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	
(Calculate f with the transit $f = \{-2^* log_{10}[((\epsilon/D)/3.7 + \epsilon/D)/3.7]$ Transition Region Frida Repeat calc of f using Repeat again if respect again if respect to the control of the c	ion region er ()+(2.51/(F ction Factor, new value or necessary: ing the fina and ΔP _f	quation and Re*(f ^{1/2}))]] f. f. al value for pgh _L)	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	
(Calculate f with the transit $f = \{-2*log_{10}[((\epsilon/D)/3.7 + 1)] $ Transition Region Frid Repeat calc of f using Repeat again if r 3. Calculate h_L and ΔP_f , us $(h_L = f(L/D)(V^2/2g)$ at Frictional Head Loss, $h_L = f(L/D)(V^2/2g)$	ion region er ()+(2.51/(F ction Factor, new value or necessary: ing the fina and ΔP _f	quation and Re*(f ^{1/2}))]] f. f. al value for pgh _L)	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	
(Calculate f with the transit $f = \{-2^*log_{10}[((\epsilon/D)/3.7 + \epsilon/D)/3.7\}]$ Transition Region Frid Repeat calc of f using Repeat again if r 3. Calculate h_L and ΔP_f , using $(h_L = f(L/D)(V^2/2g))$ and $(h_L = f(L/D)(V^2/2g))$ are frictional Head Loss, $h_L = f(L/D)(V^2/2g)$	ion region en (2)+(2.51/(Figure 1))+(2.51/(Figure 1)) in the final recessary: ing the final recessary ΔP _f 0.207	quation and Re*(f ^{1/2}))]] f. f. al value for pgh _L)	f = f =	0.0221 0.0220 0.0220	e one calcula	ated above.)	

The first part of the spreadsheet solution is entering the known input values in the upper left column. Then the spreadsheet calculations will proceed, leading to a value for the Moody frictions factor based on completely turbulent flow and a preliminary value for Reynolds number.

The second part of the spreadsheet solution is an iterative solution to calculate the Moody friction factor using the transition region equation.

The third and final part of the spreadsheet solution is calculation of frictional head loss and frictional pressure drop using the Darcy Weisbach equation.

The spreadsheet shows the final solution: $h_L = 0.207 \text{ m}$ and $\Delta P_f = 2.03 \text{ kN/m}^2$.

Calculation of required pipe diameter using S.I. units will be essentially the same as the calculations in Example #7 and calculation of fluid flow rate with S.I. units will be essentially the same as the calculations in Example #8.

13. Summary

The Darcy Weisbach equation and the Moody friction factor equations given in this course are the essential tools for calculations involving the parameters fluid flow rate, Q, through a pipe of diameter, D, length, L, and roughness, ϵ , with frictional head loss, h_L . Other parameters that are often used in the equations are: fluid density, ρ , fluid viscosity, μ , average fluid velocity, V, and Reynolds number, Re. This course included discussion of the steps used in three common types of calculations with these parameters that require one or more iterative calculations. The three types of calculations are calculation of frictional head loss, required pipe diameter, or fluid flow rate, when the other parameters are known. The use of Excel spreadsheets for the calculations was illustrated with several examples.

14. References and Websites

- 1. Munson, B. R., Young, D. F., & Okiishi, T. H., Fundamentals of Fluid Mechanics, 4th Ed., New York: John Wiley and Sons, Inc, 2002.
- 2. Darcy Weisbach equation history http://biosystems.okstate.edu/darcy/DarcyWeisbach/Darcy-WeisbachHistory.htm
- 3. Bengtson, H. H., <u>Pipe Flow Calculations with the Darcy Weisbach</u>
 <u>Equation</u>
- 4. U.S. EPA, *EPANet2 User Manual*, http://www.epa.gov/nrmrl/wswrd/dw/epanet/EN2manual.PDF