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# Flow Measurement in Pipes and Ducts

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Harlan H. Bengtson, Ph.D., P.E.

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Continuing Education and Development, Inc.  
22 Stonewall Court  
Woodcliff Lake, NJ 07677

P: (877) 322-5800  
[info@cedengineering.com](mailto:info@cedengineering.com)

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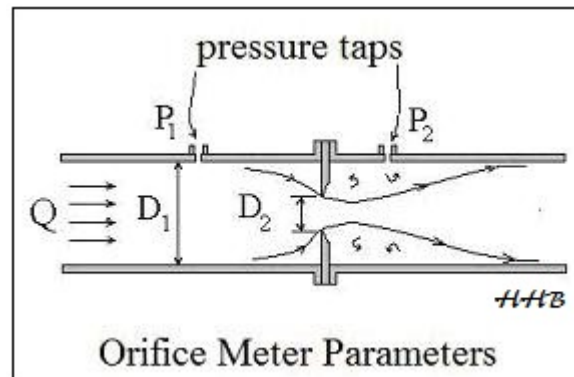
# Flow Measurement in Pipes and Ducts

*Dr. Harlan H. Bengtson, P.E.*

## COURSE CONTENT

### 1. Introduction

This course is about measurement of the flow rate of a fluid flowing under pressure in a closed conduit. The closed conduit is often circular, but also may be square or rectangular (such as a heating duct) or any other shape. The other major category of flow is open channel flow, which is the flow of a liquid with a free surface open to atmospheric pressure. Measurement of the flow rate of a fluid flowing under pressure, is carried out for a variety of purposes, such as billing for water supply to homes or businesses or, for monitoring or process control of a wide variety of industrial processes that involve flowing fluids. Several categories of pipe flow measurement devices will be described and discussed, including some associated calculations.



## 2. Learning Objectives

At the conclusion of this course, the student will

- Be able to calculate liquid flow rate from measured pressure difference, fluid properties, and meter parameters, using the provided equations for venturi, orifice, and flow nozzle meters.
- Be able to calculate gas flow rate from measured pressure difference, fluid properties, and meter parameters, using the provided equations for venturi, orifice, and flow nozzle meters.
- Be able to determine which type of ISO standard pressure tap locations are being used for a given orifice meter.
- Be able to calculate the orifice coefficient,  $C_o$ , for specified orifice and pipe diameters, pressure tap locations and fluid properties.
- Be able to estimate the density of a specified gas at specified temperature and pressure using the Ideal Gas Equation.
- Be able to calculate the velocity of a fluid for given pitot tube reading and fluid density.
- Know the general configuration and principle of operation of rotameters and positive displacement, electromagnetic, target, turbine, vortex, ultrasonic, coriolis mass, and thermal mass meters.
- Know recommended applications for each of the type of flow meter discussed in this course.
- Be familiar with the general characteristics of the types of flow meters discussed in this course, as summarized in Table 2 near the end of this document.

### **3. Types of Pipe Flow Measurement Devices**

The types of pipe flow measuring devices to be discussed in this course are as follows:

- i) Differential pressure flow meters
  - a) Venturi meter
  - b) Orifice meter
  - c) Flow nozzle meter
- ii) Velocity flow meters – pitot/pitot-static tubes
- iii) Variable area flow meters - rotameters
- iv) Positive displacement flow meters
- v) Miscellaneous
  - a) Electromagnetic flow meters
  - b) Target flow meters
  - d) Turbine flow meters
  - e) Vortex flow meters
  - f) Ultrasonic flow meters
  - g) Coriolis mass flow meters
  - h) Thermal mass flow meters

#### 4. Differential Pressure Flow meters

Three types of commonly used differential pressure flow meters are the **orifice meter**, the **venturi meter**, and the **flow nozzle meter**. These three all function by introducing a reduced area through which the fluid must flow. The decrease in area causes an increase in velocity, which in turn results in a decrease in pressure. With these flow meters, the pressure difference between the point of maximum velocity (minimum pressure) and the undisturbed upstream flow is measured and can be correlated with flow rate.

Using the principles of conservation of mass (the continuity equation) and the conservation of energy (the energy equation without friction or Bernoulli equation), the following equation can be derived for ideal flow between the upstream, undisturbed flow (subscript 1) and the downstream conditions where the flow area is constricted (subscript 2):

$$Q_{\text{ideal}} = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad (1)$$

Where:  $Q_{\text{ideal}}$  = ideal flow rate (neglecting viscosity and other friction effects), cfs

$A_2$  = constricted cross-sectional area normal to flow, ft<sup>2</sup>

$P_1$  = upstream (undisturbed) pressure in pipe, lb/ft<sup>2</sup>

$P_2$  = pressure in pipe where flow area is constricted to  $A_2$ , lb/ft<sup>2</sup>

$\beta = D_2/D_1 = (\text{diam. at } A_2)/(\text{pipe diam.})$

$\rho$  = fluid density, slugs/ft<sup>3</sup>

A discharge coefficient, C, is typically put into Equation (1) to account for friction and any other non-ideal factors, giving the following general equation for differential pressure meters:

$$Q = C A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad (2)$$

Where: Q = flow rate through the pipe and meter, cfs

C = discharge coefficient, dimensionless

All other parameters are as defined above.

**Measurement of Gas Flows:** Equations (1) and (2) apply for either liquid flow or gas flow through differential pressure flow meters. For measurement of liquid flow, the density can typically be assumed to be constant throughout the meter, however, for measurement of gas flow, with a reasonable pressure change across the meter, the density will change enough so that it can't be taken as constant in Equations (1) and (2). As a result, Equation (3), shown below, is typically used for gas flow calculations with differential pressure flow meters.

$$Q = C A_2 Y \sqrt{\frac{2ZRT_1(P_1 - P_2)}{(MW)P_1(1 - \beta^4)}} \quad (3)$$

Where: Q, C, A<sub>2</sub>, P<sub>1</sub>, P<sub>2</sub>, and β are as defined above for Equations (1) and (2).

(Note, however, that P<sub>1</sub> in the denominator must be absolute pressure in psia.)

Z = compressibility factor of the gas at P<sub>1</sub>, T<sub>1</sub>

R = Ideal Gas Law Constant = 345.23 psia-ft<sup>3</sup>/slugmole-°R

MW = molecular weight of the gas

$T_1$  = upstream absolute temperature in the pipe, °R

Y = Expansion Factor of the gas – see equation for Y below

**Gas Expansion Factor:** The expansion factor, Y, is needed for gas flow through a differential pressure flow meter in order to account for the decrease in gas density due to the decreased pressure in the constricted portion of the flow meter. For flow through an orifice meter, ISO 5167 – 2:2003 (reference #4 at the end of this course) gives Equation (4), shown below, for the expansion factor, Y:

$$Y = 1 - (0.351 + 0.265\beta^4 + 0.93\beta^8)[1 - (P_2/P_1)^{1/k}] \quad (4)$$

(for  $P_2/P_1 \geq 0.75$ )

Where  $\beta$ ,  $P_1$  and  $P_2$  are the diameter ratio, inlet pressure and pressure at constriction, as defined above, and  $k$  is the specific heat ratio ( $C_p/C_v$ ) for the gas.

For flow through a venturi meter, ISO 5167 – 4:2003 (reference #5 at the end of this course) gives Equation (5), shown below, for the expansion factor, Y. This expression for Y is often used for flow nozzle meter calculations also.

$$Y = \sqrt{\left(\frac{k}{k-1}\right) \left(\frac{1-\beta^4}{1-\beta^4 \tau^{2/k}}\right) \left(\frac{1-\tau^{(k-1)/k}}{1-\tau}\right)} \quad (5)$$

Where  $\beta$  and  $k$  are the diameter ratio and specific heat ratio as defined above, and  $\tau$  is the pressure ratio,  $P_2/P_1$ .

Each of the three types of differential pressure flow meters will now be considered separately.

**Venturi Meter:** Fluid enters a venturi meter through a converging cone of angle 15° to 20°. It then passes through the throat, which has the minimum cross-sectional area, maximum velocity, and minimum pressure in the meter. The fluid then slows down through a diverging cone of angle 5° to 7°, for the transition back to the full pipe diameter. Figure 1 shows the shape of a typical venturi meter and the parameters defined above as applied to this type of meter.  $D_2$  is the diameter of the throat and  $P_2$  is the pressure at the throat.  $D_1$  and  $P_1$  are in the pipe before entering the converging portion of the meter.

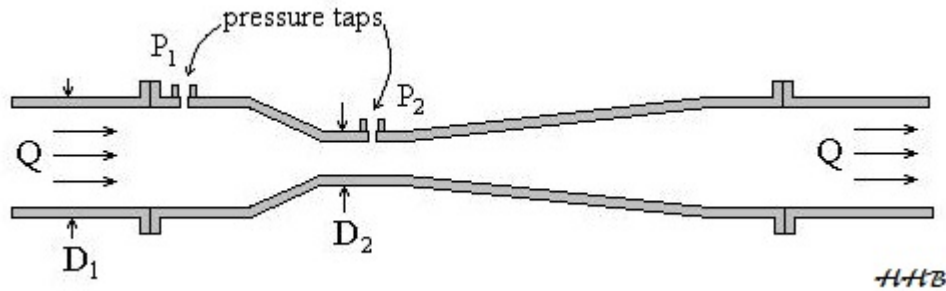


Figure 1. Venturi Meter Parameters

Due to the smooth transition to the throat and gradual transition back to full pipe diameter, the head loss through a venturi meter is quite low and the discharge coefficient is quite high. For a venturi meter, the discharge coefficient is typically called the venturi coefficient,  $C_v$ , giving the following equation for liquid flow through a venturi meter:

$$Q = C_v A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad (6)$$

The value of the venturi coefficient,  $C_v$ , will typically range from 0.95 to nearly one. In ISO 5167 ([ISO 5167-4:2003](#) – see reference #5 for this course),  $C_v$  is given as 0.995 for cast iron or machined venturi meters and 0.985 for welded sheet metal venturi meters meeting ISO specifications, all for Reynold's Number between  $2 \times 10^5$  and  $10^6$ . Information on the venturi coefficient will typically be provided by venturi meter manufacturers or vendors.

**Example #1:** Water at 50° F is flowing through a venturi meter with a 2 inch throat diameter, in a 4 inch diameter pipe. Per manufacturer's information,  $C_v = 0.984$  for this meter under these flow conditions. What is the flow rate through the meter if the pressure difference,  $P_1 - P_2$ , is measured as 8 inches of Hg?

**Solution:** The density of water in the temperature range from 32° to 70°F is 1.94 slugs/ft<sup>3</sup>, to three significant figures, so that value will be used here.  $A_2 = \pi D_2^2/4 = \pi(2/12)^2/4 = 0.02182 \text{ ft}^2$ .  $\beta = 2/4 = 0.5$ . Converting the pressure difference to lb/ft<sup>2</sup>:  $P_1 - P_2 = (8 \text{ in Hg})(70.73 \text{ lb/ft}^2/\text{in Hg}) = 565.8 \text{ lb/ft}^2$ . Substituting all of these values into Equation (6):

$$Q = (0.984)(0.02182) \sqrt{\frac{2 (565.8)}{(1.94)(1 - 0.5^4)}} = \underline{\underline{0.5355 \text{ cfs}}}$$

There is a bit more to the calculation for flow of a gas through a venturi meter, as illustrated with Example #2, which considers the flow of air through the same meter used for water flow calculation in Example #1, with the same measured pressure difference.

**Example #2:** Air at 50° F is flowing through a venturi meter with a 2 inch throat diameter, in a 4 inch diameter pipe. Per manufacturer's information,  $C_v = 0.984$  for this meter under these flow conditions. What is the flow rate through the meter if the pressure difference,  $P_1 - P_2$ , is measured as 8 inches of Hg and the pressure in the pipe upstream of the meter is 20 psia?

**Solution:** As in Example #1:  $A_2 = \pi D_2^2/4 = \pi(2/12)^2/4 = 0.02182 \text{ ft}^2$ .  $\beta = 2/4 = 0.5$ , and the pressure difference of 8 in Hg is equal to 565.8 lb/ft<sup>2</sup> for  $P_1 - P_2$ .

In order to use Equation (3) to calculate the flow rate of air through the venturi meter, values are needed for the following parameters in addition to the values identified above for  $A_2$ ,  $\beta$ , and  $P_1 - P_2$ , the value given of 20 psia for  $P_1$  and the value given above for the ideal gas law constant,  $R$  (345.23 psia-ft<sup>3</sup>/slugmole-°R).

- the compressibility factor of the air,  $Z$
- the molecular weight of the air,  $MW$
- the approach temperature of the air in °R,  $T_1$
- the expansion factor,  $Y$

For a temperature of 50°F and pressure of 20 psia, the compressibility factor for air can be taken to be one. The molecular weight of air is often rounded off to 29. The absolute temperature  $T_1 = 50 + 460 \text{ °R} = 510 \text{ °R}$ .

In order to use Equation (5) to calculate the expansion factor,  $Y$ , the parameter  $\tau$  can be calculated as:

$$\tau = P_2/P_1 = P_1 - (P_2 - P_1)/P_1 = [(20 \times 144) - 565.8]/(20 \times 144) = 0.8035.$$

Using  $k = 1.4$  for air and substituting values for  $k$ ,  $\tau$ , and  $\beta$  into Equation (5) gives  $Y = 0.881$ . Now, substituting all of the calculated parameter values into Equation (3) gives:

$$Q = (0.984)(0.02182)(0.881) \sqrt{\frac{2 \times 1 \times 345.23 \times 510 \times 565.8}{29 \times 20 \times (1 - 0.5^4)}} = \underline{\underline{11.45 \text{ cfs}}}$$

This type of calculation can be facilitated by the use of an Excel spreadsheet set up to make the calculations. An example with the solution to Example #2 is shown on the next page.

3. Calculation of Flow Rate:

Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes

| Inputs                                                      |           |                                     | Calculations                                         |           |                       |
|-------------------------------------------------------------|-----------|-------------------------------------|------------------------------------------------------|-----------|-----------------------|
| Pipe Diam, <b>D</b> (in.) =                                 | 4         | in                                  | Pipe Diam, <b>D</b> (ft.) =                          | 0.33      | ft                    |
| Throat Diam., <b>d</b> (in.) =                              | 2         | in                                  | Throat Diam., <b>d</b> (ft.) =                       | 0.167     | ft                    |
| Measured pressure                                           |           |                                     | Throat Area, <b>A<sub>2</sub></b> =                  | 0.0218    | ft <sup>2</sup>       |
| diff., <b>P<sub>1</sub> - P<sub>2</sub></b> (psi) =         | 3.9       | lb/in <sup>2</sup>                  | Pipe Area, <b>A<sub>1</sub></b> =                    | 0.0873    | ft <sup>2</sup>       |
| Abs. Press. in Pipe, <b>P<sub>1</sub></b> =                 | 20        | psia                                | Diam. Ratio, <b>β</b> =                              | 0.500     | (= d/D)               |
| Abs. Temp. in Pipe, <b>T<sub>1</sub></b> =                  | 510       | °R                                  | Discharge Coeff., <b>C</b> =                         | 0.984     |                       |
| Fluid Viscosity, <b>μ</b> =                                 | 0.0000004 | lb-sec/ft <sup>2</sup>              | Press. Diff., <b>P<sub>1</sub> - P<sub>2</sub></b> = | 565.8     | lb/ft <sup>2</sup>    |
| Gas Mol. Wt., <b>MW</b> =                                   | 29        | lb/lbmole                           | Pressure Ratio, <b>P<sub>2</sub>/P<sub>1</sub></b> = | 0.8035417 |                       |
| Sp. Ht. Ratio (C <sub>p</sub> /C <sub>v</sub> ), <b>k</b> = | 1.4       |                                     | Expansion Factor, <b>ε</b> =                         | 0.881     |                       |
| Compress. Factor, <b>Z</b> =                                | 1         |                                     | Fluid density, <b>ρ</b> =                            | 0.00329   | slugs/ft <sup>3</sup> |
| Ideal Gas Law                                               |           |                                     | Pipe Flow Rate, <b>Q</b> =                           | 11.45     | cfs                   |
| Constant, <b>R</b> =                                        | 345.23    | (psia-ft <sup>3</sup> /slugmole-°R) | Pipe Velocity, <b>V</b> =                            | 131       | ft/sec                |
|                                                             |           |                                     | Reynolds Number, <b>Re<sub>D</sub></b> =             | 360,042   | (in pipe)             |

$$Q = C A_2 \epsilon \sqrt{\frac{2 Z R T_1 (P_1 - P_2)}{(MW) P_1 (1 - \beta^4)}}$$

**Q** = flow rate through pipe and meter, cfs

**C** = discharge coefficient, dimensionless

**A<sub>2</sub>** = venturi throat area, ft<sup>2</sup>

**P<sub>1</sub>** = undisturbed upstream pressure in the pipe, lb/ft<sup>2</sup>

**P<sub>2</sub>** = pressure in the pipe at the constricted area, lb/ft<sup>2</sup>

**β** = d/D = throat diam./pipe diam., dimensionless

**Z** = compressibility factor of the gas at P<sub>1</sub>, T<sub>1</sub>

**R** = Ideal Gas Law Constant = 345.23 psia-ft<sup>3</sup>/slugmole-°R

**MW** = molecular weight of the gas

**T<sub>1</sub>** = upstream absolute temperature in the pipe, °R

**ε** = Expansibility Factor - see equation for ε below

**Orifice Meter:** The orifice meter is the simplest of the three differential pressure flow meters. It consists of a circular plate with a hole in the middle, typically held in place between pipe flanges, as shown in Figure 2.

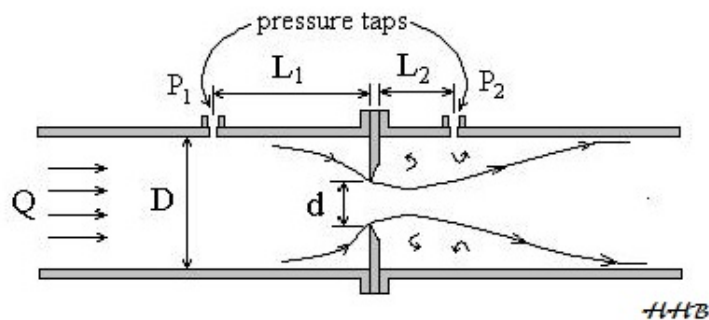


Figure 2. Orifice Meter Parameters

For an orifice meter, the diameter of the orifice,  $d$ , will be used for  $D_2$ ,  $A_2$  is typically called  $A_o$ , and the discharge coefficient is typically called an orifice coefficient,  $C_o$ , giving the following equation for liquid flow through an orifice meter:

$$Q = C_o A_o \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad (7)$$

The preferred locations of the pressure taps for an orifice meter have undergone change over time. Previously the downstream pressure tap was preferentially located at the vena-contracta, the minimum jet area, which occurs downstream of the orifice plate, as shown in Figure 2. For a vena-contracta tap, the tap location depends on the orifice hole size. This link between the tap location and the orifice size made it difficult to change orifice plates with different hole sizes in a given meter in order to alter the range of measurement. In 1991, the ISO-5167 international standard came out, in which three types of standardized differential measuring pressure taps were identified for orifice meters, as illustrated in Figure 3 below. In ISO-5167, the distance of the pressure taps from the orifice plate is specified as a fixed distance or as a function of the pipe diameter, rather than the orifice diameter as shown in Figure 3.

In ISO-5167, an equation for the orifice coefficient,  $C_o$ , is given as a function of  $\beta$ , Reynolds Number, and  $L_1$  &  $L_2$ , the distances of the pressure taps from the orifice plate, as shown in Figures 2 and 3. This equation, given in the next paragraph can be used for an orifice meter with any of the three standard pressure tap configurations.

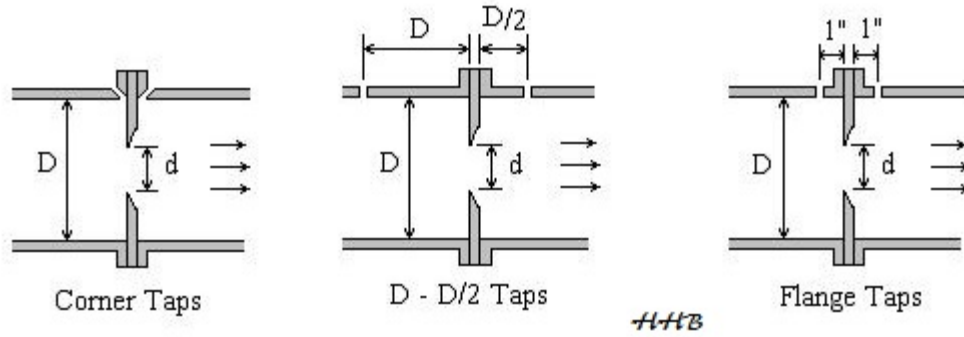


Figure 3. ISO standard orifice meter pressure tap locations

The ISO-5167 equation for  $C_o$  is shown below as Equation (8): (The earlier 2001 version of this equation is given in reference #5 for this course, U.S. Dept. of the Interior, Bureau of Reclamation, *Water Measurement Manual*).

$$\begin{aligned}
 C_o = & 0.5961 + 0.0261\beta^2 + 0.000521 (\beta 10^6/\text{Re})^{0.7} \\
 & + (0.0188 + 0.0063A)\beta^{3.5}(10^6/\text{Re})^{0.3} \\
 & + (0.043 + 0.080e^{-10L_1/D_1} - 0.123 e^{-7L_1/D_1})(1 - 0.11A)[\beta^4/(1 - \beta^4)] \\
 & - 0.031(M'_2 = 0.8M'_2)^{1.1}\beta^{1.3}
 \end{aligned} \tag{8}$$

$$A = (19,000\beta/\text{Re})^{0.8}$$

$$M'_2 = 2(L_2/D_1)/(1 - \beta)$$

If  $D_1 < 2.8$  in, then add the following term to  $C_o$ :  $0.011(0.75 - \beta)(2.8 - D_1)$

Where:  $C_o$  = orifice coefficient, as defined in equation (7), dimensionless

$L_1$  = pressure tap distance from upstream face of the plate, inches

$L_2$  = pressure tap distance from downstream face of the plate, inches

$D$  = pipe diameter, inches

$\beta$  = ratio of orifice diameter to pipe diameter =  $d/D$ , dimensionless

$\text{Re}$  = Reynolds number =  $DV/\nu = DV\rho/\mu$ , dimensionless ( $D$  in ft)

$V$  = average velocity of fluid in pipe =  $Q/(\pi D^2/4)$ , ft/sec (D in ft)

$\nu$  = kinematic viscosity of the flowing fluid, ft<sup>2</sup>/sec

$\rho$  = density of the flowing fluid, slugs/ft<sup>3</sup>

$\mu$  = dynamic viscosity of the flowing fluid, lb-sec/ft<sup>2</sup>

As shown in Figure 3:  $L_1 = L_2 = 0$  for corner taps;  $L_1 = L_2 = 1$  inch for flange taps; and  $L_1 = D$  &  $L_2 = D/2$  for  $D - D/2$  taps. Equation (8) is not intended for use with any other arbitrary values for  $L_1$  and  $L_2$ .

The ISO 5167 standard includes several conditions required for use of Equation (8) as follows:

- For all three pressure tap configurations:
  - $d \geq 0.5$  in
  - $2 \text{ in} \leq D_1 \leq 40 \text{ in}$
  - $0.1 \leq \beta \leq 0.75$
- For corner taps or  $(D - D/2)$  taps:
  - $Re \geq 5000$  for  $0.1 \leq \beta \leq 0.56$
  - $Re \geq 16,000 \beta^2$  for  $\beta > 0.56$
- For flange taps:
  - $Re > 5000$
  - $Re > 170 \beta^2 (25.4 D_1)$  ( $D_1$  in inches)

Fluid properties ( $\nu$  or  $\rho$  &  $\mu$ ) are needed in order to use Equation (8). Tables or graphs with values of  $\nu$ ,  $\rho$ , and  $\mu$  for water and other fluids over a range of temperatures are available in many handbooks and fluid mechanics or thermodynamics textbooks, as for example, in reference #1 for this course. Table 1 shows density and viscosity for water at temperatures from 32° F to 70° F.

Table 1. Density and Viscosity of Water

| <u>Temperature, °F</u> | <u>Density, slugs/ft<sup>3</sup></u> | Dynamic                               | Kinematic                            |
|------------------------|--------------------------------------|---------------------------------------|--------------------------------------|
|                        |                                      | <u>Viscosity, lb-s/ft<sup>2</sup></u> | <u>Viscosity, ft<sup>2</sup>/sec</u> |
| 32                     | 1.94                                 | $3.732 \times 10^{-5}$                | $1.924 \times 10^{-5}$               |
| 40                     | 1.94                                 | $3.228 \times 10^{-5}$                | $1.664 \times 10^{-5}$               |
| 50                     | 1.94                                 | $2.730 \times 10^{-5}$                | $1.407 \times 10^{-5}$               |
| 60                     | 1.938                                | $2.334 \times 10^{-5}$                | $1.204 \times 10^{-5}$               |
| 70                     | 1.936                                | $2.037 \times 10^{-5}$                | $1.052 \times 10^{-5}$               |

**Example #3:** What is the Reynolds number for water at 50°F, flowing at 0.35 cfs through a 4 inch diameter pipe?

**Solution:** Calculate V from  $V = Q/A = Q/(\pi D^2/4) = 0.35/[\pi(4/12)^2/4] = 4.01$  ft/s. From Table 1:  $\nu = 1.407 \times 10^{-5}$  ft<sup>2</sup>/s. From the problem statement:  $D = 4/12$  ft. Substituting into the expression for Re:  $Re = (4/12)(4.01)/(1.407 \times 10^{-5})$

$$\underline{\underline{Re = 9.50 \times 10^4}}$$

**Example #4:** Use Equation (8) to calculate  $C_o$  for orifice diameters of 0.8, 1.6, 2.0, 2.4, & 2.8 inches, each in a 4 inch diameter pipe, with  $Re = 10^5$ , for each of the standard pressure tap configurations: i) D – D/2 taps, ii) flange taps, and iii) corner taps.

**Solution:** Making all of these calculations by hand using Equation (5) would be rather tedious, but once the equation is set up in an Excel spreadsheet, the repetitive calculations are easily done. Following is a copy of the Excel spreadsheet solution to this problem.

**D - D/2 Taps:**

| <b>D, in</b> | <b>d, in</b> | <b>L<sub>1</sub>, in</b> | <b>L<sub>2</sub>, in</b> | <b><math>\beta</math></b> | <b>Re</b> | <b>C<sub>o</sub></b> |
|--------------|--------------|--------------------------|--------------------------|---------------------------|-----------|----------------------|
| 4            | 0.8          | 4                        | 2                        | 0.2                       | 100000    | <b>0.597</b>         |
| 4            | 1.6          | 4                        | 2                        | 0.4                       | 100000    | <b>0.602</b>         |
| 4            | 2            | 4                        | 2                        | 0.5                       | 100000    | <b>0.606</b>         |
| 4            | 2.4          | 4                        | 2                        | 0.6                       | 100000    | <b>0.612</b>         |
| 4            | 2.8          | 4                        | 2                        | 0.7                       | 100000    | <b>0.617</b>         |

**Flange Taps:**

| <b>D, in</b> | <b>d, in</b> | <b>L<sub>1</sub>, in</b> | <b>L<sub>2</sub>, in</b> | <b><math>\beta</math></b> | <b>Re</b> | <b>C<sub>o</sub></b> |
|--------------|--------------|--------------------------|--------------------------|---------------------------|-----------|----------------------|
| 4            | 0.8          | 1                        | 1                        | 0.2                       | 100000    | <b>0.598</b>         |
| 4            | 1.6          | 1                        | 1                        | 0.4                       | 100000    | <b>0.602</b>         |
| 4            | 2            | 1                        | 1                        | 0.5                       | 100000    | <b>0.606</b>         |
| 4            | 2.4          | 1                        | 1                        | 0.6                       | 100000    | <b>0.611</b>         |
| 4            | 2.8          | 1                        | 1                        | 0.7                       | 100000    | <b>0.614</b>         |

**Corner Taps:**

| <b>D, in</b> | <b>d, in</b> | <b>L<sub>1</sub>, in</b> | <b>L<sub>2</sub>, in</b> | <b><math>\beta</math></b> | <b>Re</b> | <b>C<sub>o</sub></b> |
|--------------|--------------|--------------------------|--------------------------|---------------------------|-----------|----------------------|
| 4            | 0.8          | 0                        | 0                        | 0.2                       | 100000    | <b>0.598</b>         |
| 4            | 1.6          | 0                        | 0                        | 0.4                       | 100000    | <b>0.603</b>         |
| 4            | 2            | 0                        | 0                        | 0.5                       | 100000    | <b>0.607</b>         |
| 4            | 2.4          | 0                        | 0                        | 0.6                       | 100000    | <b>0.610</b>         |
| 4            | 2.8          | 0                        | 0                        | 0.7                       | 100000    | <b>0.610</b>         |

Note that  $C_o$  is between 0.597 and 0.617 for all three pressure tap configurations for  $Re = 10^5$  and  $\beta$  between 0.2 and 0.7. For larger values of Reynolds number,  $C_o$  will stay within this range. For smaller values of Reynolds number,  $C_o$  will get somewhat larger, especially for higher values of  $\beta$ .

**Example #5:** Water at 50°F is flowing through an orifice meter with flange taps and a 2 inch throat diameter, in a 4 inch diameter pipe. What is the flow rate through the meter if the pressure difference,  $P_1 - P_2$ , is measured as 3.93 psi.

**Solution:** Assume  $Re$  is approximately  $10^5$ , in order to get started. Then from the solution to **Example #4**, with  $\beta = 0.5$ :  $C_o = 0.606$ .

From Table 1, the density of water at 50°F is 1.94 slugs/ft<sup>3</sup> and its viscosity is  $2.73 \times 10^{-5}$  lb-sec/ft<sup>2</sup>.  $A_2 = \pi D_2^2/4 = \pi(2/12)^2/4 = 0.02182$  ft<sup>2</sup>.  $\beta = 2/4 = 0.5$ . Converting the pressure difference to lb/ft<sup>2</sup>:  $P_1 - P_2 = (8 \text{ in Hg})(70.73 \text{ lb/ft}^2/\text{in Hg}) = 565.8 \text{ lb/ft}^2$ . Substituting all of these values into Equation (7):

$$Q = (0.606)(0.02182) \sqrt{\frac{2(565.8)}{(1.94)(1 - 0.5^4)}} = \underline{\underline{0.330 \text{ cfs}}}$$

Check on Reynolds number value:

$$V = Q/A = 0.330/[\pi(4/12)^2/4] = 3.78 \text{ ft/sec}$$

$$Re = DV/\nu = (4/12)(3.78)/(1.407 \times 10^{-5}) = 8.9 \times 10^4$$

This value is close enough to  $10^5$ , so that the value used for  $C_o$  is probably ok.

**Alternate Solution to Example #5:** The flow rate can be calculated directly without using information from Example #4, by using an iterative calculation to get the value for  $C_o$ , as illustrated in the Excel spreadsheet screenshot shown on the next page. Note that instructions are included for using Excel's Goal Seek tool to carry out the iterative calculation of  $C_o$ . Note that the value calculated for  $C_o$  here is also 0.606 to 3 significant digits and the value calculated for the flow rate  $Q$  is 0.330 cfs, the same as that calculated above.

| Liquid Flow/Orifice Calculations - Flow Rate - U.S. Units                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |           |                        |                                     |          |                    |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|------------------------|-------------------------------------|----------|--------------------|
| For Large Bore Pipes (2 in $\leq D_1 \leq 40$ in.) and $P_2/P_1 \geq 0.75$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |           |                        |                                     |          |                    |
| Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |           |                        |                                     |          |                    |
| <b>Inputs</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |           |                        | <b>Calculations</b>                 |          |                    |
| Pipe Diam, $D_1$ (in) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 4         | in                     | Pipe Diam, $D_1$ (ft) =             | 0.33     | ft                 |
| Orifice Diam., $D_o$ (in) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2         | in                     | Orifice Diam., $D_o$ (ft) =         | 0.167    | ft                 |
| Measured pressure diff., $P_1 - P_2$ (in H <sub>2</sub> O) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 3.93      | psi                    | Orifice Area, $A_o$ =               | 0.021817 | ft <sup>2</sup>    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | Pipe Area, $A_1$ =                  | 0.0873   | ft <sup>2</sup>    |
| Fluid Density, $\rho$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 1.94      | slugs/ft <sup>3</sup>  | Diam. Ratio, $\beta$ =              | 0.500    | (= $D_o/D_1$ )     |
| Fluid Viscosity, $\mu$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 0.0000273 | lb-sec/ft <sup>2</sup> | $A$ =                               | 0.1661   |                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | $M'_2$ =                            | 1.000    |                    |
| Click on the blue cell below and the arrow to the right of it. Then use the drop down list to select the pressure tap configuration*:                                                                                                                                                                                                                                                                                                                                                                                                                             |           |                        | Orifice Coeff., $C_o$ =             | 0.60646  |                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | (see eqn for $C_o$ below)***        |          |                    |
| Flange Taps                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |           |                        | Press. Diff., $P_1 - P_2$ =         | 565.9    | lb/ft <sup>2</sup> |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | Pipe Flow Rate, $Q$ =               | 0.330    | cfs                |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | Pipe Velocity, $V$ =                | 3.78     | ft/sec             |
| Assumed value of Reynolds No., $Re$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 89,592    | (in pipe)              | Upstream Press. Tap Loc., $L_1^*$ = | 1.0      | in                 |
| (Enter an initial value to start the calculation.)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |           |                        | Downstr. Press. Tap Loc., $L_2^*$ = | 1.0      | in                 |
| Diff. between assumed & calculated Reynolds Number, $\Delta Re$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 0.001     |                        | Reynolds Number, $Re$ =             | 89,592   | (in pipe)          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                        | (calculated value)                  |          |                    |
| Pipe Flow Rate, $Q$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |           |                        | 0.330                               | cfs      |                    |
| <p>NOTE: Use Excel's "Goal Seek" to find the flow rate by an iterative calculation as follows: Place the cursor on cell C34 and click on "goal seek" (in the "Tools" menu of older versions and under "Data - What If Analysis" in newer versions of Excel). Make entries to "Set cell: "C34" To value: "0" By changing cell: "C30", and click on "OK". The calculated value of Q will appear in cell F37, and cell C34 should show zero if the process worked properly. Note that the blue cell, C30, needs an initial estimate for Re to start the process.</p> |           |                        |                                     |          |                    |

Note that the calculation is already pretty extensive to calculate the flow rate of a liquid through an orifice meter, because of the complication of obtaining a value for the orifice coefficient,  $C_o$ . Additional steps are added for calculation of the flow rate of a gas through an orifice meter, as illustrated in Example #2 for gas flow through a venturi meter and in the next example for calculating the flow rate of air through an orifice meter with the same pipe and orifice diameters and same measured pressure difference as for water flow in Example #5.

**Example #6:** Air at 50° F is flowing through an orifice meter with flange taps and a 2 inch throat diameter, in a 4 inch diameter pipe. What is the flow rate through the meter if the pressure difference,  $P_1 - P_2$ , is measured as 3.93 psi and the upstream pressure in the pipe,  $P_1$ , is 20 psia?

**Solution:** The calculations will be similar to those used for Example #5, but using Equation (3) for gas flow rather than Equation (7) for liquid flow through an orifice meter. As in Example #5:  $A_2 = \pi D_2^2/4 = \pi(2/12)^2/4 = 0.02182 \text{ ft}^2$ .  $\beta = 2/4 = 0.5$ , and the pressure difference,  $P_1 - P_2$ , is 3.93 psi.

In order to use Equation (3) to calculate the flow rate of air through the venturi meter, values are needed for the following parameters in addition to the values identified above for  $A_2$ ,  $\beta$ , and  $P_1 - P_2$ , the given value of 20 psia for  $P_1$  and the value given above for the ideal gas law constant,  $R$  (345.23 psia-ft<sup>3</sup>/slugmole-°R).

- the compressibility factor of the air,  $Z$
- the molecular weight of the air,  $MW$
- the approach temperature of the air in °R,  $T_1$
- the expansion factor,  $Y$
- the viscosity of air at 50°F ( $4 \times 10^{-7} \text{ lb-sec/ft}^2$ )

For a temperature of 50°F and pressure of 20 psia, the compressibility factor for air can be taken to be one. The molecular weight of air is often rounded off to 29. The absolute temperature  $T_1 = 50 + 460 \text{ °R} = 510 \text{ °R}$ .

In order to use Equation (4) to calculate the expansion factor,  $Y$ , the ratio,  $P_2/P_1$ , can be calculated as:

$$P_2/P_1 = P_1 - (P_2 - P_1)/P_1 = [(20*144) - 565.8]/(20*144) = \mathbf{0.8035}$$

Using  $k = 1.4$  for air and substituting values for  $k$ ,  $P_2/P_1$ , and  $\beta$  into Equation (4) gives:

$$Y = 1 - (0.351 + 0.265(0.5)^4 + 0.93(0.5)^8)[1 - (0.8035)^{1/1.4}] = \mathbf{0.946}$$

Now an iterative calculation like that used in Example #5 is needed to get values for  $C_o$  and  $Q$ . Again, use of an Excel spreadsheet is a convenient way to carry out this calculation including the required iteration. The following figure is a screenshot showing the Excel spreadsheet solution to Example #6, showing the solution as:  **$Q = 7.55$  cfs.**

| Gas Flow/Orifice Calculations - Flow Rate - U.S. Units                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |                                     |                                           |         |                       |                    |        |                 |  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------|-------------------------------------------|---------|-----------------------|--------------------|--------|-----------------|--|
| For Large Bore Pipes ( $2 \text{ in} \leq D_1 \leq 40 \text{ in.}$ ) and $P_2/P_1 \geq 0.75$                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |           |                                     |                                           |         |                       |                    |        |                 |  |
| <b>Instructions:</b> Enter values in blue boxes. Spreadsheet calculates values in yellow boxes                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |           |                                     |                                           |         |                       |                    |        |                 |  |
| <b>Inputs</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |           |                                     |                                           |         | <b>Calculations</b>   |                    |        |                 |  |
| Pipe Diam, $D_1$ (in.) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 4         | in                                  | Pipe Diam, $D_1$ (ft.) =                  | 0.33    | ft                    |                    |        |                 |  |
| Orifice Diam., $D_o$ (in.) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 2         | in                                  | Orifice Diam., $D_o$ (ft.) =              | 0.167   | ft                    |                    |        |                 |  |
| Measured pressure diff., $P_1 - P_2$ (psi) =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 3.93      | lb/in <sup>2</sup>                  | Orifice Area, $A_o$ =                     | 0.0218  | ft <sup>2</sup>       | Pipe Area, $A_1$ = | 0.0873 | ft <sup>2</sup> |  |
| Abs. Press. in Pipe, $P_1$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 20        | psia                                | Diam. Ratio, $\beta$ =                    | 0.500   | (= $D_o/D_1$ )        |                    |        |                 |  |
| Temperature in Pipe, $T_1$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 50        | °F                                  | $A$ =                                     | 0.0761  |                       |                    |        |                 |  |
| Fluid Viscos., $\mu$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 0.0000004 | lb-sec/ft <sup>2</sup>              | $M'_2$ =                                  | 1.000   |                       |                    |        |                 |  |
| Gas Mol. Wt., $MW$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 29        | lb/lbmole                           | Orifice Coeff., $C_o$ =                   | 0.605   |                       |                    |        |                 |  |
| Sp. Ht. Ratio of gas ( $C_p/C_v$ ), $k$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1.4       |                                     | (see eqn for $C_o$ below)***              |         |                       |                    |        |                 |  |
| Compress. Factor of gas, $Z$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 1         |                                     | Abs. Temp. in Pipe, $T_1$ =               | 509.67  | °R                    |                    |        |                 |  |
| Ideal Gas Law Constant, $R$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 345.23    | (psia-ft <sup>3</sup> /slugmole-°R) | Press. Diff., $P_1 - P_2$ =               | 565.92  | lb/ft <sup>2</sup>    |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Pressure Ratio, $P_2/P_1$ =               | 0.8035  |                       |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Expansion Factor, $Y$ =                   | 0.946   |                       |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Fluid density, $\rho$ =                   | 0.00330 | slugs/ft <sup>3</sup> |                    |        |                 |  |
| Click on the blue cell below and the arrow to the right of it. Then use the drop down list to select the pressure tap configuration*:                                                                                                                                                                                                                                                                                                                                                                                                                                    |           |                                     | Pipe Flow Rate, $Q$ =                     | 7.5549  | cfs                   |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Pipe Velocity, $V$ =                      | 86.6    | ft/sec                |                    |        |                 |  |
| Assumed value of Reynolds No., $Re$ = 237,811 (in pipe)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |           |                                     | Upstream Press.                           |         |                       |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Tap Loc., $L_1^*$ =                       | 1.0     | in                    |                    |        |                 |  |
| (Enter an initial value to start the calculation.)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |           |                                     | Downstr. Press.                           |         |                       |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | Tap Loc., $L_2^*$ =                       | 1.0     | in                    |                    |        |                 |  |
| Diff. between assumed & calculated Reynolds Number, $\Delta Re$ = 0.000                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |           |                                     | Reynolds Number, $Re$ = 237,811 (in pipe) |         |                       |                    |        |                 |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |           |                                     | (calculated value)                        |         |                       |                    |        |                 |  |
| Pipe Flow Rate, $Q$ =                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |           |                                     | 7.5549                                    | cfs     |                       |                    |        |                 |  |
| <p><b>NOTE:</b> Use Excel's "Goal Seek" to find the flow rate by an iterative calculation as follows: Place the cursor on cell C41 and click on "goal seek" (in the "Tools" menu of older versions and under "Data - What If Analysis" in newer versions of Excel). Make entries to "Set cell: "C41" To value: "0" By changing cell: "C38", and click on "OK". The calculated value of Q will appear in cell F43, and cell C41 should show zero if the process worked properly. Note that the blue cell, C38, needs an initial estimate for Re to start the process.</p> |           |                                     |                                           |         |                       |                    |        |                 |  |

**Flow Nozzle Meter:** The flow nozzle meter is simpler and less expensive than a venturi meter, but not quite as simple as an orifice meter. It consists of a relatively short nozzle, typically held in place between pipe flanges, as shown in Figure 4.

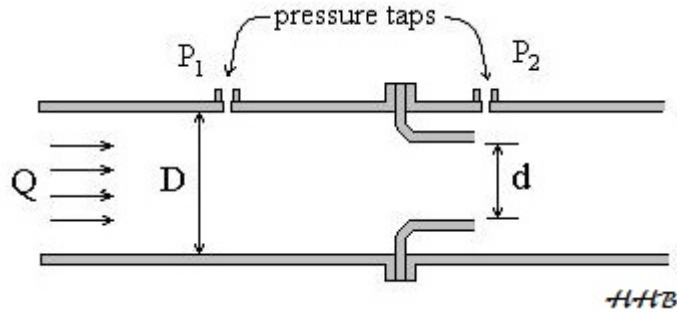


Figure 4. Flow Nozzle Meter Parameters

For a flow nozzle meter, the exit diameter of the nozzle,  $d$ , is used for  $D_2$  (giving  $A_2 = A_n$ ), and the discharge coefficient is typically called a nozzle coefficient,  $C_n$ , giving the following equation for a flow nozzle meter:

$$Q = C_n A_n \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \quad (9)$$

Due to the smoother contraction of the flow, flow nozzle coefficients are significantly higher than orifice coefficients. They are not, however as high as venturi coefficients. Flow nozzle coefficients are typically in the range from 0.94 to 0.99. There are several different standard flow nozzle designs. Information on pressure tap placement and calibration should be provided by the meter manufacturer.

## 5. Velocity Flow Meters – Pitot / Pitot-Static Tubes

Pitot tubes (also called pitot-static tubes) are an inexpensive, convenient way to measure velocity at a point in a fluid. They are used in airflow measurements in ventilation and HVAC applications. Definitions for three types of pressure and how to measure those three different kinds of pressure are given below, because understanding them helps to understand the pitot tube equation. Static pressure, dynamic pressure and total pressure are defined below and illustrated in Figure 5.

**Static pressure** is the fluid pressure relative to surrounding atmospheric pressure, measured through a flat opening, which is in parallel with the fluid flow, as shown with the first U-tube manometer in Figure 5.

**Stagnation pressure** is the fluid pressure relative to the surrounding atmospheric pressure, measured through a flat opening, which is perpendicular to and facing into the direction of fluid flow, as shown with the second U-tube manometer in Figure 5. This is also sometimes called the total pressure.

**Dynamic pressure** is the fluid pressure relative to the static pressure, measured through a flat opening, which is perpendicular to and facing into the direction of fluid flow, as shown with the third U-tube manometer in Figure 5. This is also sometimes called the velocity pressure.

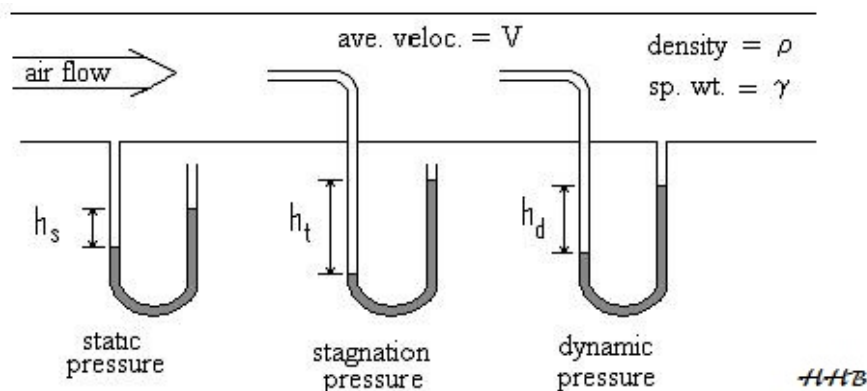


Figure 5. Various Pressure Measurements

Static pressure is typically represented by the symbol,  $p$ . Dynamic pressure is equal to  $\frac{1}{2} \rho V^2$ . Stagnation pressure, represented here by  $P_{\text{stag}}$ , is equal to static pressure plus dynamic pressure plus the pressure due to a column of fluid of height,  $h$ , equal to the elevation of the static pressure tap above the stagnation pressure tap, as shown in the following equation.

$$P_{\text{stag}} = P + \frac{1}{2} \rho V^2 + \gamma h \quad (10)$$

Where the parameters with a consistent set of units are as follows:

$P_{\text{stag}}$  = stagnation pressure, lb/ft<sup>2</sup>

$P$  = static pressure, lb/ft<sup>2</sup>

$\rho$  = density of fluid, slugs/ft<sup>3</sup>

$\gamma$  = specific weight of fluid, lb/ft<sup>3</sup>

$h$  = elevation of static pressure tap above stagnation pressure tap, ft

$V$  = average velocity of fluid, ft/sec

( $V = Q/A$  = volumetric flow rate/cross-sectional area normal to flow)

For pitot tube measurements, the static pressure tap and stagnation pressure tap are at the same elevation, so that  $h = 0$ . Then stagnation pressure minus static pressure is equal to dynamic pressure, or:

$$P_{\text{stag}} - P = \frac{1}{2} \rho V^2 \quad (11)$$

The pressure difference,  $P_{\text{stag}} - P$ , can be measured directly with a pitot tube such as the third U-tube in Figure 5, or more simply with a pitot tube like the one shown in Figure 6, which has two concentric tubes. The inner tube has a stagnation pressure opening and the outer tube has a static pressure opening parallel to the fluid flow direction. The pressure difference is equal to the dynamic pressure ( $\frac{1}{2} \rho V^2$ ) and can be used to calculate the fluid velocity for known fluid density,  $\rho$ . A consistent set of units is: pressure in lb/ft<sup>2</sup>, density in slugs/ft<sup>3</sup>, and velocity in ft/sec.

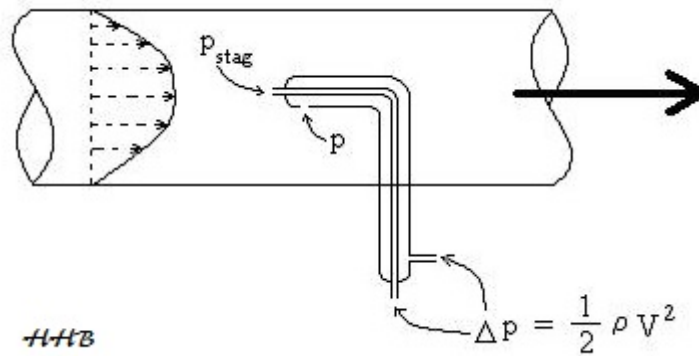


Figure 6. Pitot Tube

For use with a pitot tube, Equation (11) will typically be used to calculate the velocity of the fluid. Setting  $(P_{\text{stag}} - P) = \Delta P$ , and solving for  $V$ , gives the following equation:

$$V = \sqrt{\frac{2 \Delta P}{\rho}} \quad (12)$$

In order to use Equation (12) to calculate fluid velocity from pitot tube measurements, it is necessary to be able to obtain a value of density for the flowing fluid at its temperature and pressure. For a liquid, a value for density can typically be obtained from a table similar to Table 1 in this course. Such tables are available in handbooks and fluid mechanics or thermodynamics textbooks. Pitot tubes are used more commonly, however, to measure gas flow, as for

example, air flow in HVAC ducts, and density of a gas varies considerably with both temperature and pressure. A convenient way to obtain a value of density for a gas at known temperature and pressure is through the use of the Ideal Gas Law.

The Ideal Gas Law, as used to calculate density of a gas is as follows:

$$\rho = (MW) \left( \frac{P}{R T} \right) \quad (13)$$

Where:  $\rho$  = density of the gas at pressure, P, & temperature, T, slugs/ft<sup>3</sup>

MW = molecular weight of the gas, slugs/slug-mole (The average molecular weight typically used for air is 29.)

P = absolute pressure of the gas, psia

T = absolute temperature of the gas, °R (°F + 459.67 = °R)

R = Ideal Gas Law constant, 345.23 psia-ft<sup>3</sup>/slug-mole-°R

But, you may ask, this is the *Ideal Gas Law*, so how can we use it to find the density of *real* gases? Well, the Ideal Gas Law is a very good approximation for many real gases over a wide range of temperatures and pressures. It does not work well for very high pressures or very low temperatures (approaching the critical temperature and/or critical pressure for the gas), but for many practical, real situations, the Ideal Gas Law gives quite accurate values for density of a gas.

**Example #7:** Estimate the density of air at 16 psia and 85 °F.

**Solution:** Convert 85 °F to °R: 85 °F = 85 + 459.67 °R = 544.67 °R

Substituting values for P, T, R, & MW into Equation (13) gives:

$$\rho = (29)[16/(345.23)(544.67)] = \underline{\underline{0.002468 \text{ slugs/ft}^3}}$$

**Example #8:** A pitot tube is being used to measure air velocity in a heating duct. The air is at 85 °F and 16 psia. The pitot tube registers a pressure difference of 0.023 inches of water ( $P_{\text{stag}} - P$ ). What is the velocity of the air at that point in the duct?

**Solution:** Convert 0.023 inches of water to lb/ft<sup>2</sup> (psf) (conversion factor is: 5.204 psf/in of water):

$$0.023 \text{ in of water} = (0.023)(5.204) \text{ psf} = 0.1197 \text{ psf}$$

Air density at the given P & T is 0.002468 slugs/ft<sup>3</sup> from Example #5.

Substituting into Equation (12), to calculate the velocity, gives:

$$V = \sqrt{\frac{2(0.1197)}{0.002468}} = \underline{\underline{9.85 \text{ ft.sec}}}$$

## 6. Variable Area Flow Meter - Rotameters

A rotameter is a ‘variable area’ flow meter. It consists of a tapered glass or plastic tube with a float that moves upward to an equilibrium position determined by the flow rate of fluid going through the meter. For greater flow rate, a larger cross-sectional area is needed for the flow, so the float is moved upward until the upward force on it by the fluid is equal to the force of gravity pulling it down. Note that the ‘float’ must have a density greater than the fluid, or it would simply float to the top of the fluid. Given below, in Figure 7, is a schematic diagram of a rotameter, showing the principle of its operation.

The height of the float as measured by a graduated scale on the side of the rotameter can be calibrated for the flow rate of the fluid being measured in appropriate flow units. A few points regarding rotameters follow:

- Because of the key role of gravity, rotameters must be installed vertically.
- Typical turndown ratio is 10:1, that is, flow rates as low as 1/10 of the maximum reading can be accurately measured.
- Accuracy as good as 1% of full scale reading can be expected.
- Rotameters do not require power, so they are safer to use with flammable fluids, than an instrument using power, which would need to be explosion proof.
- A rotameter causes little pressure drop.
- It is difficult to apply machine reading and continuous recording with a rotameter.

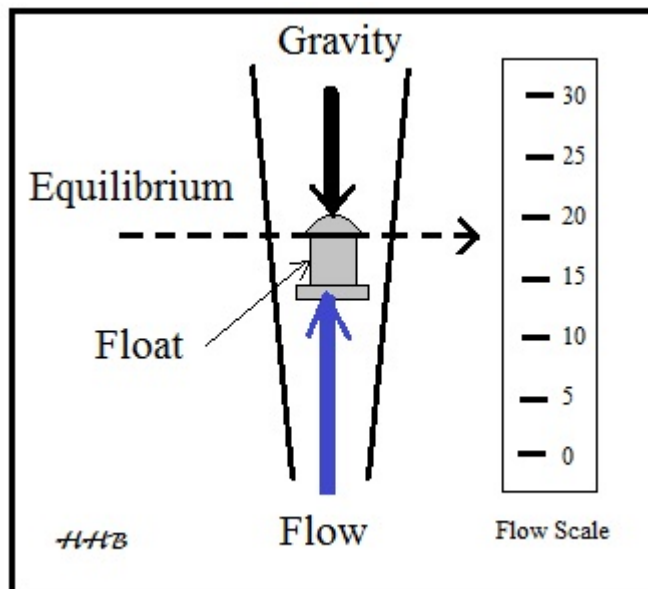


Figure 7. Rotameter Schematic diagram

## **7. Positive Displacement Flow Meters**

Positive displacement flow meters are often used in residential and small commercial applications. They are very accurate at low to moderate flow rates, which are typical of these applications. There are several types of positive displacement meters, such as reciprocating piston, nutating disk, oval gear, and rotary vane. In all of them, the water passing through the meter, physically displaces a known volume of fluid for each rotation of the moving measuring element. The number of rotations is counted electronically or magnetically and converted to the volume that has passed through the meter and/or flow rate.

Positive displacement meters can be used for any relatively nonabrasive fluid, such as heating oils, Freon, printing ink, or polymer additives. The accuracy is very good, approximately 0.1% of full flow rate with a turndown of 70:1 or more.

On the other hand, positive displacement flow meters are expensive compared to many other types of meters and produce the highest pressure drop of any flow meter type.

## **8. Miscellaneous Types of Flow meters**

In this section several more types of flow meters for use with pipe flow will each be described and discussed briefly.

### **a) Electromagnetic flow meters**

An electromagnetic flow meter (also called 'magnetic meter' or 'mag meter') measures flow rate by measuring the voltage generated by a conductive fluid passing through a magnetic field. The magnetic field is created by coils outside the flow tube, carrying electrical current. The generated voltage is proportional to the flow rate of the conductive fluid passing through the flow tube. An external sensor measures the generated voltage and converts it to flow rate.

In order to be measured by an electromagnetic flow meter, the fluid must have a conductivity of at least 5  $\mu\text{S}/\text{cm}$ . Thus, this type of meter will not work for distilled or deionized water or for most non-aqueous liquids. It works well for water that has not been distilled or deionized and many aqueous solutions. Since there is no internal sensor to get fouled, an electromagnetic flow meter is quite suitable for wastewater, other dirty liquids, corrosive liquids or slurries. Since there is no constriction or obstruction to the flow through an electromagnetic meter, it creates negligible pressure drop. It does, however, have a relatively high power consumption, in comparison with other types of flow meters.

#### b) Target flow meters

With a target flow meter, a physical target (disk) is placed directly in the path of the fluid flow. The target will be deflected due to the force of fluid striking it, and the greater the fluid flow rate, the greater the deflection will be. The deflection is measured by a sensor mounted on the pipe and calibrated to flow rate for a given fluid. Figure 8 shows a diagram of a target flow meter.

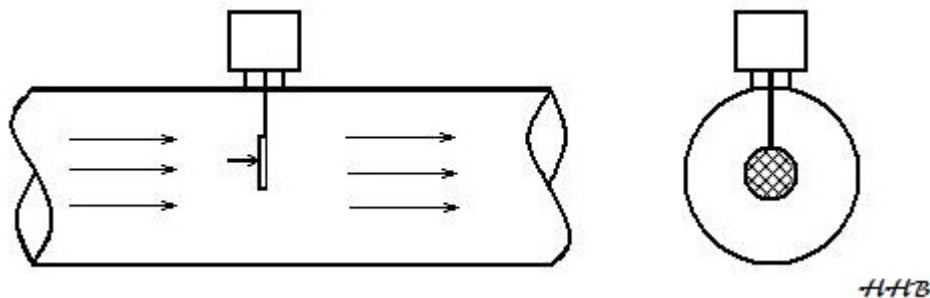


Figure 8. Target Flow Meter

A target flow meter can be used for a wide variety of liquids or gases and there are no moving parts to wear out. They typically have a turndown of 10:1 to 15:1.

c) Turbine flow meters

A turbine flow meter operates on the principle that a fluid flowing past the blades of a turbine will cause it to rotate. Increasing flow rate will cause increasing rate of rotation for the turbine. The meter thus consists of a turbine placed in the path of flow and means of measuring the rate of rotation of the turbine. The turbine's rotational rate can then be calibrated to flow rate. The turbine meter has one of the higher turndown ratios, typically 20:1 or more. Its accuracy is also among the highest at about  $\pm 0.25\%$ .

d) Vortex flow meters

An obstruction in the path of a flowing fluid will create vortices in the downstream flow if the fluid flow speed is above a critical value. A vortex flow meter (also known as vortex shedding or oscillatory flow meter), measures the vibrations of the downstream vortices caused by a barrier in the flow path, as illustrated in Figure 9. The vibrating frequency of the downstream vortices will increase with increasing flow rate, and can thus be calibrated to flow rate of the fluid.

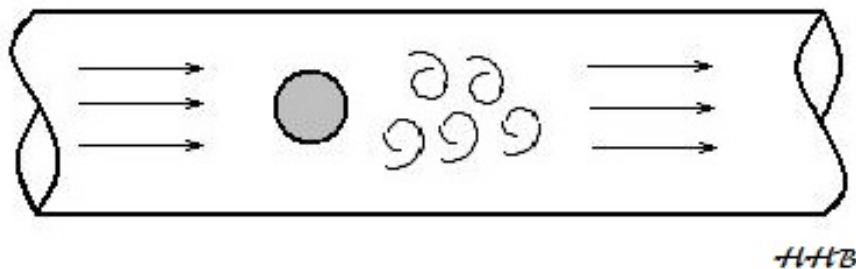


Figure 9. Vortex Flow Meter

#### e) Ultrasonic flow meters

The two major types of ultrasonic flow meters are 'Doppler' and 'transit-time' ultrasonic meters. Both use ultrasonic waves (frequency  $> 20$  kHz). Both types also use two transducers that transmit and/or receive the ultrasonic waves.

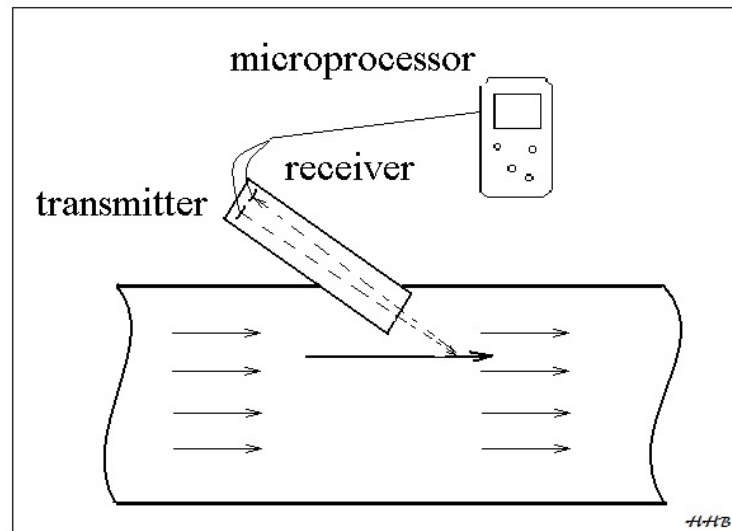


Figure 10. Doppler Ultrasonic Flowmeter

For the Doppler ultrasonic meter, one transducer transmits the ultrasonic waves and the other receives the waves. The fluid must have material in it that will reflect sonic waves, such as particles or entrained air. The frequency of the transmitted beam of ultrasonic waves will be altered, by being reflected from the particles or air bubbles. The resulting frequency shift is measured by the receiving transducer, and is proportional to the flow rate through the meter. A signal can thus be generated from the receiving transducer, which is proportional to flow rate.

Transit-time ultrasonic meters, also known as 'time-of-travel' meters, measure the difference in travel time between pulses transmitted in the direction of flow and pulses transmitted against the flow. The two transducers are mounted so that one

is upstream of the other. Both transducers serve alternately as transmitter and receiver. The upstream transducer will transmit a pulse, which is detected by the downstream transducer, acting as a receiver, giving a 'transit-time' in the direction of flow. The downstream transducer will then transmit a pulse, which is detected by the upstream transducer (acting as a receiver), to give a 'transit-time' against the flow. The difference between the upstream and downstream transit times can be correlated to flow rate through the meter.

The components of a transit-time ultrasonic flow meter are shown in Figure 10. One of the options with this type of meter is a rail-mounted set of transducers, which can be clamped onto an existing pipe without taking the pipe apart to mount the meter. It could be used in this way to check on or calibrate an existing meter, or as a permanent installation for flow measurement. Ultrasonic flow meters are also available with transducers permanently mounted on an insert that is mounted in the pipeline, much like other flow meters, such as an electromagnetic flow meter.

Like the electromagnetic flow meter, ultrasonic meters have no sensors inside the pipe nor any constrictions or obstructions in the pipe, so they are suitable for dirty or corrosive liquids or slurries. Also, they cause negligible pressure drop.

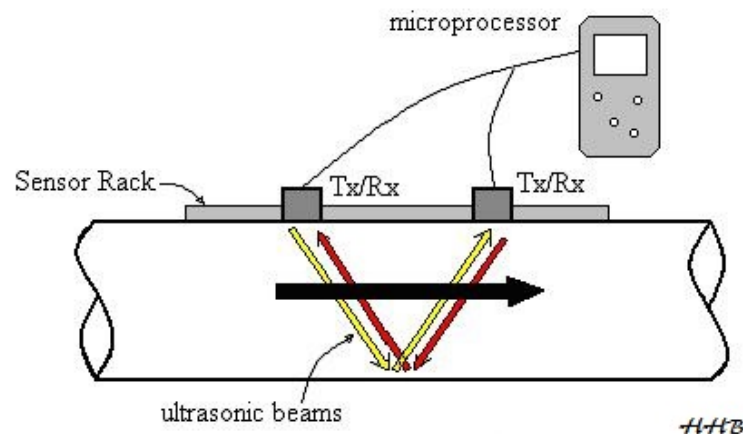


Figure 11. Transit-time Ultrasonic Flow Meter

#### f) Mass flow meters

The two types of mass flow meters will be described and discussed here. They are the coriolis mass flow meter and thermal mass flow meter. Both of these types of flow meters measure mass flow rate rather than volumetric flow rate.

**Coriolis flow meters** make use of the Coriolis effect (a coriolis force that acts on objects that are in motion relative to a rotating frame of reference. A coriolis flow meter typically functions by generating a vibration of the tube or tubes that the fluid is flowing through. Often the part of the tube that is vibrated is curved. The amount of twist caused by the coriolis force is measured and is proportional to the mass flow rate passing through the tube(s). Quite a variety of different designs are used for coriolis mass flow meters.

**Coriolis flow meters** are among the most accurate of the types of flow meters and have a very high turndown ratio (range from minimum to maximum readable flow rate for a given meter).

**Thermal mass flow meter** typically include a means of heat input to the flowing fluid and for temperature measurement at two or more points. The amount of temperature increase and rate of heat input to the fluid are measured and can be correlated with the flow rate of the fluid through its thermal properties.

**Thermal mass flow meters** are among the most accurate types of flow meters, have a very high turndown ratio (range from minimum to maximum readable flow rate for a given meter), and have a medium cost. On the other hand, they are only useable for the flow of clean gases and do not work well for gas mixtures if the gas composition varies with time

### 9. Comparison of Flow Meter Alternatives

Table 2 shows a summary of several useful characteristics of the different types of pipe flow meters described and discussed in this course. The information in Table 2 was extracted from similar tables at the Omega Engineering and ICENTA web sites at:

<http://www.omega.com/techref/table1.html>, and

<http://www.icenta.co.uk/knowledge-base/flow-selection-guide/>

The flow meter characteristics summarized in Table 2 are: recommended applications, typical turndown ratio (also called rangeability), pressure drop, typical accuracy, upstream pipe diameters (required upstream straight pipe length), effect of viscosity, and relative cost.

| Flowmeter Type        | Recomm. Application                                   | Typical Turndown Ratio | Pressure Drop | Typical Accuracy %                | Upstream Pipe Diameters | Effect of Viscosity | Relative Cost |
|-----------------------|-------------------------------------------------------|------------------------|---------------|-----------------------------------|-------------------------|---------------------|---------------|
| Orifice               | Clean, dirty liquid; some slurries                    | 4 to 1                 | Medium        | $\pm 2$ to $\pm 4$ of full scale  | 10 to 30                | High                | Low           |
| Venturi               | Clean, dirty and viscous liquids; some slurries       | 4 to 1                 | Low           | $\pm 1$ of full scale             | 5 to 20                 | High                | Medium        |
| Flow Nozzle           | Clean and dirty liquids                               | 4 to 1                 | Medium        | $\pm 1$ to $\pm 2$ of full scale  | 10 to 30                | High                | Medium        |
| Pitot Tube            | Clean liquids, gases                                  | 3 to 1                 | Very low      | $\pm 3$ to $\pm 5$ of full scale  | 20 to 30                | Low                 | Low           |
| Rotameter             | Clean, dirty and viscous liquids                      | 10 to 1                | Medium        | $\pm 1$ to $\pm 10$ of full scale | None                    | Medium              | Low           |
| Positive Displacement | Clean, viscous liquids                                | 10 to 1                | High          | $\pm 0.5$ of rate                 | None                    | High                | Medium        |
| Electromagnetic       | Clean, dirty, viscous conductive liquids and slurries | 40 to 1                | None          | $\pm 0.5$ of rate                 | 5                       | None                | High          |

| Flowmeter Type              | Recomm. Application                               | Typical Turndown Ratio | Pressure Drop | Typical Accuracy %                  | Upstream Pipe Diameters | Effect of Viscosity | Relative Cost |
|-----------------------------|---------------------------------------------------|------------------------|---------------|-------------------------------------|-------------------------|---------------------|---------------|
| Target Meter                | Clean, dirty, viscous liquids; some slurries      | 10 to 1                | Medium        | $\pm 1$ to $\pm 5$ of full scale    | 10 to 30                | Medium              | Medium        |
| Turbine                     | Clean, viscous liquids                            | 20 to 1                | High          | $\pm 0.25$ of rate                  | 5 to 10                 | High                | High          |
| Vortex                      | Clean and dirty liquids                           | 10 to 1                | Medium        | $\pm 1$ of rate                     | 10 to 20                | Medium              | High          |
| Ultrasonic (Doppler)        | Clean, dirty, viscous liquids and slurries        | 10 to 1                | None          | $\pm 5$ of full scale               | 5 to 30                 | None                | High          |
| Ultrasonic (Time-of-travel) | Clean, viscous liquids                            | 20 to 1                | None          | $\pm 1$ to $\pm 5$ of full scale    | 5 to 30                 | None                | High          |
| Coriolis Mass               | Clean, dirty, viscous liquids, gases and slurries | 100 to 1               | Low           | $\pm 0.05$ to $\pm 0.15$ of reading |                         | Low                 | High          |
| Thermal Mass                | Clean gases                                       | 50 to 1                | Low           | $\pm 0.05$ to $\pm 0.10$ of reading |                         | n.a.                | Medium        |

## 10. Summary

There are a wide variety of meter types for measuring flow rate in closed conduits. Fourteen of those types were described and discussed in this course. This included a considerable amount of detail about pressure differential flow meters (venturi, orifice and flow nozzle meters), such as equations and example calculations for liquid flow and for gas flow through differential flow meters.

Table 2 in Section 9, summarizes a comparison among those fourteen types of flow meters: Orifice meter, Venturi meter, Flow nozzle meter, Pitot tube, Rotameter, Electromagnetic flow meter, Target meter, Turbine meter, Vortex flow meter, Ultrasonic (Doppler) flow meter, Ultrasonic (time of travel) flow meter, Coriolis mass flow meter, and Thermal mass flow meter.

For each of these types of flow meters, Table 2 provides information about i) recommended applications, ii) typical turndown ratio, iii) whether its pressure drop is high, medium, low, or none, iv) typical accuracy in %, v) required upstream pipe diameters of straight pipe, vi) effect of viscosity, and vii) relative cost.

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