CEB2083 PROCESS INSTRUMENTATION & CONTROL
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Chapter 11:
Design of Feedback Controllers

BY

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Chapter Objectives

End of this chapter, you should be able to:

- Design Feedback Controllers
- Specify controller performance criteria
- Select appropriate types of controller
- Tune controllers using
  - Zeigler-Nichols
  - Cohen-Coon
- Design Model based Controllers using
  - Direct synthesis
  - Internal Model Control (IMC)

Performance Criteria

- The function of a feedback control system is to ensure that the closed-loop system has desirable dynamic and steady-state response characteristics

1. Stable closed-loop feedback control system
2. Provide good disturbance rejection
3. Good set-point tracking - rapid, smooth responses
4. No offset
5. No Excessive control action
6. Robust, i.e., insensitive to changes in process conditions and inaccuracies in process model

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Performance Criteria

- It is not possible to achieve all of these goals simultaneously, because they involve conflicts and tradeoffs
- The performance must balance performance and robustness
- A high performing controller (rapid and smooth responses with little or no oscillation) has less robustness
- A highly robust controller (satisfactory performance over a wide range of process conditions and model inaccuracies) has poor performance

Performance Criteria

- Another trade-off in controller design
- Controllers with excellent disturbance rejection can produce large overshoots for set-point changes
- Controllers that provide excellent set-point tracking can be very sluggish for disturbance changes
- Thus, a trade-off between set-point tracking and disturbance rejection occurs for standard PID controllers

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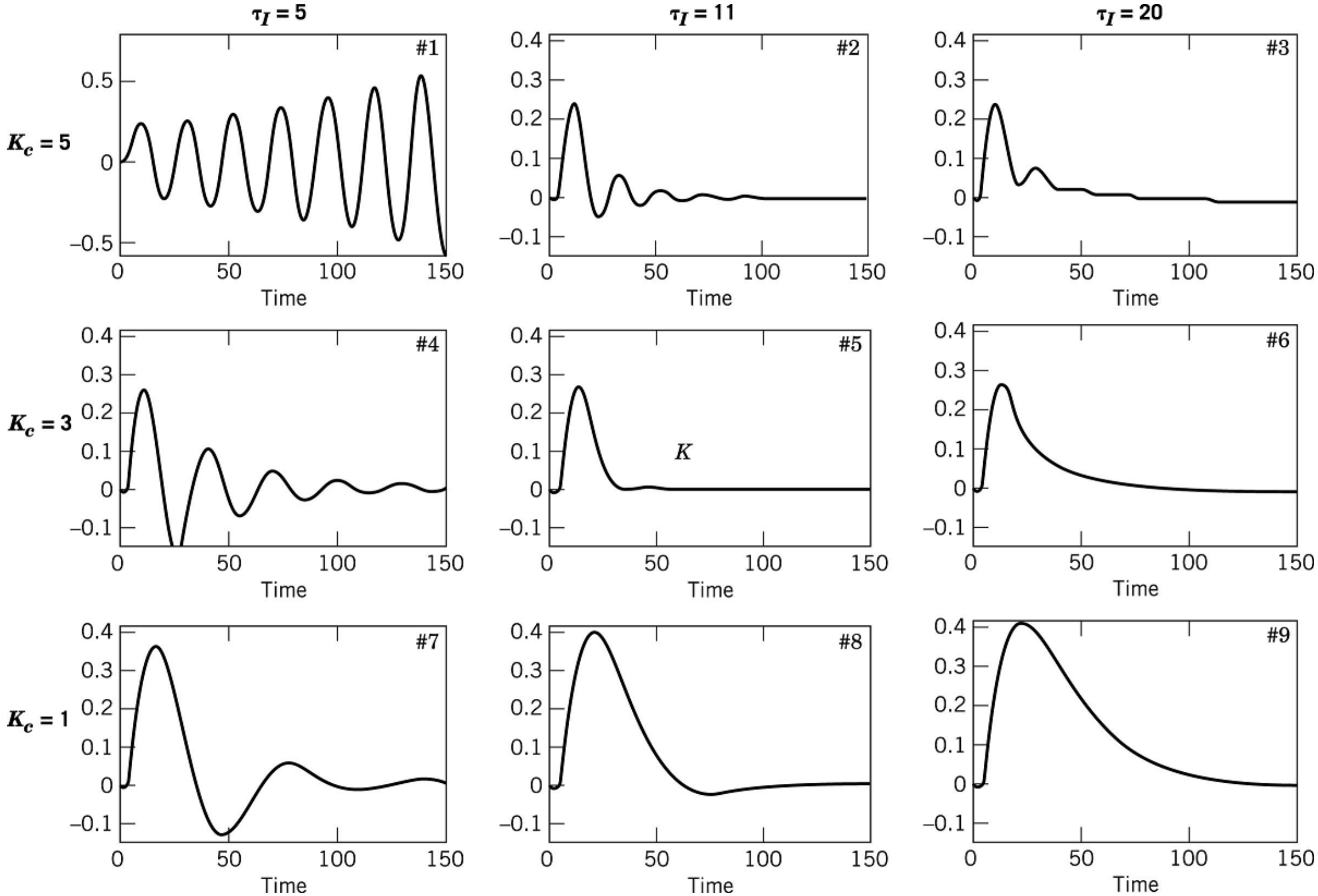
Performance Criteria



Unit-step disturbance responses for controlling a FOPTD model ($K = 1$, $\tau = 20$, $\theta = 4$)

Selection of Controller Types

- General guidelines are available for selection of controller types (P, PI or PID)
- The guidelines are useful but use with caution
- **Flow and Pressure Control**
  - Characterized by fast responses (on the order of seconds) and no time delay
  - Disturbances in flow control systems small, frequent and high frequency noises
  - PI controller is generally used with

$$0.5 < K_c < 0.7;$$

$$0.2 < \tau_I < 0.3 \text{ min}$$

Selection of Controller Types

- **Level Control**
  - A liquid storage vessel with a pump on its exit line can act as an integrating process
  - Standard P or PI controllers are widely used
  - Level control problems have unusual characteristics
    - Increasing the gain of a PI controller increases stability or reducing the gain increase oscillations
    - When K_c becomes too large, oscillations or even instability can occur
    - P control is good enough if small offsets can be tolerated – averaging control
    - Derivative controllers are not used the level measurements are noisy

Selection of Controller Types

- **Gas Pressure Control**
  - Pressure control is analogous to level control in the sense that it can also use averaging control
  - Some applications require tight control of pressure
  - High and low limits are of more serious concern for safety reasons
  - Pressure systems also can behave like integrating systems
  - PI controllers are generally used with small amount of integral action
  - Derivative action is not needed as the process response times are usually quite small

Selection of Controller Types

- **Temperature Control**
  - General guidelines are difficult to state because of variety of processes and equipment involving heat transfer and their different time scales
  - Presence of time delays and multiple thermal capacitances will place stability limits on controller gain
  - PI and PID controllers are commonly used

Selection of Controller Types

- **Composition Control**
  - Generally have characteristics similar to temperature loops but with some differences
  - Measurement noise is a more significant problem
  - Time delays associated with the analyzer and sampling system is a significant factor
  - The effectiveness of derivative action, therefore, is limited
- Because of their importance and the difficulty of control, composition and temperature control loops are prime candidates for advanced control strategies

Controller Tuning

- The stability and performance of a feedback control system highly depends on the controller settings, i.e., the values of K_c, τ_I, and τ_D
- PID controller settings can be determined by a number of alternatives techniques:
  - Controller tuning relations / Empirical tuning
  - Model-based controller design techniques
    - Direct synthesis (DS) method
    - Internal model control method
  - Frequency response techniques
  - Computer simulation
  - Online tuning

Controller Performance Criteria

Integral Error Criteria

Integral of the absolute value of the error

$$IAE = \int_0^\infty |e(t)|\,dt$$

Integral of the squared error

$$ISE = \int_0^\infty e(t)^2\,dt$$

Integral of the time-weighted absolute error

$$ITAE = \int_0^\infty t|e(t)|\,dt$$

Controller Tuning Relations / Empirical Tuning

- One of the traditional ways to design a PID controller was to use empirical tuning rules based on measurements made on the real plant

- Ziegler-Nichols (Z-N) Oscillation Method

- Reaction Curve based methods

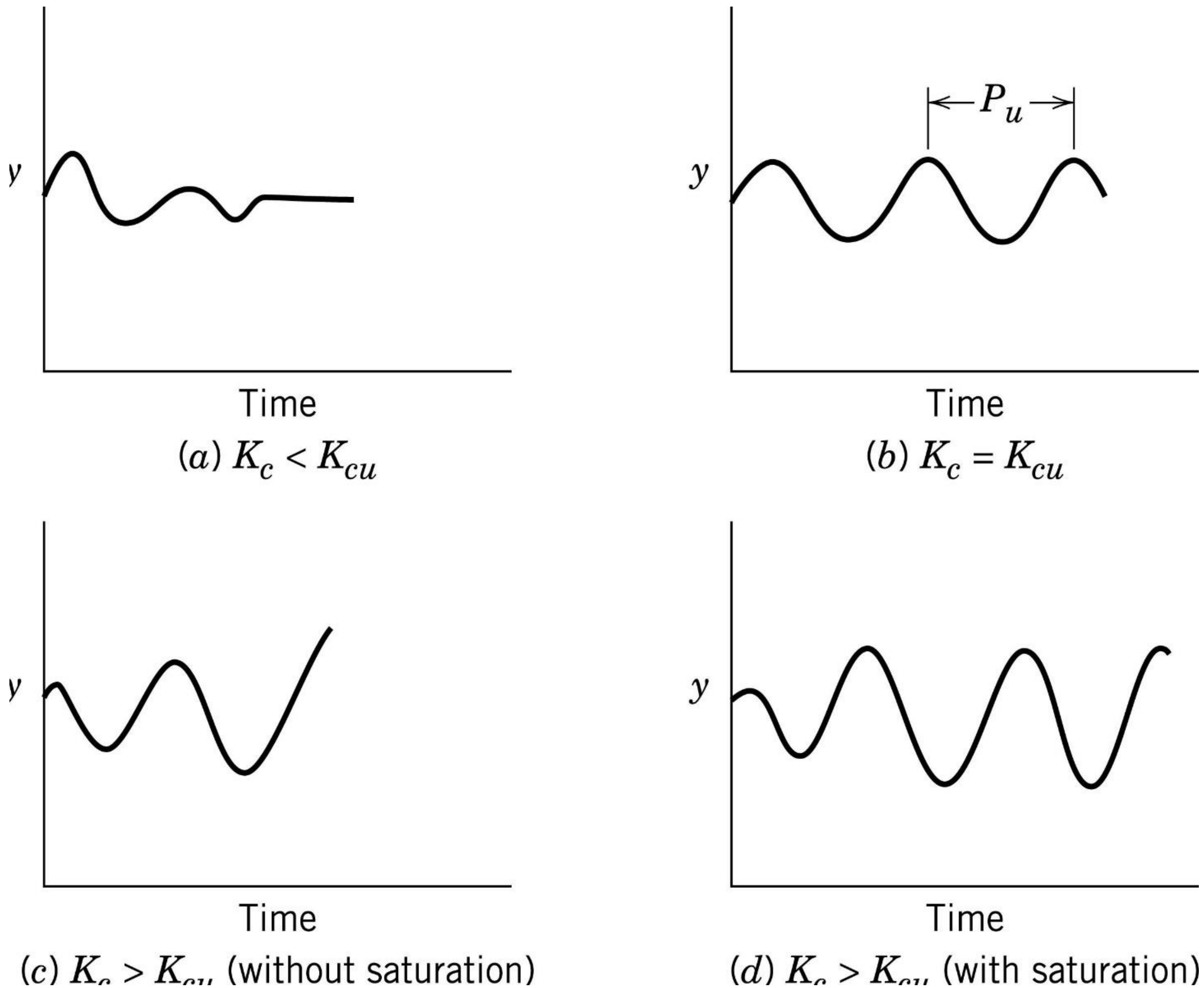
Controller Tuning Relations / Empirical Tuning

Ziegler-Nichols (Z-N) Oscillation Method

- This procedure is only valid for open loop stable plants
- It is carried out through the following steps:

1. Set the true plant under proportional control, with a very small gain and bring it to a desired operating conditions
2. Using P-control only and with the loop closed, introduce a set-point change and observe the response
3. Increase the gain until the system oscillates continuously with constant amplitude

Experimental determination of K_u



(a) $K_c < K_{cu}$

(b) $K_c = K_{cu}$

(c) $K_c > K_{cu}$ (without saturation)

(d) $K_c > K_{cu}$ (with saturation)

Controller Tuning Relations / Empirical Tuning

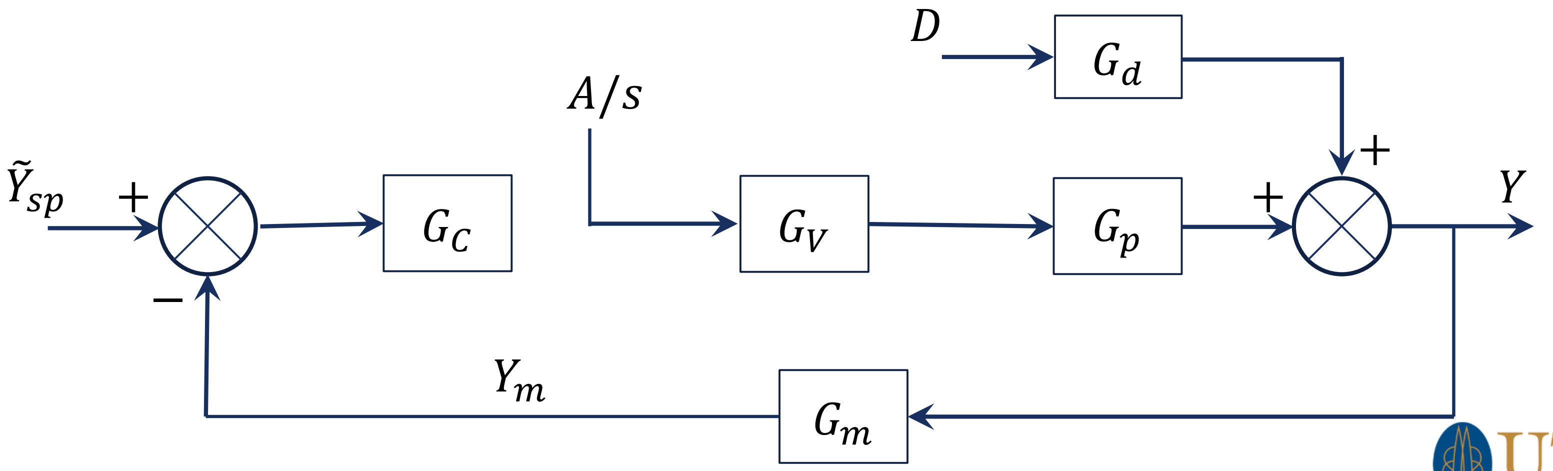
5. The value of K_c that produces continuous cycling is called the ultimate gain, K_{cu}
6. The period of corresponding sustained oscillation is referred to as ultimate period: P_u min/cycle
7. Using the values of K_{cu} and P_u, Ziegler and Nichols recommended controller settings

Controller	K_c	τ_I (min)	τ_D (min)
P	$K_{cu}/2$	-	-
PI	$K_{cu}/2.2$	$P_u/1.2$	-
PID	$K_{cu}/1.7$	$P_u/2$	$P_u/8$

Reaction Curve based method

Cohen and Coon Method

- Known as process reaction curve method
- A popular empirical tuning method
- Consider a control system as shown in Figure



Cohen – Coon Method

- Introduce a step change of magnitude M in the control valve
- Record the response with respect to time
- The reaction curve is affected by the dynamics of final control element, process and measuring sensor
- The response generally has a sigmoid shape and it is approximated by FOPDT model

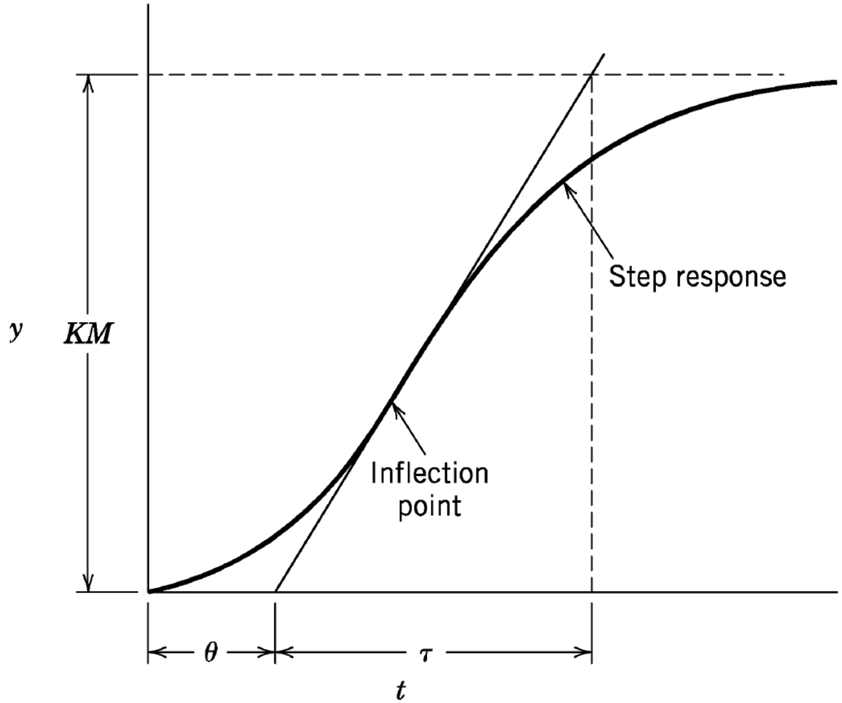


Figure 7.5 Graphical analysis of the process reaction curve to obtain parameters of a first-order-plus-time-delay model.

Cohen and Coon Method

- For load changes
- One-quarter decay ratio
- Minimum offset
- Minimum ISE

Controller	K_c	τ_I **(min)**	τ_D **(min)**
P	$\frac{1}{K}\frac{\tau}{\theta}\left(1+\frac{\theta}{3\tau}\right)$	-	-
PI	$\frac{1}{K}\frac{\tau}{\theta}\left(0.9+\frac{\theta}{12\tau}\right)$	$\theta\frac{30+3\,\theta/\tau}{9+20\,\theta/\tau}$	-
PID	$\frac{1}{K}\frac{\tau}{\theta}\left(\frac{4}{3}+\frac{\theta}{4\tau}\right)$	$\theta\frac{32+6\,\theta/\tau}{13+8\,\theta/\tau}$	$\theta\frac{4}{11+2\,\theta/\tau}$

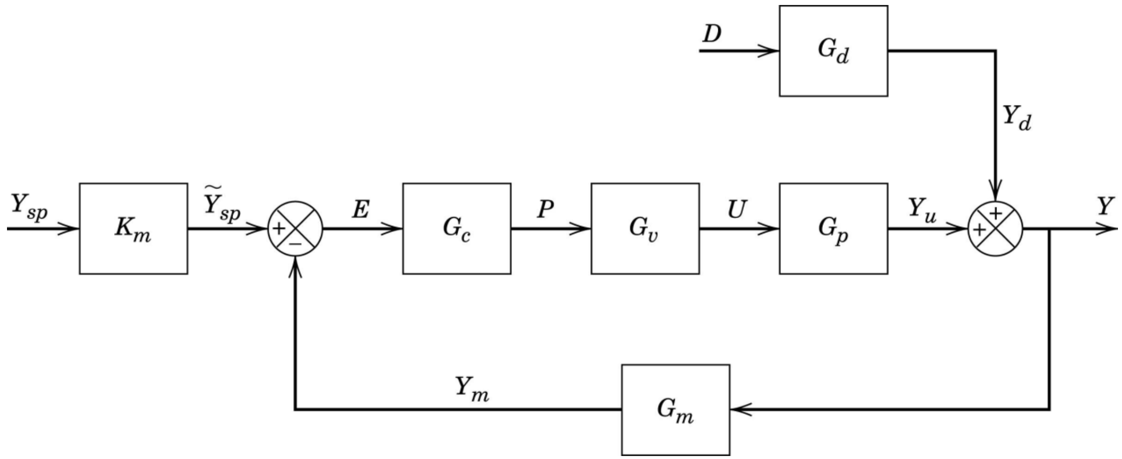
Model-Based Controller Design

Direct Synthesis Controller

- The controller design is based on a process model and a desired closed-loop transfer function
- Does not always have PID structure, however it produce PI or PID controllers for common process models

Direct Synthesis Controller

- Consider the block diagram of a feedback control system



- The closed-loop transfer function for set-point changes is

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} = \frac{G_c G}{1 + G_c G}$$

with $G = G_v G_p G_m$ and assume $G_m = K_m$

Direct Synthesis Controller

- Rearranging and solving for G_c

$$G_c = \frac{1}{G}\left(\frac{Y/Y_{sp}}{1-Y/Y_{sp}}\right)$$

- As Y/Y_{sp} is not known a priori, the above design cannot be used
- Also, distinguish between the plant G and the model $\tilde{G}$
- The practical design equation

$$G_c = \frac{1}{\tilde{G}}\left(\frac{\left(Y/Y_{sp}\right)_d}{1-\left(Y/Y_{sp}\right)_d}\right)$$

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Direct Synthesis Controller

- The selection of the desired closed-loop transfer function is the key decision
- Note: the controller transfer function has the inverse of $\tilde{G}$
- Desired closed-loop transfer functions

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1}{\tau_c s + 1}$$

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{e^{-\theta s}}{\tau_c s + 1}$$

Direct Synthesis Controller

- By substituting in the controller design equation, we get respectively,

$$G_c = \frac{1}{\tilde{G}} \frac{1}{\tau_c s}$$

$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{\tau_c s + 1 - e^{-\theta s}}$$

- Approximating the time delay in the denominator with a truncated Taylor series expansion

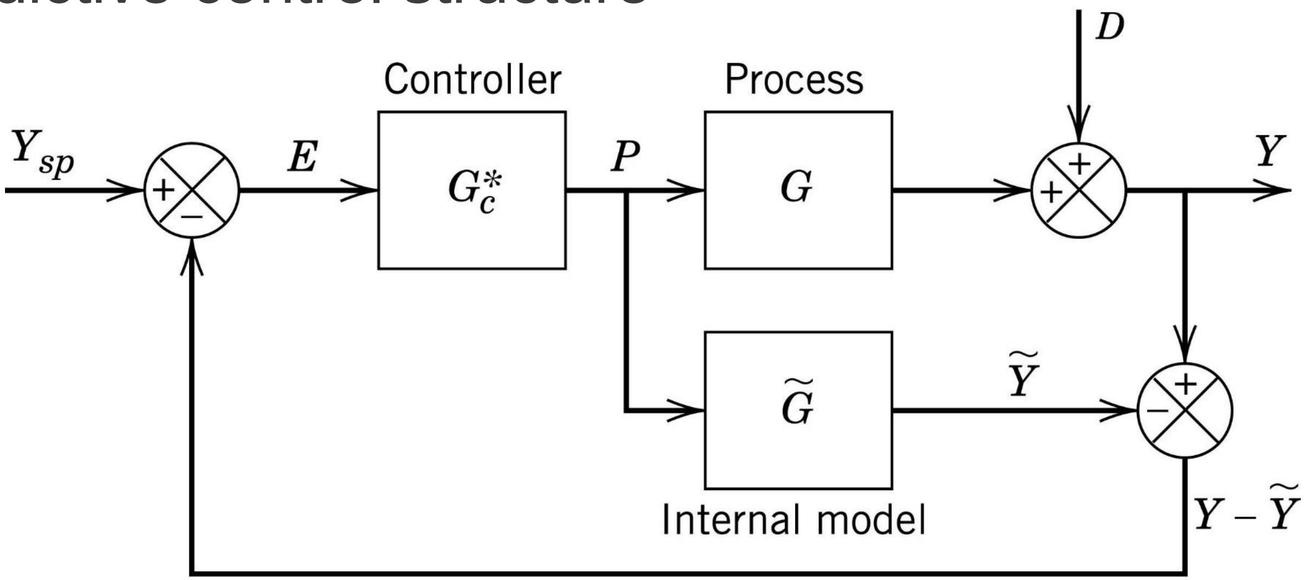
$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{(\tau_c + \theta)s}$$

- Both are integral controllers, eliminate offset

Model Based Controller Design

Internal Model Control (IMC)

- Based on Brosilow (1979), Garcia and Morari (1982), Rivera et al. (1986)
- The derivation of control algorithm is based on the predictive control structure



(b) Internal model control

Internal Model Controller

- Perfect control can be achieved if the controller could be set equal to the inverse of the process dynamic model

$$G_c^* = \frac{1}{\tilde{G}}$$

- The following are the four reasons why an exact inverse of the process is not possible:
  - Dead time
  - Numerator dynamics
  - Constraints
  - Model mismatch

Dead time

- In most physical processes, the process transfer function includes dead time in the numerator
- The IMC equation for a typical process model with dead time gives, when the model is factored into two terms:

$$G_c^*(s) = \left[\tilde{G}(s)\right]^{-1} = \left[\tilde{G}^-(s)\right]^{-1} e^{\theta s} \qquad \tilde{G}(s) = \tilde{G}^-(s)e^{-\theta s}$$

- The perfect controller would have to include the ability to use future information in determining the current manipulated variable – not physically realizable

Numerator Dynamics

- Some process models have dynamic elements in the numerators of feedback transfer function
- Application of predictive controller equation to an example gives

$$\tilde{G}(s) = K \frac{\tau_2 s + 1}{(\tau_1 s + 1)^2}$$

$$G_c^*(s) = \left[\tilde{G}(s)\right]^{-1} = \frac{1}{K} \frac{(\tau_1 s + 1)^2}{\tau_2 s + 1}$$

- The controller would not be able to provide perfect control when $\tau_2 < 0$

Constraints

- The manipulated variable must observe constraints
- There is no guarantee that the controller would observe constraints
- Thus, in some cases, values of the manipulated variables that are required to achieve perfect control performance would not be possible

Model mismatch

- The model used in the predictive system will almost certainly be different from the true process
- If the difference is large, the closed-loop system could become unstable, a situation that precludes acceptable control performance

Internal Model Controller

- Since the perfect controller is not possible, a manner for deriving an approximate inverse of the model is required
- The approximate inverse is the $G_c(s)$ that contains important features for control performance
- Many methods exist for developing approximate inverse
- Each method would result in a different control algorithm giving different control performance

IMC Controller

- Since an exact inverse is not possible, the IMC approach segregates and eliminates the aspects of the model transfer function that make the calculation of realizable inverse impossible
- The first step is to factor the model into the product of two factors

$$\tilde{G}(s) = \tilde{G}^{+}(s)\tilde{G}^{-}(s)$$

IMC Controller

- $\tilde{G}^{+}(s)$ - The noninvertible part has an inverse that is not causal or is unstable
- The steady state gain of this term must be 1.0
- $\tilde{G}^{-}(s)$ - The invertible part has an inverse that is causal and stable, leading to realizable, stable controller
- The IMC Controller (idealized)

$$\tilde{G}_c^{*} = \left[\tilde{G}^{-}(s)\right]^{-1}$$

- This design ensures the controller is realizable and the system is internally stable

IMC Controller

- Example 1: Apply the IMC procedure to design a controller for a process described by

$$\tilde{G}(s) = \frac{0.039}{(5s+1)^3}$$

$$\tilde{G}^-(s) = \frac{0.039}{(5s+1)^3} \qquad \tilde{G}^+(s) = 1.0$$

$$\tilde{G}_c^*(s) = \left[G_m^-(s)\right]^{-1} = \frac{(5s+1)^3}{0.039}$$

IMC Controller

Drawbacks of the design:

- The controller involves first, second and third order derivatives of the feedback signal
- These derivatives can not be calculated exactly, although they can be estimated numerically
- Appearance of higher-order derivatives of a noisy signal could lead to unacceptable control
- High derivatives can lead to extreme sensitivity to model errors
- The controller cannot be used without modification

IMC Controller

- All realistic processes are modeled by transfer functions having a denominator order greater than the numerator order
- Thus, the IMC controller, the inverse of the process model, will have a numerator order greater than denominator
- Results in first- or higher-order derivatives in the controller that lead to unacceptable manipulated variable behavior, and, thus, poor performance and poor robustness when model errors occur
- Achieving good control performance requires modification that modulates the manipulated variable behavior and increase the robustness of the system

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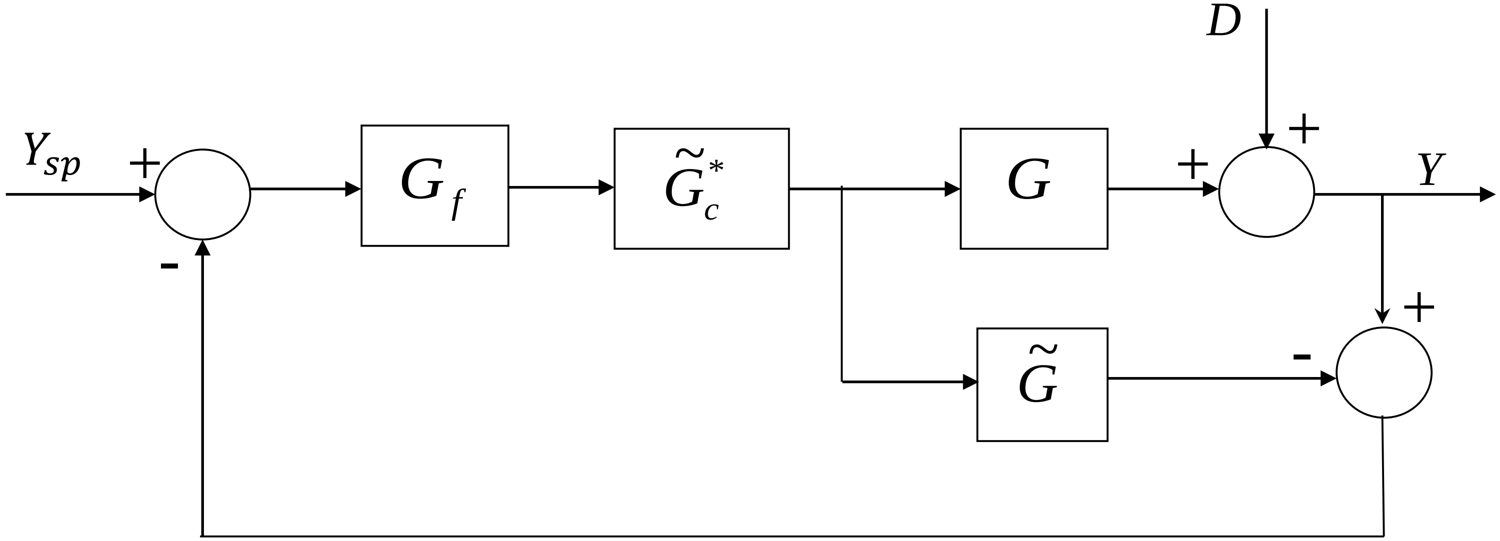
IMC Controller

- A filter of the feedback signal is used
- The filter is placed before the controller as shown in fig.



IMC Controller

- The filter can make the controller proper or semiproper,

$$G_c^*(s) = \tilde{G}_c^*(s) G_f(s) = \left[\tilde{G}^-(s)\right]^{-1} G_f(s)$$

- For tracking set-point changes,

$$G_f(s) = \frac{1}{\left(\tau_c s + 1\right)^n}$$

- Adjust the filter-tuning parameter to vary speed of the response of the closed-loop system
- τ_c : small --- response is fast

  large --- the closed loop response is more robust (insensitive to model error)

IMC Controller

- The modified IMC Controller for the example

$$G_f(s)G_c^*(s) = \frac{1}{0.039}\frac{(5s+1)^3}{(\lambda s+1)^3}$$

Example 2

Design an IMC controller using the alternative first-order-plus-dead-time approximate model for the process

$$\tilde{G}(s) = \frac{0.039e^{-5.5s}}{(10.5s+1)}$$

$$\tilde{G}^{-}(s) = \frac{0.039}{(10.5s+1)} \qquad \tilde{G}^{+}(s) = e^{-5.5s}$$

$$\tilde{G}_c^{*}(s) = \left[\tilde{G}^{-}(s)\right]^{-1} = \frac{(10.5s+1)}{0.039}$$

- The controller is proportional-derivative, which still might be too aggressive but can be modified to give acceptable performance

Example 2

- To make the controller semiproper,

$$G_c^*(s) = \frac{(10.5s + 1)}{0.039(\tau_c s + 1)}$$

- Filter tuning parameter is adjusted to provide the required performance

Example 3

- Consider the following transfer function:

$$\tilde{G}(s) = \frac{(-6s+1)}{(15s+1)(3s+1)}$$

- This system has a RHP zero and will exhibit inverse response characteristics
- An all-pass factorization of the model is to be used

$$\tilde{G}^{+}(s) = \frac{(-6s+1)}{(6s+1)} \qquad \tilde{G}^{-}(s) = \frac{(6s+1)}{(15s+1)(3s+1)}$$

Example 3

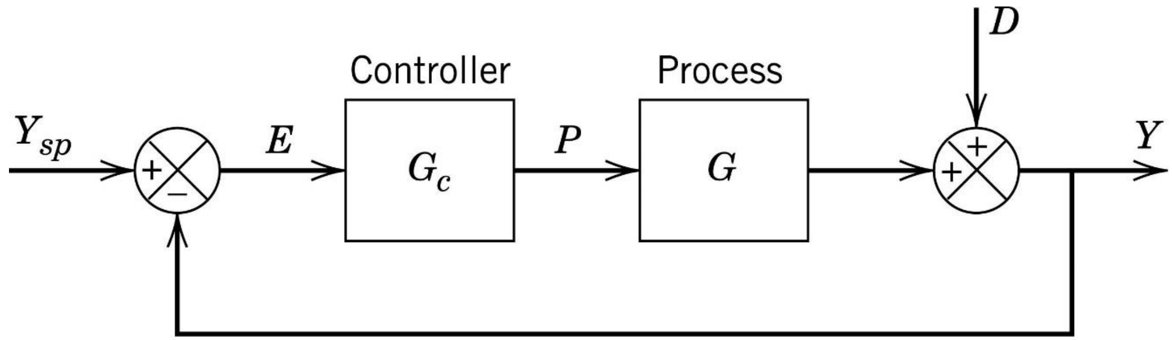
- An idealized controller would be:

$$\tilde{G}_c^*(s) = \left[\tilde{G}^-(s)\right]^{-1} = \frac{(15s+1)(3s+1)}{(6s+1)}$$

- Add the filter to make the controller semi-proper

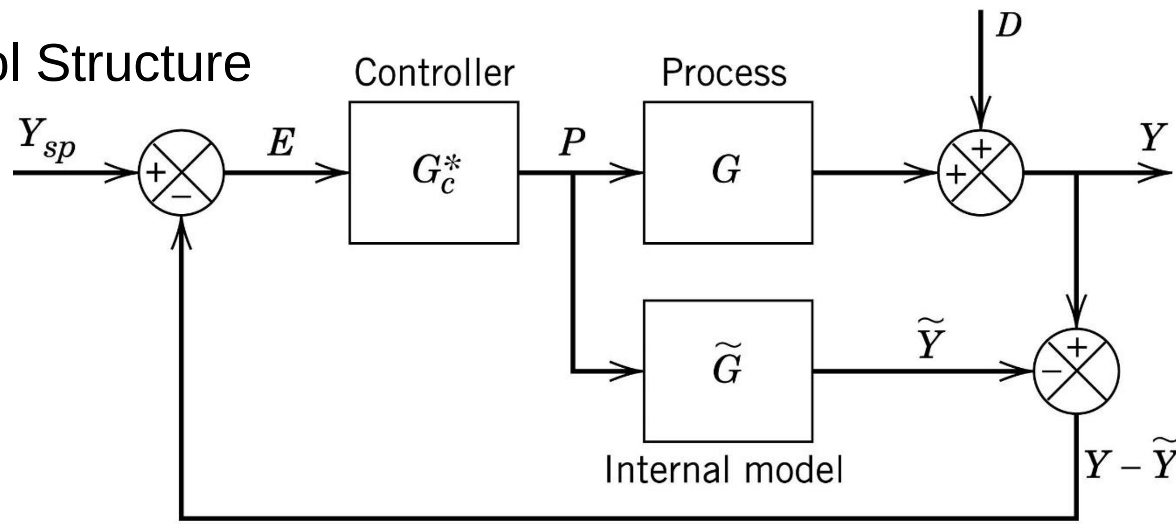
$$G_c^*(s) = \frac{(15s+1)(3s+1)}{(6s+1)(\tau_c s+1)}$$

Internal Model Control



(a) Classical feedback control

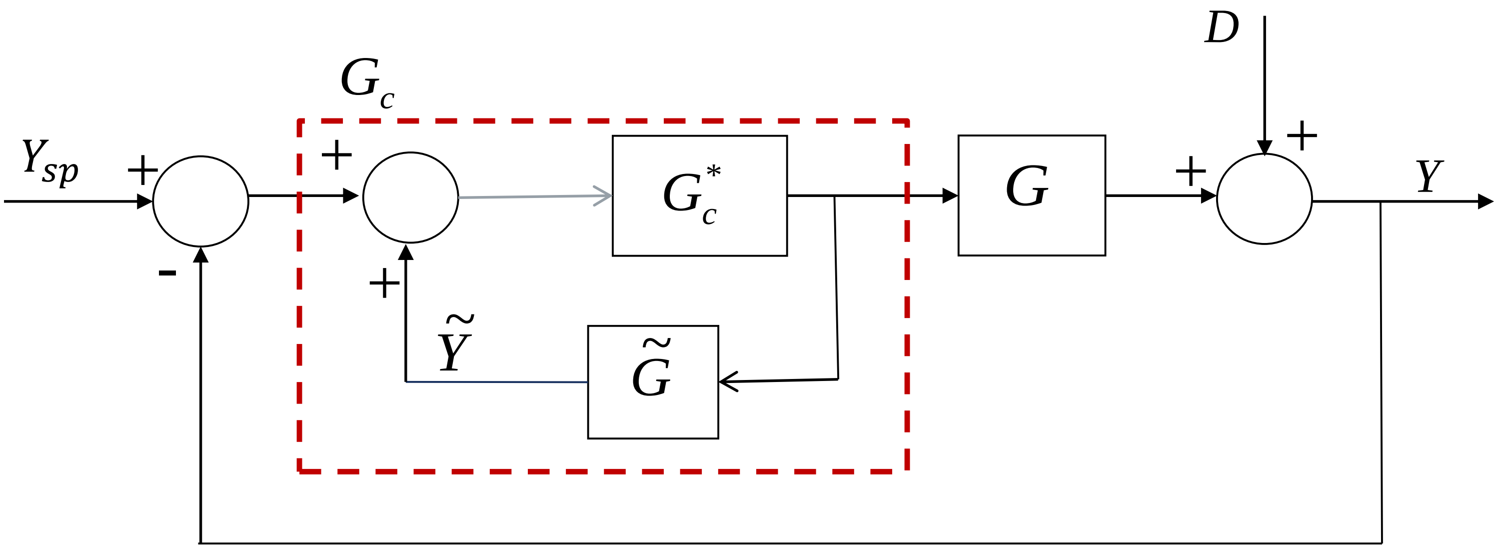
Predictive Control Structure



(b) Internal model control

Internal Model Control

- The predictive control structure can be redrawn:



By comparing the two block diagrams:

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$

Any IMC controller is equivalent to a standard feedback controller

IMC-Based PID Controller Design Procedure

1. Find the IMC controller transfer function, G_c^*, which includes a filter, $G_f = \frac{1}{(\tau_c s+1)^r}$
   - to make G_c^* semi-proper, or
   - the order of numerator of G_c^* is one order greater than the denominator of G_c^* (to give derivative action)
   - τ_c is the desired closed loop time constant
2. Find the equivalent standard feedback controller using the transformation:

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$

   Write this in the form of a ratio between two polynomials

3. Show this in PID form and find K_c, τ_I and τ_D

IMC-Based PID Controller Design for First-Order Process

- First order process: $\tilde{G}(s) = \dfrac{K_p}{\tau_p s + 1}$

$$G_C^*(s) = G_f(s)\left[\tilde{G}^-(s)\right]^{-1} = \frac{\tau_p s + 1}{K_p}\frac{1}{\tau_c s + 1}$$

$$G_c^*(s) = \frac{1}{K_p}\frac{\tau_p s + 1}{\tau_c s + 1}$$

- Find the equivalent standard feedback controller:

$$G_c(s) = \frac{G_c^*(s)}{1 - \tilde{G}(s)G_c^*(s)} = \frac{\dfrac{\tau_p s + 1}{K_p(\tau_c s + 1)}}{1 - \dfrac{K_p}{\tau_p s + 1}\dfrac{\tau_p s + 1}{K_p(\tau_c s + 1)}} = \frac{\tau_p s + 1}{K_p \tau_c s}$$

IMC-Based PID Controller Design

- Find the equivalent standard feedback controller:

$$G_c(s) = \frac{\tau_p}{K_p \tau_c} + \frac{1}{K_p \tau_c s} = \frac{\tau_p}{K_p \tau_c}\left(1 + \frac{1}{\tau_p s}\right)$$

$$K_c = \frac{\tau_p}{K_p \tau_c} \qquad \tau_I = \tau_p$$

- The IMC-based PID design procedure for a first-order process has resulted in a PI control law

IMC-Based PID Controller Design for First-Order Process with Time Delay

- First order process: $\tilde{G}(s) = \dfrac{K_p e^{-\theta s}}{\tau_p s + 1}$
- Use a first-order Pade approximation: $e^{-\theta s} \approx \dfrac{-0.5\theta s + 1}{0.5\theta s + 1}$

$$\tilde{G}(s) = \frac{K_p(-0.5\theta s + 1)}{(\tau_p s + 1)(0.5\theta s + 1)}$$

$$\tilde{G}^-(s) = \frac{K_p}{(\tau_p s + 1)(0.5\theta s + 1)} \qquad \tilde{G}^+(s) = -0.5\theta s + 1$$

- The idealized controller is

$$\tilde{G}_c^*(s) = \left[\tilde{G}^-(s)\right]^{-1} = \frac{(\tau_p s + 1)(0.5\theta s + 1)}{K_p}$$

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IMC-Based PID Controller Design for First-Order Process with Time Delay

$$G_c^*(s) = G_f(s)\left[\tilde{G}^-(s)\right]^{-1} = \frac{(\tau_p s + 1)(0.5\theta s + 1)}{K_p}\frac{1}{\tau_c s + 1}$$

- Note: The numerator order is one degree higher than the denominator to realize a PID controller
- Find the equivalent standard feedback controller:

$$G_c(s) = \frac{G_c^*(s)}{1 - \tilde{G}(s)G_c^*(s)} = \frac{\tilde{G}^-(s)G_f(s)}{1 - \tilde{G}(s)\tilde{G}_c^*(s)G_f(s)}$$

IMC-Based PID Controller Design for First-Order Process with Time Delay

$$G_c(s) = \frac{\tilde{G}_c^*(s)G_f(s)}{1-\tilde{G}^-(s)\tilde{G}^+(s)\left[\tilde{G}^-(s)\right]^{-1}G_f(s)} = \frac{\tilde{G}_c^*(s)G_f(s)}{1-\tilde{G}^+(s)G_f(s)}$$

$$= \left(\frac{1}{K_p}\right)\frac{(\tau_p s+1)(0.5\theta s+1)}{(\tau_c+0.5\theta)s}$$

$$= \left(\frac{1}{K_p}\right)\frac{0.5\tau_p\theta s^2+(\tau_p+0.5\theta)s+1}{(\tau_c+0.5\theta)s}$$

$$= \frac{(\tau_p+0.5\theta)}{K_p(\tau_c+0.5\theta)}\left[1+\frac{1}{(\tau_p+0.5\theta)s}+\frac{\tau_p\theta}{2\tau_p+\theta}s\right]$$

Summary

You have learnt

- Design Feedback Controllers
- Specify controller performance criteria
- Select appropriate types of controller
- Tune controllers using
  - Zeigler-Nichols
  - Cohen-Coon
- Design Model based Controllers using
  - Direct synthesis
  - Internal Model Control (IMC)

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