

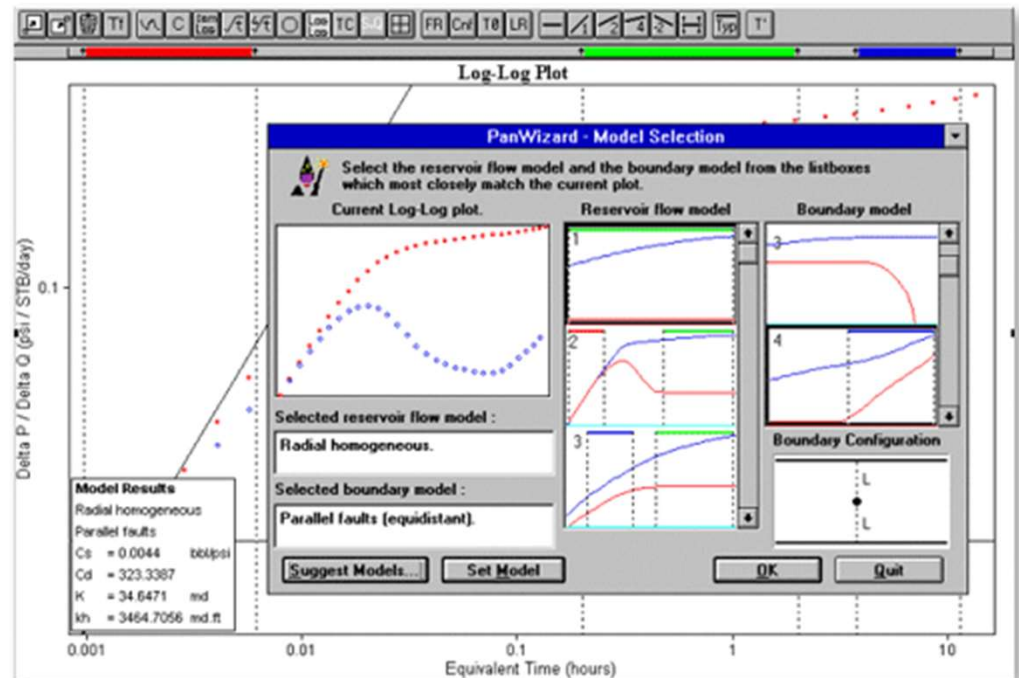


Introduction to Well Testing

By: Amin Nemati

References :

- ✓ **Pressure Transient Testing By John Lee**
- ✓ **Pressure Buildup and Flow Tests in Wells By C.S.Matthews & D.G.Russell**

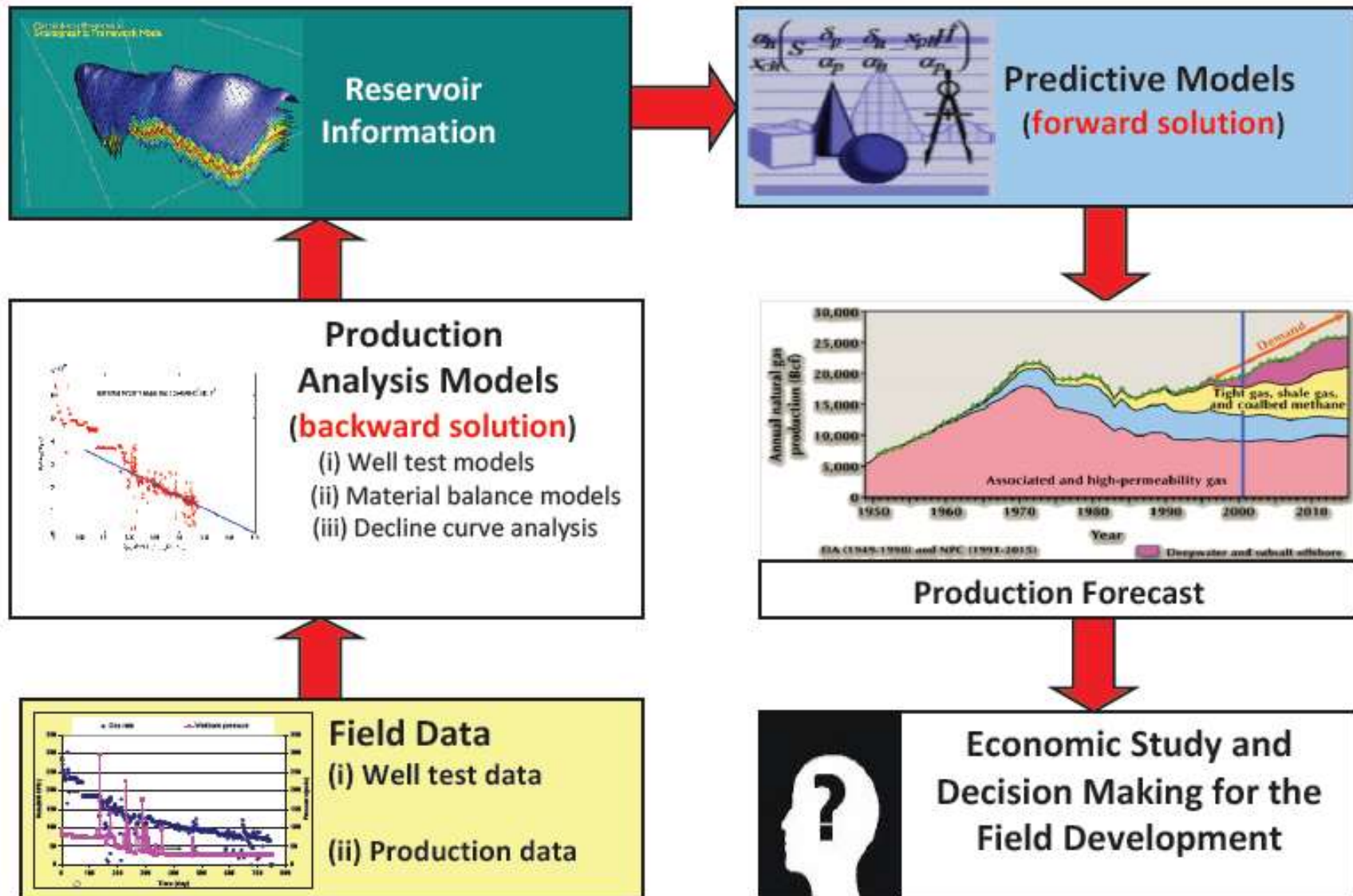




Chapter 1

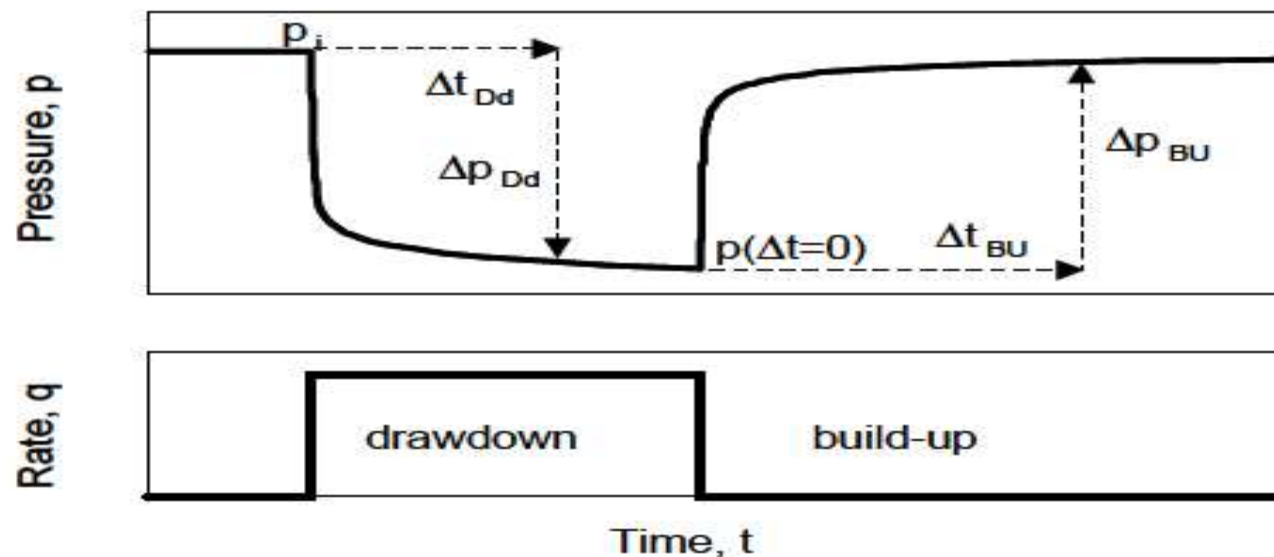
Introduction

Importance of Production Data Analysis



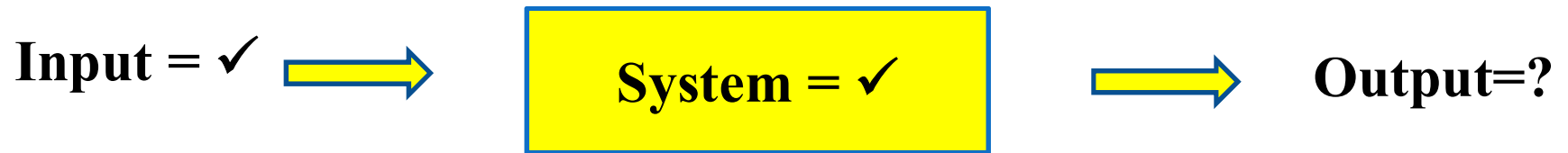
Basic Definition & Concepts

- ✓ During a well test, a transient pressure response that is created by a temporary change in production rate is measured.
- ✓ The well response is usually monitored during a relatively short period of time compared to the life of the reservoir.
- ✓ In most cases, the flow rate is measured at surface while the pressure is recorded down-hole.

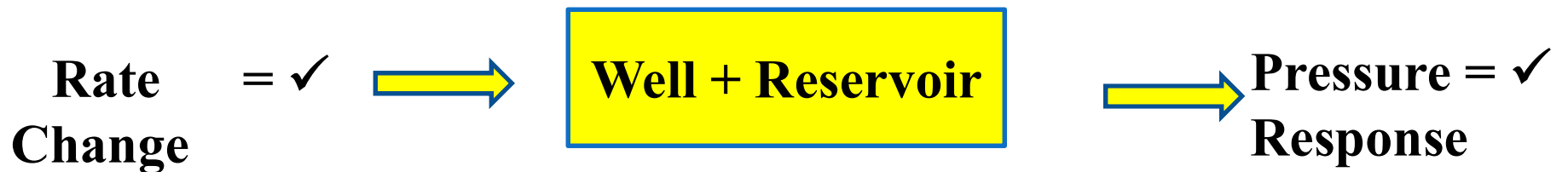




Forward Solution :



Backward Solution :





I. • Reservoir evaluation

The Objectives of Well Test

II. • Reservoir management

III. • Reservoir description

I. Reservoir evaluation

- Deliverability (conductivity; kh)
 - Design of well spacing
 - Number of wells • Wellbore stimulation**
- Properties (initial reservoir pressure)
 - Potential energy of the reservoir**
- Size (reservoir limits)
 - Closed or open (with aquifer support) reservoir boundaries**
- Near well conditions (skin, storage and turbulence)**

II. Reservoir management

- Monitoring performance and well conditions**

III. • Reservoir description

- Fault, Barriers**
- Estimation of bulk reservoir properties**

Other :

- $\frac{kh}{\mu}$ = Transmissibility**
- Fracturing parameters (ω , λ)**
- Effective permeability**
- Non-Darcy effect (D) by Multirate test**



Types of Test

Type of tests is governed by the test objective.

• Transient tests which are relatively short term tests are used to define reservoir characteristics.

- Drawdown Test**
- Build-up Test**
- Injection Test**
- Falloff Test**
- Interference Test**
- Drill Stem Test**

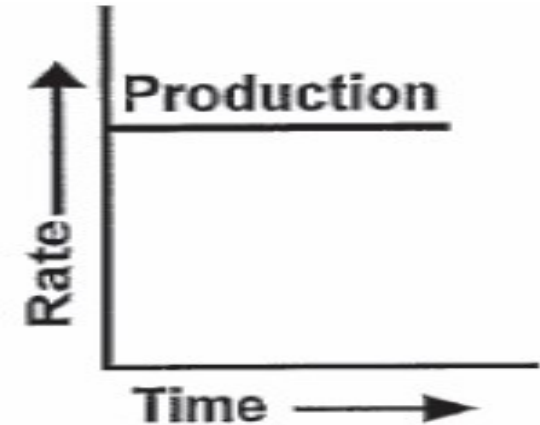
- 
- **Stabilized tests which are relatively long duration tests are used to define long term production performance.**

- **Reservoir limit test**
- **AOF (single point and multi point)**
- **IPR (Inflow Performance Relationship)**

Types of Test-Drawdown Test

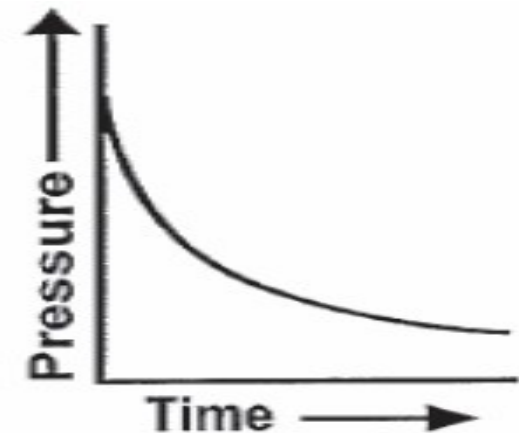
– Conditions

- An static, stable and shut-in is opened to flow
- flow rate is supposed to be constant (for using traditional analysis)



– Objective

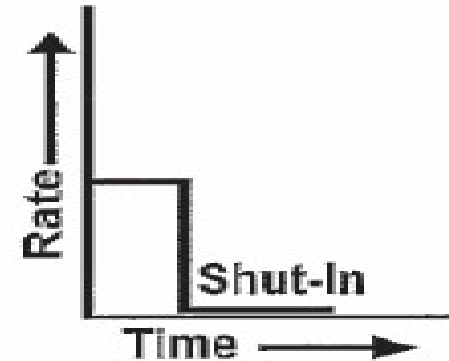
- To obtain average permeability of the reservoir rock within the drainage area of the well
- To assess the degree of damage or stimulation
- To obtain pore volume of the reservoir
- To detect reservoir in homogeneity within the drainage area of the well.



Types of Test-Buildup Test

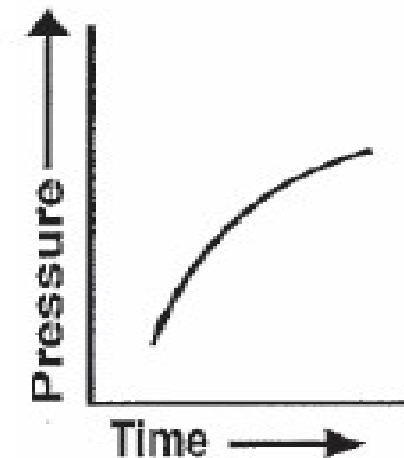
– Conditions

- A well which is already flowing (ideally constant rate) is shut-in
- Down hole pressure measured as the pressure builds up



– Objective

- To obtain average permeability of the reservoir rock within the drainage area of the well
- To assess the degree of damage or stimulation
- To obtain initial reservoir pressure during the transient state
- To obtain the average reservoir pressure over the drainage area of the well during pseudo steady state



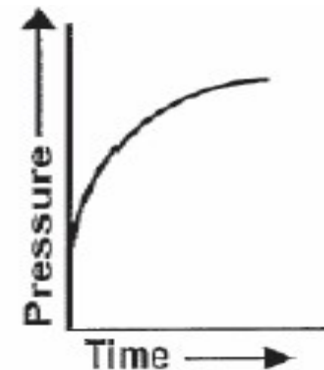
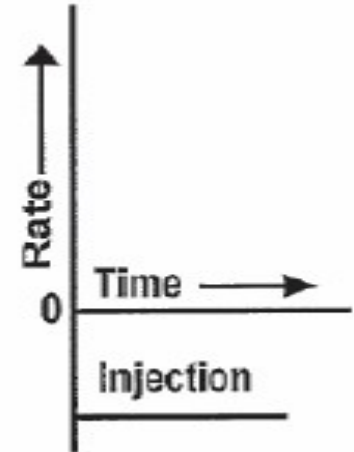
Types of Test-Injection Test

– Conditions

- An injection test is conceptually identical to a drawdown test, except flow is into the well rather than out of it.

– Objective

- Injection well testing has its application in water flooding, pressure maintenance by water or gas injection, gas recycling and EOR operations.
- In most cases the objective of the injection test is the same as those of production test (k , S , P_{avg}).
- Determination of reservoir heterogeneity and front tracing.





Types of Test

- **Falloff Test:**

- A pressure falloff test is usually preceded by an injectivity test of a long duration. Injection then is stopped while recording the pressure. Thus, the pressure falloff test is similar to the pressure buildup test.

- **Interference Test:**

- In an interference test one well is produced and pressure is observed in a different wells.
- To test reservoir continuity
- To detect directional permeability and other major reservoir heterogeneity
- Determination of reservoir volume

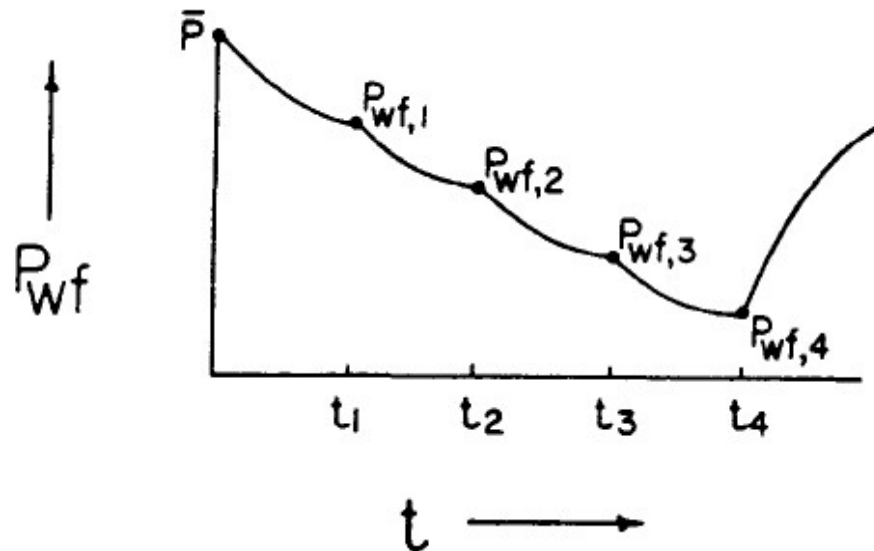
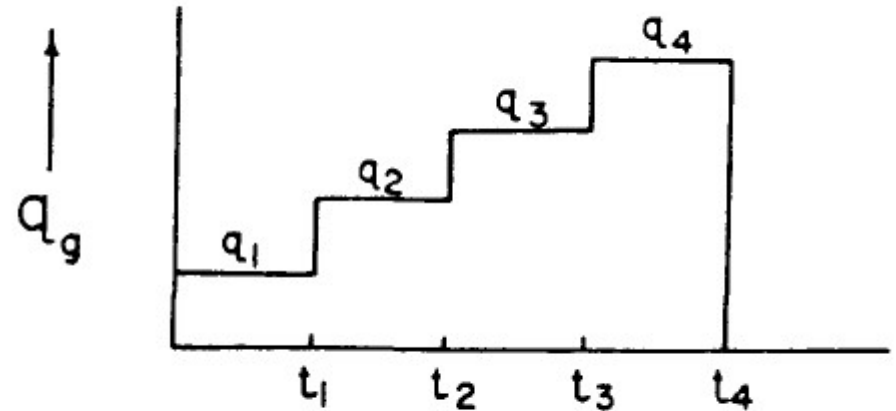
- **Drill Stem Test (DST):**

- It is a test commonly used to test a newly drilled well (since it can only be carried out while a rig is over the hole.
- In a DST, the well is opened to flow by a valve at the base of the test tool, and reservoir fluid flows up the drill string.
- Analysis of the DST requires the special techniques, since the flow rate is not constant as the fluid rises in the drill string.

• Flow-After-Flow Test:

– In this testing method, a well flows at a selected constant rate until pressure stabilizes - i.e., pseudo steady state is reached.

– The stabilized rate and pressure are recorded; rate is then changed and the well flows until the pressure stabilizes again at the new rate. The process is repeated for a total of three or four rates.

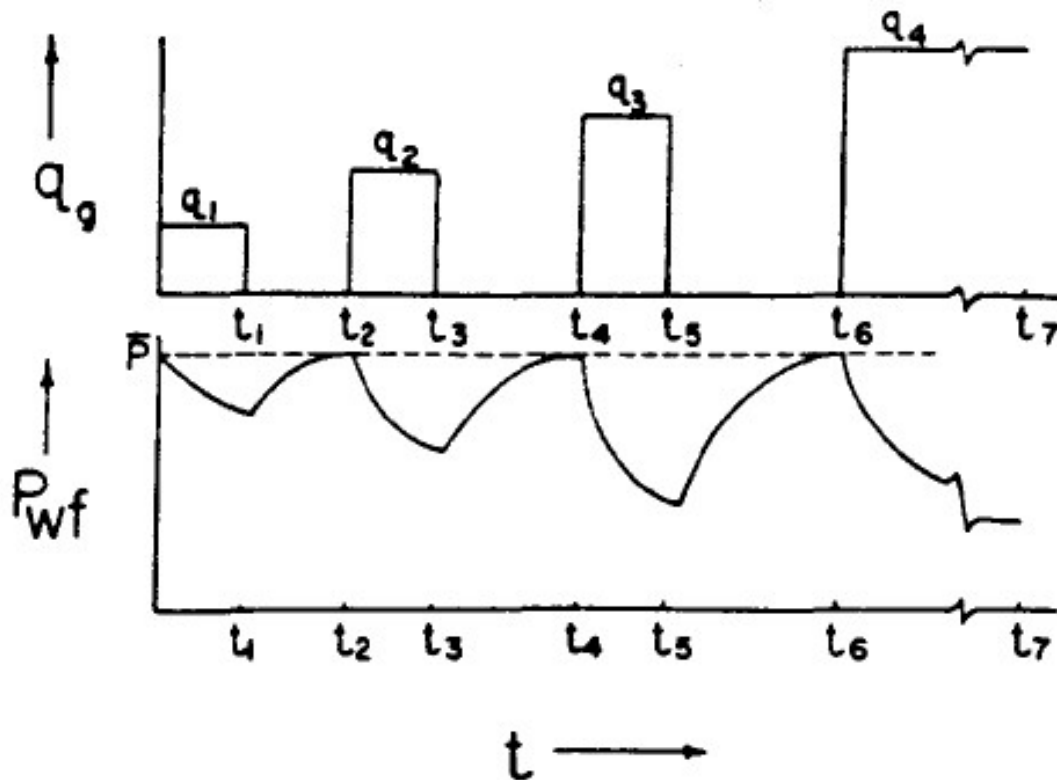


- **Isochronal Test :**

- An isochronal test is conducted by flowing a well at a fixed rate, then shutting it in until the pressure builds up to an unchanging (or almost unchanging) value, P^- .

- The well then is flowed at a second rate for the same length of time, followed by another shut-in, etc.

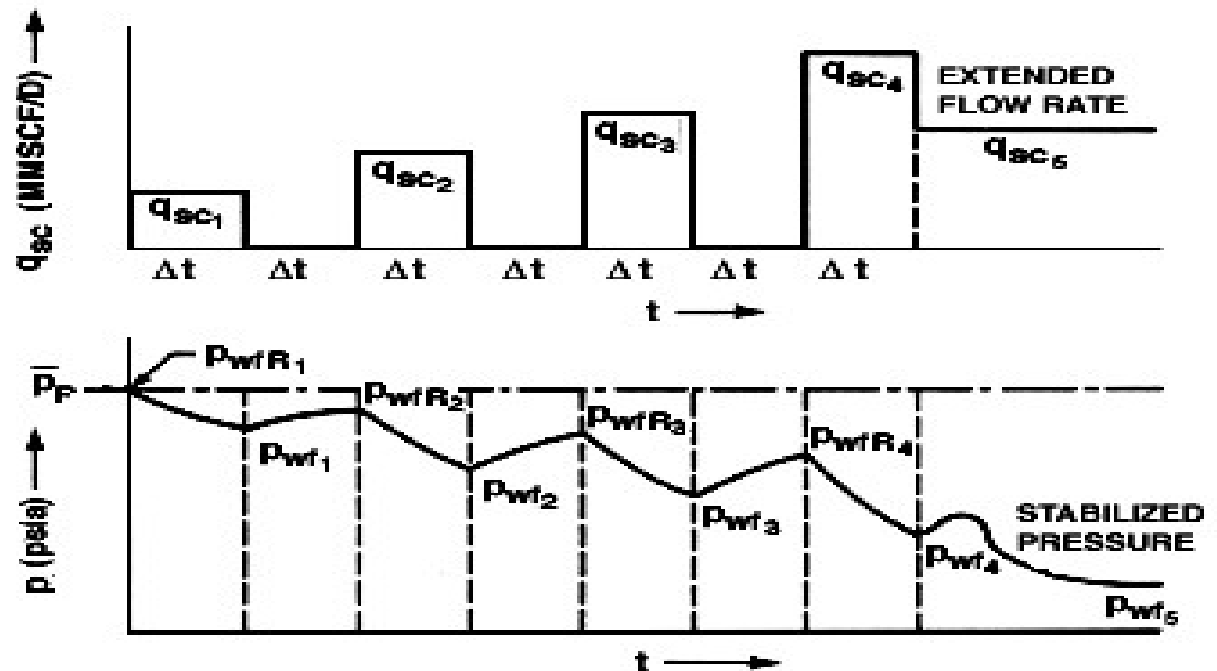
- If possible, the final flow period should be long enough to achieve stabilized flow.



• Modified Isochronal Test :

– The objective of modified isochronal tests is to obtain the same data as in an isochronal test without using the sometimes lengthy shut-in periods required for pressure to stabilize completely before each flow test is run.

– In the modified isochronal test shut-in periods of the same duration as the flow periods are used. and the final shut-in BHP (P_{ws}) before the beginning of a new flow period is used as an approximation to P^- in the test analysis procedure.



Primary reservoir characteristics

- **Types of fluids in the reservoir**

- Incompressible fluids
- Slightly compressible fluids
- Compressible fluids

- **Flow regimes**

- Steady-state flow
- Unsteady-state flow
- Pseudosteady-state flow

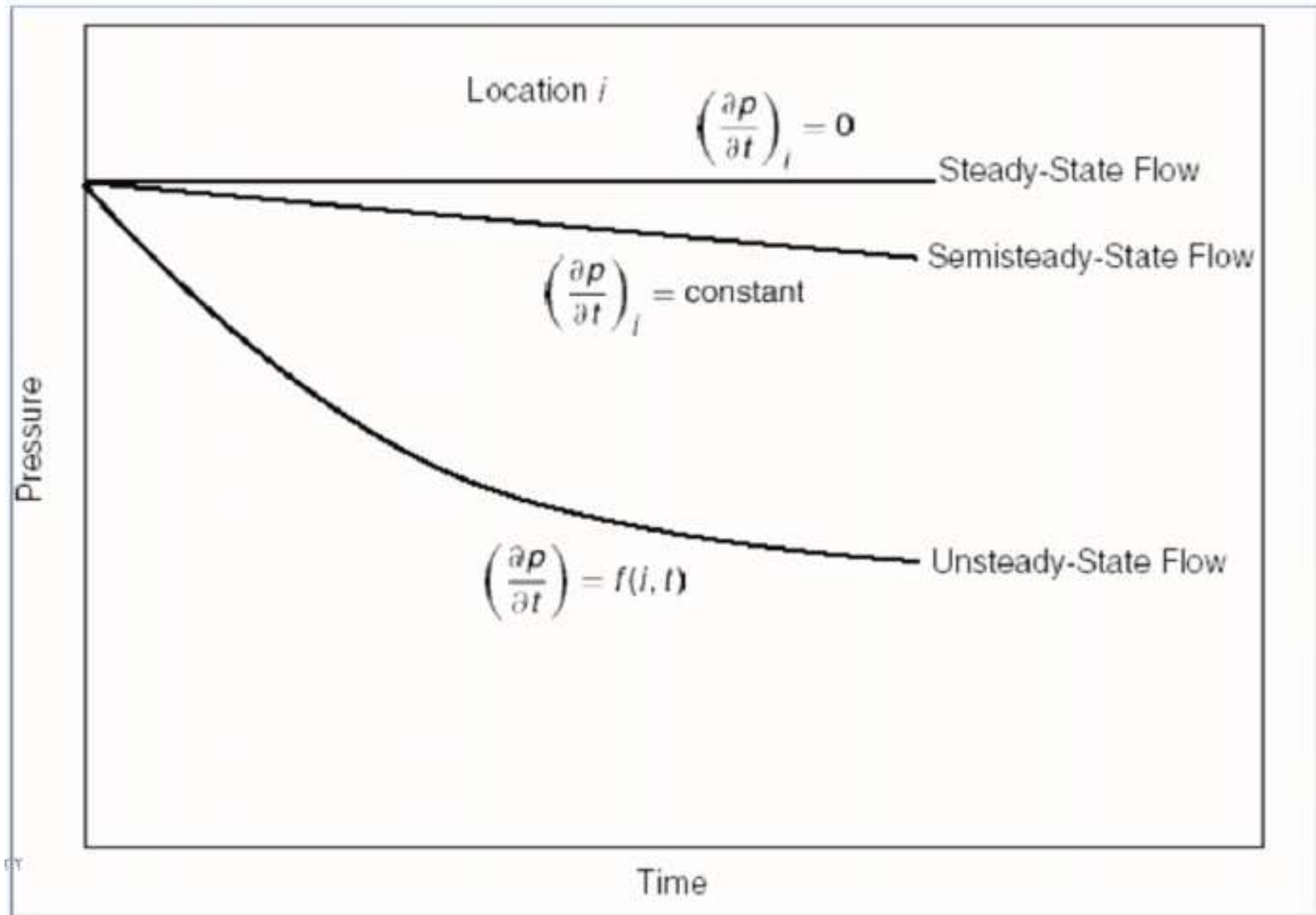
- **Reservoir geometry**

- Radial flow
- Linear flow
- Spherical and hemispherical flow

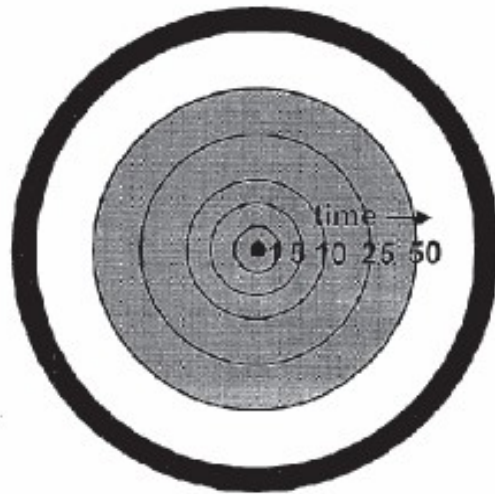
- **Number of flowing fluids in the reservoir.**

- Single-phase flow (oil, water, or gas)
- Two-phase flow (oil–water, oil–gas, or gas–water)
- Three-phase flow (oil, water, and gas)

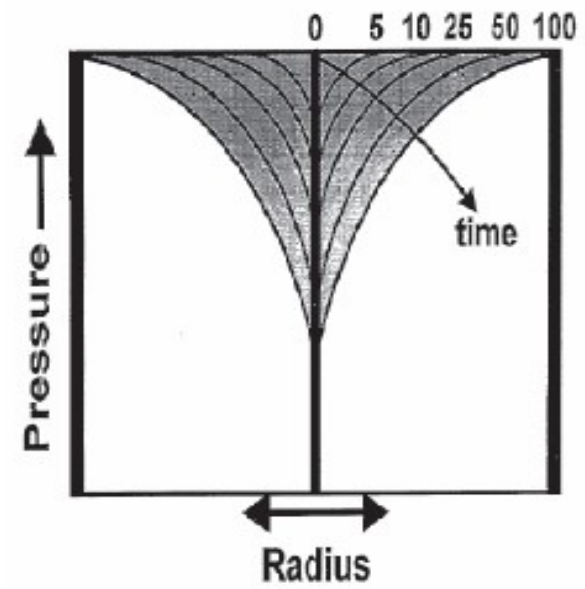
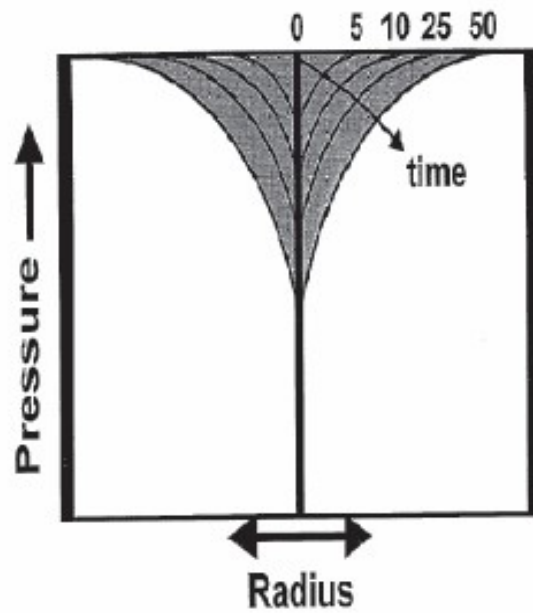
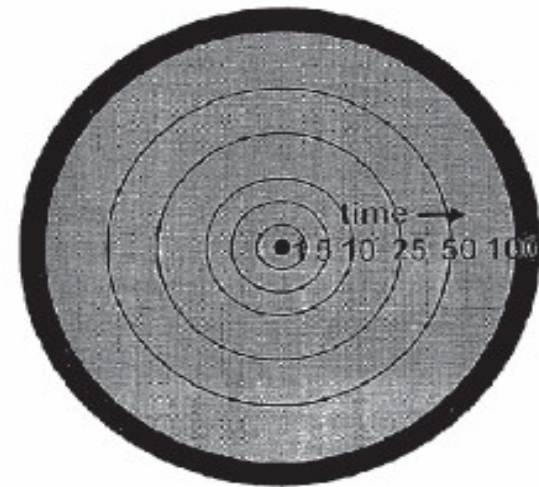
Flow Regimes



TRANSIENT



STABILIZED





Chapter 2

Fluid Flow in Porous Media



• **The Ideal Reservoir Model**

➤ **To develop analysis and design techniques for well testing. we first must make several simplifying assumptions about the well and reservoir that we are modeling.**

- ✓ **100 % saturated with single fluid**
- ✓ **Flow is radial**
- ✓ **Homogenous & Isotropic ($\phi = \text{cte}$ & $k = \text{cte}$) reservoir**
- ✓ **Infinite acting reservoir**
- ✓ **Constant rate**
- ✓ **Fully penetrated**
- ✓ **Isothermal**
- ✓ **....**



➤ These assumptions are introduced as needed. to combine

(1) The law of conservation of mass.

(2) Darcy's law

(3) Equations of state

If we combine the law of conservation of mass and Darcy's law, we obtain a partial differential equation that simplifies to

$$\frac{d^2 p}{dr^2} + \frac{1}{r} \frac{dp}{dr} = \frac{\mu C_0 \phi}{0.000264k} \frac{dp}{dt}$$

t = time, hr

k = permeability, md


$$\frac{d^2 p}{dr^2} + \frac{1}{r} \frac{dp}{dr} = \frac{\phi \mu C_0}{0.006328k} \frac{dp}{dt}$$

t = time, day

k = permeability, md

➤ **The assumptions and limitations used in developing diffusivity equation :**

- 1. Homogeneous and isotropic porous medium**
- 2. Uniform thickness**
- 3. Single phase flow**
- 4. Laminar flow**
- 5. Rock and fluid properties independent of pressure**



➤ When the reservoir contains more than one fluid, total compressibility should be computed as :

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_f$$

➤ The term $[0.006328k / \phi \mu c_t]$ and $[0.000264 k / \phi \mu c_t]$ in previous equations are called the diffusivity constant and is denoted by the symbol η , or:

$$\eta = \frac{0.006328k}{\phi \mu C_t}$$

k: milli darcy
t :day

$$\eta = \frac{0.000264k}{\phi \mu C_t}$$

k: milli darcy
t :hr

Solution to the diffusivity equation :

- **To obtain a solution to the diffusivity equation it is necessary to specify an initial condition and impose two boundary conditions.**
- **The initial condition simply states that the reservoir is at a uniform pressure p_i when production begins.**

Initial condition

$$p = p_i, \quad t = 0, \quad r \geq r_w$$

- **The two boundary conditions require that the well is producing at a constant production rate and that the reservoir behaves as if it were infinite in size, i.e., $r_e = \infty$.**

Boundary conditions

$$\begin{cases} (p)_{r_w} = p_{wf} \\ (p)_{r_e} = p_e \end{cases}$$



➤ **There are four solutions to diffusivity equation that are particularly useful in well testing:**

- A. The solution for a bounded cylindrical reservoir**
- B. The solution for an infinite reservoir with a well considered to be a line source with zero well bore radius**
- C. The pseudo steady state solution**
- D. The solution that includes well bore storage for a well in an infinite reservoir.**



A. The solution for a bounded cylindrical reservoir

➤ **A realistic and practical solution is obtained if we assume that :**

- 1) a well produces at constant rate, qB , into the wellbore (q flow rate in STB/D, and B is FVF in RB/STB).**
- 2) the well, with wellbore radius r_w ' is centered in a cylindrical reservoir of radius, r_e ' and that there is no flow across this outer boundary.**
- 3) before production begins, the reservoir is at uniform pressure, P_i .**



➤ By previous assumptions the most useful form of solution that relates flowing pressure, P_{wf} to time and to reservoir rock and fluid properties is :

$$p_{wf} = p_i - 141.2 \frac{q_{ub}}{kh} \left\{ \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4} + 2 \sum_{n=1}^{\infty} \frac{e^{-\alpha_n^2 t_D} J_1^2(\alpha_n r_{eD})}{\alpha_n^2 [J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)]} \right\}$$

✓ In this formula

$$r_{eD} = \frac{r_e}{r_w}$$

$$t_D = \frac{0.000264kt}{\phi\mu C_t r_w^2}$$

α_n are the roots of $J_1(\alpha_n r_{eD})Y_1(\alpha_n) - J_1(\alpha_n)Y_1(\alpha_n r_{eD}) = 0$

and where J_1 and Y_1 are Bessel functions.

- 
- The most important fact about previous equation is that, under the assumptions made in its development, it is an **exact solution**.
 - It sometimes is called the **van Everdingen-Hurst constant-terminal rate solution**.
 - It will not be necessary to use this equation in its complete form to calculate numerical values of P_{wf}
 - Instead, we will use limiting forms of the solution in most computations.

B. The solution for an infinite reservoir with line source well

➤ **assume that :**

- 1) a well produces at constant rate, qB ,**
- 2) The well has zero radius.**
- 3) The reservoir is at uniform pressure, P_i before production begins .**
- 4) The well drains an infinite area (ie., that $P \Rightarrow P_i$; as $r \Rightarrow \infty$).**



➤ Under these conditions, the solution to diffusivity equation is :

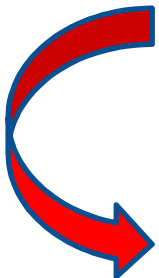
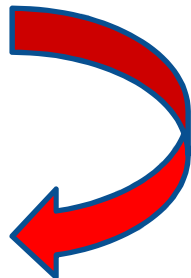
$$P(r, t) = P_i + 70.6 \frac{q \mu B_o}{kh} E_i \left(\frac{-948 \phi \mu C_t r^2}{kt} \right) \quad \begin{array}{l} \text{k: milli darcy} \\ \text{t :hr} \end{array}$$

=

$$P(r, t) = P_i + 70.6 \frac{q \mu B_o}{kh} E_i \left(-\frac{r^2}{4\eta t} \right)$$

➤ Where P is the pressure (psi) at distance r (feet) from the well at time t (hours),

- 
- E_i -function solution is an accurate approximation to the more exact solution for time:

$$\frac{3.79 \times 10^5 \phi \mu C_t r_w^2}{k} < t < 948 \frac{\phi \mu C_t r_e^2}{k}$$



For times less than , the assumption of zero well size limits the accuracy of the equation;

At times greater than, the reservoir's boundaries begin to affect the pressure distribution in the reservoir. so that the reservoir is no longer infinite acting.



The mathematical function, E_i , is called the exponential integral and is defined by:

$$E_i(-x) = -\int_x^\infty \frac{e^{-u}}{u} du = \left[\ln x - \frac{x}{1!} + \frac{x^2}{2(2!)} - \frac{x^3}{3(3!)} + \text{etc.} \right]$$

$$E_i(-x) = ? \left\{ \begin{array}{l} a) x < 0.02 \rightarrow E_i(-x) = \ln x + 0.5772 \\ b) x > 10 \rightarrow E_i(-x) = 0 \\ c) x = 0.02 \rightarrow E_i(-x) = -3.3 \\ d) 0.02 < x < 10 \rightarrow \text{Use table 7.1 or fig 7.11 Craft} \end{array} \right.$$

**Values of the $-E_i(-x)$ as a function of x
(After Craft, Hawkins, and Terry, 1991)**

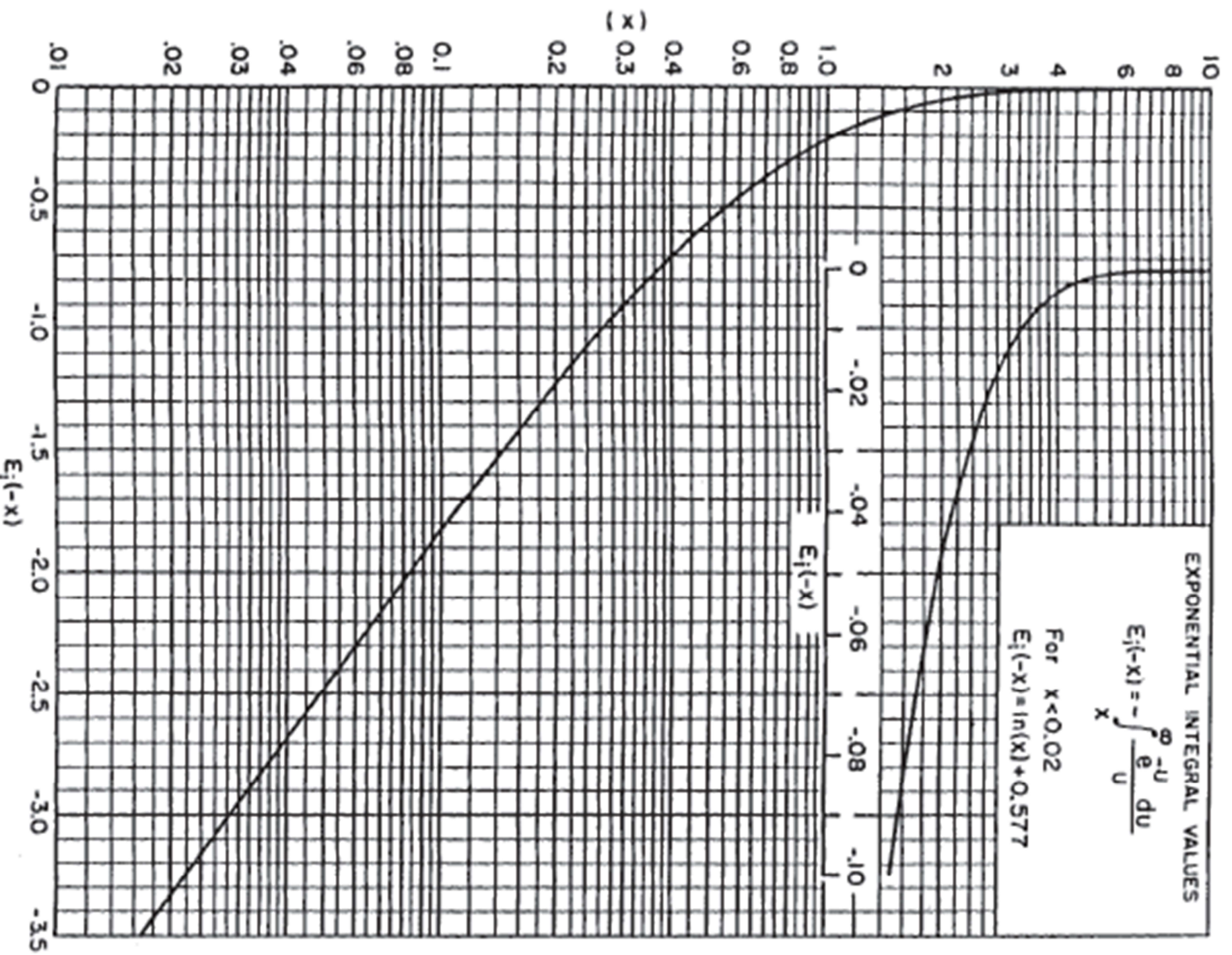
x	$-E_i(-x)$	x	$-E_i(-x)$	x	$-E_i(-x)$
0.1	1.82292	4.3	0.00263	8.5	0.00002
0.2	1.22265	4.4	0.00234	8.6	0.00002
0.3	0.90568	4.5	0.00207	8.7	0.00002
0.4	0.70238	4.6	0.00184	8.8	0.00002
0.5	0.55977	4.7	0.00164	8.9	0.00001
0.6	0.45438	4.8	0.00145	9.0	0.00001
0.7	0.37377	4.9	0.00129	9.1	0.00001
0.8	0.31060	5.0	0.00115	9.2	0.00001
0.9	0.26018	5.1	0.00102	9.3	0.00001
1.0	0.21938	5.2	0.00091	9.4	0.00001
1.1	0.18599	5.3	0.00081	9.5	0.00001
1.2	0.15841	5.4	0.00072	9.6	0.00001
1.3	0.13545	5.5	0.00064	9.7	0.00001
1.4	0.11622	5.6	0.00057	9.8	0.00001
1.5	0.10002	5.7	0.00051	9.9	0.00000
1.6	0.08631	5.8	0.00045	10.0	0.00000
1.7	0.07465	5.9	0.00040		
1.8	0.06471	6.0	0.00036		
1.9	0.05620	6.1	0.00032		
2.0	0.04890	6.2	0.00029		
2.1	0.04261	6.3	0.00026		
2.2	0.03719	6.4	0.00023		
2.3	0.03250	6.5	0.00020		
2.4	0.02844	6.6	0.00018		
2.5	0.02491	6.7	0.00016		
2.6	0.02185	6.8	0.00014		
2.7	0.01918	6.9	0.00013		
2.8	0.01686	7.0	0.00012		
2.9	0.01482	7.1	0.00010		
3.0	0.01305	7.2	0.00009		
3.1	0.01149	7.3	0.00008		
3.2	0.01013	7.4	0.00007		
3.3	0.00894	7.5	0.00007		
3.4	0.00789	7.6	0.00006		
3.5	0.00697	7.7	0.00005		
3.6	0.00616	7.8	0.00005		
3.7	0.00545	7.9	0.00004		
3.8	0.00482	8.0	0.00004		
3.9	0.00427	8.1	0.00003		
4.0	0.00378	8.2	0.00003		
4.1	0.00335	8.3	0.00003		
4.2	0.00297	8.4	0.00002		

EXPONENTIAL INTEGRAL VALUES

$$E_1(-x) = - \int_x^{\infty} \frac{e^{-u}}{u} du$$

For $x < 0.02$

$$E_1(-x) \approx \ln(x) + 0.577$$



- 
- For the damaged or stimulated zone the additional pressure drop (ΔP_s) across this zone can be modeled by :

$$\Delta P_s = 141.2 \frac{q \mu B}{kh} \times S$$

$$Skin = \left[\ln \frac{r_s}{r_w} \left(\frac{k}{k_s} - 1 \right) \right]$$

- So the total pressure drop **at the well bore** is

$$\begin{aligned} P_i - P_{wf} &= -70.6 \frac{q \mu B}{kh} E_i \left(\frac{-948 \phi \mu C_t r_w^2}{kt} \right) + \Delta P_s \\ &= -70.6 \frac{q \mu B}{kh} \left[E_i \left(\frac{-948 \phi \mu C_t r_w^2}{kt} \right) - 2 \ln \left(\frac{k}{k_s} - 1 \right) \left(\frac{r_s}{r_w} \right) \right] \end{aligned}$$

- 
- For $r = r_w$ the argument of the E_i function is sufficiently small after a short time that we can use the logarithmic approximation; thus,

$$P_i - P_{wf} = -70.6 \frac{q \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{kt} \right) \right]$$

And

$$P_i - P_{wf} = -70.6 \frac{q \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{kt} \right) - 2S \right]$$

- ✓ This equation is used only for calculation of pressures **at the sandface of a well**.

- ✓ For $r = r_w$ Previous equation
- ✓ For $r_w < r < r_s$ No simple equation
- ✓ For $r_s < r_e$ Use first equation (E_i -function)

□ H.W) Read the Example 1.1 John Lee on page 5



C. The Pseudo steady-State Solution :

- The summation involving exponentials and Bessel functions is negligible for this solution, after this time $948 \frac{\phi \mu C_t r_e^2}{k} \langle t$

$$P_{wf} = P_i - 141.2 \frac{q \mu B}{kh} \left[\frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{1}{2} \right]$$

Or

$$P_{wf} = P_i - 141.2 \frac{q \mu B}{kh} \left[\frac{0.000527kt}{\phi \mu C_t r_e^2} + \ln \frac{r_e}{r_w} - \frac{1}{2} \right]$$



✓ During this time period we find, by differentiating previous equation $\frac{\partial p}{\partial t} = cte$ and it is :

If $t : \text{hr}$
$$\frac{\partial p_{wf}}{\partial t} = - \frac{0.0744qB}{\phi\mu C_t h r_e^2}$$

If $t : \text{day}$
$$\frac{\partial p_{wf}}{\partial t} = - \frac{1.87qB}{\phi\mu C_t h r_e^2}$$



- Since the liquid-filled pore volume of the reservoir, V_p (cubic feet), is

$$V_p = \pi r_e^2 h \phi \quad \longrightarrow \quad \frac{\partial p_{wf}}{\partial t} = - \frac{0.23qB}{C_t V_p} \quad t : \text{hr}$$

- Thus, during this time period, the rate of pressure decline is inversely proportional to the liquid-filled pore volume V_p .
- This result leads to a form of well testing sometimes called **reservoir limits testing**, which seeks to determine reservoir size from the rate of pressure decline in a well bore with time.

- 
- Another useful form of equation is achieved by replacing P_i with P^- and including **skin factor**

$$P_{wf} = \bar{P} - 141.2 \frac{q \mu B}{kh} \left[\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right]$$

- ✓ Further, we can define an average permeability, k_j

$$k_j = \frac{k \left[\ln \frac{r_e}{r_w} - \frac{3}{4} \right]}{\left[\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right]}$$

- ✓ k is reservoir permeability without damage

- 
- Since we sometimes estimate the permeability of a well from productivity-index (PI) measurements, and since the **productivity index J** (STB/D/psi), of an oil well is defined as

$$PI = J = \frac{q}{\bar{P} - P_{wf}} = \frac{k_j h}{141.2 B \mu \left[\ln \frac{r_e}{r_w} - \frac{3}{4} \right]}$$

- ✓ This method does not necessarily provide a good estimate of formation permeability, k.

□ HW) Read example 1.2-Analysis of Well From PI Test on page 7 John lee



❖ Flow Equations for Generalized Reservoir Geometry

➤ Previous equation is limited to a well centered in a circular drainage area.

✓ Pseudo steady- state flow in more general reservoir shapes:

$$P_{wf} = \bar{P} - 141.2 \frac{q \mu B}{kh} \left[\frac{1}{2} \ln \left(\frac{10.06A}{C_A r_w^2} \right) - \frac{3}{4} + S \right]$$

where

A = drainage area, **sq ft**,

C_A = shape factor for specific drainage-area shape and well location,
dimensionless.

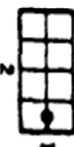
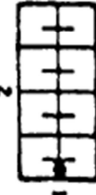
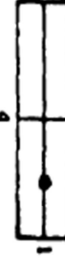
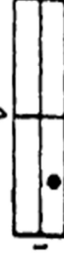
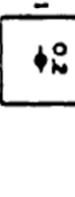
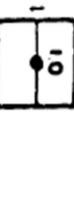
- 
- **Productivity index, J, can be expressed for general drainage-area geometry as**

$$PI = J = \frac{q}{\bar{P} - P_{wf}} = \frac{0.00708kh}{B\mu \left[\frac{1}{2} \ln \left(\frac{10.06A}{C_A r_w^2} \right) - \frac{3}{4} + s \right]}$$

- **Other numerical constants tabulated in following table allow us to calculate**

- i. the maximum elapsed time during which a reservoir is infinite acting (So that the Ei-function solution can be used)**
- ii. the time required for the pseudo steady-state solution to predict pressure drawdown within 1% accuracy**
- iii. time required for the pseudo steady-state solution to be exact.**

n Bounded Reservoirs	C_A	$\ln C_A$	$0.5 \ln \left(\frac{2.2458}{C_A} \right)$	Exact for $f_{DA} >$	Less Than 1% Error for $f_{DA} >$	Use Infinite System Solution With Less Than 1% Error for $f_{DA} <$
	31.62	3.4538	1.3224	0.1	0.06	0.10
	31.6	3.4532	-1.3220	0.1	0.06	0.10
	27.6	3.3178	-1.2544	0.2	0.07	0.09
	27.1	3.2995	-1.2452	0.2	0.07	0.09
	21.9	3.0865	-1.1387	0.4	0.12	0.08
	0.098	-2.3227	1.5659	0.9	0.60	0.015
	30.8828	3.4302	-1.3106	0.1	0.05	0.09
	12.9851	2.5638	-0.8774	0.7	0.25	0.03
	4.5132	1.5070	-0.3490	0.6	0.30	0.025
	3.3351	1.2045	-0.1977	0.7	0.25	0.01
	21.8369	3.0836	-1.1373	0.3	0.15	0.025
	10.8374	2.3830	-0.7870	0.4	0.15	0.025
	4.5141	1.5072	-0.3491	1.5	0.50	0.06
	2.0769	0.7309	0.0391	1.7	0.50	0.02
	3.1573	1.1497	-0.1703	0.4	0.15	0.005

In Bounded Reservoirs	C_A	$\ln C_A$	$0.5 \ln \left(\frac{2.2458}{C_A} \right)$	Exact for $f_{DA} >$	Less Than 1% Error for $f_{DA} >$	Use Infinite System Solution With Less Than 1% Error for $f_{DA} <$
	0.5813	-0.5425	0.6758	2.0	0.60	0.02
	0.1109	-2.1991	1.5041	3.0	0.60	0.005
	5.3790	1.6825	-0.4367	0.8	0.30	0.01
	2.6896	0.9894	-0.0902	0.8	0.30	0.01
	0.2318	-1.4619	1.1355	4.0	2.00	0.03
	0.1155	-2.1585	1.4838	4.0	2.00	0.01
	2.3606	0.8589	-0.0249	1.0	0.40	0.025
In vertically fractured reservoirs: use $(r_e/L_f)^2$ in place of A/r_w^2 for fractured systems						
	2.6541	0.9761	-0.0835	0.175	0.08	cannot use
	2.0348	0.7104	0.0493	0.175	0.09	cannot use
	1.9886	0.6924	0.0583	0.175	0.09	cannot use
	1.6620	0.5080	0.1505	0.175	0.09	cannot use
	1.3127	0.2721	0.2685	0.175	0.09	cannot use
	0.7887	-0.2374	0.5232	0.175	0.09	cannot use
In water-drive reservoirs						
	19.1	2.95	-1.07	-	-	-
In reservoirs of unknown production character						
	25.0	3.22	-1.20	-	-	-

- 
- For a given reservoir geometry, the maximum time a reservoir is infinite acting can be determined using the entry in the column
"Use Infinite-System Solution With Less Than 1% Error for $t_{DA} < "$

- Since, $t_{DA} = \frac{0.000264kt}{\phi\mu C_t A}$ this means that the time in hours is
calculated from

$$t < \frac{\phi\mu C_t A t_{DA}}{0.00264k}$$

- 
- Time required for the pseudo steady-state equation to be accurate within 1 % can be found from the entry in the column headed "**Less Than 1% Error for t_{DA}** " and the relationship

$$t > \frac{\phi \mu C_t A t_{DA}}{0.00264k}$$

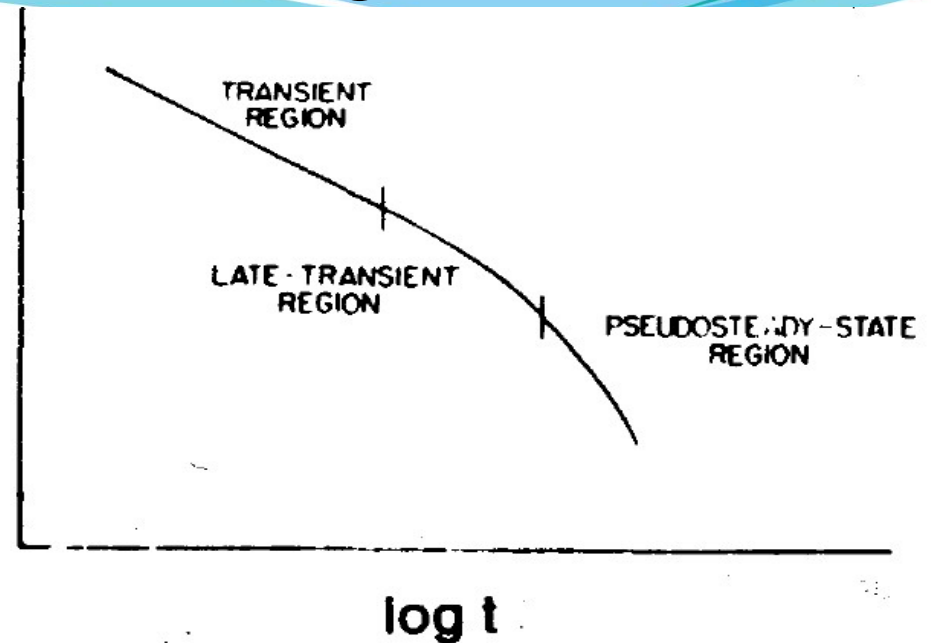
- Finally, time required for the pseudo steady-state equation to be exact is found from the entry in the column "**Exact for $t_{DA} > .$** "

Flow regimes that occur in different time ranges.

Flow regions on semi logarithmic paper



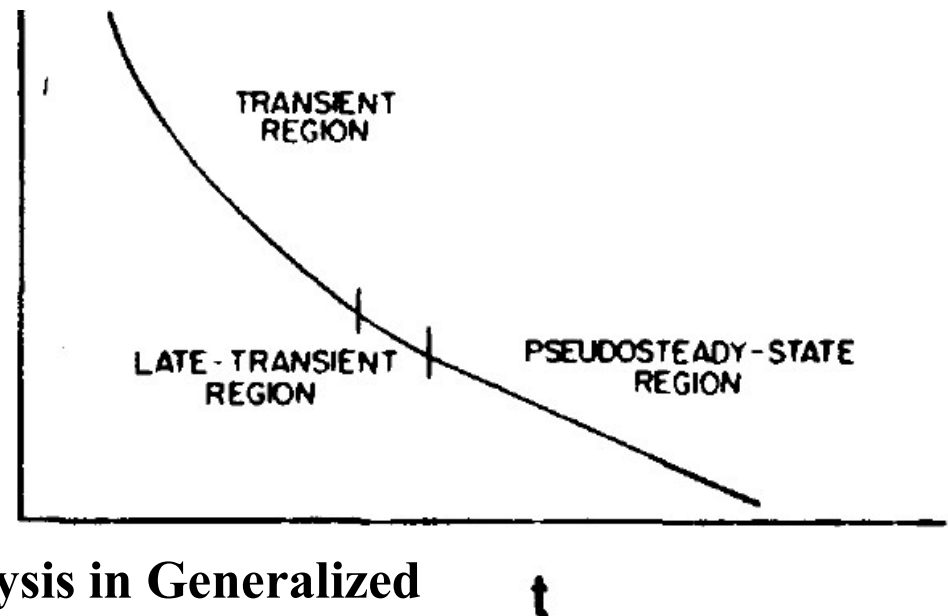
P_{wf}



Flow regions on Cartesian coordinate graph.



P_{wf}



□ HW) Read Example 1.3 - Flow Analysis in Generalized Reservoir Geometry (on page 8)



D. Radial Flow in infinite reservoir with well bore storage

Wellbore Storage

- **Distortions in the reservoir response due to the volume of wellbore.**
- **A crucial part of the transient analysis is to distinguish the effects of wellbore storage from the interpretable reservoir response .**



In Drawdown test

- On opening the valve at surface, the initial flow rate is due to wellbore unloading
- As wellbore unloading gradually decreases to zero, the flow from the formation increases from zero to q_{wh}

In Build up test

- After shut-in at the surface, flow from the formation does not stop immediately.
- Flow of fluid into the well persists for some time after shut-in due to compressibility of the fluid.
- The rate of flow changes gradually from q_{wh} at the time of shut-in to zero during a certain time period.



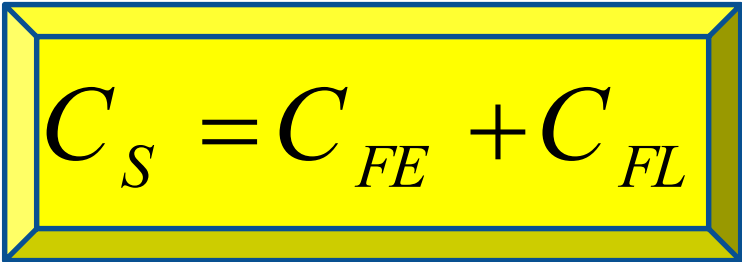
Well bore storage coefficient, C_s :

$$C_s = \frac{\Delta v_{wb}}{\Delta P}$$

C_s : Well bore storage coefficient,(bbl/Psi)

Δv_{wb} :Volume change in well bore (bbl)

ΔP :Pressure change (Psi)


$$C_s = C_{FE} + C_{FL}$$



Well bore storage effect due to **fluid expansion :**

$$C_{FE} = V_{wb} \times C_{wb}$$

V_{wb} = Volume of fluid in well bore (bbl)

C_{wb} = Average fluid compressibility in well bore Psi ⁻¹



Well bore storage effect due to **change of fluid level in annulus**

$$C_{FL} = \frac{144A_a}{5.615\rho} = 25.64 \times \frac{A_a}{\rho}$$

$$A_a = \pi \left[\left(\frac{ID_c}{2} \right)^2 - \left(\frac{OD_t}{2} \right)^2 \right] \frac{1}{144}$$

A_a : Area of annulus (ft²)

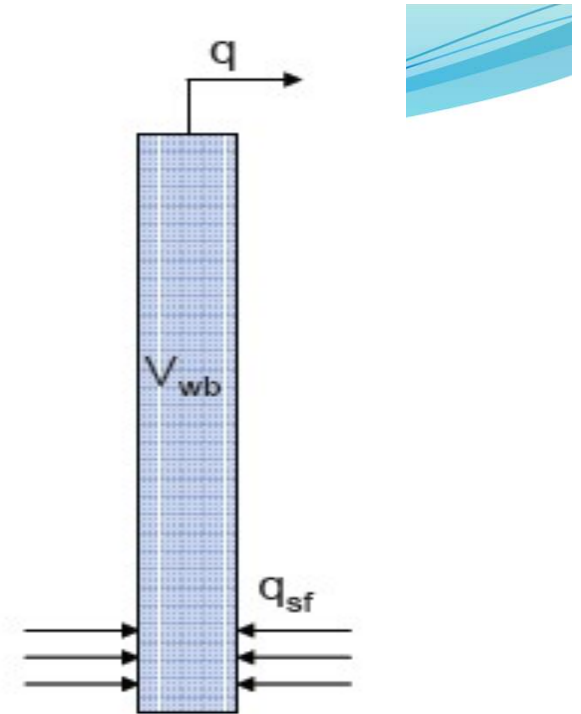
ID_c : Inner diameter of casing (in)

OD_t : Outer diameter of tubing (in)

ρ : Density of fluid in well bore (lbm/ft³)

➤ To relate **sand face flow rate** to **well head flow rate** we can use :

$$q_{sf} = q_{wh} + \frac{24C_s}{B_o} \frac{dp_w}{dt}$$

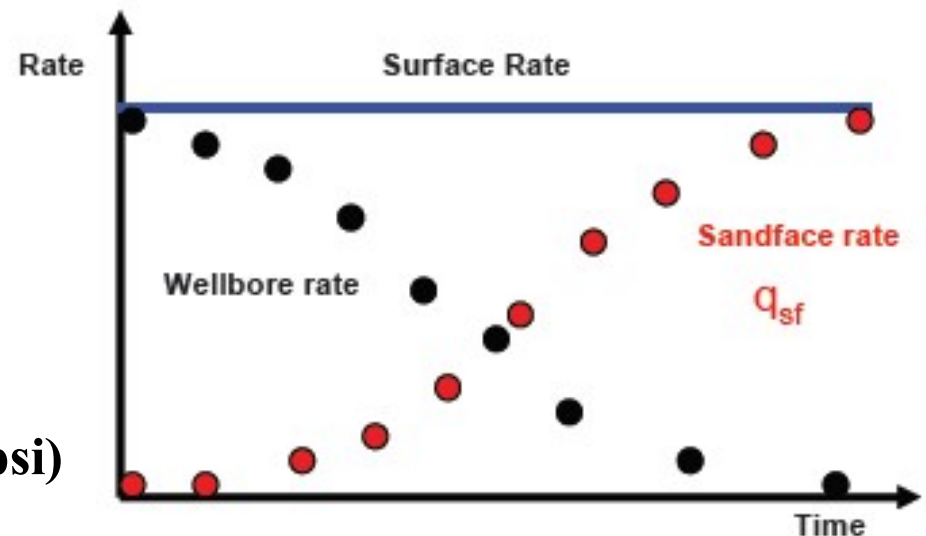


q_{wh}: Flow rate at well head (STB/Day)

q_{sf}: Flow rate at sand face (STB/Day)

B : Formation volume factor (bbl/STB)

C_s : Well bore storage coefficient (bbl/psi)



Derivation :

In – Out = Accumulation

$$In = q_{sf} B \rho$$

$$Out = q B \rho$$

$$Accum. = \frac{d(24 \rho_{wb} V_{wb})}{dt}$$

$$q_{sf} B \rho - q B \rho = 24 V_{wb} \frac{d\rho}{dt}$$

We can write

$$c = \frac{1}{\rho} \frac{d\rho}{dp} \quad \Rightarrow \quad \frac{d\rho_{wb}}{dt} = \frac{d\rho_{wb}}{dp} \frac{dp_{wb}}{dt} = \rho_{wb} c_{wb} \frac{dp_{wb}}{dt}$$

$$\rho B = C^{st} \quad \Rightarrow \quad q_{sf} = q + \frac{24 c_{wb} V_{wb}}{B} \left(\frac{\rho_{wb}}{\rho_R} \right) \frac{dp_w}{dt}$$

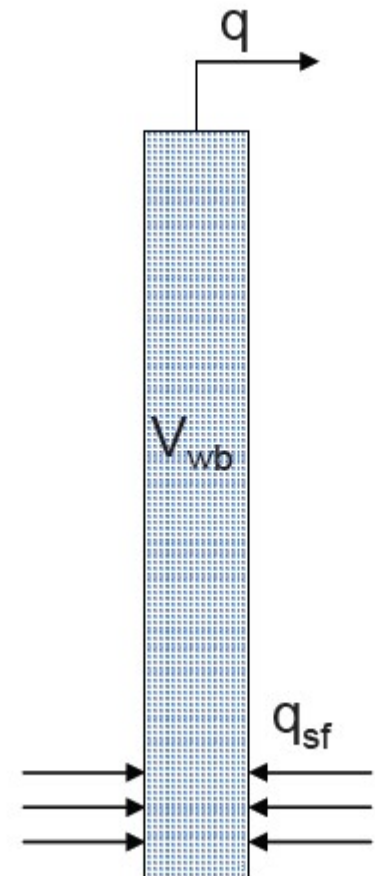
Define $C = c_{wb} V_{wb}$

Assume $\rho_{wb} \approx \rho_R$ $\Rightarrow$ $q_{sf} = q + \frac{24 C_{wb}}{B} \frac{dp_w}{dt}$

- To determine the duration of well bore storage effect it is better the well bore storage constant (Coefficient) is defined as a dimensionless variable :

$$C_{SD} = \frac{0.894C_s}{\phi c_t h r_w^2}$$

$$C_D = \frac{0.8936C}{\phi c_t h r_w^2} = \frac{0.8936c_{wb} V_{wb}}{\phi c_t h r_w^2}$$





➤ **Dimensionless time and dimensionless pressure are :**

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2}$$

$$p_D = \frac{0.00708kh (p_i - p_w)}{q_i B \mu}$$



C_{sD} : Dimensionless well bore storage constant

C_s : Well bore storage constant bbl/psi

h : Formation thickness ft

ϕ : porosity, fraction

c_t : total compressibility, psi^{-1}

r : Well bore radius ft

q : oil flow rate STB/day

t_D : Dimensionless time

k : permeability, md

t : test time hr

μ : viscosity, cp

P_D : Dimensionless pressure

B : formation volume factor bbl /psi



➤ **For constant-rate production**

$$\frac{q_{sf}}{q_{wh}} = 1 - C_{SD} \frac{dp_D}{dt_D}$$

- ✓ **Previous Eq is the inner boundary condition for the problem of constant-rate flow of a slightly compressible liquid with well bore storage.**



Presence of unit slope line :

- At the earliest time for a given value of C_{SD} and for most value of **S** , a unit slope line (i.e., line with 45° slope) is present on the graph.
- This line appears and remains as long as all production comes from the well bore and none comes from the formation.

$$q_{sf} = 0 \quad \Rightarrow \quad \frac{q_{sf}}{q_{wh}} = 0 \quad \Rightarrow \quad 1 - C_{SD} \frac{dp_D}{dt_D} = 0$$

$$\Rightarrow \quad dt_D = C_{SD} \times dp_D$$

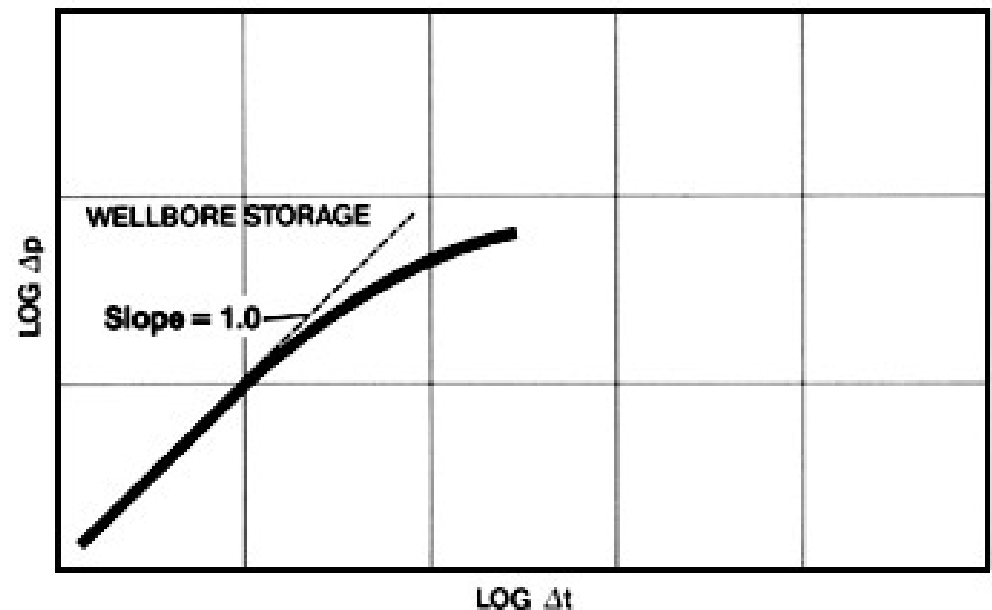
- Integrating from $t_D = 0$ (where $p_D = 0$) to t_D and p_D the result is

$$C_{SD} \times p_D = t_D$$

- Taking logarithms of both side of the equation,

$$\log C_{SD} + \log p_D = \log t_D$$

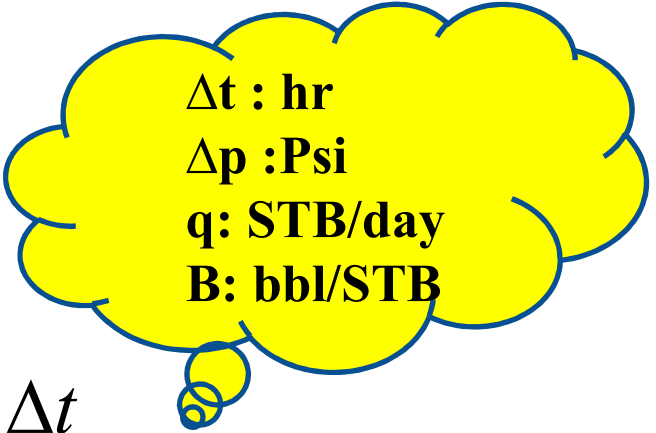
- Thus a graph of $\log p_D$ vs. $\log t_D$ will have a slope of unity.



- 
- Any point on (p_D, t_D) on this unit slope line must satisfy the following relation

$$\frac{C_{SD} \times p_D}{t_D} = 1$$

- ✓ For any point of this line (unit slope line) with its appropriate time and pressure we can find C_s from following equation

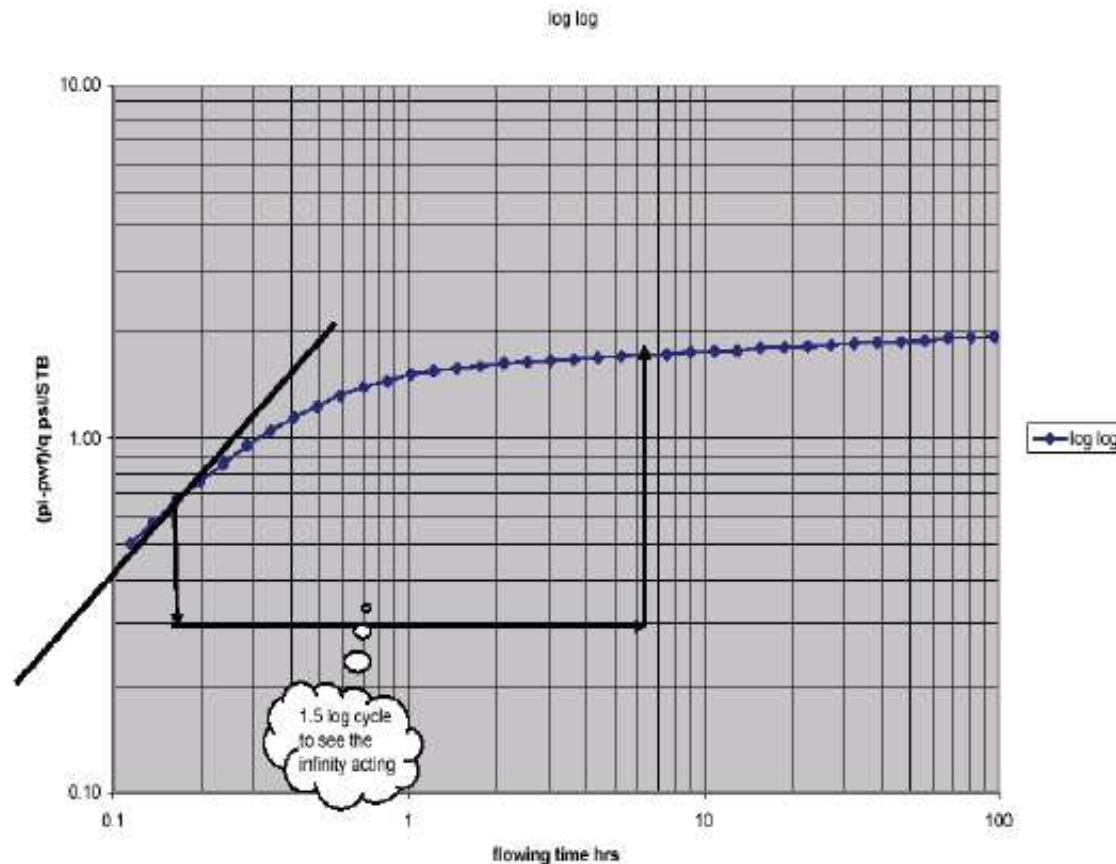


Δt : hr
 Δp : Psi
 q : STB/day
 B : bbl/STB

$$C_s = \left(\frac{qB}{24} \right) \frac{\Delta t}{\Delta P}$$

End of Wellbore Storage Distortion :

- One useful empirical observation is that end of well bore storage distortion (t_{wbs}) occurs approximately one and half log cycle after disappearance of the unit slope line.



- 
- **Another useful observation is that the dimensionless time at which well bore storage distortion ceases is given by:**

- ✓ **For positive skin**

$$t_D = (60 + 3.5s) C_{SD}$$

$$t_{wbs} = \frac{(200000 + 12000s) C_s}{\frac{kh}{\mu}}$$

- ✓ **For negative skin and No skin**

$$t_D > 60 C_{SD}$$



Example :

The following data are available for an oil well under draw down test. If the well produces with constant rate, calculate the well bore storage constant and End of Wellbore Storage distortion .

$$V_{wb} = 180 \text{ bbl}$$

$$OD_t : 2 \text{ in}$$

$$ID_c : 7.675 \text{ in}$$

$$\rho_o : 45 \text{ lbm/ft}^3$$

$$h : 50 \text{ ft}$$

$$\phi : 15 \%$$

$$r_w : 0.25 \text{ ft}$$

$$c_t : 20 \times 10^{-6} \text{ psi}^{-1}$$

$$k : 30 \text{ md}$$

$$\mu_o : 2 \text{ cp}$$

$$s : 0$$

$$c_o : 10 \times 10^{-6} \text{ psi}^{-1}$$



Well bore storage constant due to **fluid expansion** :

$$C_{FE} = V_{wb} \times C_{wb} = 180(10 \times 10^{-6}) = 0.0018 \frac{bbl}{psi}$$

Well bore storage constant due to **change of fluid level in annulus**

$$A_a = \pi \left[\left(\frac{ID_C}{2} \right)^2 - \left(\frac{OD_t}{2} \right)^2 \right] \frac{1}{144} = \pi \left[\left(\frac{7.675}{2} \right)^2 - \left(\frac{2}{2} \right)^2 \right] \frac{1}{144} = 0.2995 ft^2$$

$$C_{FL} = \frac{144 A_a}{5.615 \rho} = 25.64 \times \frac{A_a}{\rho} = 25.64 \times \frac{0.2995}{45} = 0.1707 \frac{bbl}{psi}$$



The total well bore storage constant

$$C_S = C_{FE} + C_{FL} = 0.0018 + 0.1707 = 0.1725 \frac{bbl}{psi}$$

Dimensionless well bore storage constant

$$C_{SD} = \frac{0.894 C_S}{\phi c_t h r_w^2} = \frac{0.894 \times 0.1725}{0.15 \times (20 \times 10^{-6}) \times 50 \times 0.25^2} = 16271$$



End of Wellbore Storage Distortion :

$$t_{wbs} = \frac{(200000 + 12000s)C_s}{\frac{kh}{\mu}} = \frac{(200000 + 12000 \times 0)0.1725}{\frac{30 \times 50}{2}} = 46hr$$

Or

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2} = \frac{0.000264 \times 30 \times t}{0.15 \times 2 \times (20 \times 10^{-6}) \times 0.25^2}$$

$$t_D = (60 + 3.5s)C_{SD} = (60 + 3.5 \times 0)16271 = 976260$$

$$t_D = \frac{0.000264kt}{\phi\mu c_t r_w^2} = \frac{0.000264 \times 30 \times t}{0.15 \times 2 \times (20 \times 10^{-6}) \times 0.25^2} \quad \Rightarrow \quad t = 46 \text{ hr}$$

Radius of investigation

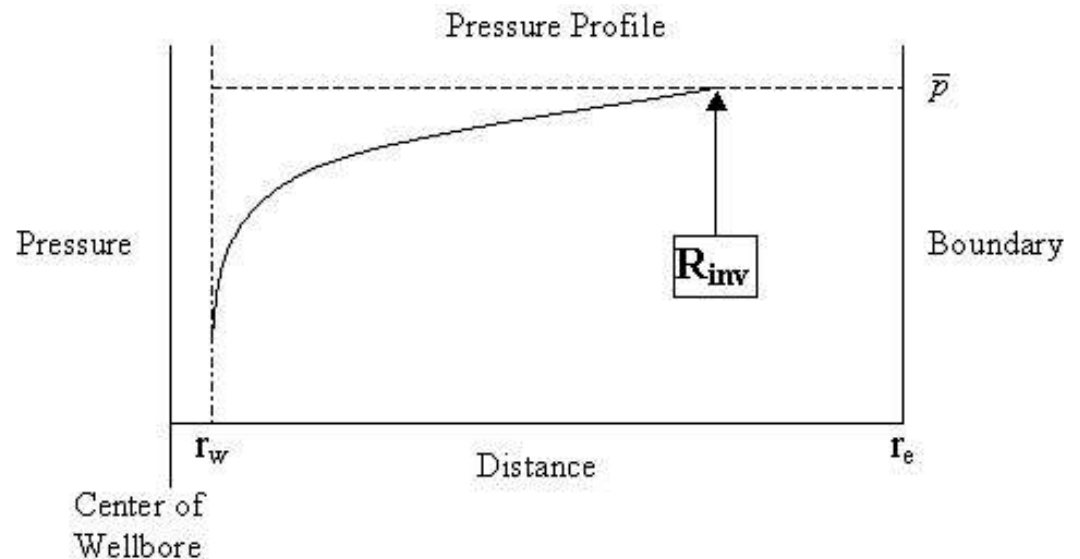
- By radius of investigation r_i we mean the distance that a pressure transient has moved into a formation.
- This distance is related to formation rock and fluid properties and time elapsed since the rate change. The rate affects only the magnitude of the pressure response.

$$r_i = \left(\frac{kt}{948\phi\mu C_t} \right)^{\frac{1}{2}}$$

k : milli darcy

t : hr

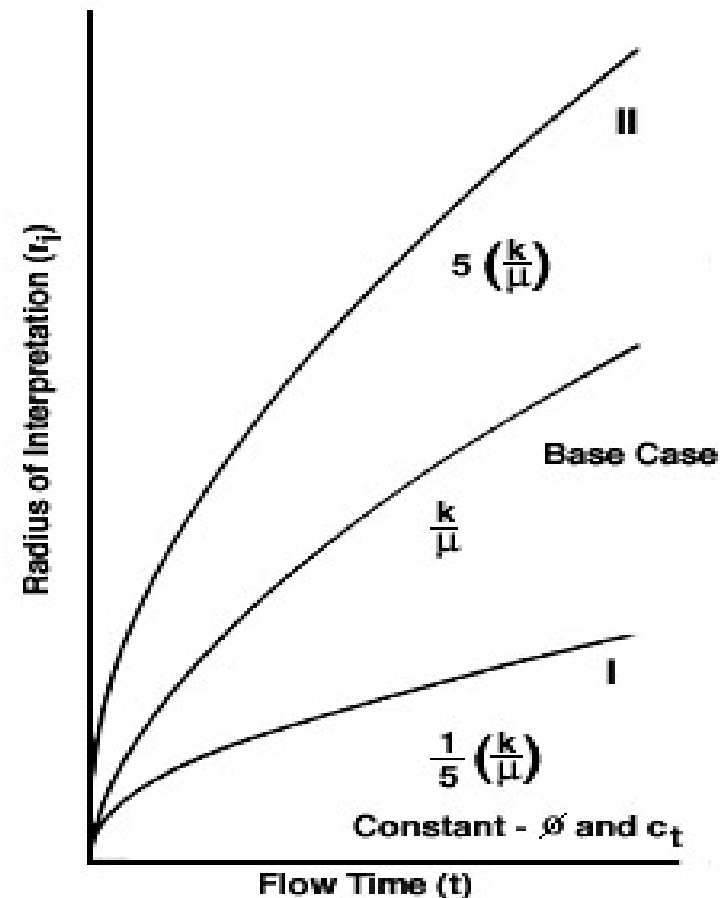
μ : cp



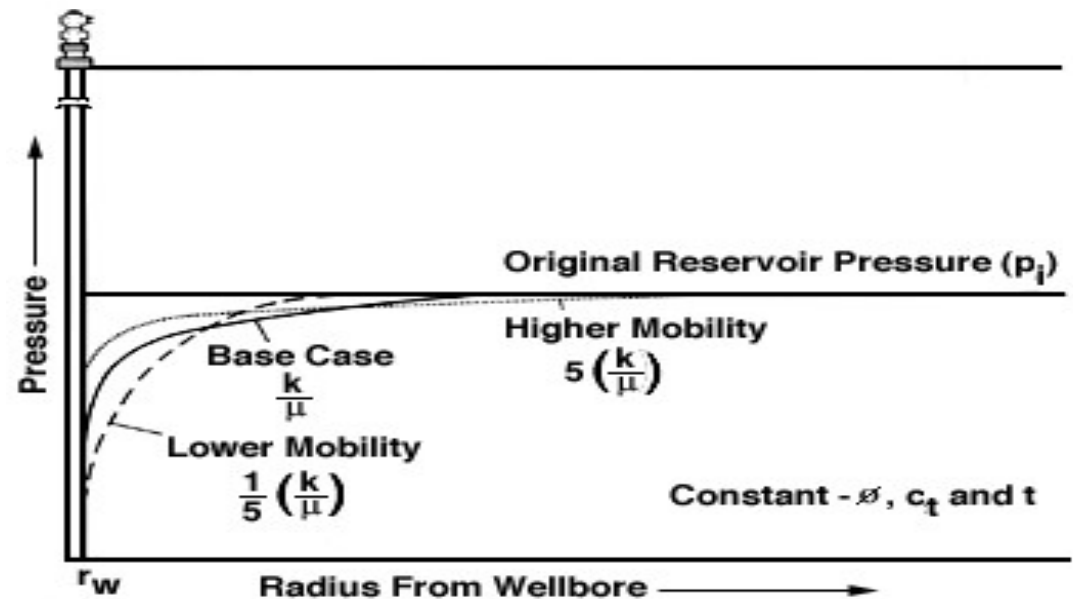
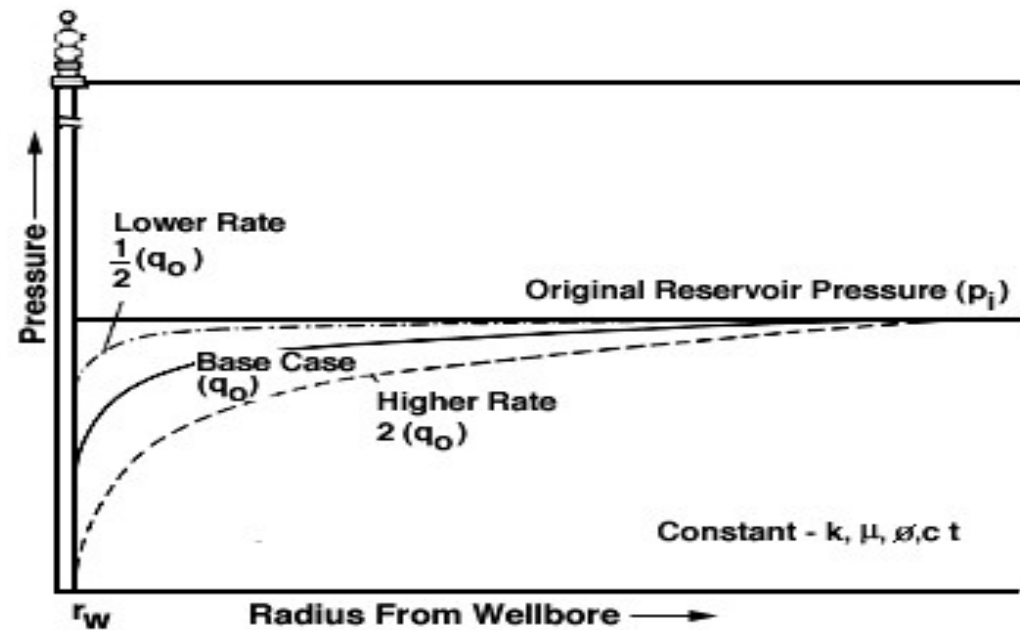
The effect-of-mobility ratio:

(the radius investigation versus flow time during a drawdown test).

- If the mobility of one reservoir is five times less than that of another, the former must be tested five times longer if the same radius is to be investigated in both cases.
- This assumes, of course, that the porosity and fluid compressibility are the same in both cases.



The effect of production rate on pressure transients during a drawdown test



❑ Read Example 1.4 -
Calculation of Radius
of Investigation on page 15



Principle of Superposition

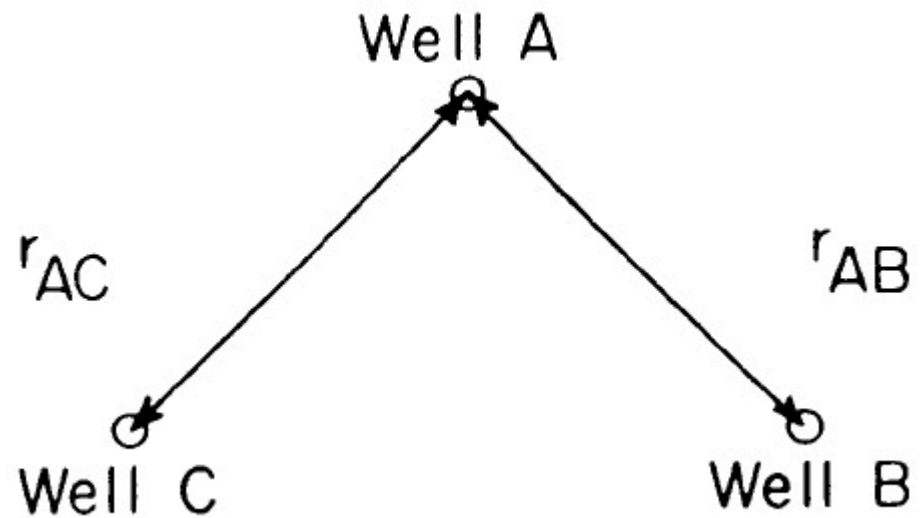
- **The superposition concept states that the total pressure drop at any point in the reservoir is the sum of the pressure changes at that point caused by flow in each of the wells in the reservoir.**

- **This concept can be applied to account for the following effects on the transient flow solution:**
 - 1. **Effects of multiple wells**
 - 2. **Effects of rate change**
 - 3. **Effects of shut-in after a flow period**
 - 4. **Effects of the boundary**



1. Effects of Multiple Wells :

- Wells A, B, and C, start to produce at the same time from an infinite reservoir



$$(p_i - p_{wf})_{\text{total at Well A}} = (p_i - p)_{\text{due to Well A}} + (p_i - p)_{\text{due to Well B}} \\ + (p_i - p)_{\text{due to Well C}}.$$



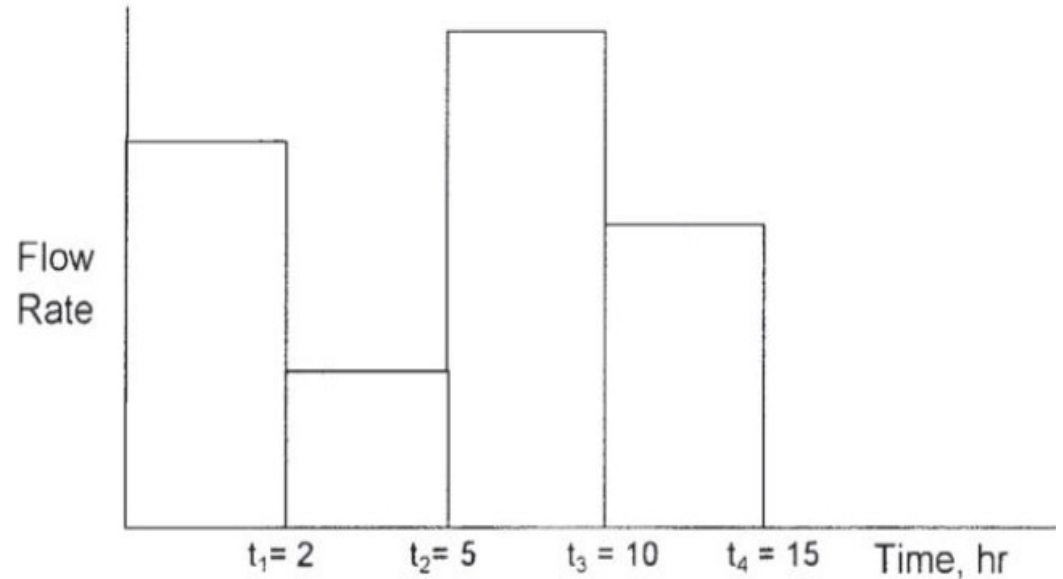
➤ In terms of Ei functions and logarithmic approximations,

$$\begin{aligned}(p_i - p_{wf})_{\text{total at Well A}} = & -70.6 \frac{q_A B \mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_{wA}^2}{kt} \right) - 2s_A \right] \\ & - 70.6 \frac{q_B B \mu}{kh} Ei \left(\frac{-948 \phi \mu c_i r_{AB}^2}{kt} \right) \\ & - 70.6 \frac{q_C B \mu}{kh} Ei \left(\frac{-948 \phi \mu c_i r_{AC}^2}{kt} \right)\end{aligned}$$

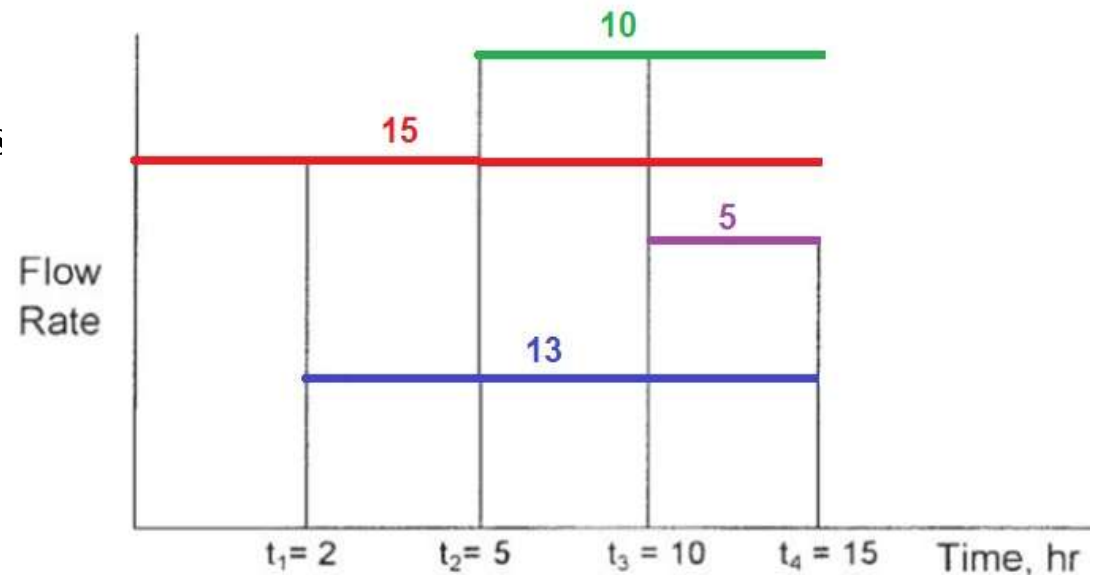
✓ Note that this equation includes a skin factor for Well A, but does not include skin factors for Wells B and C. Because most wells have a nonzero skin factor and because we are modeling pressure inside the zone of altered permeability near Well A, we must include its skin factor.

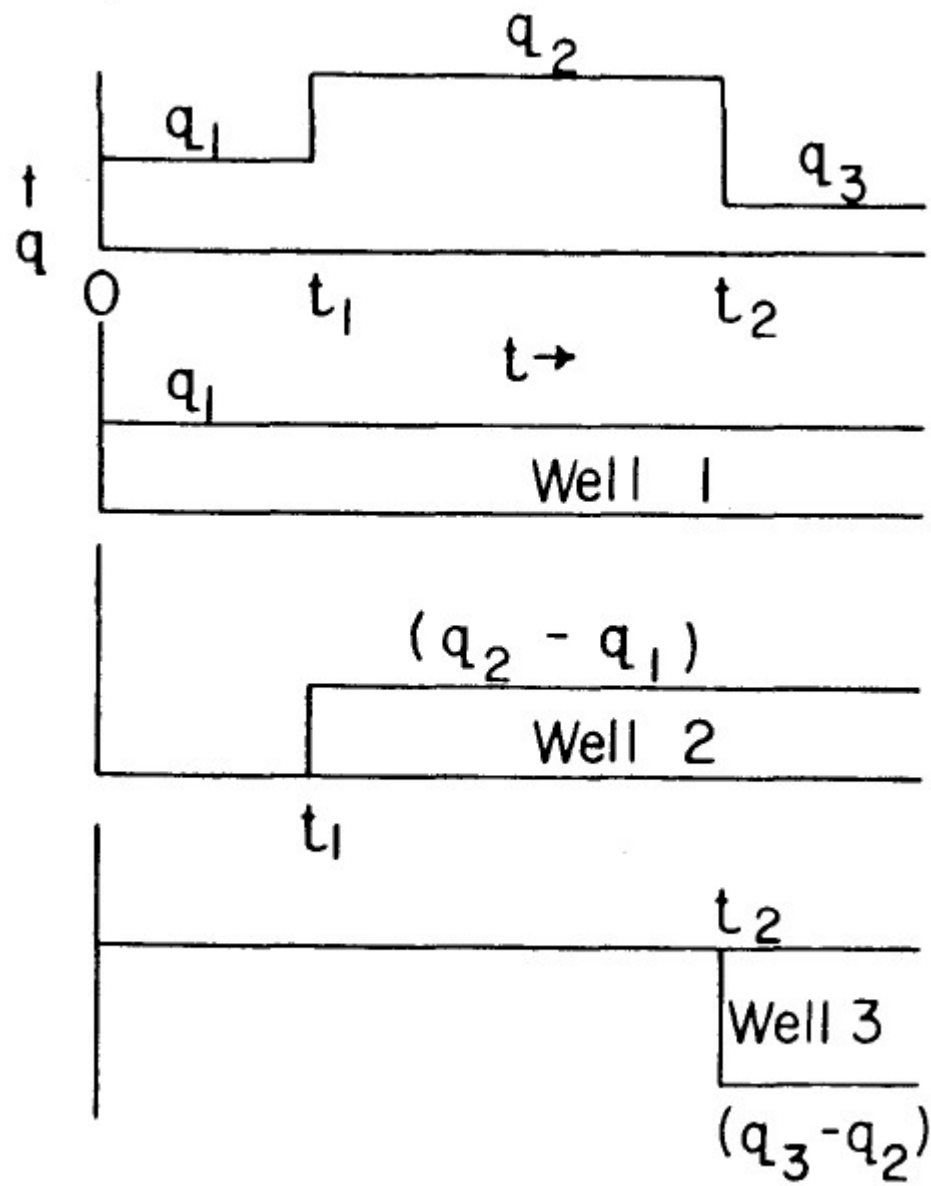
2. Effects of Variable Flow Rates :

➤ Every flow rate change in a well will result in a pressure response which is independent of the pressure responses caused by other previous rate changes.



➤ The total pressure drop that has occurred at any time is the summation of pressure changes caused separately by each net flow rate change.







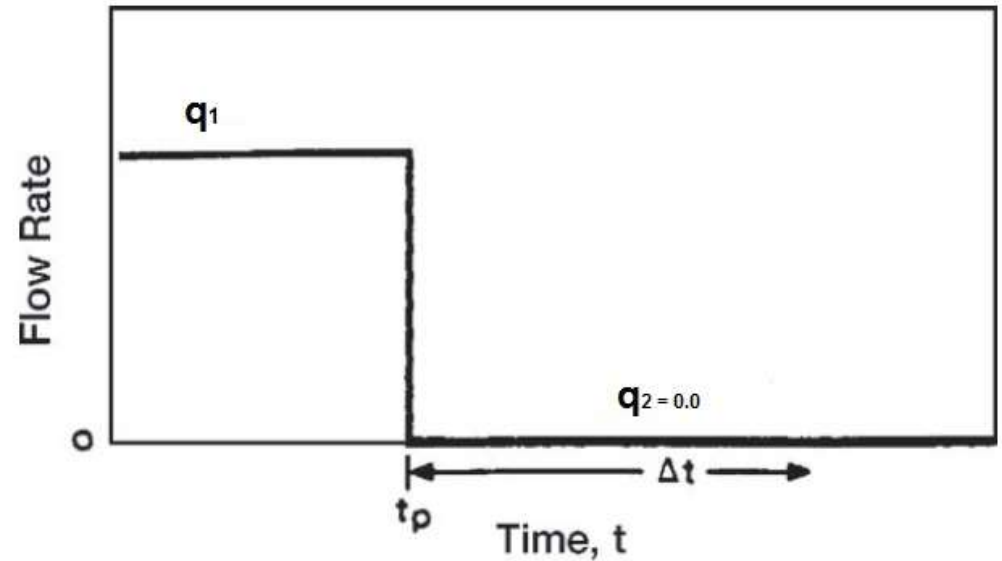
$$p_i - p_{wf} = (\Delta p)_1 + (\Delta p)_2 + (\Delta p)_3$$

$$= -70.6 \frac{\mu q_1 B}{kh} \left| \ln \left(\frac{1,688 \phi \mu c_l r_w^2}{kt} \right) - 2s \right|$$

$$- 70.6 \frac{\mu (q_2 - q_1) B}{kh} \cdot \left\{ \ln \left| \frac{1,688 \phi \mu c_l r_w^2}{k(t - t_1)} \right| - 2s \right\}$$

$$- 70.6 \frac{\mu (q_3 - q_2) B}{kh} \cdot \left\{ \ln \left| \frac{1,688 \phi \mu c_l r_w^2}{k(t - t_2)} \right| - 2s \right\}.$$

3. Effects of shut-in after a flow period



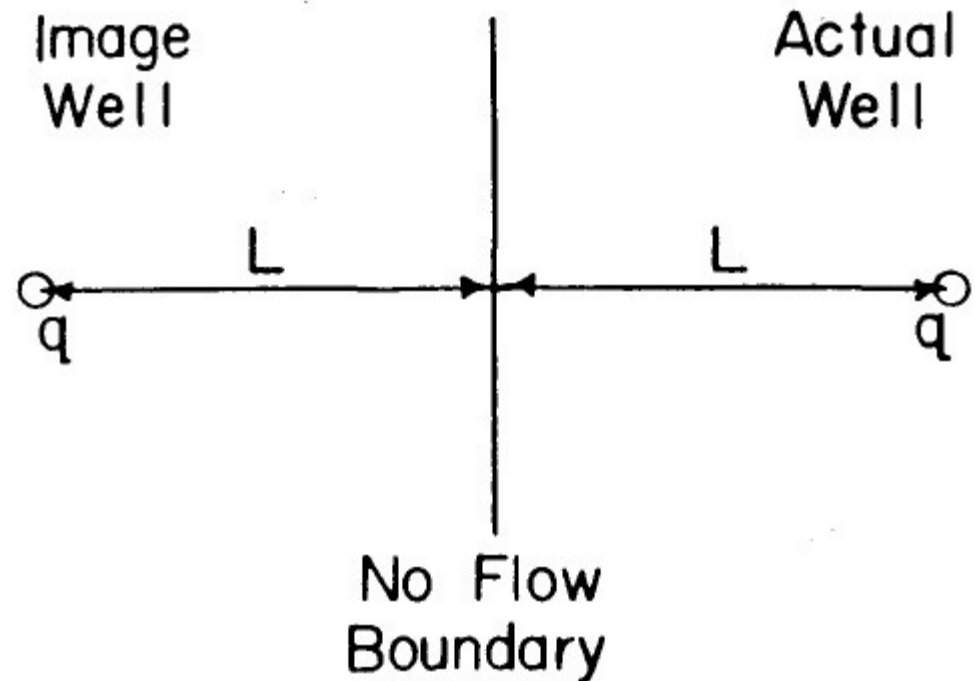
$$\Delta P_t = \Delta P_1 + \Delta P_2$$

$$= -70.6 \frac{q_1 \mu B_o}{kh} \left[\ln \left(\frac{1688 \phi \mu c_t r_w^2}{kt} \right) - 2s \right]$$

$$-70.6 \frac{(0 - q_1) \mu B_o}{kh} \left[\ln \left(\frac{1688 \phi \mu c_t r_w^2}{k(t - t_1)} \right) - 2s \right]$$

4. Effects of the boundary

➤ The effect of the boundary on the pressure behavior of a well would be the same as the effect from an image well located a distance $2L$ from the actual well.



$$p_i - p_{wf} = -70.6 \frac{qB\mu}{kh} \left(\ln \frac{1,688 \phi \mu c_i r_w^2}{kt} - 2s \right) - 70.6 \frac{qB\mu}{kh} Ei \left(\frac{-948 \phi \mu c_i (2L)^2}{kt} \right).$$

□ HW) Read example 1.5 – on page 18 John lee



Horner's Approximation

- **In 1951, Horner reported an approximation that can be used in many cases to avoid the use of superposition in modeling the production history of a variable-rate well.**
- **With this approximation, we can replace the sequence of E_i functions, reflecting rate changes, with a single E_i function that contains a single producing time and a single producing rate.**
- **The single rate is the most recent nonzero rate at which the well was produced; we call this rate q_{last} for now.**

- 
- The single producing time is found by dividing cumulative production from the well by the most recent rate; we call this producing time t_p or pseudo producing time

$$t_p(\text{hours}) = 24 \frac{\text{cumulative production from well, } N_p(\text{STB})}{\text{most recent rate, } q_{\text{last}}(\text{STB/D})}$$

- ✓ Then, to model pressure behavior at any point in a reservoir, we can use the simple equation

$$p_i - p = - \frac{70.6 \mu q_{\text{last}} B}{kh} Ei\left(\frac{-948 \phi \mu c_i r^2}{kt_p} \right).$$

- 
- **when is the approximation adequate?**

- ✓ **If the most recent rate is maintained sufficiently long for the radius of investigation achieved at this rate to reach the drainage radius of the tested well, then Horner's approximation is always sufficiently accurate.**
- ✓ **If the last constant rate for at least twice as long as the previous rate.**

Example : *Application of Horner's Approximation*

- ✓ Following completion, a well is produced for a short time and then shut in for a buildup test. The production history was as follows.

Production Time (hours)	Total Production (STB)
25	52
12	0
26	46
72	68

1. Calculate the pseudo producing time, t_p
2. Is Horner's approximation adequate for this case?
If not, how should the production history for this well be simulated?

$$1. \quad q_{\text{last}} = \frac{68 \text{ STB}}{72 \text{ hours}} \times \frac{24 \text{ hours}}{\text{day}} = 22.7 \text{ STB/D.}$$

$$t_p = \frac{24 (\text{cumulative production, STB})}{q_{\text{last, STB/D}}} = \frac{(24)(166)}{(22.7)} = 176 \text{ hours.}$$

$$2. \quad \frac{\Delta t_{\text{last}}}{\Delta t_{\text{next-to-last}}} = \frac{72}{26} = 2.77 > 2.$$

- ✓ Thus, Horner's approximation is probably adequate for this case.
- ✓ It should not be necessary to use superposition, which is required when Horner's approximation is not adequate.



Chapter 3

Pressure Buildup Tests



➤ **Basically, the test is conducted by**

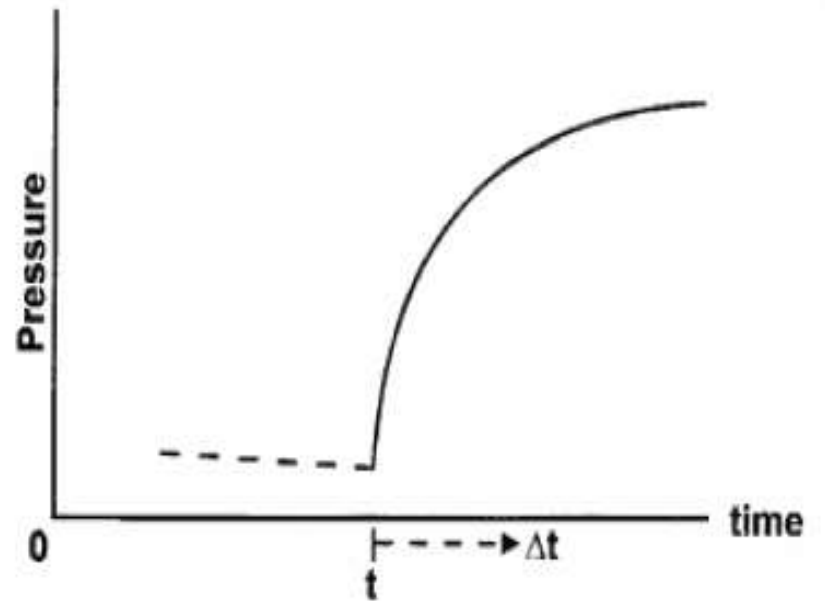
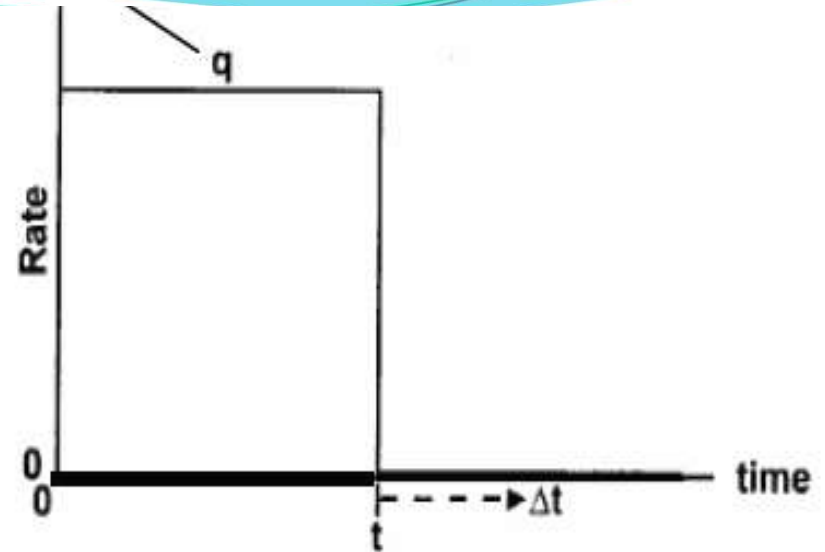
- ✓ **producing a well at constant rate for some time,**
- ✓ **shutting the well in (usually at the surface),**
- ✓ **allowing the pressure to build up in the well bore,**
- ✓ **and recording the pressure(usually down hole) in the well bore as a function of time.**

➤ **From these data, it is frequently possible to estimate**

- ✓ **formation permeability**
- ✓ **current drainage-area pressure,**
- ✓ **characterize damage or stimulation**
- ✓ **and reservoir heterogeneities or boundaries.**

Methods of analysis:

- **Horner plot (1951):**
Infinite acting reservoir
- **Matthews-Brons-Hazebroek (MBH, 1954):**
Extension of Horner plot to finite reservoir.
- **Miller-Dyes-Hutchinson (MDH plot, 1950):**
Analysis of P.S.S. flow conditions.



The Ideal Buildup Test

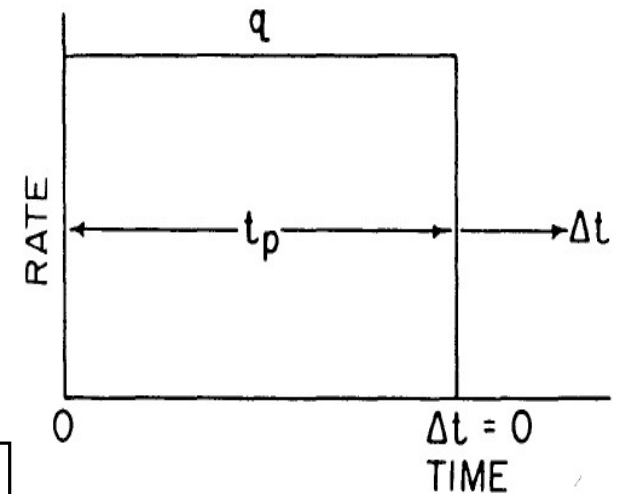
By ideal test we mean

- a test in an infinite, homogeneous, isotropic reservoir containing a slightly compressible, single-phase fluid with constant fluid properties.
- Any well bore damage or stimulation is considered to be concentrated in a skin of zero thickness at the well bore; at the instant of shut-in, flow into the well bore ceases totally.

Assume that

- 1) A well is producing from an infinite-acting reservoir**
- 2) The formation and fluids have uniform properties,**
- 3) Horner's pseudo producing time approximation is applicable.**

**By using superposition for following Fig ,
we find that :**



$$P_i - P_{wf} = -70.6 \frac{q \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{k (t_p + \Delta t)} \right) - 2S \right]$$

$$-70.6 \frac{(-q) \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{k \times \Delta t} \right) - 2S \right]$$



which becomes

$$P_{wf} = P_i - 70.6 \frac{q \mu B}{kh} \ln \left(\frac{t_p + \Delta t}{\Delta t} \right)$$

or

$$\boxed{\ln x = 2.303 \log x} \Rightarrow P_{wf} = P_i - 162.6 \frac{q \mu B}{kh} \log \left(\frac{t_p + \Delta t}{\Delta t} \right)$$

- **The form of above equation suggests that shut-in BHP, P_{ws} recorded during a pressure buildup test should plot as a straight-line function of $\log [(t_p + \Delta t) / \Delta t]$.**

- 
- Further, the slope m of this straight line should be

$$m = -162.6 \frac{q \mu B}{kh}$$

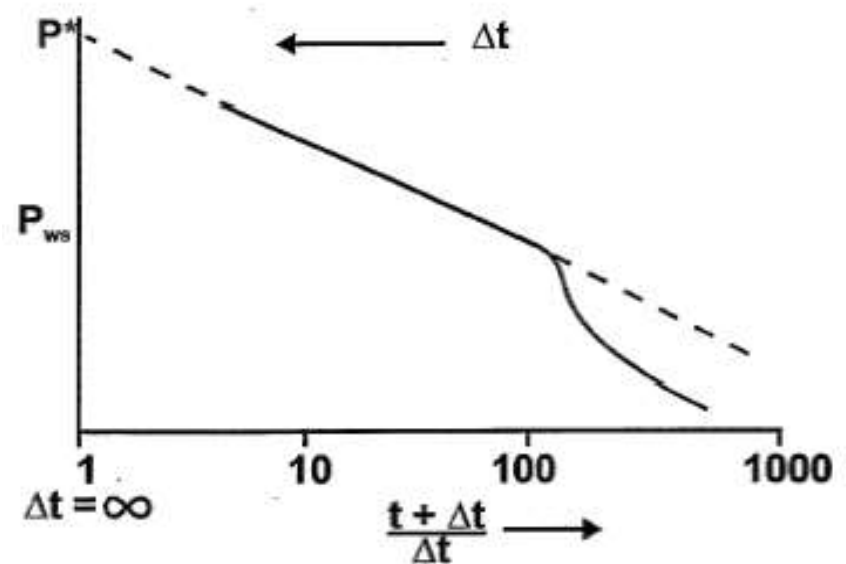
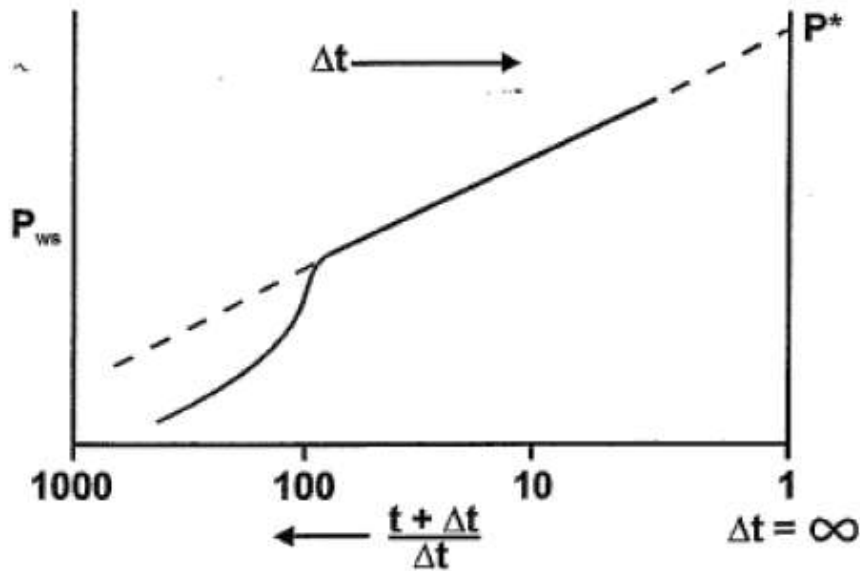
- It is convenient to use the absolute value of m in test analysis; accordingly, we will use the convention that m is considered a positive number and that

$$m = 162.6 \frac{q \mu B}{kh}$$

{	q : STB/day
	B : bbl/STB
	k : md
	μ : cp
	h : ft

➤ Thus, formation permeability, k , can be determined from a buildup test by measuring the slope m .

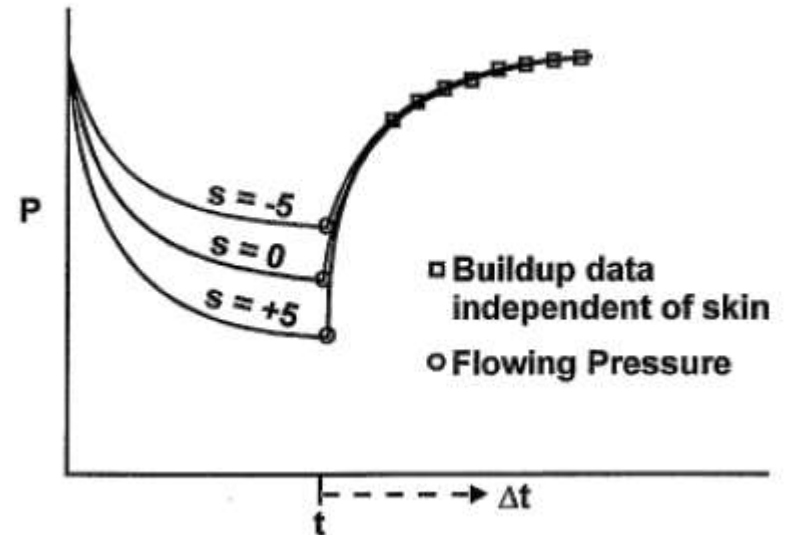
➤ If we extrapolate this straight line to infinite shut-in time [i.e., $(t_p + \Delta t) / \Delta t = 1$] the pressure at this time will be the original formation pressure P_i .



- 
- **Conventional practice in the industry is to plot P_{ws} vs $(t_p + \Delta t) / \Delta t$ on semilogarithmic paper .**
 - **The slope m on such a plot is found by simply subtracting the pressures at any two points on the straight line that are one cycle (i.e., a factor of 10) apart on the semi log paper.**

Calculation of skin factor s

- Buildup test does NOT allow for skin calculation. Skin is obtained from FLOWING pressure before shut-in.



- ✓ At the instant a well is shut in, the flowing BHP, P_{wf}' is

$$P_{wf} = P_i + 70.6 \frac{q \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{k (t_p + \Delta t)} \right) - 2S \right]$$

$$= P_i + 162.6 \frac{q \mu B}{kh} \left[\log \left(\frac{1688 \phi \mu C_t r_w^2}{k (t_p + \Delta t)} \right) - 0.869S \right]$$



$$P_{wf} = P_i + m \left[\log \left(\frac{1688 \phi \mu C_t r_w^2}{k (t_p + \Delta t)} \right) - 0.869 S \right]$$

➤ At shut-in time Δt in the buildup test,

$$P_{ws} = P_i - m \log \left[\frac{(t_p + \Delta t)}{\Delta t} \right]$$

➤ Combining these equations and solving for the skin factor S, we have

$$s = 1.151 \left(\frac{P_{ws} - P_{wf}}{m} \right) + \log \left(\frac{1688 \phi \mu C_t r_w^2}{k \Delta t} \right) + 1.151 \times \log \left(\frac{t_p + \Delta t}{t_p} \right)$$



- It is conventional practice in the petroleum industry to choose a fixed shut-in time, Δt , of 1 hour and the corresponding shut-in pressure, P_1 hr , to use in this equation.
- ✓ (although any shut-in time and the corresponding pressure would work just as well).
- The pressure, P_1 hr must be on the straight line or its extrapolation.
- We usually can assume further that $\log [(t_p + \Delta t) / t_p]$ is negligible.

$$\log \left(\frac{t_p + 1}{t_p} \right) \simeq 0$$

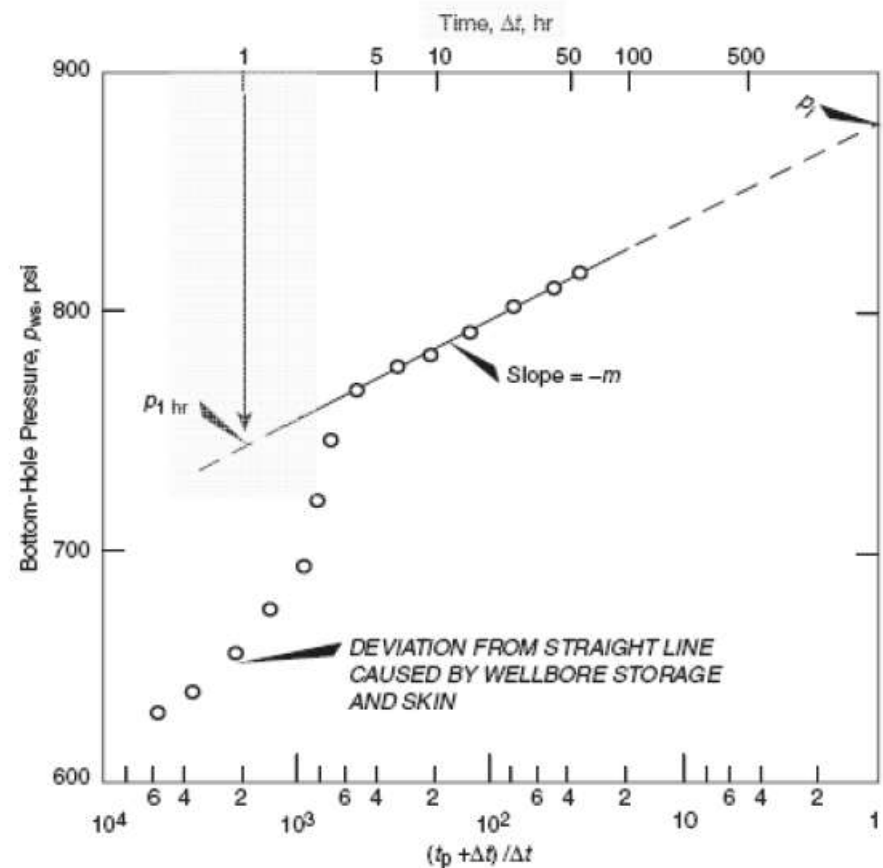
✓ With these simplifications,

$$s = 1.151 \left[\frac{(P_{1hr} - P_{wf})}{m} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$

P_{wf} : The last pressure before shut-in

P_{1hr} : The BHP 1hour after shut-in

✓ Note again that the **slope m** is considered to be a positive number in this equation.




$$\Delta P_s = 141.2 \frac{Q \mu B}{hk} S \quad \rightarrow \quad \Delta P_s = 0.87 |m| S$$

$$s = \frac{\Delta P_s}{0.87 |m|}$$

$$Skin = \left[\ln \frac{r_s}{r_w} \left(\frac{k}{k_s} - 1 \right) \right]$$

Example - Analysis of Ideal Pressure Buildup Test

- A new oil well produced 500 STB/D for 3 days; it then was shut in for a pressure buildup test, during which the data in following table were recorded.

Time After Shut-In Δt (hours)	p_{ws} (psig)
0	1,150
2	1,794
4	1,823
8	1,850
16	1,876
24	1,890
48	1,910

- ✓ For this well, net sand thickness, is 22 ft; formation volume factor, is 1.3 RB/STB; porosity, is 0.2; total compressibility, is 20×10^{-6} ; oil viscosity is 1.0 cp; and well bore radius is 0.3 ft.
- From these data, estimate **formation permeability, k , initial reservoir pressure, P_i and skin factor, s .**

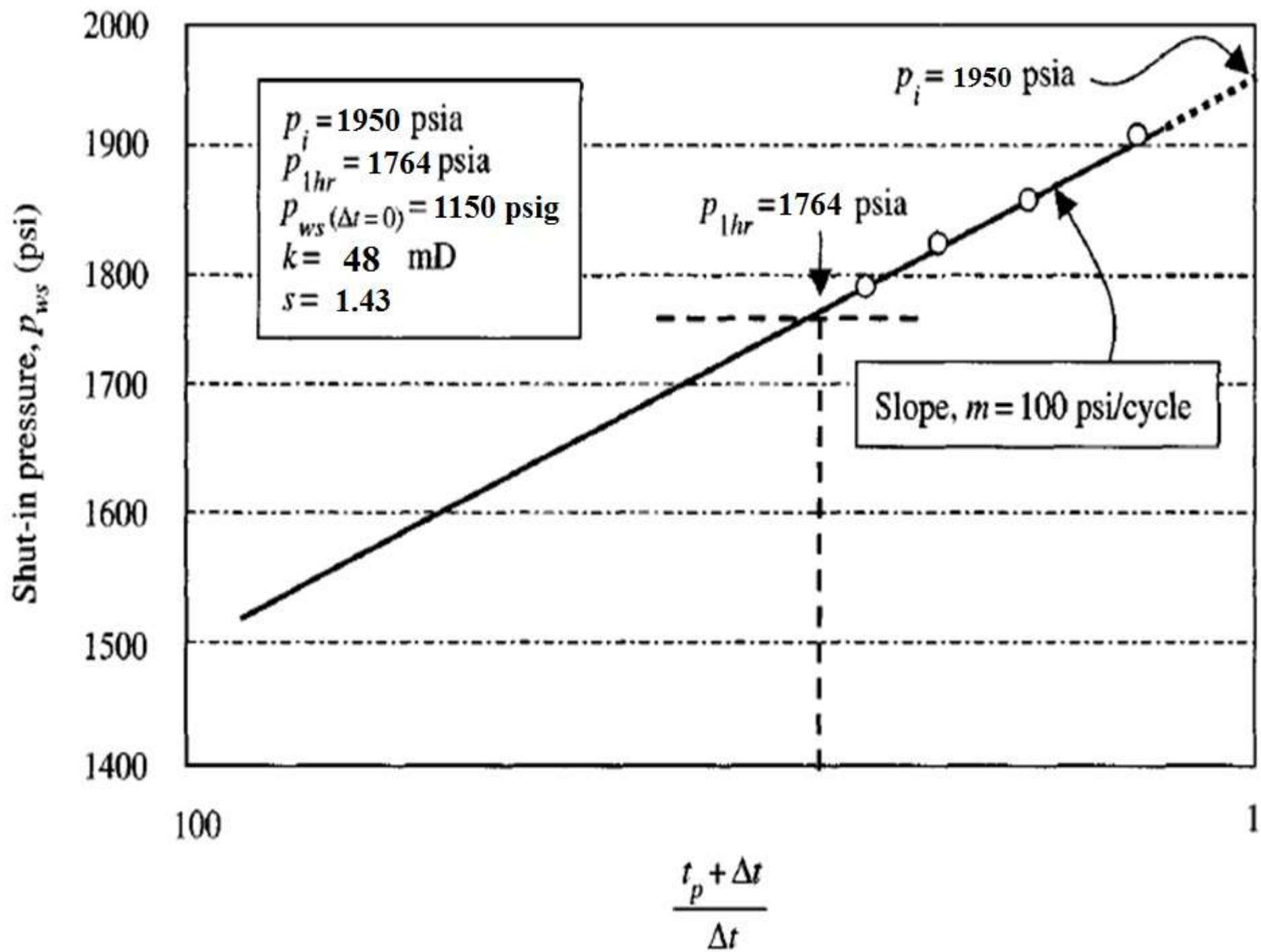
Solution

- ✓ Producing time, t_p , is given to be 3 days, or 72 hours thus, we develop following Table .

Δt (hours)	$\frac{t_p + \Delta t}{\Delta t}$	p_{ws} (psig)
2	37.0	1,794
4	19.0	1,823
8	10.0	1,850
16	5.5	1,876
24	4.0	1,890
48	2.5	1,910

- ✓ We plot these data, and they fall along a straight line suggested by ideal theory.
- ✓ The slope m of the straight line is $1,950 - 1,850 = 100$ psi (units are actually psi/cycle).

$$m = \tan \alpha = \frac{p_1 - p_2}{\log 10 - \log 1} = 100 : \left[\frac{\text{psi}}{\text{cycle}} \right]$$





$$k = 162.6 \frac{q \mu B}{m h} \rightarrow k = \frac{162.6 \times 500 \times 1.3 \times 1}{100 \times 22} \rightarrow k = 48 \text{ md}$$

✓ The skin factor s is found from

$$s = 1.151 \left[\frac{(P_{1hr} - P_{wf})}{m} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$

✓ The value for P_{ws} is P_{1hr} on the ideal straight line

at $(t_p + \Delta t) / \Delta t = (72 + 1) / 1 = 73$; this value is $P_{1hr} = 1,764$ psig. Thus,



$$s = 1.151 \left[\frac{(1764 - 1150)}{100} - \log \left(\frac{48}{(0.2)(1.0)(2 \times 10^{-5})(0.3)^2} \right) + 3.23 \right] = 1.43$$

✓ **From extrapolation of the buildup curve to $[(t_p + \Delta t) / t_p] = 1$, $P_i = 1950$ psig.**

Example 1.27^a Table 1.5 shows the pressure buildup data from an oil well with an estimated drainage radius of 2640 ft. Before shut-in, the well had produced at a stabilized rate of 4900 STB/day for 310 hours. Known reservoir data is:

depth = 10 476 ft, $r_w = 0.354$ ft, $c_t = 22.6 \times 10^{-6}$ psi⁻¹

$Q_o = 4900$ STB/D, $h = 482$ ft, $p_{wf}(\Delta t = 0) = 2761$ psig

$\mu_o = 0.20$ cp, $B_o = 1.55$ bbl/STB, $\phi = 0.09$

$t_p = 310$ hours, $r_e = 2640$ ft

Calculate:

- the average permeability k ;
- the skin factor;
- the additional pressure drop due to skin.

Δt (hr)	$t_p + \Delta t$ (hr)	$t_p + \Delta t \Delta t$	p_{ws} (psig)
0.0	—	—	2761
0.10	310.30	3101	3057
0.21	310.21	1477	3153
0.31	310.31	1001	3234
0.52	310.52	597	3249
0.63	310.63	493	3256
0.73	310.73	426	3260
0.84	310.84	370	3263
0.94	310.94	331	3266
1.05	311.05	296	3267
1.15	311.15	271	3268
1.36	311.36	229	3271
1.68	311.68	186	3274
1.99	311.99	157	3276
2.51	312.51	125	3280
3.04	313.04	103	3283
3.46	313.46	90.6	3286
4.08	314.08	77.0	3289
5.03	315.03	62.6	3293
5.97	315.97	52.9	3297
6.07	316.07	52.1	3297
7.01	317.01	45.2	3300
8.06	318.06	39.5	3303
9.00	319.00	35.4	3305
10.05	320.05	31.8	3306
13.09	323.09	24.7	3310
16.02	326.02	20.4	3313
20.00	330.00	16.5	3317
26.07	336.07	12.9	3320
31.03	341.03	11.0	3322
34.98	344.98	9.9	3323
37.54	347.54	9.3	3323

Solution

Step 1. Plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on a semilog scale as shown in Figure 1.38).

Step 2. Identify the correct straight-line portion of the curve and determine the slope m :

$$m = 40 \text{ psi/cycle}$$

Step 3. Calculate the average permeability by using Equation 1.3.8:

$$k = \frac{162.6 Q_o B_o \mu_o}{mh}$$

$$= \frac{(162.6)(4900)(1.55)(0.22)}{(40)(482)} = 12.8 \text{ md}$$

Step 4. Determine p_{wf} after 1 hour from the straight-line portion of the curve:

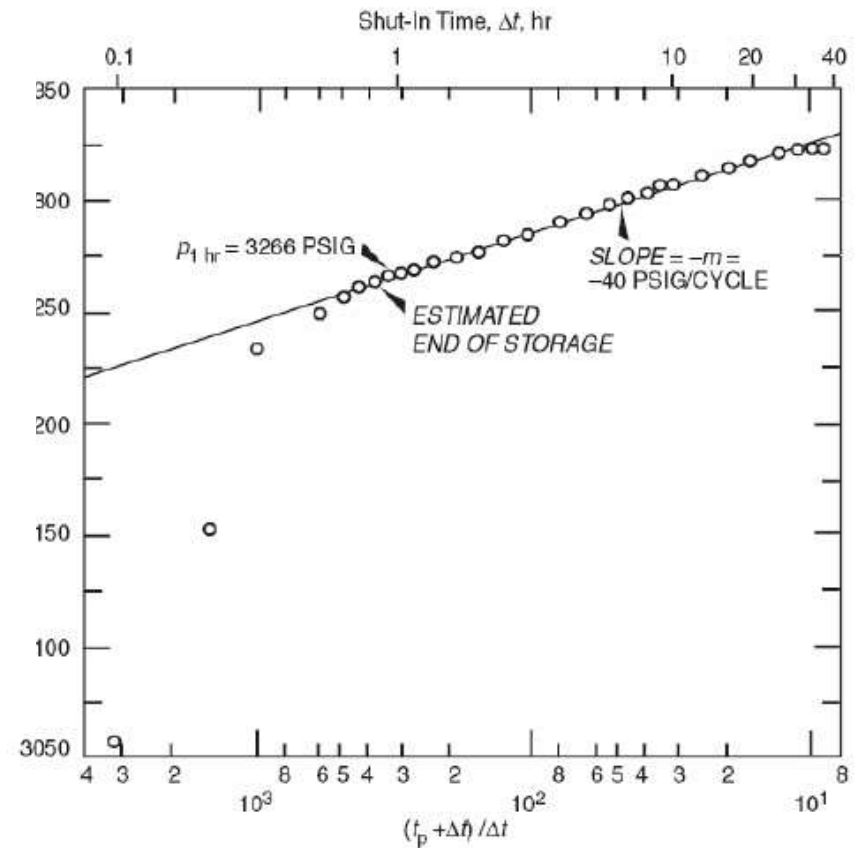
$$p_{1 \text{ hr}} = 3266 \text{ psi}$$

Step 5. Calculate the skin factor by applying Equation 1.3.9

$$s = 1.151 \left[\frac{p_{1 \text{ hr}} - p_{wf \Delta t=0}}{m} - \log \left(\frac{k}{\phi \mu c_l r_w^2} \right) + 3.23 \right]$$

$$= 1.151 \left[\frac{3266 - 2761}{40} - \log \left(\frac{(12.8)}{(0.09)(0.20)(22.6 \times 10^{-6})(0.354)^2} \right) + 3.23 \right]$$

$$= 8.6$$



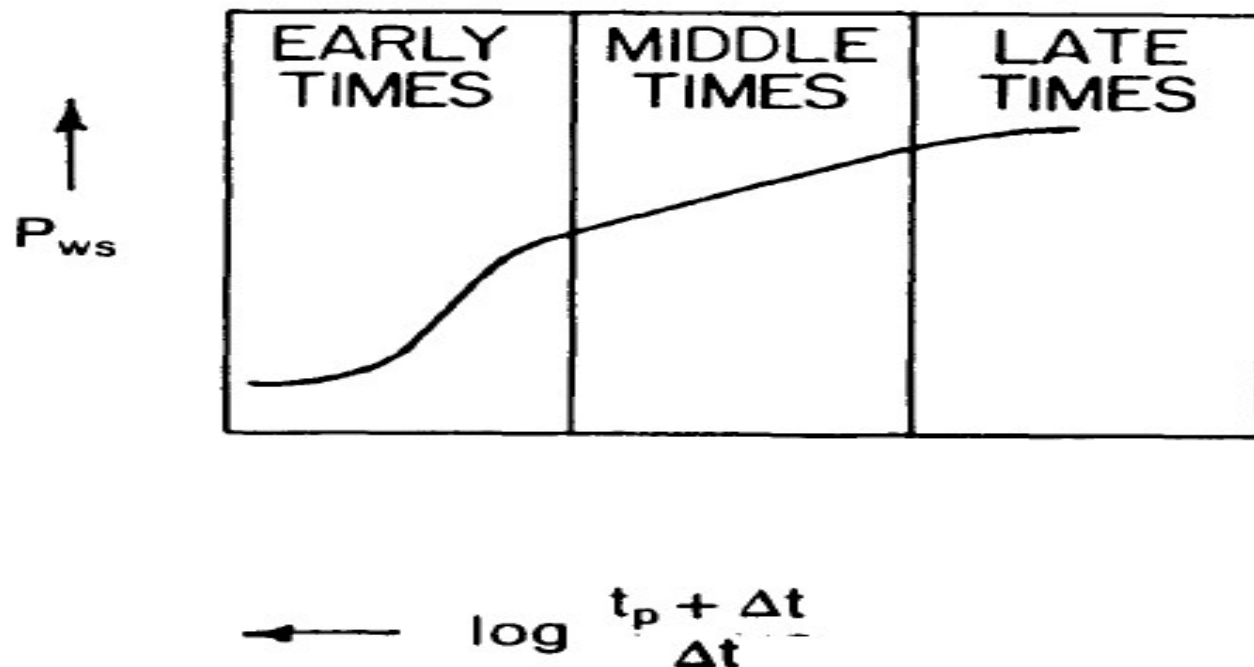
Step 6. Calculate the additional pressure drop by using:

$$\Delta p_{\text{skin}} = 0.87 |m| s$$

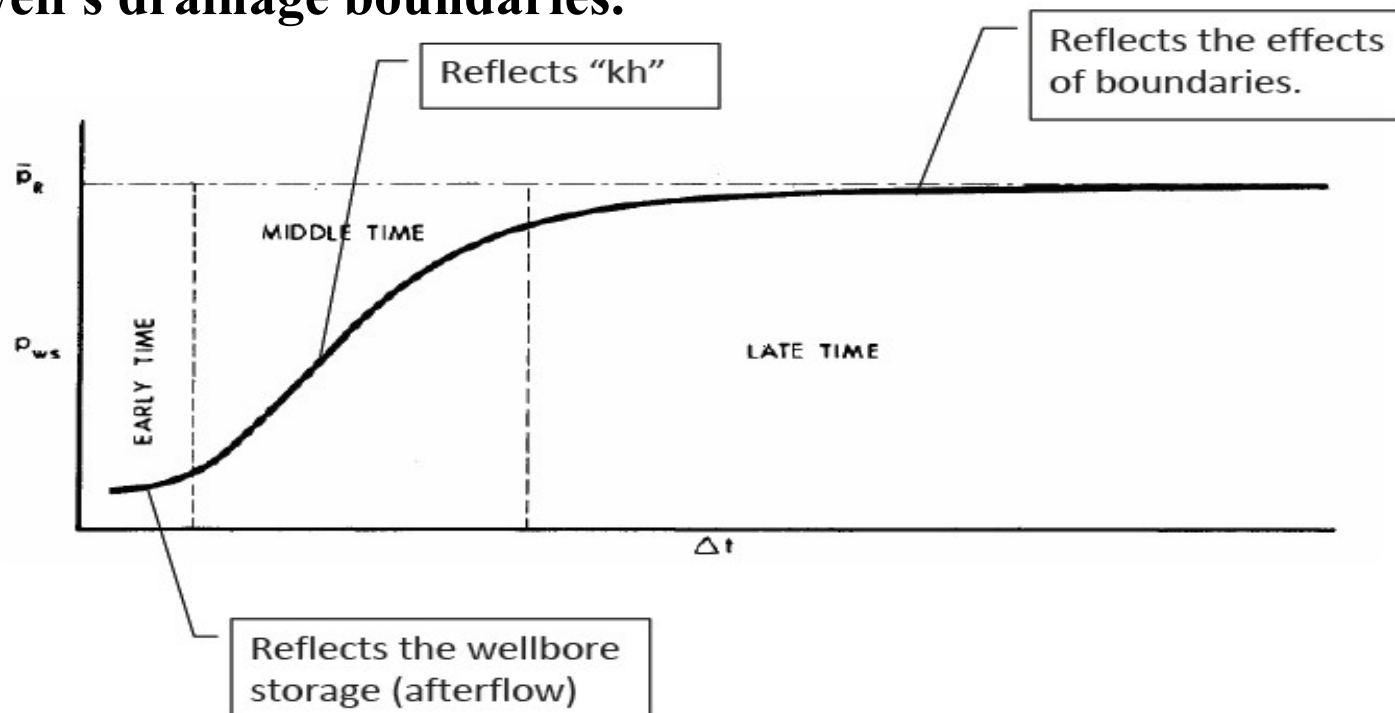
$$= 0.87(40)(8.6) = 299.3 \text{ psi}$$

Actual Buildup Tests

- In this case instead of a single straight line for all times, we obtain a curve with a complicated shape.
- Based on radius-of-investigation concept, we logically can divide a buildup curve into three regions :



- (1) **An early-time region** during which a pressure transient is moving through the formation nearest the well bore;
- (2) **A middle-time region** during which the pressure transient has moved away from the wellbore and into the bulk formation; and
- (3) **A late-time region**, in which the radius of investigation has reached the well's drainage boundaries.



Deviations From Assumptions in Ideal Test Theory

- 1. The infinite-reservoir assumption
- 2. The single-phase liquid assumption
- 3. The homogeneous reservoir assumption

1. The infinite-reservoir assumption

- Frequently, the reservoir is at pseudo steady-state before shut-in; if so, neither the Ei-function solution nor its logarithmic approximation should be used :

$$(P_i - P_{wf})_{\text{Prod well}} \neq -70.6 \frac{q \mu B}{kh} \left[\ln \left(\frac{1688 \phi \mu C_t r_w^2}{k (t_p + \Delta t)} \right) - 2S \right]$$



- Instead, if the well is centered in a cylindrical reservoir



- ✓ Thus, the Horner plot is incorrect when the reservoir is not infinite acting during the flow period preceding the buildup test.
- This difficulty is resolved in different ways by **different analysts**. In this course, we will use a method supported by the research of **Cobb** and **Smith**.

- 
- We will use the Horner plot for all tests (**even when the reservoir has reached pseudo steady-state during the production period preceding the test**) for the following reasons.

1. This method of plotting is correct theoretically for an infinite-acting reservoir (i.e., at time $t_p + \Delta t$, $r_i < r_e$).
2. The Horner plot offers a convenient means of extrapolating to $\Delta t \rightarrow \infty$ not found in some other plots.
3. For finite-acting reservoirs, formation permeability can be determined accurately at even greater shut-in times than from a plotting method developed specifically for reservoirs at pseudo steady state at shut-in.



- Other analysis methods for finite-acting reservoirs are discussed by **Miller, Dyes, and Hutchinson (MDH)** and **Slider**.
- Many analysts use the data plotting method suggested by MDH because **it is simpler than the Horner method**.
- Consider a buildup test with a middle-time region

$$P_{ws} = P_i - m \log \left[\frac{(t_p + \Delta t)}{\Delta t} \right]$$

$$P_{ws} = P_i - m \log(t_p + \Delta t) + m \log \Delta t$$



If $t_p \gg \Delta t$ during the range of shut-in time values, then

$$\log(t_p + \Delta t) \simeq \log t_p = \text{constant}$$

And

$$P_{ws} = \text{constant} + m \log \Delta t$$

- This leads to the plotting technique suggested by MDH: P_{ws} vs. $\log \Delta t$
- It has the same slope m as the Horner plot (in the time range of applicability).



2. The single-phase liquid assumption

- The assumption that a petroleum reservoir contains only a single-phase liquid must be modified.
- Even reservoirs in which only oil flows contain an immobile water saturation; many also contain an immobile gas saturation.
- These factors are taken into account if we use total compressibility, C_t

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_f$$



3. The homogeneous reservoir assumption

- **No reservoir is homogeneous, yet solutions to the flow equations are valid only for homogeneous reservoirs.**
- **The solutions prove to be adequate for most real reservoirs, particularly **early in time** while conditions nearest the tested well dominate test behavior.**
- **Modifications to the simple reservoir models have been developed for some important reservoir heterogeneities.**

Empirical relationships to verify the end of well bore storage distortion

$$t_D \cong C_{sD} e^{0.14s}$$

$$t_{wbs} \cong \frac{170000C_s e^{0.14s}}{\left(\frac{kh}{\mu} \right)}$$

➤ For a well bore containing only single-phase fluid(liquid or gas)
We define

$$\Delta t_e = \frac{\Delta t}{\left(1 + \frac{\Delta t}{t_p} \right)}$$

□ H.W) Read the Example 2.2 on page 29 john lee



Determination of Permeability

- Because **bulk-formation permeability** is obtained from the slope of the MTR line, correct selection of this region is critical.
- **Average permeability , k_J** , also can be estimated from information available in buildup tests.

Predicting the time at which the MTR ends is more difficult than predicting when it begins.

Basically, the middle-time line ends when the radius of investigation begins to detect drainage boundaries of the tested well; at this time, the pressure buildup curve begins to bend.



➤ The time at which the **middle region ends** depends on

- (1) The distance from the tested well to the reservoir boundaries
- (2) The geometry of the area drained by the well
- (3) The duration of the flow period as well as the shut-in period.

➤ If a well was at **pseudo steady-state before shut-in**, the time Δt at which the L TR begins for a well centered in a square or circular drainage area is approximately:

$$\Delta t_L \simeq \frac{380\phi\mu c_t A}{K}$$

A : the drainage area of the tested well **ft²**

✓ If the well was not at pseudo steady-state, Δt_L is larger than calculated by the rule above.

In many cases we simply assume that **the straight line spanning the times between** the end of after flow distortion and a later bend of the Horner plot constitutes the MTR.

- Average permeability, k_j from data obtained in a buildup test.
(is valid only if pseudo steady-state is reached during the production period)

$$k_j = \frac{141.2qB \mu \left[\ln \frac{r_e}{r_w} - \frac{3}{4} \right]}{h \left(\bar{P} - P_{wf} \right)}$$



For a well that is neither damaged nor stimulated $k_J = k$

For a damaged well $k_J < k$

For a stimulated well $k_J > k$

k : bulk-formation permeability, k , determined from the slope of the MTR

□ H.W) Read the Example 2.3 on page 30 john lee

Estimation of Effective (Apparent) Well bore Radius

$$r_{wa} = r_w e^{-S}$$

$$\begin{aligned} p_i - p_{wf} &= -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_w^2}{kt} \right) - 2s \right] \\ &= -70.6 \frac{qB\mu}{kh} \left| \ln \left(\frac{1,688 \phi \mu c_i r_w^2}{kt} \right) + \ln (e^{-2s}) \right| \\ &= -70.6 \frac{qB\mu}{kh} \left[\ln \left(\frac{1,688 \phi \mu c_i r_w^2 e^{-2s}}{kt} \right) \right] \\ &= -70.6 \frac{qB\mu}{kh} \ln \left(\frac{1,688 \phi \mu c_i r_{wa}^2}{kt} \right). \end{aligned}$$

- 
- Calculation of **effective well bore radius** is of special value for analyzing wells with **vertical fractures**.
 - Model studies have shown that for highly conductive vertical fractures with two equal-length wings of length L_f

$$L_f \simeq 2r_{wa}$$

- ✓ Thus, calculation of skin factor from a pressure buildup or falloff test can lead to an estimate of fracture length - useful in a post fracture analysis.



Productivity Index (PI or J) :

$$J = PI = \frac{Q}{P_i - P_{wf}} : \left[\frac{STBD}{psi} \right]$$

Specific Productivity Index (J_s) :

$$PI_s = J_s = \frac{PI}{h} = \frac{Q}{h \left(P_i - P_{wf} \right)}$$

Flow Efficiency (FE) = Productivity Ratio (PR) :

$$FE = PR = \frac{J_{act}}{J_{ideal}} = \frac{\left(\frac{Q}{P_i - P_{wf}} \right)_{act}}{\left(\frac{Q}{P_i - P_{wf}} \right)_{ideal}} = \frac{\left(P_i - P_{wf} \right)_{ideal}}{\left(P_i - P_{wf} \right)_{act}} = \frac{\left(\bar{P}_r - P_{wf} \right)_{act} - \Delta P_s}{\left(\bar{P}_r - P_{wf} \right)_{act}}$$

$$PR \approx \frac{\left(P^* - P_{wf} \right) - \Delta P_s}{\left(P^* - P_{wf} \right)}$$

- For a **damaged well**, flow efficiency is **less than one**; for a **stimulated well**, flow efficiency is **greater than one**.

Damage Ratio (DR):

$$DR = \frac{1}{FE} = \frac{J_{ideal}}{J_{act}} = \frac{\left(\bar{P}_r - P_{wf} \right)_{act}}{\left(\bar{P}_r - P_{wf} \right)_{act} - \Delta P_s}$$

Damage Factor (DF):

$$DF = 1 - FE = 1 - \frac{\left(\bar{P}_r - P_{wf} \right)_{act} - \Delta P_s}{\left(\bar{P}_r - P_{wf} \right)_{act}} = \frac{\Delta P_s}{\left(\bar{P}_r - P_{wf} \right)_{act}}$$

□ H.W) Read the Example 2.4 on page 32 john lee



Modifications for Gases

- Wattenbarger and Ramey have shown that **for some gases at pressures above 3,000 psi**, flow in **an infinite-acting reservoir** can be modeled accurately by the equation

$$P_{wf} = P_i + \frac{162.6q_g \mu_i B_{gi}}{kh} \left[\log \left(\frac{1688 \phi \mu_i C_{ti}}{kt_p} \right) - \frac{(S + Dq_g)}{1.151} \right]$$

- This equation has the same form as the equation for a slightly compressible liquid, but there are **some important differences**:



1) q_g is expressed in (Mscf / D), and B_g in (RB/Mscf), so the product $q_g B_g$ in (RB/D) as in the equation for slightly compressible liquids.

2) All gas properties (B_g , μ_g , and C_g) are evaluated at original reservoir pressure, P_i .

$$B_{gi} = \frac{178.1 z_i T p_{sc}}{p_i T_{sc}} \text{ (RB/Mscf)},$$

$$C_{fi} = C_{gi} S_g + C_w S_w + C_f \approx C_{gi} S_g.$$

3) The **factor D** is a measure of **non-Darcy or turbulent pressure loss** (i.e., a pressure drop in addition to that predicted by Darcy's law).

✓ **D** cannot be calculated separately from the skin factor from a single buildup or drawdown test; thus, the concept of **apparent skin factor**, $s' = s + Dq_g$ is sometimes convenient since it can be determined from a single test.

➤ For $p > 3000$ psi,

$$P_{ws} = P_i - \frac{162.6q_g \mu_g B_g}{kh} \left[\log \left(\frac{t_p + \Delta t}{\Delta t} \right) \right]$$

$$s' = s + D(q_g) = 1.151 \left[\frac{(P_{1hr} - P_{wf})}{m} - \log \left(\frac{k}{\phi \mu_i C_{ti} r_w^2} \right) + 3.23 \right]$$

➤ For $p < 2000$ psi,

$$P_{ws}^2 = P_i^2 - 1637 \frac{q_g \mu_i z_i T}{kh} \log \left(\frac{t_p + \Delta t}{\Delta t} \right)$$

$$s' = s + D(q_g) = 1.151 \left[\frac{(P_{1hr}^2 - P_{wf}^2)}{m''} - \log \left(\frac{k}{\phi \mu_i C_{ti} r_w^2} \right) + 3.23 \right]$$

✓ where m'' is the slope of the plot P_{ws}^2 vs. $\log [(t_p + \Delta t) / \Delta t]$

which is $1637 \frac{q_g \mu_i z_i T}{kh}$.



❑ what technique should be used to **analyze gas reservoirs** with pressures in the range **$2,000 < p < 3,000$ psi** ?

✓ One approach is to use equations written in terms of the gas pseudo pressure instead of either pressure or pressure squared.

➤ This is at least somewhat **inconvenient**, so **an alternative approach** is to use equations written in terms of either **P_{ws}** or **P_{ws}^2** and **accept the resultant inaccuracies**,

❑ Read example 2. 10- Gas Well Buildup Test Analysis on page 45

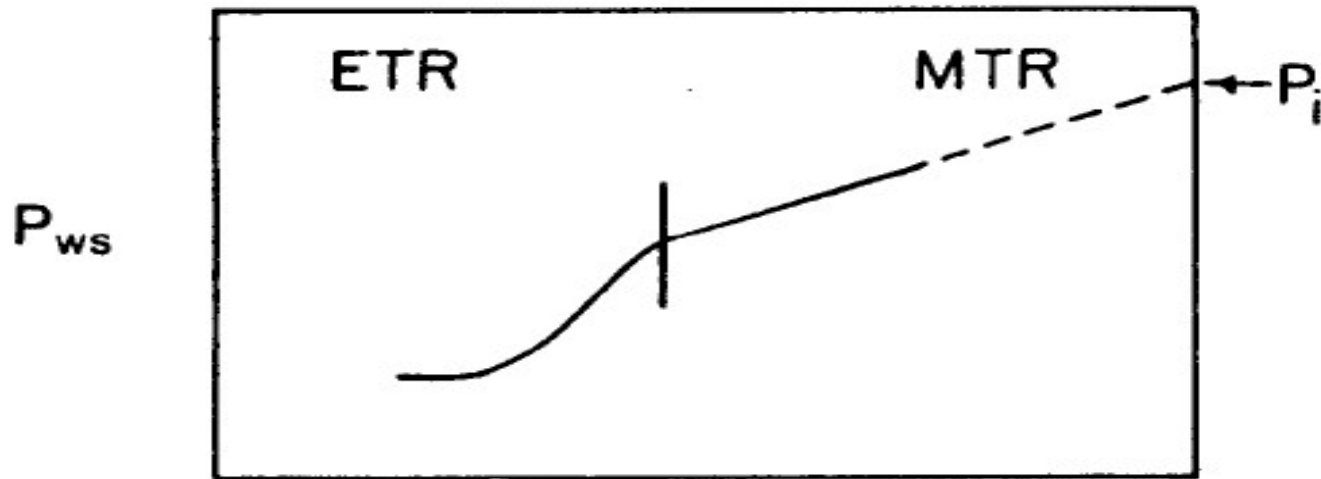
✓ Modifications for Multiphase Flow



Chapter 4

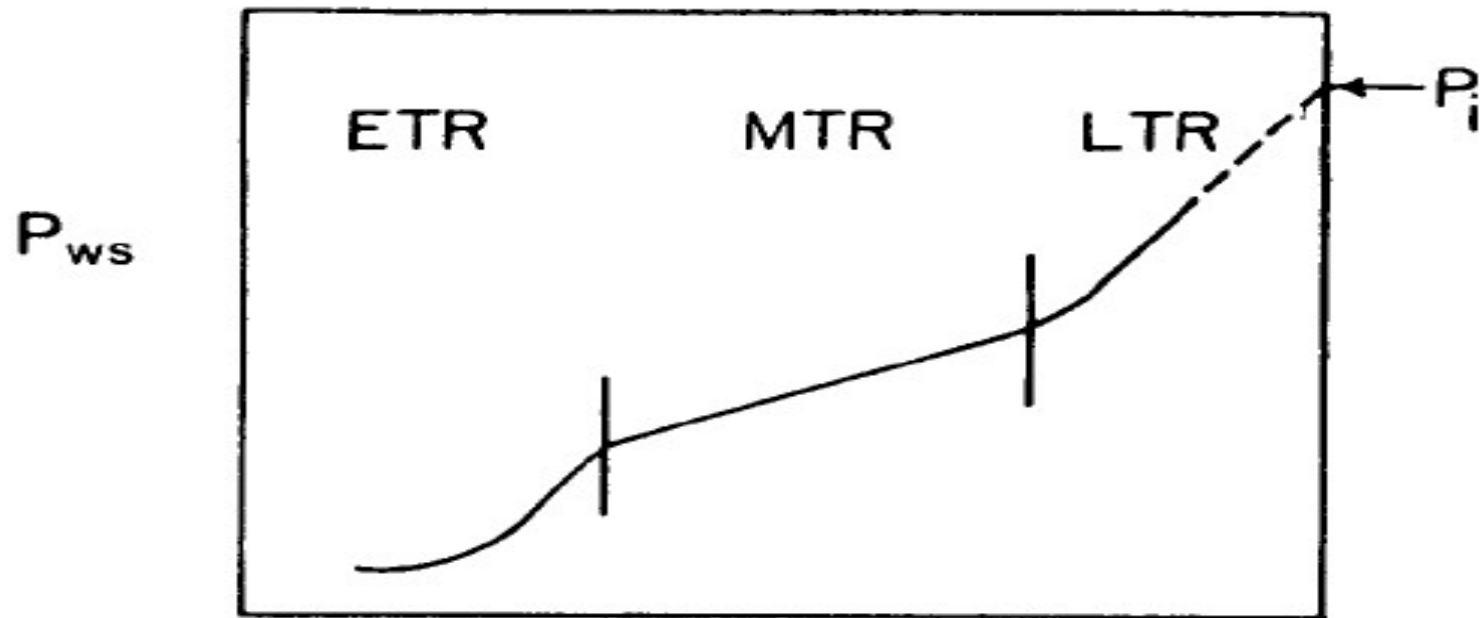
Average Reservoir Pressure

Original Reservoir Pressure



$$\log \frac{t_p + \Delta t}{\Delta t}$$

- This technique is possible only for a well in a new reservoir (ie .one in which there has been negligible pressure depletion).
- Strictly speaking, this is true only for tests in which the radius of investigation does not encounter any reservoir boundary during production.



$$\log \frac{t_p + \Delta t}{\Delta t}$$

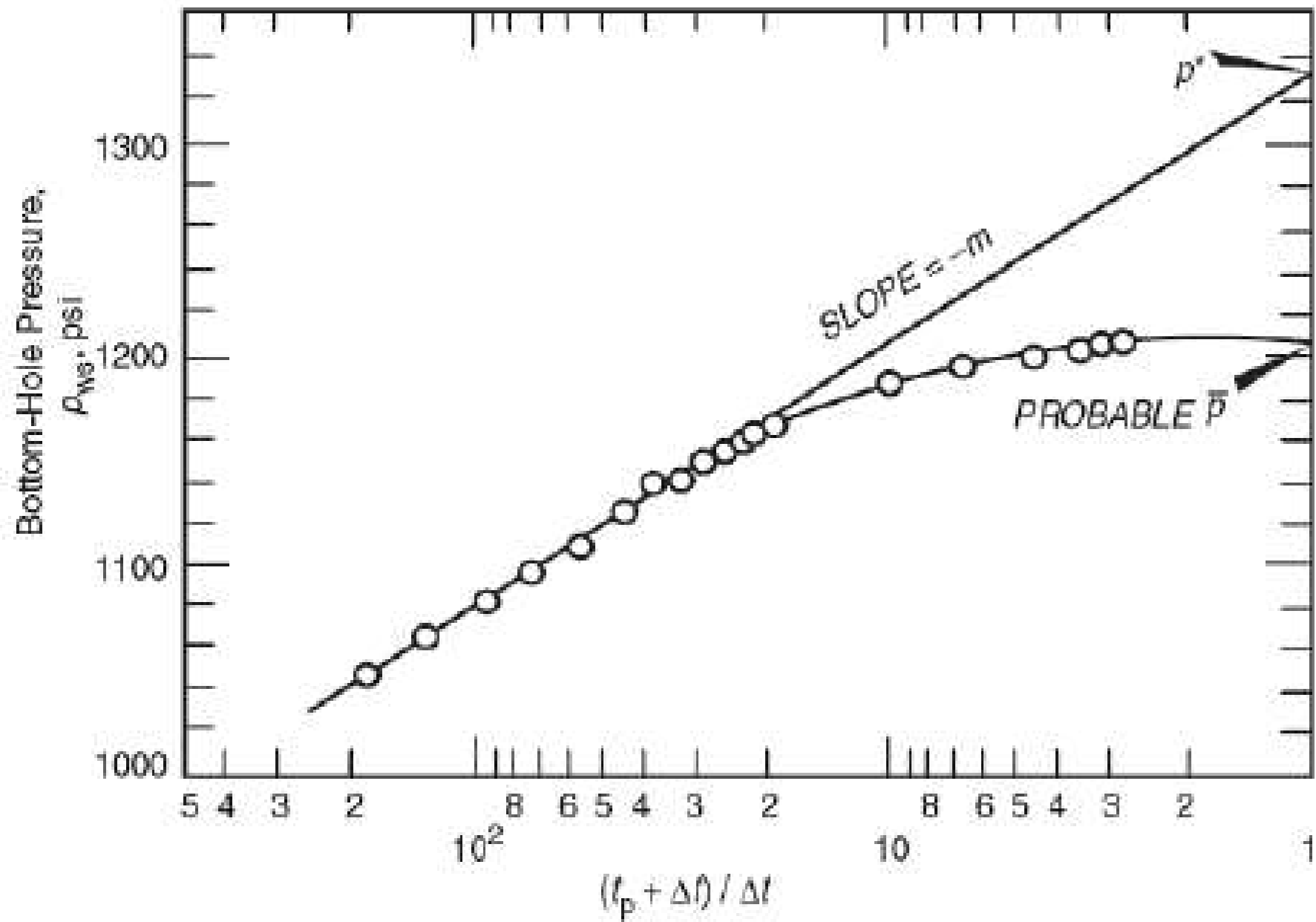
- For a reservoir with one or more boundaries relatively near a tested well the late-time line must be extrapolated

Note that our discussion is still restricted to reservoirs in which there has been negligible pressure depletion.



Static Drainage-Area Pressure

- For a well in a reservoir in which **there has been some pressure depletion**, we do not obtain an estimate of original reservoir pressure from extrapolation of a buildup curve.
- For wells with partial pressure depletion, extrapolation of a buildup test to infinite shut-in time provides an estimate of **p^*** , **which is related to, but is not equal to, current average drainage-area pressure.**
- Our usual objective is to estimate **the average pressure in the drainage area of the well**; we will call this **pressure static drainage-area pressure.**



Typical pressure buildup curve for a well in a finite reservoir

We will examine four useful methods for making these estimates:

- 1) The Mallhews-Brons-Hazebrook (MBH) p^* method**
- 2) The modified Muskat method.**
- 3) The Ramey–Cobb method**
- 4) The Dietz method**

1) the Mallhews-Brons-Hazebrook (MBH) p^* method

➤ In this method series of buildup curves were computed for wells at **various positions** in drainage areas of **various shapes** using imaging techniques and the principle of Superposition.

➤ The results of the investigation are summarized in a series of plots

of $\frac{kh(p^* - \bar{p})}{70.6q\mu B}$ vs. $\frac{0.000264kt_p}{\phi\mu c_t A}$.

$$\frac{kh(p^* - \bar{p})}{70.6q\mu B} = \frac{2.303(p^* - \bar{p})}{m}$$

$\frac{0.000264kt_p}{\phi\mu c_t A}$ is a dimensionless time and is symbolized by t_{DA}

$\frac{kh(p^* - \bar{p})}{70.6q\mu B}$ is a dimensionless pressure and is given the symbol P_{DMBH}

A: drainage area of the tested well expressed in square feet

- 
- To increase the accuracy of the p^* method use t_{pss} (producing time required to achieved pseudo steady state) in Horner plot and abscissa the MBH figures.
 - For calculation of producing time to achieved pseudo steady state t_{pss} we can use following relation

$$t_{pss} = \left[\frac{\phi \mu c_t A}{0.000264k} \right] \times (t_{DA})_{pss}$$



The following steps summarize the procedure for applying the MBH method:

Step 1. Make a Horner plot.

Step 2. Extrapolate the semilog straight line to the value of p^* at $(t_p + \Delta t) / \Delta t = 1.0$.

Step 3. Evaluate the slope of the semilog straight line m .

Step 4. Calculate the MBH dimensionless producing time t_{pDA} from Equation 1.3.14:

$$t_{pDA} = \left[\frac{0.0002637k}{\phi \mu c_t A} \right] t_p$$

Step 5. Find the closest approximation to the shape of the well drainage area in Figures 1.41 through 1.44 and identify the correction curve.

Step 6. Read the value of p_{DMBH} from the correction curve at t_{pDA}

Step 7. Calculate the value of $\bar{p}$ from Equation 1.3.13:

$$\bar{p} = p^* - \left(\frac{|m|}{2.303} \right) p_{DMBH}$$



where N_p is the cumulative volume produced since the *last pressure buildup test* and Q_o is the constant flow rate just before shut-in. Pinson (1972) and Kazemi (1974) indicate that t_p should be compared with the time required to reach the pseudosteady state, t_{pss} :

$$t_{pss} = \left[\frac{\phi \mu c_t A}{0.0002367k} \right] (t_{DA})_{pss} \quad [1.3.15]$$

For a symmetric closed or circular drainage area, $(t_{DA})_{pss} = 0.1$ as given in Table 1.4 and listed in the fifth column.

If $t_p \gg t_{pss}$, then t_{pss} should ideally replace t_p in both the Horner plot and for use with the MBH dimensionless pressure curves.

Table 1.4 Shape factors for various single-well drainage areas (After Earlougher, R. *Advances in Well Test Analysis*, permission to publish by the SPE, copyright SPE, 1977)

In bounded reservoirs	C_A	$\ln C_A$	$\frac{1}{2} \ln \left(\frac{2\pi k h}{C_A} \right)$	Exact for $t_{DA} >$	Less than 1% error for $t_{DA} >$	Use infinite system solution with less than 1% error for $t_{DA} >$
	31.62	3.4538	-1.3224	0.1	0.06	0.10
	31.6	3.4532	-1.3220	0.1	0.06	0.10
	27.6	3.3178	-1.2544	0.2	0.07	0.09
	27.1	3.2995	-1.2452	0.2	0.07	0.09
	21.9	3.0865	-1.1387	0.4	0.12	0.08
	0.098	-2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	-1.3106	0.1	0.05	0.09
	12.9851	2.5638	-0.8774	0.7	0.25	0.03
	10132	1.5070	-0.3490	0.6	0.30	0.025
	3.3351	1.2045	-0.1977	0.7	0.25	0.01
	21.8369	3.0836	-1.1373	0.3	0.15	0.025
	10.8374	2.3830	-0.7870	0.4	0.15	0.025
	10141	1.5072	-0.3491	1.5	0.50	0.06
	2.0769	0.7309	-0.0391	1.7	0.50	0.02
	3.1573	1.1497	-0.1708	0.4	0.15	0.005
	0.5813	-0.5425	+0.6758	2.0	0.60	0.02
	0.1109	-2.1991	+1.5041	3.0	0.60	0.005
	5.3790	1.6825	-0.4367	0.8	0.30	0.01
	2.6896	0.9894	-0.0902	0.8	0.30	0.01
	0.2318	-1.4619	+1.1355	4.0	2.00	0.03
	0.1155	-2.1585	+1.4838	4.0	2.00	0.01
	2.3606	0.8589	-0.0249	1.0	0.40	0.025
In vertically fractured reservoirs use $(k_e/x_f)^2$ in place of A/r_w^2 for fractured systems						
	2.6541	0.9761	-0.0835	0.175	0.08	cannot use
	2.0348	0.7104	+0.0493	0.175	0.09	cannot use
	1.9986	0.6924	+0.0583	0.175	0.09	cannot use
	1.6620	0.5080	+0.1505	0.175	0.09	cannot use
	1.3127	0.2721	+0.2685	0.175	0.09	cannot use
	0.7887	-0.2374	+0.5232	0.175	0.09	cannot use
In reservoirs of unknown production character						
	19.1	2.95	-1.07	-	-	-
	25.0	3.22	-1.20	-	-	-

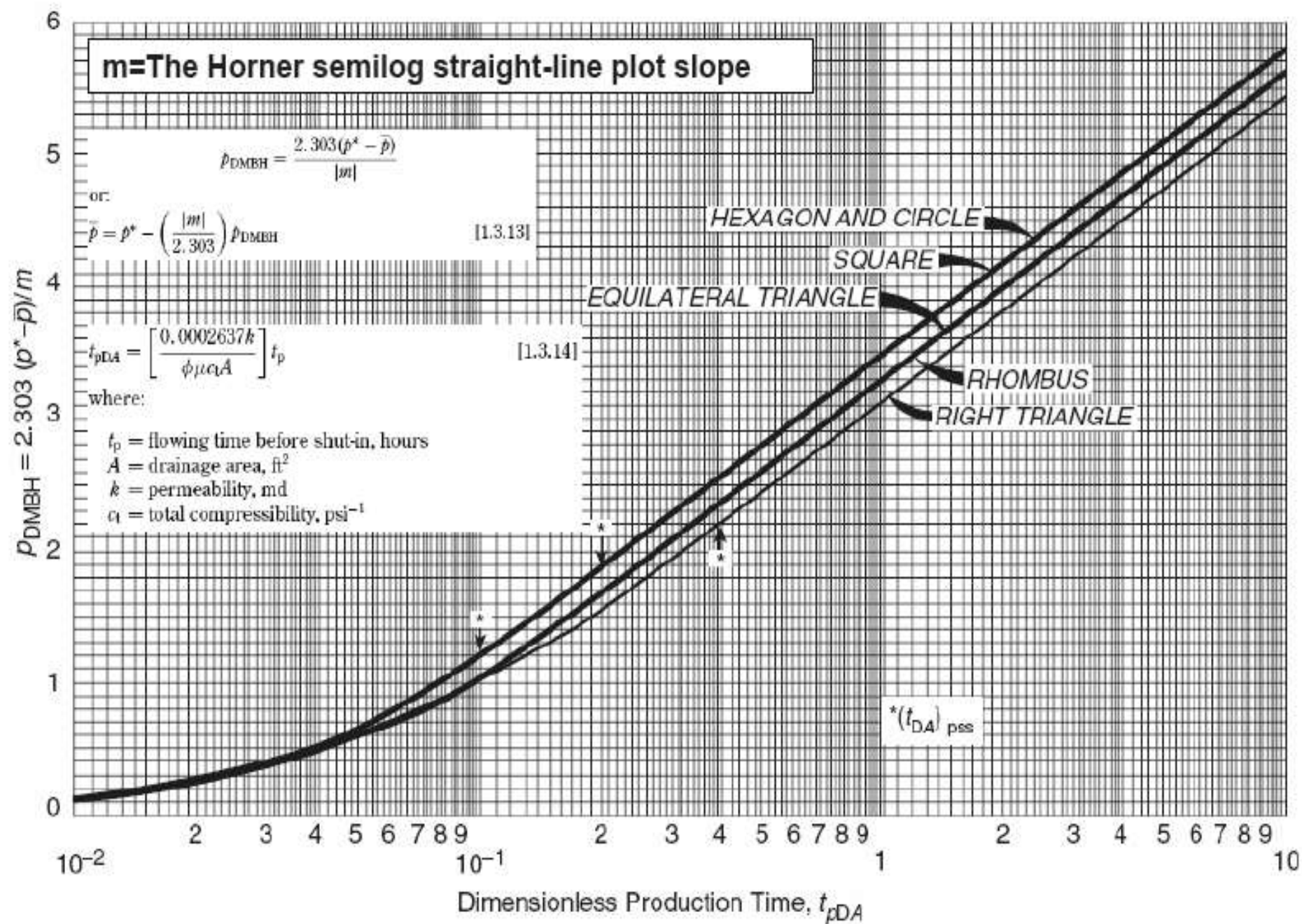


Figure 1.42 Matthews-Brons-Hazebroek dimensionless pressure for a well in the center of equilateral drainage areas

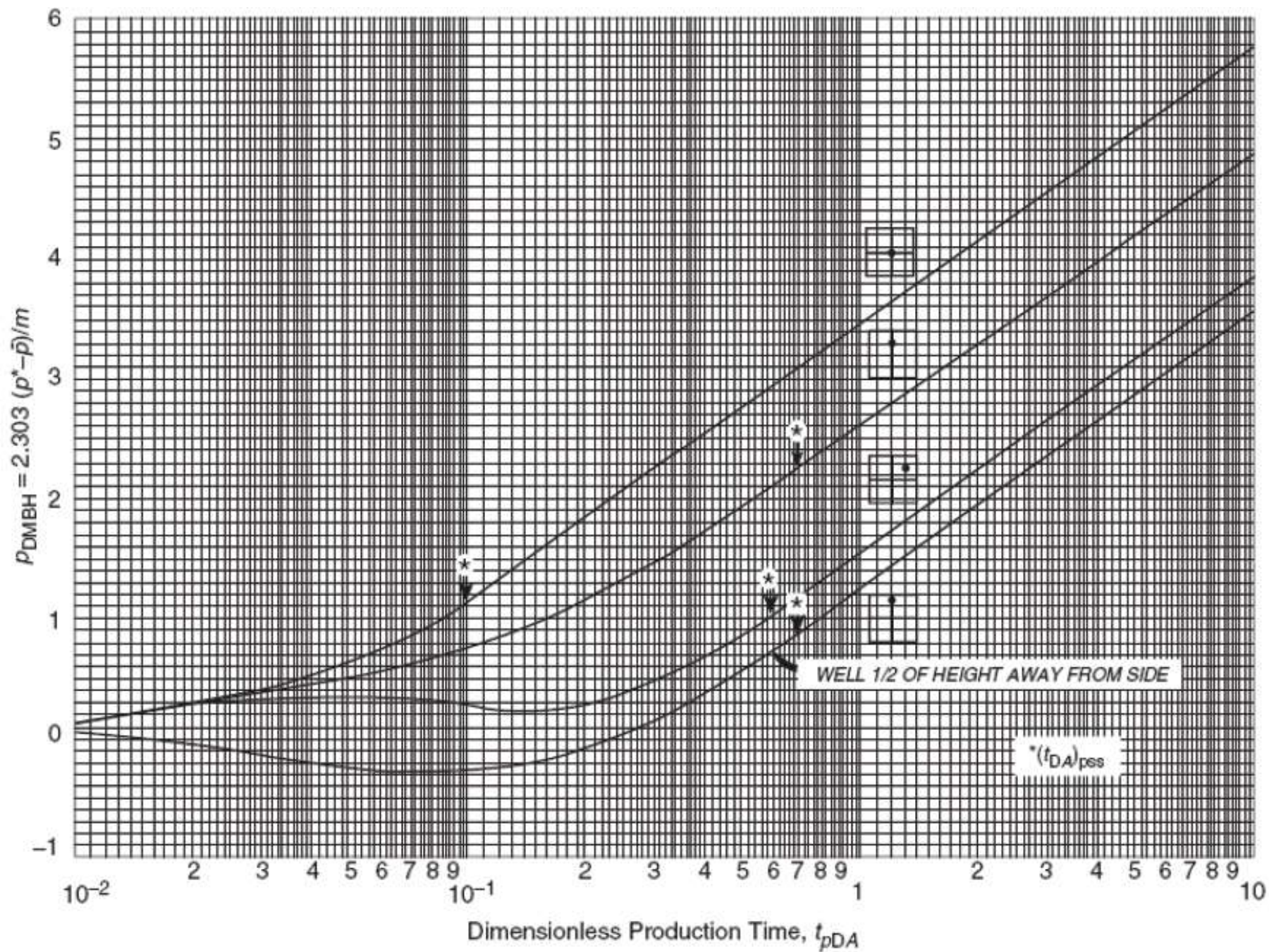


Figure 1.43 Matthews–Brons–Hazebroek dimensionless pressure for different well locations in a square drainage

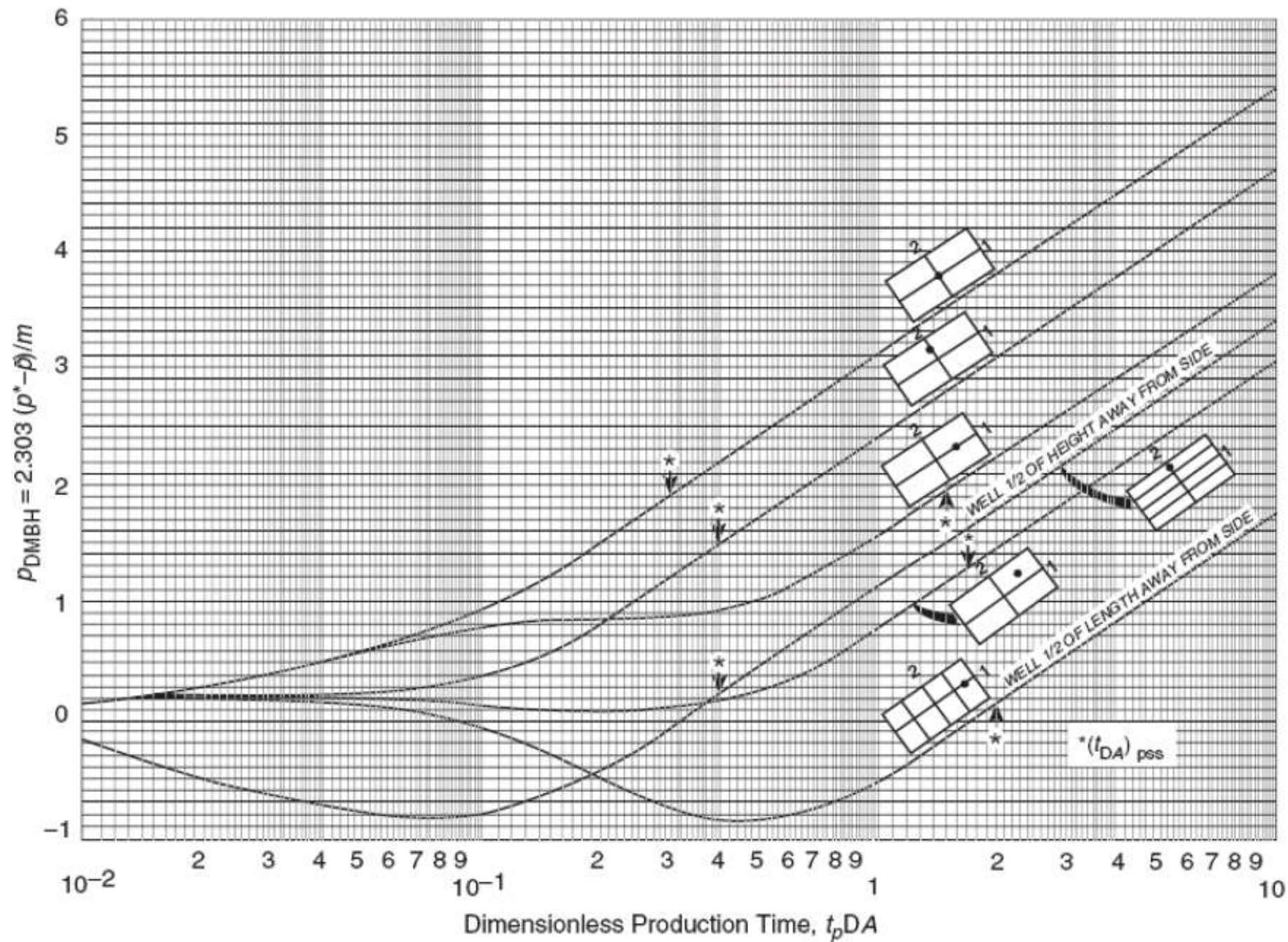
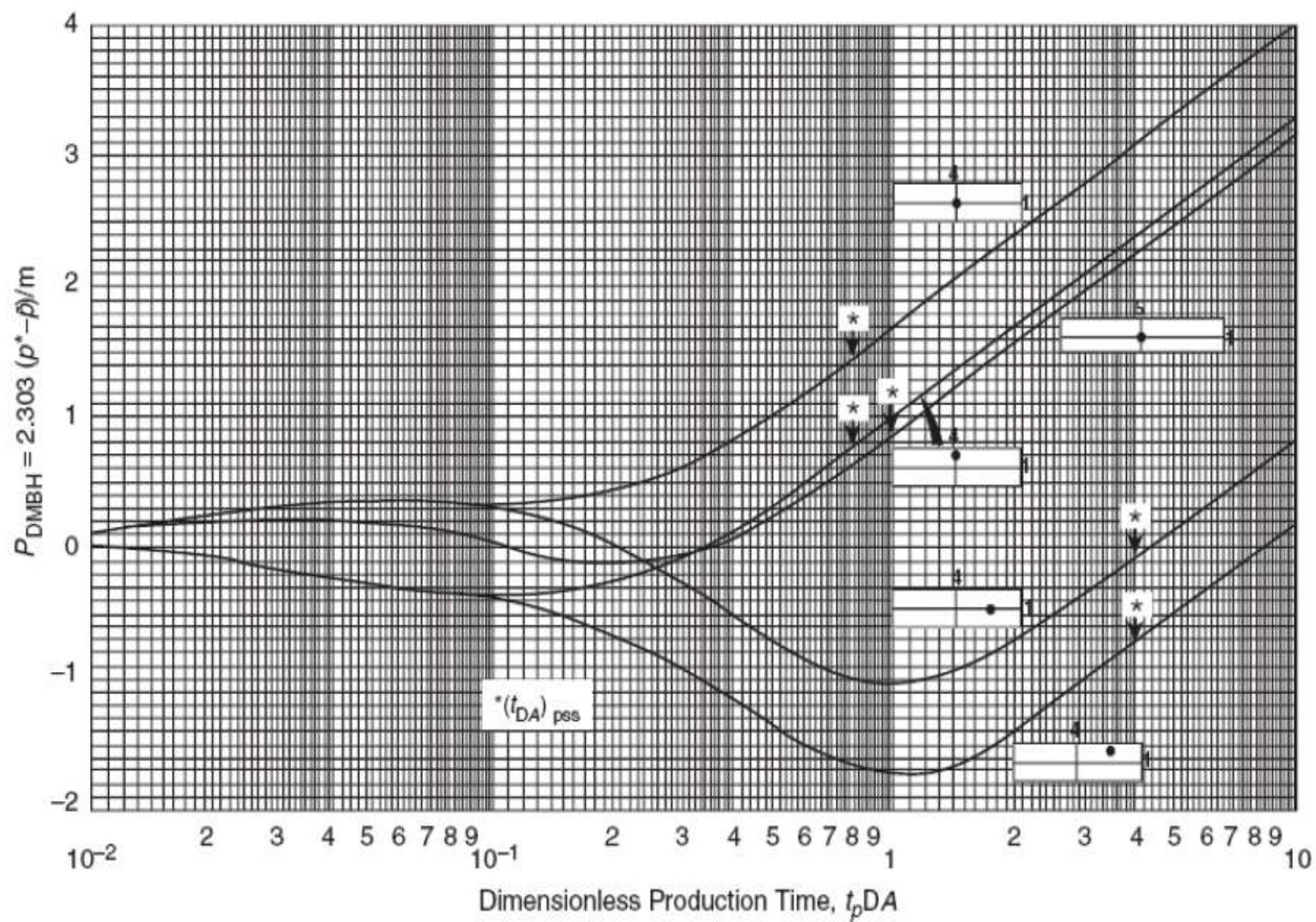


Figure 1.44 Matthews-Brons-Hazebroek dimensionless pressure for different well locations in a 2:1 rectangular



Example 1.28 Using the information given in Example 1.27 and pressure buildup data listed in Table 1.5, calculate the average pressure in the well drainage area and the drainage area by applying Equation 1.3.11. The data is listed below for convenience:

$$r_e = 2640 \text{ ft}, \quad r_w = 0.354 \text{ ft}, \quad c_t = 22.6 \times 10^{-6} \text{ psi}^{-1}$$

$$Q_o = 4,900 \text{ STB/D}, \quad k = 482 \text{ ft},$$

$$p_{wf \text{ at } \Delta t=0} = 2761 \text{ psig}$$

$$\mu_o = 0.20 \text{ cp}, \quad B_o = 1.55 \text{ bbl/STB}, \quad \phi = 0.09$$

$$t_p = 310 \text{ hours}, \quad \text{depth} = 10,476 \text{ ft},$$

$$\text{reported average pressure} = 3323 \text{ psi}$$

Table 1.5 Earlougher's pressure buildup data
(Permission to publish by the SPE, copyright
SPE, 1977.)

Δt (hr)	$t_p + \Delta t$ (hr)	$t_p + \Delta t \Delta t$	p_{ws} (psig)
0.0	—	—	2761
0.10	310.30	3101	3057
0.21	310.21	1477	3153
0.31	310.31	1001	3234
0.52	310.52	597	3249
0.63	310.63	493	3256
0.73	310.73	426	3260
0.84	310.84	370	3263
0.94	310.94	331	3266
1.05	311.05	296	3267
1.15	311.15	271	3268
1.36	311.36	229	3271
1.68	311.68	186	3274
1.99	311.99	157	3276
2.51	312.51	125	3280
3.04	313.04	103	3283
3.46	313.46	90.6	3286
4.08	314.08	77.0	3289
5.03	315.03	62.6	3293
5.97	315.97	52.9	3297
6.07	316.07	52.1	3297
7.01	317.01	45.2	3300
8.06	318.06	39.5	3303
9.00	319.00	35.4	3305
10.05	320.05	31.8	3306
13.09	323.09	24.7	3310
16.02	326.02	20.4	3313
20.00	330.00	16.5	3317
26.07	336.07	12.9	3320
31.03	341.03	11.0	3322
34.98	344.98	9.9	3323
37.54	347.54	9.3	3323

Solution

Step 1. Calculate the drainage area of the well:

$$A = \pi r_e^2 = \pi (2640)^2$$

Step 2. Compare the production time t_p , i.e., 310 hours, with the time required to reach the pseudosteady state t_{pss} by applying Equation 1.3.15. Estimate t_{pss} using $(t_{DA})_{pss} = 0.1$ to give:

$$\begin{aligned} t_{pss} &= \left[\frac{\phi \mu c_t A}{0.0002367k} \right] (t_{DA})_{pss} \\ &= \left[\frac{(0.09)(0.2)(22.6 \times 10^{-6})(\pi)(2640)^2}{(0.0002637)(12.8)} \right] 0.1 \\ &= 264 \text{ hours} \end{aligned}$$

Thus, we could replace t_p by 264 hours in our analysis because $t_p > t_{pss}$. However, since t_p is only about $1.2t_{pss}$, we use the actual production time of 310 hours in the calculation.

Step 3. Figure 1.38 does not show p^* since the semilog straight line is not extended to $(t_p + \Delta t)/\Delta t = 1.0$. However, p^* can be calculated from p_{ws} at $(t_p + \Delta t)/\Delta t = 10.0$ by extrapolating one cycle. That is:

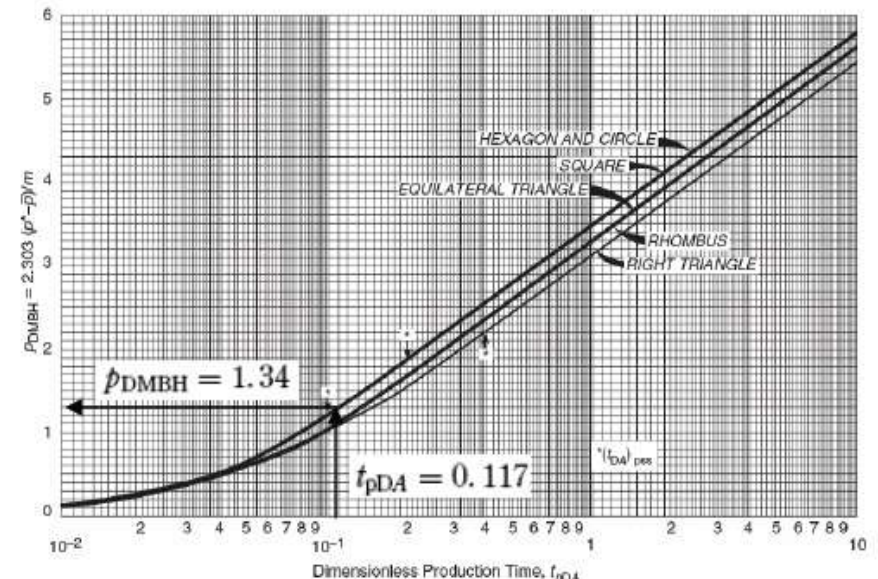
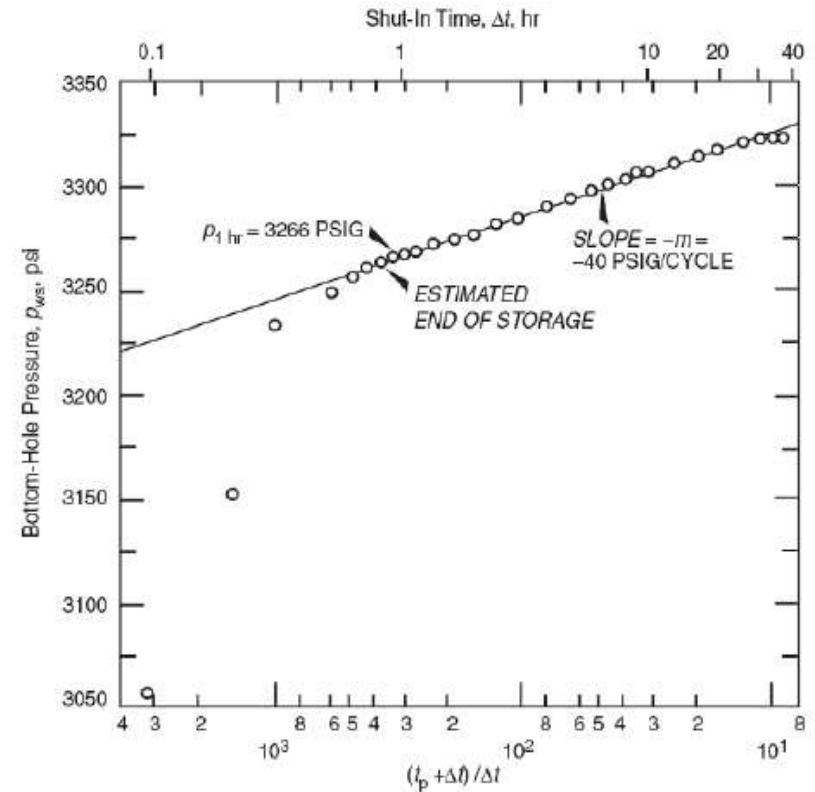
$$p^* = 3325 + (1 \text{ cycle})(40 \text{ psi/cycle}) = 3365 \text{ psig}$$

Step 4. Calculate t_{pDA} by applying Equation 1.3.14 to give:

$$\begin{aligned} t_{pDA} &= \left[\frac{0.0002637k}{\phi \mu c_t A} \right] t_p \\ &= \left[\frac{0.0002637(12.8)}{(0.09)(0.2)(22.6 \times 10^{-6})(\pi)(2640)^2} \right] 310 \\ &= 0.117 \end{aligned}$$

Step 5. From the curve of the circle in Figure 1.42, obtain the value of p_{DMBH} at $t_{pDA} = 0.117$, to give:

$$p_{DMBH} = 1.34$$





2) The modified Muskat method.

- The modified Muskat method is based on solution to the flow equations for a well producing from **a closed, cylindrical reservoir at constant rate.**
- Using superposition to simulate a buildup following stabilized flow (**depth of investigation has reached reservoir boundaries**), the equation can be approximated as

$$\bar{p} - p_{ws} = 118.6 \frac{q \mu B}{kh} \exp\left(\frac{-0.00388k \Delta t}{\phi \mu c_t r_e^2}\right)$$

$$\log(\bar{p} - p_{ws}) = \log\left(118.6 \frac{q \mu B}{kh}\right) - \frac{0.00168 k \Delta t}{\phi \mu c_t r_e^2}$$

✓ Note that above equation has the form

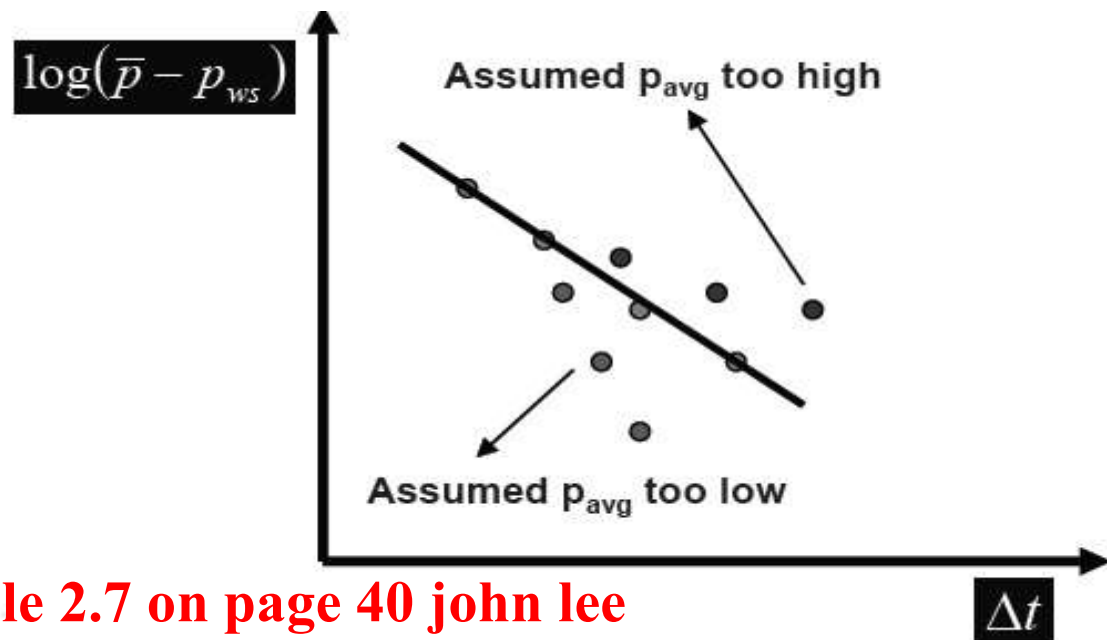
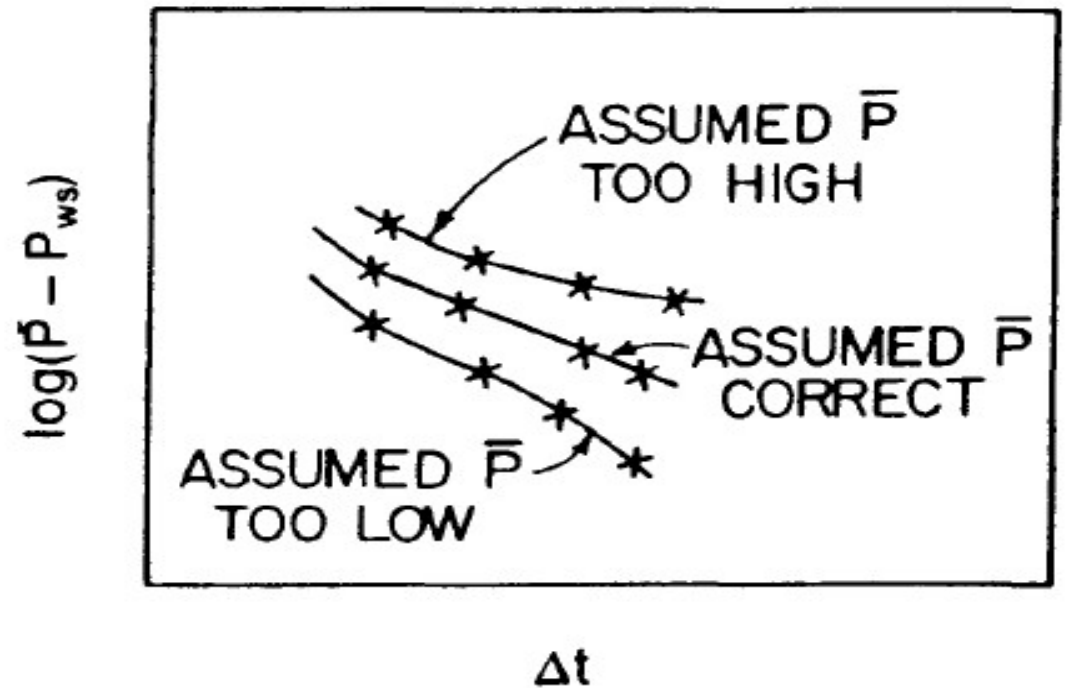
$$\log(\bar{p} - p_{ws}) = A + B \Delta t$$

✓ where A and B are constants. ✓ $\log(\bar{p} - p_{ws})$ versus Δt is linear

➤ Approximations used in developing this equation are valid in the shut-in time range.

$$\frac{250 \phi \mu c_t r_e^2}{k} \leq \Delta t \leq \frac{750 \phi \mu c_t r_e^2}{k}$$

1. Assume a value for p^- .
2. Plot $\log(p^- - p_{ws})$ versus Δt
3. Is it a straight line?
4. If the answer is yes, the assumed value is the average reservoir pressure otherwise go to 1.



□ H.W) Read the Example 2.7 on page 40 john lee



Advantages

- 1. It requires no estimate no estimates of reservoir properties when it is used to establish P_{avg} .**
- 2. It provide satisfactory estimates of P_{avg} for hydraulically fractured wells and layered reservoirs.**

Disadvantages

- 1. It fails when the tested well is not reasonably centered in its drainage area.**
- 2. The required shut-in times are frequently impractically long, particularly in low permeability reservoirs.**



3)The Ramey–Cobb method

Ramey and Cobb (1971) proposed that the average pressure in the well drainage area can be read directly from the Horner semi log straight line if the following data is available:

- **shape of the well drainage area;**
- **location of the well within the drainage area;**
- **size of the drainage area.**

The proposed methodology is based on calculating the dimensionless producing time t_{pDA} as defined by Equation 1.3.14:

$$t_{pDA} = \left[\frac{0.0002637k}{\phi \mu c_t A} \right] t_p$$

where:

t_p = producing time since the last shut-in, hours
 A = drainage area, ft²

Knowing the shape of the drainage area and well location, determine the dimensionless time to reach pseudosteady state $(t_{DA})_{pss}$, as given in Table 1.4 in the fifth column. Compare t_{pDA} with $(t_{DA})_{pss}$:

- If $t_{pDA} < (t_{DA})_{pss}$, then read the average pressure $\bar{p}$ from the Horner semilog straight line at:

$$\left(\frac{t_p + \Delta t}{\Delta t} \right) = \exp(4\pi t_{pDA}) \quad [1.3.18]$$

or use the following expression to estimate $\bar{p}$:

$$\bar{p} = p^* - m \log[\exp(4\pi t_{pDA})] \quad [1.3.19]$$

- If $t_{pDA} > (t_{DA})_{pss}$, then read the average pressure $\bar{p}$ from the Horner semilog straight-line plot at:

$$\left(\frac{t_p + \Delta t}{\Delta t} \right) = C_A t_{pDA} \quad [1.3.20]$$

where C_A is the shape factor as determined from Table 1.4.s Equivalently, the average pressure can be estimated from:

$$\bar{p} = p^* - m \log(C_A t_{pDA}) \quad [1.3.21]$$

where:

m = absolute value of the semilog straight-line slope,
 psi/cycle
 p^* = false pressure, psia
 C_A = shape factor, from Table 1.4

Example 1.29 Using the data given in Example 1.27, recalculate the average pressure using the Ramey and Cobb method.

Solution

Step 1. Calculate t_{pDA} by applying Equation (1.3.14):

$$\begin{aligned} t_{pDA} &= \left[\frac{0.0002637k}{\phi \mu c_t A} \right] t_p \\ &= \left[\frac{0.0002637(12.8)}{(0.09)(0.2)(22.6 \times 10^{-6})(\pi)(2640)^2} \right] (310) \\ &= 0.1175 \end{aligned}$$

Step 2. Determine C_A and $(t_{DA})_{pss}$ from Table 1.4 for a well located in the centre of a circle, to give:

$$\begin{aligned} C_A &= 31.62 \\ (t_{DA})_{pss} &= 0.1 \end{aligned}$$

Step 3. Since $t_{pDA} > (t_{DA})_{pss}$, calculate $\bar{p}$ from Equation 1.3.21:

$$\begin{aligned} \bar{p} &= p^* - m \log(C_A t_{pDA}) \\ &= 3365 - 40 \log[31.62(0.1175)] = 3342 \text{ psi} \end{aligned}$$

This value is identical to that obtained from the MBH method.

4) The Dietz method

- Dietz (1965) indicated that if the test well has been producing long enough to reach the pseudo steady state before shut-in, the average pressure can be read directly from the MDH semilog straight-line plot, i.e.,

P_{ws} vs. $\log(t)$, at the following shut-in time:

$$(\Delta t)_{\bar{p}} = \frac{\phi \mu c_t A}{0.0002637 C_A k}$$

where:

Δt = shut-in time, hours

A = drainage area, ft^2

C_A = shape factor

k = permeability, md

c_t = total compressibility, psi^{-1}

Example 1.30 Using the Dietz method and the buildup data given in Example 1.27, calculate the average pressure:

Solution

Step 1. Using the buildup data given in Table 1.5, construct the MDH plot of p_{ws} vs. $\log(\Delta t)$ as shown in Figure 1.40. From the plot, read the following values:

$$m = 40 \text{ psi/cycle}$$

$$p_{1 \text{ hr}} = 3266 \text{ psig}$$

Step 2. Calculate false pressure p^* from Equation 1.3.12 to give:

$$p^* = p_{1 \text{ hr}} + m \log(t_p + 1)$$

$$= 3266 + 40 \log(310 + 1) = 3365.7 \text{ psi}$$

Step 3. Calculate the shut-in time $(\Delta t)_{\bar{p}}$ from Equation 1.3.20:

$$(\Delta t)_{\bar{p}} = \frac{(0.09)(0.2)(22.6 \times 10^{-6})(\pi)(2640)^2}{(0.0002637)(12.8)(31.62)}$$

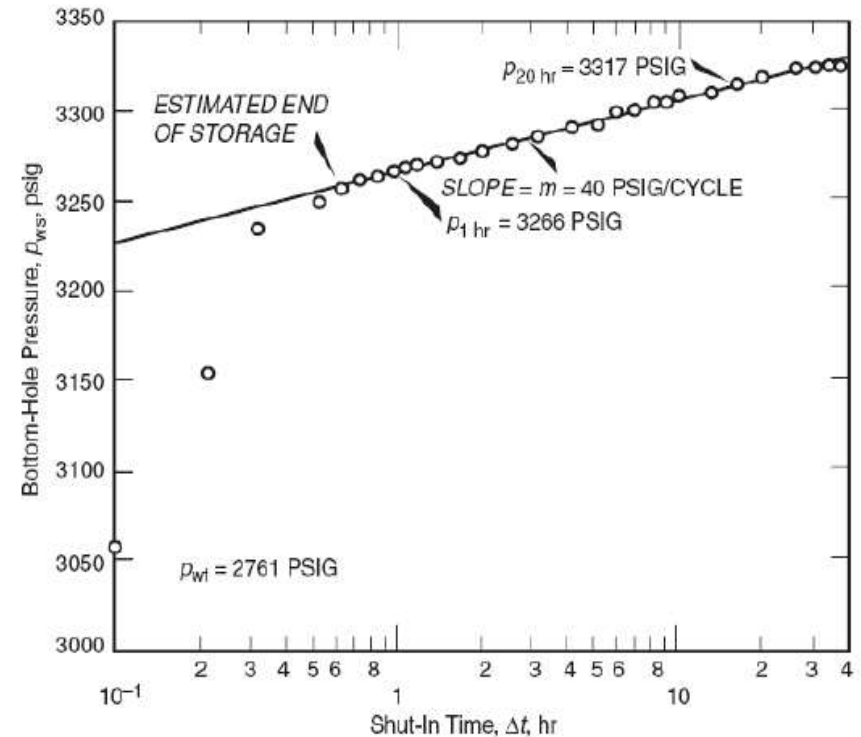
$$= 83.5 \text{ hours}$$

Step 4. Since the MDH plot does not extend to 83.5 hours, the average pressure can be calculated from the semilog straight-line equation as given by:

$$\bar{p} = p_{1 \text{ hr}} + m \log(\Delta t - 1) \quad [1.3.23]$$

or:

$$\bar{p} = 3266 + 40 \log(83.5 - 1) = 3343 \text{ psi}$$





Chapter 5

Flow Tests



- **A pressure drawdown test** is conducted by producing a well, starting ideally with uniform pressure in the reservoir.
- Rate and pressure are recorded as functions of time.

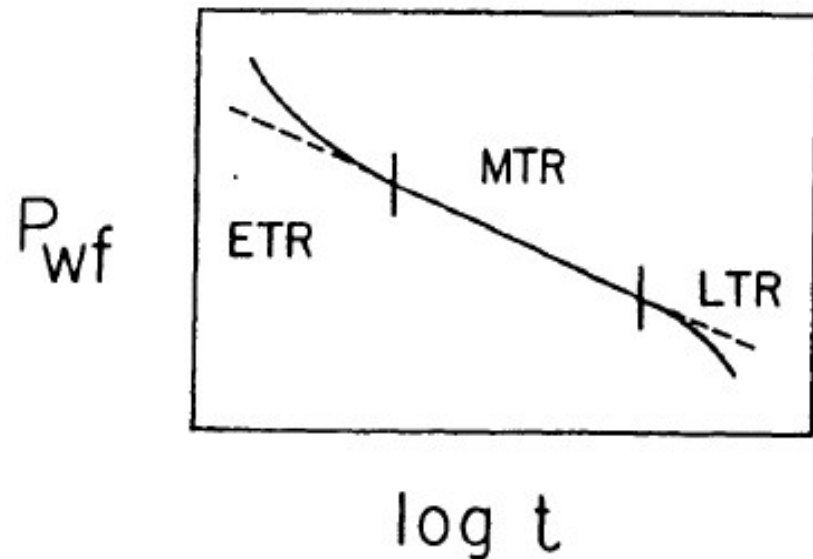
These tests are particularly applicable to

- (1) New wells**
- (2) Wells that have been shut in sufficiently long to allow the pressure to stabilize**
- (3) Wells in which loss of revenue incurred in a buildup test would be difficult to accept**

- **An idealized constant-rate drawdown test** in an infinite-acting reservoir is modeled by the logarithmic approximation to the Ei-function solution:

$$P_{wf} = P_i + \frac{162.6q \mu B}{kh} \left[\log \left(\frac{1688 \phi \mu C_t r_w^2}{kt} \right) - 0.0869s \right]$$

- Like buildup tests, drawdown tests are more complex than suggested by simple equations.
- The usual test has an **ETR**, an **MTR**, and an **LTR**.

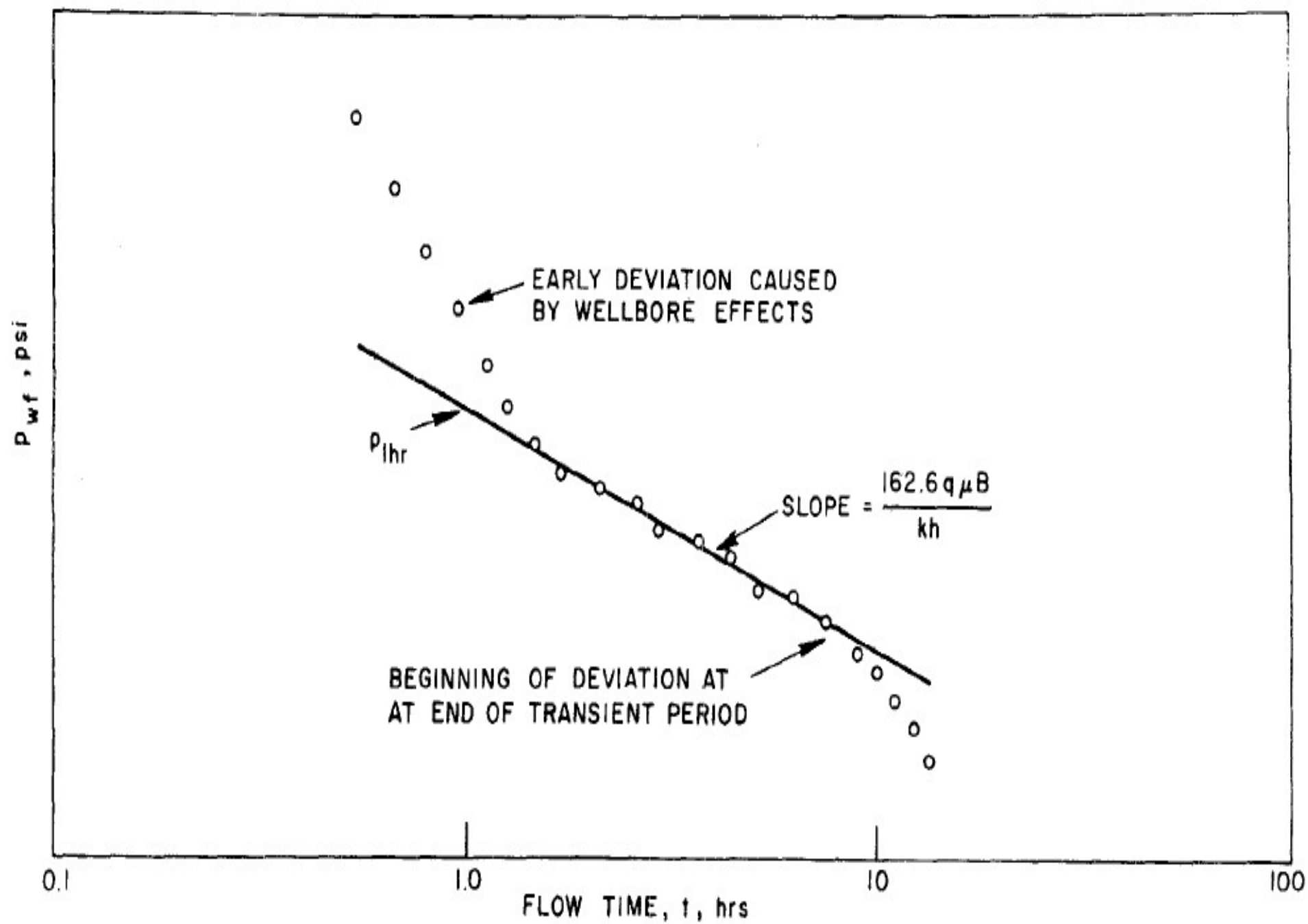


- 
- **Duration of wellbore** unloading can be estimated by qualitative comparison of a **log-log plot of $(P_i - P_{wf})$ vs. t** or with the empirical equation

$$t_D \geq (60 + 3.5s) C_{SD} \quad \text{or} \quad t_{wbs} = \frac{(200000 + 12000s) C_s}{\frac{kh}{\mu}}$$

- In the MTR, a plot of **P_{wf} vs. $\log t$** is a straight line with slope, m , give", by

$$m = 162.6 \frac{q \mu B}{kh}$$



- 
- After the MTR is identified, skin factor, s , can be determined.

$$s = 1.151 \left[\frac{(P_i - P_{1hr})}{m} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$

- The **LTR** begins when the radius of investigation reaches a portion of the reservoir influenced by reservoir boundaries or massive heterogeneities.

- 
- For a well centered in a square or circular drainage area, **LTR** occurs at a time given approximately by

$$t_{lt} \simeq \frac{380\phi\mu C_t A}{k}$$

A : ft²

- For more general drainage-area shapes, **t_{lt}** can be calculated from the number in the column "Use Infinite System Solution With Less Than 1% Error for t_{DA} <".

$$t_{lt} \simeq \frac{3800\phi\mu C_t A t_{DA}}{k}$$



➤ To analyze the typical test, the following steps are suggested.

- 1. Plot flowing BHP, P_{wf} , vs. flowing time, t , on semi log paper.**
- 2. Estimate t_{wbs} from qualitative curve matching ; this usually marks the beginning of the MTR (except for fractured wells).**
- 3. Estimate the beginning of the LTR, t_{lt} , using deviation from a match with to confirm deviation from an apparent semilog straight line**
- 4. Determine the slope m of the most probable MTR, and estimate formation permeability**
- 5. Estimate the skin factor s**

➤ Example - Constant-Rate Drawdown Test Analysis

- ✓ The data in table were recorded during a constant-rate pressure drawdown test. The wellbore had a falling liquid/gas interface throughout the drawdown test. Other pertinent data include the following.

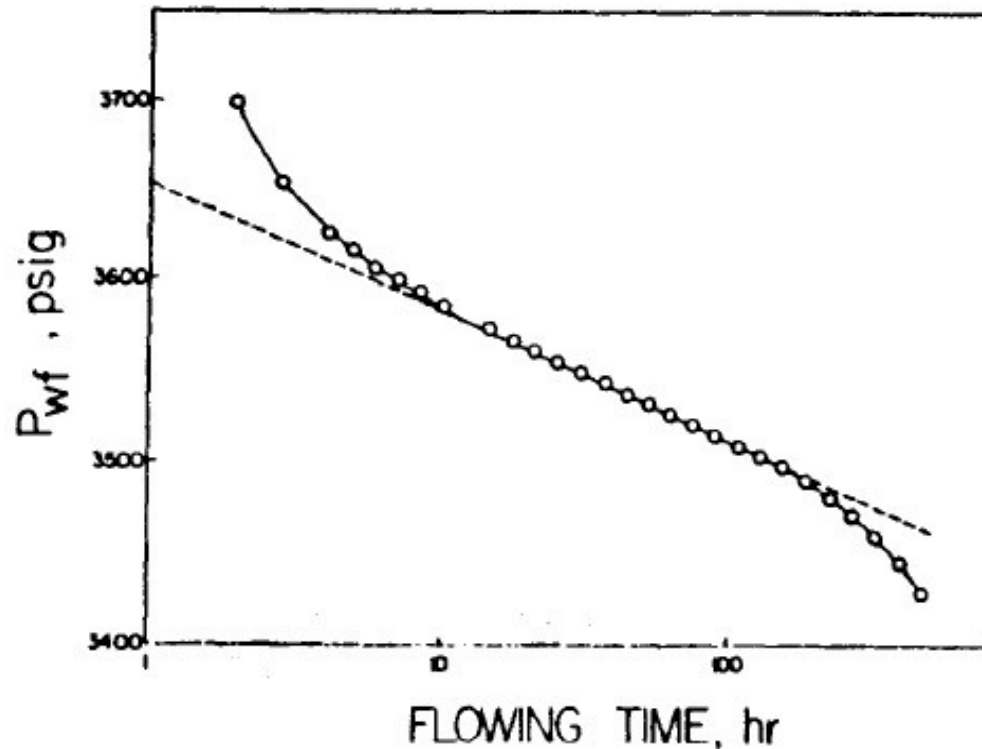
$$\begin{aligned}
 q &= 250 \text{ STB/D}, & h &= 69 \text{ ft}, \\
 B &= 1.136 \text{ bbl/STB}, & \phi &= 0.039, \text{ and} \\
 \mu &= 0.8 \text{ cp}, & c_f &= 17 \times 10^{-6} \text{ psi}^{-1}, \\
 r_w &= 0.198 \text{ ft},
 \end{aligned}$$

t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)	t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)	t (hours)	p_{wf} (psia)	$p_i - p_{wf}$ (psia)
0	4,412	0	14.4	3,573	839	89.1	3,515	897
0.12	3,812	600	17.3	3,567	845	107	3,509	903
1.94	3,699	713	20.7	3,561	851	128	3,503	909
2.79	3,653	759	24.9	3,555	857	154	3,497	915
4.01	3,636	776	29.8	3,549	863	185	3,490	922
4.82	3,616	796	35.8	3,544	868	222	3,481	931
5.78	3,607	805	43.0	3,537	875	266	3,472	940
6.94	3,600	812	51.5	3,532	880	319	3,460	952
8.32	3,593	819	61.8	3,526	886	383	3,446	966
9.99	3,586	826	74.2	3,521	891	460	3,429	983

- ✓ The tubing areas is 0.0218 sq ft; the density of the liquid in the well bore is 53 lbm/cu ft. **Determine the formation permeability and skin factor.**

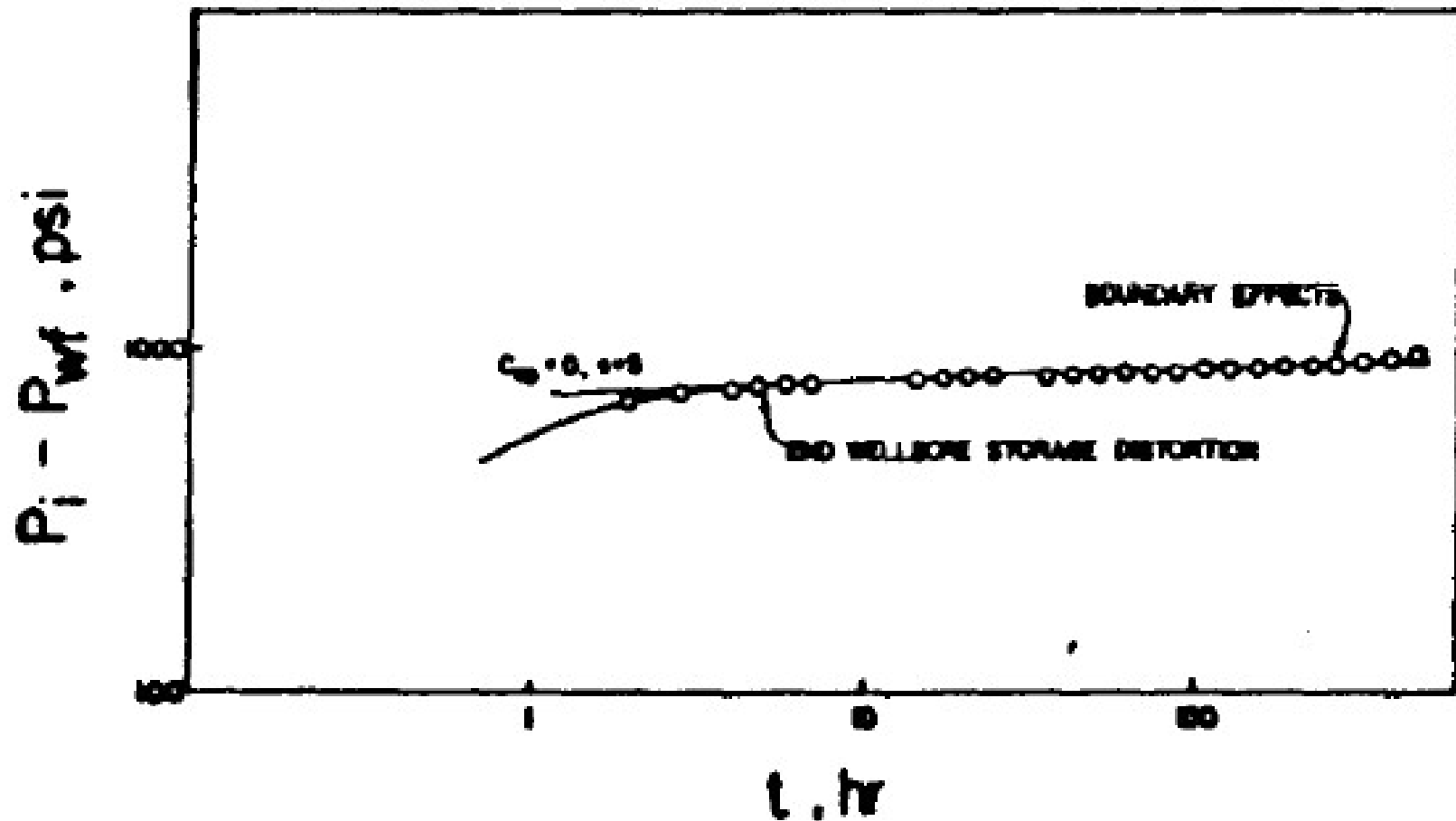
Solution

- ✓ We first plot
 - ❖ flowing BHP. P_{wf} vs. t on semilog paper
 - ❖ and $(P_i - P_{wf})$ vs. t on log-log paper.
- ✓ Then we determine when well bore effects ceased distorting the curve.



- ✓ From the shape of the semilog graph, this appears to be at about 12 hours; however, we can check this assumption with the log-log graph,

- ✓ For several values of C_D (e.g., 10^3 to 10^4), the graph shows well bore storage distortion ends at $\Delta t = 5$ hours,



- 
- The boundary effects begin when the drawdown curve begins to deviate from the established straight line on the semi log graph at a flowing time of 150 hours.
 - This is confirmed qualitatively on the less sensitive log-log graph by noticeable deviation beginning at $t \approx 260$ hours.
 - The slope of the middle-time line is

$$m = 3652 - 3582 = 70 \text{ psi / cycle}$$

$$k = \frac{162.6q \mu B}{mh} = \frac{(162.6)(250)(1.136)(0.8)}{(70)(69)} = 7.65 \text{ md}$$

✓ We next calculate the skin factor s .

$$s = 1.151 \left[\frac{(P_i - P_{1hr})}{m} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$
$$= 1.151 \left[\frac{(4412 - 3652)}{70} - \log \left(\frac{1.442 \times 10^7}{(0.198)^2} \right) + 3.23 \right] = 6.37$$

$$C_s \cong \frac{25.65 A_{wb}}{\rho} = 0.0106 [bbl / psi]$$

$$t_{wbs} = \frac{(200000 + 12000s) C_s}{\frac{kh}{\mu}} = \frac{(200000 + 12000(6.37))(0.0106)}{\frac{(7.65)(69)}{0.8}} = 4.44 [hrs]$$

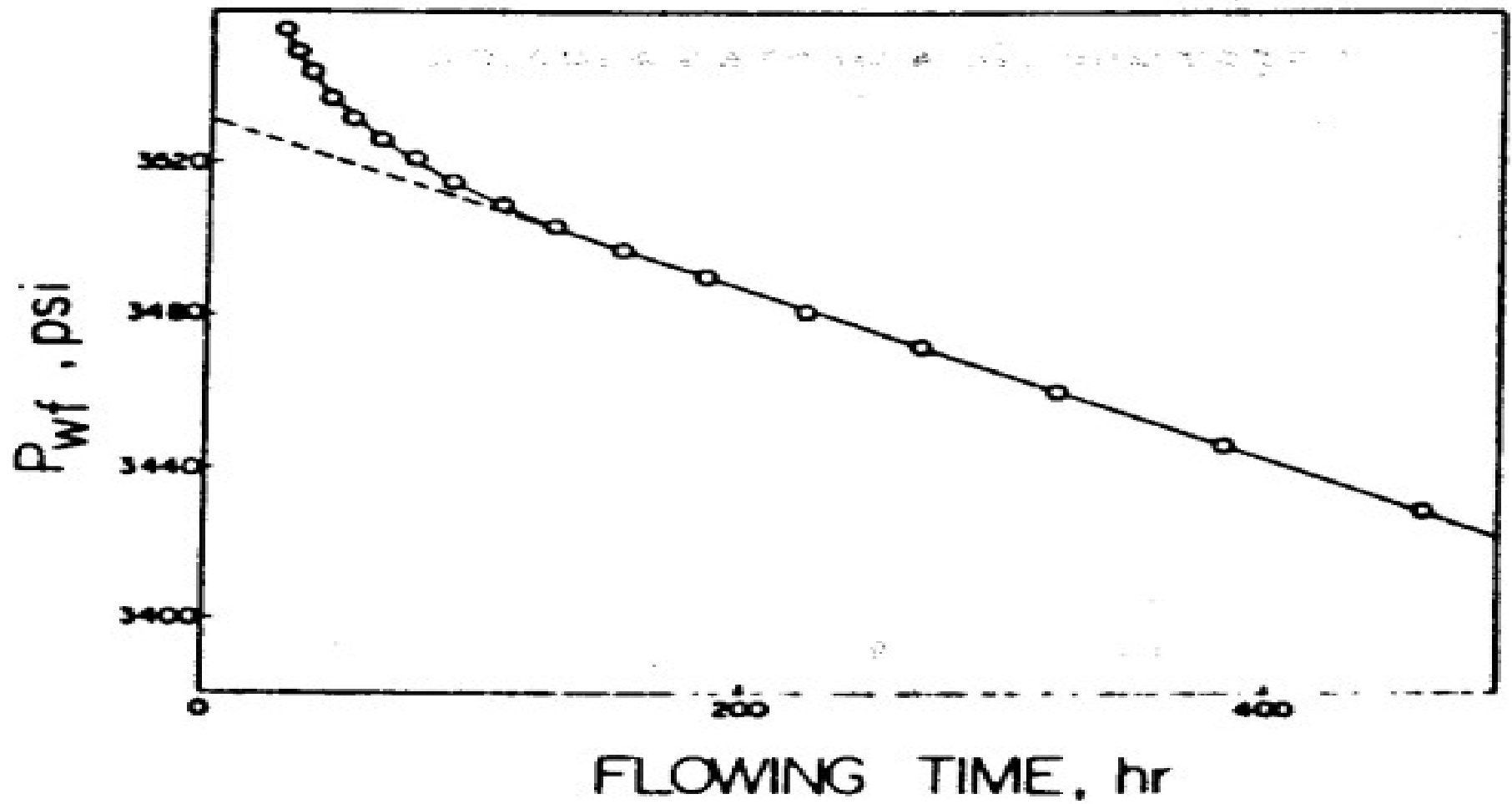
✓ This closely agrees with the result from the log-log curve fit.

Estimation of reservoir pore volume, V_p

- It is possible when the radius of investigation reaches all boundaries during a test so that pseudo steady-state flow is achieved.
- In pseudo steady-state flow, P_{wf} is related linearly to **time** and the **rate of change in P_{wf} with time** is related to the **reservoir pore volume**.

$$V_P = \frac{-0.234qB}{c_t \left(\frac{\partial p_{wf}}{\partial t} \right)}$$

$\frac{\partial p_{wf}}{\partial t}$: The slope of the straight-line P_{wf} vs. t plot on ordinary Cartesian graph paper.



- ✓ The graph of P_{wf} vs. t is a straight line once pseudo steady-state is achieved

- 
- It is important to remember, that these equations apply only to closed. or volumetric, reservoirs (i.e . **they are not valid if there is water influx or gas-cap expansion**).
 - Further they are limited to reservoirs **in which total compressibility C_t is constant** (and, specifically. in-dependent of pressure).

□ H.W) Read the Example 3.2 on page 53 john lee

Analysis of Drawdown Test with Varying Rate

- An analysis method that leads to proper interpretation is available. but it can be used **only if the producing rate is changing slowly and smoothly.**
- **Winestock** and **Colpitts** show that when rate is changing slowly and smoothly. the equation modeling the MTR of the drawdown test becomes

$$\frac{P_i - P_{wf}}{q} = \frac{162.6\mu B}{kh} \left[\log \left(\frac{1688\phi\mu C_t r_w^2}{kt} \right) + 0.0869s \right] + \text{negligible terms}$$



The analysis technique is

- Plot $(P_i - P_{wf}) / q$ vs. t on semi log paper
- Identify the **middle-time straight line**
- Measure the slope m' in psi/STB/D/cycle;
- Calculate kh from

$$kh = 162.6 \frac{\mu B}{m'}$$

and

$$s = 1.151 \left[\left(\frac{P_i - P_{wf}}{q} \right)_{1hr} \frac{1}{m'} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$

Example- Analysis of Drawdown Test with Varying Rate

□ The data in Table were obtained in a drawdown test in which the rate q was measured as a function of time.

t (hours)	p_{wf} (psi)	q (STB/D)	t (hours)	p_{wf} (psi)	q (STB/D)
0	4.412	250	8.32	3.927	147
0.105	4.332	180	9.99	3.928	145
0.151	4.302	177	14.4	3.931	143
0.217	4.264	174	20.7	3.934	140
0.313	4.216	172	29.8	3.937	137
0.450	4.160	169	43.0	3.941	134
0.648	4.099	166	61.8	3.944	132
0.934	4.039	163	74.2	3.946	130
1.34	3.987	161	89.1	3.948	129
1.94	3.952	158	107	3.950	127
2.79	3.933	155	128	3.952	126
4.01	3.926	152	154	3.954	125
5.78	3.926	150	185	3.956	123

□ Other data include the following

$$B = 1.136 \text{ bbl/STB,}$$

$$\mu = 0.8 \text{ cp,}$$

$$h = 69 \text{ ft,}$$

$$\rho = 53 \text{ lb/cu ft,}$$

$$A_{wb} = 0.0218 \text{ sq ft,}$$

$$\phi = 0.039,$$

$$c_l = 17 \times 10^{-6} \text{ psi}^{-1}$$

$$r_w = 0.198 \text{ ft.}$$

✓ Determine formation permeability and skin factor.

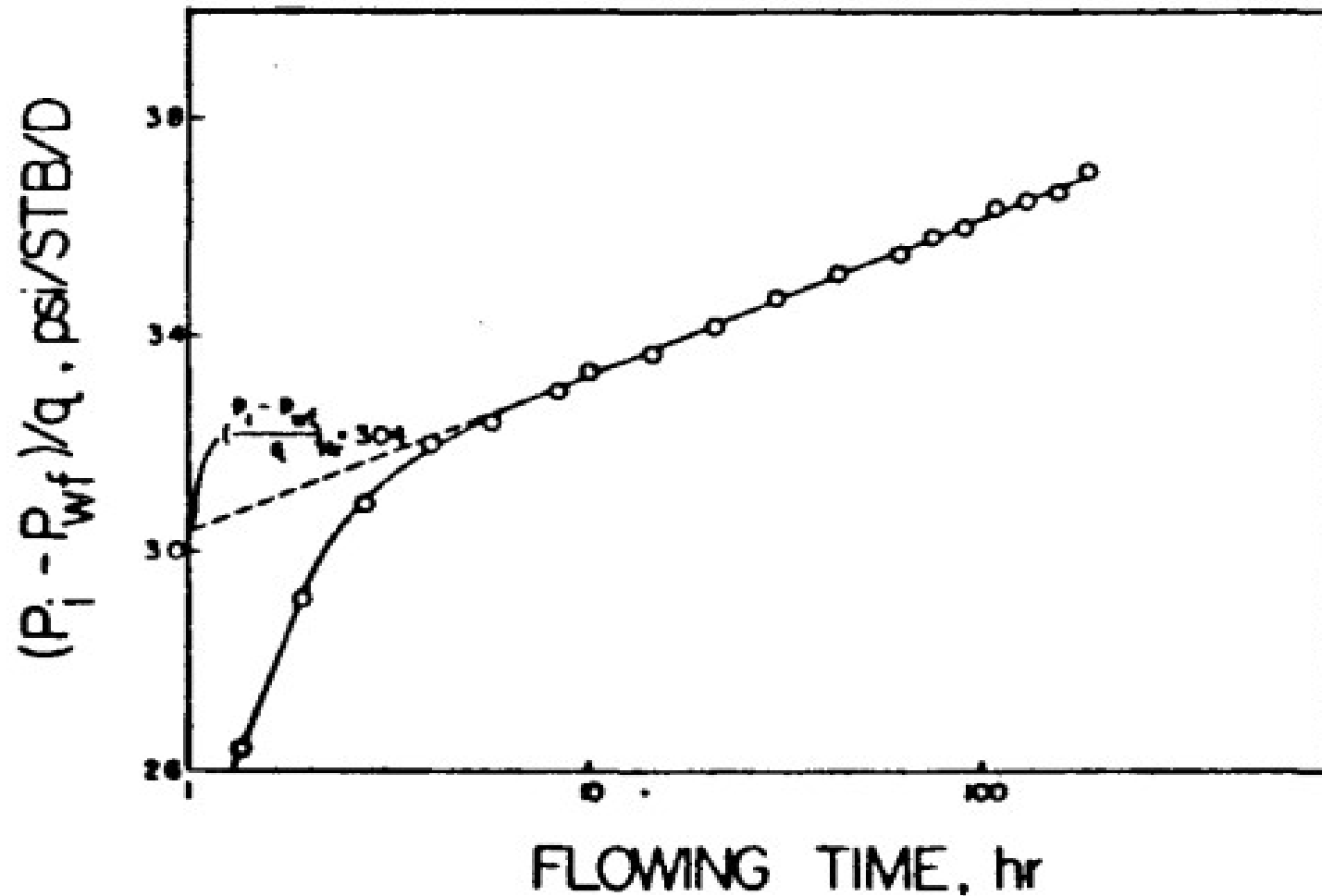
Solution

- ✓ Pressures for now times greater than about 6 hours are increasing even though production continues for another 179 hours and even though the rate decline from this time to the end of the test is only 27 STB/D (from 150 to 123 STB/D).

Thus, we must use the variable-rate analysis technique;

- ✓ the first step is to tabulate $(P_i - P_{wf}) / q$ as in Table .

t (hours)	$(p_i - p_{wf}) / q$	t (hours)	$(p_i - p_{wf}) / q$
0.105	0.444	8.32	3.299
0.151	0.621	9.99	3.338
0.217	0.851	14.4	3.364
0.313	1.140	20.7	3.414
0.450	1.491	29.8	3.467
0.648	1.886	43.0	3.515
0.934	2.288	61.8	3.545
1.34	2.640	74.2	3.585
1.94	2.911	89.1	3.597
2.79	3.090	107	3.638
4.01	3.197	128	3.651
5.78	3.240	154	3.664
		185	3.707



- ✓ On the basis of curve shape, wellbore storage appears to end at approximately 6 hours;
- ✓ There is no deviation from the straight line for $t > 6$ hours; accordingly, we assume the MTR spans the time range $6 \text{ hours} < t < 185 \text{ hours}$.



$$m' = 3.616 - 3.328 = 0.288$$

$$kh = 162.6 \frac{\mu B}{m'} = \frac{(162.6)(0.8)(1.136)}{(0.288)(69)} = 7.44 \text{ md}$$

and

$$s = 1.151 \left[\left(\frac{P_i - P_{wf}}{q} \right)_{1hr} \frac{1}{m'} - \log \left(\frac{k}{\phi \mu C_t r_w^2} \right) + 3.23 \right]$$

$$= 1.151 \left[\frac{3.04}{0.288} - \log \left(\frac{7.44}{(0.039)(0.8)(17 \times 10^{-6})(0.198)^2} \right) + 3.23 \right] = 6.02$$



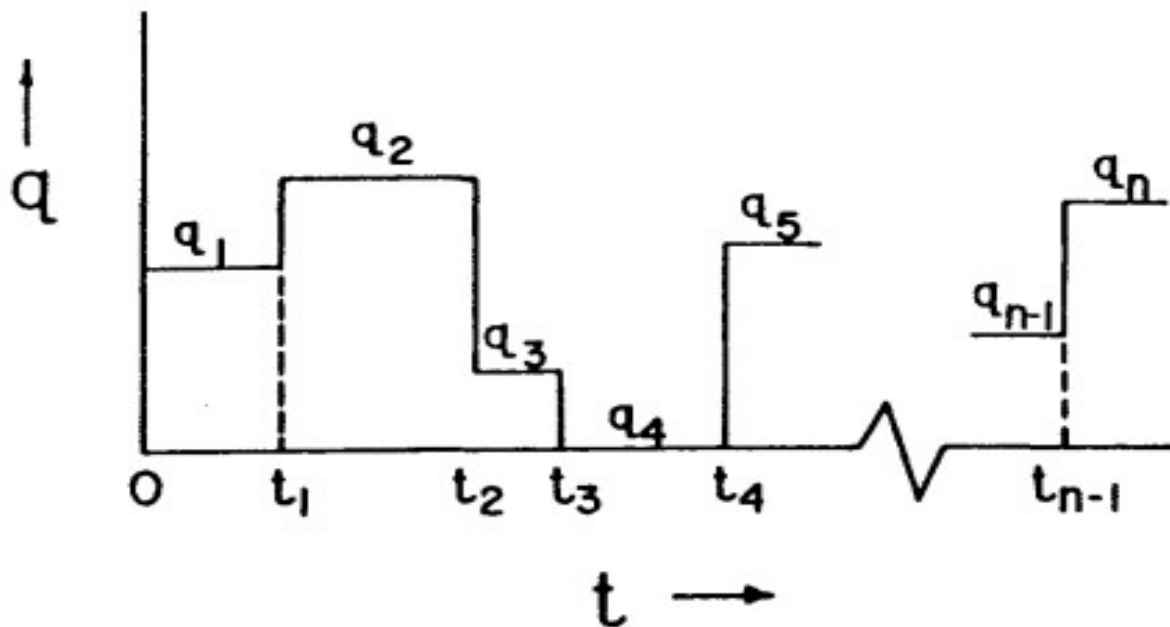
Since $C_s \approx 0.0106$ bbl/psi, as in previous Example ,

$$\begin{aligned} t_{wbs} &= \frac{(200000 + 12000s)C_s}{\frac{kh}{\mu}} \\ &= \frac{(200000 + 12000(6.02))(0.0106)}{\frac{(7.44)(69)}{0.8}} = 4.5[hrs] \end{aligned}$$

✓ This qualitatively confirms the choice of well bore storage distortion end.

Multirate Tests

- We develop a general theory for behavior of **multirate tests** in **infinite-acting** reservoirs for **slightly compressible** liquids.
- Consider a well with **n rate changes** during its production history,





➤ We use superposition of the logarithmic approximation to the E_i -function solution; to simplify the algebra.

$$\begin{aligned} P_i - P_{wf} &= \frac{162.6q \mu B}{kh} \left[\log \left(\frac{1688 \phi \mu C_t r_w^2}{kt} \right) - 0.0869s \right] \\ &= \frac{162.6q \mu B}{kh} \left(\log t + \log \frac{k}{\phi \mu C_t r_w^2} - 3.23 + 0.0869s \right) \\ &= m'q (\log t + \bar{s}) \end{aligned}$$

where

$$m' = 162.6 \frac{\mu B}{kh} \quad \& \quad \bar{s} = \log \frac{k}{\phi \mu C_t r_w^2} - 3.23 + 0.0869s$$

➤ For n rates and for $t > t_{n-1}$ application of superposition leads to

$$\begin{aligned}
 P_i - P_{wf} = & m'q_1 (\log t + \bar{s}) + m'(q_2 - q_1) [\log(t - t_1) + \bar{s}] \\
 & + m'(q_3 - q_2) [\log(t - t_2) + \bar{s}] + \dots \\
 & + m'(q_n - q_{n-1}) [\log(t - t_{n-1}) + \bar{s}]
 \end{aligned}$$

➤ This can be written more compactly as

$$\frac{P_i - P_{wf}}{q_n} = m' \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t - t_{j-1}) + m'\bar{s}, q_n \neq 0$$

In which $q_0 = 0$ and $t_0 = 0$.

- In terms of more fundamental quantities,

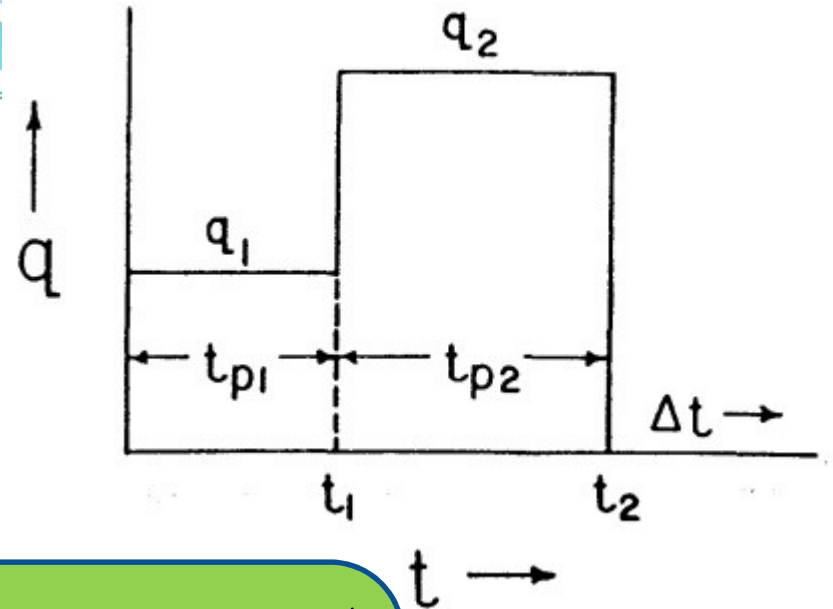
$$\frac{P_i - P_{wf}}{q_n} = m' \sum_{j=1}^n \frac{(q_j - q_{j-1})}{q_n} \log(t - t_{j-1}) + m' \left[\log \frac{k}{\phi \mu C_t r_w^2} - 3.23 + 0.0869s \right]$$

- For the special case $q_n = 0$ (**a pressure buildup test**).

$$\begin{aligned} P_i - P_{wf} &= m' q_1 (\log t + \bar{s}) + m' (q_2 - q_1) [\log(t - t_1) + \bar{s}] \\ &\quad + \dots + m' (q_{n-1} - q_{n-2}) [\log(t - t_{n-2}) + \bar{s}] \\ &\quad - m' q_{n-1} [\log(t - t_{n-1}) + \bar{s}] \\ &= 162.6 \frac{\mu B}{kh} \sum_{j=1}^n (q_j - q_{j-1}) \log(t - t_{j-1}) \end{aligned}$$

- ✓ The reservoir must be infinite acting for the total time elapsed t since the well began producing at rate q_1 .

Pressure Build up Test Preceded by Two Different flow Rates



$$P_i - P_{ws} = m' \left| q_1 \log t + (q_2 - q_1) \log (t - t_1) - q_2 \log (t - t_2) \right|$$

$$P_i - P_{ws} = 162.6 \frac{q_2 \mu B}{kh} \left| \frac{q_1}{q_2} \log \left(\frac{t}{t - t_1} \right) + \log \left(\frac{t - t_1}{t - t_2} \right) \right|$$

$$t - t_2 = \Delta t$$

$$t_1 = t_{p1}$$

$$t_2 = t_{p1} + t_{p2}$$

$$t = t_{p1} + t_{p2} + \Delta t$$



Then

$$P_i - P_{ws} = 162.6 \frac{q_2 \mu B}{kh} \left| \frac{q_1}{q_2} \log \left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t} \right) + \log \left(\frac{t_{p2} + \Delta t}{\Delta t} \right) \right|$$

- We can use this equation when the production rate is changed a short time before a buildup test begins, so that there is not sufficient time for, Horner's approximation to be valid, we frequent can consider **all production before time t_1 to have been at rate q_1 for time t_{p1}** and **production just before the test to have been at rate q_2 for time t_{p2}'**



To analyze such a test, we plot

$$p_{ws} \text{ vs. } \left[\frac{q_1}{q_2} \log \left(\frac{t_{p1} + t_{p2} + \Delta t}{t_{p2} + \Delta t} \right) + \log \left(\frac{t_{p2} + \Delta t}{\Delta t} \right) \right]$$

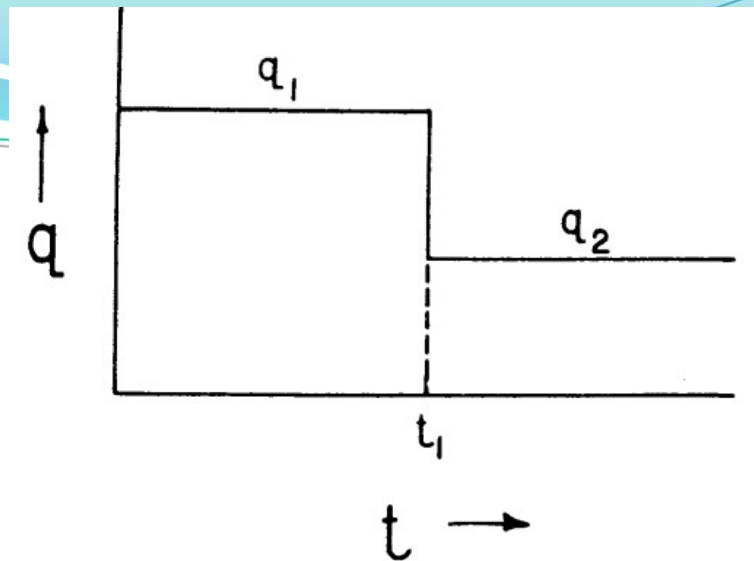
$$m = 162.6 \frac{q_2 B \mu}{kh}.$$

Note that semi log paper is not be used; instead, two logarithms is plotted on an ordinary Cartesian axis.

$$s = 1.151 \left(\frac{p_{1hr} - p_{wf}}{m} - \log \frac{k}{\phi \mu c_i r_w^2} + 3.23 \right),$$

Two-Rate Flow Test

This type of test can be used when estimates of permeability, skin factor, or reservoir pressure are needed but when the well cannot be shut in because loss of income cannot be tolerated.



$$p_i - p_{wf} = 162.6 \frac{q B_2 \mu}{k h} \left[\frac{q_1}{q_2} \log t + \frac{(q_2 - q_1)}{q_2} \cdot \log(t - t_1) + \log\left(\frac{k}{\phi \mu c_i r_w^2}\right) - 3.23 + 0.869 s \right]$$

If : $[t_1 = t_{p1}]$ & $[t - t_{p1} = \Delta t']$

$$p_{wf} = p_i - 162.6 \frac{q_2 B \mu}{k h} \left[\log\left(\frac{k}{\phi \mu c_i r_w^2}\right) - 3.23 + 0.869 s \right] - 162.6 \frac{q_1 B \mu}{k h} \left[\log\left(\frac{t_{p1} + \Delta t'}{\Delta t'}\right) + \frac{q_2}{q_1} \log(\Delta t') \right]$$



1. Plot p_{wf} vs. $\left[\log\left(\frac{p_i + \Delta t'}{\Delta t'}\right) + \frac{q_2}{q_1} \log(\Delta t') \right]$. On Cartesian paper

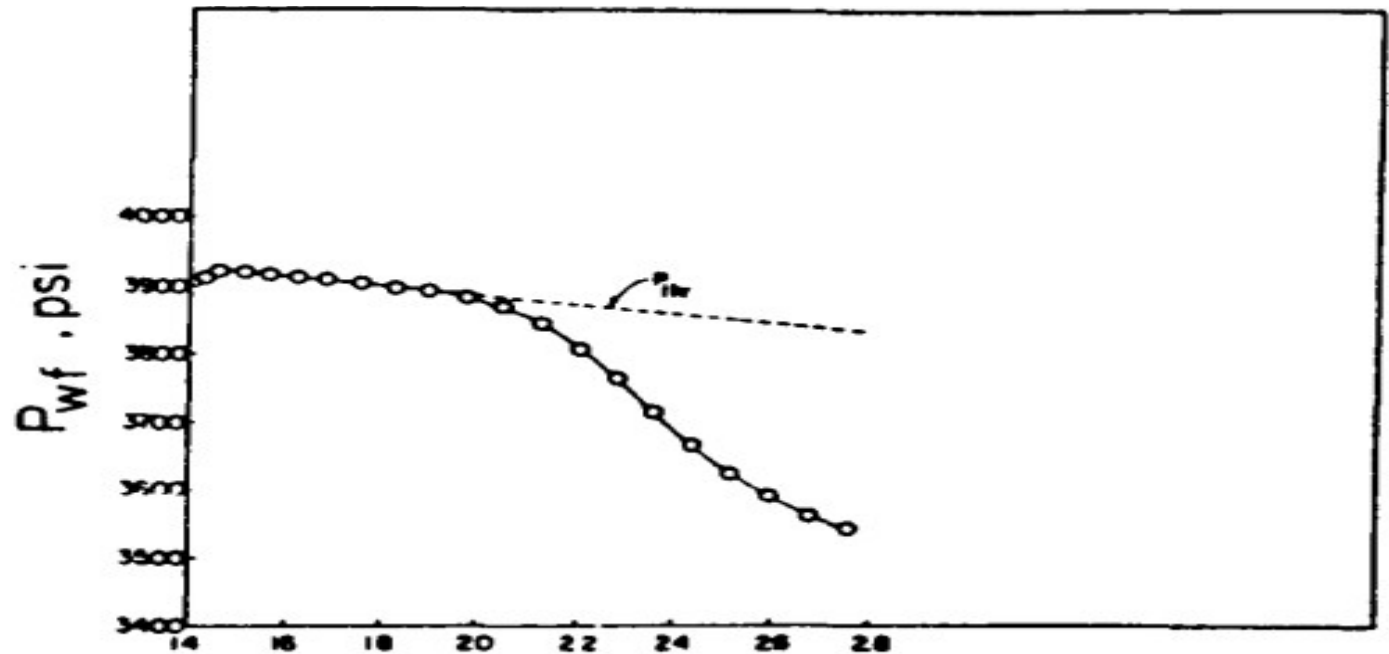
2. Determine the slope m from the plot and use it to calculate permeability, k , from the relationship

$$k = 162.6 \frac{q_1 B \mu}{m h}.$$

3. Calculate the skin factor, s , from the equation

$$s = 1.151 \left[\frac{q_1}{(q_1 - q_2)} \left(\frac{p_{1 \text{ hr}} - p_{wf1}}{m} \right) - \log\left(\frac{k}{\phi \mu c_i r_w^2} \right) + 3.23 \right].$$

✓ In above Eq. $P_{1 \text{ hr}}$ is the flowing pressure at $\Delta t' = 1$ hour on the MTR line and P_{wf} is the flowing pressure at the time the rate is changed ($\Delta t' \approx 0$).



$$\log \left(\frac{t_{pi} + \Delta t'}{\Delta t'} \right) + \frac{q_2}{q_1} \log (\Delta t')$$

4. p_i (or, more generally, p^*) is obtained by solving for p_i (p^*) from the drawdown equation written to model conditions at the time of the rate change. (It is implied that s and m are known at this point.)

$$p_i = p_{wf1} + m \left| \log \left(\frac{k h p_i}{\phi \mu c_i r_w^2} \right) - 3.23 + 0.869 s \right|.$$

□ H.W) Read the Example 3.4 on page 59 john lee





Chapter 6

Gas Well Testing



Flow tests conducted on gas wells

1. Tests designed to yield knowledge of reservoir



- **Drawdown**
- **Buildup**

2. Tests designed to measure the deliverability (down hole deliverability)



- **Back pressure tests**
- **Isochronal type tests**



Deliverability Tests

- Deliverability tests have conventionally been called **back pressure tests** because they make possible the prediction of well flow rates against any particular “back pressure” .
- Since most flowing well tests are performed to determine the deliverability of a well, the term “**deliverability tests** “ is used rather than “**back pressure tests**” .
- The purpose of these tests is to predict the manner in which the flow rate will decline with reservoir depletion

Various deliverability tests of gas well

• **Flow-after-flow (Conventional Back Pressure Test)**

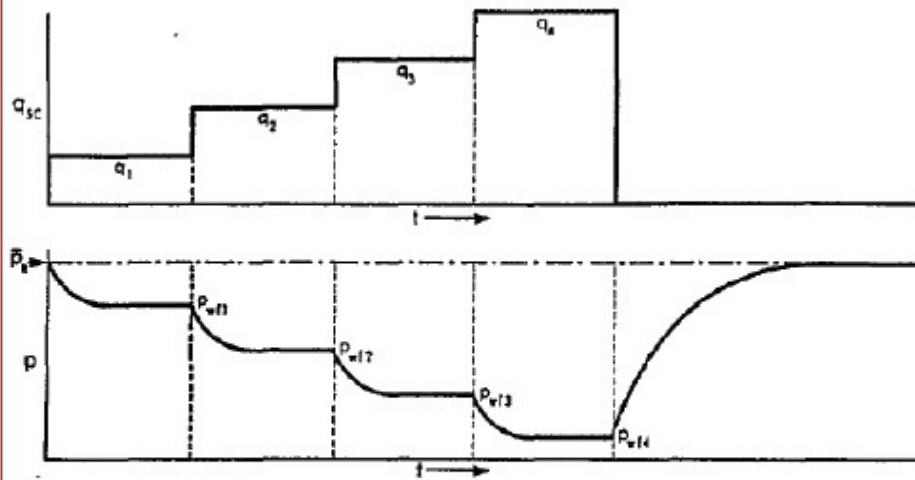
- Flowing the well at several different flow rates
- Each flow rate being continued to pressure stabilization

• **Isochronal**

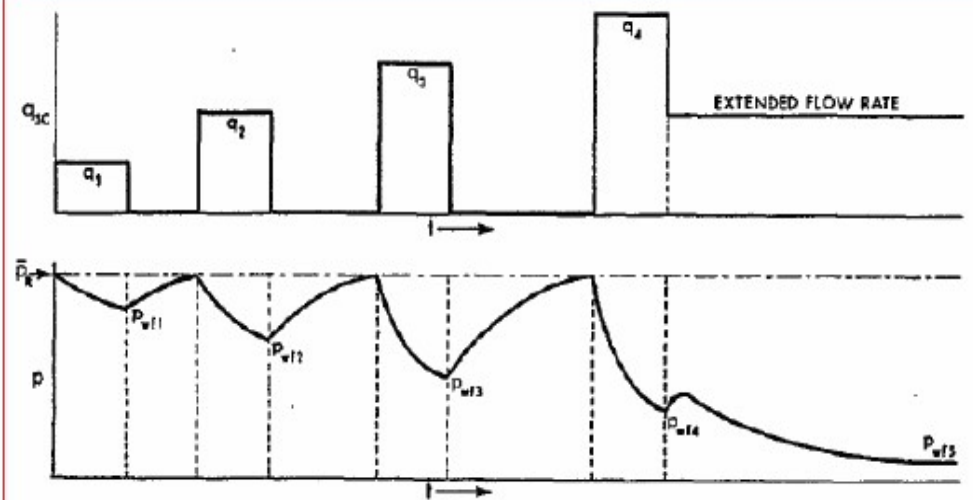
- A series flow tests at different rates for equal periods of time
- Alternately closing in the well until a stabilized flow (last flow rate is long enough to achieve stabilization)

• **Modified isochronal deliverability tests**

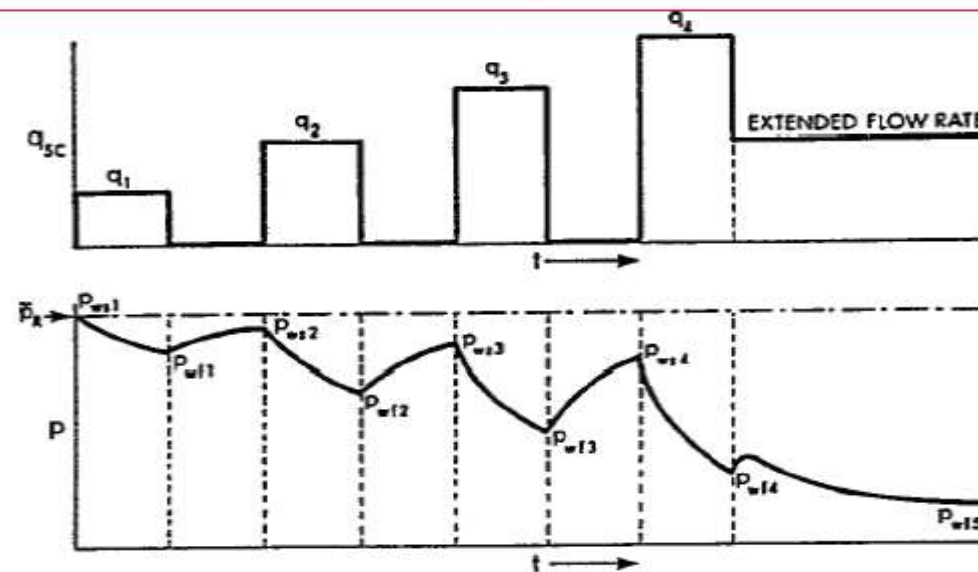
- A series tests at different rates for equal periods of flow-time and shut-in times



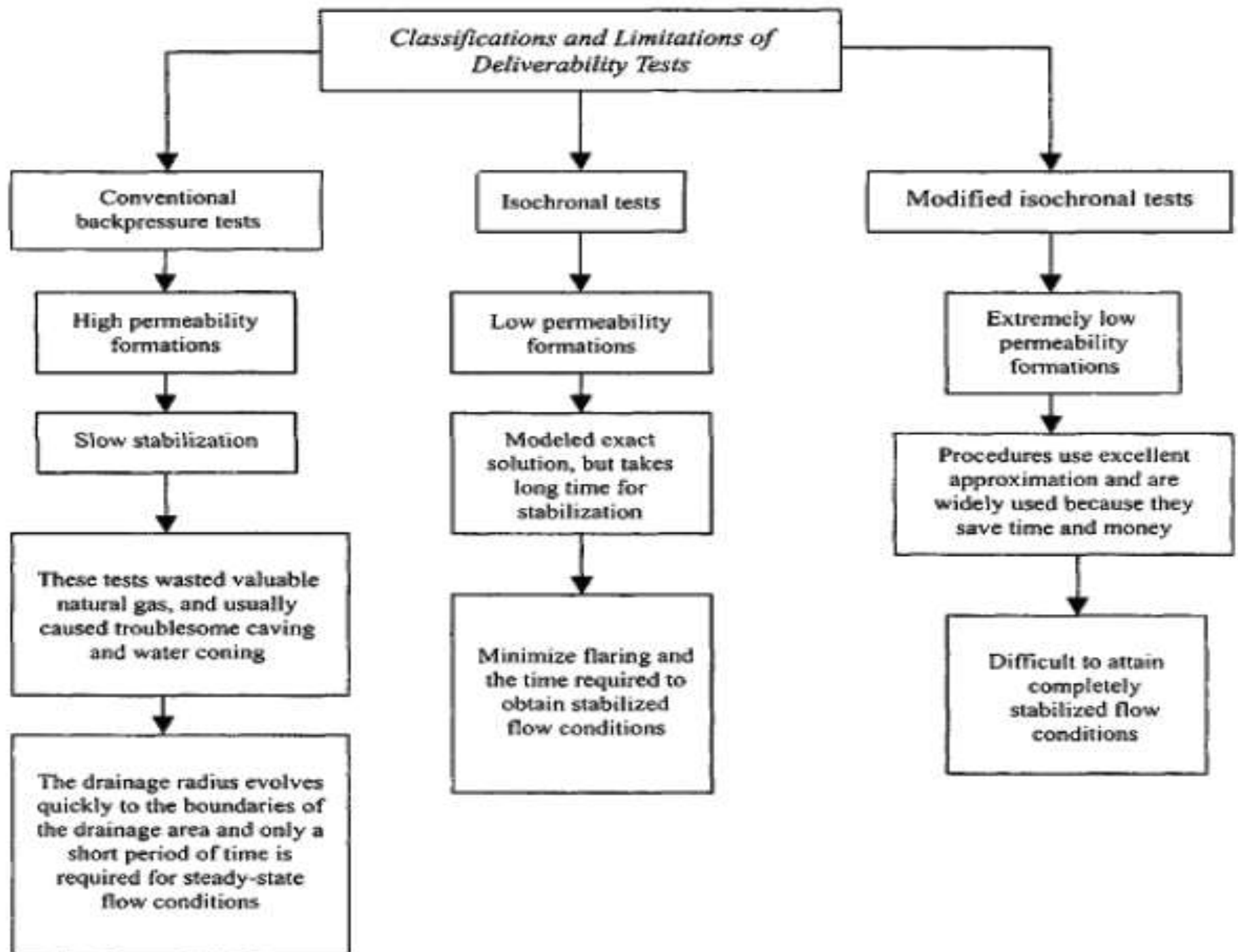
CONVENTIONAL TEST—FLOW RATE AND PRESSURE DIAGRAMS



ISOCHRONAL TEST—FLOW RATE AND PRESSURE DIAGRAMS



MODIFIED ISOCHRONAL TEST—FLOW RATE AND PRESSURE DIAGRAMS



Stabilized Flow Equations; $r_i \geq r_e$

The approximate time to stabilization

$$t_s \cong \frac{1000\phi\bar{\mu}_g r_e^2}{k\bar{p}_R}$$

$$\psi(p_{wf}) = \psi(p_R) - 1.422 \times 10^6 \frac{q_{sc} T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_{sc}| \right]$$

$$\psi(p_R) - \psi(p_{wf}) = A q_{sc} + B q_{sc}^2$$

where

$$A = 1.422 \times 10^6 \frac{T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right]$$

and

$$B = 1.422 \times 10^6 \frac{T}{kh} D$$

$$p_R^2 - p_{wf}^2 = 1.422 \times 10^6 \frac{q_{sc} \bar{\mu}_g \bar{z} T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_{sc}| \right]$$

$$p_R^2 - p_{wf}^2 = A' q_{sc} + B' q_{sc}^2$$

where

$$A' = 1.422 \times 10^6 \frac{\bar{\mu}_g \bar{z} T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right]$$

$$B' = 1.422 \times 10^6 \frac{\bar{\mu}_g \bar{z} T}{kh} D$$

The constants A , B , A' , and B' can be determined from flow tests for at least two rates in which q_{sc} and the corresponding value of p_{wf} are measured; p_R also must be known.

Transient Flow Equations; $r_i \leq r_e$

$$\psi(p_{wf}) = \psi(p_R) - 1.422 \times 10^6 \frac{q_{sc} T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_{sc}| \right]$$

$$\psi(p_R) - \psi(p_{wf}) = A_t q_{sc} + B q_{sc}^2$$

$$A_t = \frac{1.637 \times 10^6 T}{kh} \left[\log\left(\frac{kt}{\phi \mu_{gi} C_i r_w^2}\right) - 3.23 + .869s \right]$$

$$B = 1.422 \times 10^6 \frac{T}{kh} D$$

$$p_R^2 = p_{wf}^2 - 1.422 \times 10^6 \frac{q_{sc} \bar{\mu}_g \bar{z} T}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + D|q_{sc}| \right]$$

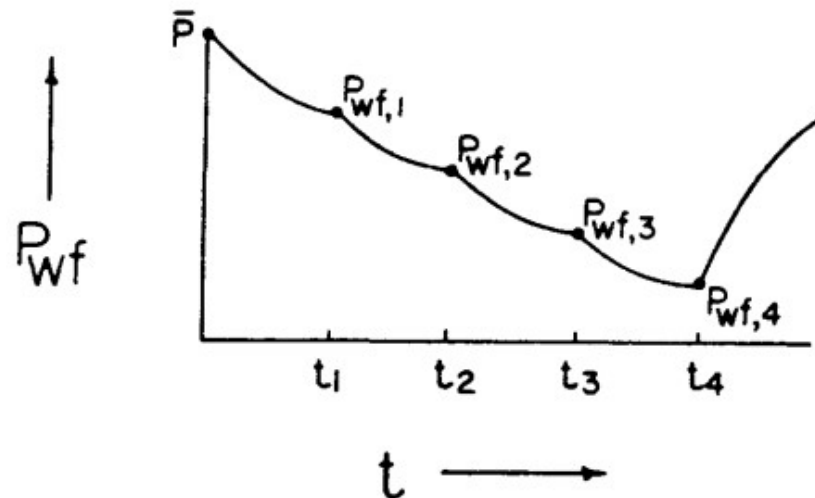
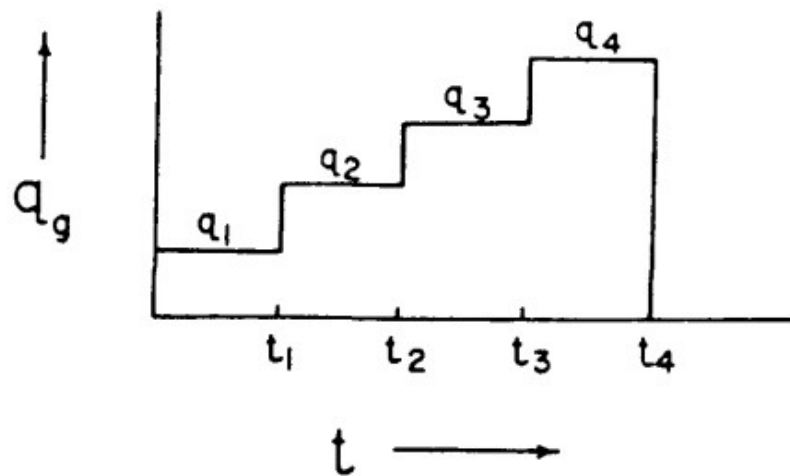
$$p_R^2 - p_{wf}^2 = A'_t q_{sc} + B' q_{sc}^2$$

$$A'_t = 1.422 \times 10^6 \frac{\bar{\mu}_g \bar{z} T}{kh} \left[\frac{1}{2} \ln\left(\frac{kt}{1.688 \phi \mu_g C_i r_w^2}\right) + s \right]$$

$$B' = 1.422 \times 10^6 \frac{\bar{\mu}_g \bar{z} T}{kh} D$$

Flow-After-Flow Tests

- In this testing method, a well flows **at a selected constant rate until pressure stabilizes** - i.e., **pseudo steady state is reached**.
- The stabilized rate and pressure are recorded; rate is then changed and the well flows **until the pressure stabilizes again at the new rate**. The process is repeated for a total of three or four rates.



- 
- Two different techniques can be used to analyze these test data.

- I. Empirical Method
- II. Theoretical Method

I. Empirical Method

- A plot of $\Delta p^2 = \bar{p}^2 - p_{wf}^2$ vs. q_g on log-log paper is approximately a straight line for many wells in which the pseudo steady state is reached at each rate in a flow-after-flow test sequence.

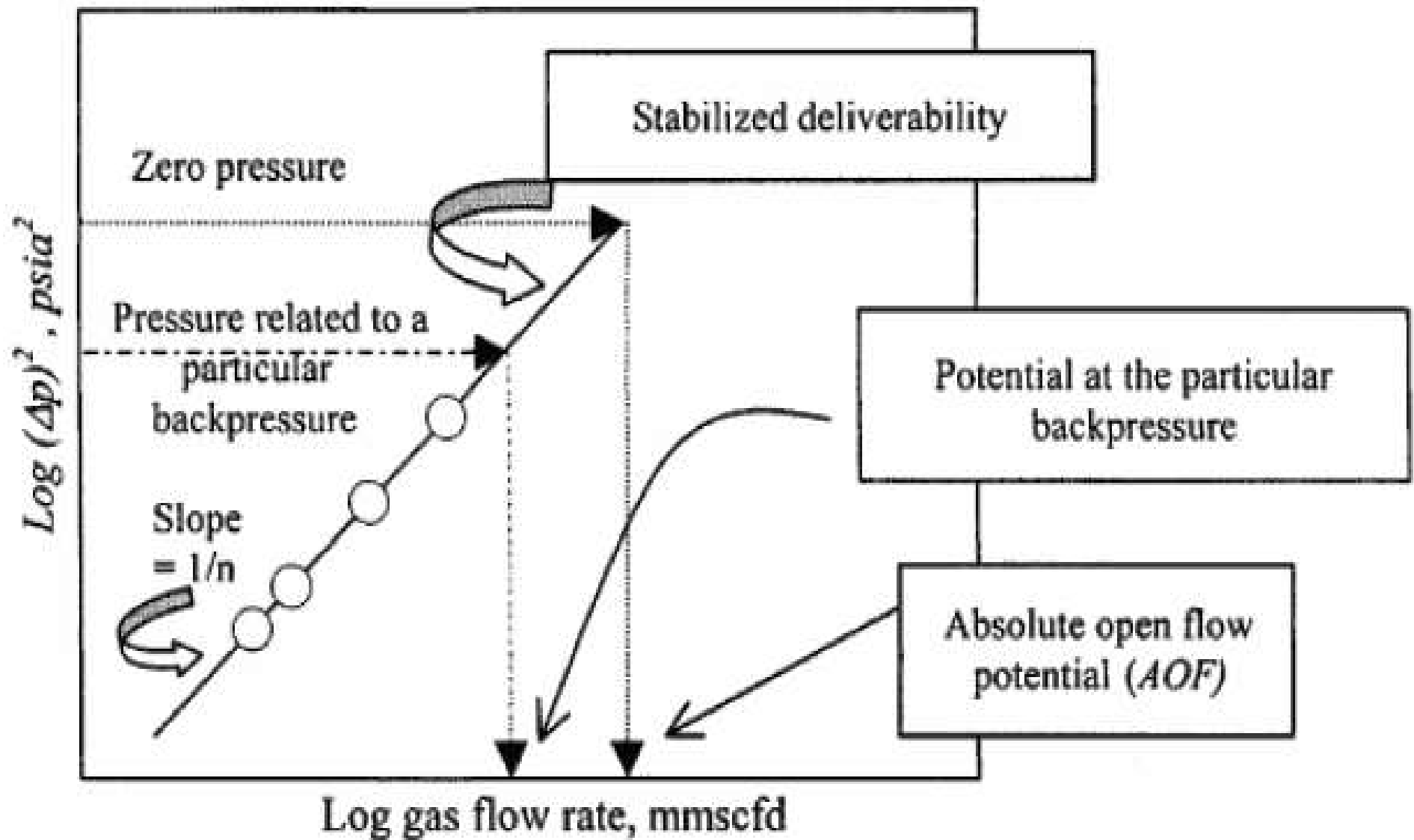


➤ The equation of the line in this plot is

$$q_g = C \left(\bar{p}^2 - p_{wf}^2 \right)^n = C \left(\Delta p^2 \right)^n$$

Where :

- q_{sc} = flow rate at standard conditions, MMscfd
(14.65 psia, 60°F)
- $\bar{p}_R$ = average reservoir pressure obtained by shut-in
of the well to complete stabilization, psia
- p_{wf} = flowing sandface pressure, psia
- Δp^2 = $\left(\bar{p}_R^2 - p_{wf}^2 \right)$
- C = a coefficient which describes the position of the
stabilized deliverability line
- n = an exponent which describes the inverse of the slope
of the stabilized deliverability line.



✓ An AOF determined from such a lengthy extrapolation may be incorrect.

- 
- The constants **C** and **n** in are not constants at all. They depend on fluid properties that are pressure (and, thus, time) dependent.
 - Accordingly, if this type of deliverability curve is used, periodic retesting of the well will show changes in **C** and perhaps in **n**.
 - We must emphasize that deliverability estimates based on this plot assume that pressures were stabilized ($r_i \geq r_e$) during the testing period used to construct the plot.



II. Theoretical Method

- We plot $\frac{(\bar{p}^2 - p_{wf}^2)}{q_g}$ vs. q_g the result (for pseudo steady-state flow) should be a straight line with **slope b** and **intercept a**.
- Because this line has theoretical basis than the log-log plot, it should be possible to **extrapolate it to determine AOF with less error**.

Example :

The data in following Table were reported for a flow-after-flow (or four-point) test . At each rate, pseudo steady state was reached. Initial (i.e., before the test) shut-in BHP, p^- , was determined to be 408.2 psia.

Estimate the AOF of the tested well using

- (1) the empirical plot and
- (2) the theoretical flow equation.

In addition plot deliver abilities estimated using the theoretical equation on the empirical curve plot.

<u>Test</u>	<u>p_{wf} (psia)</u>	<u>q_g (MMscf/D)</u>
1	403.1	4.288
2	394.0	9.265
3	378.5	15.552
4	362.6	20.177

Solution :

We prepare a table of data to be plotted for both empirical and theoretical analyses.

p_{wf} (psia)	q_g (MMscf/D)	$p^2 - p_{wf}^2$ (psia ²)	$(p^2 - p_{wf}^2) / q_g$ (psia ² /MMscf/D)
408.2	0	—	—
403.1	4.288	4,138	964.9
394.0	9.265	11,391	1,229
378.5	15.552	23,365	1,502
362.6	20.177	35,148	1,742
14.7	AOF	166,411	—



1. Empirical Method. From a plot of $(p^{-2} - P_{wf}^2)$ vs. q , on log-log paper, and extrapolation of this plot to $P^{-2} - P_{wf}^2 = 166,411$ (where $P_{wf} = 0$ psig or 14.7 psia). AOF ≈ 60 MMscf/D.

The slope of the curve, $1/n$, is

$$1/n = \frac{\log (\bar{p}^2 - p_{wf}^2)_2 - \log (\bar{p}^2 - p_{wf}^2)_1}{\log q_{g,2} - \log q_{g,1}}$$

$$= \frac{\log \left(\frac{10^5}{10^3} \right)}{\log \left(\frac{42.5}{1.77} \right)} = 1.449.$$

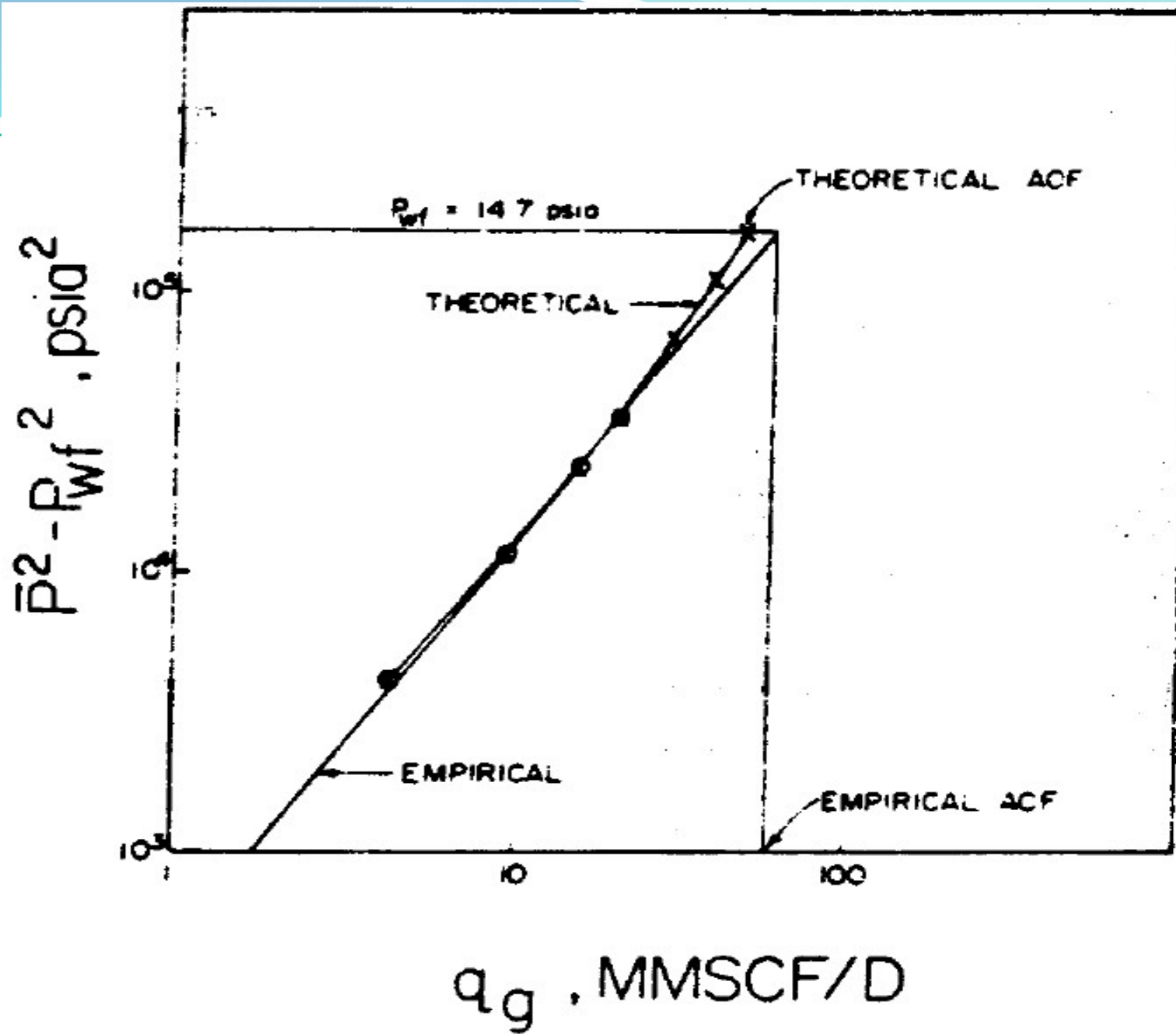


Thus, $n = 0.690$. Then,

$$C = \frac{q_g}{(\bar{p}^2 - p_{wf}^2)^n} = \frac{42.5}{(10^5)^{0.690}} = 0.01508.$$

Thus. the empirical deliverability equation is

$$q_g = 0.01508(\bar{p}^2 - p_{wf}^2)^{0.690}$$



Stabilized gas well deliverability test.

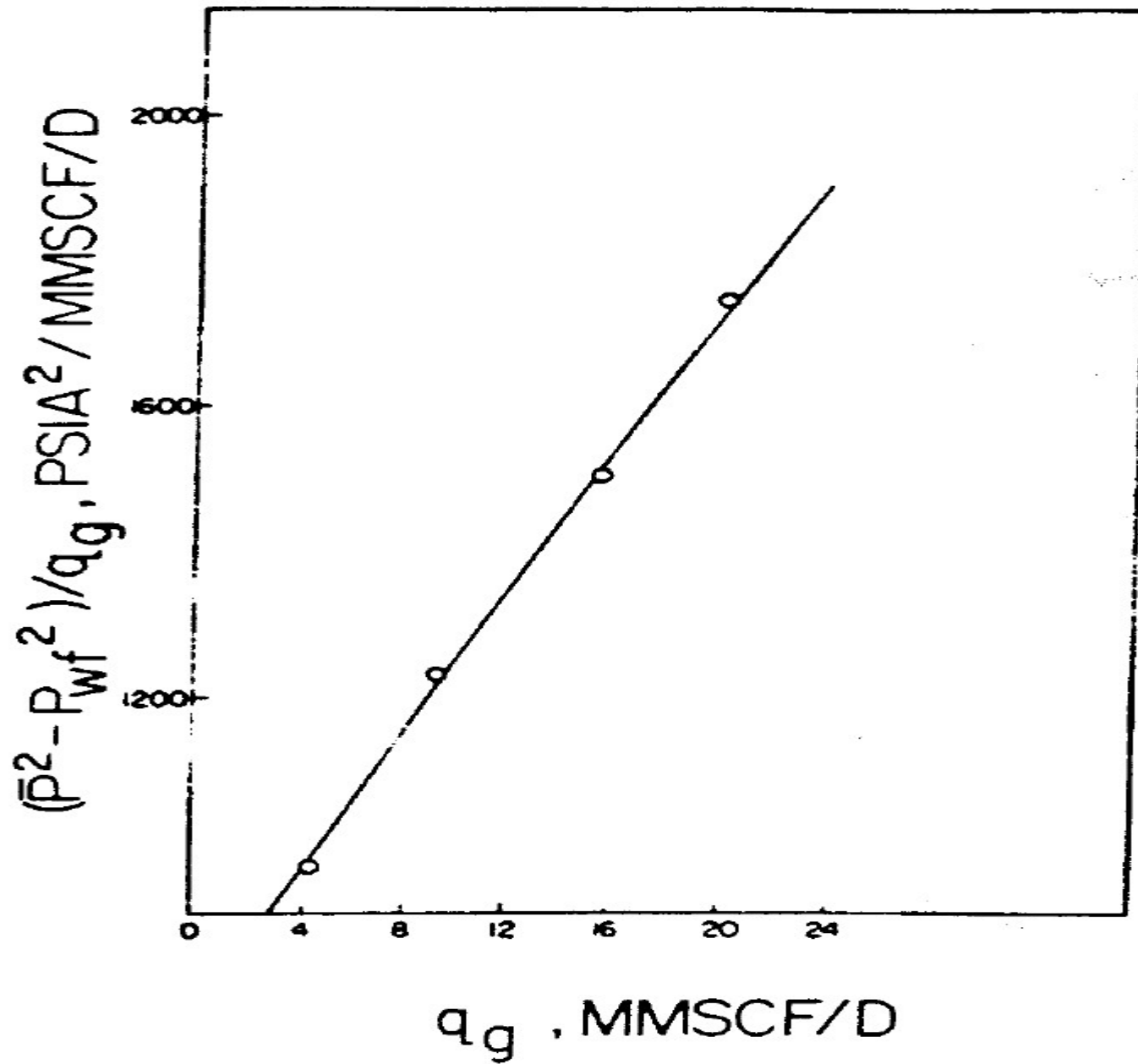


2. Theoretical Method.

The theoretical deliverability equation is

$$(\bar{p}^2 - p_{wf}^2)/q_g = a + bq_g$$

Next figure. is a plot of $(\bar{p}^2 - P_{wf}^2)/q_g$, vs. q_g for the test data. Two points on the best straight line through the data are (2.7; 900) and (23.9; 1900). Thus,



Stabilized deliverability test. theoretical flow equation.

$$900 = a + 2.7 b,$$

$$1,900 = a + 23.9 b.$$

- ✓ Solving for a and b, we find that $a = 773$ and $b = 47.17$. Thus, the theoretical deliverability equation is

$$47.17 q_g^2 + 773 q_g = (\bar{p}^2 - p_{wf}^2).$$

- ✓ We can solve this quadratic equation for the AOF:

$$47.17 q_g^2 + 773 q_g - 166,411 = 0.$$

- ✓ The solution is $q_g = \text{AOF} = 51.8 \text{ MMscf/D.}$

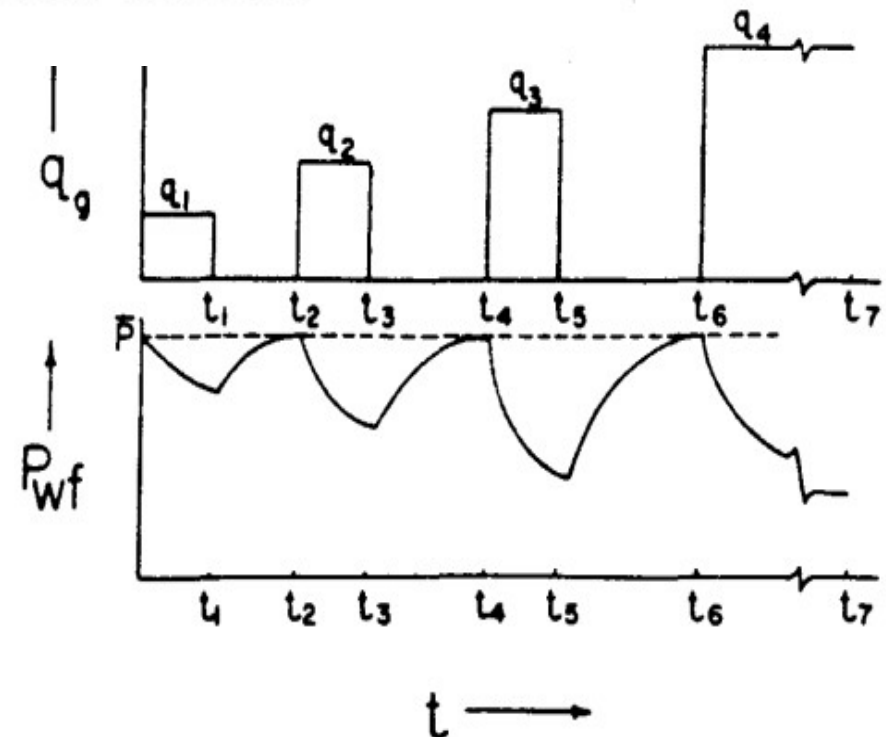


Isochronal Tests

- **The objective of isochronal testing** is to establish a stabilized deliverability curve for a gas well without flowing the well for sufficiently long to achieve stabilized conditions ($r_i \geq r_e$) at each rate.
- An isochronal test is conducted by flowing a well at **a fixed rate**, then shutting it in **until the pressure builds up to an unchanging** (or almost unchanging) **value, p^-** .
- The well then is flowed at a **second rate** for the same length of time, followed by another shut-in, etc.
- If possible, the final flow period should be long enough **to achieve stabilized flow**.

Important points

1. Flow periods, excepting the final one, are of equal length [i.e., $t_1 = (t_3 - t_2) = (t_5 - t_4) \leq (t_7 - t_6)$].
2. Shut-in periods have the objective of letting $p \approx \bar{p}$ rather than the objective of equal length. Thus, in general, $(t_2 - t_1) \neq (t_4 - t_3) \neq (t_6 - t_5)$.
3. A final flow period in which the well stabilizes (i.e., r_i reaches r_e at time t_7) is desirable but not essential.





➤ The most general theory of isochronal tests is based on **equations using pseudo pressure.**

➤ However, we will once again present the theory in terms of the low-pressure approximations to these equations (**p^2 equations**) because

(1) They are somewhat simpler and less abstract than equations in pseudo pressure

(2) They allow direct comparison with more conventional analysis methods based on plots of $(p^2 - p_{wf}^2)$ vs. q on log-log paper.

- 
- We observed previously that the **radius of investigation** achieved at a given time in a flow test is **independent of flow rate** and, **thus**

- ✓ at a given time, the same portion of the reservoir is being drained at each rate in isochronal test and, as a good approximation, stabilized flow conditions exist to a point just beyond $r=r_i$

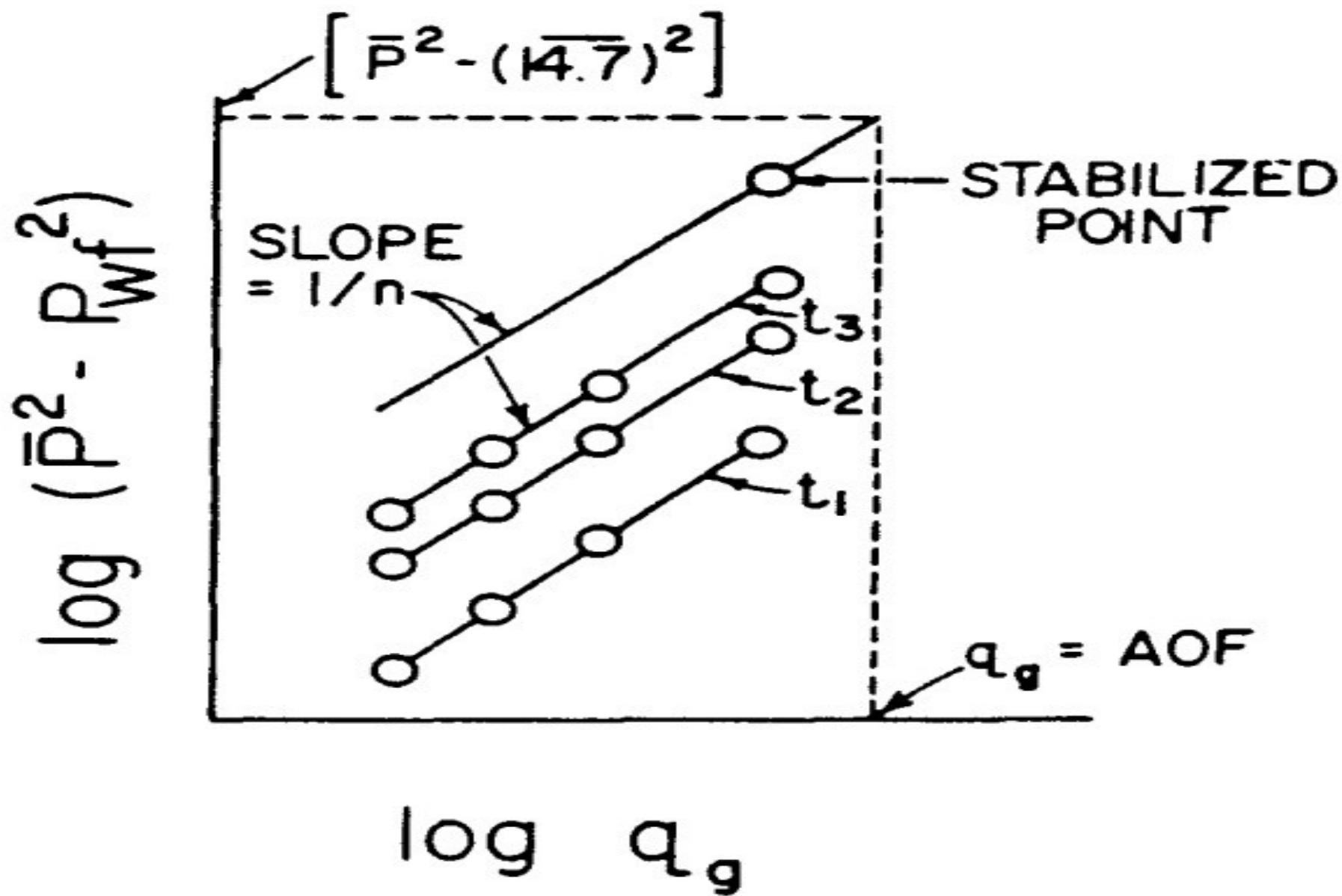
- Two different techniques can be used to analyze these test data.

- I. Empirical Method
 - II. Theoretical Method

I. Empirical Method

- 1. The $(p^{-2} - p_{wf}^2)$ vs. q should be plot on log·log paper**
- 2. Lines should be drawn for several values of time t . and the slope $\frac{1}{n}$ should be established for each isochronal deliverability curve.**
- 3. A line with the slope $\frac{1}{n}$ determined from the nonstabilized fixed-time curves· then is drawn through the single stabilized point. $(q_g , p^{-2} - p_{wf}^2)$**

This establishes the stabilized deliverability curve. Once the stabilized deliverability curve is determined. AOF is established in the usual way.



Empirical deliverability plot for isochronal test.

II. Theoretical Method

The theoretical method for analyzing isochronal test data is based on the theoretical equations for stabilized flow and transient flow .

For stabilized flow

$$\begin{aligned} \bar{p}^2 - p_{wf}^2 &= a q_g + b q_g^2 \\ r_i &\geq r_e \end{aligned} \quad \left\{ \begin{aligned} a &= 1,422 \frac{\mu \bar{p} z \bar{p}_g T}{k h} \left[\ln \left(\frac{r_e}{r_w} \right) - 0.75 + s \right] \\ b &= 1,422 \frac{\mu \bar{p} z \bar{p}_g T D}{k h} \end{aligned} \right.$$

For transient flow :

$$\bar{p}^2 - p_{wf}^2 = a_i q_g + b q_g^2 \quad \left\{ \begin{array}{l} a_i = 1,422 \frac{\mu_{\bar{p}} z_{\bar{p}} T}{kh} \left[\frac{1}{2} \ln \left(\frac{kt}{1,688 \phi \mu_{\bar{p}} c_{i\bar{p}} r_w^2} \right) + s \right] \\ b = 1,422 \frac{\mu_{\bar{p}} z_{\bar{p}} TD}{kh} \end{array} \right.$$

$r_i < r_e$

1. For a fixed value of t. determine b from a plot of $(\bar{p}^2 - p_{wf}^2)/q_g$ vs. q_g

2. Using the stabilized data point $[q_{gs}, (\bar{p}^2 - p_{wf}^2)_s]$ determine a from

$$a = \frac{\left[(\bar{p}^2 - p_{wf}^2)_s - b q_{gs}^2 \right]}{q_{gs}}$$

3. The stabilized deliverability curve uses the constants determined in Steps 1 and 2:

$$\bar{p}^2 - p_{wf}^2 = a q_g + b q_g^2$$

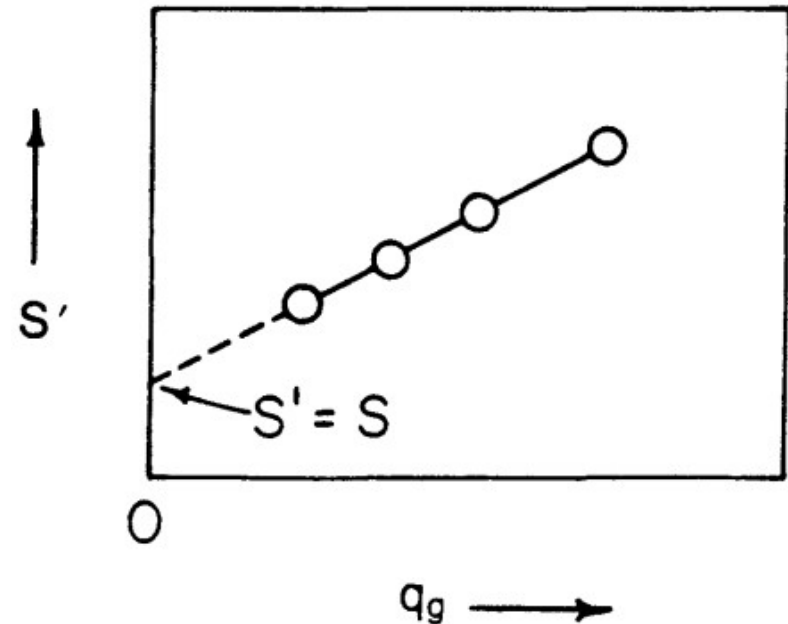
✓ This equation can be used to calculate the AOF, :

$$AOF = \frac{-a + \sqrt{a^2 + 4b(\bar{p}^2 - 14.7^2)}}{2b}$$

□ Read example 5.2 on page 82 John Lee .

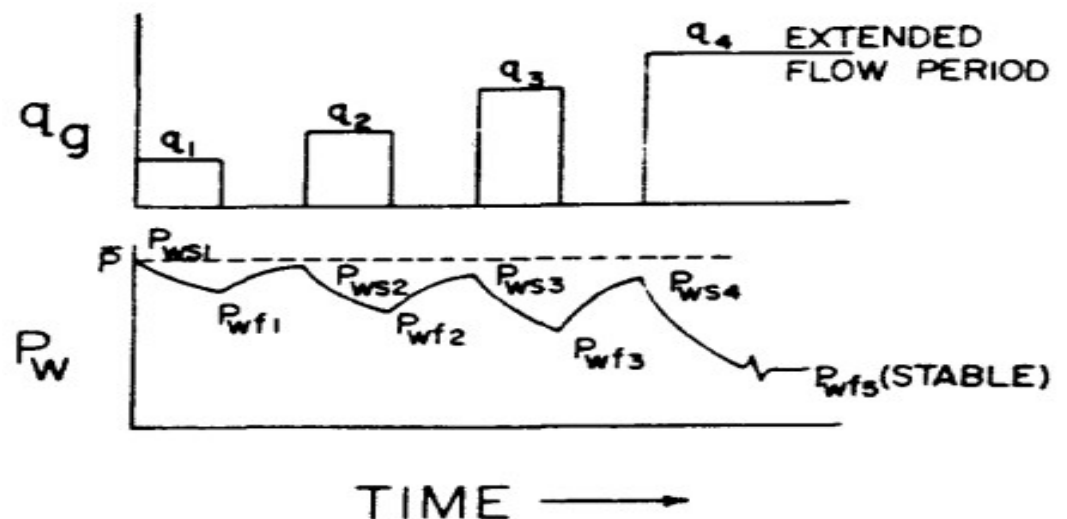
- Since an isochronal test consists of a series of draw down and buildup tests. kh and s usually can be determined from them.
- .Recall that a single test provides only an estimate of $s' = s + Dq_g$
- To determine s , we must analyze **at least two tests**: either drawdown tests run at different rates or buildup tests following drawdown tests at different rates.

- We can then plot s' vs. q_g ; extrapolation to $q_g = 0$ provides an estimate of true skin factor s

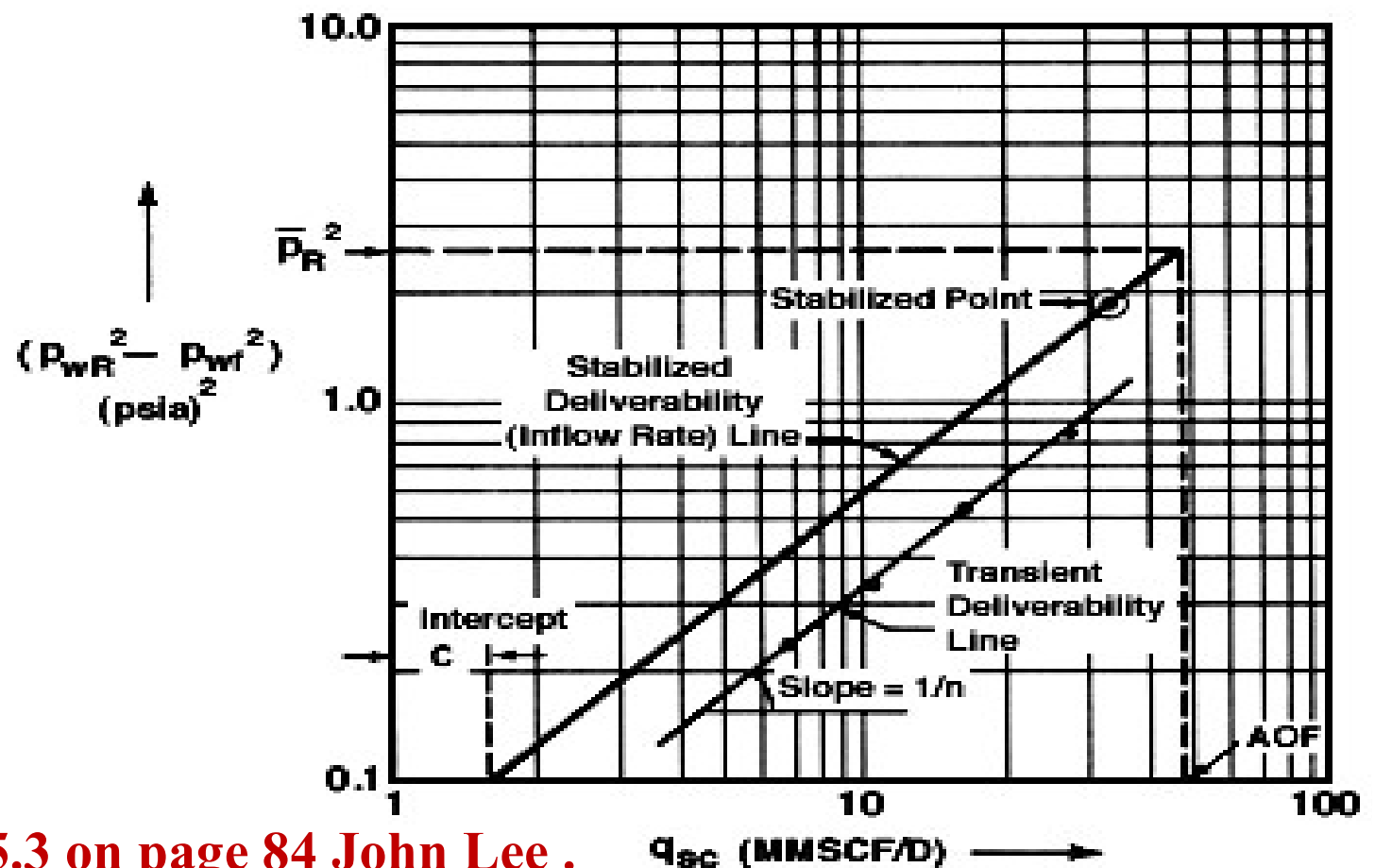


Modified Isochronal Tests

- The objective of modified isochronal tests is to obtain the same data as in an isochronal test **without lengthy shut-in periods** required for pressure to stabilize completely.
- In the modified isochronal test, **shut-in periods of the same duration as the flow periods** are used.
- The final shut-in BHP (P_{ws}) before the beginning of a new flow period is used as an approximation to $\bar{p}$ in the test analysis procedure.



For the first flow period, use $(\bar{p}^2 - P_{wf,1}^2) = (P_{ws,1}^2 - P_{wf,1}^2)$
 for the second flow period, use $(P_{ws,2}^2 - P_{wf,2}^2)$. **Otherwise, the analysis procedure is the same as for the "true" isochronal test.**



□ Read example 5.3 on page 84 John Lee .



Chapter 7

Software Review

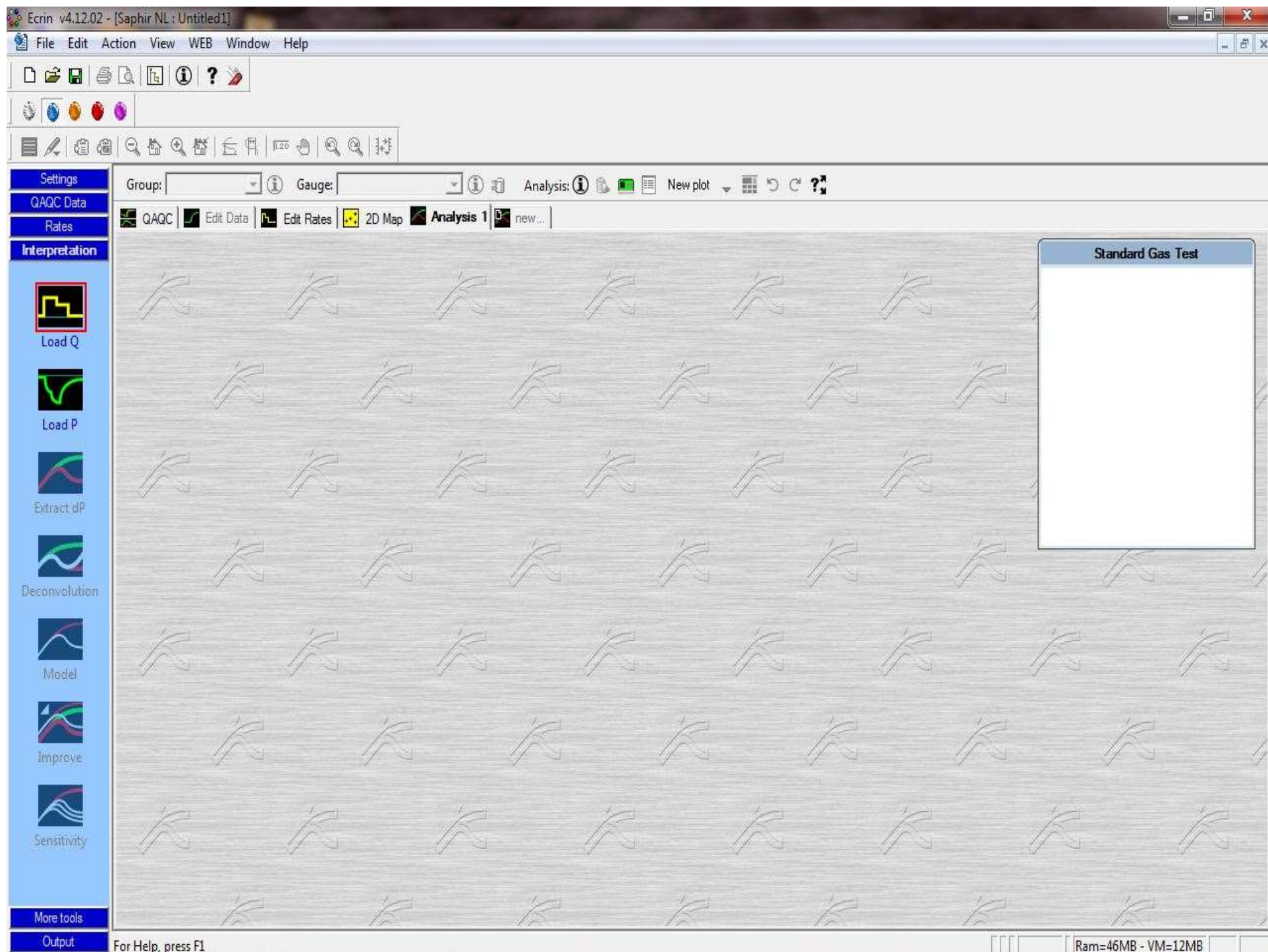
- **In well testing in general and in well test interpretation software there are several emerging trends:**

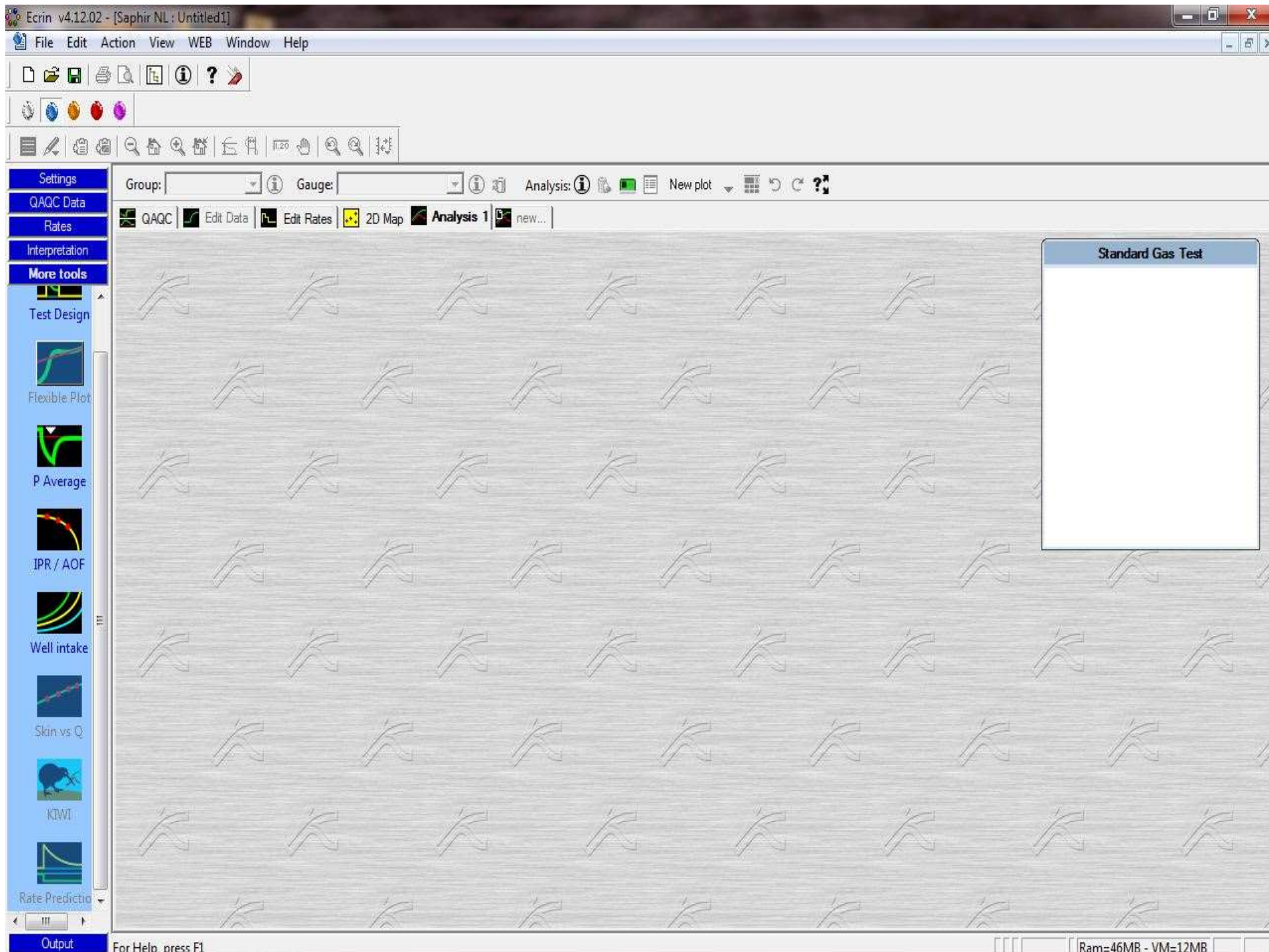
- This market segment is very competitive.**
- Popular software are easy to use and follow a common integrated interpretation methodology.**
- Ease of use at the expense of functionality (and vice versa) is not tolerated for long.**

- **The technical community is up to date with state-of-the-art technology - demands/needs the latest to be incorporated.**
- **Numerical well testing is becoming popular.**
- **Integrated interpretation methodology incorporating both analytical and numerical techniques is required.**

Saphir (Kappa)

- **One of the easiest to use and most popular well testing packages available in the market today. Has around 1400 site licenses with 200 companies(including Schlumberger).**
- **A very simple application. However the latest version - Level 3 - has a numerical option.**
- **Efficient window management. Good user interface; has a single window to display plots as you move along in your interpretation workflow.**
- **Saphir is very rich in functionality and is under active development.**

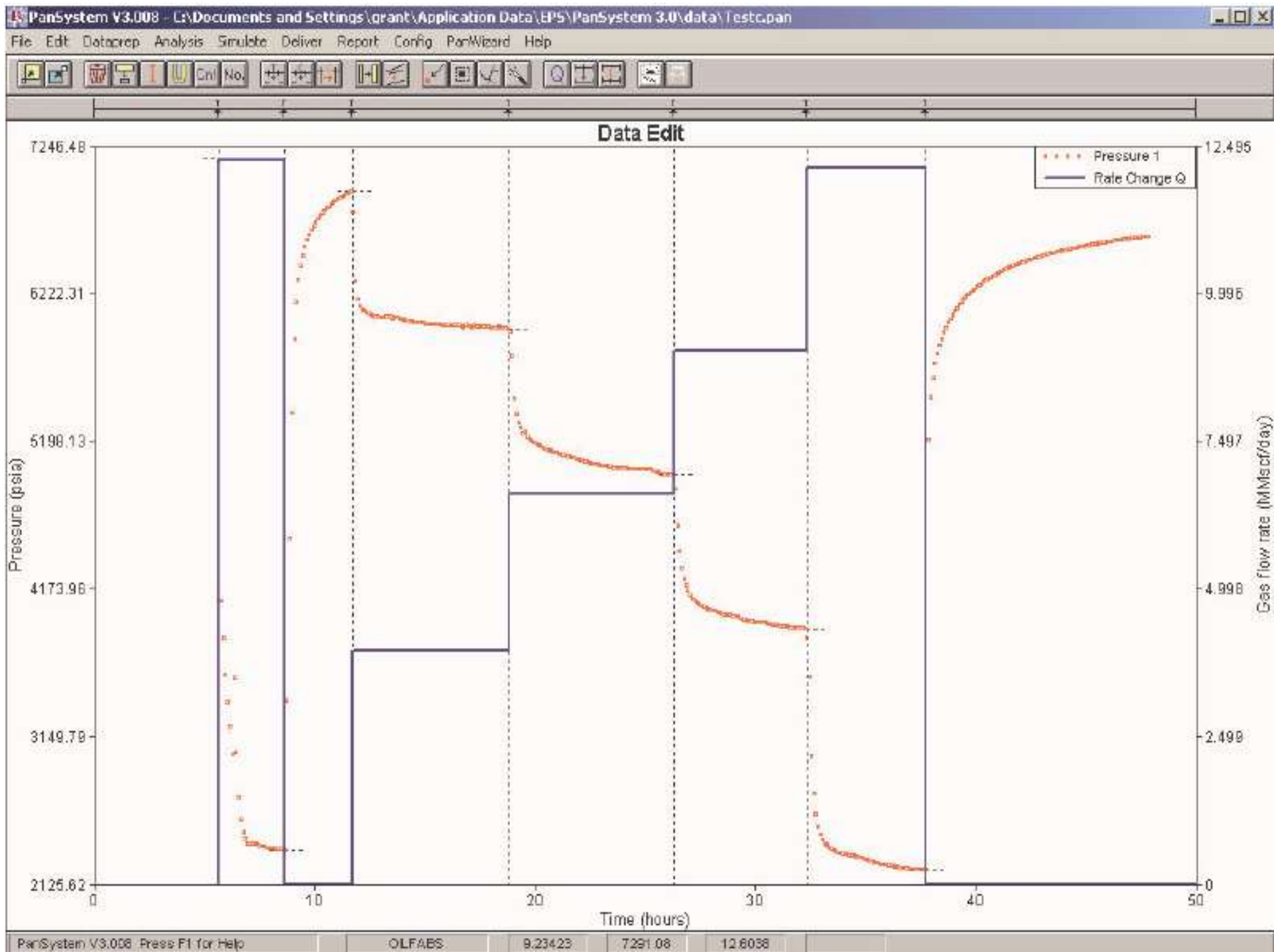






PanSystem (EPS)

- **PanSystem is a popular well test analysis software -probably more 'scientific' than Saphir.**
- **Reasonable graphical user interface and has all features of an advanced well test interpretation package.**
- **Has an integrated interpretation environment involving derivative plots, specialized analysis, and non-linear regression.**
- **PanSystem has an interface to numerical simulation PanMesh is based on finite-element technology.**





Interpret 2000 (Baker Hughes)

- **Interpret 2000, was previously known as Interpret/2(SSI).**
- **Conventional analytical interpretation and modeling application much of the original program development was by **Alain Gringarten**.**
- **No numerical options.**
- **Has a nice user interface, which appears to be very similar to Saphir.**

Zodiac (Schlumberger)

- **Has a long history, and has been around since 1992 when it replaced earlier software called Star. Now linked to GeoFrame , and in maintenance mode only.**
- **Consists of two separate programs: test design and test interpretation. Each of these is divided into a number of sub-modules.**
- **Has a useful section for layered reservoir tests and selective inflow performance (SIP) analysis.**
- **Does provide good analytical techniques for interpreting well tests, and has some functionality which is still absent in many well test analysis packages.**

BorDyn (Schlumberger)

- **BorDyn is pressure transient analysis software for test validations at the well site.**
- **Primarily used to insure that the test objectives have been reached by monitoring data integrity and providing the means for a simple interpretation of the data during its acquisition.**
- **Functionalities include:**
 - **real time plotting, transient definition, derivative and convolution derivative analysis, flow regime identification and associated specialized plots, etc.**

Well test 200 (Schlumberger)

- **Well test 200 is an integrated well test analysis package which is able to use ECLIPSE to calculate numerical solutions to well tests.**
- **It is partially integrated within the ECLIPSE suite of applications, e.g. SimOpt.**
- **Allows users to validate their raw well test data, perform conventional analytical interpretation and interactively prepare a numerical model.**
- **Has an innovative type of gridding called perpendicular bisection (PEBI) or Voronoi grids.**