

## **INSTRUMENTATION AND CONTROL**

### **TUTORIAL 4 – INSTRUMENT SYSTEM MODELS AND CALIBRATION**

This tutorial is mainly about instrument systems and simple mathematical models. It brings together the various elements covered in tutorials 2 and 3. It leads into more advance work on control system models. It is provided mainly in support of the EC module D227 – Control System Engineering.

On completion of this tutorial, you should be able to do the following.

- Explain the model of a basic instrument system.
- Calculate the relationship between input and output for complete system.
- Explain and identify the main errors that occur in instrument systems.
- Explain the basic principles of calibration.
- Explain primary and secondary standards.

In order to complete the theoretical part of this tutorial, you must be familiar with basic mechanical and electrical science.

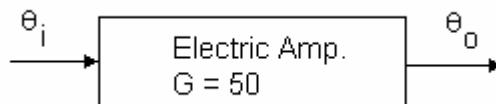
## 1. MODELS OF INSTRUMENT SYSTEMS

A mathematical model relates the input and output of a system or sub-system. In other words it is a formula relating the input and output. The instrument is usually drawn as a block with the input and output shown. The mathematical model is written inside the block. The general symbol for signals is  $\theta$  but specific symbols may be used. The suffix  $i$  denotes the input and  $o$  the output.

When the input and output is a simple ratio, the model is just a number representing the ratio of output to input. It is often denoted by  $G$ , especially if it is a gain. In such case  $G = \theta_o/\theta_i$ . If the input and output have different units, then  $G$  has units also.

### WORKED EXAMPLE No.1

Find the output if the input is 10 mW. The gain is a ratio and not in decibels.

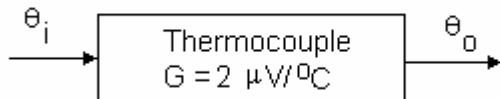


### SOLUTION

$$G = \theta_o/\theta_i = 50 \quad \theta_o = 50 \times \theta_i = 50 \times 10 = 500 \text{ mW}$$

### WORKED EXAMPLE No.2

Find the output if the input is 50°C.

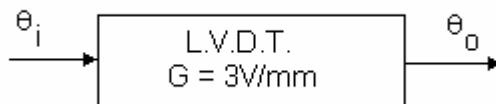


### SOLUTION

$$G = \theta_o/\theta_i = 2 \mu\text{V}/\text{°C} \quad \theta_i = \theta_o/2 = 50/2 = 25 \mu\text{V}$$

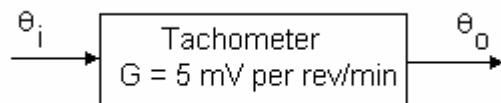
### SELF ASSESSMENT EXERCISE No.1

1. The input is 2 mm, find the output.



(Answer 6 V)

3. The input is 250 rev/min, find the output.



(Answer 1.25 V)

Some sensors have non linear equations and we cannot represent the relationship with a simple ratio so must use the full equation. For example a differential pressure flow meter has an equation

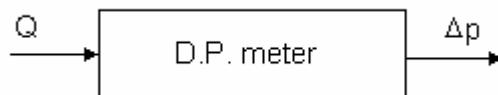
$$\text{Flow rate} = C (\Delta p)^{1/2}$$

Where  $C$  is a constant and  $\Delta p$  is the differential pressure.

### **WORKED EXAMPLE No.3**

The input and output of the D.P. meter is related by the law  $Q = C (\Delta p)^{1/2}$

Where  $Q$  is the input flow rate in  $\text{m}^3/\text{s}$ ,  $\Delta p$  is the output differential pressure and  $C$  is the meter constant. Determine the flow rate when  $\Delta p = 250 \text{ Pa}$  and  $C = 0.0004 \text{ m}^3/\text{s}$  per  $\text{Pa}$ .

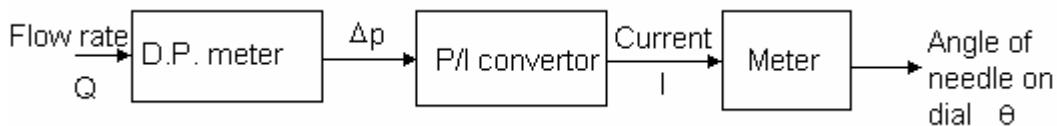


### **SOLUTION**

$$Q = C (\Delta p)^{1/2} = 0.0004 (250)^{1/2} = 0.00632 \text{ m}^3/\text{s} \text{ or } 6.32 \text{ dm}^3/\text{s}$$

## 2. MODELS FOR COMPLETE SYSTEMS

A complete instrument system is made up from several sub-systems connected in series. The best way to deduce the input or output of a complete system is a step by step analysis of the information passing through. Consider the case of a D.P. flow meter. The meter converts flow rate into differential pressure. The d.p. is then converted into current and the current is indicated on a meter.



### WORKED EXAMPLE No.4

The equations for each sub system in the above system is as follows.

Flow meter  $\Delta p = 2 \times 10^6 Q^2$  where  $p$  is in bar and  $Q$  is  $\text{m}^3/\text{s}$ .

P/I converter  $I = 20 \Delta p$  where  $I$  is mA and  $p$  is in bars.

Ammeter  $\theta = 14 I$  where  $\theta$  is in degrees and  $I$  in mA.

Calculate the output angle if the flow rate is  $0.0004 \text{ m}^3/\text{s}$ .

### SOLUTION

#### STEP 1

Calculate the differential pressure.

$$\Delta p = 2 \times 10^6 Q^2 = 2 \times 10^6 (0.0004)^2 = 0.32 \text{ bar}$$

#### STEP 2

Calculate the current.

$$I = 20 \Delta p = 20 \times 0.32 = 6.4 \text{ mA}$$

#### STEP 3

Calculate the angle of the needle on the dial.

$$\theta = 140 I = 140 \times 6.4 = 89.6^\circ$$

## SELF ASSESSMENT EXERCISE No.2

1. Find the output rotation for the temperature system below. the equations for each sub-system are as follows.

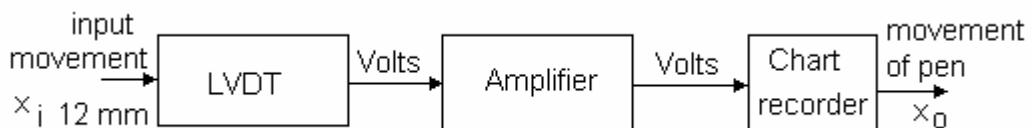
Thermocouple      3 mV per  $^{\circ}\text{C}$ .  
 Amplifier gain      5  
 Meter              0.02 $^{\circ}$  per mV



(Answer 54 $^{\circ}$ )

2. An LVDT output is amplified and connected to a chart recorder. The block diagram is shown. Calculate the output movement of the chart recorder pen. The equations for the sub- systems are

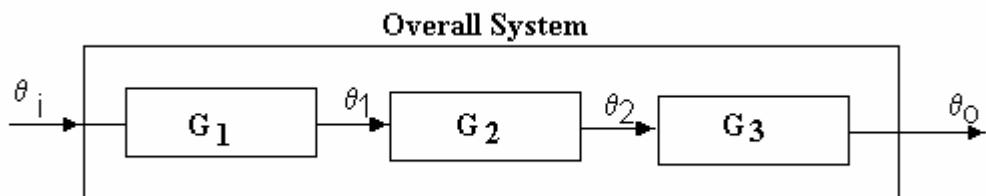
LVDT       $V = 0.2 \text{ V/mm}$   
 Amplifier      gain = 4  
 Chart recorder       $x_0 = 5 \text{ mm/V}$



(Answer 48 mm)

You have just seen how to work out problems involving instrument systems with different subsystems connected in series. The following is true for all types of systems.

In many cases each block may have a model that can be written as a ratio of output to input  $G = \theta_o/\theta_i$ . (This is not always true). In such cases we can easily work out the model for the complete system as follows. Consider three systems with model equations  $G_1$ ,  $G_2$  and  $G_3$  connected in series.



By definition

$$G_1 = \theta_1/\theta_i \quad G_2 = \theta_2/\theta_1 \quad G_3 = \theta_o/\theta_2$$

Now consider that if the three make up a single system the overall transfer function is  $G_{\text{overall}} = \theta_o/\theta_i$

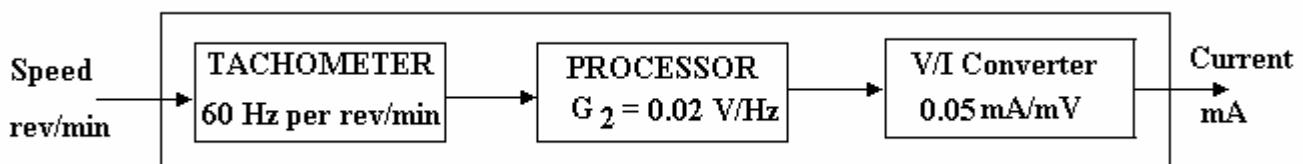
If we multiply  $G_1 \times G_2 \times G_3$  we have  $(\theta_1/\theta_i)(\theta_2/\theta_1)(\theta_o/\theta_2) = \theta_o/\theta_i = G_{\text{overall}}$

From this we conclude that the model for systems in series is obtained by multiplying the individual equations (ratios) together. Before doing this, make sure that the units are compatible.

### **SELF ASSESSMENT EXERCISE No.3**

1. A potentiometer produces 50 mV per degree of rotation of its shaft. Calculate the angle when the output is 4 V.  
**(Answer 80°)**
2. A turbine flow meter coupled to an electric voltage generator produces 4 mV for each litre/s flowing. Calculate the output when 1 V is produced.  
**(Answer 250 litres/s)**
3. An instrument system consists of pressure transducer with a range of 0 to 5 bar and a corresponding output of 0 to 10 mV. The output is connected to an electronic processor which converts the output into a current in the range 4 to 20 mA, and an analogue meter which indicates the measured pressure.
  - a. Draw a neat labelled block diagram of the system.
  - b. Deduce and write down the equation linking the output and input of the pressure transducer.  
**(Answer  $G = 2\text{mV/bar}$ )**
  - c. Deduce and write down the equation linking the input and output of the processor.  
**(Answer  $I = 4 + 1.5V$ )**
  - d. The output of the signal processor is 15 mA. Deduce the indicated pressure.  
**(Answer 3.67 bar)**
4. An instrument system comprises an a.c. tachometer connected to a processor that converts the frequency into millivolts and then another processor that converts millivolts into millamps. The data is shown on the diagram.

Write down the overall relationship between current and speed. **(Answer 0.06 mA per rev/min)**  
Calculate the output when the input speed is 400 rev/min. **(Answer 24 mA)**



### **3. INSTRUMENT ERRORS**

Any given instrument is prone to errors either due to aging or due to manufacturing tolerances. Here are some of the common terms used when describing the performance of an instrument.

#### **3.1 RANGE**

The range of an instrument is usually regarded as the difference between the maximum and minimum reading. For example a thermometer that has a scale from 20 to 100°C has a range of 80°C. This is also called the FULL SCALE DEFLECTION (f.s.d.).

#### **3.2 ACCURACY**

The accuracy of an instrument is often stated as a % of the range or full scale deflection. For example a pressure gauge with a range 0 to 500 kPa and an accuracy of plus or minus 2% f.s.d. could have an error of plus or minus 10 kPa. When the gauge is indicating 10 kPa the correct reading could be anywhere between 0 and 20 kPa and the actual error in the reading could be 100%. When the gauge indicates 500 kPa the error could be 2% of the indicated reading.

#### **3.3 REPEATABILITY**

If an accurate signal is applied and removed repeatedly to the system and it is found that the indicated reading is different each time, the instrument has poor repeatability. This is often caused by friction or some other erratic fault in the system.

#### **3.4 STABILITY**

Instability is most likely to occur in instruments involving electronic processing with a high degree of amplification. A common cause of this is adverse environment factors such as temperature and vibration. For example, a rise in temperature may cause a transistor to increase the flow of current which in turn makes it hotter and so the effect grows and the displayed reading DRIFTS. In extreme cases the displayed value may jump about. This, for example, may be caused by a poor electrical connection affected by vibration.

#### **3.5 TIME LAG ERROR**

In any instrument system, it must take time for a change in the input to show up on the indicated output. This time may be very small or very large depending upon the system. This is known as the response time of the system. If the indicated output is incorrect because it has not yet responded to the change, then we have time lag error.

A good example of time lag error is an ordinary glass thermometer. If you plunge it into hot water, it will take some time before the mercury reaches the correct level. If you read the thermometer before it settled down, then you would have time lag error. A thermocouple can respond much more quickly than a glass thermometer but even this may be too slow for some applications.

When a signal changes a lot and quite quickly, (speedometer for example), the person reading the dial would have great difficulty determining the correct value as the dial may be still going up when in reality the signal is going down again.

#### **3.6 RELIABILITY**

Most forms of equipment have a predicted life span. The more reliable it is, the less chance it has of going wrong during its expected life span. The reliability is hence a probability ranging from zero (it will definitely fail) to 1.0 (it will definitely not fail).

### **6. DRIFT**

This occurs when the input to the system is constant but the output tends to change slowly. For example when switched on, the system may drift due to the temperature change as it warms up.

## 4. INSTRUMENT CALIBRATION

Most instruments contain a facility for making two adjustments. These are

- The RANGE adjustment.
- The ZERO adjustment.

In order to calibrate the instrument an accurate gauge is required. This is likely to be a SECONDARY STANDARD. Instruments calibrated as a secondary standard have themselves been calibrated against a PRIMARY STANDARD.

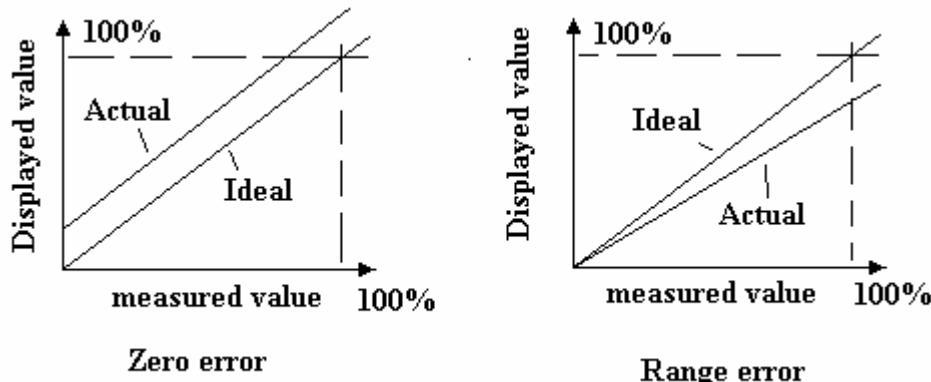
### 4.1 PROCEDURE

An input representing the minimum gauge setting should be applied. The output should be adjusted to be correct. Next the maximum signal is applied. The range is then adjusted to give the required output. This should be repeated until the gauge is correct at the minimum and maximum values.

### 4.2 CALIBRATION ERRORS

#### RANGE AND ZERO ERRO

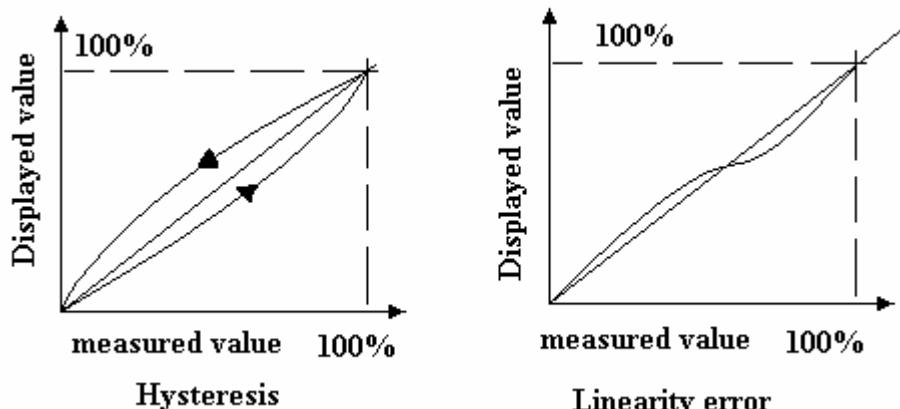
After obtaining correct zero and range for the instrument, a calibration graph should be produced. This involves plotting the indicated reading against the correct reading from the standard gauge. This should be done in about ten steps with increasing signals and then with reducing signals. Several forms of error could show up. If the zero or range is still incorrect the error will appear as shown.



#### HYSTERESIS and NON LINEAR ERRORS

Hysteresis is produced when the displayed values are too small for increasing signals and too large for decreasing signals. This is commonly caused in mechanical instruments by loose gears and linkages and friction. It occurs widely with things involving magnetisation and demagnetisation.

The calibration may be correct at the maximum and minimum values of the range but the graph joining them may not be a straight line (when it ought to be). This is a non linear error. The instrument may have some adjustments for this and it may be possible to make it correct at mid range as shown.



### **WORKED EXAMPLE No.5**

A digital thermometer reads from -120 to + 300 °C. The accuracy is guaranteed to plus or minus 2% f.s.d. Determine the possible temperature range when it indicates 80°C.

#### **SOLUTION**

The range of the instrument is the same as the f.s.d. and is 420. The accuracy is hence plus or minus 2% of 420.

$$\text{Accuracy} = (2/100)(420) = 8.4$$

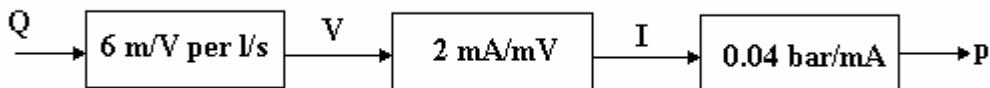
The possible temperature being indicated is hence 80 plus or minus 8.4. That means the actual temperature is between 88.4 and 71.6°C.

### SELF ASSESSMENT EXERCISE No.4

1. A digital speedometer has a range 0 to 3000 rev/min with a guaranteed accuracy of plus or minus 1%. Determine the possible correct speed when it indicates 1500 rev/min.

(Answer 1470 to 1530 rev/min)

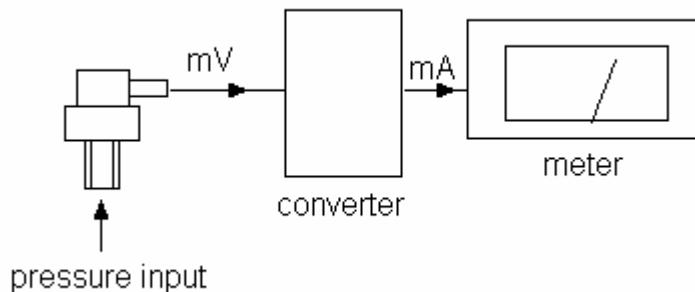
2. The block diagram shows how a flow measuring system works. The input is flow rate in litres/s and the output is air pressure in bars.



**Calculate the output of the system in bar when the input is 1.5 litres/s. (0.72 bar)**

3. A thermometer has a range from -20 to 150°C. The accuracy is guaranteed to  $\pm 3\%$  f.s.d. Determine the possible temperature range when it indicates a temperature of 80°C. (85.1 to 74.9 °C)

4. The diagram illustrates a pressure measuring system.



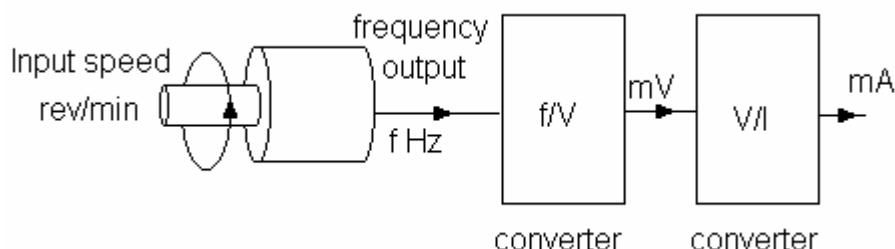
The system consists of pressure transducer connected to a converter. The output mV is related to the input pressure by  $V = 30 p$  where  $p$  is in bar and  $V$  is in mV.

The converter output in mA is related to the input by  $I = 4 + 0.2V$ .

The current is sent to a meter. What is the pressure when the meter indicates 15mA? (1.833 bar)

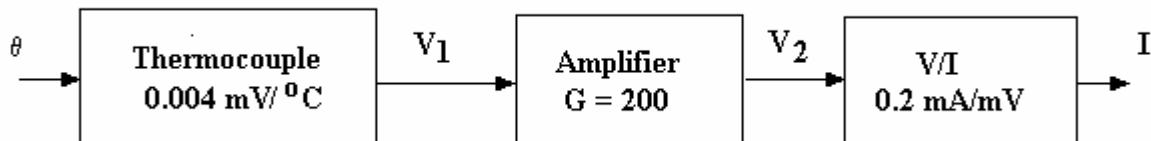
5. A pressure gauge has a range of 0 to 80 bar and is guaranteed to be accurate to within  $\pm 4\%$  f.s.d. Calculate the possible pressure range when the gauge indicates 50 bar. (46.8 to 53.2 bar)

6. The diagram shows a system for measuring rotational speed.



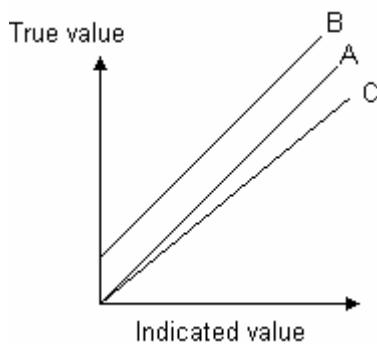
The tachometer produces one cycle of electricity for every revolution. The  $f/V$  converter converts the frequency into mV such that  $V = 0.6 f$ . The  $V/I$  converter produces an output of 0.8 mA/mV. Calculate the output when the tachometer is turned at 3000 rev/min. (24 mA)

7. The diagram shows a system for measuring temperature and producing the result in the form of a current in the range 4 to 20 mA.

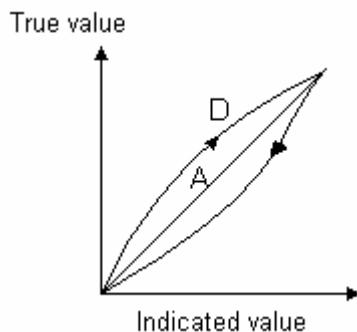


Determine the current when the temperature is 120°C. (19.2 mA)

8. An anemometer is designed to measure air speeds in the range 2 to 30 m/s. The instrument is guaranteed accurate to within  $\pm 2\%$  f.s.d. Calculate the possible range of wind speed when the instrument indicates 20 m/s. (20.56 to 19.44 m/s)
9. The diagram shows the results of a calibration of an instrument. Graph A is the ideal one. State the kind of error, which is shown on graphs B and C.



10. An experiment to calibrate a liquid expansion thermometer is carried out by placing the sensor in a vat of oil and heating it up and then cooling it down. The true temperature is measured with a high quality instrument and the indicated temperature is compared with it. The graph produced is shown below.



State two possible reasons for the kind of error shown.