10 Open-Channel Flow

Chapter Opening Photo: **Hydraulic jump:** Under certain conditions, when water flows in an open channel, even if it has constant geometry, the depth of the water may increase considerably over a short distance along the channel. This phenomenon is termed a hydraulic jump (water flow from left to right).

Learning Objectives

After completing this chapter, you should be able to:

- discuss the general characteristics of open-channel flow.
- use a specific energy diagram.
- apply appropriate equations to analyze open-channel flow with uniform depth.
- calculate key properties of a hydraulic jump.
- determine flowrates based on open-channel flow-measuring devices.

Open-channel flow involves the flow of a liquid in a channel or conduit that is not completely filled. A free surface exists between the flowing fluid (usually water) and fluid above it (usually the atmosphere). The main driving force for such flows is the fluid weight—gravity forces the fluid to flow downhill. Most open-channel flow results are based on correlations obtained from model and full-scale experiments. Additional information can be gained from various analytical and numerical efforts.

Open-channel flows are essential to the world as we know it. The natural drainage of water through the numerous creek and river systems is a complex example of open-channel flow. Although the flow geometry for these systems is extremely complex, the resulting flow properties are of considerable economic, ecological, and recreational importance. Other examples of open-channel flows include the flow of rainwater in the gutters of our houses; the flow in canals, drainage ditches, sewers, and gutters along roads; the flow of small rivulets and sheets of water across fields or parking lots; and the flow in the chutes of water rides in amusement parks.

V10.1 Off-shore oil drilling platform.

Open-channel flow involves the existence of a free surface which can distort into various shapes. Thus, a brief introduction into the properties and characteristics of surface waves is included.

The purpose of this chapter is to investigate the concepts of open-channel flow. Because of the amount and variety of material available, only a brief introduction to the topic can be presented. Further information can be obtained from the references indicated.

10.1 General Characteristics of Open-Channel Flow

Open-channel flow can have a variety of characteristics.

In our study of pipe flow (Chapter 8), we found that there are many ways to classify a flow—developing, fully developed, laminar, turbulent, and so on. For open-channel flow, the existence of a free surface allows additional types of flow. The extra freedom that allows the fluid to select its free-surface location and configuration (because it does not completely fill a pipe or conduit) allows important phenomena in open-channel flow that cannot occur in pipe flow. Some of the classifications of the flows are described below.

The manner in which the fluid depth, y, varies with time, t, and distance along the channel, x, is used to partially classify a flow. For example, the flow is *unsteady* or *steady* depending on whether the depth at a given location does or does not change with time. Some unsteady flows can be viewed as steady flows if the reference frame of the observer is changed. For example, a tidal bore (difference it water level) moving up a river is unsteady to an observer standing on the bank, but steady to an observer moving along the bank with the speed of the wave front of the bore. Other flows are unsteady regardless of the reference frame used. The complex, time-dependent, wind-generated waves on a lake are in this category. In this book we will consider only steady open-channel flows.

Uniform flow

An open-channel flow is classified as *uniform flow* (UF) if the depth of flow does not vary along the channel ($dy/dx = 0$). Conversely, it is *nonuniform flow* or *varied flow* if the depth varies with distance ($dy/dx \neq 0$). Nonuniform flows are further classified as *rapidly varying flow* (RVF) if the flow depth changes considerably over a relatively short distance; $dy/dx \sim 1$. *Gradually varying flows* (GVF) are those in which the flow depth changes slowly with distance along the channel; $dy/dx \ll 1$. Examples of these types of flow are illustrated in Fig. 10.1 and the photographs in the margin. The relative importance of the various types of forces involved (pressure, weight, shear, inertia) is different for the different types of flows.

Rapidly varying flow (photograph courtesy of Stillwater Sciences).

As for any flow geometry, open-channel flow may be *laminar*, *transitional*, or *turbulent*, depending on various conditions involved. Which type of flow occurs depends on the Reynolds number, $\mathrm{Re} = \rho V R_h/\mu$, where V is the average velocity of the fluid and R_h is the hydraulic radius of the channel (see Section 10.4). A general rule is that open-channel flow is laminar if $\mathrm{Re} < 500$, turbulent if $\mathrm{Re} > 12{,}500$, and transitional otherwise. The values of these dividing Reynolds numbers are only approximate—a precise knowledge of the channel geometry is necessary to obtain specific values. Since most open-channel flows involve water (which has a fairly small viscosity) and have relatively large characteristic lengths, it is rare to have laminar open-channel flows. For example, flow of 50 °F water ($\nu = 1.41 \times 10^{-5}\ \mathrm{ft^2/s}$) with an average velocity of $V = 1$ ft/s in a river with a hydraulic radius of $R_h = 10$ ft has $\mathrm{Re} = VR_h/\nu = 7.1 \times 10^5$. The flow is turbulent. However, flow of a thin sheet of water down a driveway with an average velocity of $V = 0.25$ ft/s such that $R_h = 0.02$ ft (in such cases the hydraulic radius is approximately equal to the fluid depth; see Section 10.4) has $\mathrm{Re} = 355$. The flow is laminar.

In some cases *stratified flows* are important. In such situations layers of two or more fluids of different densities flow in a channel. A layer of oil on water is one example of this type of flow. All of the open-channel flows considered in this book are *homogeneous flows.* That is, the fluid has uniform properties throughout.

Open-channel flows involve a free surface that can deform from its undisturbed relatively flat configuration to form waves. Such waves move across the surface at speeds that depend on

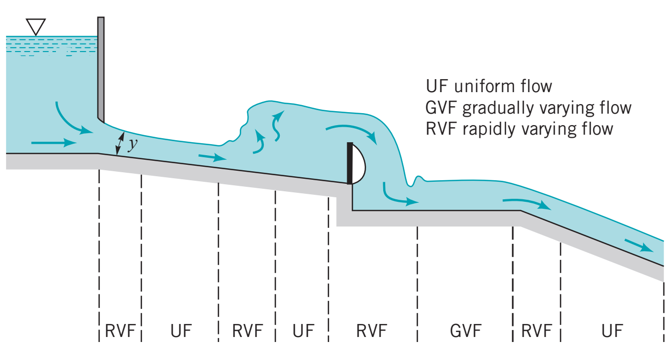
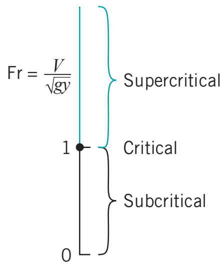


FIGURE 10.1 **Classification of open-channel flow.**



their size (height, length) and properties of the channel (depth, fluid velocity, etc.). The character of an open-channel flow may depend strongly on how fast the fluid is flowing relative to how fast a typical wave moves relative to the fluid. The dimensionless parameter that describes this behavior is termed the ***Froude number,*** $\text{Fr} = V/(g\ell)^{1/2}$, where ℓ is an appropriate characteristic length of the flow. This dimensionless parameter was introduced in Chapter 7 and is discussed more fully in Section 10.2. As shown by the figure in the margin, the special case of a flow with a Froude number of unity, $\text{Fr} = 1$, is termed a ***critical flow.*** If the Froude number is less than 1, the flow is ***subcritical*** (or *tranquil*). A flow with the Froude number greater than 1 is termed ***supercritical*** (or *rapid*).

10.2 Surface Waves

The distinguishing feature of flows involving a free surface (as in open-channel flows) is the opportunity for the free surface to distort into various shapes. The surface of a lake or the ocean is seldom "smooth as a mirror." It is usually distorted into ever-changing patterns associated with surface waves as shown in the photos in the margin. Some of these waves are very high, some barely ripple the surface; some waves are very long (the distance between wave crests), some are short; some are breaking waves that form whitecaps, others are quite smooth. Although a general study of this wave motion is beyond the scope of this book, an understanding of certain fundamental properties of simple waves is necessary for open-channel flow considerations. The interested reader is encouraged to use some of the excellent references available for further study about wave motion (Refs. 1, 2, 3).

Fluids in the News

Rogue Waves There is a long history of stories concerning giant rogue ocean *waves* that come out of nowhere and capsize ships. The movie *Poseidon* (2006) is based on such an event. Although these giant, freakish waves were long considered fictional, recent satellite observations and computer simulations prove that, although rare, they are real. Such waves are single, sharply-peaked mounds of water that travel rapidly across an otherwise relatively calm ocean. Although most ships are designed to withstand waves up to 15 meters high, satellite measurements and data from offshore oil platforms indicate that such rogue waves can reach a height of 30 meters. Although researchers still do not understand the formation of these large rogue waves, there are several suggestions as to how ordinary smaller waves can be focused into one spot to produce a giant wave. Additional theoretical calculations and wave tank experiments are needed to adequately grasp the nature of such waves. Perhaps it will eventually be possible to predict the occurrence of these destructive waves, thereby reducing the loss of ships and life because of them.

10.2.1 Wave Speed

V10.2 Filling your car's gas tank.

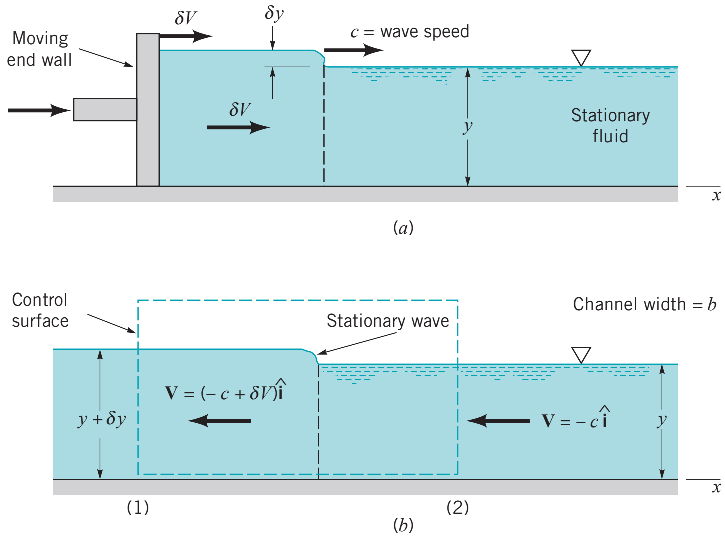
Consider the situation illustrated in Fig. 10.2*a* in which a single elementary wave of small height, δy, is produced on the surface of a channel by suddenly moving the initially stationary end wall with speed δV. The water in the channel was stationary at the initial time, $t = 0$. A stationary observer will observe a single wave move down the channel with a ***wave speed*** c, with no fluid motion ahead of the wave and a fluid velocity of δV behind the wave. The motion is unsteady for such an observer. For an observer moving along the channel with speed c, the flow will appear steady as shown in Fig. 10.2*b*. To this observer, the fluid velocity will be $\mathbf{V} = -c\hat{\mathbf{i}}$ on the observer's right and $\mathbf{V} = (-c + \delta V)\hat{\mathbf{i}}$ to the left of the observer.

The relationship between the various parameters involved for this flow can be obtained by application of the continuity and momentum equations to the control volume shown in Fig. 10.2*b* as follows. With the assumption of uniform one-dimensional flow, the continuity equation (Eq. 5.12) becomes

$$-cyb = (-c + \delta V)(y + \delta y)b$$

where b is the channel width. This simplifies to

$$c = \frac{(y + \delta y)\delta V}{\delta y}$$



■ **FIGURE 10.2** **(a) Production of a single elementary wave in a channel as seen by a stationary observer. (b) Wave as seen by an observer moving with a speed equal to the wave speed.**

or in the limit of small amplitude waves with $\delta y \ll y$

$$c = y \frac{\delta V}{\delta y} \tag{10.1}$$

Similarly, the momentum equation (Eq. 5.22) is

$$\tfrac{1}{2} \gamma y^2 b - \tfrac{1}{2} \gamma (y + \delta y)^2 b = \rho b c y [(c - \delta V) - c]$$

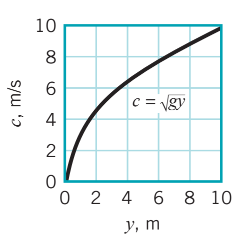
where we have written the mass flowrate as $\dot{m} = \rho bcy$ and have assumed that the pressure variation is hydrostatic within the fluid. That is, the pressure forces on the channel cross sections (1) and (2) are $F_1 = \gamma y_{c1} A_1 = \gamma(y + \delta y)^2 b/2$ and $F_2 = \gamma y_{c2} A_2 = \gamma y^2 b/2$, respectively. If we again impose the assumption of small amplitude waves [i.e., $(\delta y)^2 \ll y\, \delta y$], the momentum equation reduces to

The wave speed can be obtained from the continuity and momentum equations.

$$\frac{\delta V}{\delta y} = \frac{g}{c} \tag{10.2}$$

Combination of Eqs. 10.1 and 10.2 gives the wave speed

$$c = \sqrt{gy} \tag{10.3}$$

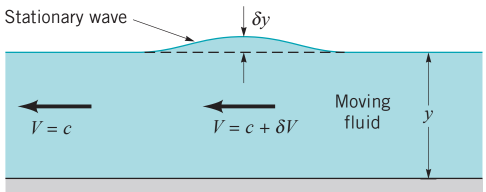


as indicated by the figure in the margin.

The speed of a small amplitude solitary wave as is indicated in Fig. 10.2 is proportional to the square root of the fluid depth, y, and independent of the wave amplitude, δy. The fluid density is not an important parameter, although the acceleration of gravity is. This is a result of the fact that such wave motion is a balance between inertial effects (proportional to ρ) and weight or hydrostatic pressure effects (proportional to $\gamma = \rho g$). A ratio of these forces eliminates the common factor ρ but retains g. For very small waves (like those produced by insects on water as shown in the photograph on the cover of the book), Eq. 10.3 is not valid because the effects of surface tension are significant.

The wave speed can also be calculated by using the energy and continuity equations rather than the momentum and continuity equations as is done above. A simple wave on the surface is shown in Fig. 10.3. As seen by an observer moving with the wave speed, c, the flow is steady. Since the pressure is constant at any point on the free surface, the Bernoulli equation for this frictionless flow is simply

$$\frac{V^2}{2g} + y = \text{constant}$$



■ **FIGURE 10.3** **Stationary simple wave in a flowing fluid.**

V10.3 Water strider

or by differentiating

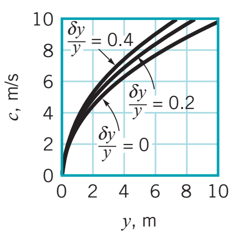
$$\frac{V\,\delta V}{g} + \delta y = 0$$

Also, by differentiating the continuity equation, $Vy = \text{constant}$, we obtain

$$y\,\delta V + V\,\delta y = 0$$

We combine these two equations to eliminate δV and δy and use the fact that $V = c$ for this situation (the observer moves with speed c) to obtain the wave speed given by Eq. 10.3.

The above results are restricted to waves of small amplitude because we have assumed one-dimensional flow. That is, $\delta y/y \ll 1$. More advanced analysis and experiments show that the wave speed for finite-sized solitary waves exceeds that given by Eq. 10.3. To a first approximation, one obtains (Ref. 4)



$$c \approx \sqrt{gy}\left(1 + \frac{\delta y}{y}\right)^{1/2}$$

As indicated by the figure in the margin, the larger the amplitude, the faster the wave travels.

A more general description of wave motion can be obtained by considering continuous (not solitary) waves of sinusoidal shape as is shown in Fig. 10.4. By combining waves of various wavelengths, λ, and amplitudes, δy, it is possible to describe very complex surface patterns found in nature, such as the wind-driven waves on a lake. Mathematically, such a process consists of using a Fourier series (each term of the series represented by a wave of different wavelength and amplitude) to represent an arbitrary function (the free-surface shape).

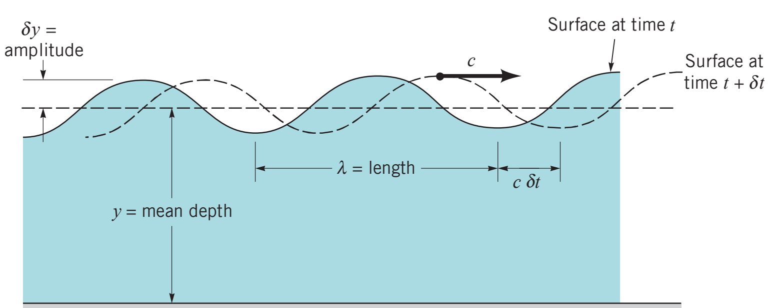
V10.4 Sinusoidal waves

A more advanced analysis of such sinusoidal surface waves of small amplitude shows that the wave speed varies with both the wavelength and fluid depth as (Ref. 1)

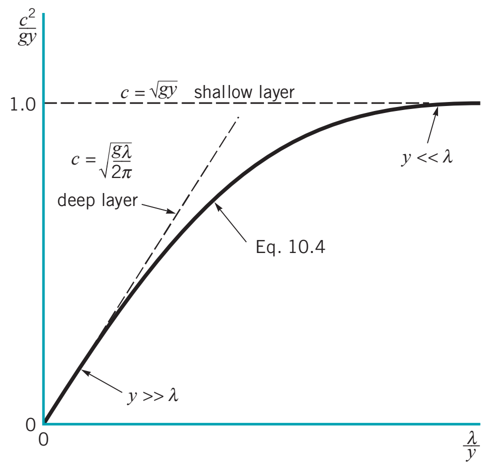
$$c = \left[\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi y}{\lambda}\right)\right]^{1/2} \tag{10.4}$$

where $\tanh(2\pi y/\lambda)$ is the hyperbolic tangent of the argument $2\pi y/\lambda$. The result is plotted in Fig. 10.5. For conditions for which the water depth is much greater than the wavelength ($y \gg \lambda$, as in the ocean), the wave speed is independent of y and given by

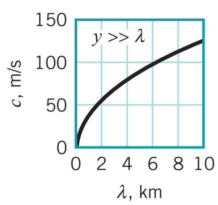
$$c = \sqrt{\frac{g\lambda}{2\pi}}$$



■ **FIGURE 10.4** **Sinusoidal surface wave.**



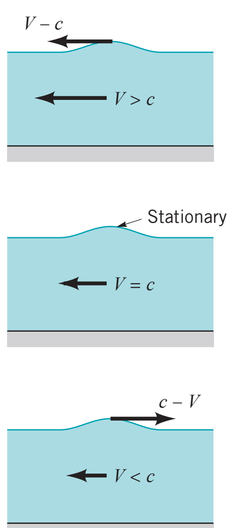
■ **FIGURE 10.5** **Wave speed as a function of wavelength.**



This result, shown in the figure in the margin, follows from Eq. 10.4, since $\tanh(2\pi y/\lambda) \to 1$ as $y/\lambda \to \infty$. Note that waves with very long wavelengths [e.g., waves created by a tsunami ("tidal wave") with wavelengths on the order of several kilometers] travel very rapidly. On the other hand, if the fluid layer is shallow ($y \ll \lambda$, as often happens in open channels), the wave speed is given by $c = (gy)^{1/2}$, as derived for the solitary wave in Fig. 10.2. This result also follows from Eq. 10.4, since $\tanh(2\pi y/\lambda) \to 2\pi y/\lambda$ as $y/\lambda \to 0$. These two limiting cases are shown in Fig. 10.5. For moderate depth layers ($y \sim \lambda$), the results are given by the complete Eq. 10.4. Note that for a given fluid depth, the long wave travels the fastest. Hence, for our purposes we will consider the wave speed to be this limiting situation, $c = (gy)^{1/2}$.

Fluids in the News

Tsunami, the nonstorm wave A tsunami, often miscalled a "tidal wave," is a wave produced by a disturbance (for example, an earthquake, volcanic eruption, or meteorite impact) that vertically displaces the water column. Tsunamis are characterized as shallow-water waves, with long periods, very long wavelengths, and extremely large wave speeds. For example, the waves of the great December 2005, Indian Ocean tsunami traveled with speeds to 500–1000 m/s. Typically, these waves were of small amplitude in deep water far from land. Satellite radar measured the wave height less than 1 m in these areas. However, as the waves approached shore and moved into shallower water, they slowed down considerably and reached heights up to 30 m. Because the rate at which a wave loses its energy is inversely related to its wavelength, tsunamis, with their wavelengths on the order of 100 km, not only travel at high speeds, they also travel great distances with minimal energy loss. The furthest reported death from the Indian Ocean tsunami occurred approximately 8000 km from the epicenter of the earthquake that produced it. (See Problem 10.14.)



10.2.2 Froude Number Effects

Consider an elementary wave traveling on the surface of a fluid, as is shown in the figure in the margin and Fig. 10.2*a*. If the fluid layer is stationary, the wave moves to the right with speed c relative to the fluid and the stationary observer. If the fluid is flowing to the left with velocity $V < c$, the wave (which travels with speed c relative to the fluid) will travel to the right with a speed of $c - V$ relative to a fixed observer. If the fluid flows to the left with $V = c$, the wave will remain stationary, but if $V > c$ the wave will be washed to the left with speed $V - c$.

The above ideas can be expressed in dimensionless form by use of the Froude number, $\mathrm{Fr} = V/(gy)^{1/2}$, where we take the characteristic length to be the fluid depth, y. Thus, the Froude number, $\mathrm{Fr} = V/(gy)^{1/2} = V/c$, is the ratio of the fluid velocity to the wave speed.

The following characteristics are observed when a wave is produced on the surface of a moving stream, as happens when a rock is thrown into a river. If the stream is not flowing, the wave spreads equally in all directions. If the stream is nearly stationary or moving in a tranquil manner (i.e., $V < c$), the wave can move upstream. Upstream locations are said to be in hydraulic communication with the downstream locations. That is, an observer upstream of a disturbance can tell that there has been a disturbance on the surface because that disturbance can propagate upstream to the observer. Viscous effects, which have been neglected in this discussion, will eventually damp out such waves far upstream. Such flow conditions, $V < c$, or $\mathrm{Fr} < 1$, are termed *subcritical*.

On the other hand, if the stream is moving rapidly so that the flow velocity is greater than the wave speed (i.e., $V > c$), no upstream communication with downstream locations is possible. Any disturbance on the surface downstream from the observer will be washed farther downstream. Such conditions, $V > c$ or $\mathrm{Fr} > 1$, are termed *supercritical*. For the special case of $V = c$ or $\mathrm{Fr} = 1$, the upstream propagating wave remains stationary and the flow is termed *critical*.

EXAMPLE 10.1

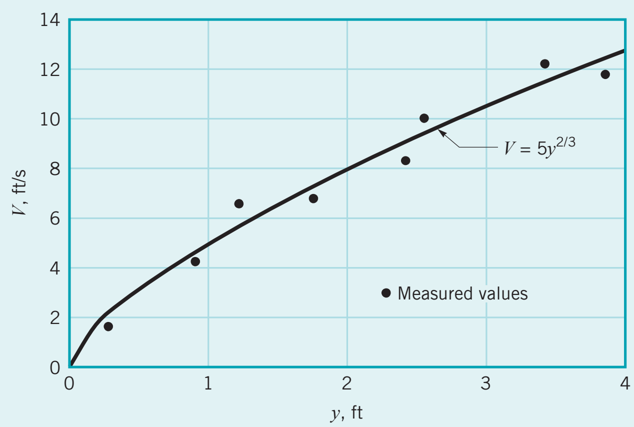
GIVEN At a certain location along the Rock River shown in Fig. E10.1*a*, the velocity, V, of the flow is a function of the depth, y, of the river as indicated in Fig. E10.1*b*. A reasonable approximation to these experimental results is

$$V = 5\,y^{2/3} \tag{1}$$

where V is in ft/s and y is in ft.

■ FIGURE E10.1*a*

FIND For what range of water depth will a surface wave on the river be able to travel upstream?



■ FIGURE E10.1*b*

SOLUTION

While the river travels to the left with speed V, the surface wave travels upstream (to the right) with speed $c = (g\,y)^{1/2}$ relative to the water (not relative to the ground). Hence relative to the stationary ground, the wave travels to the right with speed

$$\begin{aligned} c - V &= (g\,y)^{1/2} - 5\,y^{2/3} \\ &= (32.2\ \mathrm{ft/s^2}\,y)^{1/2} - 5\,y^{2/3} \end{aligned} \tag{2}$$

For the wave to travel upstream, $c - V > 0$ so that from Eq. 2,

$$(32.2\,y)^{1/2} > 5\,y^{2/3}$$

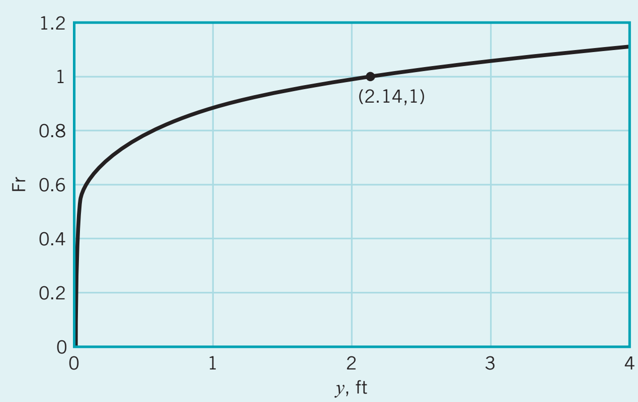
or

$$y < 2.14\ \mathrm{ft} \qquad \textbf{(Ans)}$$

COMMENT As shown above, if the river depth is less than 2.14 ft, its velocity is less than the wave speed and the wave can travel upstream against the current. This is consistent with the fact that if a wave is to travel upstream, the flow must be subcritical (i.e., $\mathrm{Fr} = V/c < 1$). For this flow

$$\begin{aligned} \mathrm{Fr} &= V/c = (5\,y^{2/3})/(g\,y)^{1/2} \\ &= 5\,y^{1/6}/(32.2\ \mathrm{ft/s^2})^{1/2} \\ &= 0.881\,y^{1/6} \end{aligned}$$

This result is plotted in Fig. E10.1*c*. Note that in agreement with the above answer, for $y < 2.14$ the flow is subcritical; the wave can travel upstream.



■ FIGURE E10.1*c*

The character of an open-channel flow may depend strongly on whether the flow is subcritical or supercritical. The characteristics of the flow may be completely opposite for subcritical flow than for supercritical flow. For example, as is discussed in Section 10.3, a "bump" on the bottom

V10.5 Bicycle through a puddle

of a river (such as a submerged log) may cause the surface of the river to dip below the level it would have had if the log were not there, or it may cause the surface level to rise above its undisturbed level. Which situation will happen depends on the value of Fr. Similarly, for supercritical flows it is possible to produce steplike discontinuities in the fluid depth (called a hydraulic jump; see Section 10.6.1). For subcritical flows, however, changes in depth must be smooth and continuous. Certain open-channel flows, such as the broad-crested weir (Section 10.6.3), depend on the existence of critical flow conditions for their operation.

As strange as it may seem, there exist many similarities between the open-channel flow of a liquid and the compressible flow of a gas. The governing dimensionless parameter in each case is the fluid velocity, V, divided by a wave speed, the surface wave speed for open-channel flow or sound wave speed for compressible flow. Many of the differences between subcritical ($\text{Fr} < 1$) and supercritical ($\text{Fr} > 1$) open-channel flows have analogs in subsonic ($\text{Ma} < 1$) and supersonic ($\text{Ma} > 1$) compressible gas flow, where Ma is the Mach number. Some of these similarities are discussed in this chapter and in Chapter 11.

10.3 Energy Considerations

The slope of the bottom of most open channels is very small; the bottom is nearly horizontal.

A typical segment of an open-channel flow is shown in Fig. 10.6. The slope of the channel bottom (or *bottom slope*), $S_0 = (z_1 - z_2)/\ell$, is assumed constant over the segment shown. The fluid depths and velocities are y_1, y_2, V_1, and V_2 as indicated. Note that the fluid depth is measured in the vertical direction and the distance x is horizontal. For most open-channel flows the value of S_0 is very small (the bottom is nearly horizontal). For example, the Mississippi River drops a distance of 1470 ft in its 2350-mi length to give an average value of $S_0 = 0.000118$. In such circumstances the values of x and y are often taken as the distance along the channel bottom and the depth normal to the bottom, with negligibly small differences introduced by the two coordinate schemes.

With the assumption of a uniform velocity profile across any section of the channel, the one-dimensional energy equation for this flow (Eq. 5.84) becomes

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \tag{10.5}$$

where h_L is the head loss due to viscous effects between sections (1) and (2) and $z_1 - z_2 = S_0\ell$. Since the pressure is essentially hydrostatic at any cross section, we find that $p_1/\gamma = y_1$ and $p_2/\gamma = y_2$ so that Eq. 10.5 becomes

$$y_1 + \frac{V_1^2}{2g} + S_0\ell = y_2 + \frac{V_2^2}{2g} + h_L \tag{10.6}$$

One of the difficulties of analyzing open-channel flow, similar to that discussed in Chapter 8 for pipe flow, is associated with the determination of the head loss in terms of other physical parameters. Without getting into such details at present, we write the head loss in terms of the slope of the energy line, $S_f = h_L/\ell$ (often termed the *friction slope*), as indicated in Fig. 10.6. Recall from

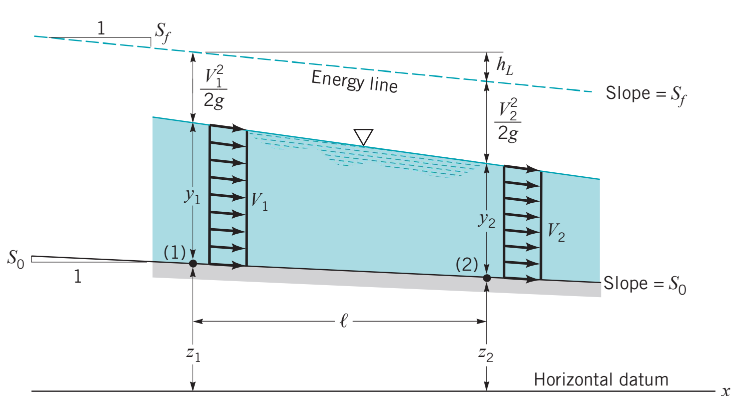


FIGURE 10.6 **Typical open-channel geometry.**

Chapter 3 that the energy line is located a distance z (the elevation from some datum to the channel bottom) plus the pressure head (p/γ) plus the velocity head $(V^2/2g)$ above the datum. Therefore, Eq. 10.6 can be written as

$$y_1 - y_2 = \frac{(V_2^2 - V_1^2)}{2g} + (S_f - S_0)\ell \qquad \textbf{(10.7)}$$

If there is no head loss, the energy line is horizontal $(S_f = 0)$, and the total energy of the flow is free to shift between kinetic energy and potential energy in a conservative fashion. In the specific instance of a horizontal channel bottom $(S_0 = 0)$ and negligible head loss $(S_f = 0)$, Eq. 10.7 simply becomes

$$y_1 - y_2 = \frac{(V_2^2 - V_1^2)}{2g}$$

10.3.1 Specific Energy

The specific energy is the sum of potential energy and kinetic energy (per unit weight).

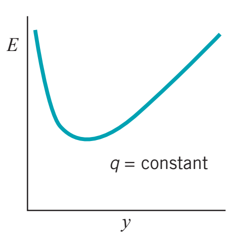
The concept of the ***specific energy*** or specific head, E, defined as

$$E = y + \frac{V^2}{2g} \qquad \textbf{(10.8)}$$

is often useful in open-channel flow considerations. The energy equation, Eq. 10.7, can be written in terms of E as

$$E_1 = E_2 + (S_f - S_0)\ell \qquad \textbf{(10.9)}$$

If head losses are negligible, then $S_f = 0$ so that $(S_f - S_0)\ell = -S_0\ell = z_2 - z_1$ and the sum of the specific energy and the elevation of the channel bottom remains constant (i.e., $E_1 + z_1 = E_2 + z_2$, a statement of the Bernoulli equation).

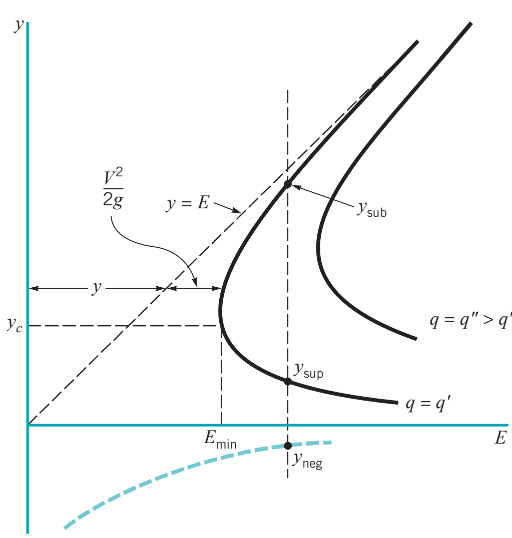


If we consider a simple channel whose cross-sectional shape is a rectangle of width b, the specific energy can be written in terms of the flowrate per unit width, $q = Q/b = Vyb/b = Vy$, as

$$E = y + \frac{q^2}{2gy^2} \qquad \textbf{(10.10)}$$

which is illustrated by the figure in the margin.

For a given channel of constant width, the value of q remains constant along the channel, although the depth, y, may vary. To gain insight into the flow processes involved, we consider the ***specific energy diagram,*** a graph of $E = E(y)$, with q fixed, as shown in Fig. 10.7. The relationship between the flow depth, y, and the velocity head, $V^2/2g$, as given by Eq. 10.8 is indicated in the figure.



■ **FIGURE 10.7** **Specific energy diagram.**

For a given value of specific energy, a flow may have alternate depths.

For given q and E, Eq. 10.10 is a cubic equation $[y^3 - Ey^2 + (q^2/2g) = 0]$ with three solutions, y_{sup}, y_{sub}, and y_{neg}. If the specific energy is large enough (i.e., $E > E_{min}$, where E_{min} is a function of q), two of the solutions are positive and the other, y_{neg}, is negative. The negative root, represented by the curved dashed line in Fig. 10.7, has no physical meaning and can be ignored. Thus, for a given flowrate and specific energy there are two possible depths, unless the vertical line from the E axis does not intersect the specific energy curve corresponding to the value of q given (i.e., $E < E_{min}$). These two depths are termed *alternate depths.*

For large values of E the upper and lower branches of the specific energy diagram (y_{sub} and y_{sup}) approach $y = E$ and $y = 0$, respectively. These limits correspond to a very deep channel flowing very slowly ($E = y + V^2/2g \rightarrow y$ as $y \rightarrow \infty$ with $q = Vy$ fixed), or a very high-speed flow in a shallow channel ($E = y + V^2/2g \rightarrow V^2/2g$ as $y \rightarrow 0$).

As is indicated in Fig. 10.7, $y_{sup} < y_{sub}$. Thus, since $q = Vy$ is constant along the curve, it follows that $V_{sup} > V_{sub}$, where the subscripts "sub" and "sup" on the velocities correspond to the depths so labeled. The specific energy diagram consists of two portions divided by the E_{min} "nose" of the curve. We will show that the flow conditions at this location correspond to critical conditions (Fr = 1), those on the upper portion of the curve correspond to subcritical conditions (hence, the "sub" subscript), and those on the lower portion of the curve correspond to supercritical conditions (hence, the "sup" subscript).

To determine the value of E_{min}, we use Eq. 10.10 and set $dE/dy = 0$ to obtain

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0$$

or

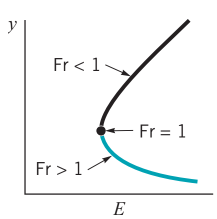
$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{10.11}$$

where the subscript "c" denotes conditions at E_{min}. By substituting this back into Eq. 10.10 we obtain

$$E_{min} = \frac{3y_c}{2}$$

By combining Eq. 10.11 and $V_c = q/y_c$, we obtain

$$V_c = \frac{q}{y_c} = \frac{(y_c^{3/2}g^{1/2})}{y_c} = \sqrt{gy_c}$$



or $\mathrm{Fr}_c \equiv V_c/(gy_c)^{1/2} = 1$. Thus, critical conditions (Fr = 1) occur at the location of E_{min}. Since the layer is deeper and the velocity smaller for the upper part of the specific energy diagram (compared with the conditions at E_{min}), such flows are subcritical (Fr < 1). Conversely, flows for the lower part of the diagram are supercritical. This is shown by the figure in the margin. Thus, for a given flowrate, q, if $E > E_{min}$ there are two possible depths of flow, one subcritical and the other supercritical.

It is often possible to determine various characteristics of a flow by considering the specific energy diagram. Example 10.2 illustrates this for a situation in which the channel bottom elevation is not constant.

EXAMPLE 10.2 Specific Energy Diagram—Quantitative

GIVEN Water flows up a 0.5-ft-tall ramp in a constant width rectangular channel at a rate $q = 5.75\ \mathrm{ft^2/s}$ as is shown in Fig. E10.2*a*. (For now disregard the "bump.") The upstream depth is 2.3 ft and viscous effects are negligible.

FIND Determine the elevation of the water surface downstream of the ramp, $y_2 + z_2$.

SOLUTION

With $S_0 \ell = z_1 - z_2$ and $h_L = 0$, conservation of energy (Eq. 10.6 which, under these conditions, is actually the Bernoulli equation) requires that

$$y_1 + \frac{V_1^2}{2g} + z_1 = y_2 + \frac{V_2^2}{2g} + z_2$$

For the conditions given ($z_1 = 0, z_2 = 0.5$ ft, $y_1 = 2.3$ ft, and $V_1 = q/y_1 = 2.5$ ft/s), this becomes

$$1.90 = y_2 + \frac{V_2^2}{64.4} \qquad \textbf{(1)}$$

where V_2 and y_2 are in ft/s and feet, respectively. The continuity equation provides the second equation

$$y_2 V_2 = y_1 V_1$$

or

$$y_2 V_2 = 5.75 \text{ ft}^2/\text{s} \qquad \textbf{(2)}$$

Equations 1 and 2 can be combined to give

$$y_2^3 - 1.90 y_2^2 + 0.513 = 0$$

which has solutions

$$y_2 = 1.72 \text{ ft}, \qquad y_2 = 0.638 \text{ ft}, \quad \text{or} \quad y_2 = -0.466 \text{ ft}$$

Note that two of these solutions are physically realistic, but the negative solution is meaningless. This is consistent with the previous discussions concerning the specific energy (recall the three roots indicated in Fig. 10.7). The corresponding elevations of the free surface are either

$$y_2 + z_2 = 1.72 \text{ ft} + 0.50 \text{ ft} = 2.22 \text{ ft}$$

or

$$y_2 + z_2 = 0.638 \text{ ft} + 0.50 \text{ ft} = 1.14 \text{ ft}$$

The question is which of these two flows is to be expected? This can be answered by use of the specific energy diagram obtained from Eq. 10.10, which for this problem is

$$E = y + \frac{0.513}{y^2}$$

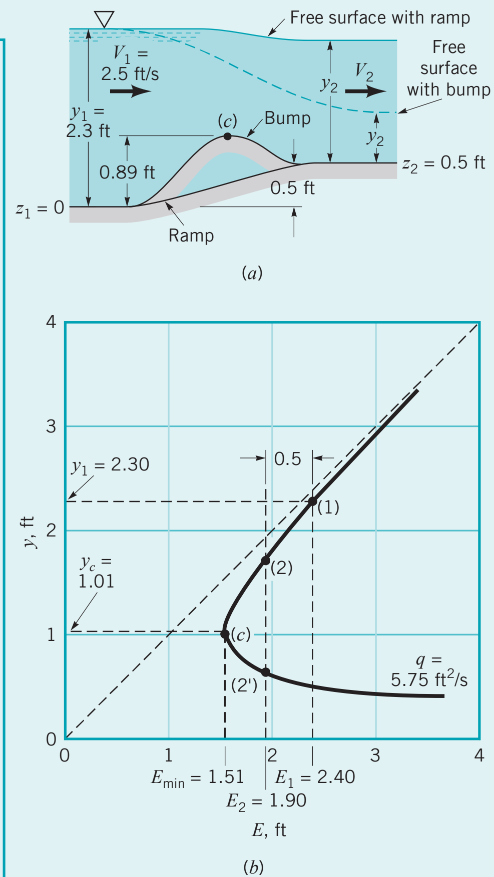
where E and y are in feet. The diagram is shown in Fig. E10.2*b*. The upstream condition corresponds to subcritical flow; the downstream condition is either subcritical or supercritical, corresponding to points 2 or 2′. Note that since $E_1 = E_2 + (z_2 - z_1) = E_2 + 0.5$ ft, it follows that the downstream conditions are located 0.5 ft to the left of the upstream conditions on the diagram.

With a constant width channel, the value of q remains the same for any location along the channel. That is, all points for the flow from (1) to (2) or (2′) must lie along the $q = 5.75$ ft^2/s curve shown. Any deviation from this curve would imply either a change in q or a relaxation of the one-dimensional flow assumption. To stay on the curve and go from (1) around the critical point (point c) to point (2′) would require a reduction in specific energy to $E_{\min}$. As is seen from Fig. E10.2*a*, this would require a specified elevation (bump) in the channel bottom so that critical conditions would occur above this bump. The height of this bump can be obtained from the energy equation (Eq. 10.9) written between points (1) and (c) with $S_f = 0$ (no viscous effects) and $S_0 \ell = z_1 - z_c$. That is, $E_1 = E_{\min} - z_1 + z_c$. In particular, since $E_1 = y_1 + 0.513/y_1^2 = 2.40$ ft and $E_{\min} = 3y_c/2 = 3(q^2/g)^{1/3}/2 = 1.51$ ft, the top of this bump would need to be $z_c - z_1 = E_1 - E_{\min} = 2.40 \text{ ft} - 1.51 \text{ ft} = 0.89$ ft above the channel bottom at section (1). The flow could then accelerate to supercritical conditions ($\text{Fr}_{2'} > 1$) as is shown by the free surface represented by the dashed line in Fig. E10.2*a*.

Since the actual elevation change (a ramp) shown in Fig. E10.2*a* does not contain a bump, the downstream conditions will correspond to the subcritical flow denoted by (2), not the supercritical condition (2′). Without a bump on the channel bottom, the state (2′) is inaccessible from the upstream condition state (1). Such considerations are often termed the *accessibility of flow regimes*. Thus, the surface elevation is

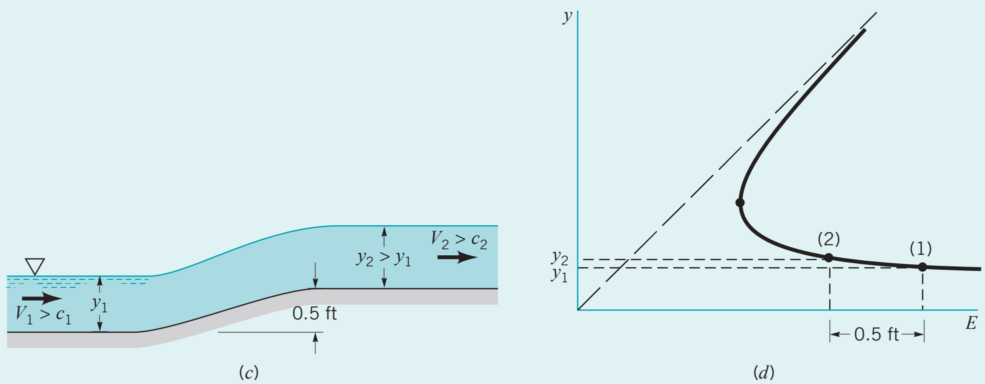
$$y_2 + z_2 = 2.22 \text{ ft} \qquad \textbf{(Ans)}$$

Note that since $y_1 + z_1 = 2.30$ ft and $y_2 + z_2 = 2.22$ ft, the elevation of the free surface decreases as it goes across the ramp.



■ FIGURE E10.2

COMMENT If the flow conditions upstream of the ramp were supercritical, the free-surface elevation and fluid depth would increase as the fluid flows up the ramp. This is indicated in Fig. E10.2*c* along with the corresponding specific energy diagram, as is shown in Fig. E10.2*d*. For this case the flow starts at (1) on the lower (supercritical) branch of the specific energy curve and ends at (2) on the same branch with $y_2 > y_1$. Since both y and z increase from (1) to (2), the surface elevation, $y + z$, also increases. Thus, flow up a ramp is different for subcritical than it is for supercritical conditions.



■ FIGURE E10.2 (*Continued*)

10.3.2 Channel Depth Variations

By using the concepts of the specific energy and critical flow conditions (Fr = 1), it is possible to determine how the depth of a flow in an open channel changes with distance along the channel. In some situations the depth change is very rapid so that the value of dy/dx is of the order of 1. Complex effects involving two- or three-dimensional flow phenomena are often involved in such flows.

In this section we consider only gradually varying flows. For such flows, $dy/dx \ll 1$ and it is reasonable to impose the one-dimensional velocity assumption. At any section the total head is $H = V^2/2g + y + z$ and the energy equation (Eq. 10.5) becomes

$$H_1 = H_2 + h_L$$

where h_L is the head loss between sections (1) and (2).

As is discussed in the previous section, the slope of the energy line is $dH/dx = dh_L/dx = S_f$ and the slope of the channel bottom is $dz/dx = S_0$. Thus, since

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{V^2}{2g} + y + z\right) = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

we obtain

$$\frac{dh_L}{dx} = \frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} + S_0$$

or

$$\frac{V}{g}\frac{dV}{dx} + \frac{dy}{dx} = S_f - S_0 \qquad \textbf{(10.12)}$$

For a given flowrate per unit width, q, in a rectangular channel of constant width b, we have $V = q/y$ or by differentiation

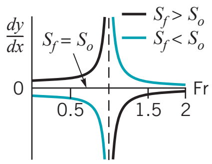
$$\frac{dV}{dx} = -\frac{q}{y^2}\frac{dy}{dx} = -\frac{V}{y}\frac{dy}{dx}$$

so that the kinetic energy term in Eq. 10.12 becomes

$$\frac{V}{g}\frac{dV}{dx} = -\frac{V^2}{gy}\frac{dy}{dx} = -\mathrm{Fr}^2\frac{dy}{dx} \tag{10.13}$$

where $\mathrm{Fr} = V/(gy)^{1/2}$ is the local Froude number of the flow. Substituting Eq. 10.13 into Eq. 10.12 and simplifying gives

$$\frac{dy}{dx} = \frac{(S_f - S_0)}{(1 - \mathrm{Fr}^2)} \tag{10.14}$$



It is seen that the rate of change of fluid depth, dy/dx, depends on the local slope of the channel bottom, S_0, the slope of the energy line, S_f, and the Froude number, Fr. As shown by the figure in the margin, the value of dy/dx can be either negative, zero, or positive, depending on the values of these three parameters. That is, the channel flow depth may be constant or it may increase or decrease in the flow direction, depending on the values of S_0, S_f, and Fr. The behavior of subcritical flow may be the opposite of that for supercritical flow, as seen by the denominator, $1 - \mathrm{Fr}^2$, of Eq. 10.14.

Although in the derivation of Eq. 10.14 we assumed q is constant (i.e., a rectangular channel), Eq. 10.14 is valid for channels of any constant cross-sectional shape, provided the Froude number is interpreted properly (Ref. 3). In this book we will consider only rectangular cross-sectional channels when using this equation.

10.4 Uniform Depth Channel Flow

V10.6 Merging channels

Many channels are designed to carry fluid at a uniform depth all along their length. Irrigation canals are frequently of uniform depth and cross section for considerable lengths. Natural channels such as rivers and creeks are seldom of uniform shape, although a reasonable approximation to the flowrate in such channels can often be obtained by assuming uniform flow. In this section we will discuss various aspects of such flows.

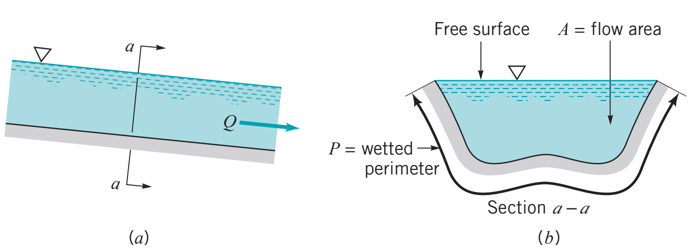
Uniform depth flow ($dy/dx = 0$) can be accomplished by adjusting the bottom slope, S_0, so that it precisely equals the slope of the energy line, S_f. That is, $S_0 = S_f$. This can be seen from Eq. 10.14. From an energy point of view, uniform depth flow is achieved by a balance between the potential energy lost by the fluid as it coasts downhill and the energy that is dissipated by viscous effects (head loss) associated with shear stresses throughout the fluid. Similar conclusions can be reached from a force balance analysis as discussed in the following section.

10.4.1 Uniform Flow Approximations

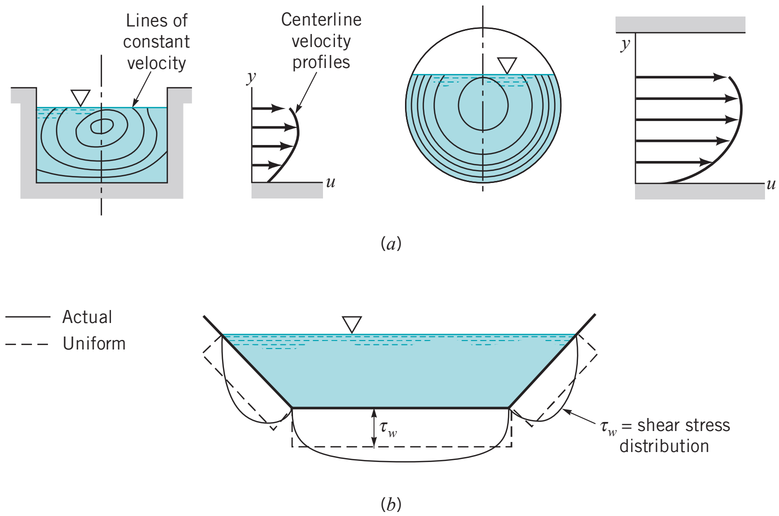
The wall shear stress acts on the wetted perimeter of the channel.

We consider fluid flowing in an open channel of constant cross-sectional size and shape such that the depth of flow remains constant as is indicated in Fig. 10.8. The area of the section is A and the ***wetted perimeter*** (i.e., the length of the perimeter of the cross section in contact with the fluid) is P. The interaction between the fluid and the atmosphere at the free surface is assumed negligible so that this portion of the perimeter is not included in the definition of the wetted perimeter.

Since the fluid must adhere to the solid surfaces, the actual velocity distribution in an open channel is not uniform. Some typical velocity profiles measured in channels of various shapes are indicated in Fig. 10.9*a*. The maximum velocity is often found somewhat below the free surface,



■ **FIGURE 10.8** **Uniform flow in an open channel.**



■ FIGURE 10.9 Typical velocity and shear stress distributions in an open channel: (*a*) velocity distribution throughout the cross section, (*b*) shear stress distribution on the wetted perimeter.

and the fluid velocity is zero on the wetted perimeter, where a wall shear stress, τ_w, is developed. This shear stress is seldom uniform along the wetted perimeter, with typical variations as are indicated in Fig. 10.9*b*.

Fortunately, reasonable analytical results can be obtained by assuming a uniform velocity profile, V, and a constant wall shear stress, τ_w. Similar assumptions were made for pipe flow situations (Chapter 8), with the friction factor being used to obtain the head loss.

Fluids in the News

Plumbing the Everglades Because of all of the economic development that has occurred in southern Florida, the natural drainage pattern of that area has been greatly altered during the past century. Previously there was a vast network of surface flow southward from the Orlando area, to Lake Okeechobee, through the Everglades, and out to the Gulf of Mexico. Currently a vast amount of freshwater from Lake Okeechobee and surrounding waterways (1.7 billion gallons per day) is sluiced into the ocean for flood control, bypassing the Everglades. A new long-term Comprehensive Everglades Restoration Plan is being implemented to restore, preserve, and protect the south Florida ecosystem. Included in the plan are the use of numerous aquifer-storage-and-recovery systems that will recharge the ecosystem. In addition, surface water reservoirs using artificial wetlands will clean agricultural runoff. In an attempt to improve the historical flow from north to south, old levees will be removed, parts of the Tamiami Trail causeway will be altered, and stored water will be redirected through miles of new pipes and rebuilt *canals.* Strictly speaking, the Everglades will not be "restored." However, by 2030, 1.6 million acres of national parkland will have cleaner water and more of it. (See Problem 10.77.)

10.4.2 The Chezy and Manning Equations

The basic equations used to determine the uniform flowrate in open channels were derived many years ago. Continual refinements have taken place to obtain better values of the empirical coefficients involved. The result is a semiempirical equation that provides reasonable engineering results. A more refined analysis is perhaps not warranted because of the complexity and uncertainty of the flow geometry (i.e., channel shape and the irregular makeup of the wetted perimeter, particularly for natural channels).

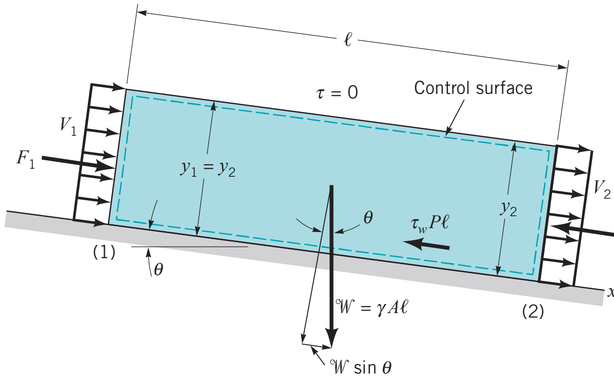
Under the assumptions of steady uniform flow, the x component of the momentum equation (Eq. 5.22) applied to the control volume indicated in Fig. 10.10 simply reduces to

$$\Sigma F_x = \rho Q(V_2 - V_1) = 0$$

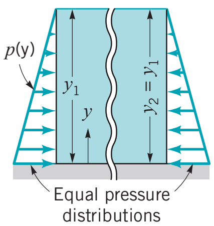
For steady, uniform depth flow in an open channel there is no fluid acceleration.

since $V_1 = V_2$. There is no acceleration of the fluid, and the momentum flux across section (1) is the same as that across section (2). The flow is governed by a simple balance between the forces in the direction of the flow. Thus, $\Sigma F_x = 0$, or

$$F_1 - F_2 - \tau_w P\ell + \mathcal{W} \sin\theta = 0 \quad \textbf{(10.15)}$$



■ **FIGURE 10.10** **Control volume for uniform flow in an open channel.**



where F_1 and F_2 are the hydrostatic pressure forces across either end of the control volume, as shown by the figure in the margin. Because the flow is at a uniform depth ($y_1 = y_2$), it follows that $F_1 = F_2$ so that these two forces do not contribute to the force balance. The term $\mathscr{W} \sin\theta$ is the component of the fluid weight that acts down the slope, and $\tau_w P\ell$ is the shear force on the fluid, acting up the slope as a result of the interaction of the water and the channel's wetted perimeter. Thus, Eq. 10.15 becomes

$$\tau_w = \frac{\mathscr{W} \sin\theta}{P\ell} = \frac{\mathscr{W} S_0}{P\ell}$$

where we have used the approximation that $\sin\theta \approx \tan\theta = S_0$, since the bottom slope is typically very small (i.e., $S_0 \ll 1$). Since $\mathscr{W} = \gamma A\ell$ and the ***hydraulic radius*** is defined as $R_h = A/P$, the force balance equation becomes

$$\tau_w = \frac{\gamma A\ell S_0}{P\ell} = \gamma R_h S_0 \tag{10.16}$$

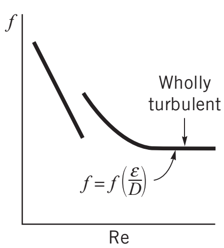
For uniform depth, channel flow is governed by a balance between friction and weight.

Most open-channel flows are turbulent rather than laminar. In fact, typical Reynolds numbers are quite large, well above the transitional value and into the wholly turbulent regime. As was discussed in Chapter 8, and shown by the figure in the margin, for very large Reynolds number pipe flows (wholly turbulent flows), the friction factor, f, is found to be independent of Reynolds number, dependent only on the relative roughness, ε/D, of the pipe surface. For such cases, the wall shear stress is proportional to the dynamic pressure, $\rho V^2/2$, and independent of the viscosity. That is,

$$\tau_w = K\rho \frac{V^2}{2}$$

where K is a constant dependent upon the roughness of the pipe.

It is not unreasonable that similar shear stress dependencies occur for the large Reynolds number open-channel flows. In such situations, Eq. 10.16 becomes



$$K\rho \frac{V^2}{2} = \gamma R_h S_0$$

or

$$V = C\sqrt{R_h S_0} \tag{10.17}$$

where the constant C is termed the Chezy coefficient and Eq. 10.17 is termed the ***Chezy equation.*** This equation, one of the oldest in the area of fluid mechanics, was developed in 1768 by A. Chezy (1718–1798), a French engineer who designed a canal for the Paris water supply. The value of the Chezy coefficient, which must be determined by experiments, is not dimensionless but has the dimensions of (length)$^{1/2}$ per time (i.e., the square root of the units of acceleration).

From a series of experiments it was found that the slope dependence of Eq. 10.17 ($V \sim S_0^{1/2}$) is reasonable, but that the dependence on the hydraulic radius is more nearly $V \sim R_h^{2/3}$ rather than $V \sim R_h^{1/2}$. In 1889, R. Manning (1816–1897), an Irish engineer, developed the following somewhat modified equation for open-channel flow to more accurately describe the R_h dependence:

$$V = \frac{R_h^{2/3} S_0^{1/2}}{n} \tag{10.18}$$

■ **TABLE 10.1**

Values of the Manning Coefficient, *n* (Ref. 6)

Wetted Perimeter	*n*	Wetted Perimeter	*n*
A. Natural channels		***D. Artificially lined channels***	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. Floodplains		Steel, painted	0.014
Pasture, farmland	0.035	Steel, riveted	0.015
Light brush	0.050	Cast iron	0.013
Heavy brush	0.075	Concrete, finished	0.012
Trees	0.15	Concrete, unfinished	0.014
C. Excavated earth channels		Planed wood	0.012
Clean	0.022	Clay tile	0.014
Gravelly	0.025	Brickwork	0.015
Weedy	0.030	Asphalt	0.016
Stony, cobbles	0.035	Corrugated metal	0.022
		Rubble masonry	0.025

Equation 10.18 is termed the ***Manning equation***, and the parameter n is the ***Manning resistance coefficient***. Its value is dependent on the surface material of the channel's wetted perimeter and is obtained from experiments. It is not dimensionless, having the units of $s/m^{1/3}$ or $s/ft^{1/3}$.

As is discussed in Chapter 7, any correlation should be expressed in dimensionless form, with the coefficients that appear being dimensionless coefficients, such as the friction factor for pipe flow or the drag coefficient for flow past objects. Thus, Eq. 10.18 should be expressed in dimensionless form. Unfortunately, the Manning equation is so widely used and has been used for so long that it will continue to be used in its dimensional form with a coefficient, n, that is not dimensionless. The values of n found in the literature (such as Table 10.1) were developed for SI units. Standard practice is to use the same value of n when using the BG system of units, and to insert a conversion factor into the equation.

The Manning equation is used to obtain the velocity or flowrate in an open channel.

Thus, uniform flow in an open channel is obtained from the Manning equation written as

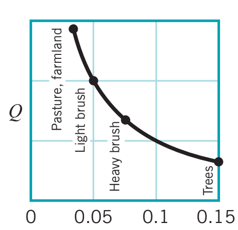
$$\boxed{V = \frac{\kappa}{n} R_h^{2/3} S_0^{1/2}} \qquad \textbf{(10.19)}$$

and

$$\boxed{Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}} \qquad \textbf{(10.20)}$$

where $\kappa = 1$ if SI units are used, and $\kappa = 1.49$ if BG units are used. The value 1.49 is the cube root of the number of feet per meter: $(3.281\ \text{ft/m})^{1/3} = 1.49$. Thus, by using R_h in meters, A in m^2, and $\kappa = 1$, the average velocity is m/s and the flowrate m^3/s. By using R_h in feet, A in ft^2, and $\kappa = 1.49$, the average velocity is ft/s and the flowrate ft^3/s.

Typical values of the Manning coefficient are indicated in Table 10.1. As expected, the rougher the wetted perimeter, the larger the value of n. For example, the roughness of floodplain surfaces increases from pasture to brush to tree conditions. So does the corresponding value of the Manning coefficient. Thus, for a given depth of flooding, the flowrate varies with floodplain roughness as indicated by the figure in the margin.



Precise values of n are often difficult to obtain. Except for artificially lined channel surfaces like those found in new canals or flumes, the channel surface structure may be quite complex and variable. There are various methods used to obtain a reasonable estimate of the value of n for a given situation (Ref. 5). For the purpose of this book, the values from Table 10.1 are sufficient. Note that the error in Q is directly proportional to the error in n. A 10%

error in the value of n produces a 10% error in the flowrate. Considerable effort has been put forth to obtain the best estimate of n, with extensive tables of values covering a wide variety of surfaces (Ref. 7). It should be noted that the values of n given in Table 10.1 are valid only for water as the flowing fluid.

Both the friction factor for pipe flow and the Manning coefficient for channel flow are parameters that relate the wall shear stress to the makeup of the bounding surface. Thus, various results are available that describe n in terms of the equivalent pipe friction factor, f, and the surface roughness, ε (Ref. 8). For our purposes we will use the values of n from Table 10.1.

V10.7 Uniform channel flow

10.4.3 Uniform Depth Examples

A variety of interesting and useful results can be obtained from the Manning equation. The following examples illustrate some of the typical considerations.

The main parameters involved in uniform depth open-channel flow are the size and shape of the channel cross section (A, R_h), the slope of the channel bottom (S_0), the character of the material lining the channel bottom and walls (n), and the average velocity or flowrate (V or Q). Although the Manning equation is a rather simple equation, the ease of using it depends in part on which variables are given and which are to be determined.

Determination of the flowrate of a given channel with flow at a given depth (often termed the *normal flowrate* for *normal depth*, sometimes denoted y_n) is obtained from a straightforward calculation as is shown in Example 10.3.

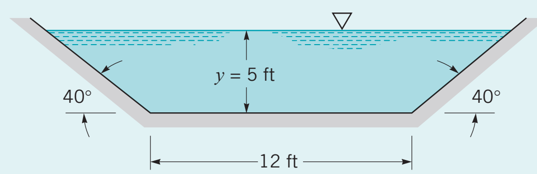
EXAMPLE 10.3 Uniform Flow, Determine Flow Rate

GIVEN Water flows in the canal of trapezoidal cross section shown in Fig. E10.3*a*. The bottom drops 1.4 ft per 1000 ft of length. The canal is lined with new finished concrete.

FIND Determine

(a) the flowrate and

(b) the Froude number for this flow.



■ FIGURE E10.3*a*

SOLUTION

(a) From Eq. 10.20,

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2} \qquad \textbf{(1)}$$

where we have used $\kappa = 1.49$, since the dimensions are given in BG units. For a depth of $y = 5$ ft, the flow area is

$$A = 12 \text{ ft } (5 \text{ ft}) + 5 \text{ ft}\left(\frac{5}{\tan 40°}\text{ ft}\right) = 89.8 \text{ ft}^2$$

so that with a wetted perimeter of $P = 12 \text{ ft} + 2(5/\sin 40° \text{ ft}) = 27.6$ ft, the hydraulic radius is determined to be $R_h = A/P = 3.25$ ft. Note that even though the channel is quite wide (the free-surface width is 23.9 ft), the hydraulic radius is only 3.25 ft, which is less than the depth.

Thus, with $S_0 = 1.4 \text{ ft}/1000 \text{ ft} = 0.0014$, Eq. 1 becomes

$$Q = \frac{1.49}{n}(89.8 \text{ ft}^2)(3.25 \text{ ft})^{2/3}(0.0014)^{1/2} = \frac{10.98}{n}$$

where Q is in ft^3/s.

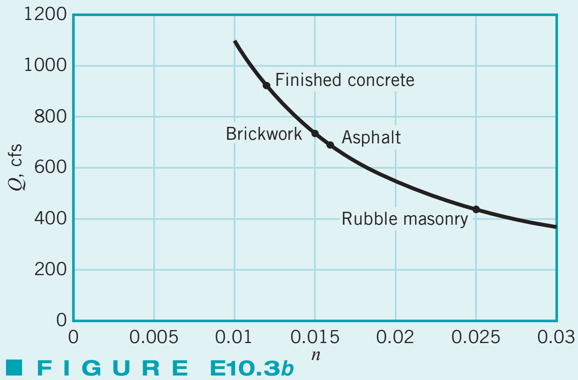
From Table 10.1, we obtain $n = 0.012$ for the finished concrete. Thus,

$$Q = \frac{10.98}{0.012} = 915 \text{ cfs} \qquad \textbf{(Ans)}$$

COMMENT The corresponding average velocity, $V = Q/A$, is 10.2 ft/s. It does not take a very steep slope ($S_0 = 0.0014$ or $\theta = \tan^{-1}(0.0014) = 0.080°$) for this velocity.

By repeating the calculations for various surface types (i.e., various Manning coefficient values), the results shown in Fig. E10.3*b* are obtained. Note that the increased roughness causes a decrease in the flowrate. This is an indication that for the turbulent flows involved, the wall shear stress increases with surface roughness. [For water at 50 °F, the Reynolds number based on the 3.25-ft hydraulic radius of the channel and a smooth concrete surface is $\text{Re} = R_h V/\nu = 3.25 \text{ ft } (10.2 \text{ ft/s})/(1.41 \times 10^{-5} \text{ ft}^2\text{/s}) = 2.35 \times 10^6$, well into the turbulent regime.]

(b) The Froude number based on the maximum depth for the flow can be determined from $\text{Fr} = V/(gy)^{1/2}$. For the finished



■ FIGURE E10.3b

concrete case,

$$\mathrm{Fr} = \frac{10.2\ \mathrm{ft/s}}{(32.2\ \mathrm{ft/s^2} \times 5\ \mathrm{ft})^{1/2}} = 0.804 \quad \textbf{(Ans)}$$

The flow is subcritical.

COMMENT The same results would be obtained for the channel if its size were given in meters. We would use the same value of n but set $\kappa = 1$ for this SI units situation.

In some instances a trial-and-error or iteration method must be used to solve for the dependent variable. This is often encountered when the flowrate, channel slope, and channel material are given, and the flow depth is to be determined as illustrated in the following examples.

EXAMPLE 10.4 Uniform Flow, Determine Flow Depth

GIVEN Water flows in the channel shown in Fig. E10.3*a* at a rate of $Q = 10.0\ \mathrm{m^3/s}$. The canal lining is weedy.

FIND Determine the depth of the flow.

SOLUTION

In this instance neither the flow area nor the hydraulic radius are known, although they can be written in terms of the depth, y. Since the flowrate is given in $\mathrm{m^3/s}$, we will solve this problem using SI units. Hence, the bottom width is (12 ft) (1 m/3.281 ft) = 3.66 m and the area is

$$A = y\left(\frac{y}{\tan 40°}\right) + 3.66y = 1.19y^2 + 3.66y$$

where A and y are in square meters and meters, respectively. Also, the wetted perimeter is

$$P = 3.66 + 2\left(\frac{y}{\sin 40°}\right) = 3.11y + 3.66$$

so that

$$R_h = \frac{A}{P} = \frac{1.19y^2 + 3.66y}{3.11y + 3.66}$$

where R_h and y are in meters. Thus, with $n = 0.030$ (from Table 10.1), Eq. 10.20 can be written as

$$Q = 10 = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}$$
$$= \frac{1.0}{0.030}(1.19y^2 + 3.66y)\left(\frac{1.19y^2 + 3.66y}{3.11y + 3.66}\right)^{2/3} \times (0.0014)^{1/2}$$

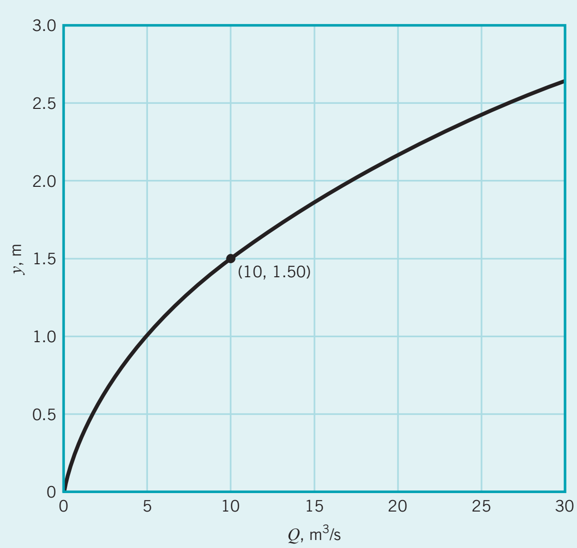
which can be rearranged into the form

$$(1.19y^2 + 3.66y)^5 - 515(3.11y + 3.66)^2 = 0 \qquad \textbf{(1)}$$

where y is in meters. The solution of Eq. 1 can be easily obtained by use of a simple rootfinding numerical technique or by trial-and-error methods. The only physically meaningful root of Eq. 1 (i.e., a positive, real number) gives the solution for the normal flow depth at this flowrate as

$$y = 1.50\ \mathrm{m} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for various flowrates, the results shown in Fig. E10.4 are obtained. Note that the water depth is not linearly related to the flowrate. That is, if the flowrate is doubled, the depth is not doubled.



■ FIGURE E10.4

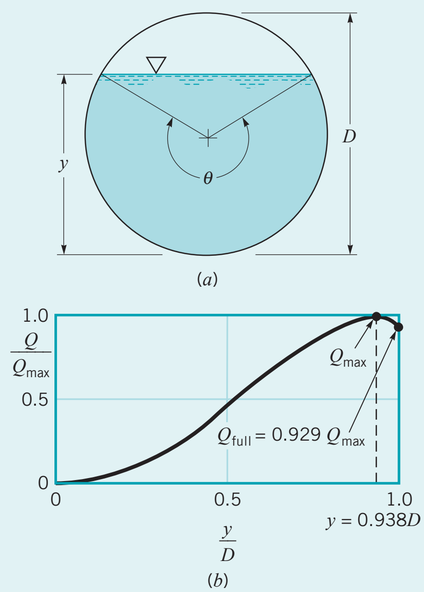
In Example 10.4 we found the flow depth for a given flowrate. Since the equation for this depth is a nonlinear equation, it may be that there is more than one solution to the problem. For a given channel there may be two or more depths that carry the same flowrate. Although this is not normally so, it can and does happen, as is illustrated by Example 10.5.

EXAMPLE 10.5 Uniform Flow, Maximum Flow Rate

GIVEN Water flows in a round pipe of diameter D at a depth of $0 \leq y \leq D$, as is shown in Fig. E10.5*a*. The pipe is laid on a constant slope of S_0, and the Manning coefficient is n.

FIND **(a)** At what depth does the maximum flowrate occur?

(b) Show that for certain flowrates there are two depths possible with the same flowrate. Explain this behavior.



■ FIGURE E10.5

SOLUTION

(a) According to the Manning equation (Eq. 10.20) the flowrate is

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2} \tag{1}$$

where S_0, n, and κ are constants for this problem. From geometry it can be shown that

$$A = \frac{D^2}{8}(\theta - \sin\theta)$$

where θ, the angle indicated in Fig. E10.5*a*, is in radians. Similarly, the wetted perimeter is

$$P = \frac{D\theta}{2}$$

so that the hydraulic radius is

$$R_h = \frac{A}{P} = \frac{D(\theta - \sin\theta)}{4\theta}$$

Therefore, Eq. 1 becomes

$$Q = \frac{\kappa}{n} S_0^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}\right]$$

This can be written in terms of the flow depth by using $y = (D/2)[1 - \cos(\theta/2)]$.

A graph of flowrate versus flow depth, $Q = Q(y)$, has the characteristic indicated in Fig. E10.5*b*. In particular, the maximum flowrate, Q_{max}, does not occur when the pipe is full; $Q_{full} = 0.929 Q_{max}$. It occurs when $y = 0.938D$, or $\theta = 5.28$ rad $= 303°$. Thus,

$$Q = Q_{max} \text{ when } y = 0.938D \qquad \textbf{(Ans)}$$

(b) For any $0.929 < Q/Q_{max} < 1$ there are two possible depths that give the same Q. The reason for this behavior can be seen by considering the gain in flow area, A, compared to the increase in wetted perimeter, P, for $y \approx D$. The flow area increase for an increase in y is very slight in this region, whereas the increase in wetted perimeter, and hence the increase in shear force holding back the fluid, is relatively large. The net result is a decrease in flowrate as the depth increases.

COMMENT For most practical problems, the slight difference between the maximum flowrate and full pipe flowrates is negligible, particularly in light of the usual inaccuracy of the value of n.

Fluids in the News

Done without GPS or lasers Two thousand years before the invention of such tools as the GPS or laser surveying equipment, Roman engineers were able to design and construct structures that made a lasting contribution to Western civilization. For example, one of the best surviving examples of Roman aqueduct construction is the Pont du Gard, an aqueduct that spans the Gardon River near Nîmes, France. This aqueduct is part of a circuitous, 50 km long open channel that transported water to Rome from a spring located 20 km from Rome. The spring is only 14.6 m above the point of delivery, giving an average *bottom slope* of only 3×10^{-4}. It is obvious that to carry out such a project, the Roman understanding of hydraulics, surveying, and construction was well advanced. (See Problem 10.59.)

For many open channels, the surface roughness varies across the channel.

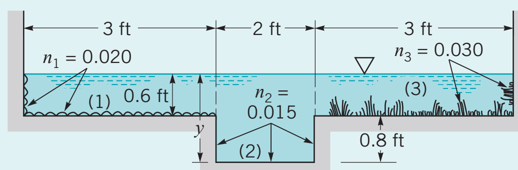
In many man-made channels and in most natural channels, the surface roughness (and hence the Manning coefficient) varies along the wetted perimeter of the channel. A drainage ditch, for example, may have a rocky bottom surface with concrete side walls to prevent erosion. Thus, the effective n will be different for shallow depths than for deep depths of flow. Similarly, a river channel may have one value of n appropriate for its normal channel and another very different value of n during its flood stage when a portion of the flow occurs across fields or through floodplain woods. An ice-covered channel usually has a different value of n for the ice than for the remainder of the wetted perimeter (Ref. 7). (Strictly speaking, such ice-covered channels are not "open" channels, although analysis of their flow is often based on open-channel flow equations. This is acceptable, since the ice cover is often thin enough so that it represents a fixed boundary in terms of the shear stress resistance, but it cannot support a significant pressure differential as in pipe flow situations.)

A variety of methods has been used to determine an appropriate value of the effective roughness of channels that contain subsections with different values of n. Which method gives the most accurate, easy-to-use results is not firmly established, since the results are nearly the same for each method (Ref. 5). A reasonable approximation is to divide the channel cross section into N subsections, each with its own wetted perimeter, P_i, area, A_i, and Manning coefficient, n_i. The P_i values do not include the imaginary boundaries between the different subsections. The total flowrate is assumed to be the sum of the flowrates through each section. This technique is illustrated by Example 10.7.

EXAMPLE 10.6 Uniform Flow, Variable Roughness

GIVEN Water flows along the drainage canal having the properties shown in Fig. E10.6*a*. The bottom slope is $S_0 = 1 \text{ ft}/500 \text{ ft} = 0.002$.

FIND Estimate the flowrate when the depth is $y = 0.8 \text{ ft} + 0.6 \text{ ft} = 1.4 \text{ ft}$.



■ FIGURE E10.6*a*

SOLUTION

We divide the cross section into three subsections as is indicated in Fig. E10.6*a* and write the flowrate as $Q = Q_1 + Q_2 + Q_3$, where for each section

$$Q_i = \frac{1.49}{n_i} A_i R_{h_i}^{2/3} S_0^{1/2}$$

The appropriate values of A_i, P_i, R_{hi}, and n_i are listed in Table E10.6. Note that the imaginary portions of the perimeters between sections (denoted by the vertical dashed lines in Fig. E10.6*a*) are not included in the P_i. That is, for section (2)

$$A_2 = 2 \text{ ft } (0.8 + 0.6) \text{ ft} = 2.8 \text{ ft}^2$$

and

$$P_2 = 2 \text{ ft} + 2(0.8 \text{ ft}) = 3.6 \text{ ft}$$

■ **TABLE E10.6**

i	A_i (ft^2)	P_i (ft)	R_{hi} (ft)	n_i
1	1.8	3.6	0.500	0.020
2	2.8	3.6	0.778	0.015
3	1.8	3.6	0.500	0.030

so that

$$R_{h_2} = \frac{A_2}{P_2} = \frac{2.8 \text{ ft}^2}{3.6 \text{ ft}} = 0.778 \text{ ft}$$

Thus, the total flowrate is

$$Q = Q_1 + Q_2 + Q_3 = 1.49(0.002)^{1/2} \times \left[\frac{(1.8 \text{ ft}^2)(0.500 \text{ ft})^{2/3}}{0.020} + \frac{(2.8 \text{ ft}^2)(0.778 \text{ ft})^{2/3}}{0.015} + \frac{(1.8 \text{ ft}^2)(0.500 \text{ ft})^{2/3}}{0.030}\right]$$

or

$$Q = 16.8 \text{ ft}^3/\text{s} \qquad \textbf{(Ans)}$$

COMMENTS If the entire channel cross section were considered as one flow area, then $A = A_1 + A_2 + A_3 = 6.4 \text{ ft}^2$ and $P = P_1 + P_2 + P_3 = 10.8 \text{ ft}$, or $R_h = A/P = 6.4 \text{ ft}^2/10.8 \text{ ft} = 0.593 \text{ ft}$. The flowrate is given by Eq. 10.20, which can be written as

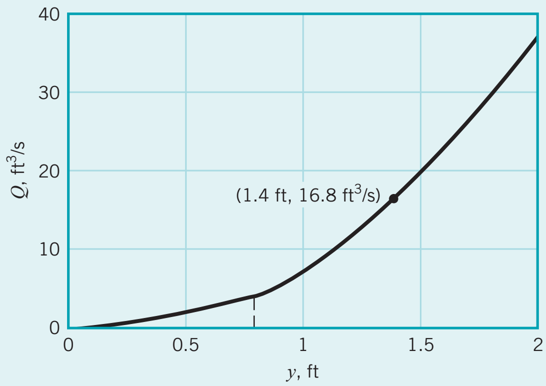
$$Q = \frac{1.49}{n_{\text{eff}}} A R_h^{2/3} S_0^{1/2}$$

where n_{eff} is the effective value of n for this channel. With $Q = 16.8 \text{ ft}^3/\text{s}$ as determined above, the value of n_{eff} is found to be

$$n_{\text{eff}} = \frac{1.49 A R_h^{2/3} S_0^{1/2}}{Q} = \frac{1.49(6.4)(0.593)^{2/3}(0.002)^{1/2}}{16.8} = 0.0179$$

As expected, the effective roughness (Manning n) is between the minimum ($n_2 = 0.015$) and maximum ($n_3 = 0.030$) values for the individual subsections.

By repeating the calculations for various depths, y, the results shown in Fig. E10.6b are obtained. Note that there are two distinct portions of the graph—one when the water is contained entirely within the main, center channel ($y < 0.8$ ft); the other when the water overflows into the side portions of the channel ($y > 0.8$ ft).



■ FIGURE E10.6b

One type of problem often encountered in open-channel flows is that of determining the *best hydraulic cross section* defined as the section of the minimum area for a given flowrate, Q, slope, S_0, and roughness coefficient, n. By using $R_h = A/P$ we can write Eq. 10.20 as

For a given flowrate, the channel of minimum area is denoted as the best hydraulic cross section.

$$Q = \frac{\kappa}{n} A \left(\frac{A}{P}\right)^{2/3} S_0^{1/2} = \frac{\kappa}{n} \frac{A^{5/3} S_0^{1/2}}{P^{2/3}}$$

which can be rearranged as

$$A = \left(\frac{nQ}{\kappa S_0^{1/2}}\right)^{3/5} P^{2/5}$$

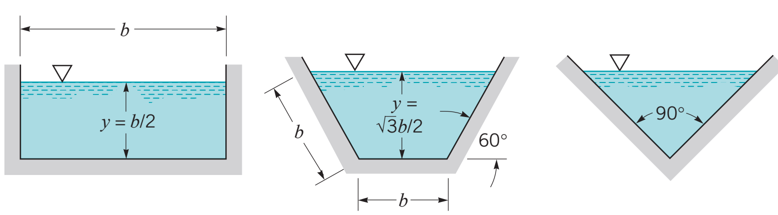
where the quantity in the parentheses is a constant. Thus, a channel with minimum A is one with a minimum P, so that both the amount of excavation needed and the amount of material to line the surface are minimized by the best hydraulic cross section.

The best hydraulic cross section possible is that of a semicircular channel. No other shape has as small a wetted perimeter for a given area. It is often desired to determine the best shape for a class of cross sections. The results (given here without proof) for rectangular, trapezoidal (with 60° sides), and triangular shapes are shown in Fig. 10.11. For example, the best hydraulic cross section for a rectangle is one whose depth is half its width; for a triangle it is a 90° triangle.

10.5 Gradually Varied Flow

In many situations the flow in an open channel is not of uniform depth (y = constant) along the channel. This can occur because of several reasons: The bottom slope is not constant, the cross-sectional shape and area vary in the flow direction, or there is some obstruction across a portion of the channel. Such flows are classified as gradually varying flows if $dy/dx \ll 1$.

If the bottom slope and the energy line slope are not equal, the flow depth will vary along the channel, either increasing or decreasing in the flow direction. In such cases $dy/dx \neq 0$, $dV/dx \neq 0$, and the right-hand side of Eq. 10.10 is not zero. Physically, the difference between the



■ FIGURE 10.11 **Best hydraulic cross sections for a rectangle, a 60° trapezoid, and a triangle.**

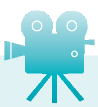
component of weight and the shear forces in the direction of flow produces a change in the fluid momentum that requires a change in velocity and, from continuity considerations, a change in depth. Whether the depth increases or decreases depends on various parameters of the flow, with a variety of surface profile configurations [flow depth as a function of distance, $y = y(x)$] possible (Refs. 5, 9).

10.6 Rapidly Varied Flow

In many open channels, flow depth changes occur over a relatively short distance so that $dy/dx \sim 1$. Such ***rapidly varied flow*** conditions are often quite complex and difficult to analyze in a precise fashion. Fortunately, many useful approximate results can be obtained by using a simple one-dimensional model along with appropriate experimentally determined coefficients when necessary. In this section we discuss several of these flows.

In many cases the flow depth may change significantly in a short distance.

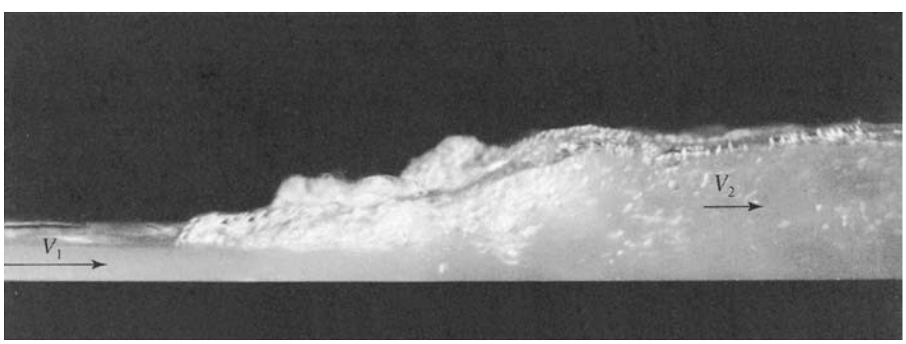
Some rapidly varied flows occur in constant area channels for reasons that are not immediately obvious. The hydraulic jump is one such case. As is indicated in Fig. 10.12, the flow may change from a relatively shallow, high-speed condition into a relatively deep, low-speed condition within a horizontal distance of just a few channel depths. Other rapidly varied flows may be due to a sudden change in the channel geometry such as the flow in an expansion or contraction section of a channel as is indicated in Fig. 10.13.



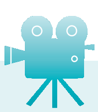
V10.8 Erosion in a channel

In such situations the flow field is often two- or three-dimensional in character. There may be regions of flow separation, flow reversal, or unsteady oscillations of the free surface. For the purpose of some analyses, these complexities can be neglected and a simplified analysis can be undertaken. In other cases, however, it is the complex details of the flow that are the most important property of the flow; any analysis must include their effects. The scouring of a river bottom in the neighborhood of a bridge pier, as is indicated in Fig. 10.14, is such an example. A one- or two-dimensional model of this flow would not be sufficient to describe the complex structure of the flow that is responsible for the erosion near the foot of the bridge pier.

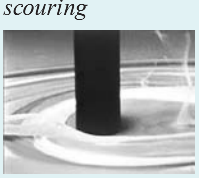
Many open-channel flow-measuring devices are based on principles associated with rapidly varied flows. Among these devices are broad-crested weirs, sharp-crested weirs, critical flow flumes, and sluice gates. The operation of such devices is discussed in the following sections.

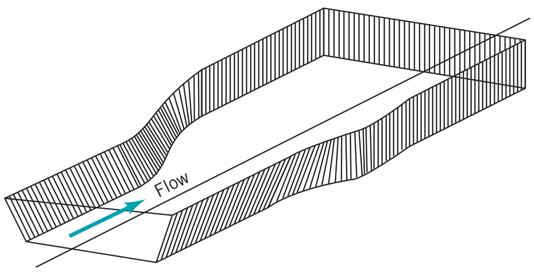


■ FIGURE 10.12 Hydraulic jump.

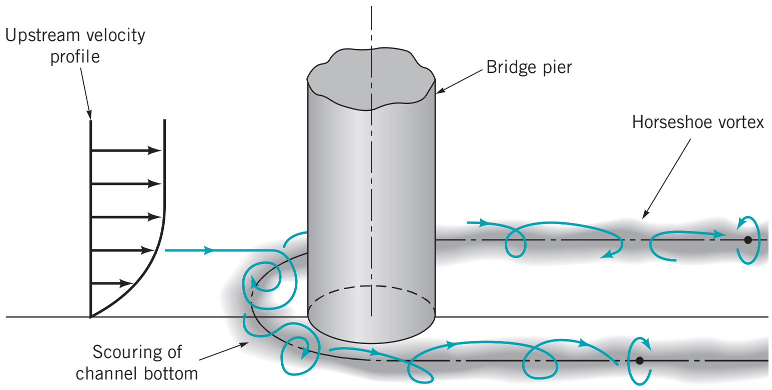


V10.9 Bridge pier scouring





■ FIGURE 10.13 Rapidly varied flow may occur in a channel transition section.



■ FIGURE 10.14 The complex three-dimensional flow structure around a bridge pier.

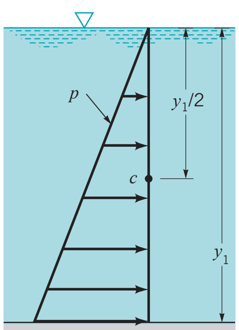
10.6.1 The Hydraulic Jump

Observations of flows in open channels show that under certain conditions it is possible that the fluid depth will change very rapidly over a short length of the channel without any change in the channel configuration. Such changes in depth can be approximated as a discontinuity in the free-surface elevation ($dy/dx = \infty$). For reasons discussed below, this step change in depth is always from a shallow to a deeper depth—always a step up, never a step down.

A hydraulic jump is a steplike increase in fluid depth in an open channel.

Physically, this near discontinuity, called a ***hydraulic jump***, may result when there is a conflict between the upstream and downstream influences that control a particular section (or reach) of a channel. For example, a sluice gate may require that the conditions at the upstream portion of the channel (downstream of the gate) be supercritical flow, while obstructions in the channel on the downstream end of the reach may require that the flow be subcritical. The hydraulic jump provides the mechanism (a nearly discontinuous one at that) to make the transition between the two types of flow.

The simplest type of hydraulic jump occurs in a horizontal, rectangular channel as is indicated in Fig. 10.15. Although the flow within the jump itself is extremely complex and agitated, it is reasonable to assume that the flow at sections (1) and (2) is nearly uniform, steady, and one-dimensional. In addition, we neglect any wall shear stresses, τ_w, within the relatively short segment between these two sections. Under these conditions the x component of the momentum equation (Eq. 5.22) for the control volume indicated can be written as



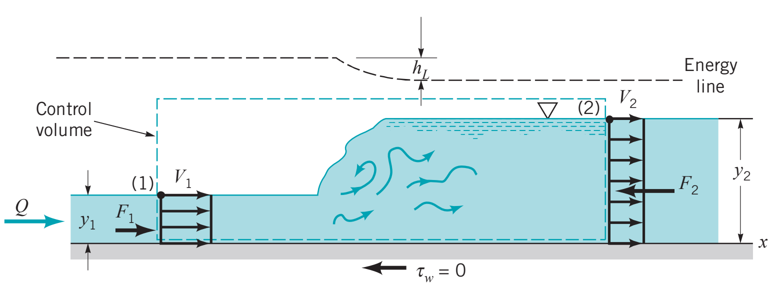
$$F_1 - F_2 = \rho Q(V_2 - V_1) = \rho V_1 y_1 b(V_2 - V_1)$$

where, as indicated by the figure in the margin, the pressure force at either section is hydrostatic. That is, $F_1 = p_{c1}A_1 = \gamma y_1^2 b/2$ and $F_2 = p_{c2}A_2 = \gamma y_2^2 b/2$, where $p_{c1} = \gamma y_1/2$ and $p_{c2} = \gamma y_2/2$ are the pressures at the centroids of the channel cross sections and b is the channel width. Thus, the momentum equation becomes

$$\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1) \tag{10.21}$$

In addition to the momentum equation, we have the conservation of mass equation (Eq. 5.12)

$$y_1 b V_1 = y_2 b V_2 = Q \tag{10.22}$$



■ FIGURE 10.15 Hydraulic jump geometry.

V10.11 Hydraulic jump in a river

and the energy equation (Eq. 5.84)

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \tag{10.23}$$

The head loss, h_L, in Eq. 10.23 is due to the violent turbulent mixing and dissipation that occur within the jump itself. We have neglected any head loss due to wall shear stresses.

Clearly Eqs. 10.21, 10.22, and 10.23 have a solution $y_1 = y_2$, $V_1 = V_2$, and $h_L = 0$. This represents the trivial case of no jump. Since these are nonlinear equations, it may be possible that more than one solution exists. The other solutions can be obtained as follows. By combining Eqs. 10.21 and 10.22 to eliminate V_2 we obtain

$$\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}\left(\frac{V_1 y_1}{y_2} - V_1\right) = \frac{V_1^2 y_1}{g y_2}(y_1 - y_2)$$

which can be simplified by factoring out a common nonzero factor $y_1 - y_2$ from each side to give

$$\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - 2\,\mathrm{Fr}_1^2 = 0$$

where $\mathrm{Fr}_1 = V_1/\sqrt{g y_1}$ is the upstream Froude number. By using the quadratic formula we obtain

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 \pm \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$$

The depth ratio across a hydraulic jump depends on the Froude number only.

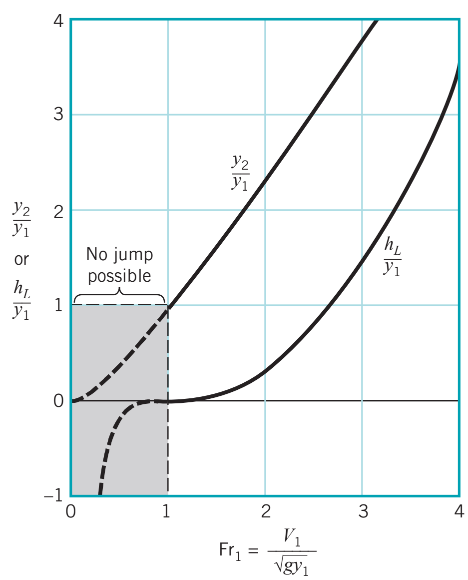
Clearly the solution with the minus sign is not possible (it would give a negative y_2/y_1). Thus,

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right) \tag{10.24}$$

This depth ratio, y_2/y_1, across the hydraulic jump is shown as a function of the upstream Froude number in Fig. 10.16. The portion of the curve for $\mathrm{Fr}_1 < 1$ is dashed in recognition of the fact that to have a hydraulic jump the flow must be supercritical. That is, the solution as given by Eq. 10.24 must be restricted to $\mathrm{Fr}_1 \geq 1$, for which $y_2/y_1 \geq 1$. This can be shown by consideration of the energy equation, Eq. 10.23, as follows. The dimensionless head loss, h_L/y_1, can be obtained from Eq. 10.23 as

$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\mathrm{Fr}_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right] \tag{10.25}$$

where, for given values of Fr_1, the values of y_2/y_1 are obtained from Eq. 10.24. As is indicated in Fig. 10.16, the head loss is negative if $\mathrm{Fr}_1 < 1$. Since negative head losses violate the second law



■ **FIGURE 10.16** **Depth ratio and dimensionless head loss across a hydraulic jump as a function of upstream Froude number.**

of thermodynamics (viscous effects dissipate energy, they cannot create energy; see Section 5.3), it is not possible to produce a hydraulic jump with $\mathrm{Fr}_1 < 1$. The head loss across the jump is indicated by the lowering of the energy line shown in Fig. 10.15.

(Photograph courtesy of U.S. Army Corps of Engineers.)

A flow must be supercritical (Froude number > 1) to produce the discontinuity called a hydraulic jump. This is analogous to the compressible flow ideas discussed in Chapter 11 in which it is shown that the flow of a gas must be supersonic (Mach number > 1) to produce the discontinuity called a normal shock wave. However, the fact that a flow is supercritical (or supersonic) does not guarantee the production of a hydraulic jump (or shock wave). The trivial solution $y_1 = y_2$ and $V_1 = V_2$ is also possible.

Hydraulic jumps dissipate energy.

The fact that there is an energy loss across a hydraulic jump is useful in many situations. For example, the relatively large amount of energy contained in the fluid flowing down the spillway of a dam like that shown in the figure in the margin could cause damage to the channel below the dam. By placing suitable flow control objects in the channel downstream of the spillway, it is possible (if the flow is supercritical) to produce a hydraulic jump on the apron of the spillway and thereby dissipate a considerable portion of the energy of the flow. That is, the dam spillway produces supercritical flow, and the channel downstream of the dam requires subcritical flow. The resulting hydraulic jump provides the means to change the character of the flow.

Fluids in the News

Grand Canyon rapids building Virtually all of the rapids in the Grand Canyon were formed by rock debris carried into the Colorado River from side canyons. Severe storms wash large amounts of sediment into the river, building debris fans that narrow the river. This debris forms crude dams which back up the river to form quiet pools above the rapids. Water exiting the pool through the narrowed channel can reach supercritical conditions and produce *hydraulic jumps* downstream. Since the configuration of the jumps is a function of the flowrate, the difficulty in running the rapids can change from day to day. Also, rapids change over the years as debris is added to or removed from the rapids. For example, Crystal Rapid, one of the notorious rafting stretches of the river, changed very little between the first photos of 1890 and those of 1966. However, a debris flow from a severe winter storm in 1966 greatly constricted the river. Within a few minutes the configuration of Crystal Rapid was completely changed. The new, immature rapid was again drastically changed by a flood in 1983. While Crystal Rapid is now considered full grown, it will undoubtedly change again, perhaps in 100 or 1000 years. (See Problem 10.100.)

EXAMPLE 10.7 Hydraulic Jump

GIVEN Water on the horizontal apron of the 100-ft-wide spillway shown in Fig. E10.7*a* has a depth of 0.60 ft and a velocity of 18 ft/s.

FIND Determine the depth, y_2, after the jump, the Froude numbers before and after the jump, Fr_1 and Fr_2, and the power dissipated, $\mathcal{P}_d$, within the jump.

SOLUTION

Conditions across the jump are determined by the upstream Froude number

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{18 \text{ ft/s}}{[(32.2 \text{ ft/s}^2)(0.60 \text{ ft})]^{1/2}} = 4.10 \qquad \textbf{(Ans)}$$

Thus, the upstream flow is supercritical, and it is possible to generate a hydraulic jump as sketched.

From Eq. 10.24 we obtain the depth ratio across the jump as

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\,\mathrm{Fr}_1^2}\right)$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8(4.10)^2}\right] = 5.32$$

or

$$y_2 = 5.32\,(0.60 \text{ ft}) = 3.19 \text{ ft} \qquad \textbf{(Ans)}$$

Since $Q_1 = Q_2$, or $V_2 = (y_1 V_1)/y_2 = 0.60 \text{ ft}\,(18 \text{ ft/s})/3.19 \text{ ft} = 3.39 \text{ ft/s}$, it follows that

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{3.39 \text{ ft/s}}{[(32.2 \text{ ft/s}^2)(3.19 \text{ ft})]^{1/2}} = 0.334 \qquad \textbf{(Ans)}$$

As is true for any hydraulic jump, the flow changes from supercritical to subcritical flow across the jump.

The power (energy per unit time) dissipated, $\mathcal{P}_d$, by viscous effects within the jump can be determined from the head loss

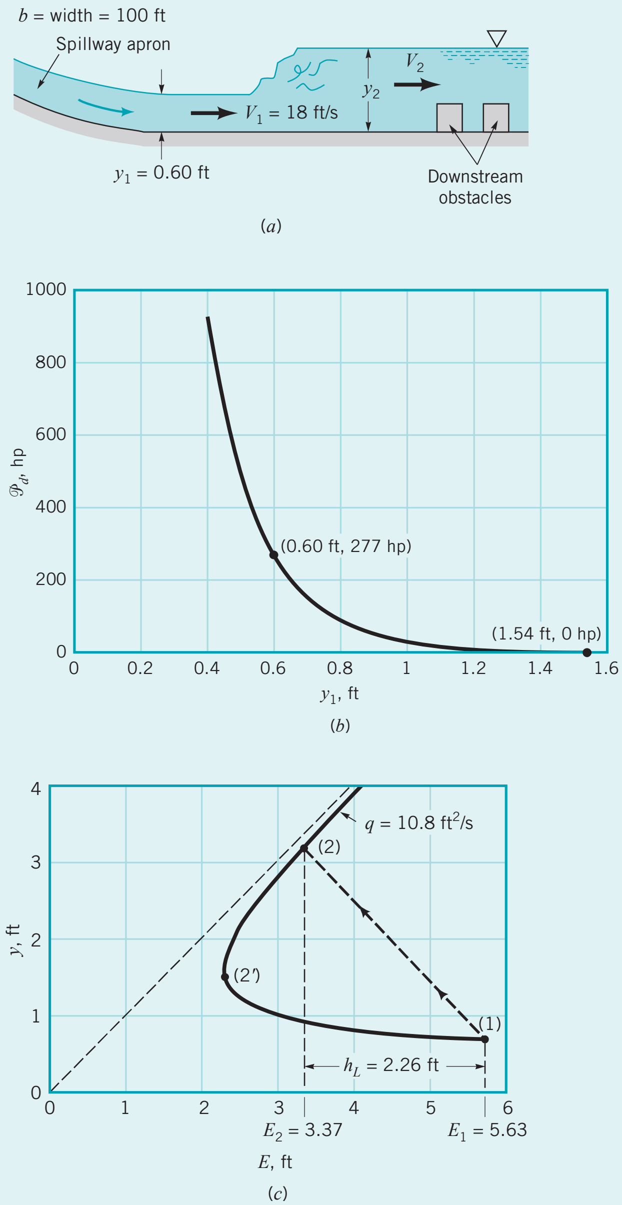


FIGURE E10.7

as (see Eq. 5.85)

$$\mathcal{P}_d = \gamma Q h_L = \gamma b y_1 V_1 h_L \tag{1}$$

where h_L is obtained from Eqs. 10.23 or 10.25 as

$$h_L = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) = \left[0.60 \text{ ft} + \frac{(18.0 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)}\right] - \left[3.19 \text{ ft} + \frac{(3.39 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)}\right]$$

or

$$h_L = 2.26 \text{ ft}$$

Thus, from Eq. 1,

$$\mathcal{P}_d = (62.4 \text{ lb/ft}^3)(100 \text{ ft})(0.60 \text{ ft})(18.0 \text{ ft/s})(2.26 \text{ ft}) = 1.52 \times 10^5 \text{ ft} \cdot \text{lb/s}$$

or

$$\mathcal{P}_d = \frac{1.52 \times 10^5 \text{ ft} \cdot \text{lb/s}}{550[(\text{ft} \cdot \text{lb/s})/\text{hp}]} = 277 \text{ hp} \qquad \textbf{(Ans)}$$

COMMENTS This power, which is dissipated within the highly turbulent motion of the jump, is converted into an increase in water temperature, T. That is, $T_2 > T_1$. Although the power dissipated is considerable, the difference in temperature is not great because the flowrate is quite large.

By repeating the calculations for the given flowrate $Q_1 = A_1V_1 = b_1y_1V_1 = 100 \text{ ft } (0.6 \text{ ft})(18 \text{ ft/s}) = 1080 \text{ ft}^3/\text{s}$ but with various upstream depths, y_1, the results shown in Fig. E10.7*b* are obtained. Note that a slight change in water depth can produce a considerable change in energy dissipated. Also, if $y_1 > 1.54$ ft the flow is subcritical ($\text{Fr}_1 < 1$) and no hydraulic jump can occur.

The hydraulic jump flow process can be illustrated by use of the specific energy concept introduced in Section 10.3 as follows. Equation 10.23 can be written in terms of the specific energy, $E = y + V^2/2g$, as $E_1 = E_2 + h_L$, where $E_1 = y_1 + V_1^2/2g = 5.63$ ft and $E_2 = y_2 + V_2^2/2g = 3.37$ ft. As is discussed in Section 10.3, the specific energy diagram for this flow can be obtained by using $V = q/y$, where

$$q = q_1 = q_2 = \frac{Q}{b} = y_1V_1 = 0.60 \text{ ft } (18.0 \text{ ft/s}) = 10.8 \text{ ft}^2/\text{s}$$

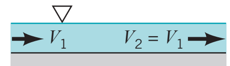
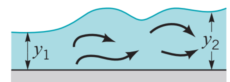
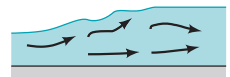
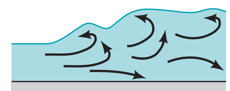
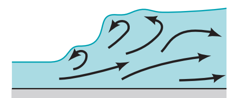
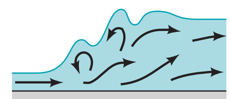
Thus,

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(10.8 \text{ ft}^2/\text{s})^2}{2(32.2 \text{ ft/s}^2)y^2} = y + \frac{1.81}{y^2}$$

where y and E are in feet. The resulting specific energy diagram is shown in Fig. E10.7*c*. Because of the head loss across the jump, the upstream and downstream values of E are different. In going from state (1) to state (2) the fluid does not proceed along the specific energy curve and pass through the critical condition at state 2′. Rather, it jumps from (1) to (2) as is represented by the dashed line in the figure. From a one-dimensional consideration, the jump is a discontinuity. In actuality, the jump is a complex three-dimensional flow incapable of being represented on the one-dimensional specific energy diagram.

The actual structure of a hydraulic jump is a complex function of Fr_1, even though the depth ratio and head loss are given quite accurately by a simple one-dimensional flow analysis (Eqs. 10.24 and 10.25). A detailed investigation of the flow indicates that there are essentially five types of surface and jump conditions. The classification of these jumps is indicated in Table 10.2, along with sketches of the structure of the jump. For flows that are barely supercritical, the jump is more like a standing wave, without a nearly step change in depth. In some Froude number ranges the jump is

■ **TABLE 10.2**
Classification of Hydraulic Jumps (Ref. 12)

Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

The actual structure of a hydraulic jump depends on the Froude number.

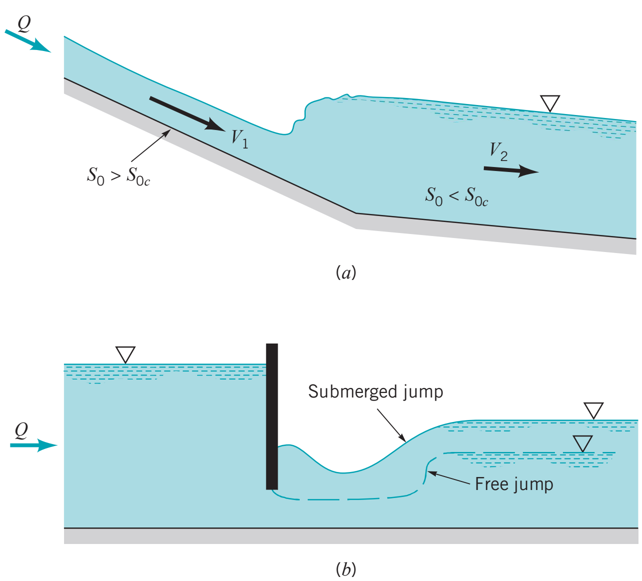
V10.12 Hydraulic jump in a sink

unsteady, with regular periodic oscillations traveling downstream. (Recall that the wave cannot travel upstream against the supercritical flow.)

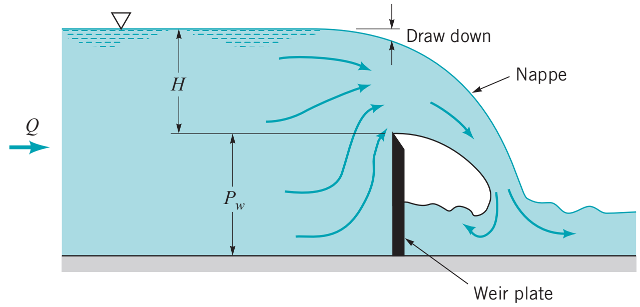
The length of a hydraulic jump (the distance between the nearly uniform upstream and downstream flows) may be of importance in the design of channels. Although its value cannot be determined theoretically, experimental results indicate that over a wide range of Froude numbers, the jump is approximately seven downstream depths long (Ref. 5).

Hydraulic jumps can occur in a variety of channel flow configurations, not just in horizontal, rectangular channels as discussed above. Jumps in nonrectangular channels (i.e., circular pipes, trapezoidal canals) behave in a manner quite like those in rectangular channels, although the details of the depth ratio and head loss are somewhat different from jumps in rectangular channels.

Other common types of hydraulic jumps include those that occur in sloping channels as is indicated in Fig. 10.17*a* and the submerged hydraulic jumps that can occur just downstream of a



■ **FIGURE 10.17**
Hydraulic jump variations: (*a*) jump caused by a change in channel slope, (*b*) submerged jump.



■ FIGURE 10.18 Sharp-crested weir geometry.

sluice gate as is indicated in Fig. 10.17*b*. Details of these and other jumps can be found in standard open-channel flow references (Refs. 3 and 5).

10.6.2 Sharp-Crested Weirs

A weir is an obstruction on a channel bottom over which the fluid must flow. It provides a convenient method of determining the flowrate in an open channel in terms of a single depth measurement. A ***sharp-crested weir*** is essentially a vertical sharp-edged flat plate placed across the channel in a way such that the fluid must flow across the sharp edge and drop into the pool downstream of the weir plate, as is shown in Fig. 10.18. The specific shape of the flow area in the plane of the weir plate is used to designate the type of weir. Typical shapes include the rectangular weir, the triangular weir, and the trapezoidal weir, as indicated in Fig. 10.19.

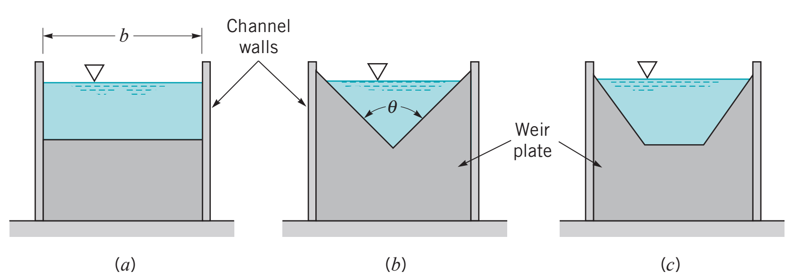
A sharp-crested weir can be used to determine the flowrate.

The complex nature of the flow over a weir makes it impossible to obtain precise analytical expressions for the flow as a function of other parameters, such as the weir height, P_w, ***weir head,*** H, the fluid depth upstream, and the geometry of the weir plate (angle θ for triangular weirs or aspect ratio, b/H, for rectangular weirs). The flow structure is far from one-dimensional, with a variety of interesting flow phenomena obtained.

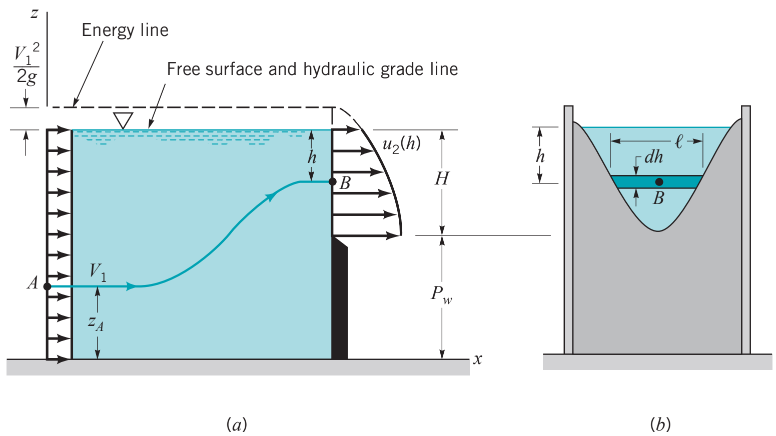
The main mechanisms governing flow over a weir are gravity and inertia. From a highly simplified point of view, gravity accelerates the fluid from its free-surface elevation upstream of the weir to larger velocity as it flows down the hill formed by the nappe. Although viscous and surface tension effects are usually of secondary importance, such effects cannot be entirely neglected. Generally, appropriate experimentally determined coefficients are used to account for these effects.

As a first approximation, we assume that the velocity profile upstream of the weir plate is uniform and that the pressure within the nappe is atmospheric. In addition, we assume that the fluid flows horizontally over the weir plate with a nonuniform velocity profile, as indicated in Fig. 10.20. With $p_B = 0$ the Bernoulli equation for flow along the arbitrary streamline A–B indicated can be written as

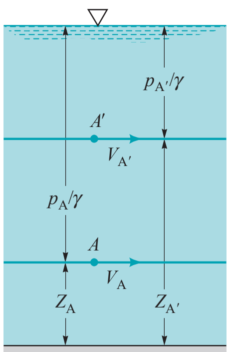
$$\frac{p_A}{\gamma} + \frac{V_1^2}{2g} + z_A = (H + P_w - h) + \frac{u_2^2}{2g} \tag{10.26}$$



■ FIGURE 10.19 Sharp-crested weir plate geometry: (*a*) rectangular, (*b*) triangular, (*c*) trapezoidal.



■ FIGURE 10.20 **Assumed flow structure over a weir.**



where h is the distance that point B is below the free surface. We do not know the location of point A from which came the fluid that passes over the weir at point B. However, since the total head for any particle along the vertical section (1) is the same, $z_A + p_A/\gamma + V_1^2/2g = H + P_w + V_1^2/2g$, the specific location of A (i.e., A or A' shown in the figure in the margin) is not needed, and the velocity of the fluid over the weir plate is obtained from Eq. 10.26 as

$$u_2 = \sqrt{2g\left(h + \frac{V_1^2}{2g}\right)}$$

The flowrate can be calculated from

$$Q = \int_{(2)} u_2 \, dA = \int_{h=0}^{h=H} u_2 \ell \, dh \qquad \textbf{(10.27)}$$

where $\ell = \ell(h)$ is the cross-channel width of a strip of the weir area, as is indicated in Fig. 10.20*b*. For a rectangular weir ℓ is constant. For other weirs, such as triangular or circular weirs, the value of ℓ is known as a function of h.

For a rectangular weir, $\ell = b$, and the flowrate becomes

$$Q = \sqrt{2g}\, b \int_0^H \left(h + \frac{V_1^2}{2g}\right)^{1/2} dh$$

or

$$Q = \frac{2}{3}\sqrt{2g}\, b \left[\left(H + \frac{V_1^2}{2g}\right)^{3/2} - \left(\frac{V_1^2}{2g}\right)^{3/2}\right] \qquad \textbf{(10.28)}$$

Equation 10.28 is a rather cumbersome expression that can be simplified by using the fact that with $P_w \gg H$ (as often happens in practical situations) the upstream velocity is negligibly small. That is, $V_1^2/2g \ll H$ and Eq. 10.28 simplifies to the basic rectangular weir equation

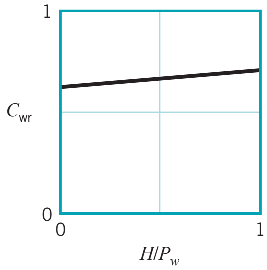
$$Q = \tfrac{2}{3}\sqrt{2g}\, b\, H^{3/2} \qquad \textbf{(10.29)}$$

Note that the weir head, H, is the height of the upstream free surface above the crest of the weir. As is indicated in Fig. 10.18, because of the drawdown effect, H is not the distance of the free surface above the weir crest as measured directly above the weir plate.

A weir coefficient is used to account for nonideal conditions excluded in the simplified analysis.

Because of the numerous approximations made to obtain Eq. 10.29, it is not unexpected that an experimentally determined correction factor must be used to obtain the actual flowrate as a function of weir head. Thus, the final form is

$$Q = C_{\text{wr}}\, \tfrac{2}{3}\sqrt{2g}\, b\, H^{3/2} \qquad \textbf{(10.30)}$$



where C_{wr} is the rectangular weir coefficient. From dimensional analysis arguments, it is expected that C_{wr} is a function of Reynolds number (viscous effects), Weber number (surface tension effects), and H/P_w (geometry). In most practical situations, the Reynolds and Weber number effects are negligible, and the following correlation, shown in the figure in the margin, can be used (Refs. 4, 7):

$$C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right) \tag{10.31}$$

More precise values of C_{wr} can be found in the literature, if needed (Refs. 3, 14).

The triangular sharp-crested weir is often used for flow measurements, particularly for measuring flowrates over a wide range of values. For small flowrates, the head, H, for a rectangular weir would be very small and the flowrate could not be measured accurately. However, with the triangular weir, the flow width decreases as H decreases so that even for small flowrates, reasonable heads are developed. Accurate results can be obtained over a wide range of Q.

V10.13 Triangular weir

The triangular weir equation can be obtained from Eq. 10.27 by using

$$\ell = 2(H - h)\tan\left(\frac{\theta}{2}\right)$$

where θ is the angle of the V-notch (see Figs. 10.19 and 10.20). After carrying out the integration and again neglecting the upstream velocity ($V_1^2/2g \ll H$), we obtain

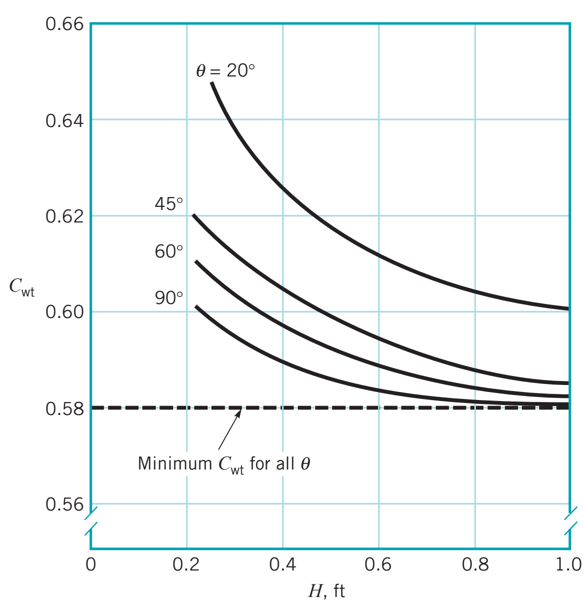
$$Q = \frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2}$$

An experimentally determined triangular weir coefficient, C_{wt}, is used to account for the real-world effects neglected in the analysis so that

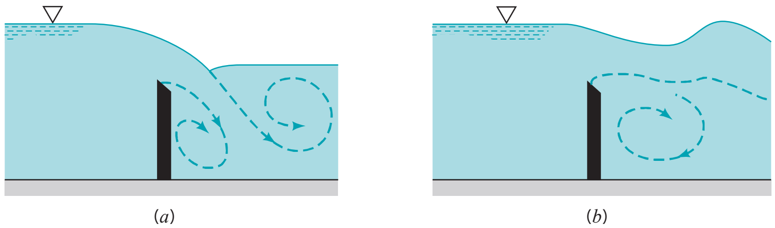
$$Q = C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2} \tag{10.32}$$

V10.14 Low-head dam

Typical values of C_{wt} for triangular weirs are in the range of 0.58 to 0.62, as is shown in Fig. 10.21. Note that although C_{wt} and θ are dimensionless, the value of C_{wt} is given as a function of the weir head, H, which is a dimensional quantity. Although using dimensional parameters is not recommended (see the dimensional analysis discussion in Chapter 7), such parameters are often used for open-channel flow.



■ **FIGURE 10.21** **Weir coefficient for triangular sharp-crested weirs (Ref. 10).**



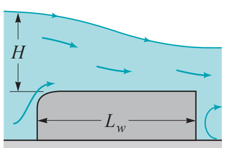
■ FIGURE 10.22 Flow conditions over a weir without a free nappe: (a) plunging nappe, (b) submerged nappe.

Flowrate over a weir depends on whether the nappe is free or submerged.

The above results for sharp-crested weirs are valid provided the area under the nappe is ventilated to atmospheric pressure. Although this is not a problem for triangular weirs, for rectangular weirs it is sometimes necessary to provide ventilation tubes to ensure atmospheric pressure in this region. In addition, depending on downstream conditions, it is possible to obtain submerged weir operation, as is indicated in Fig. 10.22. Clearly the flowrate will be different for these situations than that given by Eqs. 10.30 and 10.32.

10.6.3 Broad-Crested Weirs

$H/L_w = 0.08$



$H/L_w = 0.50$

A ***broad-crested weir*** is a structure in an open channel that has a horizontal crest above which the fluid pressure may be considered hydrostatic. A typical configuration is shown in Fig. 10.23. Generally, to ensure proper operation, these weirs are restricted to the range $0.08 < H/L_w < 0.50$. These conditions are drawn to scale in the figure in the margin. For long weir blocks (H/L_w less than 0.08), head losses across the weir cannot be neglected. On the other hand, for short weir blocks (H/L_w greater than 0.50) the streamlines of the flow over the weir block are not horizontal. Although broad-crested weirs can be used in channels of any cross-sectional shape, we restrict our attention to rectangular channels.

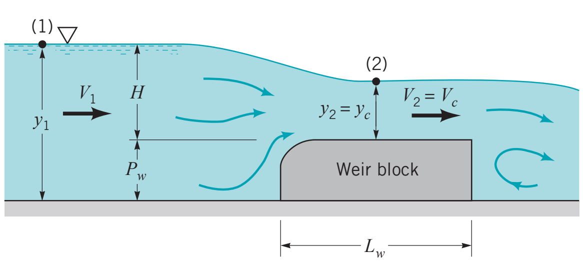
The operation of a broad-crested weir is based on the fact that nearly uniform critical flow is achieved in the short reach above the weir block. (If $H/L_w < 0.08$, viscous effects are important, and the flow is subcritical over the weir.) If the kinetic energy of the upstream flow is negligible, then $V_1^2/2g \ll y_1$ and the upstream specific energy is $E_1 = V_1^2/2g + y_1 \approx y_1$. Observations show that as the flow passes over the weir block, it accelerates and reaches critical conditions, $y_2 = y_c$ and $\text{Fr}_2 = 1$ (i.e., $V_2 = c_2$), corresponding to the nose of the specific energy curve (see Fig. 10.7). The flow does not accelerate to supercritical conditions ($\text{Fr}_2 > 1$). To do so would require the ability of the downstream fluid to communicate with the upstream fluid to let it know that there is an end of the weir block. Since waves cannot propagate upstream against a critical flow, this information cannot be transmitted. The flow remains critical, not supercritical, across the weir block.

The Bernoulli equation can be applied between point (1) upstream of the weir and point (2) over the weir where the flow is critical to obtain

$$H + P_w + \frac{V_1^2}{2g} = y_c + P_w + \frac{V_c^2}{2g}$$

or, if the upstream velocity head is negligible

$$H - y_c = \frac{(V_c^2 - V_1^2)}{2g} = \frac{V_c^2}{2g}$$



■ FIGURE 10.23 Broad-crested weir geometry.

However, since $V_2 = V_c = (gy_c)^{1/2}$, we find that $V_c^2 = gy_c$ so that we obtain

The broad-crested weir is governed by critical flow across the weir block.

$$H - y_c = \frac{y_c}{2}$$

or

$$y_c = \frac{2H}{3}$$

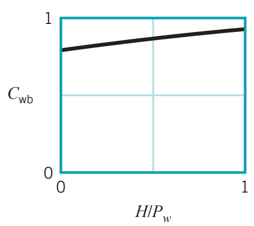
Thus, the flowrate is

$$Q = by_2V_2 = by_cV_c = by_c(gy_c)^{1/2} = b\sqrt{g}\,y_c^{3/2}$$

or

$$Q = b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

Again an empirical weir coefficient is used to account for the various real-world effects not included in the above simplified analysis. That is



$$Q = C_{wb}\, b\sqrt{g}\left(\frac{2}{3}\right)^{3/2} H^{3/2} \tag{10.33}$$

where approximate values of C_{wb}, the broad-crested weir coefficient shown in the figure in the margin, can be obtained from the equation (Ref. 6)

$$C_{wb} = 1.125\left(\frac{1 + H/P_w}{2 + H/P_w}\right)^{1/2} \tag{10.34}$$

EXAMPLE 10.8 Sharp-Crested and Broad-Crested Weirs

GIVEN Water flows in a rectangular channel of width $b = 2$ m with flowrates between $Q_{min} = 0.02$ m^3/s and $Q_{max} = 0.60$ m^3/s. This flowrate is to be measured by using either (a) a rectangular sharp-crested weir, (b) a triangular sharp-crested weir with $\theta = 90°$, or (c) a broad-crested weir. In all cases the bottom of the flow area over the weir is a distance $P_w = 1$ m above the channel bottom.

FIND Plot a graph of $Q = Q(H)$ for each weir and comment on which weir would be best for this application.

SOLUTION

(a) For the rectangular weir with $P_w = 1$ m, Eqs. 10.30 and 10.31 give

$$Q = C_{wr}\frac{2}{3}\sqrt{2g}\,bH^{3/2}$$
$$= \left(0.611 + 0.075\frac{H}{P_w}\right)\frac{2}{3}\sqrt{2g}\,bH^{3/2}$$

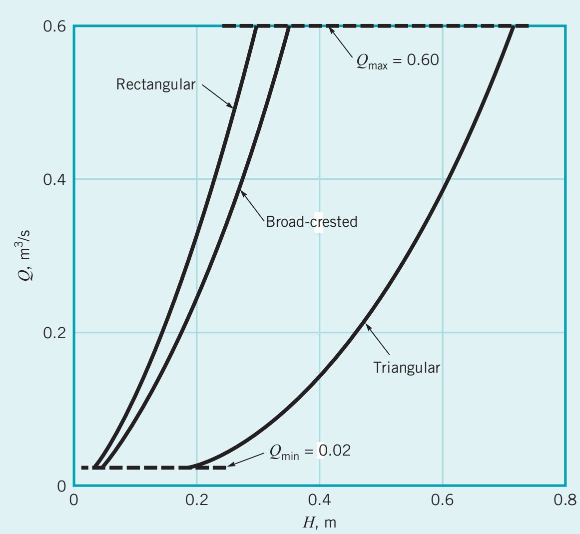
Thus,

$$Q = (0.611 + 0.075H)\frac{2}{3}\sqrt{2(9.81 \text{ m/s}^2)}\,(2 \text{ m})\,H^{3/2}$$

or

$$Q = 5.91(0.611 + 0.075H)H^{3/2} \tag{1}$$

where H and Q are in meters and m^3/s, respectively. The results from Eq. 1 are plotted in Fig. E10.8.



■ FIGURE E10.8

(b) Similarly, for the triangular weir, Eq. 10.32 gives

$$Q = C_{wt}\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\,H^{5/2}$$

$$= C_{wt}\frac{8}{15}\tan(45°)\sqrt{2(9.81\text{ m/s}^2)}\,H^{5/2}$$

or

$$Q = 2.36C_{wt}H^{5/2} \tag{2}$$

where H and Q are in meters and m^3/s and C_{wt} is obtained from Fig. 10.21. For example, with $H = 0.20$ m, we find $C_{wt} = 0.60$, or $Q = 2.36\,(0.60)(0.20)^{5/2} = 0.0253\text{ m}^3/\text{s}$. The triangular weir results are also plotted in Fig. E10.8.

(c) For the broad-crested weir, Eqs. 10.28 and 10.29 give

$$Q = C_{wb}\,b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2}$$

$$= 1.125\left(\frac{1 + H/P_w}{2 + H/P_w}\right)^{1/2} b\sqrt{g}\left(\frac{2}{3}\right)^{3/2}H^{3/2}$$

Thus, with $P_w = 1$ m

$$Q = 1.125\left(\frac{1 + H}{2 + H}\right)^{1/2}(2\text{ m})\sqrt{9.81\text{ m/s}^2}\left(\frac{2}{3}\right)^{3/2}H^{3/2}$$

or

$$Q = 3.84\left(\frac{1 + H}{2 + H}\right)^{1/2}H^{3/2} \tag{3}$$

where, again, H and Q are in meters and m^3/s. This result is also plotted in Fig. E10.8.

COMMENTS Although it appears as though any of the three weirs would work well for the upper portion of the flowrate range, neither the rectangular nor the broad-crested weir would be very accurate for small flowrates near $Q = Q_{min}$ because of the small head, H, at these conditions. The triangular weir, however, would allow reasonably large values of H at the lowest flowrates. The corresponding heads with $Q = Q_{min} = 0.02\text{ m}^3/\text{s}$ for rectangular, triangular, and broad-crested weirs are 0.0312, 0.182, and 0.0375 m, respectively.

In addition, as discussed in this section, for proper operation the broad-crested weir geometry is restricted to $0.08 < H/L_w < 0.50$, where L_w is the weir block length. From Eq. 3 with $Q_{max} = 0.60\text{ m}^3/\text{s}$, we obtain $H_{max} = 0.349$. Thus, we must have $L_w > H_{max}/0.5 = 0.698$ m to maintain proper critical flow conditions at the largest flowrate in the channel. However, with $Q = Q_{min} = 0.02\text{ m}^3/\text{s}$, we obtain $H_{min} = 0.0375$ m. Thus, we must have $L_w < H_{min}/0.08 = 0.469$ m to ensure that frictional effects are not important. Clearly, these two constraints on the geometry of the weir block, L_w, are incompatible.

A broad-crested weir will not function properly under the wide range of flowrates considered in this example. The sharp-crested triangular weir would be the best of the three types considered, provided the channel can handle the $H_{max} = 0.719$-m head.

10.6.4 Underflow Gates

(Photograph courtesy of Pend Oreille Public Utility District.)

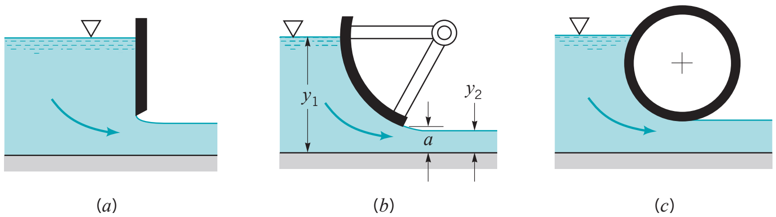
A variety of ***underflow gate*** structures is available for flowrate control at the crest of an overflow spillway (as shown by the figure in the margin), or at the entrance of an irrigation canal or river from a lake. Three types are illustrated in Fig. 10.24. Each has certain advantages and disadvantages in terms of costs of construction, ease of use, and the like, although the basic fluid mechanics involved are the same in all instances.

The flow under a gate is said to be free outflow when the fluid issues as a jet of supercritical flow with a free surface open to the atmosphere as shown in Fig. 10.24. In such cases it is customary to write this flowrate as the product of the distance, a, between the channel bottom and the bottom of the gate times the convenient reference velocity $(2gy_1)^{1/2}$. That is,

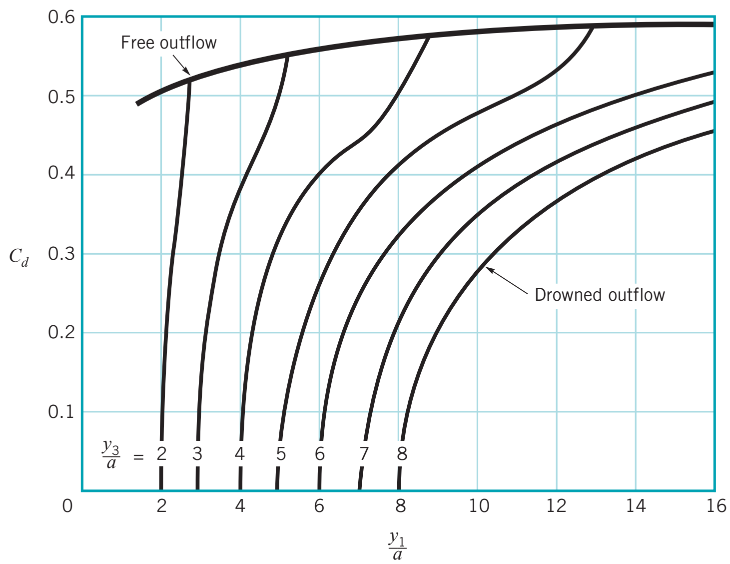
$$q = C_d\,a\sqrt{2gy_1} \tag{10.35}$$

V10.15 Spillway gate

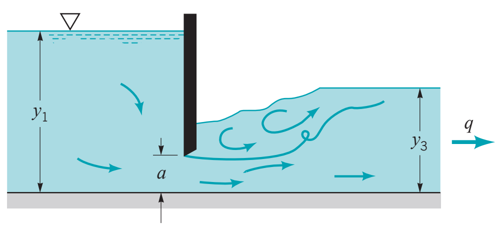
where q is the flowrate per unit width. The discharge coefficient, C_d, is a function of the contraction coefficient, $C_c = y_2/a$, and the depth ratio y_1/a. Typical values of the discharge coefficient for free



■ **FIGURE 10.24** **Three variations of underflow gates: (*a*) vertical gate, (*b*) radial gate, (*c*) drum gate.**



■ **FIGURE 10.25** **Typical discharge coefficients for underflow gates (Ref. 3).**



■ **FIGURE 10.26** **Drowned outflow from a sluice gate.**

The flowrate from an underflow gate depends on whether the outlet is free or drowned.

V10.16 Unsteady under and over

outflow (or free discharge) from a vertical sluice gate are on the order of 0.55 to 0.60 as indicated by the top line in Fig. 10.25 (Ref. 3).

As indicated in Fig. 10.26, in certain situations the depth downstream of the gate is controlled by some downstream obstacle and the jet of water issuing from under the gate is overlaid by a mass of water that is quite turbulent.

The flowrate for a submerged (or drowned) gate can be obtained from the same equation that is used for free outflow (Eq. 10.35), provided the discharge coefficient is modified appropriately. Typical values of C_d for drowned outflow cases are indicated as the series of lower curves in Fig. 10.25. Consider flow for a given gate and upstream conditions (i.e., given y_1/a) corresponding to a vertical line in the figure. With $y_3/a = y_1/a$ (i.e., $y_3 = y_1$) there is no head to drive the flow so that $C_d = 0$ and the fluid is stationary. For a given upstream depth (y_1/a fixed), the value of C_d increases with decreasing y_3/a until the maximum value of C_d is reached. This maximum corresponds to the free discharge conditions and is represented by the free outflow line so labeled in Fig. 10.25. For values of y_3/a that give C_d values between zero and its maximum, the jet from the gate is overlaid (drowned) by the downstream water and the flowrate is therefore reduced when compared with a free discharge situation. Similar results are obtained for the radial gate and drum gate.

EXAMPLE 10.9 Sluice Gate

GIVEN Water flows under the sluice gate shown in Fig. E10.9. The channel width is $b = 20$ ft, the upstream depth is $y_1 = 6$ ft, and the gate is $a = 1.0$ ft off the channel bottom.

FIND Plot a graph of flowrate, Q, as a function of y_3.

SOLUTION

From Eq. 10.35 we have

$$Q = bq = baC_d \sqrt{2gy_1}$$
$$= 20 \text{ ft } (1.0 \text{ ft})\, C_d \sqrt{2(32.2 \text{ ft/s}^2)(6.0 \text{ ft})}$$

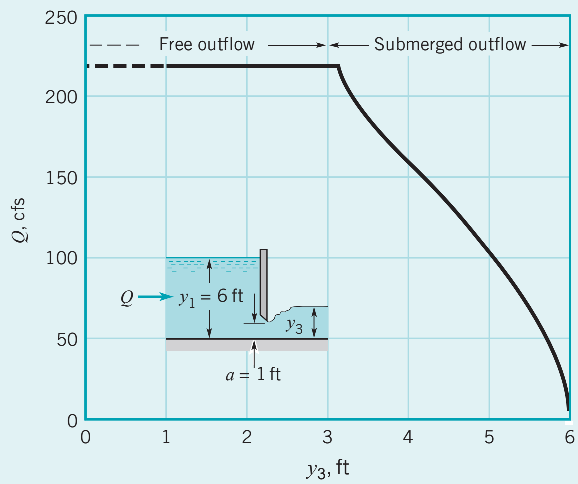
or

$$Q = 393C_d \text{ cfs} \qquad \textbf{(1)}$$

The value of C_d is obtained from Fig. 10.25 along the vertical line $y_1/a = 6 \text{ ft}/1 \text{ ft} = 6$. For $y_3 = 6$ ft (i.e., $y_3/a = 6 = y_1/a$) we obtain $C_d = 0$, indicating that there is no flow when there is no head difference across the gate. The value of C_d increases as y_3/a decreases, reaching a maximum of $C_d = 0.56$ when $y_3/a = 3.2$. Thus, with $y_3 = 3.2a = 3.2$ ft

$$Q = 393\,(0.56) \text{ cfs} = 220 \text{ cfs}$$

The flowrate for 3.2 ft $\leq y_3 \leq$ 6 ft is obtained from Eq. 1 and the C_d values of Fig. 10.24 with the results as indicated in Fig. E10.9.



■ FIGURE E10.9

COMMENT For $y_3 < 3.2$ ft the flowrate is independent of y_3, and the outflow is a free (not submerged) outflow. For such cases the inertia of the water flowing under the gate is sufficient to produce free outflow even with $y_3 > a$.

10.7 Chapter Summary and Study Guide

open-channel flow
Froude number
critical flow
subcritical flow
supercritical flow
wave speed
specific energy
specific energy diagram
uniform depth flow
wetted perimeter
hydraulic radius
Chezy equation
Manning equation
Manning coefficient
rapidly varied flow
hydraulic jump
sharp-crested weir
weir head
broad-crested weir
underflow gate

This chapter discussed various aspects of flows in an open channel. A typical open-channel flow is driven by the component of gravity in the direction of flow. The character of such flows can be a strong function of the Froude number, which is ratio of the fluid speed to the free-surface wave speed. The specific energy diagram is used to provide insight into the flow processes involved in open-channel flow.

Uniform depth channel flow is achieved by a balance between the potential energy lost by the fluid as it coasts downhill and the energy dissipated by viscous effects. Alternately, it represents a balance between weight and friction forces. The relationship among the flowrate, the slope of the channel, the geometry of the channel, and the roughness of the channel surfaces is given by the Manning equation. Values of the Manning coefficient used in the Manning equation are dependent on the surface material roughness.

The hydraulic jump is an example of nonuniform depth open-channel flow. If the Froude number of a flow is greater than one, the flow is supercritical, and a hydraulic jump may occur. The momentum and mass equations are used to obtain the relationship between the upstream Froude number and the depth ratio across the jump. The energy dissipated in the jump and the head loss can then be determined by use of the energy equation.

The use of weirs to measure the flowrate in an open channel is discussed. The relationships between the flowrate and the weir head are given for both sharp-crested and broad-crested weirs.

The following checklist provides a study guide for this chapter. When your study of the entire chapter and end-of-chapter exercises has been completed you should be able to

- write out meanings of the terms listed here in the margin and understand each of the related concepts. These terms are particularly important and are set in ***italic, bold, and color*** type in the text.
- determine the Froude number for a given flow and explain the concepts of subcritical, critical, and supercritical flows.
- plot and interpret the specific energy diagram for a given flow.

- use the Manning equation to analyze uniform depth flow in an open channel.
- calculate properties such as the depth ratio and the head loss for a hydraulic jump.
- determine the flowrates over sharp-crested weirs, broad-crested weirs, and under underflow gates.

Some of the important equations in this chapter are:

Froude number	$\mathrm{Fr} = V/(gy)^{1/2}$	
Wave speed	$c = \sqrt{gy}$	**(10.3)**
Specific energy	$E = y + \dfrac{V^2}{2g}$	**(10.8)**
Manning equation	$V = \dfrac{\kappa}{n} R_h^{2/3} S_0^{1/2}$	**(10.19)**
Hydraulic jump depth ratio	$\dfrac{y_2}{y_1} = \dfrac{1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$	**(10.24)**
Hydraulic jump head loss	$\dfrac{h_L}{y_1} = 1 - \dfrac{y_2}{y_1} + \dfrac{\mathrm{Fr}_1^2}{2}\left[1 - \left(\dfrac{y_1}{y_2}\right)^2\right]$	**(10.25)**
Rectangular sharp-crested weir	$Q = C_{\mathrm{wr}} \dfrac{2}{3} \sqrt{2g}\, b\, H^{3/2}$	**(10.30)**
Triangular sharp-crested weir	$Q = C_{\mathrm{wt}} \dfrac{8}{15} \tan\left(\dfrac{\theta}{2}\right) \sqrt{2g}\, H^{5/2}$	**(10.32)**
Broad-crested weir	$Q = C_{\mathrm{wb}}\, b \sqrt{g} \left(\dfrac{2}{3}\right)^{3/2} H^{3/2}$	**(10.33)**
Underflow gate	$q = C_d\, a \sqrt{2gy_1}$	**(10.35)**

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Review Problems

Go to Appendix G for a set of review problems with answers. Detailed solutions can be found in *Student Solution Manual and Study Guide for Fundamentals of Fluid Mechanics*, by Munson et al. (© 2009 John Wiley and Sons, Inc.).

Problems

Note: Unless otherwise indicated, use the values of fluid properties found in the tables on the inside of the front cover. Problems designated with an (*) are intended to be solved with the aid of a programmable calculator or a computer. Problems designated with a (†) are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide the necessary data. There is not a unique answer to these problems.

Answers to the even-numbered problems are listed at the end of the book. Access to the videos that accompany problems can be obtained through the book's web site, www.wiley.com/college/munson. The lab-type problems can also be accessed on this web site.

Section 10.2 Surface Waves

10.1 Obtain a photograph/image of surface waves. Print this photo and write a brief paragraph that describes the similarities and differences between these waves and those depicted in Fig. 10.4.

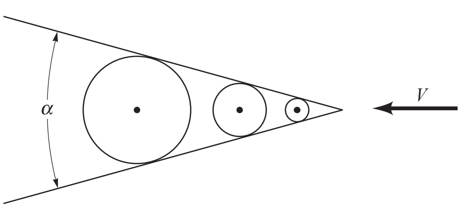
10.2 On a distant planet small amplitude waves travel across a 1-m-deep pond with a speed of 5 m/s. Determine the acceleration of gravity on the surface of that planet.

10.3 The flowrate in a 50-ft-wide, 2-ft-deep river is $Q = 190$ cfs. Is the flow subcritical or supercritical?

10.4 The flowrate per unit width in a wide channel is $q = 2.3\ \text{m}^2/\text{s}$. Is the flow subcritical or supercritical if the depth is **(a)** 0.2 m, **(b)** 0.8 m, or **(c)** 2.5 m?

10.5 A rectangular channel 3 m wide carries 10 m^3/s at a depth of 2 m. Is the flow subcritical or supercritical? For the same flowrate, what depth will give critical flow?

10.6 Consider waves made by dropping objects (one after another from a fixed location) into a stream of depth y that is moving with speed V as shown in Fig. P10.6 (see **Video V10.5**). The circular wave crests that are produced travel with speed $c = (gy)^{1/2}$ relative to the moving water. Thus, as the circular waves are washed downstream, their diameters increase and the center of each circle is fixed relative to the moving water. **(a)** Show that if the flow is supercritical, lines tangent to the waves generate a wedge of half-angle $\alpha/2 = \arcsin(1/\text{Fr})$, where $\text{Fr} = V/(gy)^{1/2}$ is the Froude number. **(b)** Discuss what happens to the wave pattern when the flow is subcritical, $\text{Fr} < 1$.

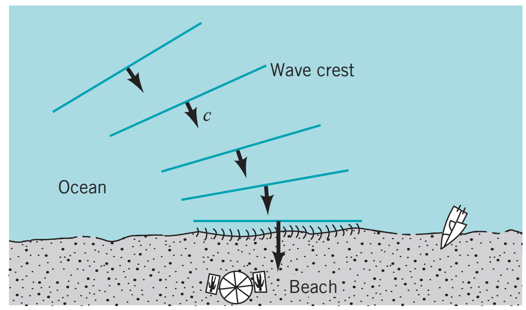


■ FIGURE P10.6

10.7 Waves on the surface of a tank are observed to travel at a speed of 2 m/s. How fast would these waves travel if **(a)** the tank were in an elevator accelerating downward at a rate of 4 m/s^2, **(b)** the tank accelerates horizontally at a rate of 9.81 m/s^2, **(c)** the tank were aboard the orbiting Space Shuttle? Explain.

10.8 In flowing from section (1) to section (2) along an open channel, the water depth decreases by a factor of two and the Froude number changes from a subcritical value of 0.5 to a supercritical value of 3.0. Determine the channel width at (2) if it is 12 ft wide at (1).

10.9 Observations at a shallow sandy beach show that even though the waves several hundred yards out from the shore are not parallel to the beach, the waves often "break" on the beach nearly parallel to the shore as indicated in Fig. P10.9. Explain this behavior based on the wave speed $c = (gy)^{1/2}$.

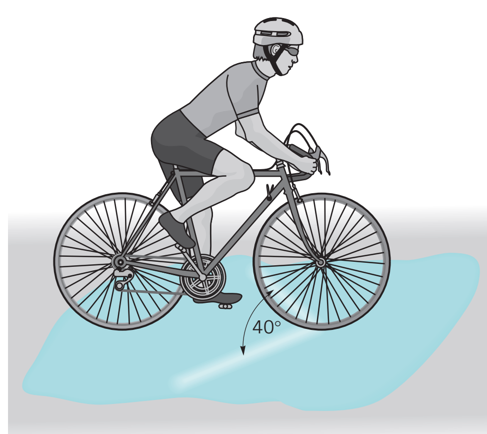


■ FIGURE P10.9

†10.10 Explain, physically, why surface tension increases the speed of surface waves.

10.11 Often when an earthquake shifts a segment of the ocean floor, a relatively small amplitude wave of very long wavelength is produced. Such waves go unnoticed as they move across the open ocean; only when they approach the shore do they become dangerous (a tsunami or "tidal wave"). Determine the wave speed if the wavelength, λ, is 6000 ft and the ocean depth is 15,000 ft.

10.12 A bicyclist rides through a 3-in.-deep puddle of water as shown in **Video V10.5** and Fig. P10.12. If the angle made by the V-shaped wave pattern produced by the front wheel is observed to be 40°, estimate the speed of the bike through the puddle. *Hint:* Make a sketch of the current location of the bike wheel relative to where it was Δt seconds ago. Also indicate on this sketch the current location of the wave that the wheel made Δt seconds ago. Recall that the wave moves radially outward in all directions with speed c relative to the stationary water.



■ FIGURE P10.12

10.13 Determine the minimum depth in a 3-m-wide rectangular channel if the flow is to be subcritical with a flowrate of $Q = 60\ \text{m}^3/\text{s}$.

10.14 (See Fluids in the News article titled "Tsunami, the nonstorm wave," Section 10.2.1.) An earthquake causes a shift in the ocean floor that produces a tsunami with a wavelength of 100 km. How fast will this wave travel across the ocean surface if the ocean depth is 3000 m?

Section 10.3 Energy Considerations

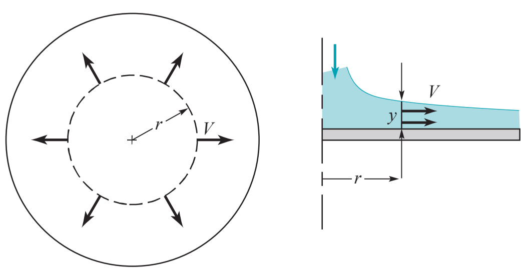
10.15 Water flows in a 10-m-wide open channel with a flowrate of 5 m^3/s. Determine the two possible depths if the specific energy of the flow is $E = 0.6$ m.

10.16 Water flows in a rectangular channel with a flowrate per unit width of $q = 2.5\ m^2/s$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 2.5$ m.

10.17 Water flows radially outward on a horizontal round disk as shown in Video V10.12 and Fig. P10.17. **(a)** Show that the specific energy can be written in terms of the flowrate, Q, the radial distance from the axis of symmetry, r, and the fluid depth, y, as

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) For a constant flowrate, sketch the specific energy diagram. Recall Fig. 10.7, but note that for the present case r is a variable. Explain the important characteristics of your sketch. **(c)** Based on the results of Part **(b)**, show that the water depth increases in the flow direction if the flow is subcritical, but that it decreases in the flow direction if the flow is supercritical.

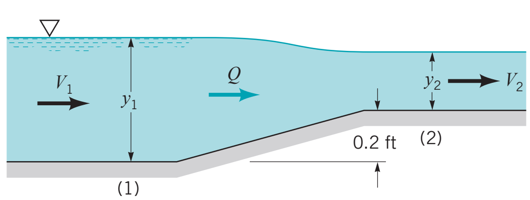


■ FIGURE P10.17

10.18 Water flows in a 10-ft-wide rectangular channel with a flowrate of 200 ft^3/s. Plot the specific energy diagram for this flow. Determine the two possible flowrates when the specific energy is 6 ft.

10.19 Water flows in a rectangular channel at a rate of $q = 20$ cfs/ft. When a Pitot tube is placed in the stream, water in the tube rises to a level of 4.5 ft above the channel bottom. Determine the two possible flow depths in the channel. Illustrate this flow on a specific energy diagram.

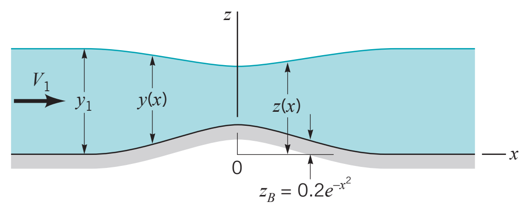
10.20 Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q = 30\ ft^3/s$ and an upstream depth of $y_1 = 2.5$ ft as is shown in Fig. P10.20. Determine the flow depth and the surface elevation at section (2).



■ FIGURE P10.20

10.21 Repeat Problem 10.20 if the upstream depth is $y_1 = 0.5$ ft.

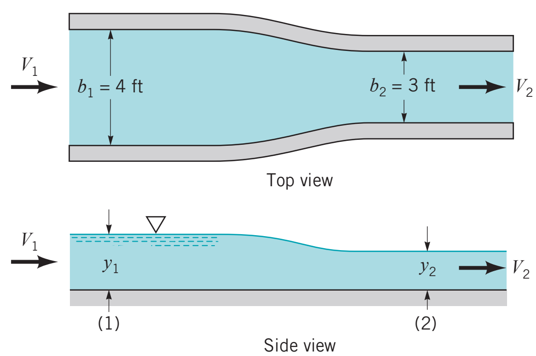
***10.22** Water flows over the bump in the bottom of the rectangular channel shown in Fig. P10.22 with a flowrate per unit width of $q = 4\ m^2/s$. The channel bottom contour is given by $z_B = 0.2e^{-x^2}$, where z_B and x are in meters. The water depth far upstream of the bump is $y_1 = 2$ m. Plot a graph of the water depth, $y = y(x)$, and the surface elevation, $z = z(x)$, for $-4\text{ m} \leq x \leq 4$ m. Assume one-dimensional flow.



■ FIGURE P10.22

***10.23** Repeat Problem 10.22 if the upstream depth is 0.4 m.

10.24 Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.24. If the flowrate is $Q = 25\ ft^3/s$ and the upstream depth is $y_1 = 2$ ft, determine the downstream depth, y_2.



■ FIGURE P10.24

10.25 Sketch the specific energy diagram for the flow of Problem 10.24 and indicate its important characteristics. Note that $q_1 \neq q_2$.

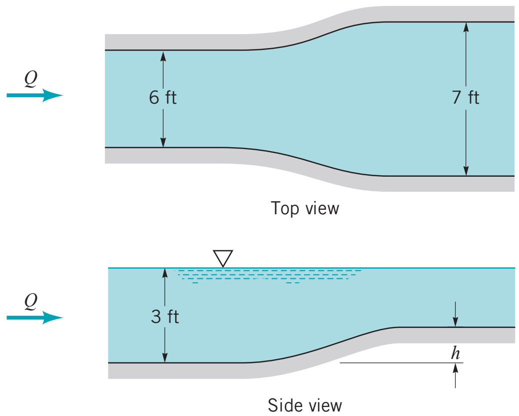
10.26 Repeat Problem 10.24 if the upstream depth is $y_1 = 0.5$ ft. Assume that there are no losses between sections (1) and (2).

10.27 Water flows in a rectangular channel with a flowrate per unit width of $q = 1.5\ m^2/s$ and a depth of 0.5 m at section (1). The head loss between sections (1) and (2) is 0.03 m. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram. Is it possible to have a head loss of 0.06 m? Explain.

10.28 Water flows in a horizontal rectangular channel with a flowrate per unit width of $q = 10\ ft^2/s$ and a depth of 1.0 ft at the downstream section (2). The head loss between section (1) upstream and section (2) is 0.2 ft. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram.

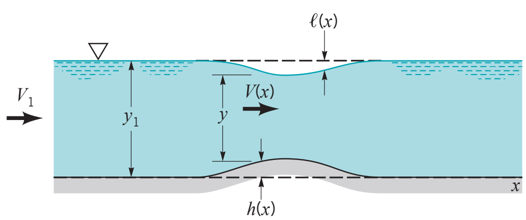
10.29 Water flows in a horizontal, rectangular channel with an initial depth of 1 m and an initial velocity of 4 m/s. Determine the depth downstream if losses are negligible. Note that there may be more than one solution.

10.30 A smooth transition section connects two rectangular channels as shown in Fig. P10.30. The channel width increases from 6.0 to 7.0 ft and the water surface elevation is the same in each channel. If the upstream depth of flow is 3.0 ft, determine h, the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.



■ FIGURE P10.30

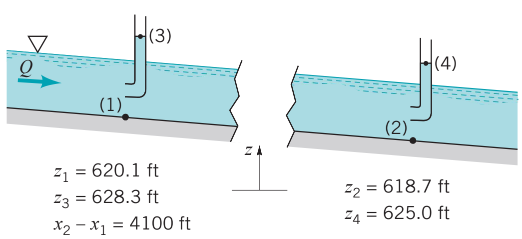
10.31 Water flows over a bump of height $h = h(x)$ on the bottom of a wide rectangular channel as is indicated in Fig. P10.31. If energy losses are negligible, show that the slope of the water surface is given by $dy/dx = -(dh/dx)/[1 - (V^2/gy)]$, where $V = V(x)$ and $y = y(x)$ are the local velocity and depth of flow. Comment on the sign (i.e., <0, $= 0$, or >0) of dy/dx relative to the sign of dh/dx.



■ FIGURE P10.31

10.32 Integrate the differential equation obtained in Problem 10.31 to determine the draw-down distance, $\ell = \ell(x)$, indicated in Fig. P10.31. Comment on your results.

10.33 Water flows in the river shown in Fig. P10.33 with a uniform bottom slope. The total head at each section is measured by using Pitot tubes as indicated. Determine the value of dy/dx at the location where the Froude number is 0.357.



■ FIGURE P10.33

10.34 Repeat Problem 10.33 if the Froude number is 2.75.

10.35 Water flows in a horizontal rectangular channel at a depth of 0.5 ft and a velocity of 8 ft/s. Determine the two possible depths at a location slightly downstream. Viscous effects between the water and the channel surface are negligible.

Section 10.4.2 The Manning Equation

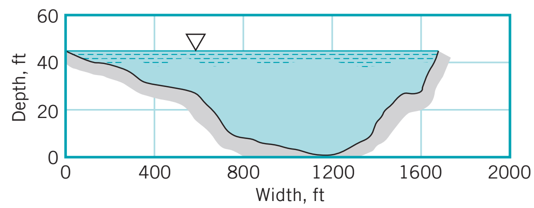
10.36 Water flows in a 5-m-wide channel with a speed of 2 m/s and a depth of 1 m. The channel bottom slopes at a rate of 1 m per 1000 m. Determine the Manning coefficient for this channel.

10.37 Fluid properties such as viscosity or density do not appear in the Manning equation (Eq. 10.20). Does this mean that this equation is valid for any open-channel flow such as that involving mercury, water, oil, or molasses? Explain.

10.38 The following data are taken from measurements on Indian Fork Creek: $A = 26\ \text{m}^2$, $P = 16$ m, and $S_0 = 0.02$ m/62 m. Determine the average shear stress on the wetted perimeter of this channel.

10.39 The following data are obtained for a particular reach of the Provo River in Utah: $A = 183\ \text{ft}^2$, free-surface width $= 55$ ft, average depth $= 3.3$ ft, $R_h = 3.22$ ft, $V = 6.56$ ft/s, length of reach $= 116$ ft, and elevation drop of reach $= 1.04$ ft. Determine **(a)** the average shear stress on the wetted perimeter, **(b)** the Manning coefficient, n, and **(c)** the Froude number of the flow.

10.40 At a particular location the cross section of the Columbia River is as indicated in Fig. P10.40. If on a day without wind it takes 5 min to float 0.5 mi along the river, which drops 0.46 ft in that distance, determine the value of the Manning coefficient, n.

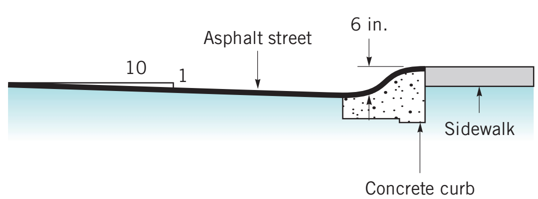


■ FIGURE P10.40

Section 10.4.3 Uniform Depth Examples—Determine Flowrate

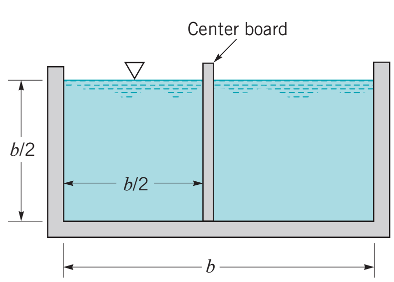
10.41 A 2-m-diameter pipe made of finished concrete lies on a slope of 1 m elevation change per 1000 m horizontal distance. Determine the flowrate when the pipe is half full.

10.42 Rainwater flows down a street whose cross section is shown in Fig. P10.42. The street is on a hill at an angle of 2°. Determine the maximum flowrate possible if the water is not to overflow onto the sidewalk.



■ FIGURE P10.42

10.43 By what percent is the flowrate reduced in the rectangular channel shown in Fig. P10.43 because of the addition of the thin center board? All surfaces are of the same material.



■ FIGURE P10.43

10.44 The great Kings River flume in Fresno County, California, was used from 1890 to 1923 to carry logs from an elevation of 4500 ft where trees were cut to an elevation of 300 ft at the railhead. The flume was 54 miles long, constructed of wood, and had a V-cross section as indicated in Fig. P10.44. It is claimed that logs would travel the length of the flume in 15 hours. Do you agree with this claim? Provide appropriate calculations to support your answer.

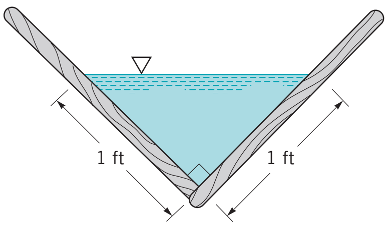


FIGURE P10.44

10.45 Water flows in a channel as shown in Fig. P10.45. The velocity is 4.0 ft/s when the channel is half full with depth d. Determine the velocity when the channel is completely full, depth $2d$.

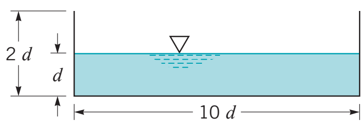


FIGURE P10.45

10.46 A trapezoidal channel with a bottom width of 3.0 m and sides with a slope of 2 : 1 (horizontal:vertical) is lined with fine gravel ($n = 0.020$) and is to carry 10 m^3/s. Can this channel be built with a slope of $S_0 = 0.00010$ if it is necessary to keep the velocity below 0.75 m/s to prevent scouring of the bottom? Explain.

10.47 Water flows in a 2-m-diameter finished concrete pipe so that it is completely full and the pressure is constant all along the pipe. If the slope is $S_0 = 0.005$, determine the flowrate by using open-channel flow methods. Compare this result with that obtained by using pipe flow methods of Chapter 8.

10.48 Water flows in a weedy earthen channel at a rate of 30 m^3/s. What flowrate can be expected if the weeds are removed and the depth remains constant?

10.49 A round concrete storm sewer pipe used to carry rainfall runoff from a parking lot is designed to be half full when the rainfall rate is a steady 1 in./hr. Will this pipe be able to handle the flow from a 2-in./hr rainfall without water backing up into the parking lot? Support your answer with appropriate calculations.

10.50 A 10-ft-wide rectangular channel is built to bypass a dam so that fish can swim upstream during their migration. During normal conditions when the water depth is 4 ft, the water velocity is 5 ft/s. Determine the velocity during a flood when the water depth is 8 ft.

†10.51 Overnight a thin layer of ice forms on the surface of a river. Estimate the percent reduction in flowrate caused by this condition. List all assumptions and show all calculations.

***10.52** Water flows in the painted steel rectangular channel with rounded corners shown in Fig. P10.52. The bottom slope is 1 ft/200 ft. Plot a graph of flowrate as a function of water depth for $0 \le y \le 1$ ft with corner radii of $r = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0 ft.

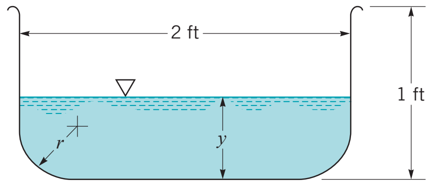


FIGURE P10.52

***10.53** The cross section of a long tunnel carrying water through a mountain is as indicated in Fig. P10.53. Plot a graph of flowrate as a function of water depth, y, for $0 \le y \le 18$ ft. The slope is 2 ft/mi and the surface of the tunnel is rough rock (equivalent to rubble masonry). At what depth is the flowrate maximum? Explain.

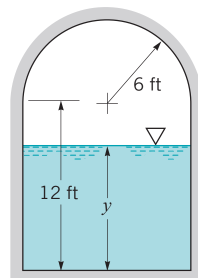


FIGURE P10.53

10.54 The smooth concrete-lined channel shown in Fig. P10.54 is built on a slope of 2 m/km. Determine the flowrate if the depth is $y = 1.5$ m.

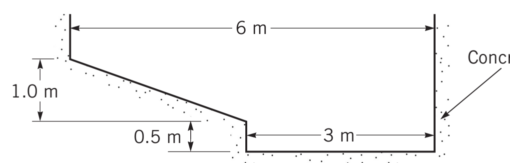


FIGURE P10.54

***10.55** At a given location, under normal conditions a river flows with a Manning coefficient of 0.030 and a cross section as indicated in Fig. P10.55*a*. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of trees and brush in the floodplain) and a cross section as shown in Fig. P10.55*b*. Determine the ratio of the flowrate during flood conditions to that during normal conditions.

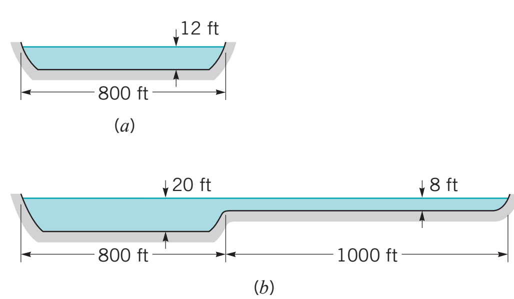
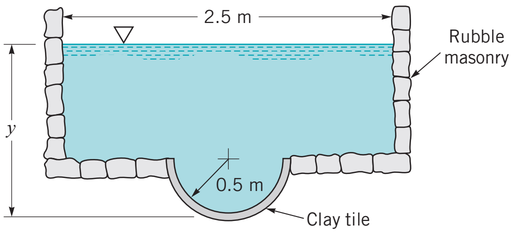


FIGURE P10.55

10.56 Repeat Problem 10.54 if the surfaces are smooth concrete as is indicated, except for the diagonal surface, which is gravelly with $n = 0.025$.

***10.57** Water flows through the storm sewer shown in Fig. P10.57. The slope of the bottom is 2 m/400 m. Plot a graph of the flowrate as a function of depth for $0 \le y \le 1.7$ m. On the same graph, plot the flowrate expected if the entire surface were lined with material similar to that of a clay tile.



■ FIGURE P10.57

10.58 Determine the flowrate for the symmetrical channel shown in Fig. P10.80 if the bottom is smooth concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$.

10.59 (See Fluids in the News article titled "Done without a GPS or lasers," Section 10.4.3.) Determine the number of gallons of water delivered per day by a rubble masonry, 1.2-m-wide aqueduct laid on an average slope of 14.6 m per 50 km if the water depth is 1.8 m.

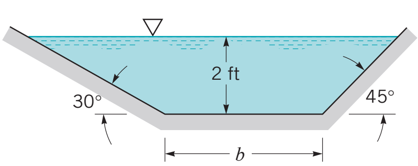
Section 10.4.3 Uniform Depth Examples—Determine Depth or Size

10.60 Water flows in a rectangular, finished concrete channel at a rate of 2 m^3/s. The bottom slope is 0.001. Determine the channel width if the water depth is to be equal to its width.

10.61 An old, rough-surfaced, 2-m-diameter concrete pipe with a Manning coefficient of 0.025 carries water at a rate of 5.0 m^3/s when it is half full. It is to be replaced by a new pipe with a Manning coefficient of 0.012 that is also to flow half full at the same flowrate. Determine the diameter of the new pipe.

10.62 Four sewer pipes of 0.5-m diameter join to form one pipe of diameter D. If the Manning coefficient, n, and the slope are the same for all of the pipes, and if each pipe flows half-full, determine D.

10.63 The flowrate in the clay-lined channel ($n = 0.025$) shown in Fig. P10.63 is to be 300 ft^3/s. To prevent erosion of the sides, the velocity must not exceed 5 ft/s. For this maximum velocity, determine the width of the bottom, b, and the slope, S_0.



■ FIGURE P10.63

10.64 Overnight a thin layer of ice forms on the surface of a 40-ft-wide river that is essentially of rectangular cross-sectional shape. Under these conditions the flow depth is 3 ft. During the following day the sun melts the ice cover. Determine the new depth if the flowrate remains the same and the surface roughness of the ice is essentially the same as that for the bottom and sides of the river.

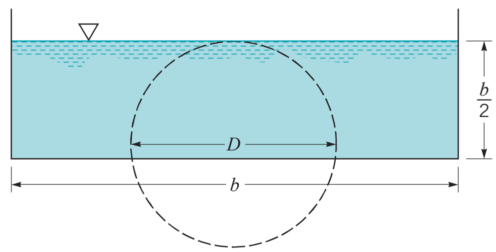
10.65 A rectangular, unfinished concrete channel of 28-ft-width is laid on a slope of 8 ft/mi. Determine the flow depth and Froude number of the flow if the flowrate is 400 ft^3/s.

10.66 An engineer is to design a channel lined with planed wood to carry water at a flowrate of 2 m^3/s on a slope of 10 m/800 m. The channel cross section can be either a 90° triangle or a rectangle with a cross section twice as wide as its depth. Which would require less wood and by what percent?

10.67 A circular finished concrete culvert is to carry a discharge of 50 ft^3/s on a slope of 0.0010. It is to flow not more than half-full. The culvert pipes are available from the manufacture with diameters that are multiples of 1 ft. Determine the smallest suitable culvert diameter.

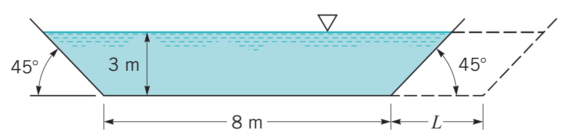
10.68 At what depth will 50 ft^3/s of water flow in a 6-ft-wide rectangular channel lined with rubble masonry set on a slope of 1 ft in 500 ft? Is a hydraulic jump possible under these conditions? Explain.

10.69 The rectangular canal shown in Fig. P10.69 changes to a round pipe of diameter D as it passes through a tunnel in a mountain. Determine D if the surface material and slope remain the same and the round pipe is to flow completely full.



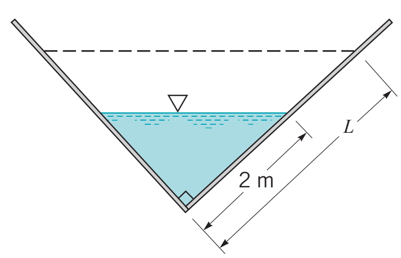
■ FIGURE P10.69

10.70 The flowrate through the trapezoidal canal shown in Fig. P10.70 is Q. If it is desired to double the flowrate to $2Q$ without changing the depth, determine the additional width, L, needed. The bottom slope, surface material, and the slope of the walls are to remain the same.



■ FIGURE P10.70

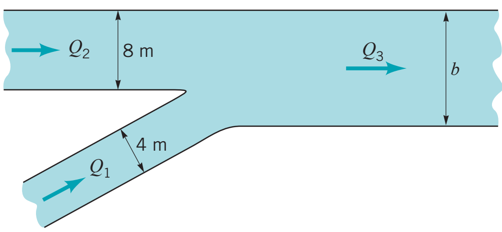
10.71 When the channel of triangular cross section shown in Fig. P10.71 was new, a flowrate of Q caused the water to reach $L = 2$ m up the side as indicated. After considerable use, the walls of the channel became rougher and the Manning coefficient, n, doubled. Determine the new value of L if the flowrate stayed the same.



■ FIGURE P10.71

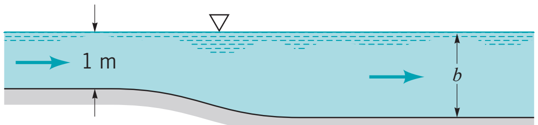
10.72 A smooth steel water slide at an amusement park is of semicircular cross section with a diameter of 2.5 ft. The slide descends a vertical distance of 35 ft in its 420-ft length. If pumps supply water to the slide at a rate of 6 cfs, determine the depth of flow. Neglect the effects of the curves and bends of the slide.

10.73 Two canals join to form a larger canal as shown in **Video V10.6** and Fig. P10.73. Each of the three rectangular canals is lined with the same material and has the same bottom slope. The water depth in each is to be 2 m. Determine the width of the merged canal, *b*. Explain physically (i.e., without using any equations) why it is expected that the width of the merged canal is less than the combined widths of the two original canals (i.e., $b < 4 \text{ m} + 8 \text{ m} = 12 \text{ m}$).



■ **FIGURE P10.73**

10.74 Water flows uniformly at a depth of 1 m in a channel that is 5 m wide as shown in Fig. P10.74. Further downstream the channel cross section changes to that of a square of width and height *b*. Determine the value of *b* if the two portions of this channel are made of the same material and are constructed with the same bottom slope.

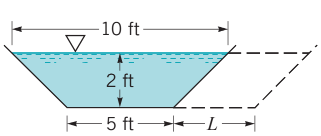


■ **FIGURE P10.74**

10.75 Determine the flow depth for the channel shown in Fig. P10.54 if the flowrate is $15 \text{ m}^3/\text{s}$.

10.76 Rainwater runoff from a 200-ft by 500-ft parking lot is to drain through a circular concrete pipe that is laid on a slope of 3 ft/mi. Determine the pipe diameter if it is to be full with a steady rainfall of 1.5 in./hr.

10.77 (See Fluids in the News article titled "Plumbing the Everglades," Section 10.4.1.) The canal shown in Fig. P10.77 is to be widened so that it can carry twice the amount of water. Determine the additional width, *L*, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.

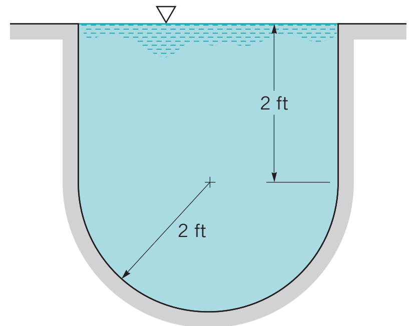


■ **FIGURE P10.77**

Section 10.4.3 Uniform Depth Examples—Determine Slope

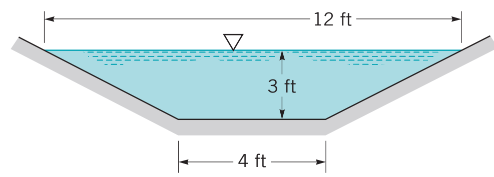
10.78 Water flows 1 m deep in a 2-m-wide finished concrete channel. Determine the slope if the flowrate is $3 \text{ m}^3/\text{s}$.

10.79 Water flows in the channel shown in Fig. P10.79 at a rate of $90 \text{ ft}^3/\text{s}$. Determine the minimum slope that this channel can have so that the water does not overflow the sides. The Manning coefficient for this channel is $n = 0.014$.



■ **FIGURE P10.79**

10.80 To prevent weeds from growing in a clean earthen-lined canal, it is recommended that the velocity be no less than 2.5 ft/s. For the symmetrical canal shown in Fig. P10.80, determine the minimum slope needed.



■ **FIGURE P10.80**

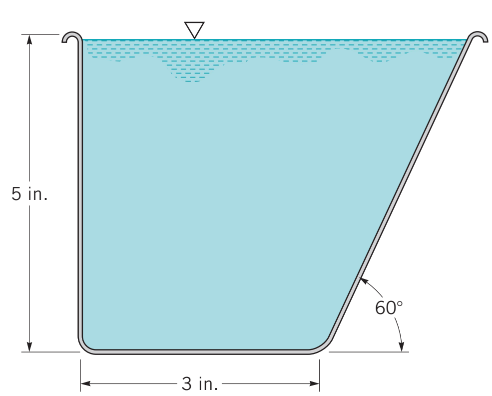
10.81 The smooth, concrete-lined, symmetrical channel shown in **Video V10.7** and Fig. P10.80 carries water from the silt-laden Colorado River. If the velocity must be 4.0 ft/s to prevent the silt from settling out (and eventually clogging the channel), determine the minimum slope needed.

10.82 The symmetrical channel shown in Fig. P10.80 is dug in sandy loam soil with $n = 0.020$. For such surface material it is recommended that to prevent scouring of the surface the average velocity be no more than 1.75 ft/s. Determine the maximum slope allowed.

10.83 The depth downstream of a sluice gate in a rectangular wooden channel of width 5 m is 0.60 m. If the flowrate is $18 \text{ m}^3/\text{s}$, determine the channel slope needed to maintain this depth. Will the depth increase or decrease in the flow direction if the slope is **(a)** 0.02; **(b)** 0.01?

10.84 Water in a painted steel rectangular channel of width $b = 1$ ft and depth y is to flow at critical conditions, $\text{Fr} = 1$. Plot a graph of the critical slope, S_{0c}, as a function of y for $0.05 \text{ ft} \le y \le 5 \text{ ft}$. What is the maximum slope allowed if critical flow is not to occur regardless of the depth?

10.85 A 50-ft-long aluminum gutter (Manning coefficient $n = 0.011$) on a section of a roof is to handle a flowrate of $0.15 \text{ ft}^3/\text{s}$



■ **FIGURE P10.85**

during a heavy rain storm. The cross section of the gutter is shown in Fig. P10.85. Determine the vertical distance that this gutter must be pitched (i.e., the difference in elevation between the two ends of the gutter) so that the water does not overflow the gutter. Assume uniform depth channel flow.

Section 10.6.1 The Hydraulic Jump (Also see Lab Problems 10.116 and 10.117.)

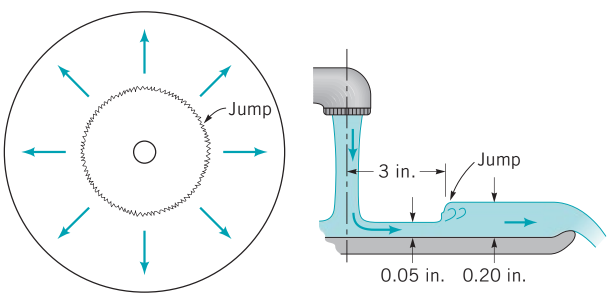
10.86 Obtain a photograph/image of a situation that involves a hydraulic jump. Print this photo and write a brief paragraph that describes the flow.

10.87 Water flows upstream of a hydraulic jump with a depth of 0.5 m and a velocity of 6 m/s. Determine the depth of the water downstream of the jump.

10.88 A 2.0-ft standing wave is produced at the bottom of the rectangular channel in an amusement park water ride. If the water depth upstream of the wave is estimated to be 1.5 ft, determine how fast the boat is traveling when it passes through this standing wave (hydraulic jump) for its final "splash."

10.89 The water depths upstream and downstream of a hydraulic jump are 0.3 and 1.2 m, respectively. Determine the upstream velocity and the power dissipated if the channel is 50 m wide.

10.90 Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. P10.90 and Video V10.12. Consider a situation where a jump forms 3.0 in. from the center of the plate with depths upstream and downstream of the jump of 0.05 in. and 0.20 in., respectively. Determine the flowrate from the faucet.



■ FIGURE P10.90

10.91 Show that the Froude number downstream of a hydraulic jump in a rectangular channel is $(y_1/y_2)^{3/2}$ times the Froude number upstream of the jump, where (1) and (2) denote the upstream and downstream conditions, respectively.

10.92 Water flows in a 2-ft-wide rectangular channel at a rate of 10 ft^3/s. If the water depth downstream of a hydraulic jump is 2.5 ft, determine **(a)** the water depth upstream of the jump, **(b)** the upstream and downstream Froude numbers, and **(c)** the head loss across the jump.

10.93 A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively (see Video V10.11). If the spillway is 10 m wide, what is the flowrate over the spillway?

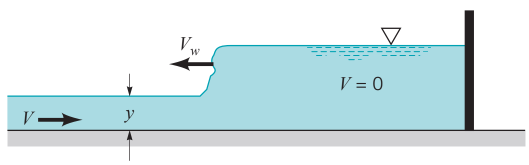
10.94 Determine the head loss and power dissipated by the hydraulic jump of Problem 10.93.

10.95 A hydraulic jump occurs in a 4-m-wide rectangular channel at a point where the slope changes from 3 m per 100 m upstream of the jump to h m per 100 m downstream of the jump. The depth and velocity of the uniform flow upstream of the jump are 0.5 m and 8 m/s, respectively. Determine the value of h if the flow downstream of the jump is to be uniform flow.

10.96 At a given location in a 12-ft-wide rectangular channel the flowrate is 900 ft^3/s and the depth is 4 ft. Is this location upstream or downstream of the hydraulic jump that occurs in this channel? Explain.

***10.97** A rectangular channel of width b is to carry water at flowrates from $30 \le Q \le 600$ cfs. The water depth upstream of the hydraulic jump that occurs (if one does occur) is to remain 1.5 ft for all cases. Plot the power dissipated in the jump as a function of flowrate for channels of width $b = 10$, 20, 30, and 40 ft.

10.98 Water flows in a rectangular channel at a depth of $y = 1$ ft and a velocity of $V = 20$ ft/s. When a gate is suddenly placed across the end of the channel, a wave (a moving hydraulic jump) travels upstream with velocity V_w as is indicated in Fig. P10.98. Determine V_w. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.



■ FIGURE P10.98

10.99 Water flows in a rectangular channel with velocity $V = 6$ m/s. A gate at the end of the channel is suddenly closed so that a wave (a moving hydraulic jump) travels upstream with velocity $V_w = 2$ m/s as is indicated in Fig. P10.98. Determine the depths ahead of and behind the wave. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w, the flow appears as a steady hydraulic jump.

10.100 (See Fluids in the News article titled "Grand Canyon rapids building," Section 10.6.1.) During the flood of 1983, a large hydraulic jump formed at "Crystal Hole" rapid on the Colorado River. People rafting the river at that time report "entering the rapid at almost 30 mph, hitting a 20-ft-tall wall of water, and exiting at about 10 mph." Is this information (i.e., upstream and downstream velocities and change in depth) consistent with the principles of a hydraulic jump? Show calculations to support your answer.

Section 10.6.2,3 Sharp-Crested and Broad-Crested Weirs (Also see Lab Problems 10.114 and 10.115.)

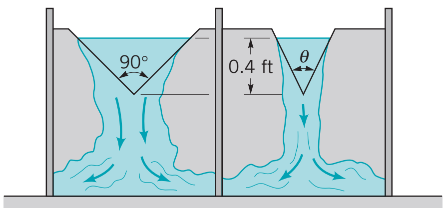
10.101 Obtain a photograph/image of a situation that involves a weir. Print this photo and write a brief paragraph that describes the flow.

10.102 Water flows over a 2-m-wide rectangular sharp-crested weir. Determine the flowrate if the weir head is 0.1 m and the channel depth is 1 m.

10.103 Water flows over a 5-ft-wide, rectangular sharp-crested weir that is $P_w = 4.5$ ft tall. If the depth upstream is 5 ft, determine the flowrate.

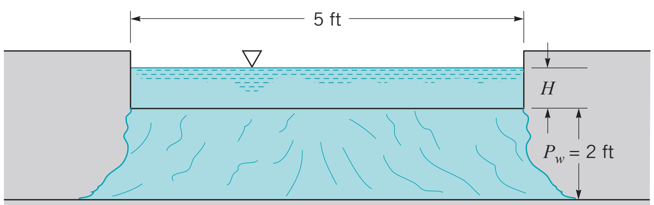
10.104 A rectangular sharp-crested weir is used to measure the flowrate in a channel of width 10 ft. It is desired to have the channel flow depth be 6 ft when the flowrate is 50 cfs. Determine the height, P_w, of the weir plate.

10.105 Water flows from a storage tank, over two triangular weirs, and into two irrigation channels as shown in **Video V10.13** and Fig. P10.105. The head for each weir is 0.4 ft, and the flowrate in the channel fed by the 90°-V-notch weir is to be twice the flowrate in the other channel. Determine the angle θ for the second weir.



■ FIGURE P10.105

10.106 Rain water from a parking lot flows into a 2-acre (8.71×10^4 ft^2) retention pond. After a heavy rain when there is no more inflow into the pond, the rectangular weir shown in Fig. P10.106 at the outlet of the pond has a head of $H = 0.6$ ft. **(a)** Determine the rate at which the level of the water in the pond decreases, dH/dt, at this condition. **(b)** Determine how long it will take to reduce the pond level by half a foot; that is, to $H = 0.1$ ft.

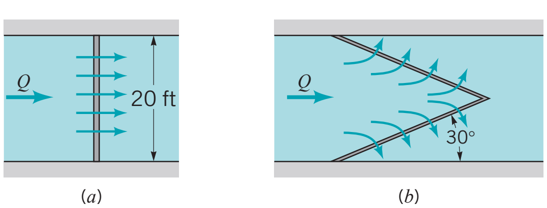


■ FIGURE P10.106

10.107 A basin at a water treatment plant is 60 ft long, 10 ft wide, and 5 ft deep. Water flows from the basin over a 3-ft-long, rectangular weir whose crest is 4 ft above the bottom of the basin. Estimate how long it will take for the depth of the water in the basin to change from 4.5 ft to 4.4 ft if there is no flow into the basin.

10.108 Water flows over a sharp-crested triangular weir with $\theta = 90°$. The head range covered is $0.20 \leq H \leq 1.0$ ft and the accuracy in the measurement of the head, H, is $\delta H = \pm 0.01$ ft. Plot a graph of the percent error expected in Q as a function of Q.

10.109 **(a)** The rectangular sharp-crested weir shown in Fig. P10.109*a* is used to maintain a relatively constant depth in the channel upstream of the weir. How much deeper will the water be upstream of the weir during a flood when the flowrate is 45 ft^3/s compared to normal conditions when the flowrate is 30 ft^3/s? Assume the weir coefficient remains constant at $C_{wr} = 0.62$. **(b)** Repeat the calculations if the weir of part **(a)** is replaced by a rectangular sharp-crested "duck bill" weir which is oriented at an angle of 30° relative to the channel centerline as shown in Fig. P10.109*b*. The weir coefficient remains the same.

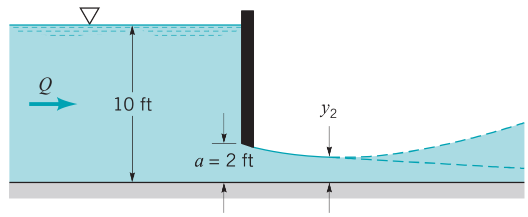


■ FIGURE P10.109

10.110 Water flows in a rectangular channel of width $b = 20$ ft at a rate of 100 ft^3/s. The flowrate is to be measured by using either a rectangular weir of height $P_w = 4$ ft or a triangular ($\theta = 90°$) sharp-crested weir. Determine the head, H, necessary. If measurement of the head is accurate to only ± 0.04 ft, determine the accuracy of the measured flowrate expected for each of the weirs. Which weir would be the most accurate? Explain.

Section 10.6.4 Underflow Gates

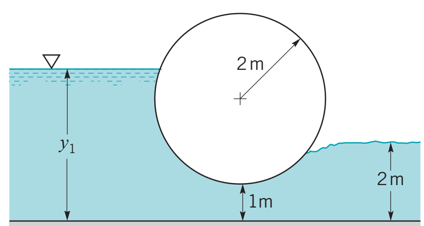
10.111 Water flows under a sluice gate in a 60-ft-wide finished concrete channel as is shown in Fig. P10.111. Determine the flowrate. If the slope of the channel is 2.5 ft/200 ft, will the water depth increase or decrease downstream of the gate? Assume $C_c = y_2/a = 0.65$. Explain.



■ FIGURE P10.111

10.112 Water flows under a sluice gate in a channel of 10-ft width. If the upstream depth remains constant at 5 ft, plot a graph of flowrate as a function of the distance between the gate and the channel bottom as the gate is slowly opened. Assume free outflow.

10.113 A water-level regulator (not shown) maintains a depth of 2.0 m downstream from a 10-m-wide drum gate as shown in Fig. P10.113. Plot a graph of flowrate, Q, as a function of water depth upstream of the gate, y_1, for $2.0 \leq y_1 \leq 5.0$ m.



■ FIGURE P10.113

■ Lab Problems

10.114 This problem involves the calibration of a triangular weir. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

10.115 This problem involves the calibration of a rectangular weir. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

10.116 This problem involves the depth ratio across a hydraulic jump. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

10.117 This problem involves the head loss across a hydraulic jump. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

Life Long Learning Problems

10.118 With the increased usage of low-lying coastal areas and the possible rise in ocean levels because of global warming, the potential for widespread damage from tsunamis (i.e., "tidal waves") is increasing. Obtain information about new and improved methods available to predict the occurrence of these damaging waves and how to better use coastal areas so that massive loss of life and property does not occur. Summarize your findings in a brief report.

10.119 Recent photographs from NASA's Mars Orbiter Camera on the Mars Global Surveyor provide new evidence that water may still flow on the surface of Mars. Obtain information about the possibility of current or past open-channel flows on Mars and other planets or their satellites. Summarize your findings in a brief report.

10.120 Hydraulic jumps are normally associated with water flowing in rivers, gullies, and other such relatively high-speed open channels. However, recently, hydraulic jumps have been used in various manufacturing processes involving fluids other than water (such as liquid metal solder) in relatively small-scale flows. Obtain information about new manufacturing processes that involve hydraulic jumps as an integral part of the process. Summarize your findings in a brief report.

FE Exam Problems

Sample FE (Fundamentals of Engineering) exam questions for fluid mechanics are provided on the book's web site, www.wiley.com/college/munson.