Stress Analysis

Pipe Stress Analysis

Purpose

- To ensure that piping is routed and supported so that no damage occurs to either the pipe or associated equipment due to the effects of thermal expansion or contraction, or loads resulting from weight, pressure, wind earthquake, pulsation, shock, foundation settlement, etc.

WHAT MAKES PIPING CRITICAL?

- **Temperature**
- **Pressure**
- **Occasional loads (Seismic & Wind)**
- **Dynamics of the flowing fluid**

WHAT CAUSES STRESS IN THE PIPE?

- **Live and Dead load (Piping component, Insulation, Flowing medium etc.).**
- **Line pressure**
- **Restraining of pipe expansion or Temperature gradient**
- **Discharge reaction (E.g. Relief valves)**
- **Seismic / Wind load**
- **Dynamics of flowing fluid.**

Cont….

WHY STRESS ANALYSIS IS TO BE DONE?

- **To ensure that all stresses in the piping system are within allowable limits.**
- **To limit the loads on the equipment nozzle.**
- **To limit the thermal displacement and sagging**
- **To limit the loads on the supporting structures & flanged joints.**
- **To avoid excessive piping vibration**

STRESS / FLEXIBILITY ANALYSIS

Why do we perform Pipe stress analysis?

- To keep stresses within the code allowable limits
- To keep nozzle loadings on equipment within manufacturer's or code allowable (API 610,API617,Nema SM 23 etc.)
- To keep vessel stresses at nozzles within ASME SEC.V111 allowable
- To calculate design loads on supports and structures
- To determine piping displacements for interference check
- To solve dynamic problems such as those due to
  - Harmonic/Cyclic loading in mechanical or acoustic vibrations/pulsations
  - Impulse loading in fluid hammer, transient flow, relief valve discharge
  - Random loadings as in earthquake or wind

When & Why Perform Pipe Stress Analysis

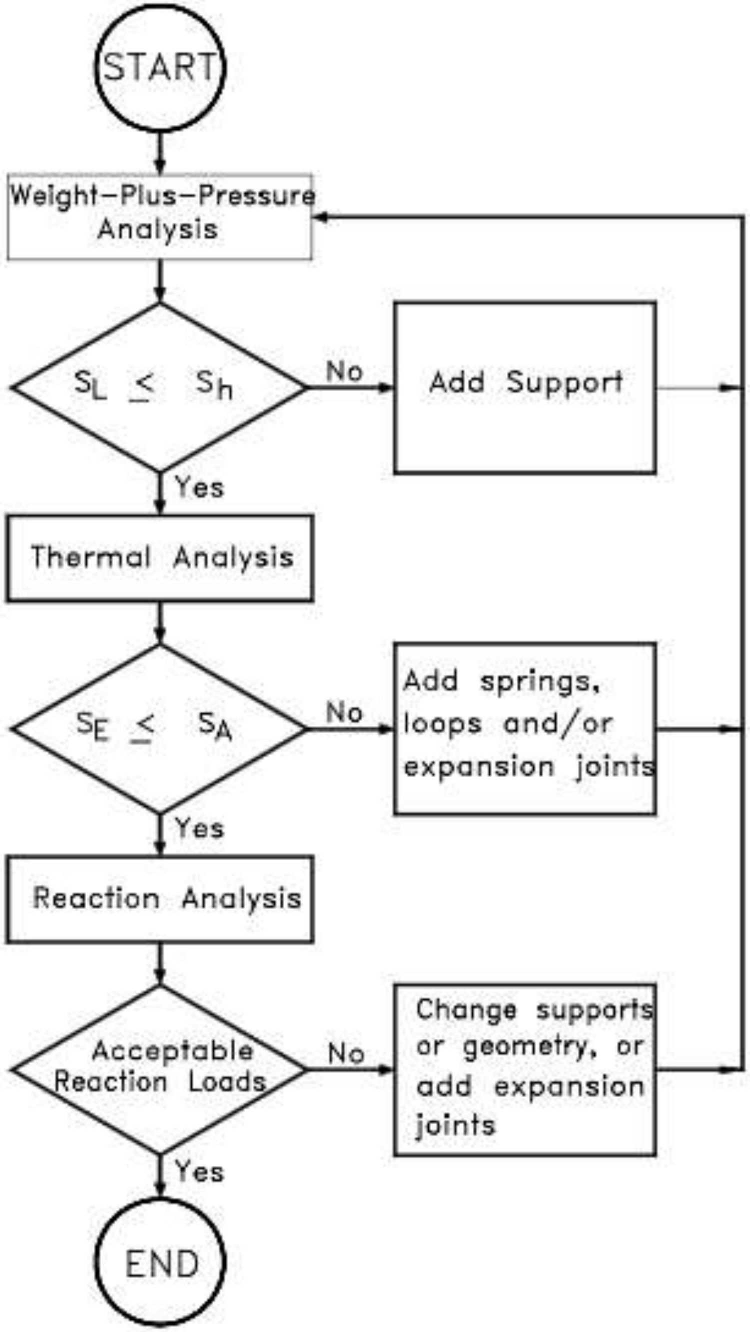
- Lines 75 mm and larger that are:
  - connected to rotating equipment
  - subject to differential settlement of connected equipment and/or supports, or
  - with temperatures less than -5 C.

When & Why Perform Pipe Stress Analysis

- Lines connected to reciprocating equipment such as suction and discharge lines to and from reciprocating compressors.
- Lines 100 mm and larger connected to air coolers, steam generators, or fired heater tube sections
- All size lines with temperatures of 300 C and higher
- Welded lines 150 mm and larger at a design temperature of 175 C or higher

When & Why Perform Pipe Stress Analysis

- High pressure lines (over 14,000 kPa). Although systems over 10,000 kPa are sometimes a problem, particularly with restraint arrangements.
- Lines subject to external pressure
- Thin-walled pipe or duct of 450 mm diameter and over, having an outside diameter over wall thickness ratio (d/t) of more than 90
- Pressure relief systems. Also relief valve stacks with an inlet pressure greater than 1100 kPa.



1. THE PIPING FLEXIBILITY ANALYSIS PROCESS

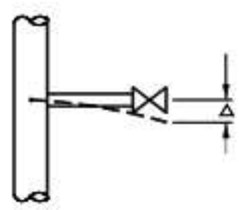
S	Basic allowable stress from Appendix A of B31.3, ps
S_A	Allowable stress range, psi.
S_b	Resultant bending stress, psi.
	$= \dfrac{\left((i_i M_i)^2 + (i_o M_o)^2\right)^{0.5}}{Z}$
S_c	Basic allowable stress at minimum metal temperature
S_E	Computed displacement stress range, psi.
S_h	Basic allowable stress at maximum metal temperature
S_L	Sum of longitudinal stresses caused by pressure, weig
S_t	Torsional stress, psi = $M_t / 2Z$
T_p	Pressure thrust of bellows expansion joint, lbf.
t	Pipewall thickness, inch.
y	Deflection in y-direction, inch.
Z	Section modulus of pipe, inch^3.

SUSTAINED AND DISPLACEMENT STRESSES

- Piping flexibility analysis in accordance with the basic assumptions and requirements of B31.3 is concerned with two types of stress called sustained stress and displacement stress.
- Each type of stress must be considered separately; these are the two criteria by which the adequacy of a piping system is evaluated.
- They are considered separately because sustained stresses are associated with sustained forces while displacement stresses are associated with fixed displacements

SUSTAINED STRESSES

- Sustained Stresses are defined as stresses caused by loads that are not relieved as the piping system deflects.
- An example is the stress induced by the weight of the valve at the end of the cantilevered pipe segment shown below



– SUSTAINED STRESS

SUSTAINED STRESSES

- Regardless of the magnitude of the displacement Δ, the magnitude of the load (the weight of the valve) which causes the stress is unchanged.
- Therefore, to avoid catastrophic failure, the magnitude of any sustained stress must not exceed the yield strength of the material.
- Another example of a sustained stress is the hoop and longitudinal stresses induced by pressure.
- The loadings, which induce sustained stresses, are termed sustained loadings

SUSTAINED STRESSES

- The sustained stress principle is expressed as a Code requirement

This requirement is written as :

$$S_L \leq S_h \qquad (1)$$

S_L is computed by the following equation :

$$S_L = S_b + \left| \frac{F_A}{A_w} \right| \qquad (2)$$

S_b Resultant bending stress, psi.

$$= \frac{\left[(i_i M_i)^2 + (i_o M_o)^2 \right]^{0.5}}{Z}$$

F_A Axial force, lbf.

A_w Cross-sectional area of corroded pipe wall, $inch^2$

DISPLACEMENT STRESSES :

- Displacement stresses are defined as those stresses caused by fixed displacements, i.e., caused by loads that are relieved as the piping system deforms. Consider the cantilevered beam shown below

– DISPLACEMENT STRESS

DISPLACEMENT STRESSES :

- Assume that the end of the beam is displaced, its elastic limit and its elastic range is δ. As long as any displacement cycle is within the elastic range of the beam, no yielding will take place

- In Figure 5A, if the pipe end at position (1) is assumed free, then when the piping is heated it would move to the unrestrained hot shape with the free end at position (2).
- Figure 5B illustrates how an increase in temperature of the restrained piping system is equivalent to displacing the unrestrained hot pipe from position (2) to position (1).
- Therefore, stresses caused by thermal expansion are displacement stresses.

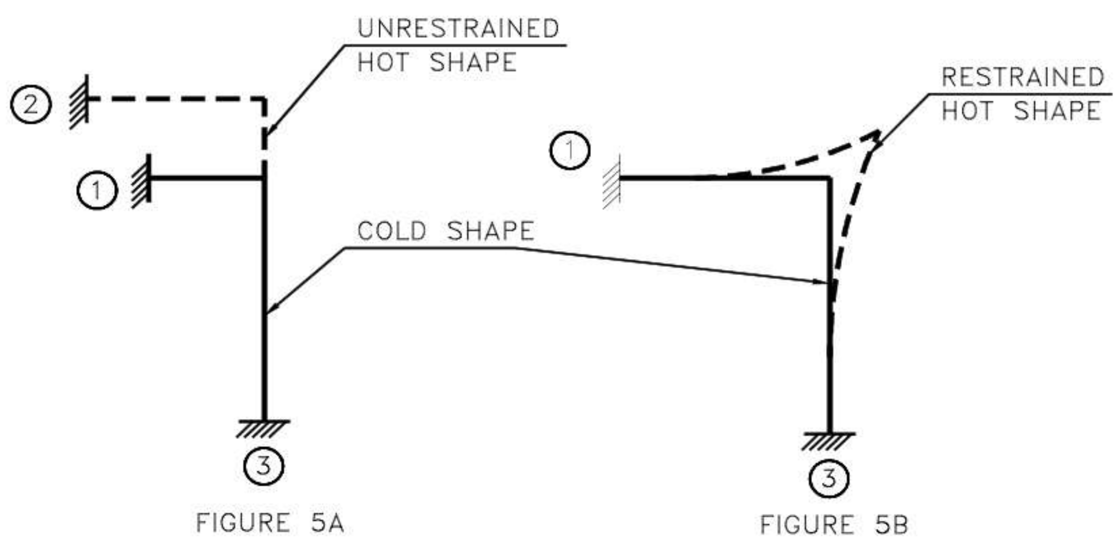


FIGURE 5 — RESTRAINED PIPING SYSTEM

$$S_E \le S_A \qquad (3)$$

- When piping system satisfies Eq.3, it is judged to be adequately flexible against thermal expansion and restraint displacement because the elastic range of the system will never be exceeded even though the system may yield.
- Since the inherent flexibility is most piping systems is provided by changes in direction, the code considers only bending and torsional stresses significant in the calculation of S_E and gives the following equation for its computation.

$$S_E = (S_b^2 + 4S_t^2)^{1/2}$$

ALLOWABLE STRESSES

- The stresses computed by the program must be compared to Code allowable stresses to determine the adequacy of piping systems.
- The Code differentiates between stresses caused by pressure and other sustained loads and stresses caused by displacement strains.
- These allowable values are a function of the basic allowable stresses.

BASIC ALLOWABLE STRESSES

- The code sets forth the rules for deriving basic allowable stresses for metals in the clause, 302.3.2 - Bases for design stresses.
- The significant aspect of these rules is that B31.3 permits the use of one third the tensile strength, rather than one fourth the tensile strength as used by Section VIII, Division I, of the ASME Code, using section VIII allowable stresses for piping is unduly conservative.

ALLOWABLE STRESS AS PER CODE

ALLOWABLE STRESS IN CODE ARE BASES ON MATERIAL PROPERTIES

As per the ASME B31.1 (Minimum of)

- ✓ 1/4TH OF UTS OF MATERIAL AT OPERATING TEMPERATURE
- ✓ 1/4TH OF UTS OF MATERIAL AT ROOM TEMPERATURE.
- ✓ 5/8TH OF YIELD STRENGTH OF THE MATERIAL AT OPE TEMPERATURE
- ✓ 5/8TH IF YIELD STRENGTH OF THE MATERIAL AT ROOM TEMPERATURE.

As per the ASME B31.3 (Minimum of)

- ✓ 1/3RD OF UTS OF MATERIAL AT OPERATING TEMPERATURE
- ✓ 1/3RD OF UTS OF MATERIAL AT ROOM TEMPERATURE.
- ✓ 2/3TH OF YIELD STRENGTH OF THE MATERIAL AT OPE TEMPERATURE.
- ✓ 2/3TH IF YIELD STRENGTH OF THE MATERIAL AT ROOM TEMPERATURE.

ALLOWABLE DISPLACEMENT STRESS RANGE

$S_A = f\,(1.25\,S_c + 0.25\,S_h)$

S_c = Cold allowable stress.

S_h = Hot allowable stress.

f = Stress range reduction factor for cyclic condition.

Cycles	f
7000 and less	1.0
7000 to 14000	0.9
14000 to 22000	0.8
22000 to 45000	0.7
45000 to 100000	0.6
100000 and over	0.5

STRESS / FLEXIBILITY ANALYSIS

Code equation for condition equilibrium:

$$S_E < S_A$$

$$S_E < (S_{b2} + 4\ S_{t2})^{1\backslash 2}$$

$P = S_A = f\,(1.25\ S_c + 0.25\ S_h)$

$S_b = i\ M_b / Z$ (Resultant bending stress due to thermal expansion)

$S_t = M_t / 2Z$ (Torsional stress due to thermal expansion)

M_b = Resultant bending moment due to thermal restraint

M_t = Torsional moment due to the thermal restraint

i = Stress intensification factor (SIF).

z = Section modulus of pipe

S_c = Basic allowable stress at minimum metal temperature.

S_h = Basic allowable stress at maximum metal temperature.

f = Stress range reduction factor for cyclic conditions., 1.0 for 7000 Cycles

BEAM BENDING

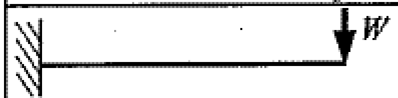
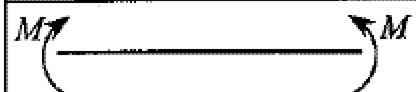
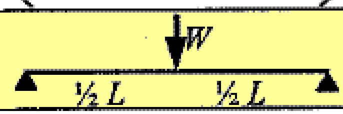
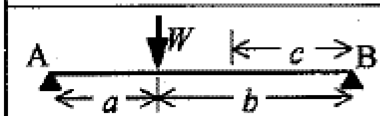
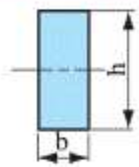
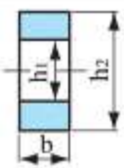
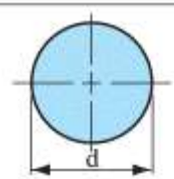
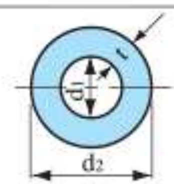
L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \le b, \quad c = \sqrt{\frac{1}{3}b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

Table 2. Typical Equations to Calculate Section Modulus

Cross Section	Section Modulus Z
	$\frac{1}{6}bh^2$
	$\frac{1}{6} \cdot \frac{b(h2^3-h1^3)}{h2}$
	$\frac{\pi}{32}d^3$
	$\frac{\pi}{32} \cdot \frac{d2^4-d1^4}{d2}$

STRESS / FLEXIBILITY ANALYSIS

What is primary stress and secondary stress?

• Primary stress: normal or shear stress due to imposed loading such as pressure and sustained loads. Could cause permanent deformation and failure of the piping system.

• Code equation for condition equilibrium:

$$S_L = S_{L1} + S_{L2}$$

$$S_L = (P \times D / 4 \times t) + (M_A / Z) < 1.0\ S_h$$

P = Internal design pressure

D = outside diameter of pipe

t = pressure design thickness

M_A = resultant moment loading due to weight and sustained loads

Z = section modulus

S_h = basic allowable stress at maximum metal temperature.

• Secondary stress: Normal or shear stress caused when a system's flexibility is constrained as due to thermal loads. Failure will not occur and local yielding or distortion can satisfy this condition.

- For B31.1 / B31.3 and similar codes, the piping system is evaluated, from a *stress* point of view by considering failure modes. This leads to the following load cases for evaluation:
  Sustained - for primary, force based loads
- Expansion - for secondary, displacement based self limiting loads
- Occasional - for primary extreme (wind, seismic) loads
-

  For each of these conditions, the piping codes provide equations to determine both the *code stress* and the *allowable stress*. From a *pipe stress* point of view, the system passes the code requirements if all of the *code stresses* are less than the corresponding *allowable stresses*.

  The B31.1 / B31.3 codes do not consider the Operating Condition to be a *stress case*, therefore, there are no equations provided to compute a *code stress* or an *allowable stress*. **CAESAR II** computes an operating stress using a basic strength of materials equation, *but the allowable is set to zero* since there is nothing to compare this stress to.

  The Operating case is important and it must be evaulated for:
  The determination of maximum displacments.
- The determination of equipment and support loads.
- The computation of the "Expansion Stress Range".
-

  There are other piping codes (B31.4 / B31.8) that do consider the Operating stress condition as a *code case*. For these codes there is an allowable stress computed and compared to the actual stress in the pipe.

  Take a look at the **CAESAR II Quick Reference Guide**, for a list of the piping codes supported, the load cases addressed, and the equations for the *code stress* and the *allowable stress*. Note, these are the load cases that must be run for *stress evaluation*. There may be other load cases necessary for structural or operational reasons.

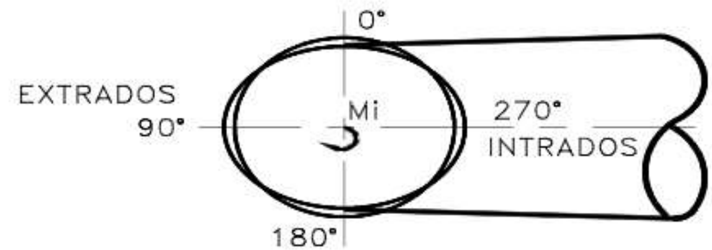
STRESS INTENSIFICATION

- Local stresses in fittings such as tees and elbows are generally higher than stresses in the adjoining pipe segments.
- The code allows these stresses to be calculated by multiplying the stresses in the adjoining pipe segments by a Stress Intensification Factor (SIF)

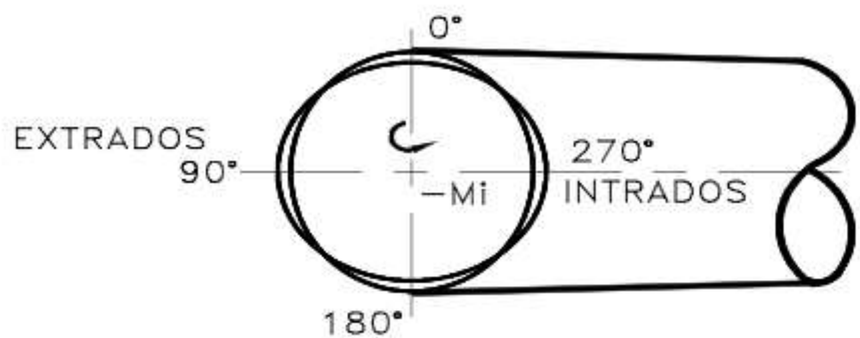
STRESS INTENSIFICATION

- The stress analysis of elbows has been the subject of many studies since 1910 when a German named Bantlin demonstrated that curved pipe segments behaved differently than predicted by simple curved beam theory.
- The curved beam theory assumes the elbow cross section remains circular when subjected to bending moments.
- In fact, the cross section becomes oval when subjected to load, increasing the flexibility and the stress magnitude in the curved portion of the elbow

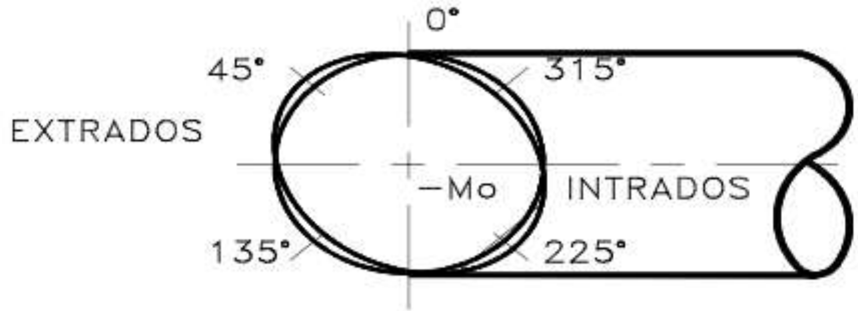
The figure below illustrates the cross section distortion of an elbow when subjected to bending moments.



(a) IN–PLANE BENDING, TENDING TO OPEN THE ELBOW



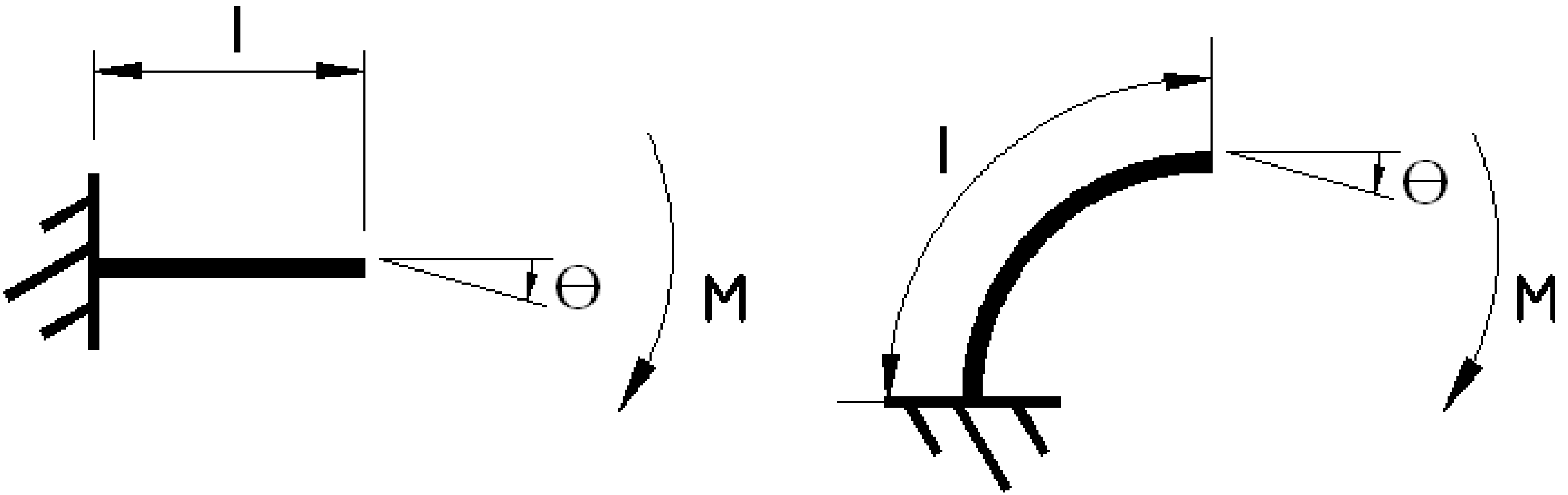
(b) IN–PLANE BENDING, TENDING TO CLOSE THE ELBOW



(C) OUT-OF-PLANE BENDING

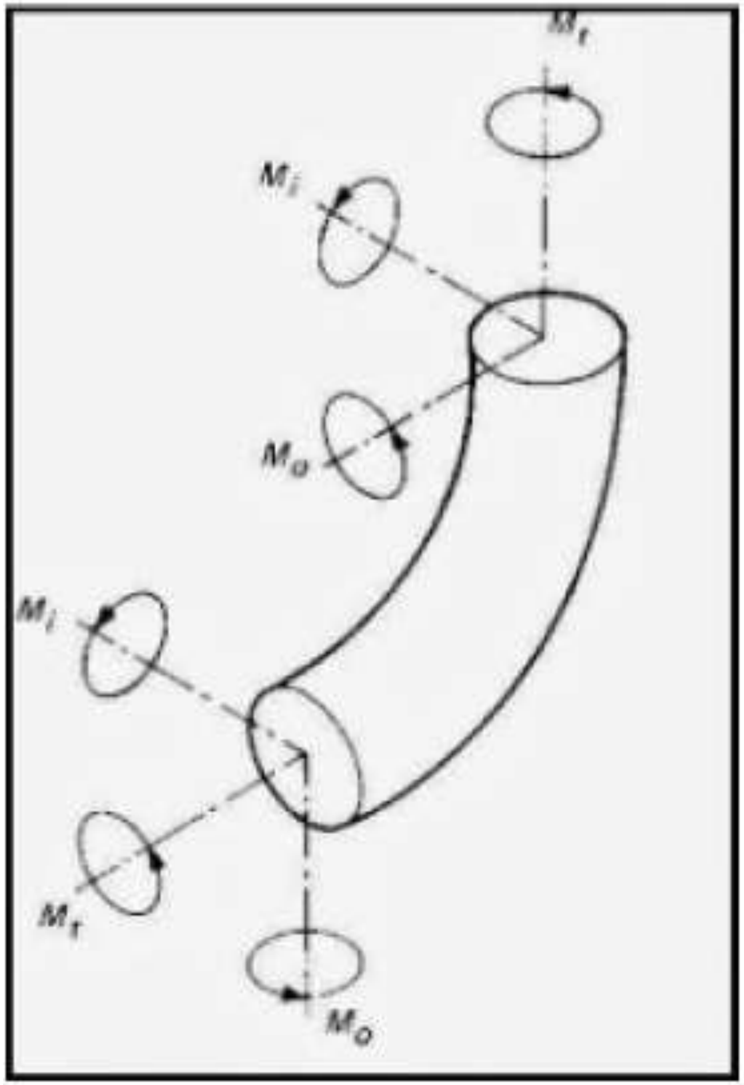
FIGURE 7 – DISTORTION OF ELBOW

- Consider a straight pipe (refer Figure 1) with length "l‟ which will produce rotation "Ɵ‟ under the action of bending moment „M‟.
- A bend having same diameter and thickness with same arc length "l‟ under the action of same bending moment "M‟ will exhibit "k Ɵ‟ rotation.
- In nutshell, bend shows k times flexibility than the straight pipe, called as Flexibility factor.



Stress Intensification Factor for a Piping Bend/Elbow

- In lay man’s language the SIF of a bend or elbow can be defined as the ratio of bending stress of an elbow to that of straight pipe of same diameter and thickness when subjected to same bending moment.
- Whenever the same bending moment is applied to a bend because of ovalization the bending stress of the elbow will be much higher than that of strainght pipe.
- That is why the SIF value will always be greater than or equal to 1 .0 (for straight pipe).



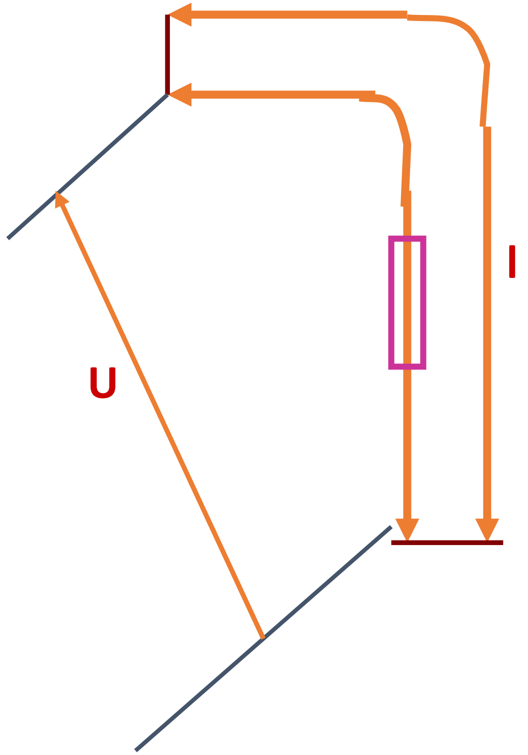
M_t
M_i
M_o
M_i
M_t
M_o

- THe process piping code ASME B 31 .3 provides a simple formula to calculate the SIF of a bend or elbow. As per that co de
- SIFin-plane = 0 . 9 / h^(2/ 3)
- SIF out-plane = 0 . 7 5 / h^(2/ 3)
- Here h = T R1 / r 2^2
- h =Flexibility characteristics, dimensionless
- T =Nominal wall thickness o f bend, in
- R1 =Bend radius, in
- r 2 =Mean radius of matching pipe, in

Stress Analysis

- **The need for formal stress analysis should be determined by designers with extensive flexibility experience.**
- **Hot piping connecting to strain-sensitive equipment such as pumps, compressors and turbines shall be closely reviewed for possible full analysis.**
- **For other systems full stress analysis is required when the following criteria is not satisfied :**

Emprical formula for finding flexibility of the system having only two terminal points and pipe of uniform size.



$$\frac{D\,Y}{(L-U)^2} \leq 208.3$$

D = Outside dia of pipe

Y = Resultant expansion in mm

L = Developed length of line axis between anchors (m)

U = Anchor distance (m).

where

In SI Units

D = nominal pipe size, in (mm)

Y = resultant of movement to be absorbed by piping system, in (mm)

L = developed length of piping system between two anchors, ft (m)

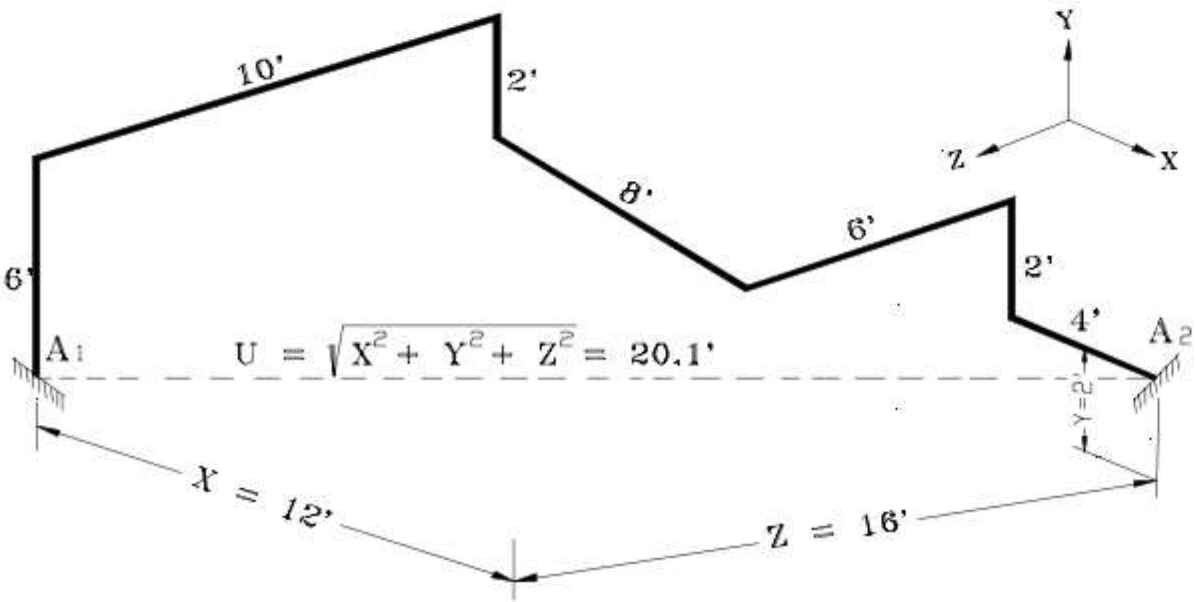
U = anchor distance (length of straight line joining anchors), ft (m)

$$\frac{D\,E}{(L-U)^2} \leq 0.03$$

Where D = Nominal pipe size, in inches.
E = Expansion to be absorbed, in inches (E = U e)
L = Developed length of line axis, in feet
U = Anchor distance, in feet = Length of straight line joining the anchors
e = Coefficient of expansion

In Imperial Units

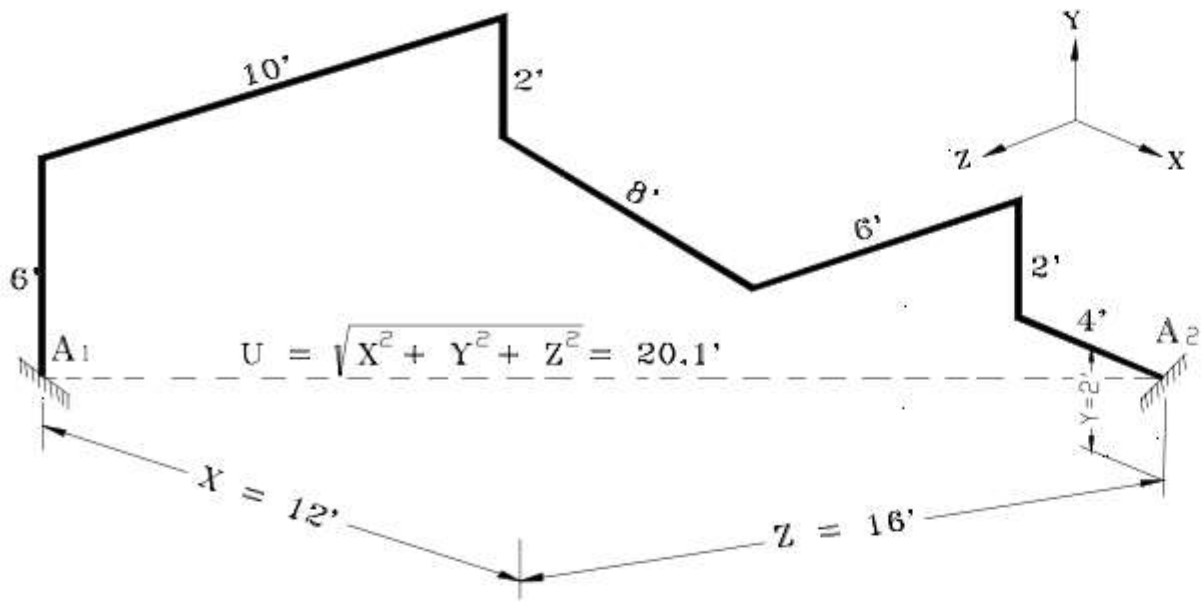
For quick-check stress analysis



Example for quick-check stress analysis

See Figure (5-9):
Pipe = 6" Sch. 40 Carbon Steel, D = 6"
Design Temp. = 400 °F.

$$\frac{DE}{(L-U)^2} \leq 0.03$$



STEP 1

Establish the distance between anchors in plan and elevation, in feet and decimals of a foot.

X = Total line length away from A_2 = 8' + 4' = 12'
Y = Vertical elevation difference = 6' - 2' - 2' = 2'
(Difference in elevation between A_1 and A_2)
Z = Total line length away from A_1 = 10' + 6' = 16'

STEP 2

Determine length U, the straight length between points A_1 and A_2.

$$U = \sqrt{x^2 + y^2 + z^2} = \sqrt{12^2 + 2^2 + 16^2} = 20.1'$$ **(say 20')**

STEP 3

Determine the expansion to be absorbed, E = U e,

where U = Anchor distance, in feet = Length of straight line joining the anchors = 20'

e = Coefficient of expansion (See Table 5-5)

From Table 5-5 @ 400 °F e = 2.7 inch per 100 ft,

so e = 0.027 inch per ft.

Then E = U e = 20 x 0.027 = 0.54"

STEP 4

Determine value of L, the total length of line.

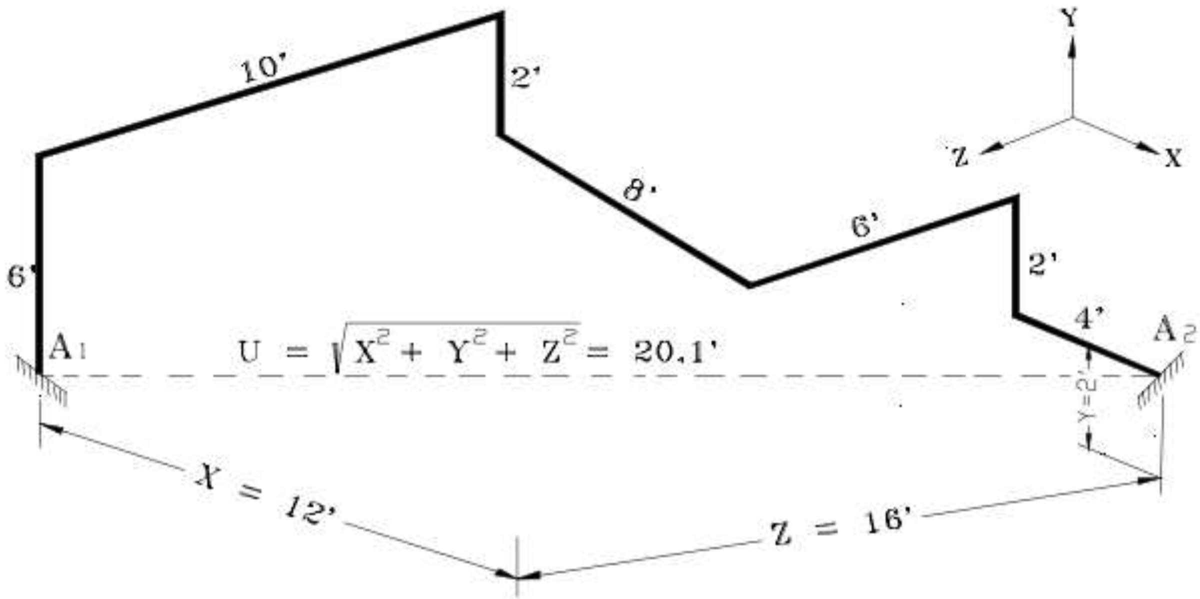
L = 6 + 10 + 2 + 8 + 6 + 2 + 4 = 38'

STEP 5

Solve the formula which must be equal to or less than 0.03 or full stress analysis is needed.

$$\frac{D\,E}{(L-U)^2} \leq 0.03$$

$$\frac{6 \text{x} 0.54}{(38-20)^2} = \frac{3.24}{18^2} = 0.01 \leq 0.03,$$ **So the configuration is satisfactory.**



When piping connects to equipment nozzles which expand and contract due to temperature, the nozzle movement must be considered and added to expansion (E) calculations in the direction they occur.

Referring to Figure (5-9), should anchor point A_1 become an equipment nozzle and expand downward 0.375" and in direction Z toward A_2 by 2" , the calculations must be modified. Expansion must be figured for net lengths of X, Y, and Z and anchor movements applied.

STEP I:

Calculate expansion in direction X : $\sum X = 12 \text{ x } 0.027" = 0.324"$

Since there is no anchor movement in direction X : $\sum X = 0.324" + 0 = 0.324"$

STEP II:

Calculate expansion in direction Y: $\sum Y = 2 \text{ x } 0.027" = 0.054"$

Since A_1 is moving downward 0.375” $\sum Y = 0.054" - 0.375" = -0.321"$

So use 0.321” as this becomes the net anchor movement.

STEP III:

Calculate expansion in direction Z : $\sum Z = 16 \text{ x } 0.027" = 0.432"$

Since A_1 is moving 2” in the direction Z toward A_2: $\sum Z = 0.432" + 2" = 2.432"$

STEP IV:

Calculate expansion in direction U:

$$E = \sqrt{\sum X^2 + \sum Y^2 + \sum Z^2} = \sqrt{0.324^2 + 0.321^2 + 2.432^2} = 2.45"$$

STEP V: **Solve the basic formula using the value for E:**

$$\frac{DE}{(L-U)^2} \leq 0.03 \qquad \frac{6\text{x}2.45}{(38-20)^2} = \frac{14.7}{324} = 0.045 > 0.03$$

which is larger than 0.03, so a stress analysis required.

Table 5-5 Coefficients of Expansion

Linear Thermal Expansion Between 70 ^{O}F and Indicated Temperature, inches per 100 feet*

Temp. ^{o}F	Material												
	Carbon Steel Carbon-Moly Low Chrome (thru 3Cr-Mo)	5Cr-Mo thru 9Cr-Mo	Austenitic Stainless Steels 18Cr-18Ni	12 Cr 17 Cr 27 Cr	25Cr-20Ni	Monel 67Ni – 30Cu	3 ½ Nickel	Aluminum	Gray Cast Iron	Bronze	Brass	Wrought Iron	70Cu-30Ni
-325	-2.37	-2.22	-3.85	-2.04	-3.00	-2.62	-2.22	-4.68		-3.98	-3.88	-2.70	-3.15
-300	-2.24	-2.10	-3.63	-1.92	-2.83	-2.50	-2.10	-4.46		-3.74	-3.64	-2.55	-2.87
-275	-2.11	-1.98	-3.41	-1.80	-2.66	-2.38	-1.98	-4.21		-3.50	-3.40	-2.40	-2.70
-250	-1.98	-1.86	-3.19	-1.68	-2.49	-2.26	-1.86	-3.97		-3.26	-3.16	-2.25	-2.53
-225	-1.85	-1.74	-2.96	-1.57	-2.32	-2.14	-1.74	-3.71		-3.02	-2.93	-2.10	-2.36
-200	-1.71	-1.62	-2.73	-1.46	-2.15	-2.02	-1.62	-3.44		-2.78	-2.70	-1.95	-2.19
-175	-1.58	-1.50	-2.50	-1.35	-1.98	-1.90	-1.50	-3.16		-2.54	-2.47	-1.81	-2.12
-150	-1.45	-1.37	-2.27	-1.24	-1.81	-1.79	-1.38	-2.88		-2.31	-2.24	-1.67	-1.95
-125	-1.30	-1.23	-2.01	-1.11	-1.60	-1.59	-1.23	-2.57		-2.06	-2.00	-1.49	-1.74
-100	-1.15	-1.08	-1.75	-0.98	-1.39	-1.38	-1.08	-2.27		-1.81	-1.76	-1.31	-1.53
-75	-1.00	-0.94	-1.50	-0.85	-1.18	-1.18	-0.93	-1.97		-1.56	-1.52	-1.13	-1.33
-50	-0.84	-0.79	-1.24	-0.72	-0.98	-0.98	-0.78	-1.67		-1.32	-1.29	-0.96	-1.13
-25	-0.68	-0.63	-0.98	-0.57	-0.78	-0.77	-0.62	-1.32		-1.25	-1.02	-0.76	-0.89
0	-0.49	-0.46	-0.72	-0.42	-0.57	-0.57	-0.46	-0.97		-0.77	-0.75	-0.56	-0.66
25	-0.32	-0.30	-0.46	-0.27	-0.37	-0.37	-0.30	-0.63		-0.49	-0.48	-0.36	-0.42
50	-0.14	-0.13	-0.21	-0.12	-0.16	-0.20	-0.14	-0.28		-0.22	-0.21	-0.16	-0.19
70	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
100	0.23	0.22	0.34	0.20	0.28	0.28	0.22	0.46	0.21	0.36	0.35	0.26	0.31
125	0.42	0.40	0.62	0.36	0.51	0.52	0.40	0.85	0.38	0.66	0.64	0.48	0.56
150	0.61	0.58	0.90	0.53	0.74	0.75	0.58	1.23	0.55	0.96	0.94	0.70	0.82
175	0.80	0.76	1.18	0.69	0.98	0.99	0.76	1.62	0.73	1.26	1.23	0.92	1.07
200	0.99	0.94	1.46	0.86	1.21	1.22	0.94	2.00	0.90	1.56	1.52	1.14	1.33
225	1.21	1.13	1.75	1.03	1.45	1.46	1.13	2.41	1.08	1.86	1.83	1.37	1.59
250	1.40	1.33	2.03	1.21	1.70	1.71	1.32	2.83	1.27	2.17	2.14	1.60	1.86
275	1.61	1.52	2.32	1.38	1.94	1.96	1.51	3.24	1.45	2.48	2.45	1.83	2.13
300	1.82	1.71	2.61	1.56	2.18	2.21	1.69	3.67	1.64	2.79	2.76	2.06	2.40
325	2.04	1.90	2.90	1.74	2.43	2.44	1.88	4.09	1.83	3.11	3.08	2.29	2.68
350	2.26	2.10	3.20	1.93	2.69	2.68	2.08	4.52	2.03	3.42	3.41	2.53	2.96
375	2.48	2.30	3.50	2.11	2.94	2.91	2.27	4.95	2.22	3.74	3.73	2.77	3.24
400	**2.7**	2.50	3.80	2.30	3.20	3.25	2.47	5.39	2.42	4.05	4.05	3.01	3.52

Table 5-5 Coefficients of Expansion

Linear Thermal Expansion Between 70 °F and Indicated Temperature, inches per 100 feet*

Temp. °F	Material												
	Carbon Steel Carbon-Moly Low Chrome (thru 3Cr-Mo)	5Cr-Mo thru 9Cr-Mo	Austenitic Stainless Steels 18Cr-18Ni	12 Cr 17 Cr 27 Cr	25Cr-20Ni	Monel 67Ni – 30Cu	3 ½ Nickel	Aluminum	Gray Cast Iron	Bronze	Brass	Wrought Iron	70Cu-30Ni
425	2.93	2.72	4.10	2.50	3.46	3.52	2.69	5.83	2.62	4.37	4.38	3.25	
450	3.16	2.93	4.41	2.69	3.72	3.79	2.91	6.28	2.83	4.69	4.72	3.50	
475	3.39	3.14	4.71	2.89	3.98	4.06	3.13	6.72	3.03	5.01	5.06	3.74	
500	3.62	3.35	5.01	3.08	4.24	4.33	3.34	7.17	3.24	5.33	5.40	3.99	
525	3.86	3.58	5.31	3.28	4.51	4.61	3.57	7.63	3.46	5.65	5.75	4.25	
550	4.11	3.80	5.62	3.49	4.79	4.90	3.80	8.10	3.67	5.98	6.10	4.50	
575	4.35	4.02	5.93	3.69	5.06	5.18	4.03	8.56	3.89	6.31	6.45	4.76	
600	4.60	4.24	6.24	3.90	5.33	5.46	4.27	9.03	4.11	6.64	6.80	5.01	
625	4.86	4.47	6.55	4.10	5.60	5.75	4.51		4.34	6.96	7.16	5.27	
650	5.11	4.69	6.87	4.31	5.88	6.05	4.75		4.57	7.29	7.53	5.53	
675	5.37	4.92	7.18	4.52	6.16	6.34	4.99		4.80	7.62	7.89	5.80	
700	5.63	5.14	7.50	4.73	6.44	6.64	5.24		5.03	7.95	8.26	6.06	
725	5.90	5.38	7.82	4.94	6.73	6.94	5.50		5.26	8.28	8.64	6.32	
750	6.16	5.62	8.15	5.16	7.02	7.25	5.76		5.50	8.62	9.02	6.59	
775	6.43	5.86	8.47	5.38	7.31	7.55	6.02		5.74	8.96	9.40	6.85	
800	6.70	6.10	8.80	5.60	7.60	7.85	6.27		5.98	9.30	9.78	7.12	
825	6.97	6.34	9.13	5.82	7.89	8.16	6.54		6.22	9.64	10.17	7.40	
850	7.25	6.59	9.46	6.05	8.19	8.48	6.81		6.47	9.99	10.57	7.69	
875	7.53	6.83	9.79	6.27	8.48	8.80	7.08		6.72	10.33	10.96	7.91	
900	7.81	7.07	10.12	6.49	8.70	9.12	7.35		6.97	10.68	11.35	8.26	
925	8.08	7.31	10.46	6.71	9.07	9.44	7.72		7.23	11.02	11.75	8.53	
950	8.35	7.56	10.80	6.94	9.37	9.77	8.09		7.50	11.37	12.16	8.81	
975	8.62	7.81	11.14	7.17	9.66	10.09	8.46		7.76	11.71	12.57	9.08	
1000	8.89	8.06	11.48	7.40	9.95	10.42	8.83		8.02	12.05	12.98	9.39	
1025	9.17	8.30	11.82	7.62	10.24	10.75	8.98			12.40	13.39		
1050	9.46	8.55	12.16	7.95	10.54	11.09	9.14			12.76	13.81		
1075	9.75	8.80	12.50	8.18	10.83	11.43	9.29			13.11	14.23		
1100	10.04	9.05	12.84	8.31	11.12	11.77	9.45			13.47	14.65		
1125	10.31	9.28	13.18	8.53	11.41	12.11	9.78						
1150	10.57	9.52	13.52	8.76	11.71	12.47	10.11						
1175	10.83	9.76	13.86	8.98	12.01	12.81	10.44						
1200	11.10	10.00	14.20	9.20	12.31	13.15	10.78						
1225	11.38	10.26	14.54	9.42	12.59	13.50							
1250	11.66	10.53	14.88	9.65	12.88	13.86							
1275	11.94	10.79	15.22	9.88	13.17	14.22							
1300	12.22	11.06	15.56	10.11	13.46	14.58							

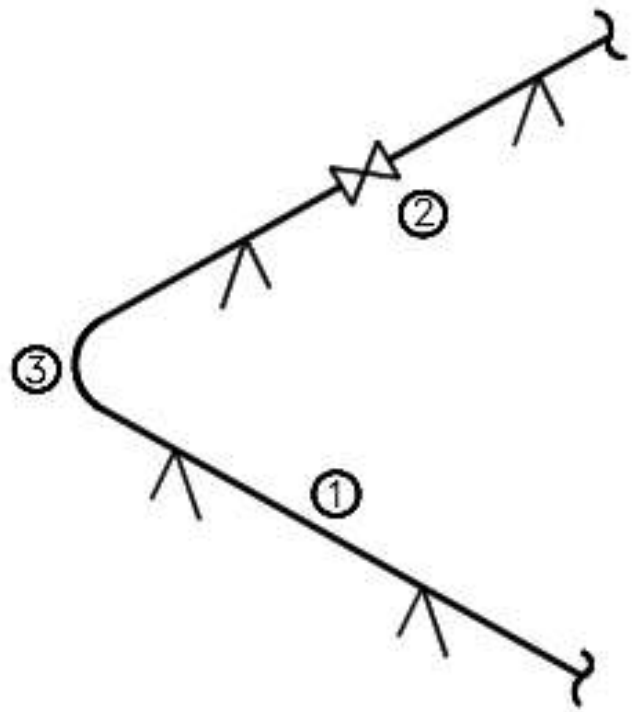


FIGURE 9 — SECTIONING A PIPING SYSTEM

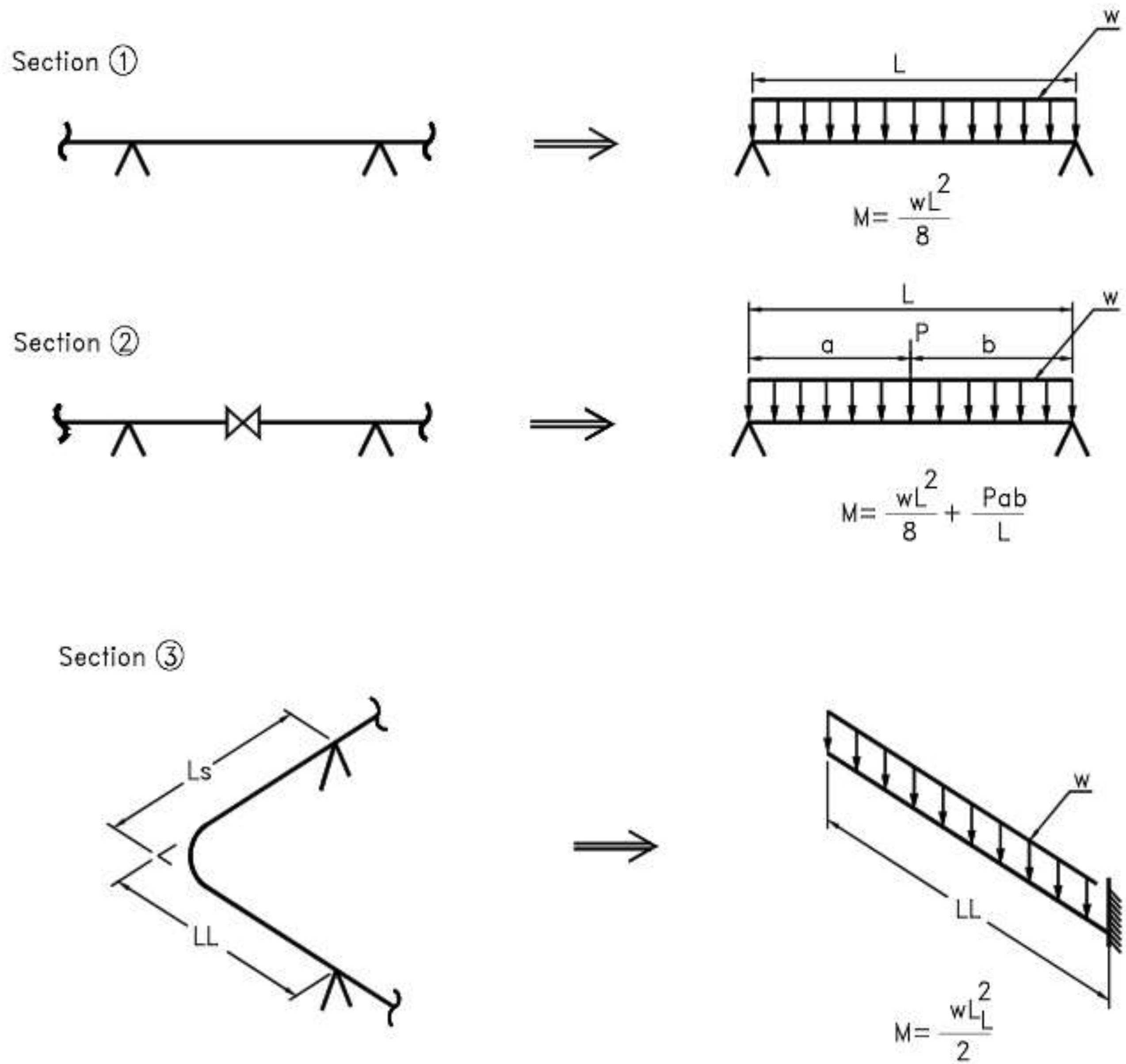
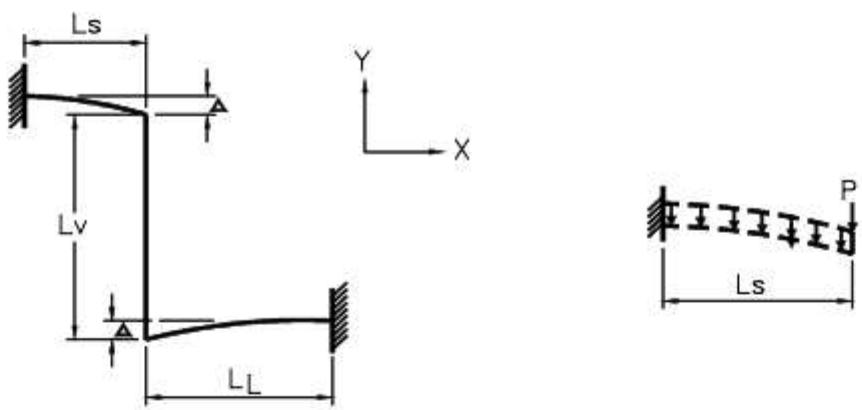


FIGURE 10 – SECTION MODELS



$$M = \frac{wL_s^2 + PL_s}{2}$$

Where : $P = w\,(cL_L + L_v)$

$c = 0.5$

FIGURE 11 — VERTICAL PIPING

Figure 11, illustrates the conservative consideration of another situation frequently encountered in piping system analysis.

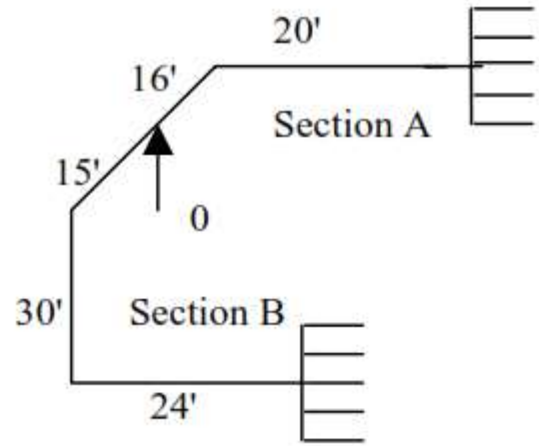
Because both cantilever legs deflect the same due to the weight of the vertical section, the moment is larger at the anchor for the shorter cantilever.

Therefore, to be conservative, only the moment on the leg is considered for calculating the stress in the piping system. To conservatively calculate that moment, it is modelled as a uniform load on the shorter cantilever plus a concentrate load acting at its end.

The concentrated load P is composed of two components, viz, the actual weight of the vertical leg and the conservatively estimated effective weight of the longer cantilever leg.

Experience has shown that including half the weight of the longer leg is sufficient to assure a conservative analysis

Figure 12 shows the application of the foregoing concepts to an actual piping system numerical values.



Data
Pipe NPS 12 Sch 10 S
w = 25 lb / ft
Z=22.0 $inch^3$
P = 200 psi

FIGURE 12 - EXAMPLE PROBLEM

To simplify the problem, the analyst separated the system into two sections at Point 0. The two ends at Point 0 are both considered to be anchored. The maximum longitudinal stress in each section is calculated.

FOR SECTION A (Figure 12A) :

In this case, the weight of the 16 feet section is ignored, and the maximum moment for the 20 feet length is calculated on the basis of uniformly loaded cantilever beam.

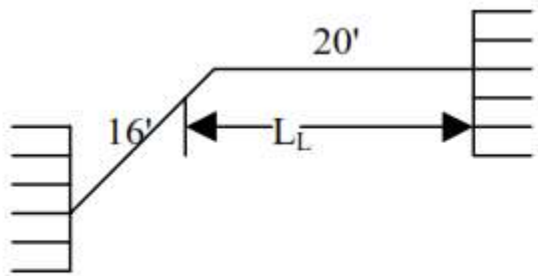


FIGURE 12 A - SECTION A

$$M = \frac{wL_L^2}{2}$$

$$\frac{25\ (\ 20^2\ )}{2} = 5000 \text{ FT-lb}$$

$$S_w = \frac{M}{Z} = \frac{5000\ (12 \text{ in./ft.})}{22}$$

$$= 2727 \text{ psi}$$

FOR SECTION B (Figure 12 B) :

The weights of the 30 feet vertical leg Lv and the 24 feet horizontal leg L_L are combined into a concentrated load P acting at the end of the shorter 15 feet cantilever leg L_S. In this case the analyst considered the effective weight of L_L to be one half its actual weight.

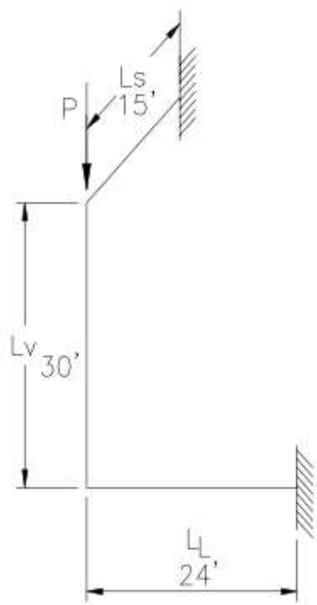


FIGURE 12B – SECTION B

$$P = w\,(L_v + L_L/2)$$
$$= 25\,(30 + 24/2) = 1050 \text{ lb}$$
$$M = PL_s \quad + w\,(L_s/2)\,L_s$$
$$= \text{(conc. load)} \quad \text{(uniform load)}$$
$$= 1050\,(15) + 25\,(15/2)\,15$$
$$= 18563 \text{ ft - lb}$$

$$S_w = M/Z = \frac{18{,}563 \times 12 \text{ in / ft}}{22} = 10{,}125 \text{ psi}$$

The longitudinal stress resulting from internal pressure in the system is then calculated by the formula.

$$Sp = \frac{P_r D}{4\,t} = \frac{200\,(12.75)}{4(0.18)} = 3542 \text{ psi}$$

This value is added to the largest calculated weight stress to give the total longitudinal stress in the system.

$S_L = S_w$ (max.) + Sp = 10,125 + 3542 = 13,667 psi.

S_L is then compared to the allowable stress for the material used at operating temperature Sh to decide whether a formal analysis is required because of sustained loads.

DISPLACEMENT LOAD STRESSES :

Stresses produced by displacement loads are calculated by the guided cantilever method. Consider the

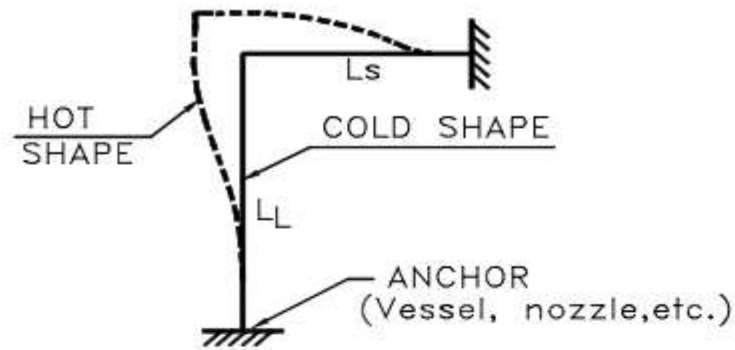


FIGURE 13 – SIMPLE PIPING SYSTEM

where L_S = length of shorter leg

L_L = length of longer leg

This system may be conservatively modelled as follows :

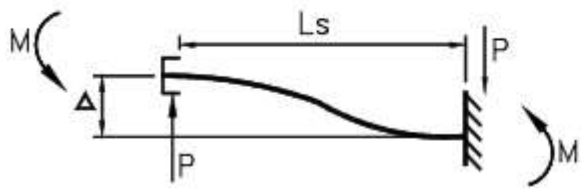


FIGURE 14 – MODEL FOR GUIDED CANTILEVER

where M = bending moment at end of L_S
P = force exerted by expansion of L_L.
Δ = free expansion of L_L.

This is the "guided cantilever" model - a beam with one end fixed and other free to deflect but locked against rotation.

8.2.1 **EXAMPLES** :

The following illustrate the application of the guided cantilever method for evaluating flexibility.

Example in Figure 15

Header : NPS 6, A53 Gr.B
Branches : NPS 2, A53 Gr.B
$S_A = 29.6$ ksi
$T_{COLD} = 70\,^{\circ}$ F
$T_{HOT} = 275^{\circ}$ F
Expansion = 1.61 in/100 ft.
SIF at stub-ins (2), (3) & (4) = 3.36

$$f = \frac{3\,E\,D\,\Delta}{144\,L^2} \quad \text{where}$$

f = Stress. lb / in^2
E = 30×10^6 lb / in^2
D = Pipe OD, inches
Δ = Deflection, inches
L = Span, ft.

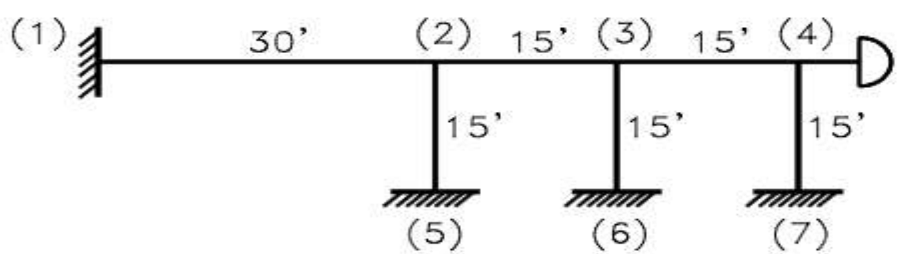


FIGURE 15 — EXAMPLE

The point on the system which will undergo the largest deflection is node (4). The horizontal deflection will be :

$4 = (30 + 15 + 15)\ (0.0161) = 0.966$ inch

Considering branch 4-7 to be a guided cantilever, the approximate stress is $f_4 = 6370$ psi from above formulae including the SIF raises the stress to 21.4 ksi. This stress is less than S_A, therefore the system is adequately flexible and need not be formally analysed for displacement stresses.

Example in figure 16 :
Pipe : NPS 4, Sch 40, A53 Gr.B.
$S_A = 29.0$ ksi
$T_{COLD} = 70\ °F$
$T_{HOT} = 550°F$
Expansion = 4.11 in/100 ft.
SIF at LR ells = 1.95

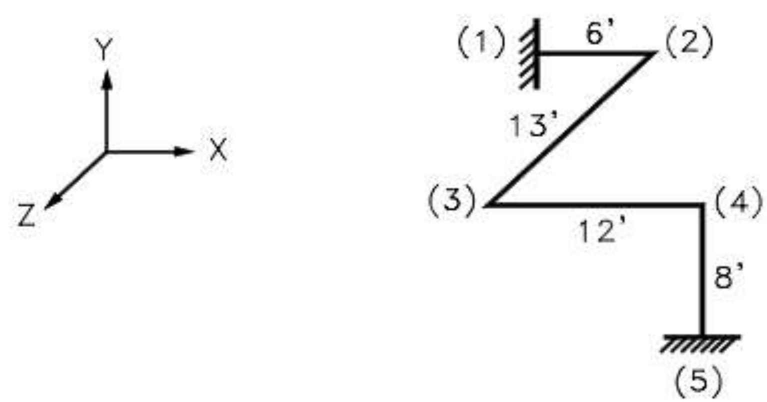


FIGURE 16 — EXAMPLE

Since the system is of uniform pipe size, an imaginary anchor may be assumed to exist at node (3). This breaks the system into two parts and each part can be further simplified into a guided cantilever. The horizontal movement at node (2) can be approximated as $\Delta_2 = (13)\ (0.0.411) = 0.53"$ and at node (4) it can be approximated as $\Delta_4 = (12)(0.0411) = 0.49"$. From above formulae, the guided cantilever stresses are S_{E2} = 80,740 psi and S_{E4} = 45,420 psi. Because S_{E2} and S_{E4} is higher than the allowable stress S_A, a formal analysis is required.

PROBLEM 1

90° Bend, Both Ends Fixed

12-in. pipe, A.S.T.M. Specification A—106

Steam temperature = 710°F

Pressure = 850 psi

Wall thickness required:

Stress value S = 11,650 psi

$$t_{min} = 0.518 \text{ in.}$$

$$^{*}t_{\min} + 0.125t_{\text{nom}} = t_{\text{nom}}, \quad t_{\text{nom}} = \frac{t_{\min}}{0.875} = 0.595 \text{ in.}$$

Use 12-in. pipe, Schedule 80 $\quad t = 0.687$ in.

$I = 475 \text{ in.}^4$

$S = 74.5 \text{ in.}^3$

$$\text{Expansion} = 5.75 \text{ in. per } 100 \text{ ft.}$$
$$E = 25 \times 10^6 \text{ psi}$$

$$\Delta x = \frac{5.75}{100} \text{ in.} \times 80 \text{ ft} = 4.6 \text{ in.}$$
$$\Delta y = \frac{5.75}{100} \text{ in.} \times 50 \text{ ft} = 2.88 \text{ in.}$$
$$\Delta x\, EI = \frac{4.6}{12} \text{ in.} \times 25 \times 10^6 \times 12^2 \times \frac{475}{12^4} = 31{,}600{,}000 \text{ lb ft}^3$$
$$\Delta y\, EI = \frac{2.88}{12} \text{ in.} \times 25 \times 10^6 \times 12^2 \times \frac{475}{12^4} = 19{,}800{,}000 \text{ lb ft}^3$$

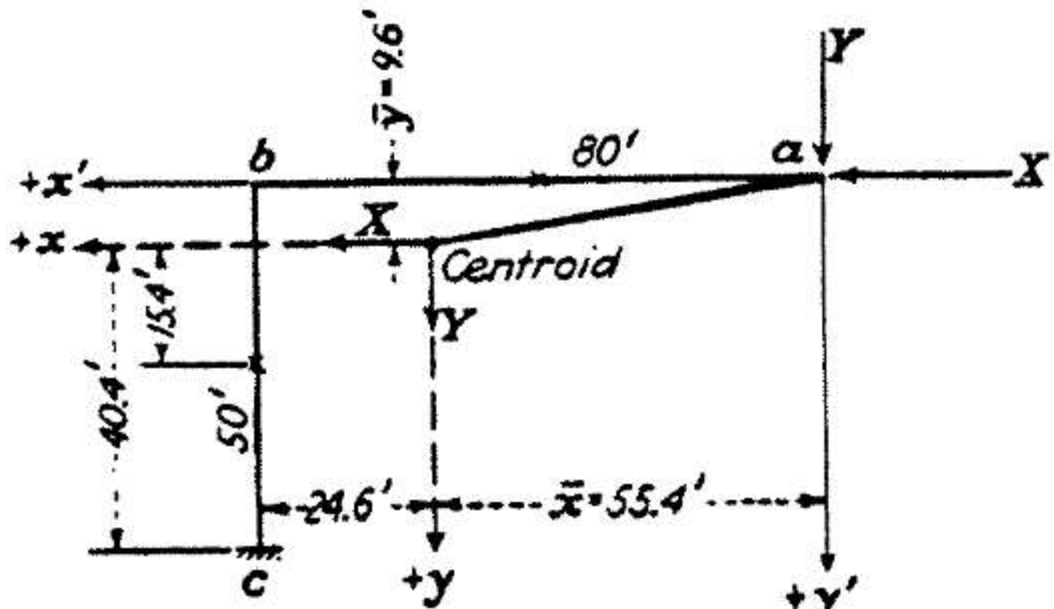


FIG. 8.

	Centroid					I_{xy}			
	l, ft	x', ft	lx'	y', ft	ly'	l, ft	x, ft	y, ft	lxy
ab	80	40	3,200	0	0	80	−15.4	−9.6	+11,820
bc	50	80	4,000	25	1,250	50	24.6	15.4	+18,990
	130		7,200		1,250			I_{xy} =	+30,810

$$\bar{x} = \frac{7,200}{130} = 55.4 \text{ ft} \qquad \bar{y} = \frac{1,250}{130} = 9.6 \text{ ft}$$

	I_x	I_y
ab	$80 \times 9.6^2 = 7,370$	$\frac{80^3}{12} + 80 \times 15.4^2 = 61,540$
bc	$\frac{50^3}{12} + 50 \times 15.4^2 = 22,230$	$50 \times 24.6^2 = 30,200$
	$I_x' = 29,600$	$I_y = 91,740$

$$29,600X - 30,810Y = 31,600,000$$
$$-30,810X + 91,740Y = 19,800,000$$

$$X = \frac{I_y(\Delta x\ EI) + I_{xy}(\Delta y\ EI)}{I_x I_y - I_{xy}^2}$$

$$Y = \frac{I_x(\Delta y\ EI) + I_{xy}(\Delta x\ EI)}{I_x I_y - I_{xy}^2}$$

$$X = \frac{91{,}740 \times 31{,}600{,}000 + 30{,}810 \times 19{,}800{,}000}{29{,}600 \times 91{,}740 - 30{,}810^2}$$

$$= 1{,}990 \text{ lb}$$

$$Y = \frac{29{,}600 \times 19{,}800{,}000 + 30{,}810 \times 31{,}600{,}000}{29{,}600 \times 91{,}740 - 30{,}810^2}$$

$$= 890 \text{ lb}$$

Reacting moment at *a*:

$+1{,}990 \text{ lb} \times 9.6 \text{ ft} - 890 \text{ lb} \times 55.4 \text{ ft}$

$= -30{,}200 \text{ ft lb}$

Bending moment at *b*:

$+1{,}990 \text{ lb} \times 9.6 \text{ ft} + 890 \text{ lb} \times 24.6 \text{ ft}$

$= +41{,}000 \text{ ft lb}$

Bending moment at *c*:

$-1{,}990 \text{ lb} \times 40.4 \text{ ft} + 890 \text{ lb} \times 24.6 \text{ ft}$

$= -58{,}500 \text{ ft lb}$

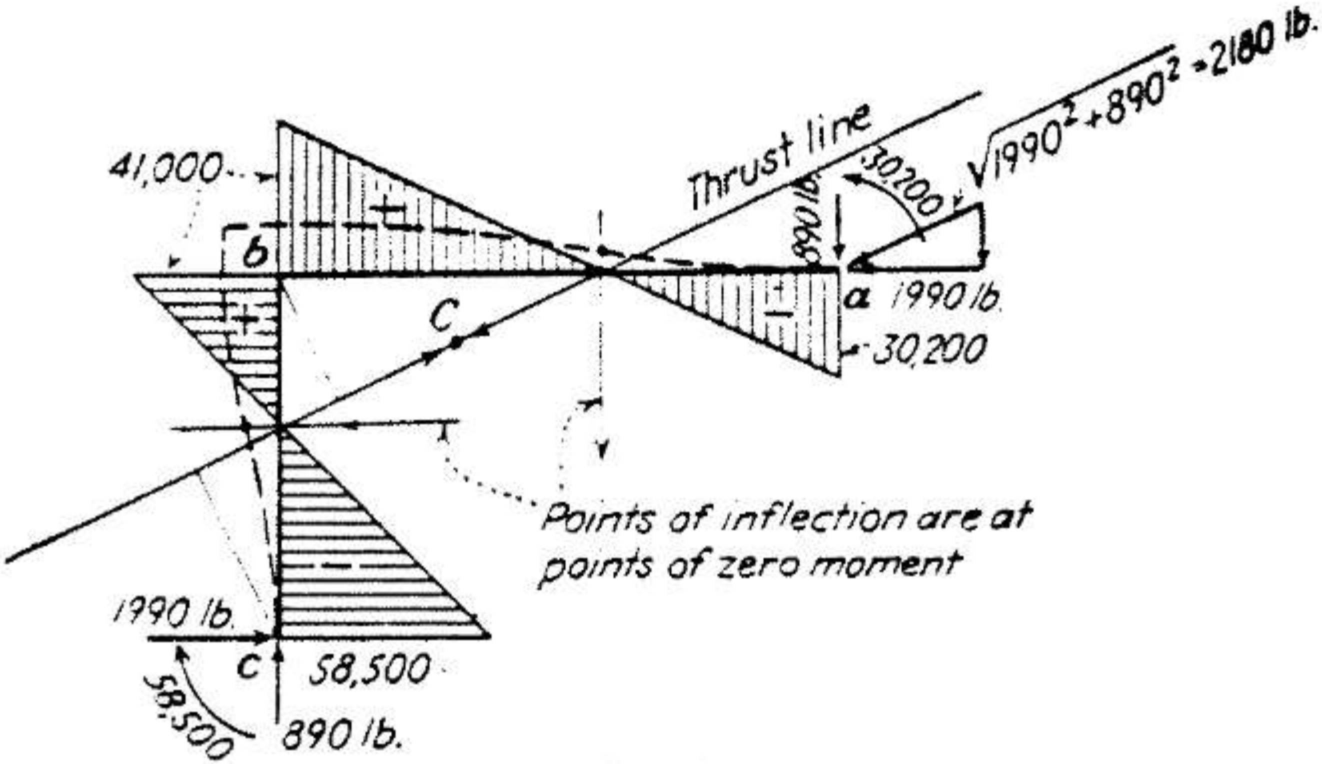

41,000
b
Thrust line
30200
$\sqrt{1990^2+890^2}=2180$ lb.
890 lb.
a
1990 lb.
C
-30,200
Points of inflection are at
points of zero moment
1990 lb.
c
58,500
58,500
890 lb.

FIG. 9.

SIMPLIFIED TABULATION
FOR
PROBLEM 1

$I_o = \frac{80^3}{12}$ $\quad$ $I_o = \frac{50^3}{12}$

Branch	Length l	x	y	$lx^2 + I_o$	$ly^2 + I_o$	lxy
ab	80	-15.4	-9.6	18,940 42,600	7,370	+ 11,820
bc	50	+ 24.6	+15.4	30,200	11,814 10,416	+ 18,990

$I_y = 91,740$

$I_x = 29,600$

$I_{xy} = + 30,810$

PROBLEM 2

90° Bend, One End Hinged

This problem shows the procedure for determining the end reactions when one of the two ends is held so as to permit rotation but stop translatory motions. This end then acts as if it were hinged and therefore offers no resistance to moments. Let end a be the hinged end; then $M_a = 0$.

In this case the origin of the coordinate system is placed at a, and the axes are assumed in directions opposite to the anticipated expansions. The moments and products of inertia of the line about these axes when divided by EI represent, respectively, the deflections produced by a unit force in its own direction and normal to its own direction.

The calculation follows the pattern of problem 1 except that the axes are laid through the hinged end. The dimensions and the temperature change are the same as in problem 1.

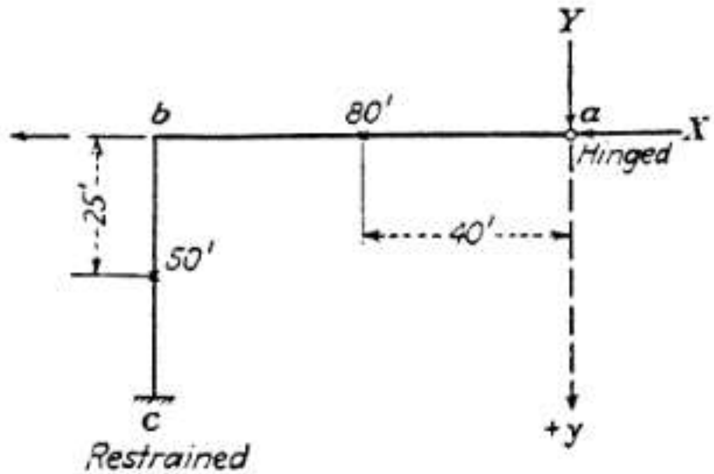
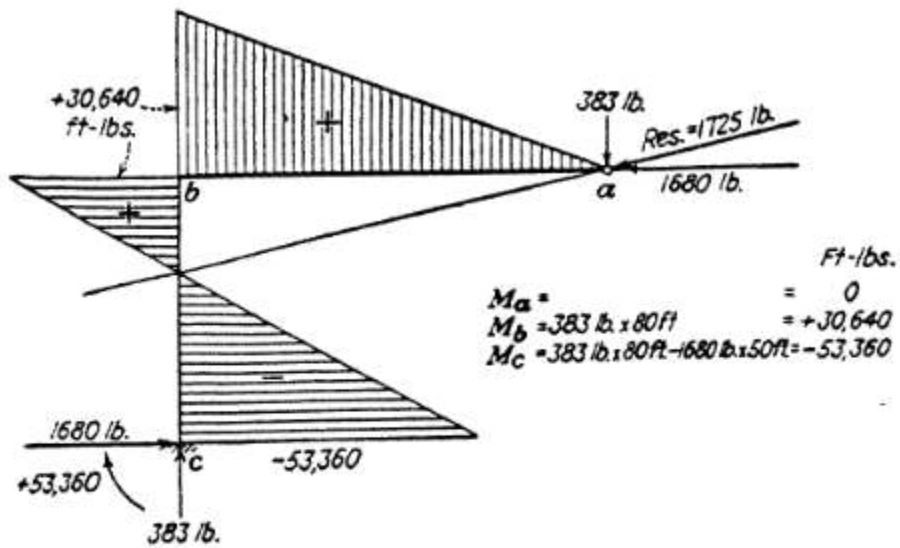


FIG. 10.

	I_{xy}				I_x	I_y
	l, ft	x, ft	y, ft	lxy		
ab	80	40	0	0	0	$\frac{80^3}{12} + 80 \times 40^2 = 170{,}660$
bc	50	80	25	100,000	$\frac{50^3}{12} + 50 \times 25^2 = 41{,}650$	$50 \times 80^2 = 320{,}000$
				$I_{xy} = 100{,}000$	$I_x = 41{,}650$	$I_y = 490{,}660$

$$41{,}650X - 100{,}000Y = 31{,}600{,}000$$
$$-100{,}000X + 490{,}660Y = 19{,}800{,}000$$
$$X = 1{,}680 \text{ lb} \qquad Y = 383 \text{ lb}$$



+30,640 ft-lbs.
383 lb.
Res. = 1725 lb.
b
a
1680 lb.
Ft-lbs.
Ma = = 0
Mb = 383 lb. x 80 ft = +30,640
Mc = 383 lb. x 80 ft - 1680 lb. x 50 ft = -53,360
1680 lb.
c
-53,360
+53,360
383 lb.

FIG. 11.

PROBLEM 3

90° Bend, One End Hinged

In this problem end c is hinged, and end a is restrained. The procedure follows that of problem 2 except that the origin is at c, and the axes are opposite to the directions of problem 2.

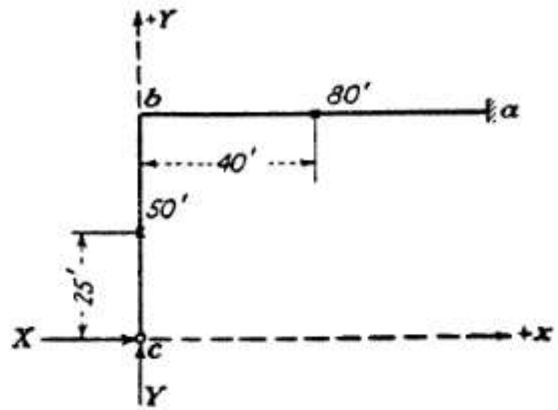


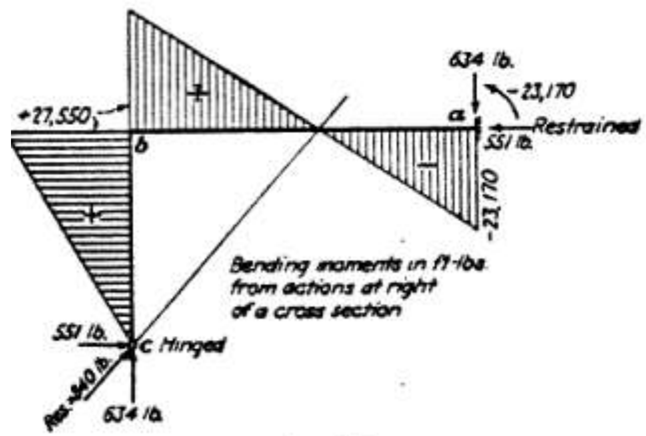
FIG. 12.

	I_{xy}				I_x	I_y
	l, ft	x, ft	y, ft	lxy		
ab	80	40	50	160,000	$80 \times 50^2 = 200{,}000$	$\frac{80^3}{12} + 80 \times 40^2 = 170{,}660$
bc	50	0	25	0	$\frac{50^3}{12} + 50 \times 25^2 = 41{,}650$	0
				$I_{xy} = 160{,}000$	$I_x = 241{,}650$	$I_y = 170{,}660$

$$241{,}650X - 160{,}000Y = 31{,}600{,}000$$

$$-160{,}000X + 170{,}660Y = 19{,}800{,}000$$

$$X = 551 \text{ lb} \qquad Y = 634 \text{ lb}$$



634 lb.
-23,170
+27,550
a
Restrained
b
551 lb.
-23,170
Bending moments in ft-lbs.
from actions at right
of a cross section
551 lb.
c Hinged
634 lb.

Fig. 13.

PROBLEM 4

90° Bend, Both Ends Hinged

Figure 14 shows the solution when both ends are held in such a manner as to permit rotation but stop expansion. In this case the terminals act as if they were hinged, and, in order to satisfy the laws of equilibrium, the reacting forces must be situated in a common line of action.

The process of solution is as follows: Make one end free and place it on rollers that permit expansion along line *ab* only. A unit force applied in the opposite direction will cause a deflection $\delta_x = I_x/EI$. The unknown reaction X causes X times as much deflection. If the expansion of length L is ΔL, then

$$X\delta_x = \Delta L$$

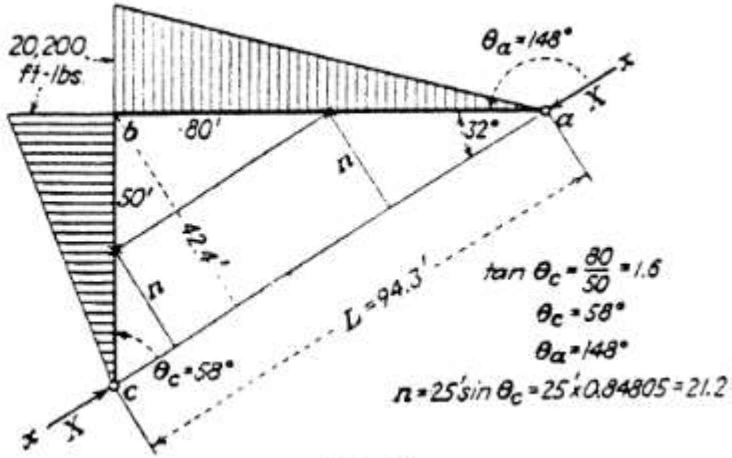


FIG. 14.

$$I_x = \frac{l^3 \sin^2 \theta}{12} + ln^2$$

$$I_x \text{ of } ab\text{: } \frac{80^3}{12} \sin^2 148° + 80 \times 21.2^2 = 47{,}900$$

$$I_x \text{ of } bc\text{: } \frac{50^3}{12} \sin^2 58° + 50 \times 21.2^2 = 29{,}950$$

$$I_x = \overline{77{,}850}$$

$$\text{Expansion } \Delta L = \frac{5.75 \text{ in.}}{100 \text{ ft}} \times 94.3 \text{ ft} = 5.43 \text{ in.} = 0.452 \text{ ft}$$

$$E = 25 \times 10^6 \text{ psi} \qquad I = 475 \text{ in.}^4$$

$$EI = 25 \times 10^6 \times 12^2 \times \frac{475}{12^4} = 82{,}500{,}000 \text{ lb ft}^2$$

$$\text{Deflection due to unit force} = \delta_x = \frac{I_x}{EI}$$

$$X\delta_x = \Delta L \qquad X = \frac{\Delta L}{\delta_x} = \frac{0.452 \times 82{,}500{,}000}{77{,}850} = 478 \text{ lb}$$

$$M_b = 478 \text{ lb} \times 42.4 \text{ ft} = 20{,}200 \text{ ft lb}$$