Mechanical Engineering Courses

Ce n t r i f u g a l P u M P

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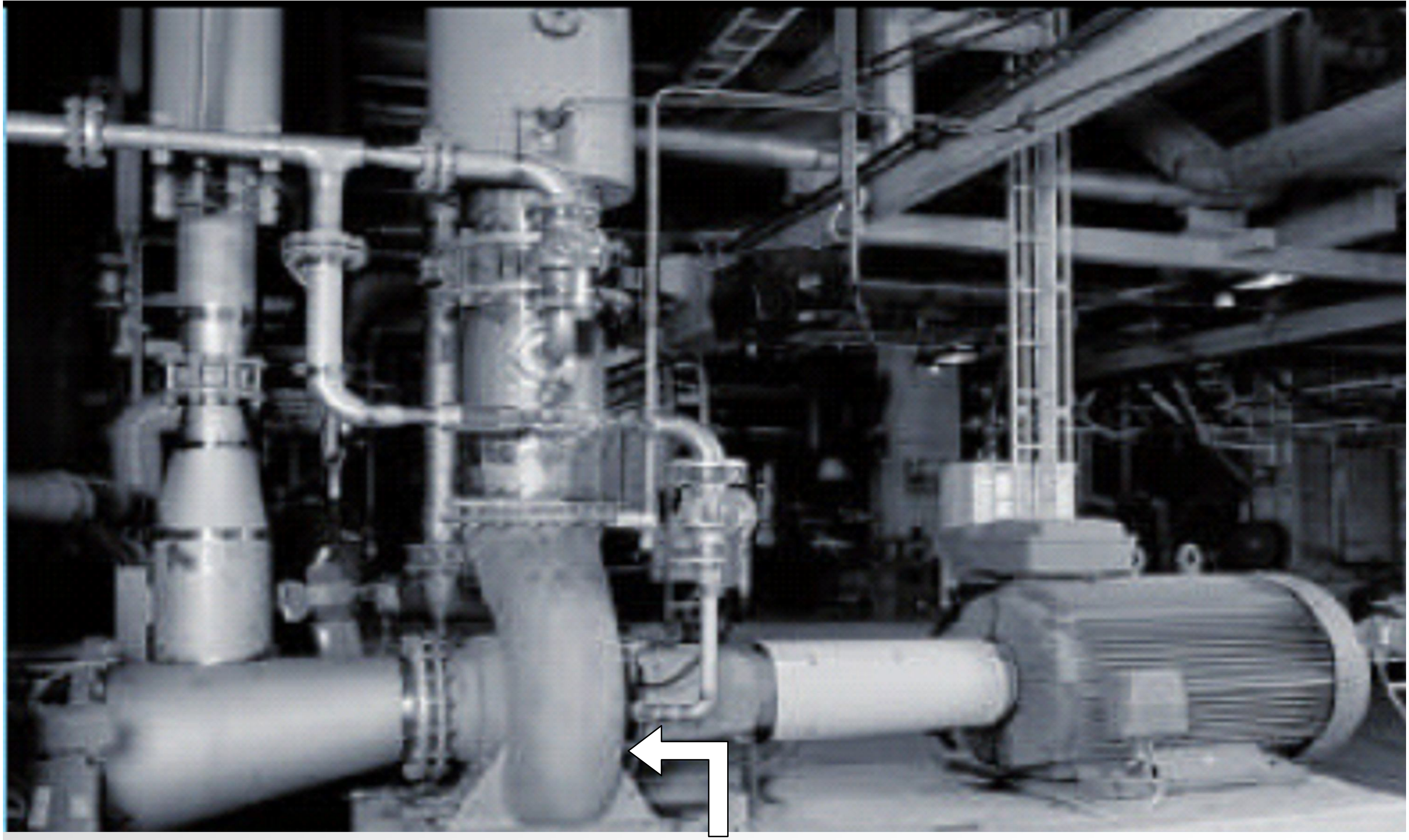
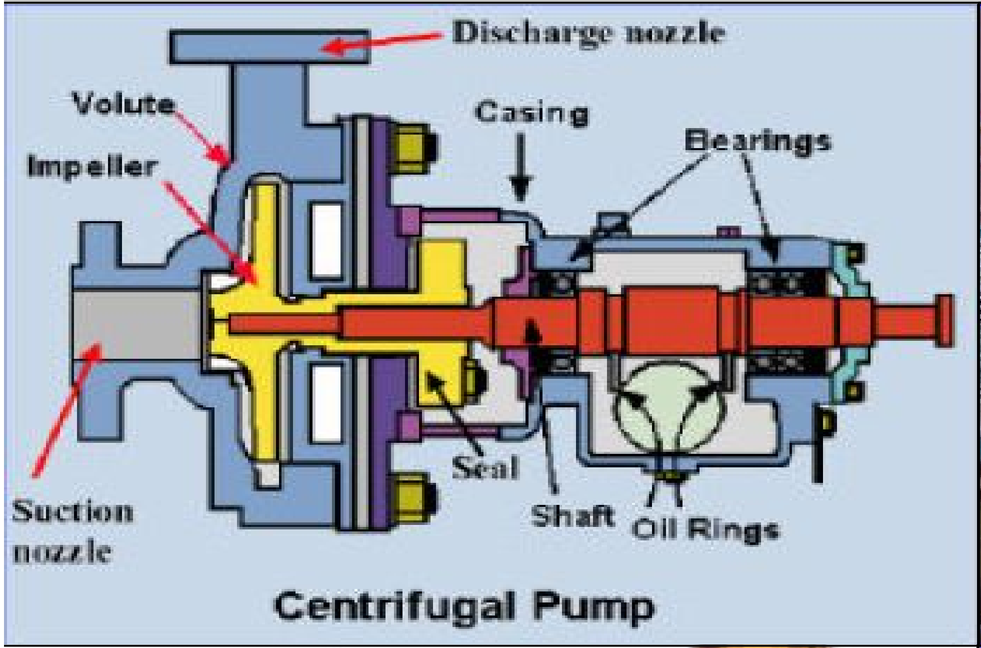
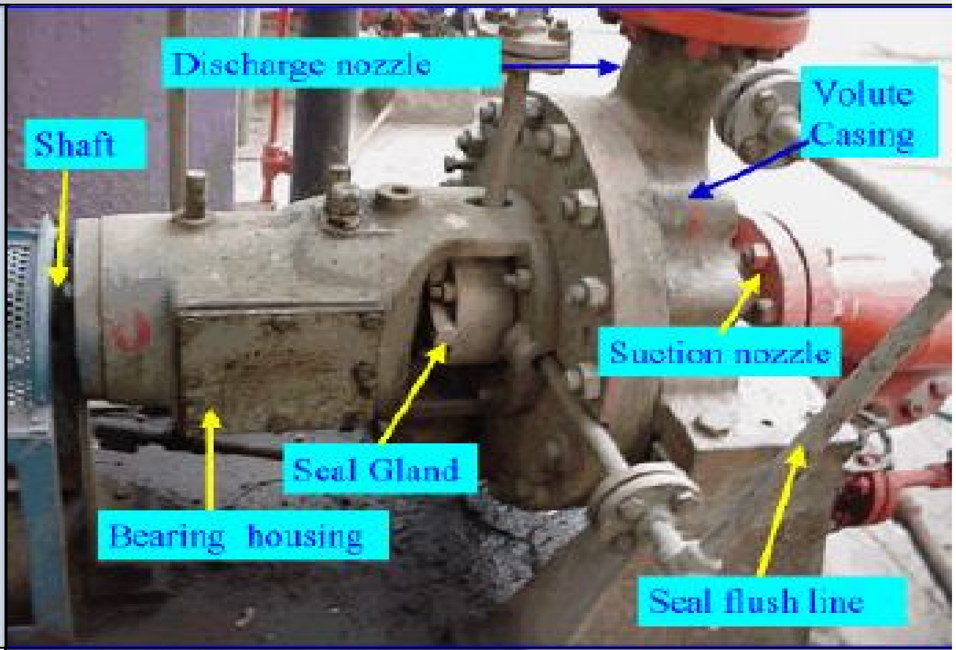
Pumping System in an Industry

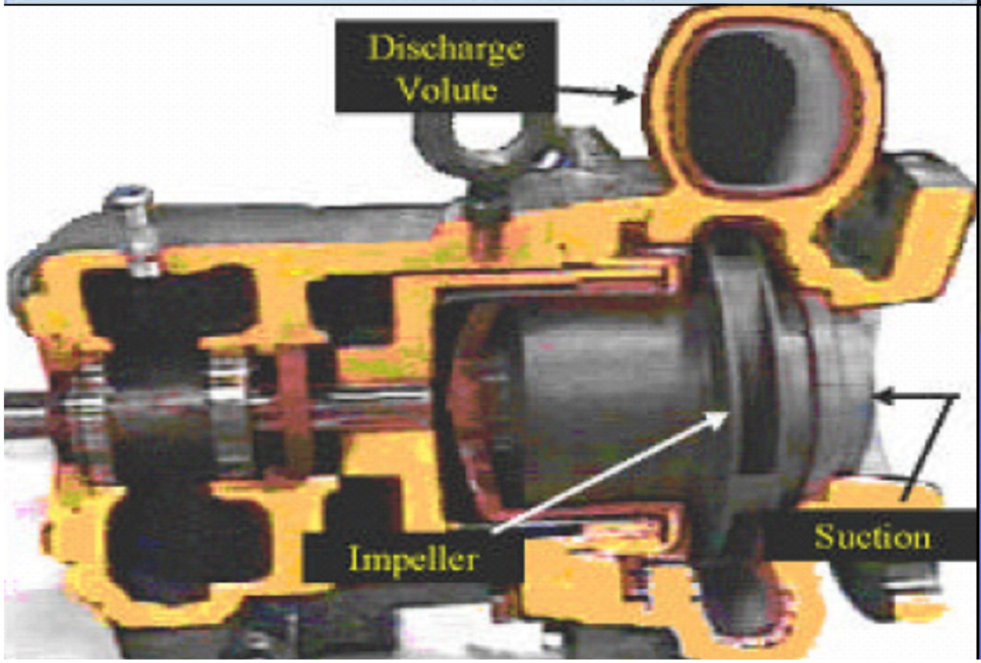


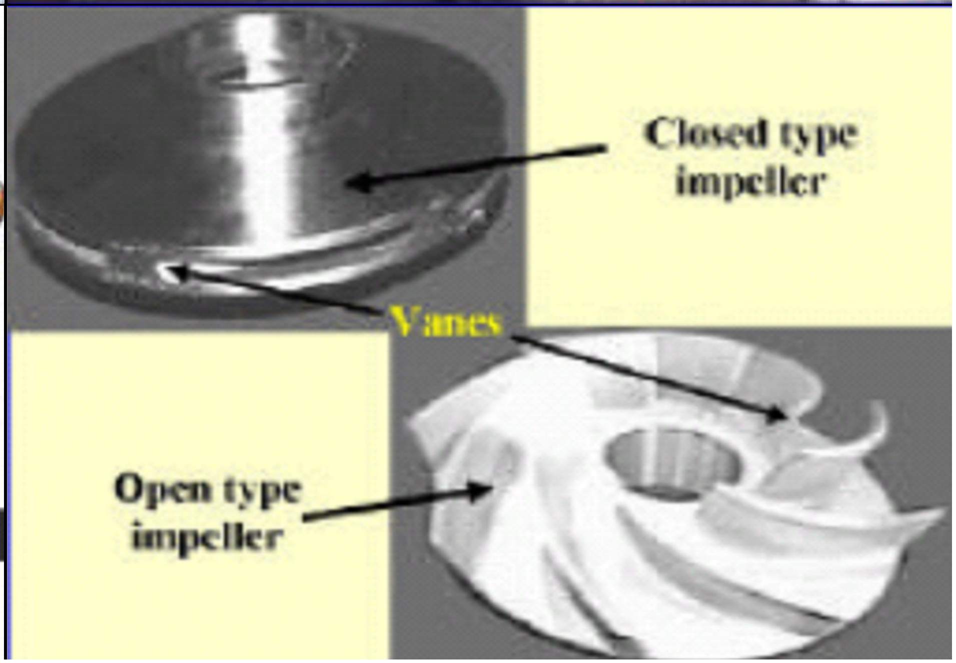
Centrifugal Pump

Construction and Component

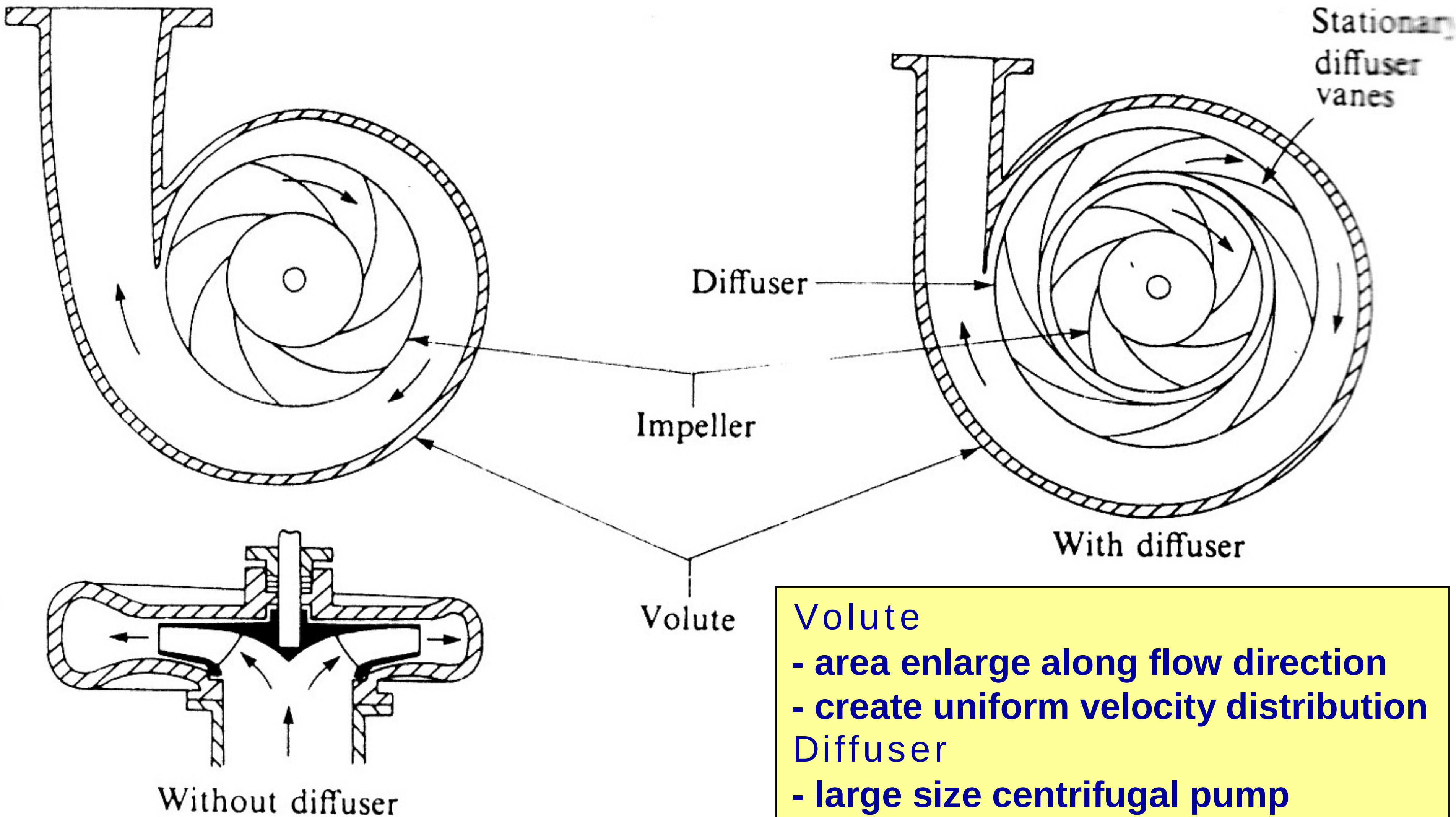








Casing



Volute

- area enlarge along flow direction

- create uniform velocity distribution

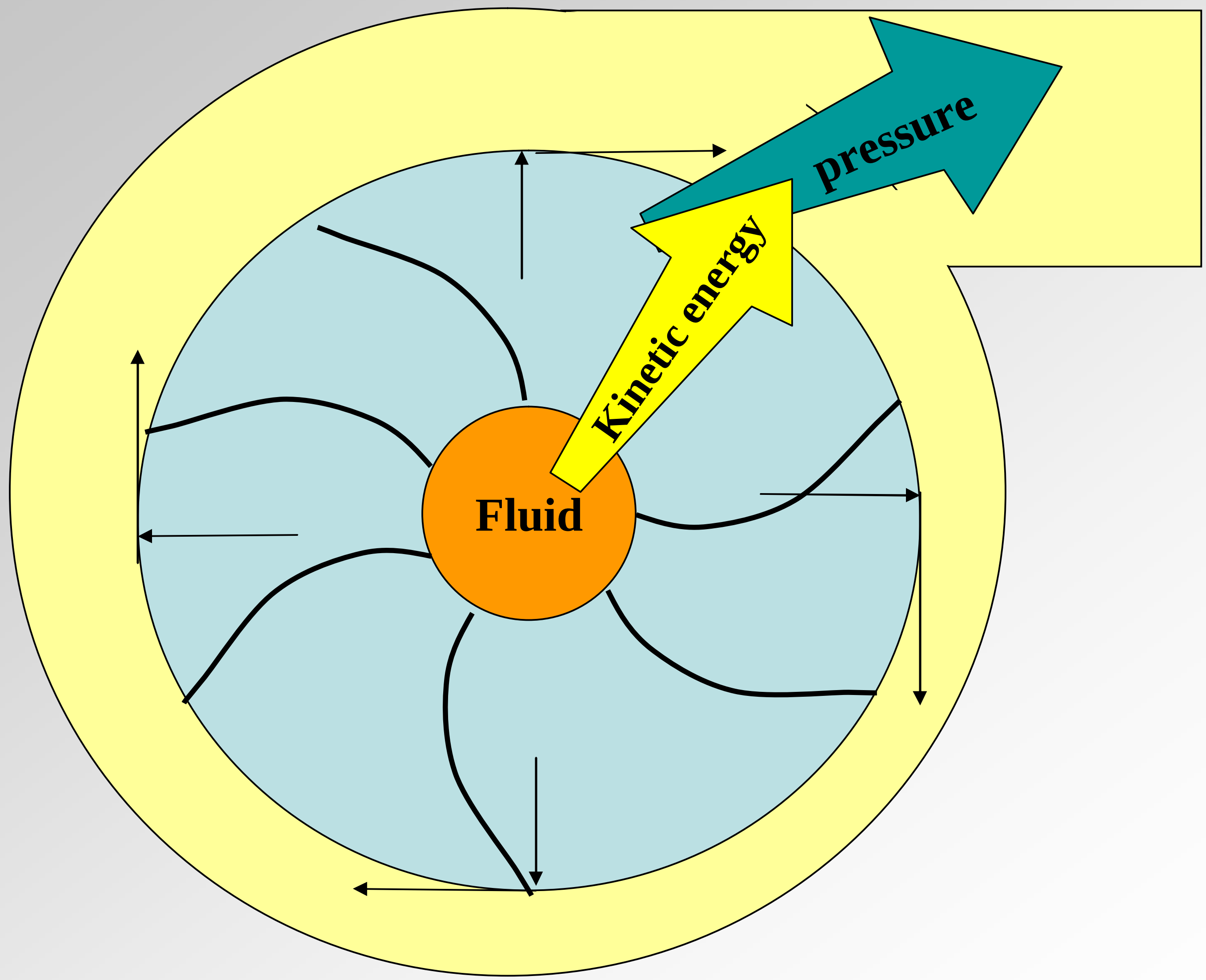
Diffuser

- large size centrifugal pump

- guide vanes surround the impeller

- fluid flow decelerated while directed to enter the volute

Working Principles



Installation

Inlet head :

$$\frac{p_i}{\rho g}+\frac{V_i^2}{2g}+Z_i$$

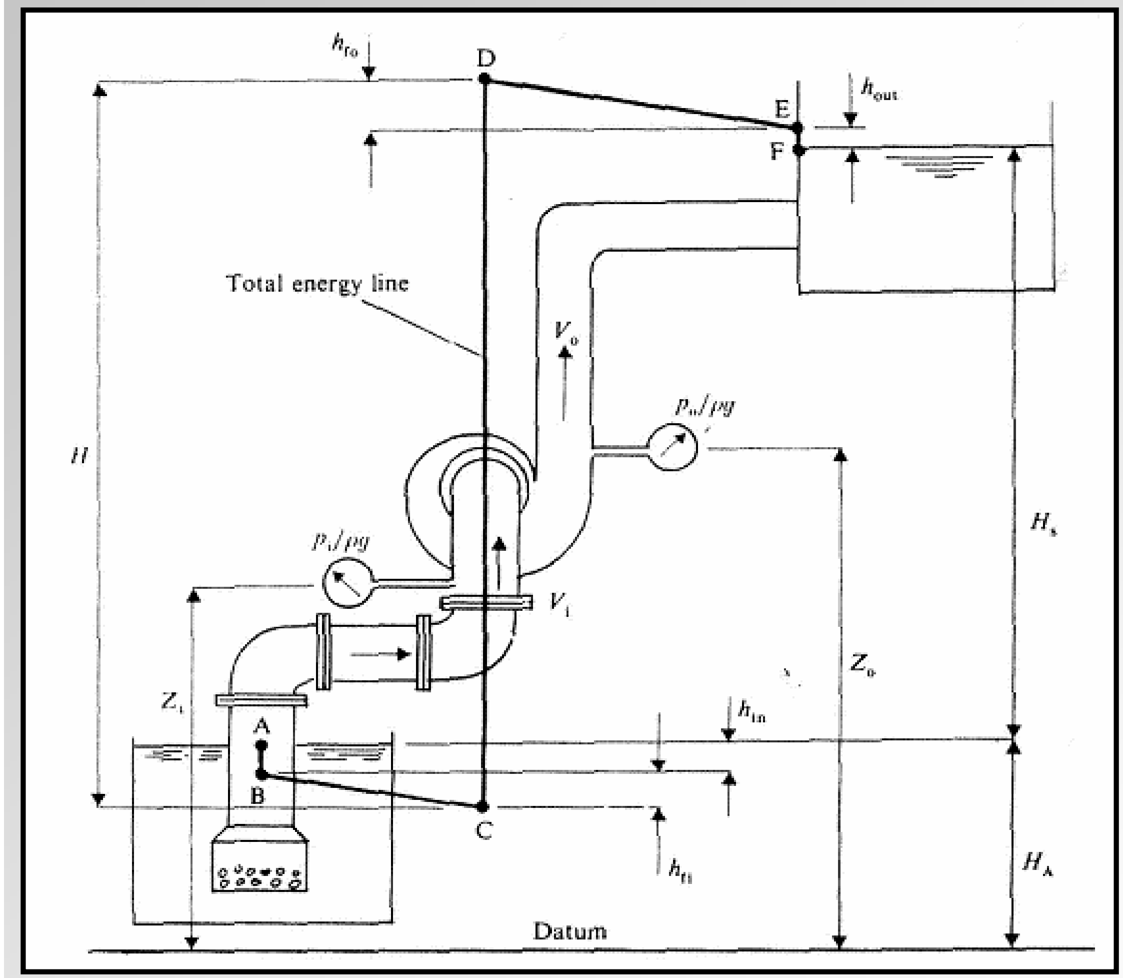
Outlet head :

$$\frac{p_o}{\rho g}+\frac{V_o^2}{2g}+Z_o$$

Total head developed by the pump:

$$H=\frac{p_o-p_i}{\rho g}+\frac{V_o^2-V_i^2}{2g}+\left(Z_o-Z_i\right)$$

$$H=H_s+\sum losses$$
$$=H_s+h_{fi}+h_{fo}+h_{in}+h_{out}$$



H = manometric head

h_{fi} = friction loss at inlet

h_{fi} = friction loss at outlet

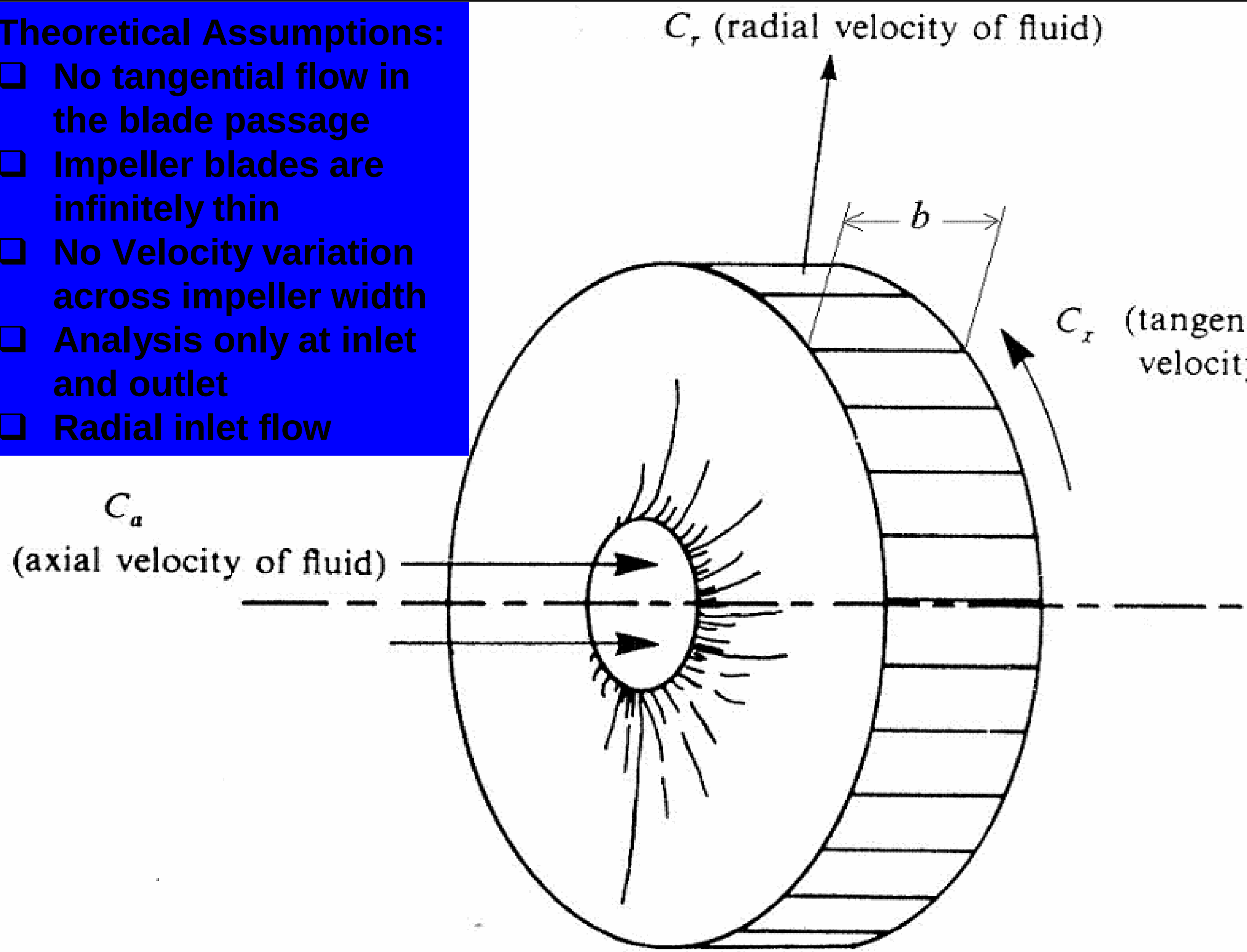
h_{in}= inlet loss

h_{out} = outlet loss

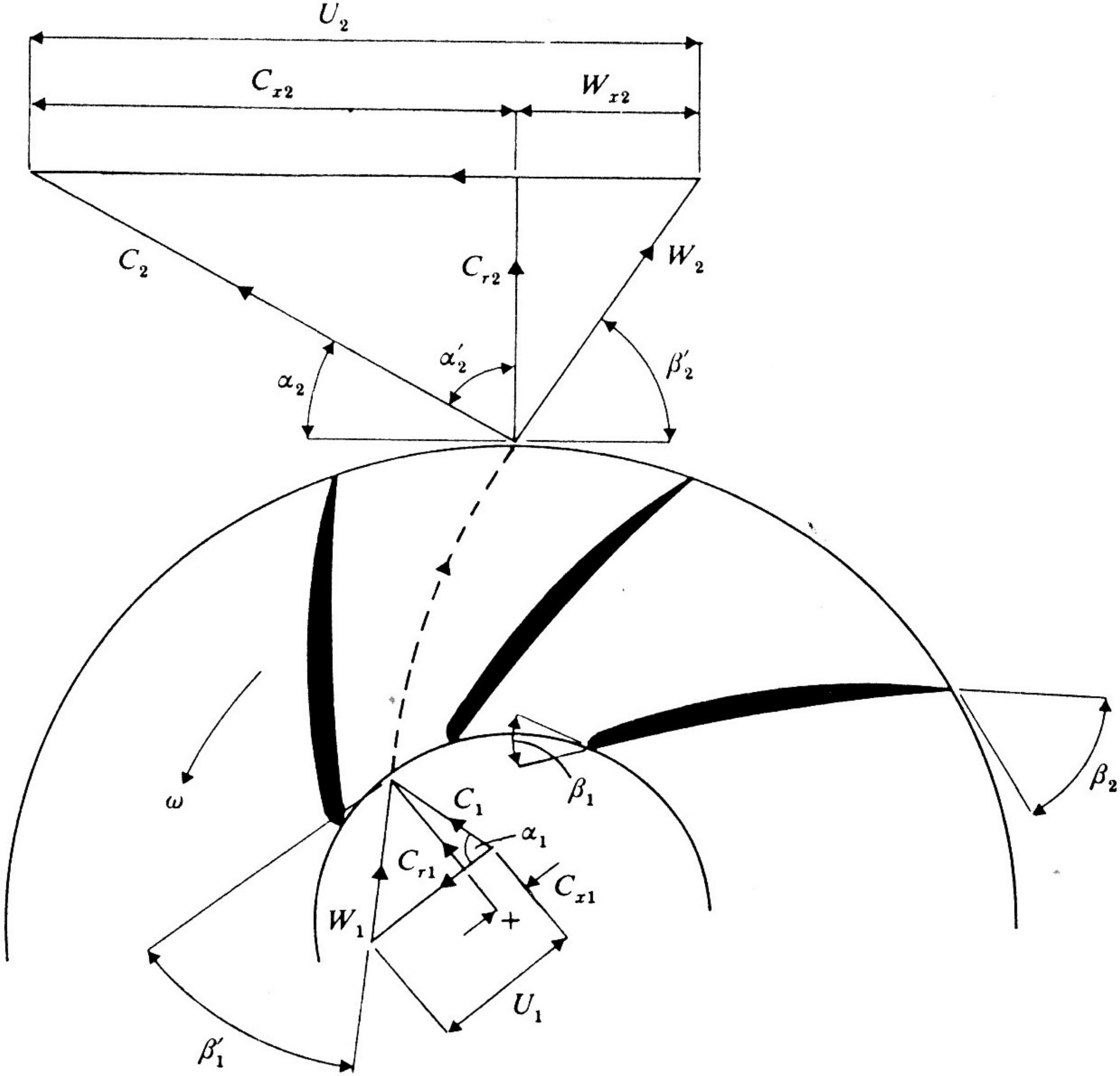
Impeller

Theoretical Assumptions:

- **No tangential flow in the blade passage**
- **Impeller blades are infinitely thin**
- **No Velocity variation across impeller width**
- **Analysis only at inlet and outlet**
- **Radial inlet flow**



VELOCITY TRIANGLE



Head and Flow Capacity H - Q

Theoretical Head Rise / Euler Head

$$\Delta h = E = \frac{\left(U_2 C_{x2}\right) - \left(U_1 C_{x1}\right)}{g}$$

$$= \frac{1}{2g}\left[\left(C_2^2 - C_1^2\right) + \left(U_2^2 - U_1^2\right) - \left(W_2^2 - W_1^2\right)\right]$$

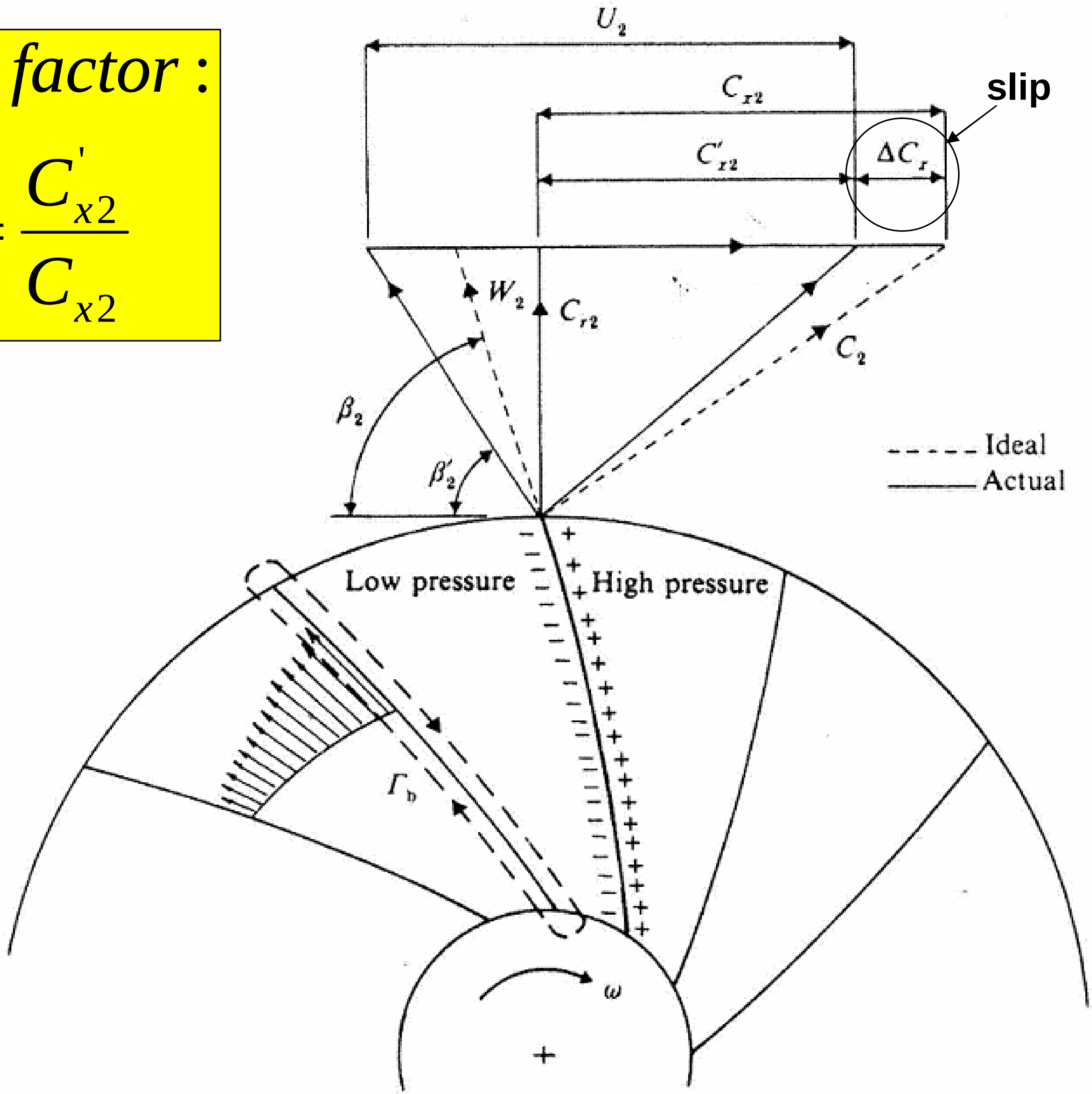
Flow Capacity/Flow Rate

$$Q = 2\pi r_1 C_{r1} b_1 = 2\pi r_2 C_{r2} b_2$$

S L I P

slip factor :

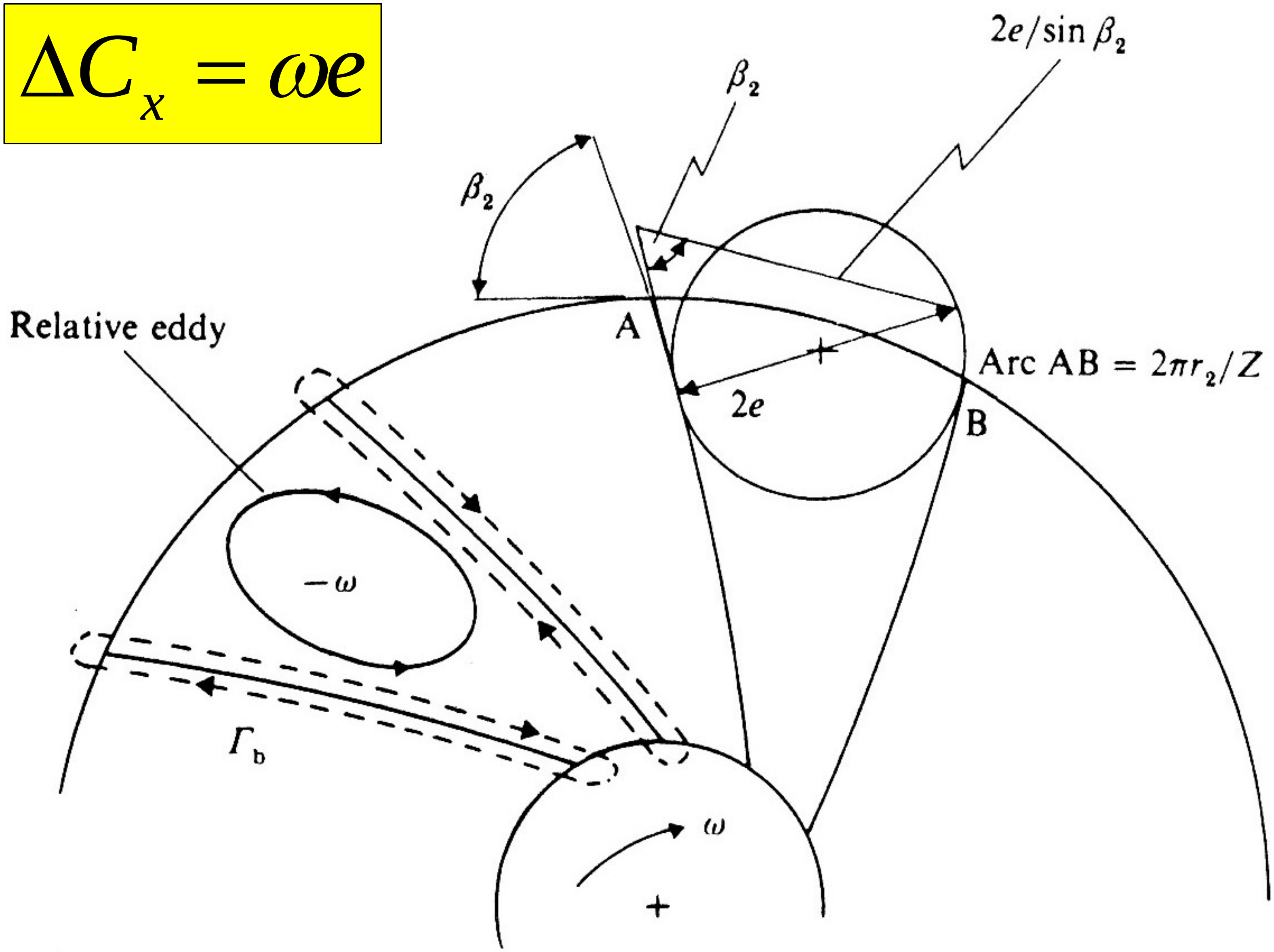
$$\sigma_s = \frac{C'_{x2}}{C_{x2}}$$



STODOLA PROPOSAL

SLIP FACTOR

$$\Delta C_x = \omega e$$



STODOLA PROPOSAL

If the number of blades is Z, and impeller circumference is $2\pi r_2$ then the distance between blades is $2\pi r_2/Z = 2e/\sin\beta_2$

Then :

$$e = (\pi r_2/Z)\sin\beta_2$$

$$\Delta C_x = (U_2/Zr_2)(\pi r_2 \sin\beta_2)$$
$$= (U_2 \pi \sin\beta_2)/Z$$

$$C_{x2} = U_2 - C_{r2}\cot\beta_2$$

$$\text{Slip factor} = (C_{x2} - \Delta C_x)/C_{x2}$$
$$= 1 - \Delta C_x/C_{x2}$$
$$= 1 - (U_2\pi\sin\beta_2)/[Z(U_2 - C_{r2}\cot\beta_2)]$$
$$= 1 - (\pi\sin\beta_2)/\{Z[1 - (C_{r2}/U_2)\cot\beta_2]\}$$

Other Slip Factor

Stodola → 20º < β_2 < 30º

$$\sigma_s = 1 - \frac{(\pi \sin \beta_2)}{\{Z[1-(C_{r2}/U_2)\cot \beta_2]\}}$$

Buseman → 30º < β_2 < 80º

$$\sigma_s = \frac{[A - B(C_{r2}/U_2)\cot \beta_2]}{[1-(C_{r2}/U_2)\cot \beta_2]}$$

A and B are function of β_2, Z and r_2/r_1

Stanitz → 80º < β_2 < 90º

$$\sigma_s = 1 - \frac{0.63\pi}{\{Z[1-(C_{r2}/U_2)\cot \beta_2]\}}$$

Example

The impeller of a centrifugal pump has backward-facing blades inclined at 30º to the tangent at impeller outlet. The blades are 20 mm in depth at the outlet, the impeller is 250 mm in diameter and it rotates at 1450 rpm. The flow rate through the pump is 0.028 m3/s and a slip factor of 0.77 may be assumed. Assume also the blades of infinitesimal thickness. Determine the theoretical and actual head developed by the impeller, and the number of impeller blades

Solution:

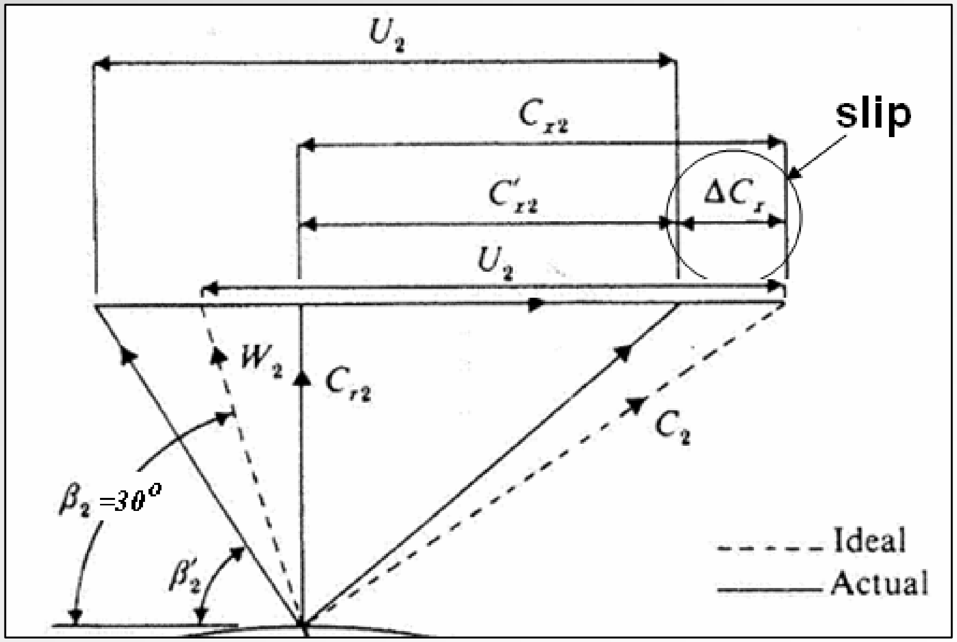
Flow Capacity/Flow Rate

$$Q = \pi D_2 C_{r2} b_2$$

$$\Rightarrow C_{r2} = Q / \pi D_2 b_2$$

$$C_{r2} = 0.028 / \pi (0.25)(0.02)$$

$$C_{r2} = 1.78 \text{ m/s}$$



For ideal outlet velocity triangle β_2= 30º

$$W_{x2} = C_{r2} / \tan 30^o = (1.78) / \tan 30^o = 3.08 \text{ m/s}$$

$$U_2 = \pi D_2 N/60 = \pi(0.25)(1450)/60 = 19\ \text{m/s}$$
$$C_{x2} = U_2 - W_{x2} = 19 - 3.08 = 15.92\ \text{m/s}$$

Theoretical (Euler) head

$$E = \frac{U_2 C_{x2} - U_1 C_{x1}}{g} \quad \Rightarrow C_{x1} = 0(\textit{flow enters radially at inlet})$$
$$E = \frac{(19)(15.92)}{9.81} = 30.83\,\text{m} \quad (\text{ans.})$$

Actual head with slip

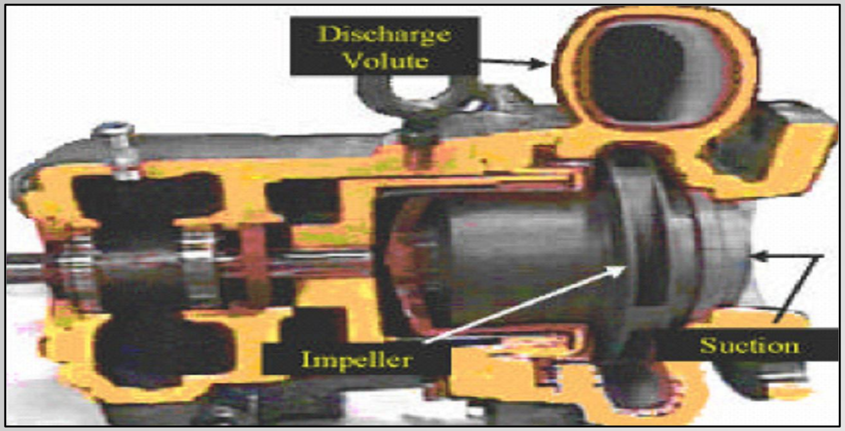
$$C'_{x2} = \sigma_s . C_{x2}$$
$$\Rightarrow E_N = \sigma_s . E = (0.77)(30.83) = 23.74\ \text{m} \quad (\text{ans.})$$

Number of blade

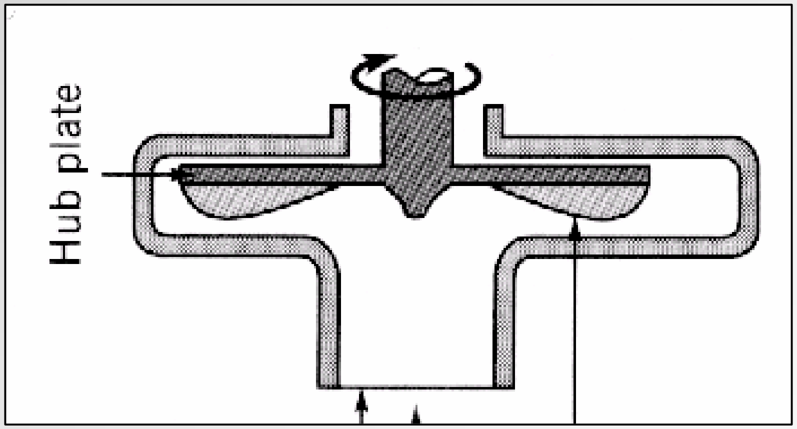
$$\sigma_s = 1 - (\pi \sin\beta_2)/\{Z[1-(C_{r2}/U_2)\cot\beta_2]\}$$
$$0.77 = 1 - (\pi \sin 30^o)/\{Z[1-(1.78/19)\cot 30^o]\}$$
$$\Rightarrow Z = 8.15 \approx 8 \quad (\text{ans.})$$

Pump Losses

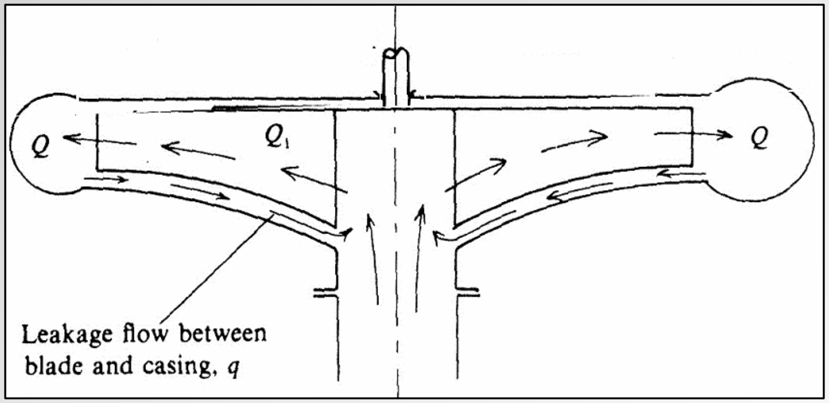
1. Mechanical friction power loss, P_m



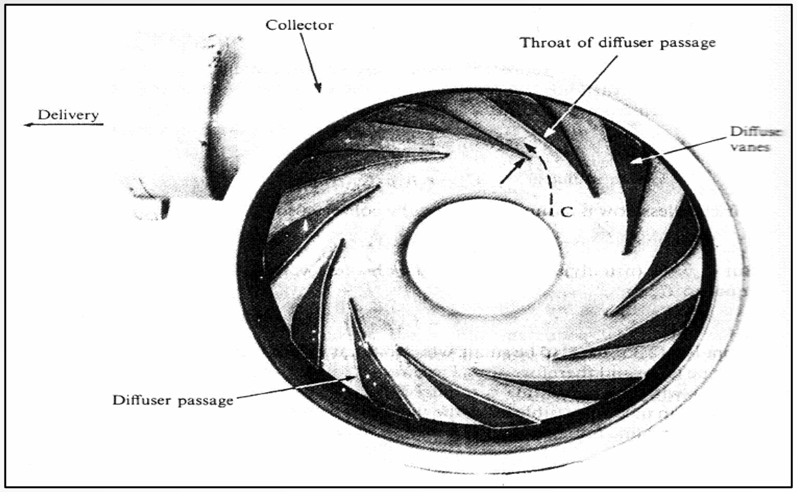
2. Impeller (Disc) friction power loss, P_i



3. Leakage and recirculation power loss, P_l

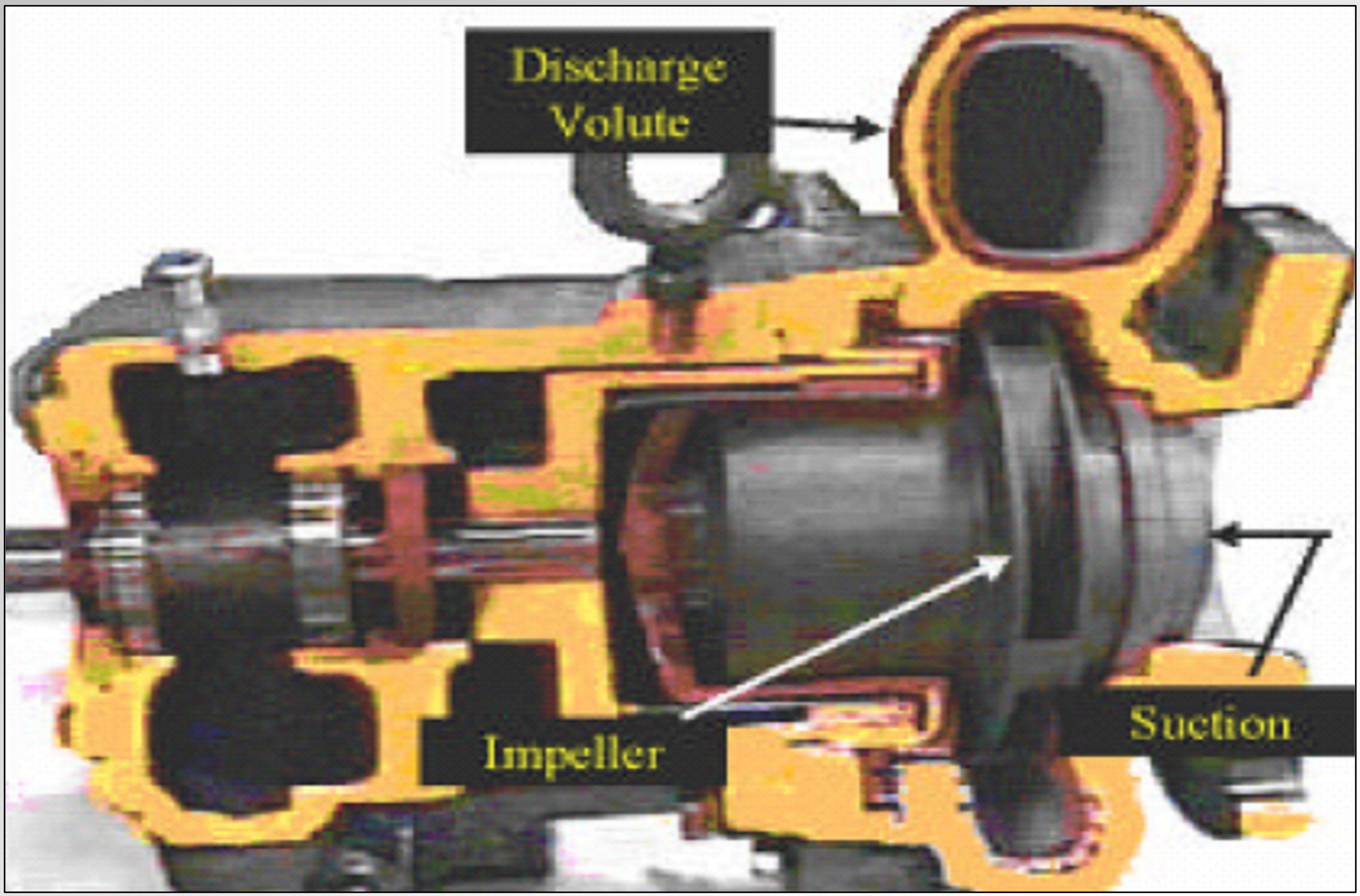


4. Casing power loss, P_c



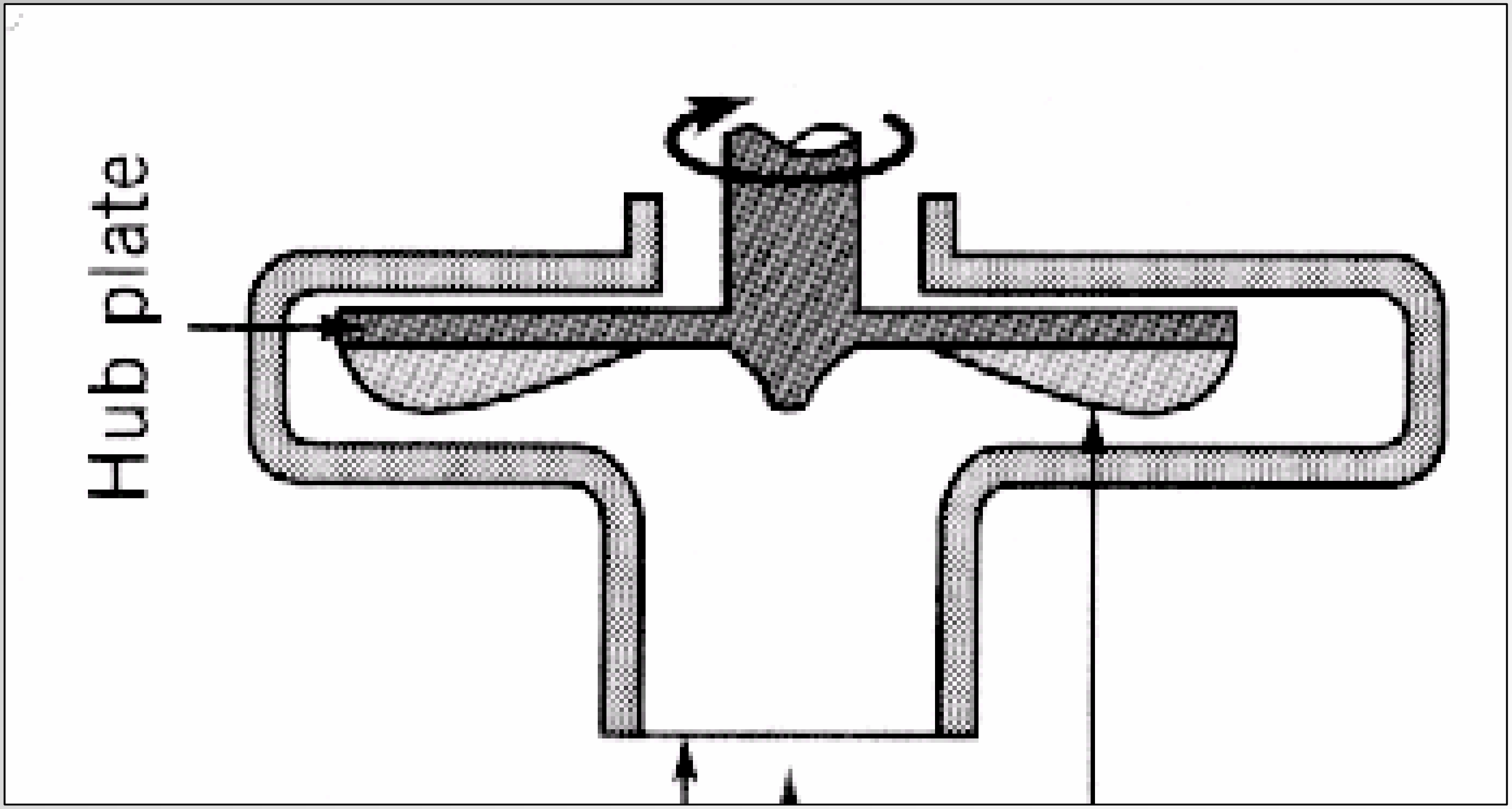
Pump Losses

1. Mechanical friction power loss, P_m



Pump Losses

2. Impeller (Disc) friction power loss, P_i

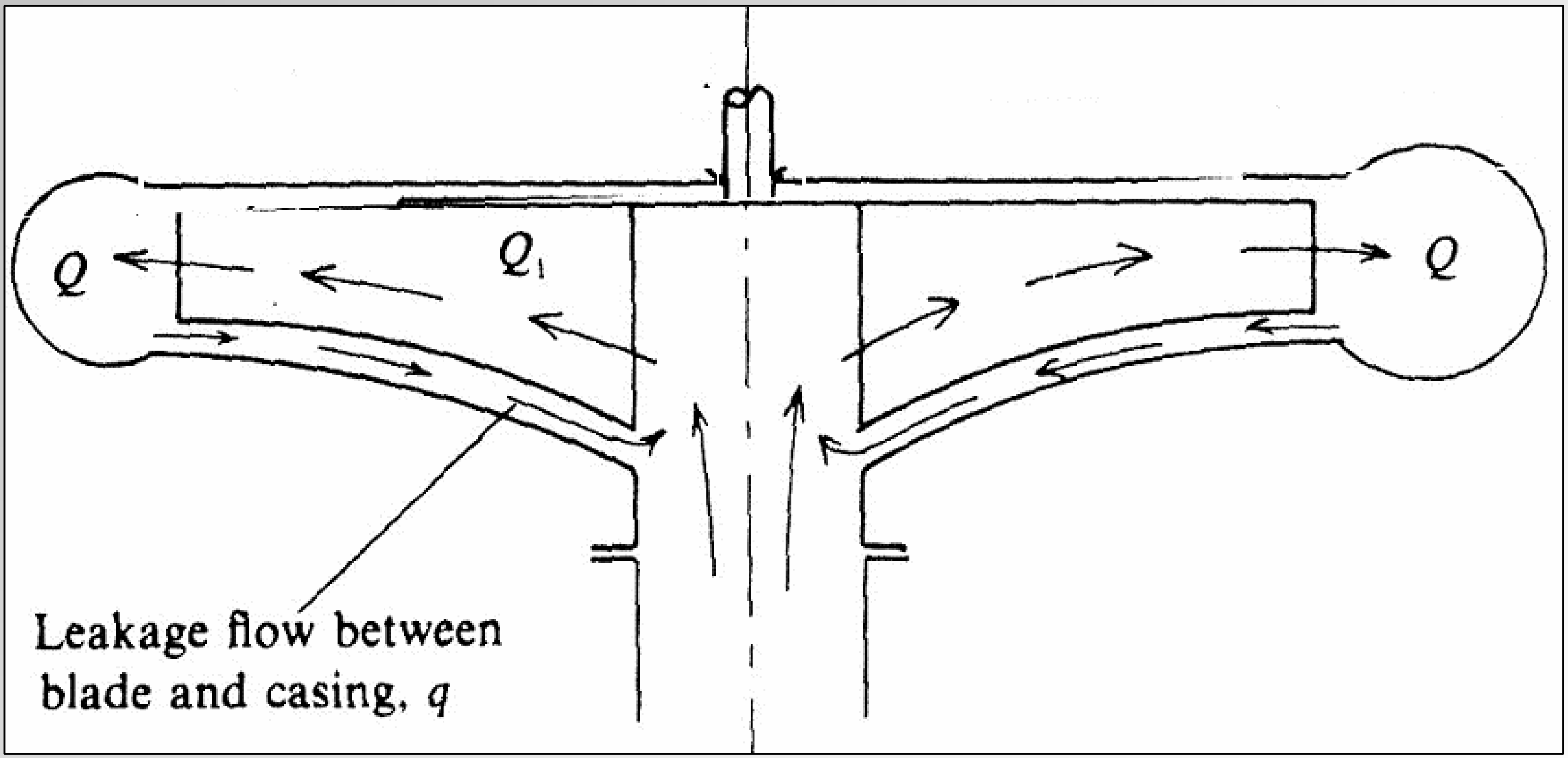


Head loss : h_i
Flow rate : Q_i

$$P_i = \rho\, g\, Q_i\, h_i$$

Pump Losses

3. Leakage and recirculation power loss, P_l

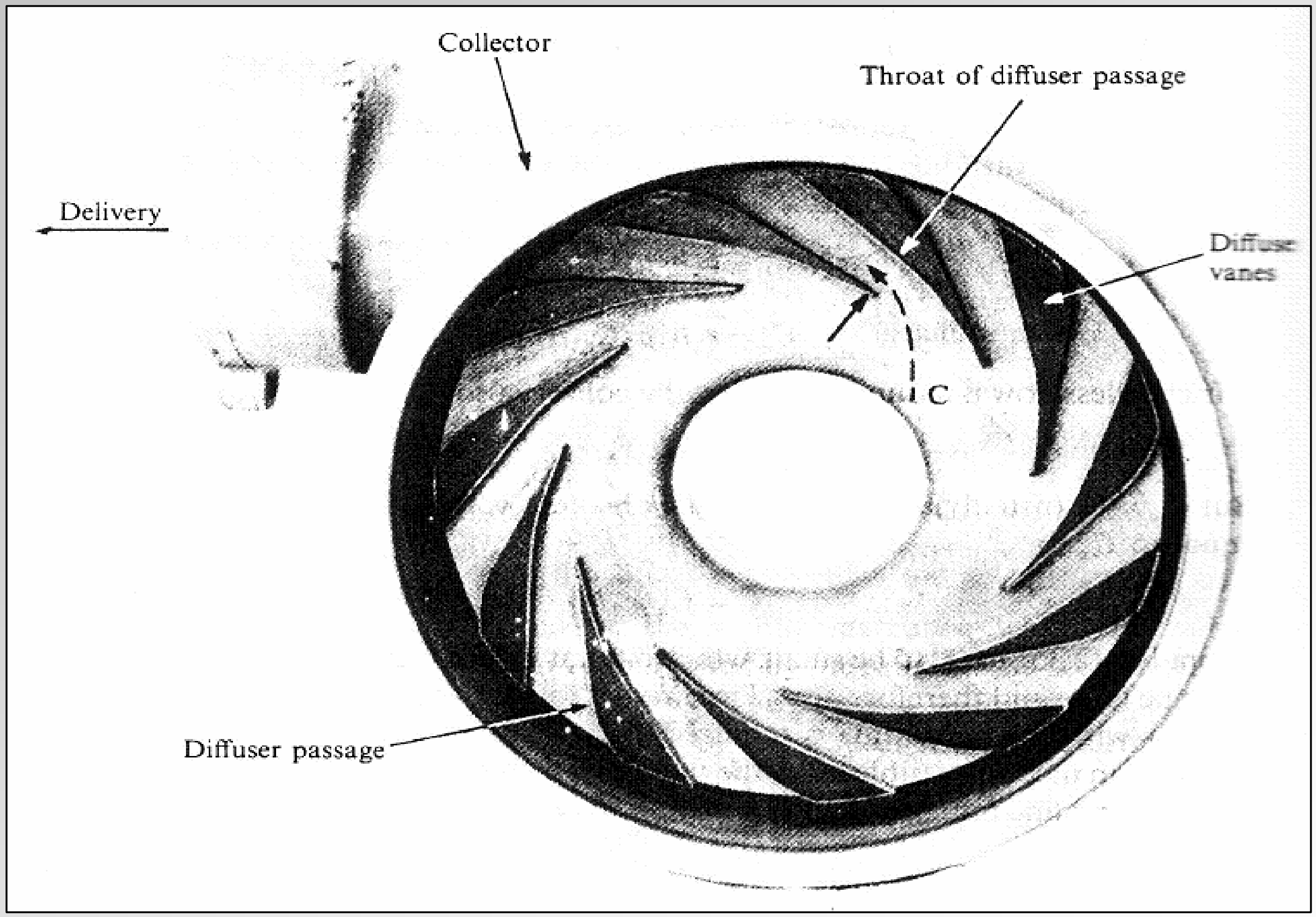


Head across impeller : H_i
Leakage flow rate : $q = Q_i - Q$

$$P_l = \rho\, g\, q_i\, H_i$$

Pump Losses

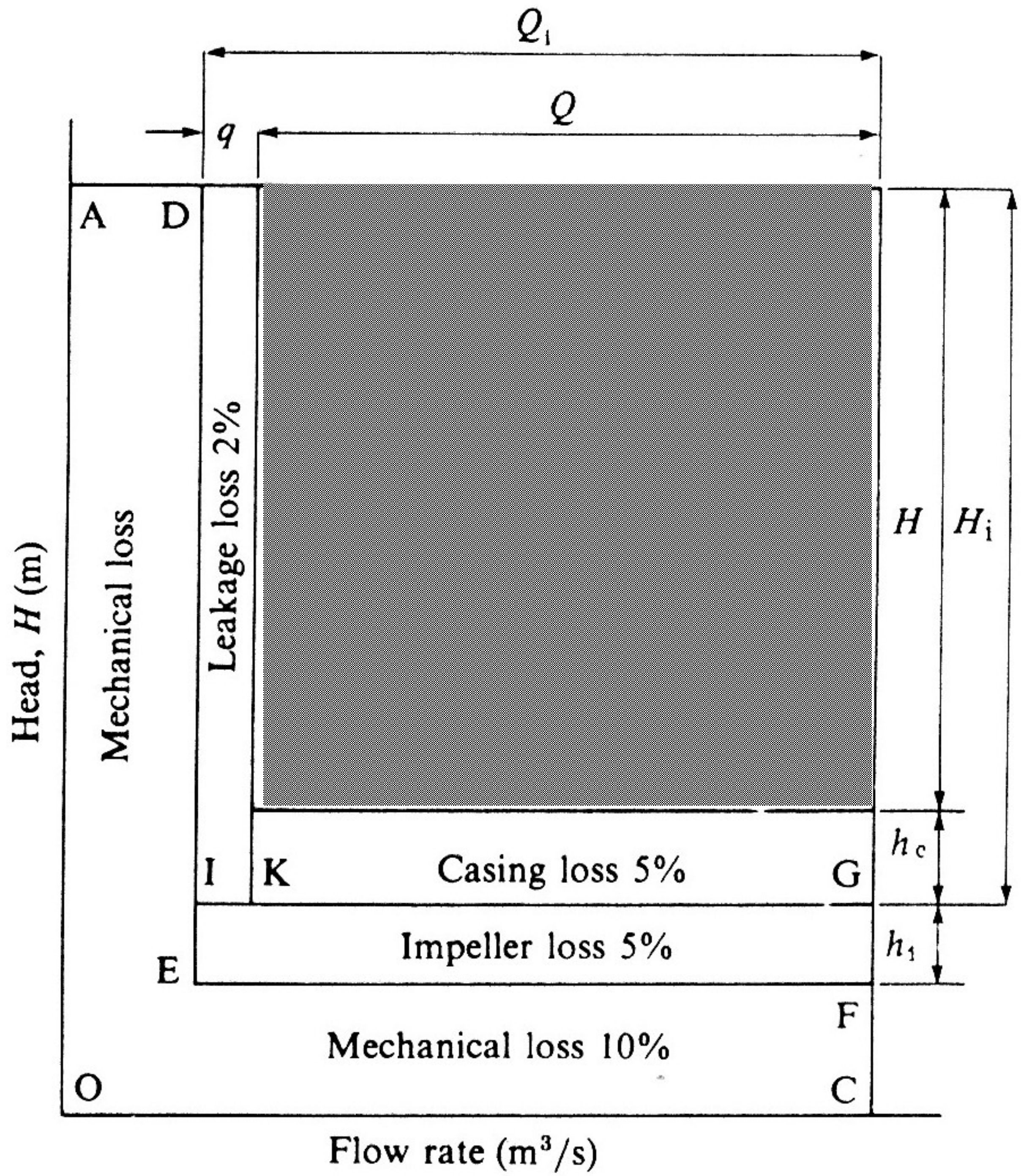
4. Casing power loss, P_c



Head loss : h_c
Flow rate : Q

$$P_c = \rho g Q h_c$$

Pump Losses H-Q Diagram



Efficiency

$$\text{Overall Efficiency} = \frac{\text{Fluid power developed by pump}}{\text{shaft power input}} = \eta_o = \frac{\rho g Q H}{P_s}$$

$$\text{Casing Efficiency} = \frac{\text{Fluid power at casing outlet}}{\text{Fluid power at casing inlet}}$$

$$= \frac{\text{Fluid power at casing outlet}}{\text{Fluid power developed by impeller - Leakage loss}}$$

$$= \eta_c = \frac{\rho g Q H}{\rho g Q H_i} = \frac{H}{H_i}$$

$$\text{Volumetric Efficiency} = \frac{\text{Flow rate through pump}}{\text{Flow rate through impeller}} = \eta_v = \frac{Q}{Q+q} = \frac{Q}{Q_i}$$

Efficiency

$$\text{Impeller Efficiency} = \frac{\text{Fluid power at impeller exit}}{\text{Fluid power supplied to impeller}}$$

$$= \frac{\text{Fluid power at impeller exit}}{\text{Fluid power developed by impeller} + \text{impeller loss}}$$

$$= \eta_i = \frac{\rho g Q_i H_i}{\rho g Q_i \left(H_i + h_i\right)} = \frac{H_i}{\left(H_i + h_i\right)}$$

$$\text{Mechanical Efficiency} = \frac{\text{Fluid power supplied to impeller}}{\text{Power input to the shaft}}$$

$$= \eta_m = \frac{\rho g Q_i \left(h_i + H_i\right)}{P_s}$$

$$\text{Hydraulic Efficiency} = \frac{\text{Actual head developed by pump}}{\text{Theoretical head developed by impeller}}$$

$$= \eta_H = \frac{H}{\left(H_i + h_i\right)} = \frac{H}{E}$$

Efficiency Relation

$$\eta_H = \eta_c \eta_i$$

$$\eta_o = \eta_c \eta_i \eta_v \eta_m = \eta_H \eta_v \eta_m$$

Pump Shaft Power, P_s

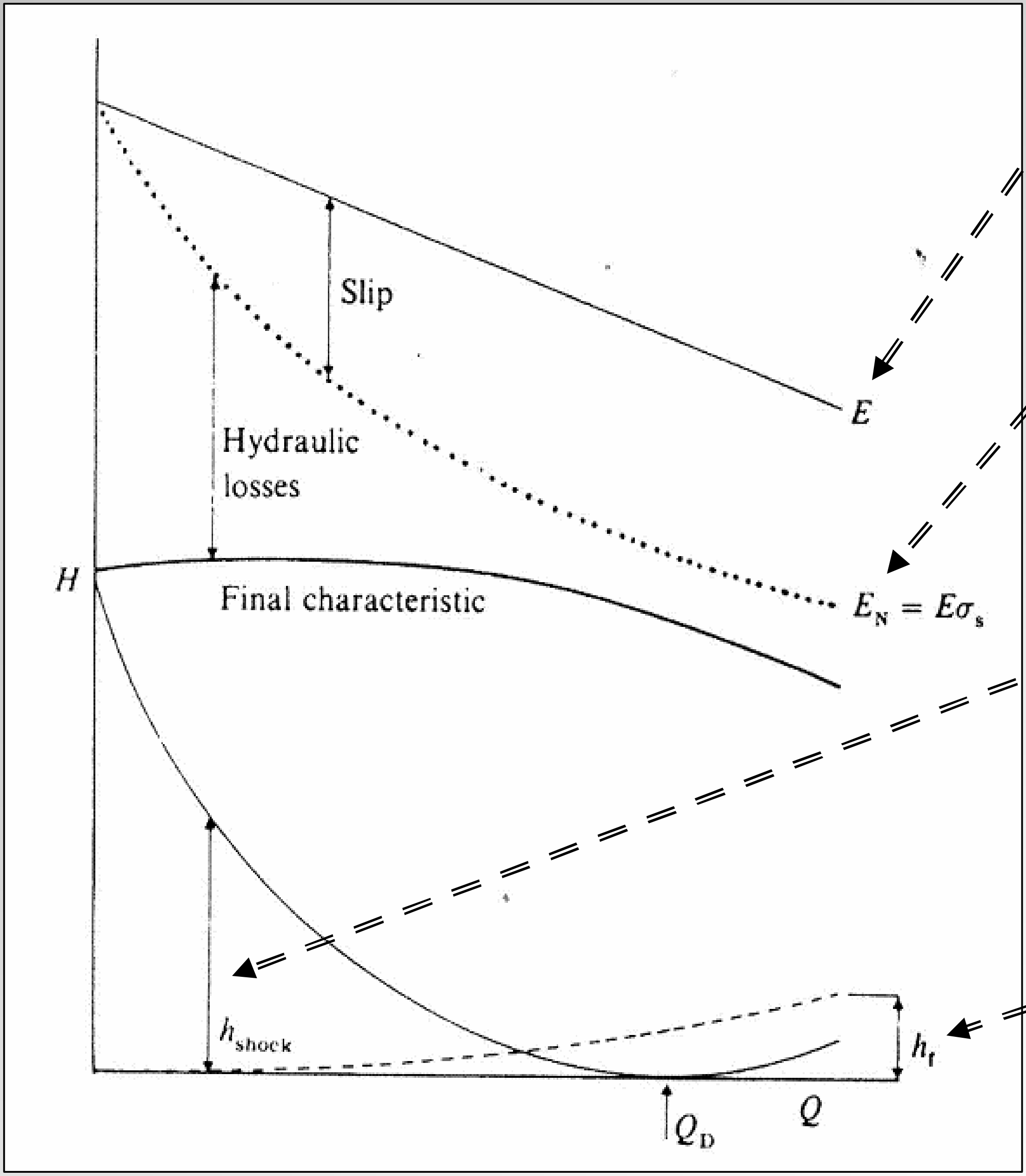
$$P_s = P_m + \rho g\left(h_i Q_i + h_c Q + H_i q + QH\right)$$

Driven Motor Shaft Power, P_M

Transmission Efficiency, η_T

$$P_s = \eta_T P_M \quad \Rightarrow \quad P_M = \frac{P_S}{\eta_T}$$

Pump's Characteristic Curve



$$E = U_2\left[U_2 - (Q/A)\cot\beta_2\right]/g$$

$$E = K_1 - K_2 Q$$

$$E_N = (K_1 - K_2 Q)\sigma_s$$

$$h_{shock} = K_3 (Q - Q_D)^2$$

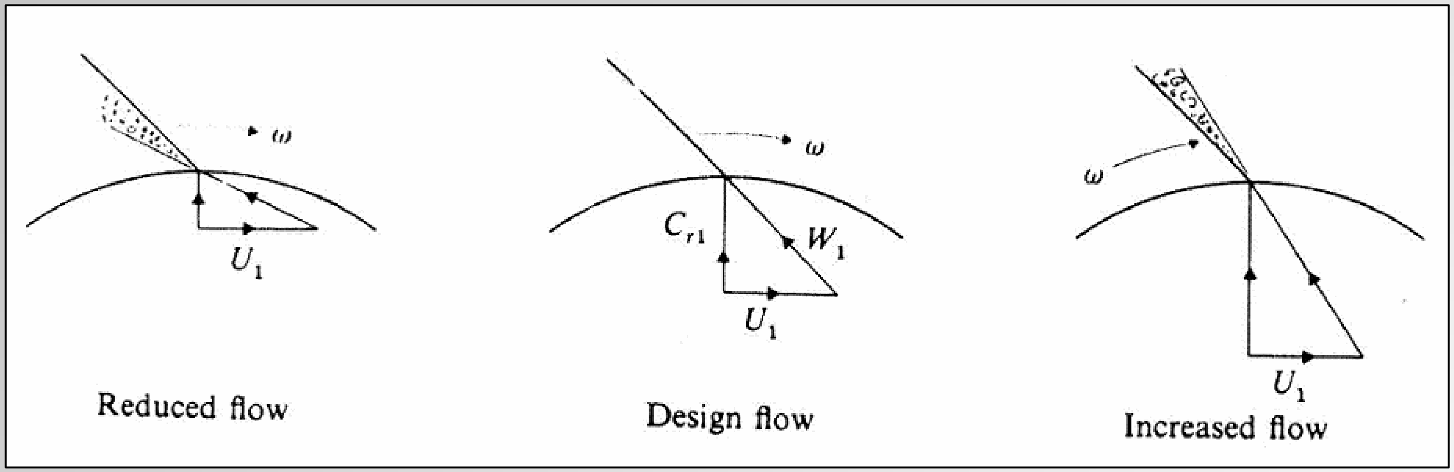
where:

Q_D is design flowrate

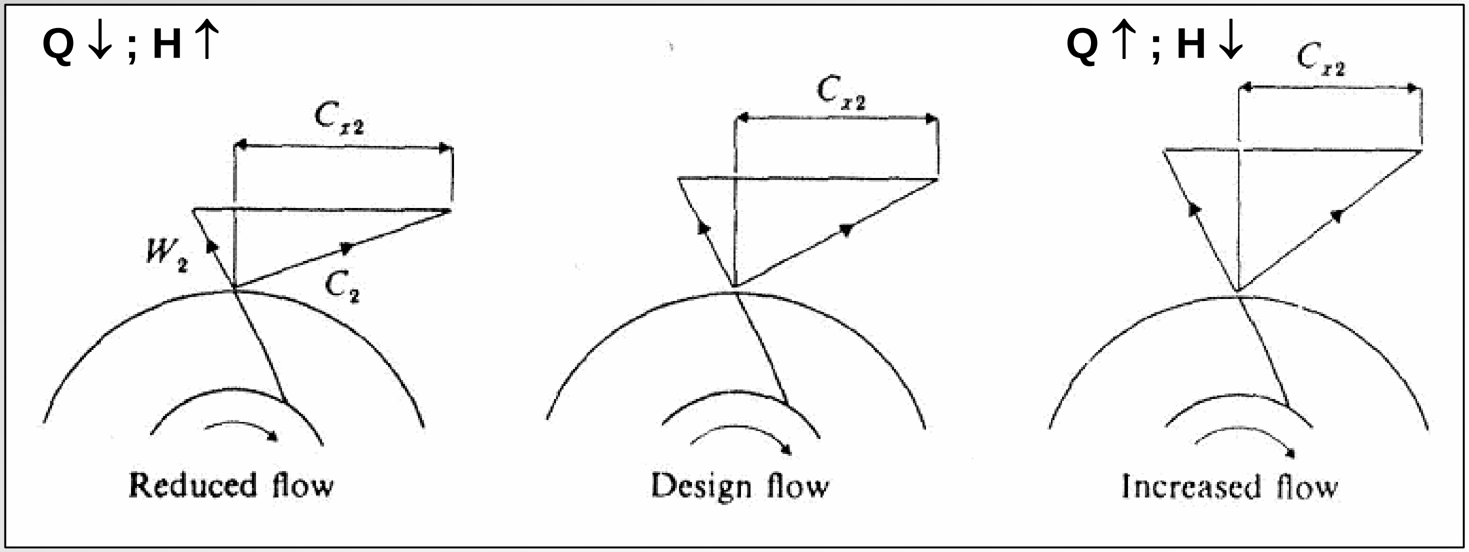
$$h_f = K_4 Q^2$$

Effect of Flow Rate Variation

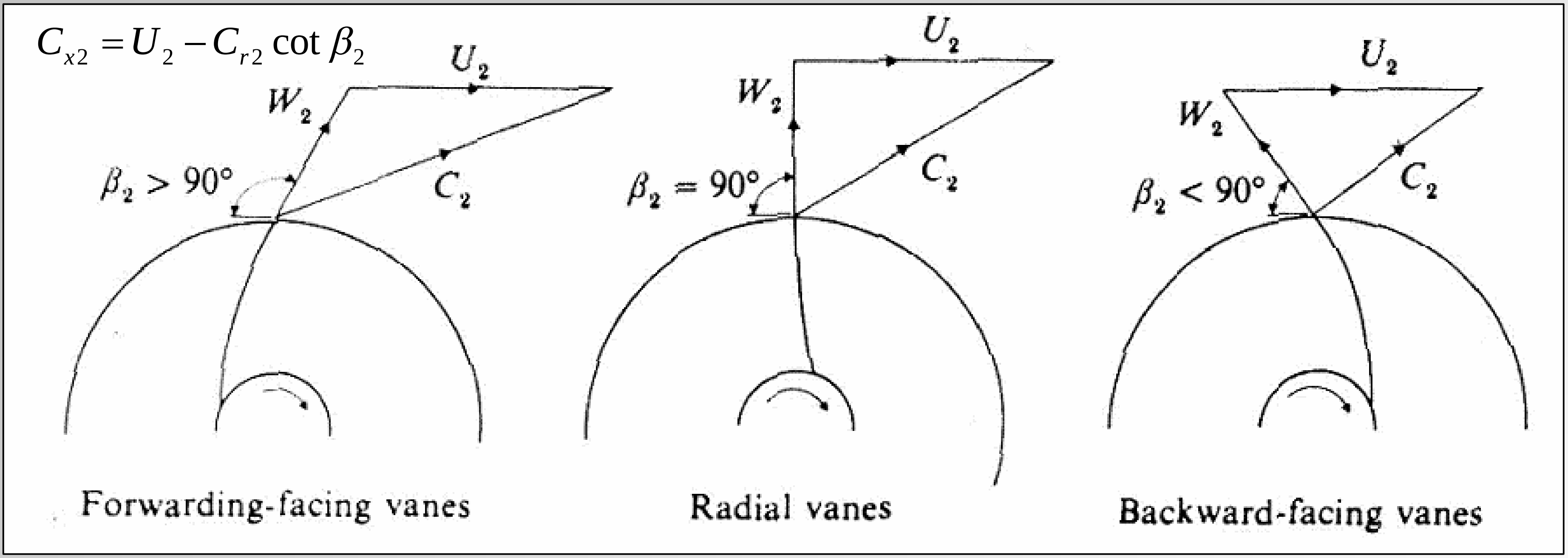
Inlet velocity



Outlet velocity



Effect of Blade Outlet Angle



$for\ \beta_2 < 90^o$

$$E = U_2\left(U_2 - C_{r2} \cot \beta_2\right)/g$$

$$E = \left(U_2^2/g\right) - \left(QU_2 \cot \beta_2/gA\right) \Rightarrow H = a - bQ$$

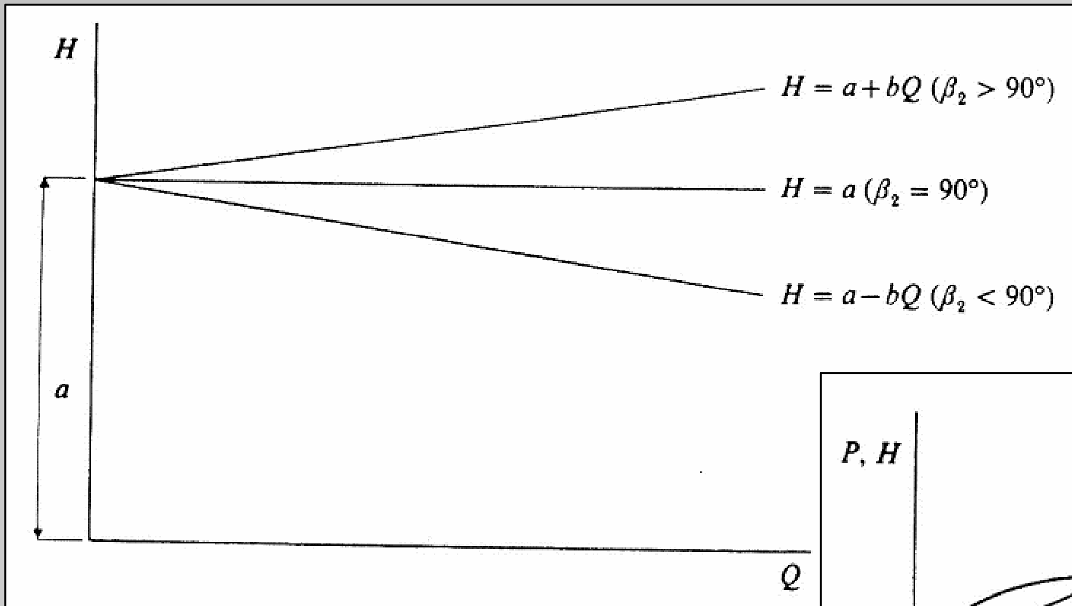
$for\ \beta_2 = 90^o$

$$H = a$$

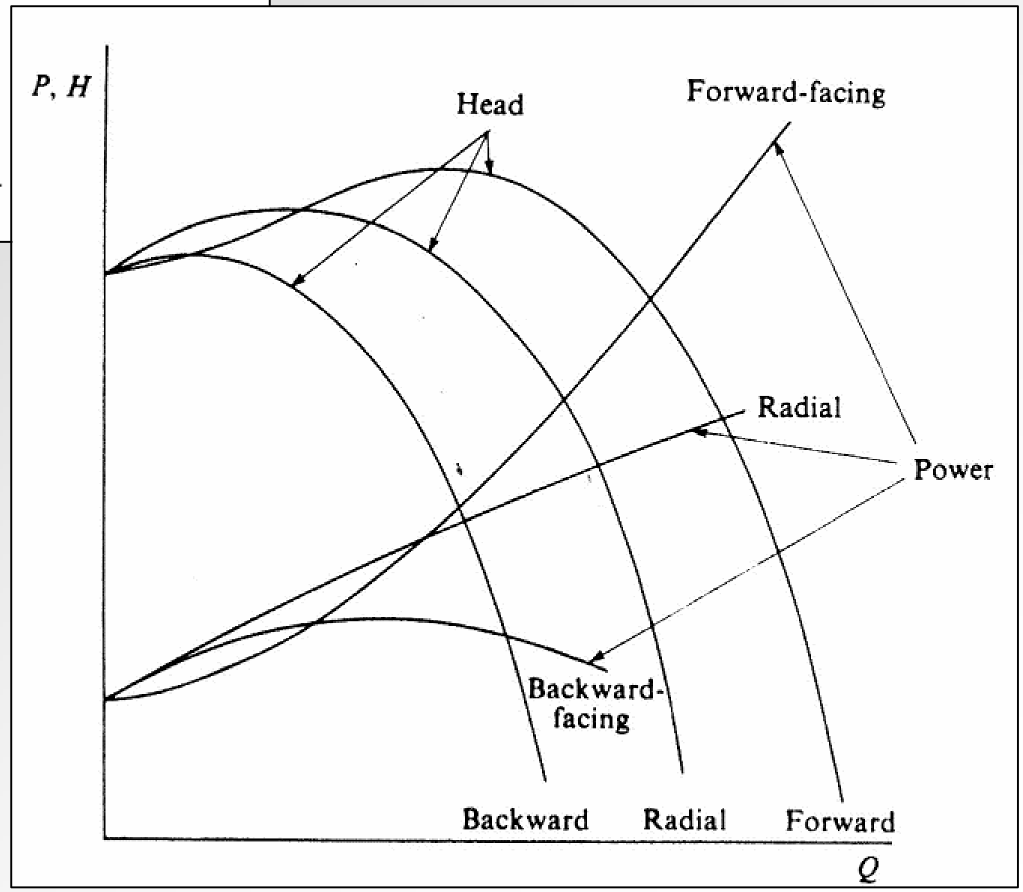
$for\ \beta_2 > 90^o$

$$H = a + bQ$$

Effect of Blade Outlet Angle



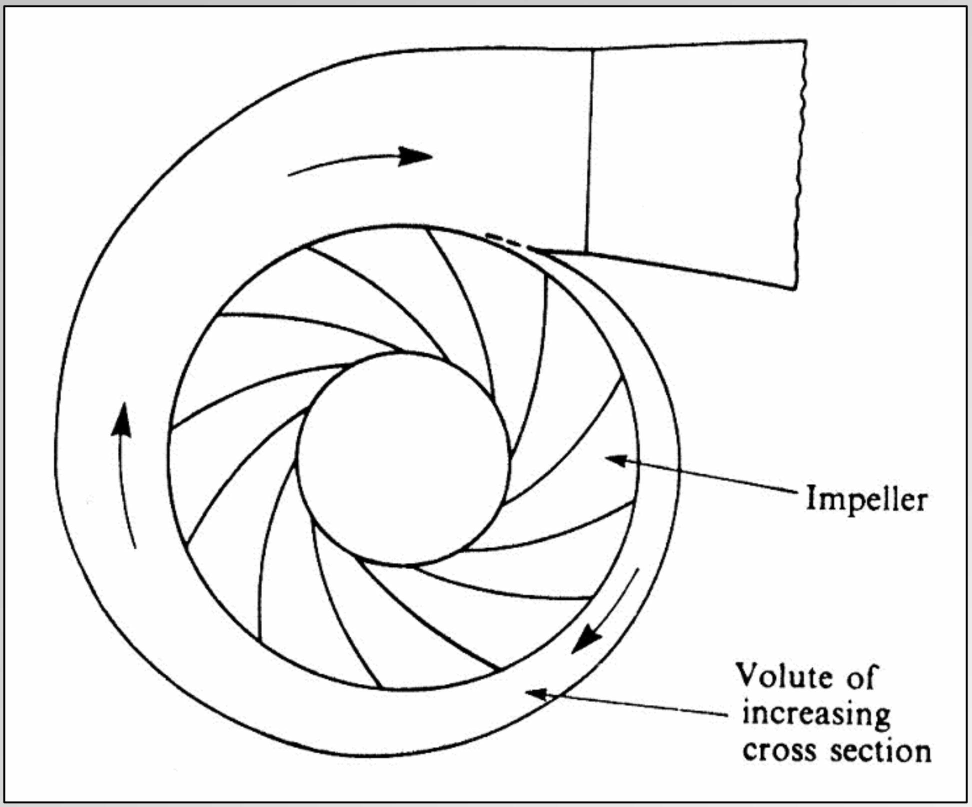
Theoretical characteristic curves

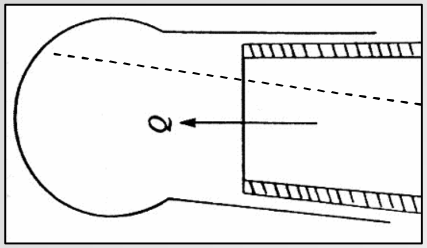


Actual characteristic curves

Flow in the Discharge Casing

Volute Casing





Circular section to reduce losses due to friction and impact

Function:

1. **Collector**
2. **Diffuser**

Deviation in capacity from the design condition will result in a radial thrust (*P*):

$$P = 495 K H D_2 B_2$$

$$where: K = 0.36\left[1-\left(Q/Q_D\right)^2\right]$$

Function:

P **= radial force (N)**

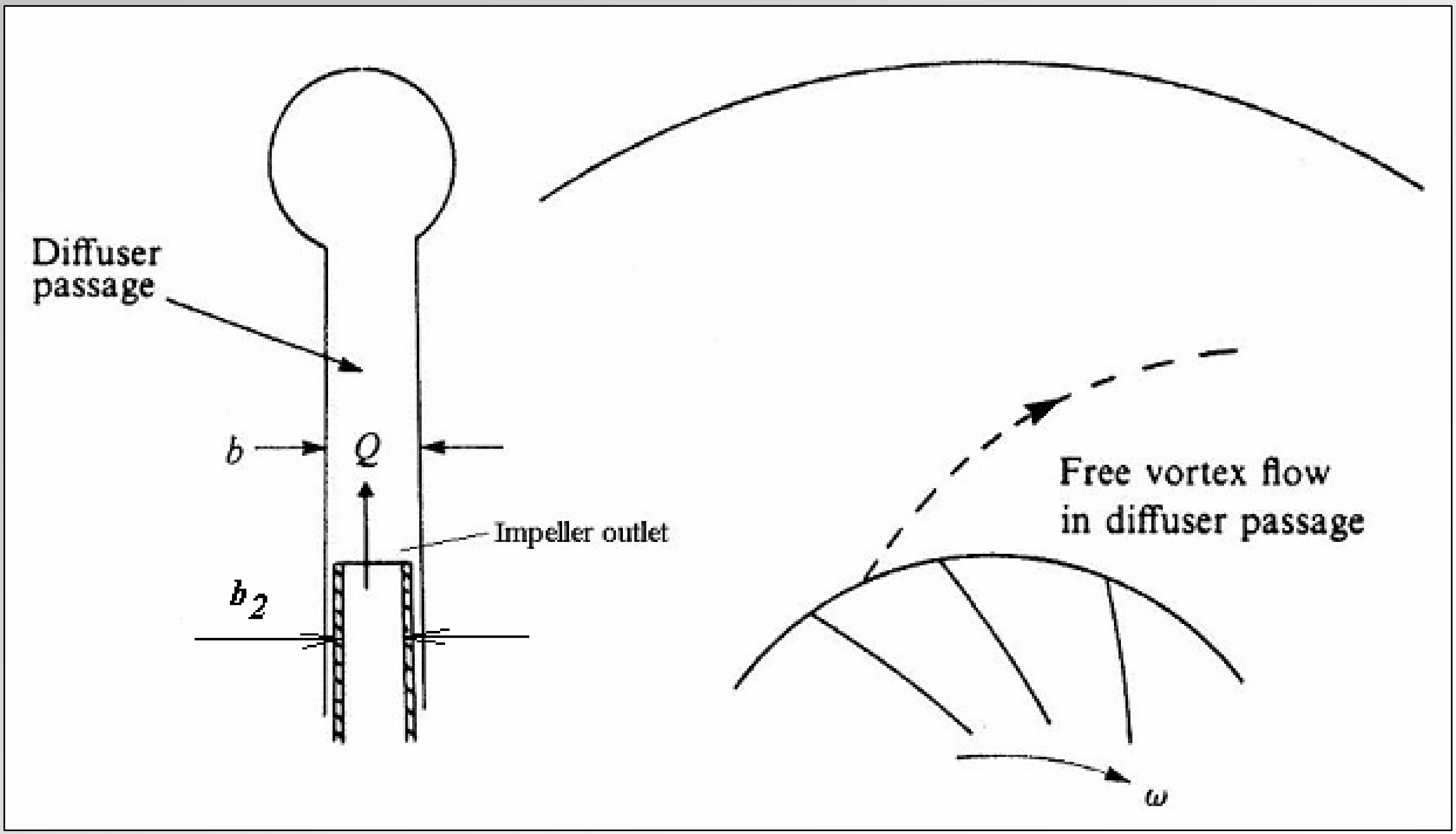
H **= Head (m)**

D_2 = peripheral diameter (m)

B_2 = impeller width (m)

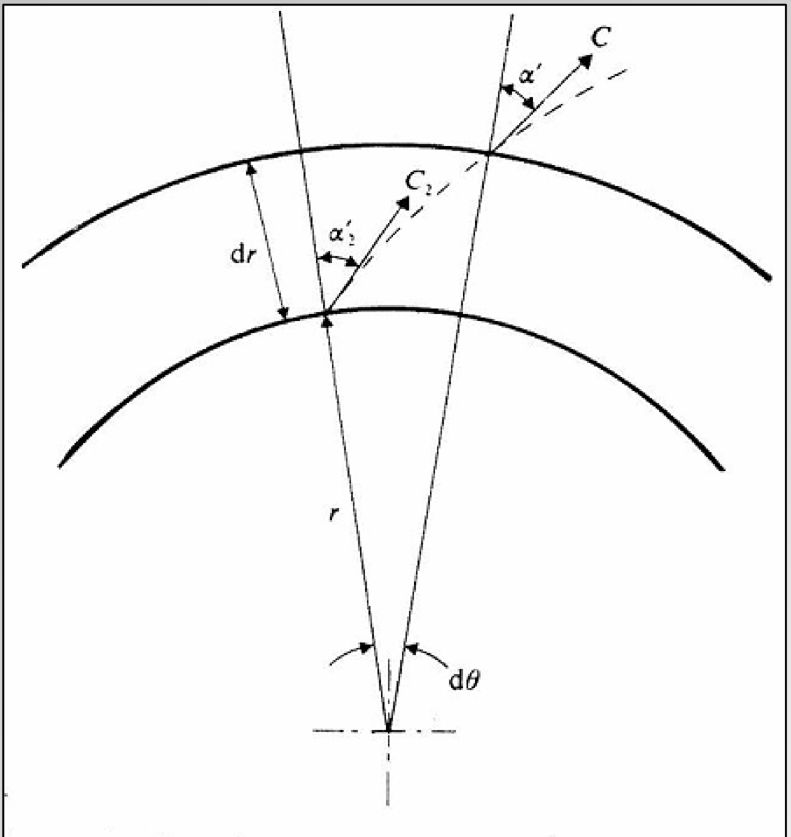
Flow in the Discharge Casing

Vaneless Diffuser



Flow in the Discharge Casing

Vaneless Diffuser



Logarithmic spiral flow in vaneless space

Continuity:

$$m = \rho A C_r = 2\pi r b \rho C_r = 2\pi r_2 b_2 \rho_2 C_{r2}$$

$$C_r = r_2 b_2 \rho_2 C_{r2} / rb\rho$$

Conservation of angular momentum:

$$C_x = C_{x2} r_2 / r \Rightarrow \; usually \;\; C_x >> C_r$$

$$C \approx C_x$$

Then:

$$C = C_{x2} r_2 / r$$

Radius, r ↑

Outlet kinetic energi ↓

$$\tan\alpha'_2 = C_{x2}/C_{r2} = cons = \tan\alpha'$$

$$\tan\alpha' = \frac{rd\theta}{dr}$$

Then:

$$\theta - \theta_2 = \tan\alpha' \ln\left(r/r_2\right)$$

$$\theta = angle\; of \;\; diffuser$$

Flow in the Discharge Casing

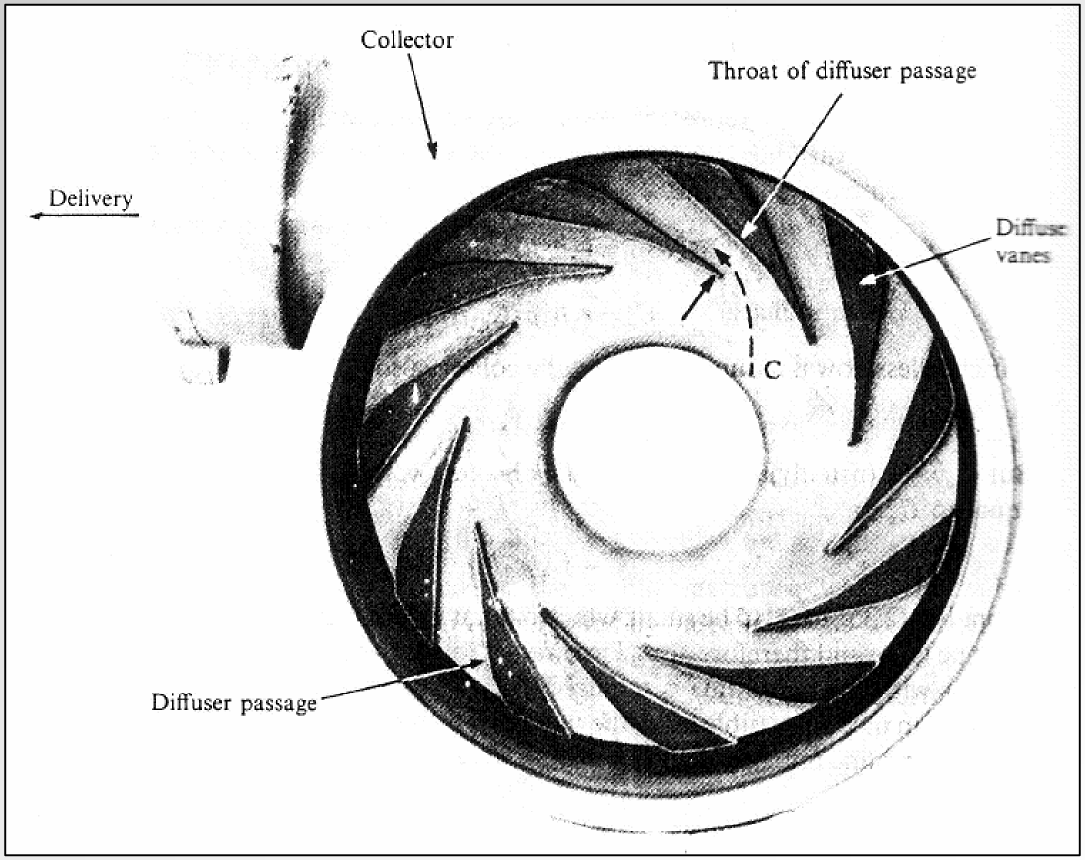
Vaned Diffuser

Able to diffuse the outlet kinetic energy at:

- ❑ **Higher rate**
- ❑ **Shorter length**
- ❑ **Higher efficiency**

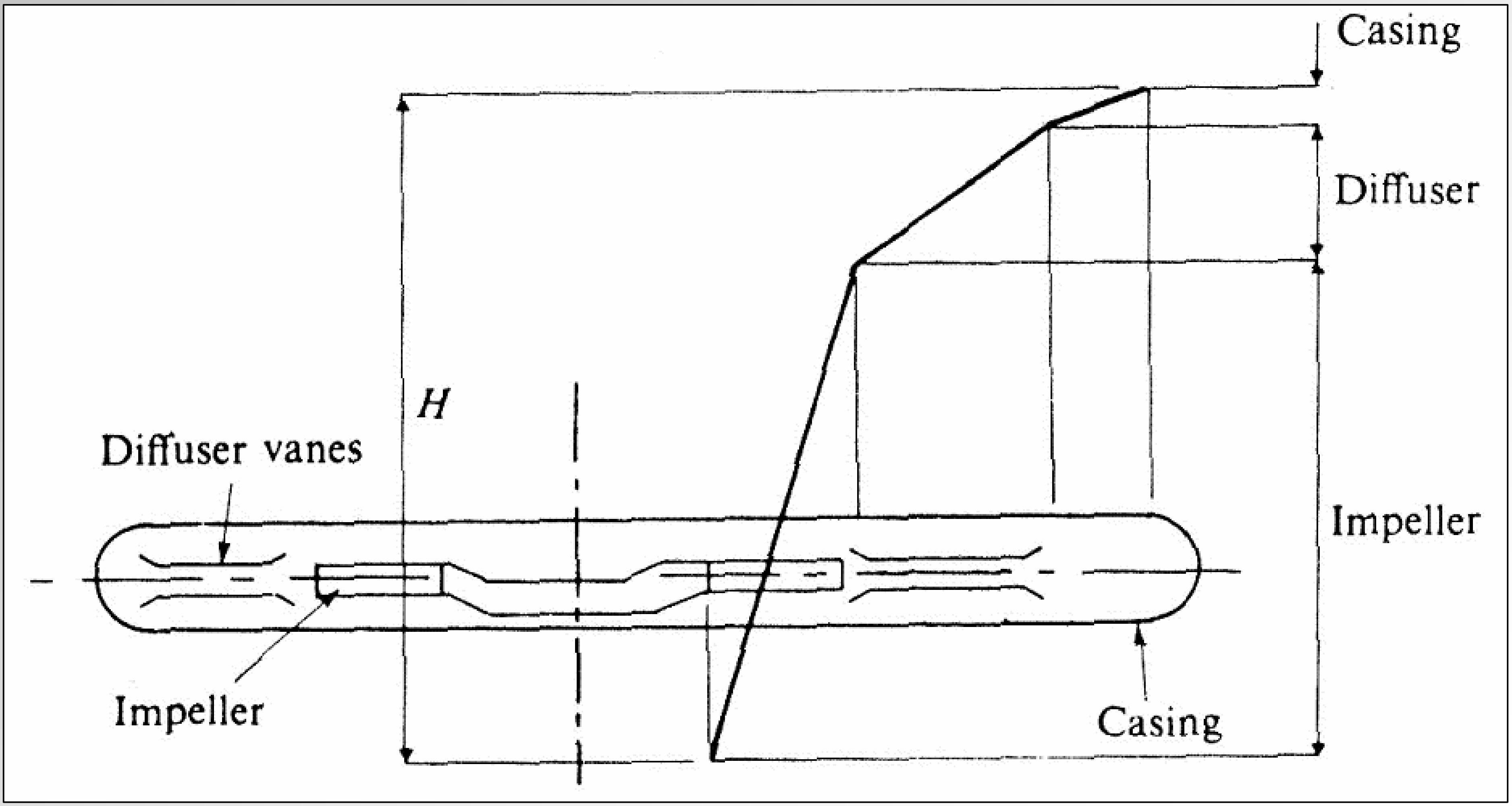
Number of vanes on the diffuser ring:

- ❑ **Greater number → better diffusion but more friction loss**
- ❑ **Square cross section of diffuser channel → max r_h**
- ❑ **Number of diffuser vanes have no common factor with the number of impeller**



Flow in the Discharge Casing

Contribution of each section of the pump to total head



Cavitation in Pumps

Vapour bubbles formation of the liquid as the local absolute static pressure of a liquid falls below the vapour pressure

- **occurs mainly at the suction side (at the eye of impeller as the velocity increases and pressure decreases)**
- **Local pitting of impeller → cavitation erosion**
- **Noise**
- **Decrease pump efficiency**

Net Positive Suction Head (NPSH)

The difference of total suction head in the impeller inlet side (impeller eye) above the vapour pressure

$$NPSH = \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} - \frac{p_{vap}}{\rho g} \right) \quad (all\ \ pressures\ are\ absolute)$$

- **A measure of the energy available on the suction side of the pump**
- **A measure to indicate the occurrence of cavitation**

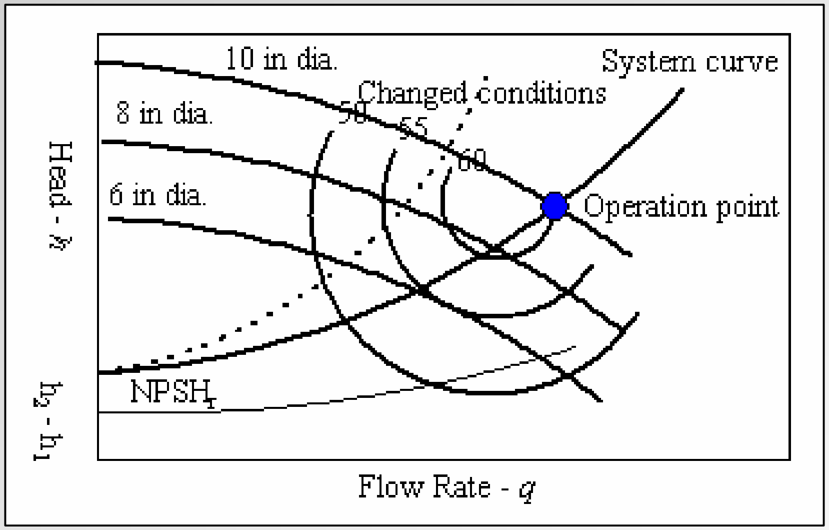
Cavitation Parameter (Toma Cavitation Number)

$$\sigma = \frac{NPSH}{Head\ Developed\ by\ \ pump} = \frac{\left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} - \frac{p_{vap}}{\rho g} \right)}{H}$$

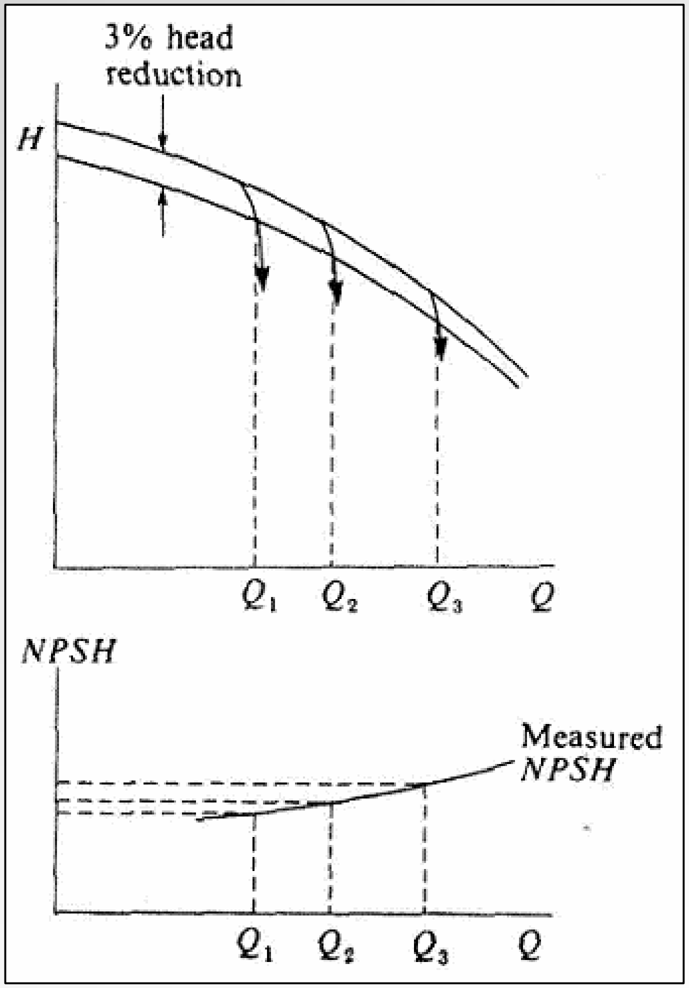
Net Positive Suction Head (NPSH)

NPSH Required ($NPSH_R$)

- Net Suction Head as required by the pump in order to prevent cavitation for safe and reliable operation of the pump.
- The required $NPSH_R$ for a particular pump is in general determined experimentally by the **pump manufacturer** (will vary depending on the size and speed of the pump) and a part of the documentation of the pump.



Example of pump documentation

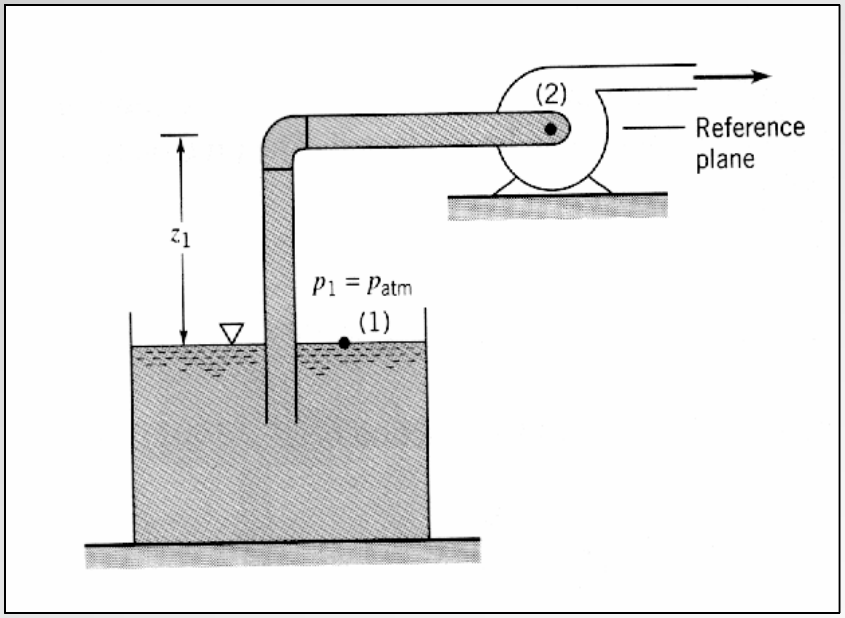


Measurement of $NPSH_R$ by 3% head reduction

Net Positive Suction Head (NPSH)

NPSH Available ($NPSH_A$)

- The Net Positive Suction Head made available the suction system for the pump.
- The $NPSH_A$ can be determined during design and construction, or determined experimentally from the actual physical system and calculated with the Energy Equation



Energy at 1 = Energy at 2 + Energy lost between 1 and 2

$$\frac{p_1}{\rho g} - z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \sum losess_{inlet} \Rightarrow \frac{p_2}{\rho g} + \frac{V_2^2}{2g} = \frac{p_1}{\rho g} - z_1 - \sum losses_{inlet}$$

At inlet $p_2 = p_i$; $V_2 = V_i$ and Σ $losses_{inlet} = h_{in} + h_{fi}$, then:

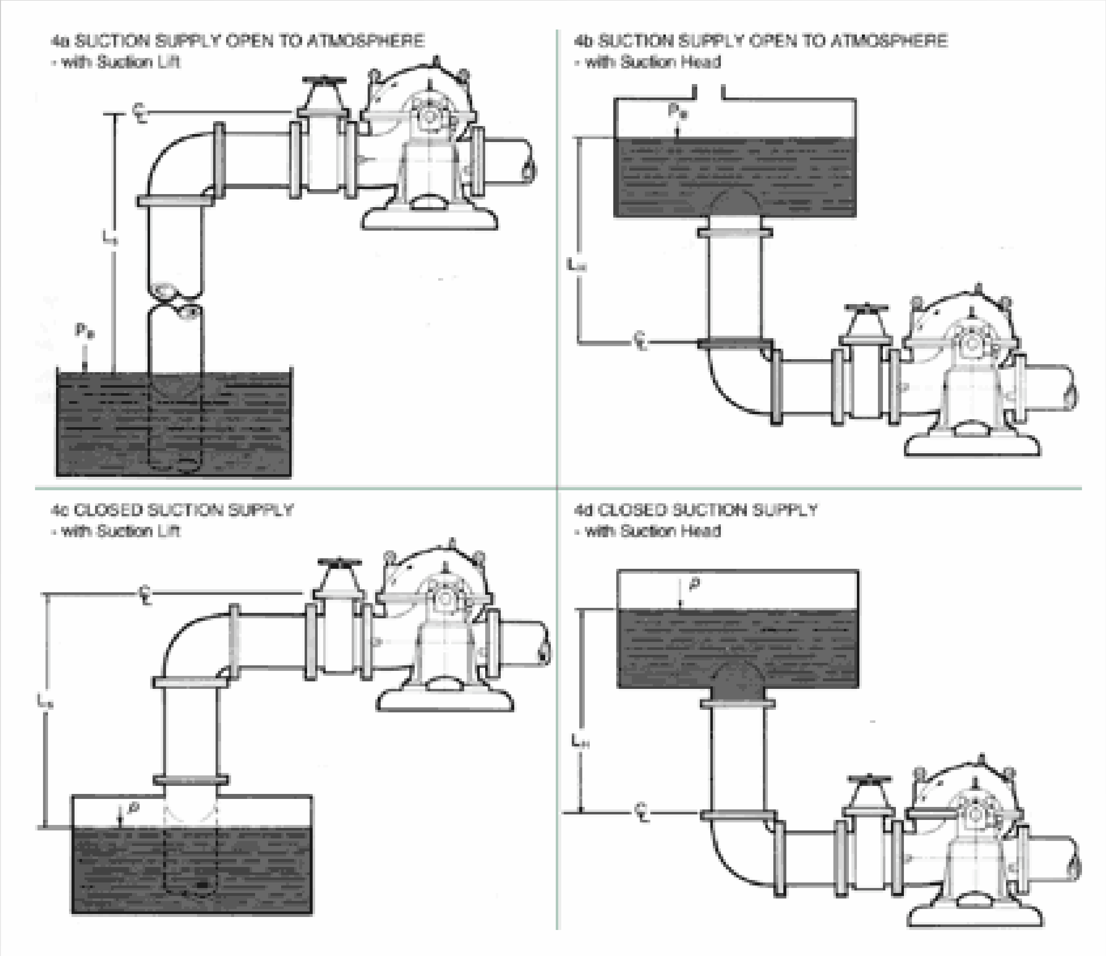
NPSH available at impeller inlet :

$$NPSH_A = \frac{p_1}{\rho g} - \frac{p_{vap}}{\rho g} - z_1 - h_i - h_{fi}$$

Cavitation ~ NPSH

To avoid cavitation in a pump operation

$NPSH_A > NPSH_R$ **or** $\sigma_A > \sigma_R$



Suction Specific Speed

A function due to cavitation that influences the efficiency

Dimensionless suction specific speed

$$N_{suc} = \frac{NQ^{1/2}}{\left[g\left(NPSH\right)\right]^{3/4}} \quad \Rightarrow \quad \eta = f\left(\phi, N_{suc}\right)$$

Cavitation parameter

$$\frac{N_s}{N_{suc}} = \frac{\left(NPSH\right)^{3/4}}{H^{3/4}} = \sigma^{3/4}$$

Similarity Laws

$$\frac{NPSH_1}{NPSH_2} = \left(N_1/N_2\right)^2\left(D_1/D_2\right)^2 = \sigma_1/\sigma_2$$

Example

When a laboratory test was carried out on a pump, it was found that, for a pump total head of 36 m at discharde of 0.05 m^3/s, cavitation began when the sum of the static pressure plus the velocity head at inlet was reduced to 3.5 m. The atmospheric pressure was 750 mmHg and the vapour pressure of water 1.8 kPa. If the pump is to operate at a location where atmospheric pressure is reduced to 620 mmHg and the vapour pressure of water is 830 Pa, what is the value of the cavitation parameter when the pump develops the same total head and discharge? Is it necessary to reduce the height of the pump above the supply, and if so by how much?