Lecture 12

Introduction to Environmental Engineering

Effect of Organic Wastes on Stream Ecosystems

- **Streeter-Phelps Model – DO sag curve**
  - Many equations and computer programs are available today to describe the quality of water in streams, rivers and lakes
  - The most prevalent is the Streeter Phelps equation.
  - Addition of wastewater (BOD) typically causes a slow decrease in O_2, followed by a gradual increase close to the dissolved oxygen (D.O.) saturation concentration

Streeter-Phelps Model

- Assumptions of the Model
  - stream is an ideal plug flow reactor
  - steady-state flow and BOD and DO reaction conditions
  - The only reactions of interest are BOD exertion and transfer of oxygen from air to water across air-water interface

Streeter-Phelps Model

- Mass Balance for the Model
  - Not a Steady-state situation

rate O_2 accum. = rate O_2 in – rate O_2 out + prod. – cons.

rate O_2 accum. = rate O_2 in – 0 + 0 – rate O_2 cons.

  - Both reoxygenation and deoxygenation are 1st order

rate of deoxygenation = $-k_1C$

k_1= deoxygenation constant, function of waste type and temperature

Streeter-Phelps Model

rate of reoxygenation = k_2D

D = deficit in D.O. or difference between saturation and current D.O.

k_2 = reoxygenation constant

$$k_2 = \frac{3.9 v^{1/2} \left([1.025]^{(T-20)} \right)^{1/2}}{H^{3/2}}$$

Streeter-Phelps Model

- Where
  - T = temperature of water, ºC
  - H= average depth of flow, m
  - v = mean stream velocity, m/s
- Oxygen Deficit
  - D = S – C

D.O. deficit = saturation D.O. – D.O. in the water

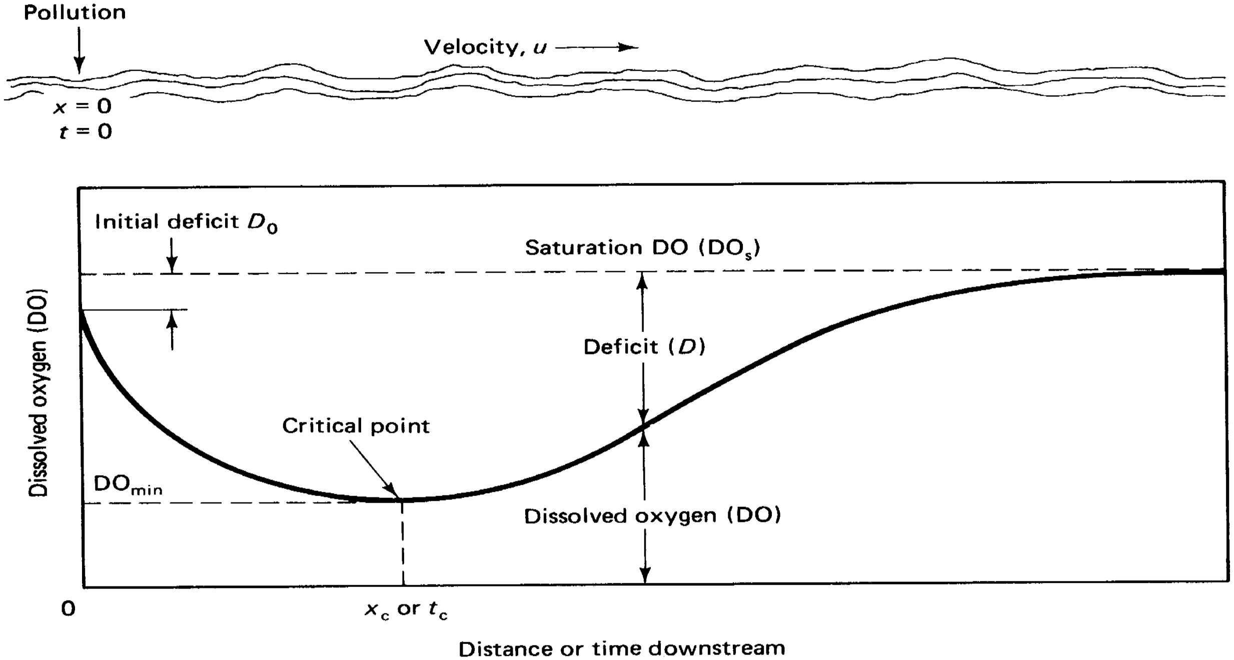

Figure 4.7 Streeter–Phelps oxygen-sag curve.

for the critical time yields

$$t_c = \frac{1}{k_r - k_d} \ln \left\{ \frac{k_r}{k_d} \left[1 - \frac{D_0(k_r - k_d)}{k_d L_0} \right] \right\} \tag{4.28}$$

The maximum deficit can then be found by substituting the value obtained for the critical time t_c into (4.24).

Streeter-Phelps Model

- Deoxygenation rate is equivalent to BOD of waste
  - $r_o = k_1 L_t$
  - $L_t = L_o e^{-Kt}$

L_o or L = ultimate BOD of the wastewater and stream water mixture

Streeter-Phelps Model

- In terms of the deficit with time

$$\frac{dD}{dt} = k_1 z - k_2 D$$

$$z = Le^{-k_1 t}$$

Streeter-Phelps Model

- Substiting and integrating yields the following equations

$$D = \frac{k_1 L_o}{k_2 - k_1}(e^{-k_1 t} - e^{-k_2 t}) + D_o e^{-k_2 t}$$

$$\frac{dD}{dt} = k_1 L_o e^{-k_1 t} - k_2 D = 0$$

$$D_c = \frac{k_1}{k_2} L_o e^{-k_1 t}$$

$$t_c = \frac{1}{k_2 - k_1} \ln\left\{\frac{k_2}{k_1}\left[1 - \frac{D_o(k_2 - k_1)}{k_1 L_o}\right]\right\}$$

Streeter-Phelps Model

- Example:
  - Wastewater mixes with a river resulting in a

BOD = 10.9 mg/L,   DO = 7.6 mg/L

The mixture has a temp. = 20 °C

Deoxygenation const.= 0.2 day^{-1}

Average flow = 0.3 m/s,   Average depth = 3.0 m

DO saturated = 9.1 mg/L

- Find the time and distance downstream at which the oxygen deficit is a maximum?
- Find the minimum value of DO?

Streeter-Phelps Model

- Initial Deficit

D_o = 9.1 – 7.6 = 1.5 mg/L

- Estimate the reaeration constant

$$k_2 = \frac{3.9v^{1/2}}{H^{3/2}} = \frac{3.9(0.3m/s)^{1/2}\left(\left[1.025\right]^{(20-20)}\right)^{1/2}}{(3.0m)^{3/2}} = 0.41\,\text{day}^{-1}$$

Streeter-Phelps Model

- Calculate the time at which the maximum deficit is reached, with t_c:

$$t_c = \frac{1}{k_2 - k_1} \ln\left\{\frac{k_2}{k_1}\left[1 - \frac{DO_o(k_2 - k_1)}{k_1 L_o}\right]\right\}$$

$$= \frac{1}{(0.41 - 0.2)} \ln\left\{\frac{0.41}{0.2}\left[1 - \frac{1.5(0.41 - 0.2)}{0.2 \times 10.9}\right]\right\}$$

$$= 2.67 days$$

$$x_c = vt_c = 0.3 m/s \times 86{,}400 s/day \times 2.67 days = 69{,}300 m$$

Streeter-Phelps Model

- The maximum DO deficit is:

$$D_c = \frac{k_1}{k_2} L_o e^{-k_1 t}$$

$$= \frac{0.2}{0.41}(10.9\,\mathrm{mg/L})\ \mathrm{e}^{-(0.2\mathrm{day}^{-1})(2.67\mathrm{days})}$$

$$= 3.1\,\mathrm{mg/L}$$