

6

Aerobic Wastewater Treatment in Activated Sludge Systems

6.1

Introduction

Since the first experiments performed by Arden and Locket (1914; see Section 1.8), many questions have arisen concerning the activated sludge process:

- What is the best type of reactor?
- Which aeration system is most effective?
- What are the real microbial processes which occur during the decrease in organics and the growth of bacteria?
- What parameters influence the rate of these processes?
- How can we determine the right volumes for reactor and settler to reduce the concentration of organics for a given flow rate?
- How can we feasibly optimize the process?
- Is it possible to treat industrial wastewater containing specific synthetic compounds?

In the first 50 years following Arden's and Locket's invention, the technical realization, and particularly research into the fundamentals, were supported only in a few industrialized countries and to a relatively low extent. Only in the past 40 years has biological wastewater treatment been realized to a greater extent. One of the first textbooks having presented fundamentals of the aerobic activated sludge process was written by Metcalf and Eddy 1972. The pollution of our rivers and lakes caused by society's industrialization endangered nature and reduced the quality of drinking water. Only then were the fundamentals of the activated sludge process studied in more detail. The most important aspects are presented in the next sections.

6.2

Kinetic and Reaction Engineering Models With and Without Oxygen Limitation

6.2.1

Batch Reactors

6.2.1.1 With High Initial Concentration of Bacteria

Monod published the first kinetic studies on the limitation of bacterial growth by substrate concentration. For these experiments he used an aerobic batch reactor,

glucose as substrate, and a culture of *Escherichia coli* (Monod 1942, 1950). His inspiration came from Michaelis–Menten kinetics (see Section 3.2.1.4), which are only valid for a system consisting of one substrate and one enzyme. Although the biodegradation of one substrate by a pure culture of bacteria and, of course, by a mixed culture is a more complicated process, the standard form of the growth kinetics of bacteria shows the same mathematical structure if the growth is not limited by the concentration of dissolved oxygen:

$$\mu = \mu_{\max} \frac{S}{K_S + S} \quad (6.1)$$

and:

$$r_x = \mu X \quad (6.2)$$

where μ is the specific growth rate (d^{-1}), $\mu_{\max}$ is the maximal specific growth rate (d^{-1}), S is the substrate concentration ($g\ L^{-1}$), K_S is the saturation coefficient ($g\ L^{-1}$), r_x is the growth rate ($g\ L^{-1}\ d^{-1}\ MLSS$) and X is the concentration of bacteria ($g\ L^{-1}\ MLSS$).

Monod's first experiments were carried out in batch reactors. He described his measurements of S and X with respect to time by using the bacterial balance:

$$\frac{dX}{dt} = \mu_{\max} \frac{S}{K_S + S} X \quad (6.3)$$

and the substrate balance:

$$\frac{dS}{dt} = - \frac{\mu_{\max}}{Y_{X/S}^o} \frac{S}{K_S + S} X \quad (6.4)$$

where the substrate consumption rate is:

$$r_s = \frac{r_x}{Y_{X/S}^o} \quad (6.5)$$

and $Y_{X/S}^o$ is the true yield coefficient.

The initial conditions of Eqs. (6.3) and (6.4) are:

$$t = 0, X = X_0 \quad \text{and} \quad t = 0, S = S_0$$

The simplest solution can be obtained for a relatively high X_0 and a relatively low S_0 , resulting in a nearly constant $X \approx X_0$. Only Eq. (6.4) must be solved, which is performed by the separation of variables:

$$\int dS \left(\frac{K_S}{S} + 1 \right) = - \frac{\mu_{\max}}{Y_{X/S}^o} X_0 \int dt + C \quad (6.6)$$

$$K_S \ln \frac{S_0}{S} + (S_0 - S) = r_{S,\max} t \quad (6.7)$$

with:

$$r_{S,\max} = \frac{\mu_{\max}}{Y_{X/S}^o} X_0 \quad (6.8)$$

The results of an experiment can be tested as a function of $S = f(t)$ and the coefficients K_S and $r_{S,\max}$ can be determined by transformation of Eq. (6.7) into:

$$\frac{\ln S_0/S}{S_0-S} = \frac{r_{S,\max}}{K_S} \frac{t}{S_0-S} - \frac{1}{K_S} \quad (6.9)$$

In order to confirm that the results of the batch experiment correspond with the model used to develop Eq. (6.4), we should plot, e.g.:

$$\frac{\ln S_0/S}{S_0-S} \quad \text{versus} \quad \frac{t}{S_0-S}$$

and the points should form a straight line with a positive slope $r_{S,\max}/K_S$ and a negative ordinate intercept $-1/K_S$.

For substrate *and* oxygen limitation, Eq. (6.1) must be expanded:

$$\mu = \mu_{\max} \frac{S}{K_S + S} \frac{c'}{K' + c'} \quad (6.10)$$

The region where oxygen limitation is significant can be estimated using $K' = 0.2 \text{ mg L}^{-1} \text{ O}_2$ at $T = 20^\circ\text{C}$ (Putnaerglis 1987). For $S \gg K_S$ the point of limitation may be given as $\mu = 0.9 \mu_{\max}$ (90% of maximal growth rate):

$$\frac{\mu}{\mu_{\max}} = 0.9 = \frac{c'}{K' + c'} \quad (6.11)$$

$$c' = 9 \cdot 0.2 = 1.8 \text{ mg L}^{-1} \text{ O}_2$$

For $c' \geq 1.8 \text{ mg L}^{-1}$ we do not expect O_2 limitation ($T = 20^\circ\text{C}$, domestic sewage, no influences of mass transfer at the surface or inside of flocs formed by bacteria).

With $K_S = 50\text{--}120 \approx 85 \text{ mg L}^{-1} \text{ BOD}_5$ at $T = 20^\circ\text{C}$ (Sundstrom and Klei 1979), the region of substrate limitation can be estimated roughly to be:

$$\frac{\mu}{\mu_{\max}} = 0.9 = \frac{S}{K_S + S} \quad (6.12)$$

$$S = 9 \cdot 85 = 765 \text{ mg L}^{-1} \text{ BOD}_5$$

If both concentrations S and c' decrease to low values during a batch process, the kinetic evaluation is complicated. Therefore, kinetics should be studied considering only under substrate limitation or oxygen limitation. But if we want to study oxygen limitation during a batch process without aeration, the substrate concentration (BOD_5 , COD , DOC) decreases only about a few milligrams per liter if c' decreases from 8 mg L^{-1} to 0 mg L^{-1} . This kind of batch experiment can be carried out even for low S values.

If we avoid substrate limitation

$$\mu = \mu_{\max} \cdot \frac{c'}{K' + c'} \quad (6.13)$$

can be tested.

For $X \approx \text{const}$, the oxygen balance is:

$$\frac{dc'}{dt} = -\frac{\mu_{\max}}{Y_{X/S}^o} \frac{c'}{K' + c'} X \quad (6.14)$$

and a corresponding solution to Eq. (6.9) is obtained which can be linearized as shown before.

6.2.1.2 With Low Initial Concentration of Bacteria

It can be advantageous to start an experiment with a low bacterial concentration. In this case, we must consider the change in X by bacterial growth.

Introducing $r_s = \frac{S_0 - S}{\Delta t}$ and $r_x = \frac{X - X_0}{\Delta t}$ into Eq. (6.5):

$$X = X_0 + Y_{X/S}^o (S_0 - S) \quad (6.15)$$

is obtained which can be used to eliminate X in the substrate balance (Eq. 6.4):

$$\frac{dS}{dt} = -\frac{\mu_{\max}}{Y_{X/S}^o} \frac{S (X_0 + Y_{X/S}^o (S_0 - S))}{K_s + S} \quad (6.16)$$

By integrating Eq. (6.16) with $t = 0$, $S = S_0$ and by separating the variables X and t , and by using:

$$a = X_0 + Y_{X/S}^o S_0 \quad (6.17)$$

we obtain the solution (Winkler 1981):

$$\frac{K_s}{a} \ln \frac{S_0}{S} + \frac{a + K_s}{a} \ln \left(1 + \frac{(S_0 - S) Y_{X/S}^o}{X_0} \right) = \mu_{\max} t \quad (6.18a)$$

After transforming Eq. (6.18a) into:

$$\underbrace{\ln \left(\frac{S}{S_0} + 1 + \frac{(S_0 - S) Y_{X/S}^o}{X_0} \right)}_y = \underbrace{\frac{\mu_{\max} a}{(a + K_s)}}_m \underbrace{\frac{t}{\ln \frac{S_0}{S}}}_x - \underbrace{\frac{K_s}{a + K_s}}_b \quad (6.18b)$$

we obtain the coefficients $\mu_{\max}$ and K_s which are determined from the slope and ordinate intercept if a straight line with a positive slope m and a negative ordinate intercept b follows.

6.2.2

Chemostat

The simple continuous process takes place in a CSTR (completely stirred tank reactor) as a steady state process with a bacteria-free influent and with no recycling of settled sludge. This reactor is called a chemostat. The growth of bacteria was initiated by inoculation. After a non-steady period, a constant concentration of bac-

teria and substrate are achieved for mean retention times which are long enough to sustain bacterial growth. Different steady states can be obtained by changing the flow rate, substrate influent concentration or dissolved oxygen concentration at different aeration rates. A very simple solution of the problem follows from a bacterial balance:

$$0 = \underbrace{-Q_0 X}_{\text{flow rate out of the reactor}} + \underbrace{\mu X V}_{\text{growth rate}} \quad (6.19)$$

$$\text{With Eq. (6.1) and the mean retention time: } t_R = \frac{V}{Q_0} \quad (6.20)$$

or the dilution rate:

$$D = \frac{Q_0}{V} = \frac{1}{t_R} \quad (6.21)$$

we obtain from Eq. (6.19):

$$S = \frac{K_S}{\mu_{\max} t_R - 1} \quad (6.22)$$

or:

$$S = \frac{DK_S}{\mu_{\max} - D} \quad (6.23)$$

With Eq. (6.15), the corresponding equation for X is:

$$X = Y_{X/S}^o \left(S_0 - \frac{K_S}{\mu_{\max} t_{RC} - 1} \right) \quad (6.24)$$

with the mean retention time t_R used as a parameter (Fig. 6.1).

It is typical for all bioreactors with suspended microorganisms in continuous operation that the organisms can be washed out at a critical mean retention time (t_{RC}) which follows from Eq. (6.24) for $X = 0$ as:

$$t_{RC} = \frac{K_S + S_0}{\mu_{\max} S_0} \quad (6.25)$$

and for $S_0 \gg K_S$ as:

$$t_{RC} = \frac{1}{\mu_{\max}} \quad (6.26)$$

Using Eq. (6.21), a well known solution is obtained:

$$D_C = \mu_{\max} \quad (6.27)$$

All suspended organisms are washed out if the dilution rate is increased beyond a critical value, which corresponds to the maximum specific growth rate, $\mu_{\max}$. From a theoretical point of view, a suspended culture of two organisms (1 and 2) with different $\mu_{\max}$ can be separated by increasing D up to a value greater than D_{C1} and less than D_{C2} .

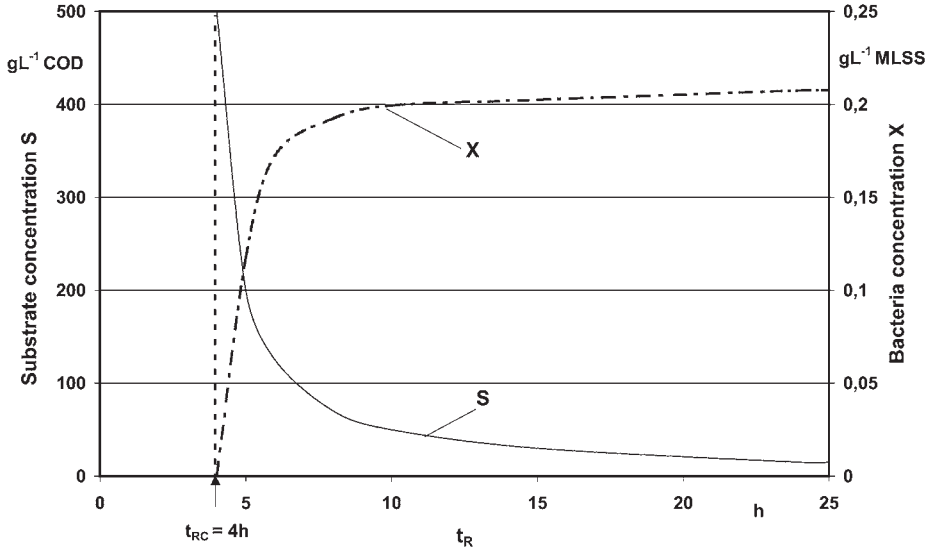


Fig. 6.1 Effluent COD S and bacterial concentration X of a chemostat versus mean retention time t_R . t_{RC} = critical mean retention time, calculated with $\mu_{\max} = 7.2 \text{ d}^{-1}$, $K_S = 100 \text{ mg L}^{-1} \text{ COD}$.

The growth rate follows directly from Eqs. (6.2) and (6.24):

$$r_x = \mu X \quad (6.28)$$

it increases with increasing $t_R > t_{RC}$, get a maximal value $r_{x,\max}$, which can be obtained from: $dr_x/dt_x = 0$ and decreases.

Because the chemostat maintains only a low concentration of microorganisms, it has no practical significance, and as a consequence, it exhibits only a low substrate consumption rate. But experiments can be used to study the reaction kinetics and to determine the coefficients. It can be used to determine S for different t_R values while Eq. (6.22) is linearized as follows:

$$\frac{1}{S} = \frac{\mu_{\max}}{K_S} t_R - \frac{1}{K_S} \quad (6.29)$$

A plot of $1/S$ versus t_R must give a straight line with a negative ordinate intercept $-K_S^{-1}$.

As discussed in Section 6.2.1.1, oxygen limitation must be avoided during these experiments and the dissolved oxygen concentration must be $c' \geq 1.8 \text{ mg L}^{-1}$ ($T = 20^\circ\text{C}$, municipal wastewater). The region of oxygen limitation can be studied if an oxygen balance is used:

$$0 = Q_G (c_{O_2,0} - c_{O_2,e}) - \frac{\mu_{\max} X}{Y_{X/O_2}} \frac{c'}{K' + c'} V \quad (6.30)$$

and if S is high enough, whereby $S/(K_S + S) \approx 1$ during aeration.

With:

$$r' = \frac{Q_G (c_{O_2,0} - c_{O_2,e})}{V} \quad (6.31)$$

it follows that:

$$\frac{r'}{X} = \frac{\mu_{\max}}{Y_{X/O_2}^\circ} \frac{c'}{K' + c'} \quad (6.32)$$

Equation (6.32) can be linearized using the Langmuir plot:

$$\frac{c'X}{r'} = \frac{Y_{X/O_2}^\circ K'}{\mu_{\max}} + \frac{Y_{X/O_2}^\circ}{\mu_{\max}} c' \quad (6.33)$$

Figure 6.2 presents results from measurements at high bacterial concentration X_0 and high substrate concentration S_0 for 17 °C and 24 °C. X_0 was added to the influent wastewater (not chemostat operation).

A good linearization was obtained with $K' = 0.19 \text{ mg L}^{-1}$ (17 °C) and $K' = 0.06 \text{ mg L}^{-1}$ (24 °C; Putnaerglis 1987).

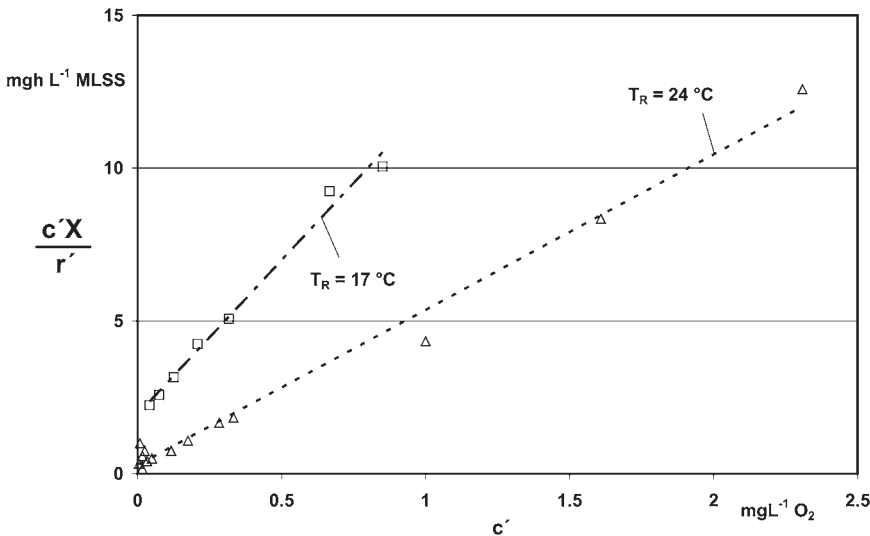


Fig. 6.2 Plot of Eq. (6.33) in order to test this equation and to determine K' and $\mu_{\max}/Y_{X/O_2}^\circ$ (Putnaerglis 1987).

6.2.3

Completely Mixed Activated Sludge Reactor

6.2.3.1 Preliminary Remarks

In the late 1960s and early 1970s, the first models for the activated sludge process were published, which were based on the balances of bacteria and substrate.

Initially, a first-order reaction with regard to substrate concentration S was used (Eckenfelder 1967). Later on, Monod kinetics were considered in most models (Reynolds and Yang 1966; Benedek and Horvath 1967; Lawrence and McCarty 1970; Peil and Gaudy 1971; Chow et al. 1979; Grady et al. 1999).

In some papers, other kinetics were suitable to describe experimental results, for example Contois kinetics (Chen and Hashimoto 1980):

$$\mu = \mu_{\max} \frac{S}{KX + S} \quad (6.34)$$

However, we will restrict ourselves to Monod kinetics. In most papers, sludge age (mean retention time of bacteria t_{RX}) is the key process parameter, but we want to show that the model first contains three other process parameter (mean retention time of wastewater t_R , recycle ratio n_R and thickening ratio n_E), which can be replaced by t_{RX} .

6.2.3.2 Mean Retention Time, Recycle Ratio and Thickening Ratio as Process Parameters

Figure 6.3 presents the flow sheet of an activated sludge plant.

The balance of bacteria X for the CSTR in steady state is:

$$0 = Q_M (X_M - X) + (\mu - k_d) X V \quad (6.35)$$

Corresponding to Eq. (6.35), the balance of substrate S is:

$$0 = Q_M (S_M - S) - \frac{\mu}{Y_{X/S}} X V \quad (6.36)$$

with: $k_d = k_s + k_e$ as the decay coefficient.

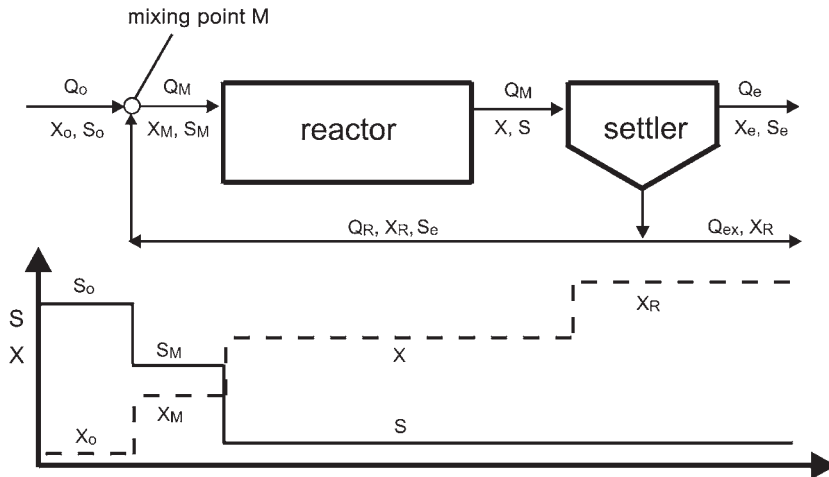


Fig. 6.3 Flow sheet for a completely mixed (stirred) activated sludge plant (CSTR).

k_d includes both the specific rate of death k_s and that of mass decrease by endogenous respiration k_e . The decay coefficient was first proposed by Herbert (1958). k_d and k_e were already introduced in Chapter 4.23 (see Eq. 4.27). Dividing Eq. (6.36) by Eq. (6.35), we obtain:

$$X = X_M + (S_M - S) Y_{X/S}^o \left(1 - \frac{k_d}{\mu} \right) \quad (6.37)$$

For $\mu \gg k_d$, Eq. (6.15) follows from Eq. (6.37), which can be interpreted as a relationship between the substrate used and the bacterial mass formed. After introducing Eq. (6.37) and Monod kinetics (Eq. 6.1) in Eq. 6.36, S can be calculated.

Furthermore, a bacterial balance at the mixing point M is necessary (Fig. 6.3). Neglecting $Q_0 X_0$, we obtain:

$$Q_M X_M = Q_R X_R \quad (6.38)$$

With the recycle ratio:

$$n_R = \frac{Q_R}{Q_0} \quad (6.39)$$

and the thickening ratio:

$$n_E = \frac{X_R}{X} \quad (6.40)$$

then we can calculate:

$$X_M = \frac{n_R n_E}{1 + n_R} X \quad (6.41)$$

After introducing Eqs. (6.1), (6.20), (6.37) and (6.41) into Eq. (6.36), we obtain:

$$S = \frac{K_S (1 + n_R - n_E n_R + k_d t_R)}{t_R (\mu_{\max} - k_d) - (1 + n_R - n_E n_R)} \quad (6.42)$$

It is interesting that S is not influenced directly by S_0 .

For $k_d \approx 0$, it follows that:

$$S = \frac{K_S (1 + n_R - n_E n_R)}{t_R \mu_{\max} - (1 + n_R - n_E n_R)} \quad (6.43)$$

If we are interested in the bacterial concentration X , we must introduce Eqs. (6.1) and (6.43) into Eq. (6.37).

For $S = S_0$, Eq. (6.43) yields:

$$t_{RC} = \frac{K_S + S_0}{\mu_{\max} S_0} (1 + n_R - n_E n_R) \quad (6.44)$$

t_{RC} is the critical mean retention time which was already introduced in Section 6.2.2 for the discussion on the chemostat (see Eq. 6.25). For $n_R = 0$, Eq. (6.44) can be transformed into Eq. (6.25). The higher n_R is, the lower t_{RC} is for $n_E > 1$. This model was published by Mehring (1979) and by Sundstrom and Klei (1979) among others.

6.2.3.3 Sludge Age as Parameter

In activated sludge plants the mean retention time of bacteria (sludge age) t_{RX} is decoupled from that of the water retention time. The sludge age is defined by Eq. (6.45):

$$t_{RX} = \frac{VX}{Q_R X_R + Q_{ex} X_R} \quad (6.45)$$

Because of $Q_R X_R \gg Q_{ex} X_R$, it is often allowed to write:

$$t_{RX} = \frac{VX}{Q_R X_R} \quad (6.46)$$

This is the mass of solids inside the aeration tank applied to the production rate of excess solids. The steady-state balance of solids around the settler gives (see Fig. 6.3):

$$(Q_0 + Q_R)X = Q_R X_R + Q_{ex} X_R + Q_e X_e \quad (6.47)$$

Neglecting $Q_e X_e$, transforming Eq. (6.47) and considering Eqs. (6.39), (6.40), (6.46) and (6.20), it follows that:

$$1 + n_R - n_E n_R = \frac{t_R}{t_{RX}} \quad (6.48)$$

If we introduce this result into the solutions with bacterial decay [Eq. (6.42)] and without bacterial decay (Eq. 6.43) we obtain:

$$S = \frac{K_S (1 + k_d t_{RX})}{t_{RX} (\mu_{max} - k_d) - 1} \quad (6.49)$$

and for $k_d = 0$:

$$S = \frac{K_S}{t_{RX} \mu_{max} - 1} \quad (6.50)$$

Substrate removal is only influenced by sludge age t_{RX} , which replaces the three parameters t_R , n_R and n_E . We see here the importance of the parameter “sludge age”, which will be used more and more not only in theoretical considerations but also in the practice of activated sludge system control.

The critical sludge age t_{RXC} is the mean retention time of bacteria which must be exceeded in the activated sludge process. Otherwise, the bacteria (the sludge) will be washed out. t_{RXC} follows from Eqs. (6.49) and (6.50) for $S = S_0$:

$$t_{RXC} = \frac{K_S + S_0}{S_0 (\mu_{max} - k_d) - K_S k_d} \quad (6.51)$$

and for $k_d = 0$:

$$t_{RXC} = \frac{K_S + S_0}{\mu_{max} S_0} \quad (6.52)$$

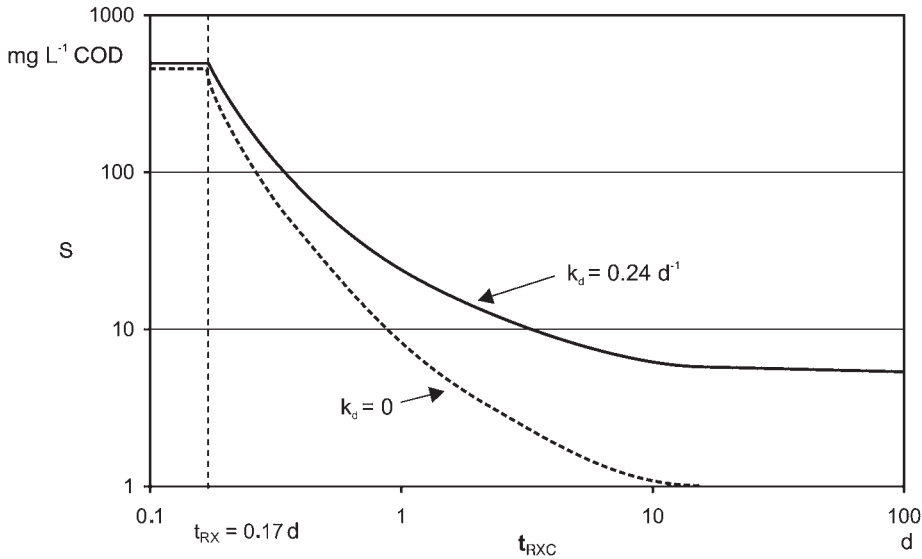


Fig. 6.4 Effluent COD concentration of a CSTR versus sludge age t_{RX} ; with bacterial decay ($k_d = 0.24 \text{ d}^{-1}$) and without bacterial decay ($k_d = 0 \text{ d}^{-1}$) for a one-stage CSTR; $\mu_{max} = 7.2 \text{ d}^{-1}$, $K_S = 100 \text{ mg L}^{-1} \text{ COD}$; $S_0 = 500 \text{ mg L}^{-1} \text{ COD}$.

If we introduce Eq. (6.52) into Eq. (6.44), then:

$$1 + n_R - n_E n_R = \frac{t_{RC}}{t_{RXC}} \quad (6.53)$$

follows, the same relationship as calculated for $t_R t_{RX}^{-1}$. After this model with t_{RX} was presented as the only process parameter in the early 1970s, it was published, discussed and applied very frequently (Sunstrom and Klei 1979; Benefield and Randall 1980; Winkler 1981; Nasaroff and Alvarez-Cohen 2001). Figure 6.4 presents some theoretical results which were calculated for $\mu_{max} = 7.2 \text{ d}^{-1}$, $K_S = 100 \text{ mg L}^{-1} \text{ COD}$, $k_d = 0.24 \text{ d}^{-1}$ and $S_0 = 500 \text{ mg L}^{-1} \text{ COD}$.

For these conditions nearly 90%, that is $S_0 - S = 500 - 50 = 450 \text{ mg L}^{-1} \text{ COD}$ are removed, if a sludge age of about $t_{RX} = 0.5 \text{ d} = 12 \text{ h}$ is selected. The real sludge production is obtained from:

$$Y_{X/S} = Y_{X/S}^o \left(1 - \frac{k_d}{\mu} \right) \quad (4.27)$$

for $t_{RX} = 0.5 \text{ d}$, $Y_{X/S}^o = 0.43 \text{ g MLSS (g COD)}^{-1}$ and $\mu_{max} = 7.2 \text{ d}^{-1}$

with

$$\mu = \mu_{max} \frac{S}{K_S + S} = 7.2 \frac{50}{100 + 50} = 2.4 \text{ d}^{-1}$$

it follow a real yield:

$$Y_{X/S} = 0.43 \left(1 - \frac{0.24}{2.4} \right) = 0.39 \frac{\text{g MLSS}}{\text{g COD}}$$

If we want to consider oxygen limitation, Eq. (6.10) must be introduced into Eqs. (6.41) and (6.42) and we have to add an oxygen balance:

$$0 = \underbrace{Q_G (c_{O_2,0} - c_{O_2,e})}_{\text{O}_2 \text{ uptake by exhaust air measurements}} - \underbrace{\frac{\mu_{\max}}{Y_{X/O_2}^o} X \frac{S}{K_S + S} \cdot \frac{c'}{K' + c'}}_{\text{O}_2 \text{ needed for growth}} V - \underbrace{\frac{k_e X V}{Y_{X/O_2}^o}}_{\text{O}_2 \text{ needed for endogenous respiration}} \quad (6.54)$$

The difference of dissolved O_2 in the influent and effluent water is neglected. We want to continue the calculation of activated sludge plants in Chapter 11.

6.2.4

Plug Flow Reactor

The main characteristic of a plug flow reactor (PFR) is that no mixing occurs in the direction of flow; however, complete mixing is assumed within a cross-sectional area of the reactor. Water and all suspended flocs of bacteria move with the same velocity along the tube reactor. In contrast to the CSTR, the PFR exhibits a continuous decrease in substrate concentration and an increase in bacterial concentration in the direction of flow (Fig. 6.5).

Because of the continuous change of concentrations, the balances for bacteria and substrate must be written as differential balances. They can be obtained by writing first an integral balance for the volume element $\Delta x A$ in a steady state:

$$0 = Q_M (S_1 - S_2) - r_S \Delta x A \quad (6.55)$$

with A as cross-sectional area, which is not correct, because of the change of S within Δx ; but it can be corrected, by forming the differential:

$$\lim_{\Delta x \rightarrow 0} \frac{S_1 - S_2}{x_1 - x_2} = - \lim_{\Delta x \rightarrow 0} \frac{S_1 - S_2}{x_2 - x_1} = - \lim_{\Delta x \rightarrow 0} \frac{\Delta S}{\Delta x} = - \frac{dS}{dx} \quad (6.56)$$

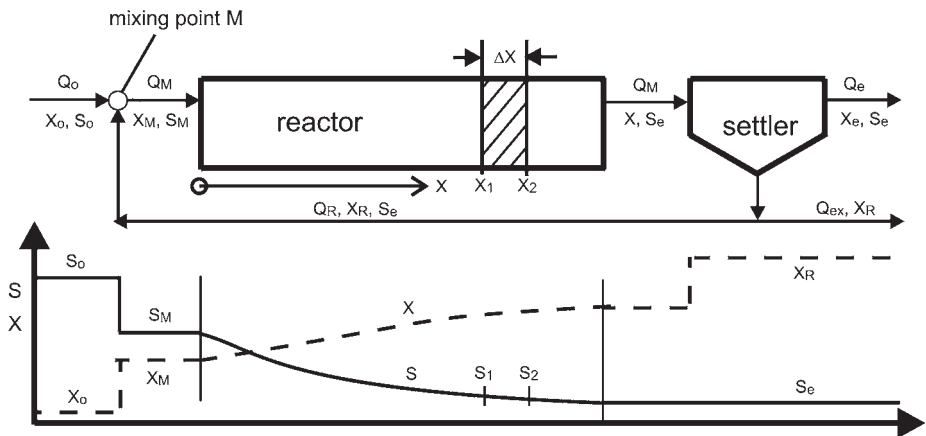


Fig. 6.5 Flow sheet for a plug flow activated sludge plant (PFR).

with:

$$\bar{w}_M = \frac{Q_M}{A} = \bar{w} (1 + n_R) \quad (6.57)$$

and Eq. (6.56), then Eq. (6.55) yields:

$$0 = -\bar{w} (1 + n_R) \frac{dS}{dx} + r_S \quad (6.58)$$

or for the bacterial balance, it yields:

$$0 = -\bar{w} (1 + n_R) \frac{dX}{dx} + r_X \quad (6.59)$$

Assuming Monod kinetics for bacterial growth, then:

$$0 = -\bar{w} (1 + n_R) \frac{dS}{dx} - \frac{\mu_{\max} X}{Y_{X/S}^o} \frac{S}{K_S + S} \quad (6.60)$$

and:

$$0 = -\bar{w} (1 + n_R) \frac{dX}{dx} + \mu_{\max} X \frac{S}{K_S + S} \quad (6.61)$$

result with the conditions $x = 0, S = S_M$ and $x = 0, X = X_M$.

If we move along the tube reactor with the flow rate $\bar{w}(1 + n_R)$ and measure S and X at time t , the process could be described with Eqs. (6.3) and (6.4). Therefore, by using:

$$\bar{w} (1 + n_R) = \frac{dX}{dt} \quad (6.62)$$

Equations (6.60) and (6.61) can be transformed into Eqs. (6.3) and (6.4). With the appropriate initial conditions, the solution in Eq. (6.18) is achieved. Batch reactor and plug flow reactor can be described by the same balances if we use the transformation in Eq. (6.62). After consideration of Eq. (6.15) and integration:

$$\frac{K_S}{a} \ln \frac{S_M}{S} + \frac{a + K_S}{a} \ln \left(1 + \frac{(S_M - S) Y_{X/S}^o}{X_M} \right) = \frac{\mu_{\max} t_R}{1 + n_R} \quad (6.63)$$

with:

$$t_R = \frac{x}{\bar{w}} \quad (6.64)$$

$$S_M = \frac{n_R S_e}{1 + n_R} \quad (6.65)$$

and:

$$X_M = \frac{n_R n_E X_e}{1 + n_R} \quad (6.66)$$

With Eq. (6.58) the critical sludge age t_{RXC} can be calculated. The critical mean retention time t_{RX} follows from Eq. (6.67):

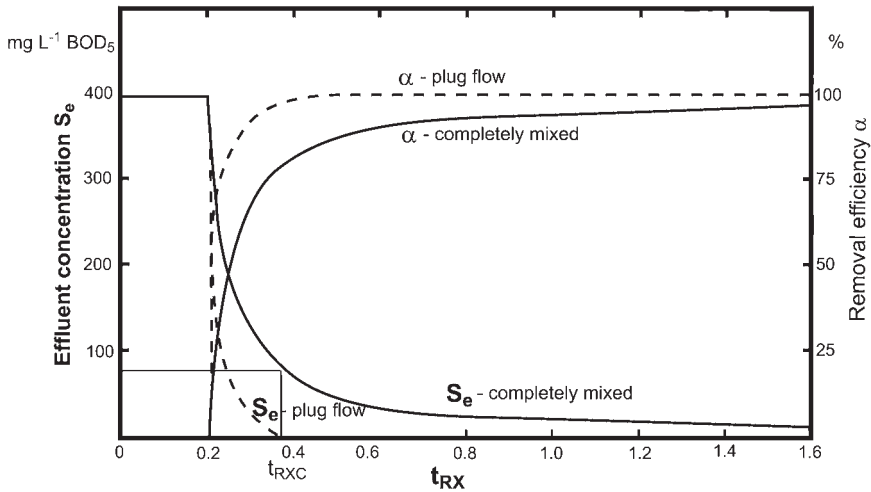


Fig. 6.6 Comparison of a CSTR with a PFR; effluent concentrations S and treatment efficiency α versus sludge age t_{RX} ; $\mu_{max} = 16.7 \text{ d}^{-1}$, $K_S = 0.1 \text{ g L}^{-1} \text{ BOD}_5$, $Y_{X/S} = 0.6 \text{ g MLSS (g BOD}_5\text{)}^{-1}$, $k_d = 0.05 \text{ d}^{-1}$, $S_0 = 400 \text{ mg L}^{-1} \text{ BOD}_5$ (Benefield and Randall 1980; after Lawrence and McCarty 1970).

$$t_{RC} = (1 + n_R) \left(\frac{1 + K_S/S_0}{\mu_{max}} \right) \ln \left(\frac{1 + n_R}{n_E n_R} \right) \quad (6.67)$$

Equation (6.67) was published by Fan et al. (1970), as the mathematical limit of a model of CSTRs in series for $n \rightarrow \infty$, with n as the total number of reactors in the system. This must correspond with the plug flow model. It follows from Eq. (6.67) that t_{RC} and t_{RX} increase with decreasing n_R ; for $n_R = 0$ (no recycle) t_{RC} is infinitely large. This result is to be expected because there is neither recycle of bacteria into the inlet feed nor mixing of fluid.

Figure 6.6 compares the results of PFR substrate removal and efficiency with those of a CSTR ($S_0 = 400 \text{ mg L}^{-1} \text{ BOD}_5$, $\mu_{max} = 16.7 \text{ d}^{-1}$, $K_S = 100 \text{ mg L}^{-1} \text{ BOD}_5$, $k_d = 0.05 \text{ d}^{-1}$, $Y_{X/S} = 0.6 \text{ g MLSS (g BOD}_5\text{)}^{-1}$). These results were published by Benefield and Randall (1980). Equation (6.49) was used for the CSTR and Eq. (6.63) for the PFR. The advantages of a PFR are clear: at $t_{RX} = 0.4 \text{ d}$, the effluent of the PFR S_e is already very low. By comparison, the CSTR shows an effluent concentration of $S = 70 \text{ mg L}^{-1} \text{ BOD}_5$. Only at $t_{RX} = 1.6 \text{ d}$ does the CSTR show S as low as nearly $10 \text{ mg L}^{-1} \text{ BOD}_5$.

In Fig. 6.6 t_{RXC} for CSTR and PFR are nearly the same, because of the relative high sludge recycle n_R . For lower n_R , t_{RC} and t_{RXC} increase in PFR systems.

6.2.5

Completely Mixed Tank Cascades With Sludge Recycle

CSTRs and PFRs are reactors idealized as either completely mixed or not mixed at all. In a technical scale, all reactors in continuous operation differ from these extreme cases. In a longitudinal open basin, dispersion effects dominate which

can be reflected, for example, by a dispersion model or by a model of n completely stirred tank reactors coupled in series, which is called a reactor cascade. Frequently, a reactor with longitudinal dispersion can be described as a cascade with a specific number of completely mixed tanks (see Section 6.2.8). In the following, this model will be discussed shortly.

The model consists of n balances each for substrate and bacteria. It is sufficient to write only the balances for stage number n , which shows the structure of all the other balances for stages $< n$. We write according to Eqs. (6.35) and (6.36) for stage number n and steady-state conditions:

$$0 = Q_M (S_{n-1} - S_n) - \frac{\mu_n}{Y_{X/S}^o} X_n V_n \quad (6.68)$$

$$0 = Q_M (X_{n-1} - X_n) + \mu_n X_n V_n - k_d X_n V_n \quad (6.69)$$

X_n in Eq. (6.68) can be replaced by:

$$X_n = X_{n-1} + (S_{n-1} - S_n) Y_{X/S}^o \left(1 \frac{k_d}{\mu_n} \right) \quad (6.37)$$

With:

$$\mu_n = \mu_{\max} \frac{S_n}{K_S + S_n} \quad (6.70)$$

$$t_R^n = \frac{V_N}{Q_0} \quad (6.71)$$

$$Q_M = Q_0 + Q_R \quad (6.72)$$

as well as Eqs. (6.38) to (6.41). S_n and X_n can be calculated by coupling the n substrate balances.

The most important property of such a system is the step by step decrease in substrate concentration and increase in bacterial concentration it exhibits. For certain conditions of influent substrate concentration and kinetic coefficients, the n -stage cascade is characterized by a higher total reaction rate, compared to a CSTR.

Let us discuss the behavior of a cascade with two stages, without presenting a special figure, but looking at Fig. 6.6 showing a somewhat higher difference because of its plug flow characteristic (infinite number of stages). If S is relative high, e.g. for low mean retention times t_R , substrate concentrations S are equivalent in both systems showing nearly the same low removal efficiency because of the zero-order region of Monod kinetics. But for low S values and/or higher t_{RX} , the two-stage cascade shows remarkably lower S as a result of the first-order region. In other words: a substrate removal from 5000 mg L⁻¹ COD down to 500 mg L⁻¹ COD is performed as a reaction with nearly zero order (for $K_S = 100$ mg L⁻¹ COD); and a two-stage cascade is not advantageous. A substrate removal from 100 mg L⁻¹ COD to 10 mg L⁻¹ COD, however, takes place in the region of the first-order reaction with a remarkably higher reaction rate in a cascade of two or more stages! The influent substrate concentration is 500 mg L⁻¹ COD. Here, the region of Monod kinetics is effective, starting from nearly zero order down to a first-order reaction.

For $S_2 = S_0$, the bacteria will be washed out and a critical sludge age t_{RXC} or a critical mean retention time t_{RC} follows from the model for a cascade of n stages:

$$t_{\text{RC}}^n = \frac{n \left(1 + \frac{K_S}{S_0} \right) \left[1 - \frac{n_E n_R}{1 + n_R} \right]^{1/n} (1 + n_R)}{\mu_{\text{max}} - k_d \left(1 + \frac{K_S}{S_0} \right)} \quad (6.73)$$

With Eq. (6.48), the critical sludge age results:

$$t_{\text{RXC}}^n = \frac{t_{\text{RC}}^n}{1 + n_R - n_E n_R} \quad (6.74)$$

For $n \rightarrow \infty$, Eq. (6.73) transforms to Eq. (6.67) and the critical mean retention time of the PFR with bacterial recycle is:

$$t_{\text{RC}} = \frac{(1 + n_R)(S_0 - K_S)}{\mu_{\text{max}} S_0} \ln \frac{1 + n_R}{n_R n_E} \quad (6.75)$$

For $n_R = 0$ (no bacterial recycle), t_{RC} is infinitely large: for all mean retention times t_R bacteria are washed out (Fan et al. 1970; Sundstrom and Klei 1979). This property differentiates the cascade with recycle from a chemostat or a cascade system without sludge recycle. In both systems (one stage = chemostat; and n stages = cascade) washing-out situations can occur. With increasing number of stages t_{RC} increases starting from the lowest value for $n = 1$ (chemostat) to the highest value for $n = \infty$ (plug flow reactor).

6.2.6

Flow Reactor With Axial Dispersion

In addition to the transport of substrate S and bacteria X by the flow of the water (see Eqs. 6.60 and 6.61), a transport by dispersion will be considered:

$$0 = -\bar{w}(1 + n_R) \frac{dS}{dx} + D_x \frac{d^2 S}{dx^2} - \frac{\mu_{\text{max}} X}{Y_{\text{B/S}}^o} \frac{S}{K_S + S} \quad (6.76)$$

$$0 = -\bar{w}(1 + n_R) \frac{dX}{dx} + D_x \frac{d^2 X}{dx^2} + \mu_{\text{max}} X \frac{S}{K_S + S} \quad (6.77)$$

The decay rate is neglected here. Two boundary conditions are needed for each balance. Normally, the following conditions are used for a “closed tube” (Danckwerths 1953). They will be written here only for Eq. (6.76) (see Fig. 6.7):

$$x = 0: \bar{w}(1 + n_R) S_M = \bar{w}(1 + n_R) S - D_x \frac{dS}{dx} \quad (6.78)$$

$$x = L: \frac{dS}{dx} = 0 \quad (6.79)$$

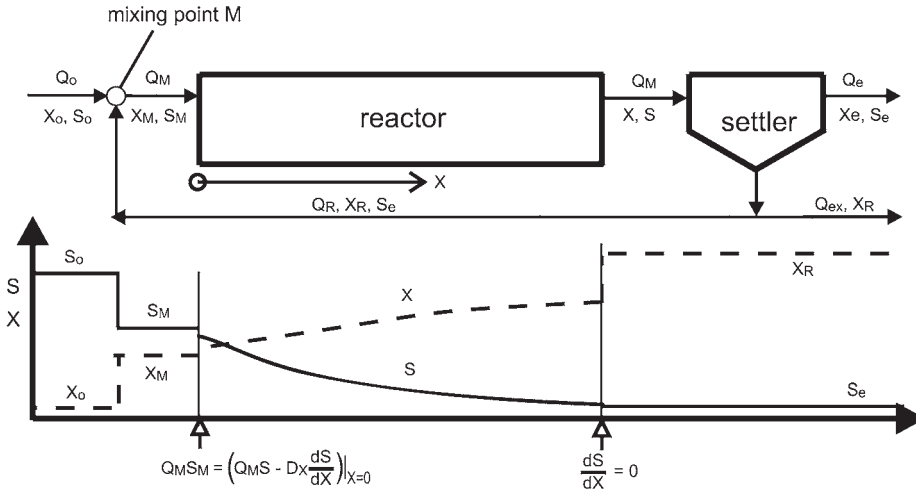


Fig. 6.7 Flow sheet for a real flow tube reactor (closed tube model).

The same conditions for X can be used to solve Eq. (6.77). As Fig. 6.7 shows at $x = 0$, a discontinuity occurs in S . For very low D_x values, this discontinuity disappears ($\rightarrow$ PFR), while for high D_x values, it reaches a maximum ($\rightarrow$ CSTR). Dispersion in the tube for $x < 0$ is neglected in this model. Under real conditions, a $dS/dx < 0$ would be correct at $x = L$. But for this “boundary condition” Eq. (6.76) cannot be solved. Danckwerths (1953) introduced Eq. (6.79) and expected a very small error at this point.

Equations (6.76) and (6.77) can be rewritten with dimensionless Peclet or Bodenstein numbers:

$$Pe = \frac{\bar{w}(1+n_R)L}{D_x} \quad (6.80)$$

$$0 = -\frac{dS^*}{d(x/L)} + \frac{1}{Pe} \frac{d^2 S^*}{d(x/L)^2} - \frac{L}{\bar{w}(1+n_R)} \cdot \frac{\mu_{\max} X^*}{Y_{X/S}^o} \frac{S^*}{1+S^*} \quad (6.81)$$

$$0 = -\frac{dX^*}{d(x/L)} + \frac{1}{Pe} \frac{d^2 X^*}{d(x/L)^2} + \frac{L}{\bar{w}(1+n_R)} \mu_{\max} X^* \frac{S^*}{1+S^*} \quad (6.82)$$

For $Pe \gg 1$ (high $\bar{w}(1+n_R)$, high L , low D_x), Eq. (6.81) and Eq. (6.82) come close to the balances for a PFR (Eqs. 6.60 and 6.61), for $Pe \ll 1$ (low $\bar{w}(1+n_R)$, low L , high D_x) the ideal CSTR system is nearly reached.

For $S \ll K_s$, the equations can be solved analytically (Lin 1979). For Monod kinetics, a numerical solution is needed (Fan 1970; Vasilin and Vasilyev 1978) even for the case of constant biomass concentration X .

6.2.7

Stoichiometric and Kinetic Coefficients

Since Monod's first experiments and kinetic studies in 1942 and 1950 and those by Herbert (1958), a large number of papers concerning the kinetics of bacterial growth and carbon removal have been published. The yield and kinetic coefficients are again typically defined by Eqs. (6.83) to (6.85):

$$r_X = \mu_{\max} \frac{S}{K_S + S} \frac{c'}{K' + c'} X - (k_s + k_e) X \quad (6.83)$$

$$r_S = \frac{\mu_{\max}}{Y_{X/S}^o} \frac{S}{K_S + S} \frac{c'}{K' + c'} X \quad (6.84)$$

$$r_{O_2} = \frac{\mu_{\max}}{Y_{X/O_2}^o} \frac{S}{K_S + S} \frac{c'}{K' + c'} X + \frac{k_e}{Y_{X/O_2}^o} X \quad (6.85)$$

with:

$$k_d = k_s + k_e \quad (6.86)$$

and:

$$Y_{X/O_2}^o = Y_{X/S}^o \cdot Y_{S/O_2}^o \quad (6.87)$$

In contrast to Herbert, Pirt (1965) introduced a kinetic expression considering the endogenous mass consumption into the balance for substrates. Here, we follow the concept of Herbert, which is usually used in kinetic descriptions of biological wastewater treatment.

Some measurements led to results which are listed in Table 6.1.

Although different substrates and mixed cultures were used, the $\mu_{\max}$ values do not differ considerably. However, K_S and K' scatter remarkably. Because of the low values of K' , an exact measurement of low dissolved oxygen concentration is necessary. In last time, O_2 electrodes with higher precision have become available, so that more exact measurements are possible. We do not want to present here all these results.

Nearly all yield coefficients are influenced by bacterial decay (see Eq. 4.27) or endogenous respiration. More exact studies are needed for us to be able to distinguish the true yield coefficient Y^o from the real yield coefficient Y .

Some published ranges of data from several textbooks are compiled in Table 6.2. On the basis of these values, new mean data will be recommended.

Table 6.1 Published coefficients and recommended ranges of values from some papers for laboratory-scale CSTR measurements.

Reference	Basis	T (°C)	$\mu_{\max}$ (h ⁻¹)	K_s ^{b)}	K' ^{b)}	$Y_{x/s}$	$Y_{O_2/s}$	k_d (h ⁻¹)	k_e (h ⁻¹)
Reynolds and Yang (1966)	COD ^{c)}	20	0.80	345	—	0.39	—	0.0071	—
Benedek and Horvath (1967)	COD ^{d)}	18	0.16	22	—	0.67	—	0.0029	—
Peil and Gaudy (1971)	COD ^{d)}	23	0.45	55	—	—	—	—	—
Strnivasaraghanan and Gaudy (1974) ^{a)}	COD ^{e)}	25	0.49	115	—	0.59	—	0.0058	—
Button and Gurver (1966)	— ^{f)}	35	0.506	—	0.45	—	1.2–1.8	—	—
Sinclair and Ryder (1972)	— ^{f)}	35	0.54	4.6	0.084	0.55	1.41	—	0.01
Putnaerglis (1987)	DOC ^{g)}	17	0.32	—	0.22	—	—	—	0.03

a) Batch, b) Values given in mg L⁻¹, c) Synthetic, d) Municipal, e) Synthetic,

f) Glycerine, g) Filtrate of excess sludge after thermal conditioning.

Table 6.2 Ranges of measured kinetic and stoichiometric coefficients for aerobic degradation of organic compounds in municipal and domestic wastewater.

Reference	Basis	T (°C)	$\mu_{\max}$ (d ⁻¹)	K_s (mg L ⁻¹)	$Y_{x/s}$	k_d (d ⁻¹)	K' (mg L ⁻¹ O ₂)
Sundstrom and Klei (1979)	BOD ₅	—	9.6–13.2	50–120	0.5–0.67 ^{a)}	0.048–0.072	—
Henze et al. (2002)	COD	20	4.0–8.0	3–30	0.5–0.7 ^{b)}	0.1–0.2	0.5–1.0
Chow et al. (1979)	BOD ₅	—	0.8–8.0	25–100	0.4–0.8 ^{c)}	0.025–0.075	—

a) Value given in g MLSS (gBOD₅)⁻¹

b) Value given in g COD (gCOD)⁻¹

c) Value given in g MLVSS (gBOD₅)⁻¹

6.2.8

Comparison of Reactors

Batch reactors are used on a technical scale for carbon removal as sequencing batch reactors (SBRs) with periods of filling, aerating, settling, part-emptying, filling and so on. We will come back to this system when we discuss the extended process with nitrogen and phosphorus removal (see Chapter 10).

For a completely mixed tank, substrate and bacterial concentrations at the effluent are nearly the same as inside the reactor. The mixed culture of bacteria always comes in contact with a low substrate concentration. The removal rate is relatively low and we do not need to consider large influences of transport processes into

flocs and oxygen limitation. However, in reality, large tanks are frequently not completely mixed. The rotation speed of surface aerators is often adjusted to a needed concentration of dissolved oxygen and not to a certain degree of mixing. Nevertheless, models can help to understand the process and the behavior of the reactor much better.

In PFRs, tube reactors with axial dispersion or cascades, the bacteria returned by the recycled sludge come into contact with substrate of a higher concentration, which decreases during the transport through the tank. Therefore, in the first part, the rate of oxygen consumption is relatively high, resulting in oxygen limitation, particularly inside the flocs. These effects were not described by the relative simple models discussed in Sections 6.2.2 to 6.2.6.

A CSTR is generally more stable to toxic substances as shock loads than all reactors with reduced backmixing, because the toxic material is dispersed rapidly throughout the reactor and the concentration is reduced. In general, a CSTR gives a more uniform effluent for varying loads, but the reactor volume must be larger, because of the lower substrate concentration and the lower carbon removal rate throughout. In axial flow reactors, aeration of the water increases the axial mixing. For a locally constant distribution of aerators (porous ceramic or membrane tubes), the dispersion model may be used successfully. However, to save energy the distribution of aerators should depend on the task: with higher density in the first part at the higher substrate concentration which goes down to a lower density at the end where the concentration and oxygen uptake rate are lower. The model for tanks in series can be here used with more success because of the possibility to change the size of the stages and aeration rates.

6.3

Retention Time Distribution in Activated Sludge Reactors

6.3.1

Retention Time Distribution

To understand and mathematically describe the activated sludge process in continuous operation, information is needed regarding the degree of mixing inside the reactor. Is the reactor completely mixed? Are there short-circuit flows or regions of eddy currents? What is the dispersion coefficient in a selected type of reactor? Such questions can be answered by measurement and mathematical analysis of retention time distributions (Pippel 1978).

Measurement of the distribution is realized by adding tracers at the influent of the investigated system. In principle, any substance can be used as a tracer, as long as it dissolves completely, does not react with other components and does not change state by desorption, adsorption, precipitation or crystallization. In reality, only a few substances are applicable which are cheap enough, can be measured continuously and are not toxic. Frequently, salts, dyes and other substances not found in wastewater are used which can be measured at very low concentrations.

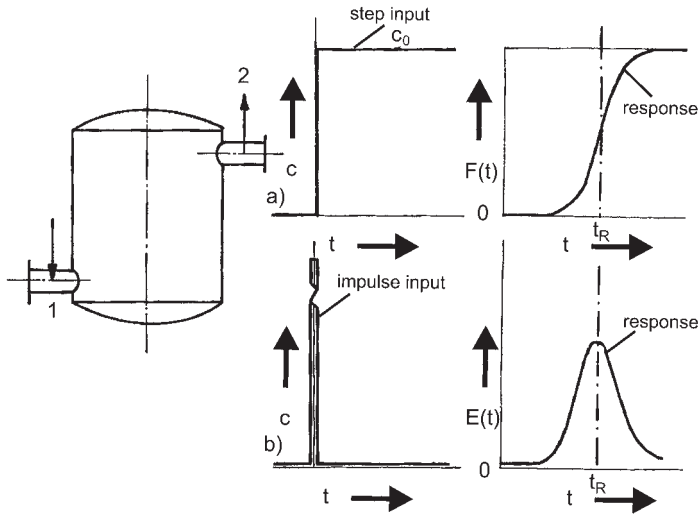


Fig. 6.8 Explanation for measurement of retention time distribution.
 (a) Step input and distribution of retention times $F(t)$ as response.
 (b) Impulse input and density distribution of retention time
 $E(t) = dF/dt$ as response.

The tracer is added at point 1 (Fig. 6.8) as a step signal or as an impulse signal. At point 2, the signal is deformed in a characteristic way. This response to a step signal is called distribution of retention time and is defined as:

$$F = \frac{c(\tau)}{c_0} = C(\tau) \quad (6.88)$$

with:

$$\tau = \frac{t}{t_R} \quad (6.89)$$

The response to an impulse signal $c_i(\tau)$ is called density distribution of retention time and is defined as:

$$E = \frac{c_i(\tau)}{c_0} \quad (6.90)$$

c_0 is here a theoretical concentration, which follows from the mass of the tracer and the volume of the apparatus.

But there is a simple relationship between F and E :

$$E(\tau) = \frac{dF(\tau)}{d\tau} \quad (6.91)$$

The density distribution results from measurements (Fig. 6.8b) and from the distribution by differentiating the dimensionless time.

In the next section we will model some idealized systems and calculate $F(\tau)$.

6.3.2

Completely Mixed Tank

The tank is free of tracer at $t = 0$ and the transport of the tracer concentration starts at $t = 0$. The tracer balance for the completely mixed tank is:

$$V \frac{dc}{dt} = Q_0(c_0 - c) \quad (6.92)$$

After separating the variables and considering $t = 0, c = 0$, the solution is:

$$F(\tau) = \frac{c}{c_0} = 1 - \exp(-\tau) \quad (6.93)$$

or:

$$E(\tau) = \frac{dF(\tau)}{d\tau} = \exp(-\tau) \quad (6.94)$$

Figure 6.9 presents an example of a completely and an incompletely mixed tank reactor with a short circuit flow.

In the incompletely mixed tank, the tracer was measured after only relatively short times at high concentrations in the effluent, as could be expected from a tank (with a short circuit flow) and at a lower concentration after longer times because of the slow transport into eddy flow regions.

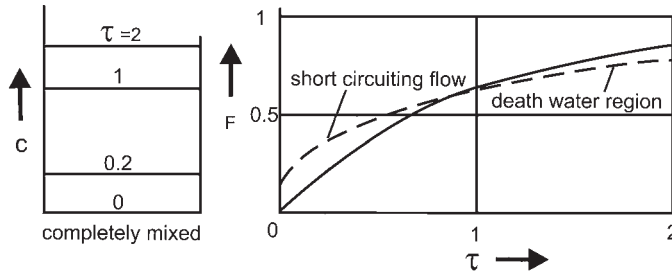


Fig. 6.9 Retention time distribution $F(\tau)$ for a CSTR compared with one from a reactor with a short circuiting flow and eddy water regions.

6.3.3

Completely Mixed Tank Cascade

This type of reactor is only rarely utilized in biological wastewater treatment. But we can compare the responses shown in retention time distribution measurements of a tube reactor with axial dispersion with those for a tank cascade. This enables us to calculate the theoretical number of stages which a real tube reactor has (see Section 6.3.5).

The model for n CSTR tanks consists of n tracer balances:

$$\begin{aligned} V_1 \frac{dc_1}{dt} &= Q_0(c_0 - c_1) \\ V_2 \frac{dc_2}{dt} &= Q_0(c_1 - c_2) \\ &\dots \\ V_n \frac{dc_n}{dt} &= Q_0(c_{n-1} - c_n) \end{aligned} \quad (6.95)$$

with the solution for the probability distribution of residence times:

$$F(\tau) = 1 - \exp(-n\tau) \left[1 + n\tau + \frac{(n\tau)^2}{2!} + \dots + \frac{(n\tau)^{n-1}}{(n-1)!} \right] \quad (6.96)$$

Some results are given in Fig. 6.10.

With increasing n , the probability distribution approaches that of a plug flow reactor, which is the same as a cascade of $n = \infty$ stages (Hill 1977; Chow et al. 1979). By differentiation of Eq. (6.96), the probability density follows:

$$E(\tau) = \frac{dF(\tau)}{d\tau} = n \exp[-n\tau] \frac{(n\tau)^{n-1}}{(n-1)!} \quad (6.97)$$

(Froment and Bischoff 1979; Chow et al. 1979). With increasing n , the peak becomes more narrow and higher.

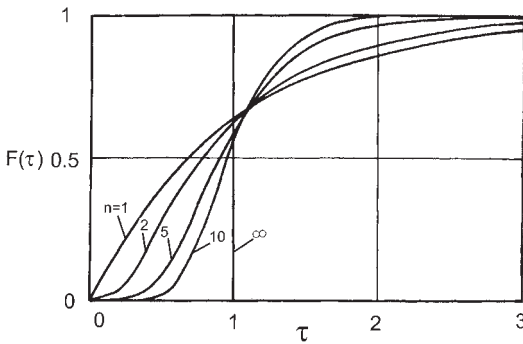


Fig. 6.10 Retention time distribution $F(\tau)$ for a CSTR ($n = 1$) compared with one from a tank cascade system ($n = 2 \dots \infty$) (Wen and Fan 1975).

6.3.4

Tube Flow Reactor With Axial Dispersion

The model for the tank cascade starts with one CSTR coupled with a second stage and so on. Alternatively, we shall begin with a PFR model which is extended by a dispersion term:

$$\frac{dc}{d\tau} = -\bar{w} \frac{dc}{dx} + D_x \frac{d^2c}{dx^2} \quad (6.98)$$

By using the Peclet number:

$$Pe' = \frac{\bar{w} L}{D_x} \quad (6.99)$$

the tracer balance can be written in dimensionless form:

$$\frac{dC}{d\tau} = -\frac{dC}{d(x/L)} + \frac{1}{Pe'} \frac{d^2C}{d(x/L)^2} \quad (6.100)$$

For a flow reactor with sludge recycle:

$$Pe = \frac{\bar{w} (1 + n_R) L}{D_x} \quad (6.80)$$

must be introduced instead of Eq. (6.99). One initial and two boundary conditions are needed.

Several different types of boundary conditions exist (Wen and Fan 1975). We want to limit the discussion only to the type of boundary condition “closed, closed”. Closed boundary means, that in both tubes at the left and right side of the reactor (see Fig. 6.7) are no concentration gradient because of the low section area compared with that inside the reactor.

These boundary conditions are:

$$t = 0, \quad x \geq 0, \quad c = 0 \quad (6.101)$$

$$t > 0, \quad x = 0, \quad w c_0 = w c - D_x \frac{dc}{dx} \quad (6.102)$$

$$t > 0, \quad x = L, \quad \frac{dc}{dx} = 0 \quad (6.103)$$

The first analytical solution of Eq. (6.98) respectively Eq. (6.100) was published by Danckwerts 1955. We only want to present this solution in graphical form (Wen and Fan 1975):

$$Pe' = \frac{wL}{D_x} = 0$$

characterizes the limit for $D_x \gg wL$, which agrees with the signal of a CSTR, In contrast to that, $Pe' \approx \infty$ shows the signal of a ideal plug flow reactor without an axial mixture.

6.3.5

Comparison Between Tank Cascades and Tube Flow Reactors

How we can compare a tube flow reactor with axial dispersion with a series of tanks in cascade? How many stages of a cascade correspond to the same degree of backmixing of a system with a known Peclet number?

Figure 6.12 presents the response to an impulse-formed input. This response is given by Eq. (6.90). $t = t_R$ follows from the correspondence of $A_1 = A_2$.

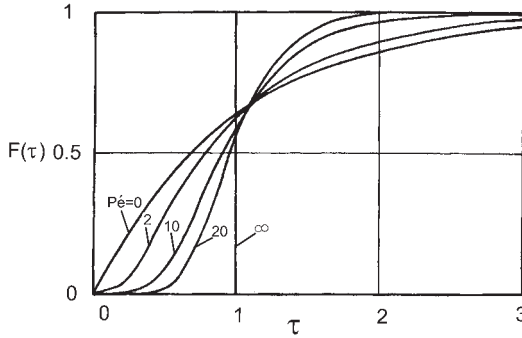


Fig. 6.11 Retention time distribution $F(\tau)$ for a CSTR ($Pe' = 0$) compared with one of real tube flow reactors ($Pe' = 2 \dots \infty$) (Wen and Fan 1975).

The variance according to the property t being investigated is (Froment and Bischoff 1979):

$$\sigma_t^2 = \int_0^\infty E(t) (t - t_R)^2 dt \quad (6.104)$$

Using Eq. (6.104):

$$\sigma^2 = \frac{\sigma_t^2}{t_R^2} = \frac{2}{Pe'} - \frac{2(1 - \exp(-Pe'))}{Pe'^2} \quad (6.105)$$

is obtained for a “closed, closed” vessel (Wen and Fan 1975; Levenspiel 1979).

At high Pe' numbers low dispersion results:

$$\sigma^2 = \frac{2}{Pe'} \quad (6.106)$$

If we consider a tank cascade, Eq. (6.97) must be introduced into Eq. (6.104). A simplified result can be estimated:

$$\sigma^2 = \frac{\sigma_t^2}{t_R^2} = \frac{1}{n} \quad (6.107)$$

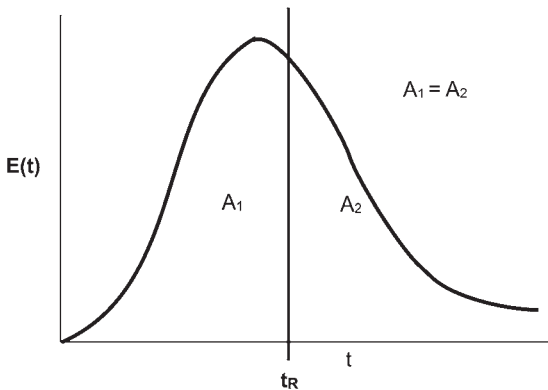


Fig. 6.12 Retention time density distribution $E(t)$ for a tube flow reactor (qualitative drawing).

Combining Eq. (6.105) and (6.107), we obtain

$$n = \frac{Pe'^2}{2(Pe' - 1 + \exp(-Pe'))} \quad (6.108)$$

For $Pe' \gg 1$, then

$$n = \frac{Pe'}{2} \quad (6.109)$$

results. This is an approximation and results f.e. in $n = 50$ for $Pe' = 100$. The exact solution from Eq. (6.108) is $n = 50.55$.

6.4

Technical Scale Activated Sludge Systems for Carbon Removal

Based on hydrodynamic properties and the special models needed for the description of bacterial growth, carbon removal and oxygen consumption, activated sludge reactors can be divided into three groups: (a) nearly completely mixed tank reactors (Fig. 6.13a), (b) nearly completely mixed tank reactor cascades (Fig. 6.13b) and (c) tube flow reactors with axial dispersion (Fig. 6.13c).

A large number of reactor types belong to group (a). Figure 6.13a presents a selection of three systems. However, measurements of residence time distribu-

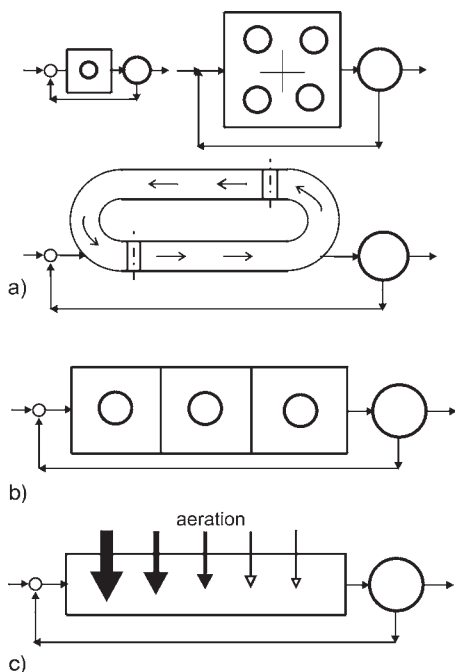


Fig. 6.13 Standard activated sludge systems. (a) Completely mixed tank reactors. (b) Closed tanks in series, frequently used for pure oxygen. (c) Flow tube reactor with diminishing aeration intensity.

tions are typically not performed. As a result, considerable deviations from the CSTR occasionally exist. In Fig. 6.14, two special types of these reactors are shown: a Bio-Hochreaktor for the treatment of industrial effluents with a height of 20–24 m (Fig. 6.14a) and a Putox-Belebungsanlage for the treatment of municipal wastewater from small housing areas, which is built into the ground (Fig. 6.14b).

Tank reactors connected in series have frequently been used for “pure oxygen WWTPs”. The greatest interest was often not the reduction of the degree of mixing, but the more efficient utilization of oxygen. Therefore, the waste gas of the first stage (O_2 , N_2 , CO_2) is dispersed again to take advantage of the relatively high residual oxygen concentration. Normally, cascades with three or four stages are operated; and both the wastewater and the gas flow in the same direction, resulting in a high removal rate in the first stage and a low removal rate in the last stage, where only a small decrease in BOD_5 occurs (see Fig. 6.13b).

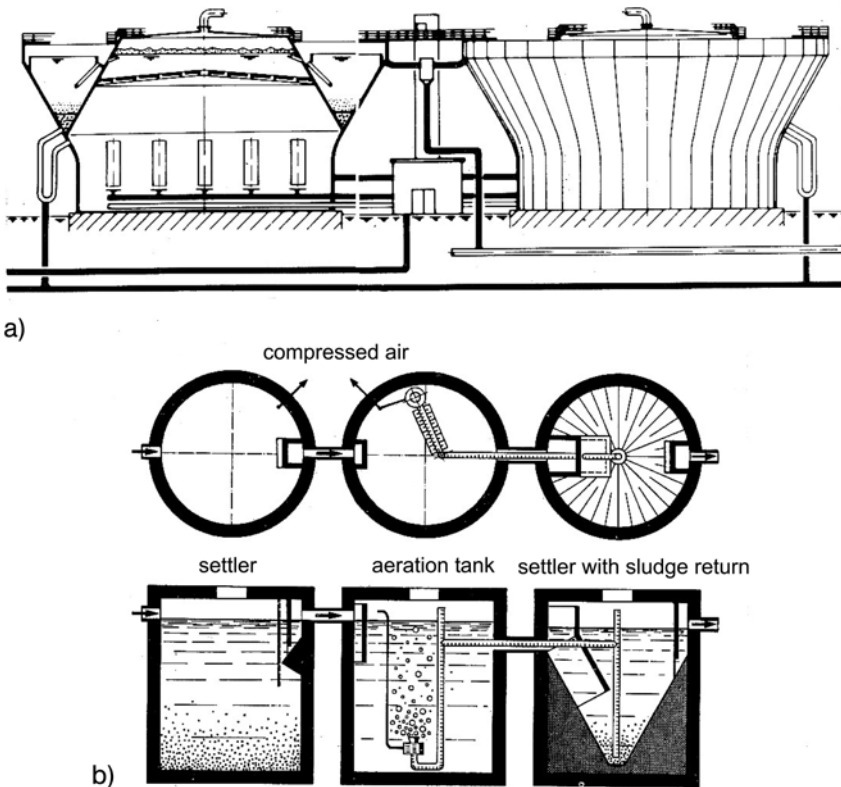


Fig. 6.14 Special types of completely mixed tank reactors in technical scale. (a) Bio-Hochreaktor for the treatment of high-load effluents (Hoechst AG; Leistner et al. 1979). (b) Putox-Belebungsanlage for domestic effluents (5–1500 inhabitants) (Purator, Vienna).

Tube flow reactors, in contrast, are open systems (Fig. 6.13c). They typically have a length of up to 300 m and are constructed in a meandering form to make better use of the available area and to reduce pipe lengths for sludge return lines. A pressurized aeration system is often used (porous ceramics or membranes) and the number of the aerators per basin area is changed from higher values near the wastewater influent to lower values near the effluent.

PROBLEM 6.1

A municipal wastewater ($Q_0 = 10^4 \text{ m}^3 \text{ d}^{-1}$, $S_0 = 500 \text{ mg L}^{-1} \text{ COD}$, $X_e = 0 \text{ mg L}^{-1}$) is to be treated by an activated sludge process. The aerobic reactor is completely mixed. For a bacterial concentration of $X = 2 \text{ g L}^{-1} \text{ MLSS}$, an effluent COD concentration of $S = 25 \text{ mg L}^{-1} \text{ COD}$ of dissolved biodegradable substances should be reached.

Coefficients:

$$Y_{X/S}^o = 0.43 \text{ g MLSS/g COD}$$

$$\mu_{\max} = 7.2 \text{ d}^{-1}$$

$$K_S = 100 \text{ mg L}^{-1} \text{ COD}$$

$$k_d = 0.24 \text{ d}^{-1}$$

1. Calculate the sludge age t_{RX} and the critical sludge age t_{RXC} .
2. Calculate the reactor volume V .
3. What is the mass flow rate of the surplus sludge which must be removed daily?
4. What is the critical mass flow rate?
5. Calculate the mass of bacteria which will be reduced by bacterial decay daily.

Solution

(1) First we have to calculate the sludge age t_{RX} needed, using:

$$S = \frac{K_S (1 + k_d t_{RX})}{t_{RX} (\mu_{\max} - k_d) - 1} \quad (6.49)$$

and eliminating:

$$t_{RX} = \frac{K_S + S}{S (\mu_{\max} - k_d) - K_S k_d} = 0.833 \text{ d} = 20 \text{ h}$$

The critical sludge age is:

$$t_{RXC} = \frac{K_S + S_0}{S_0 (\mu_{\max} - k_d) - K_S k_d} = 0.173 \text{ d} = 4.16 \text{ h}$$

(2) The reactor volume V follows from the substrate balance:

$$Q_0(S_0 - S) = \frac{\mu_{\max}}{Y_{X/S}^o} X \frac{S}{K_S + S} V$$

$$V = \frac{Q_0(S_0 - S) Y_{X/S}^o (K_S + S)}{\mu_{\max} X S} = 709 \text{ m}^3$$

(3) The concentration of excess sludge (without considering sludge thickening by sedimentation) follows from balances for S

$$0 = Q_0(S_0 - s) - \frac{\mu X V}{Y_{X/S}^o}$$

and for X

$$0 = -Q_0 X + \mu X V - k_d X V$$

After elimination of μ :

$$\begin{aligned} Q_0 X &= Y_{X/S}^o \frac{Q_0(S_0 - S)}{1 + k_d} \\ &= 0.43 \frac{10^4 \cdot 475}{1 + 0.24} \text{ mg MLSS} \cdot \text{L}^{-1} \\ &= 1647 \text{ kg d}^{-1} \text{ MLSS} \end{aligned}$$

(4) For $k_d = 0$ and $X_e = 0$ (no bacteria in the overflow of the settler) the mass flow rate for the surplus sludge follows

$$Q_0 X = 2042 \text{ kg m}^{-3} \text{ MLSS}$$

During biodegradation of $Q_0(S_0 - S) = 4750 \text{ kg h}^{-1} \text{ MLSS}$ a mass of $2042 - 1647 = 395 \text{ kg d}^{-1} \text{ MLSS}$ was removed by bacteria decay

(5) Using

$$t_{\text{RXC}} = \frac{VX}{Q_{\text{ex},c} X}$$

the critical removal rate of the excess sludge follows:

$$Q_{\text{ex},c} X_R = \frac{VX}{t_{\text{RXC}}} = \frac{709 \cdot 2}{0.173} = 8197 \text{ kg d}^{-1} \text{ MLSS}$$

For a sludge removal of more than 8200 during some days all bacteria of the activated sludge plant were washed out.

PROBLEM 6.2

The same municipal wastewater as described in Problem 6.1 must be treated in an activated sludge reactor by extended aeration. The amount of surplus sludge is to be reduced by a factor of 2, compared to the maximum value given by $Y_{X/S}^o = 0.43 \text{ g MLSS (g COD)}^{-1}$.

The sludge thickening ratio is $n_E = 2.5$.

The recycle ratio is $n_R = 0.8$.

1. Which sludge age must be selected?
2. Calculate the necessary reactor volume.
3. Compare these results with those of Problem 6.1.

Solution

$$(1) \text{ From: } Y_{X/S} = Y_{X/S}^o \left(1 - \frac{k_d}{\mu}\right) \quad (4.27)$$

and:

$$Y_{X/S} = 0.5 Y_{X/S}^o$$

it follows that: $\mu = 2 k_d = 0.48 \text{ d}^{-1}$

$$\mu = \mu_{\max} \frac{S}{K_S + S}$$

The sludge age follows from:

$$t_{RX} = \frac{K_S + S}{S (\mu_{\max} - k_d) - K_S k_d} = 4.17 \text{ d}$$

- (2) The reactor volume V follows from the substrate balance:

$$Q_0(S_0 - S) = \frac{\mu_{\max}}{Y_{X/S}^o} X \frac{S}{K_S + S} V$$

$$V = \frac{Q_0(S_0 - S) Y_{X/S}^o (K_S + S)}{\mu_{\max} S X} = 2208 \text{ m}^3$$

- (3) Comparison with the results from Problem 6.1:

Problem 6.1: $t_{RX} = 0.833 \text{ d}$, $S = 25 \text{ mg L}^{-1} \text{ COD}$ and $V = 709 \text{ m}^3$
excess sludge: $1702 \text{ kg d}^{-1} \text{ MLSS}$

Problem 6.2: $t_{RX} = 4.217 \text{ d}$, $S = 7.14 \text{ mg L}^{-1} \text{ COD}$ and $V = 2208 \text{ m}^3$
excess sludge: $1065 \text{ kg d}^{-1} \text{ MLSS}$

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