Heat and Mass Transfer: Fundamentals & Applications
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Chapter 10
BOILING AND CONDENSATION

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Objectives

- Differentiate between evaporation and boiling, and gain familiarity with different types of boiling
- Develop a good understanding of the boiling curve, and the different boiling regimes corresponding to different regions of the boiling curve
- Calculate the heat flux and its critical value associated with nucleate boiling, and examine the methods of boiling heat transfer enhancement
- Derive a relation for the heat transfer coefficient in laminar film condensation over a vertical plate
- Calculate the heat flux associated with condensation on inclined and horizontal plates, vertical and horizontal cylinders or spheres, and tube bundles
- Examine dropwise condensation and understand the uncertainties associated with them

BOILING HEAT TRANSFER

- **Evaporation** occurs at the *liquid–vapor interface* when the vapor pressure is less than the saturation pressure of the liquid at a given temperature.
- **Boiling** occurs at the *solid–liquid interface* when a liquid is brought into contact with a surface maintained at a temperature sufficiently above the saturation temperature of the liquid.

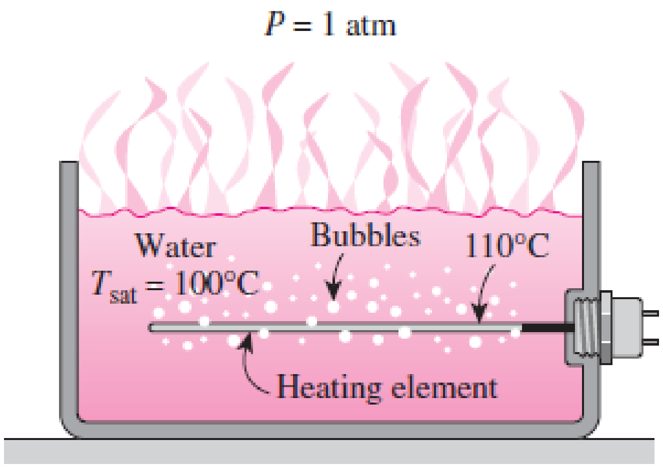


FIGURE 10–2
Boiling occurs when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid.

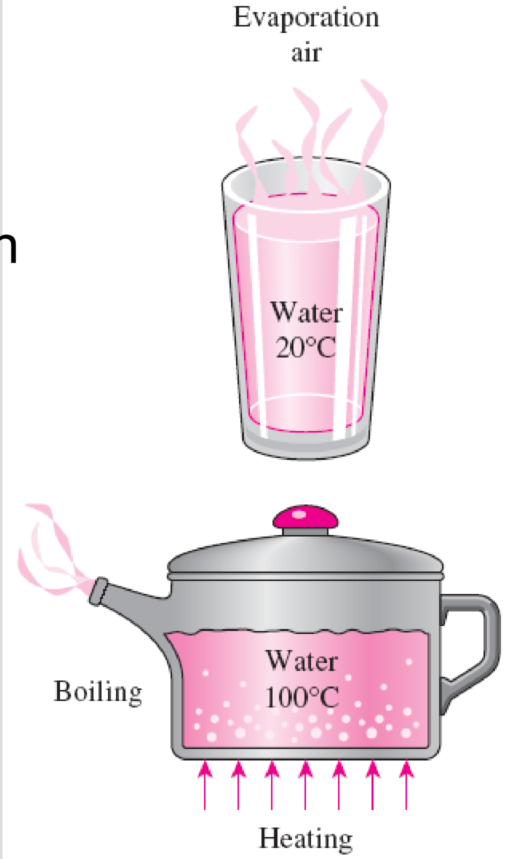


FIGURE 10–1
A liquid-to-vapor phase change process is called *evaporation* if it originates at a liquid–vapor interface and *boiling* if it occurs at a solid–liquid interface.

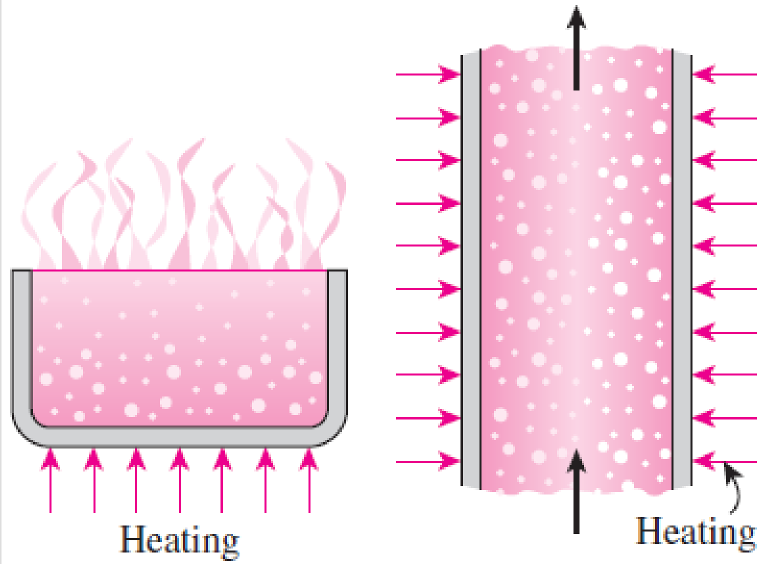
Boiling heat flux from a solid surface to the fluid

$$\dot{q}_{\text{boiling}} = h(T_s - T_{\text{sat}}) = h\Delta T_{\text{excess}} \qquad (\text{W/m}^2)$$

$$\Delta T_{\text{excess}} = T_s - T_{\text{sat}}$$ *excess temperature*

Classification of boiling

- Boiling is called **pool boiling** in the absence of bulk fluid flow.
- Any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy.
- Boiling is called **flow boiling** in the presence of bulk fluid flow.
- In flow boiling, the fluid is forced to move in a heated pipe or over a surface by external means such as a pump.



(*a*) Pool boiling (*b*) Flow boiling

FIGURE 10–3
Classification of boiling on the basis of the presence of bulk fluid motion.

Subcooled Boiling

- When the temperature of the main body of the liquid is below the saturation temperature.

Saturated Boiling

- When the temperature of the liquid is equal to the saturation temperature.

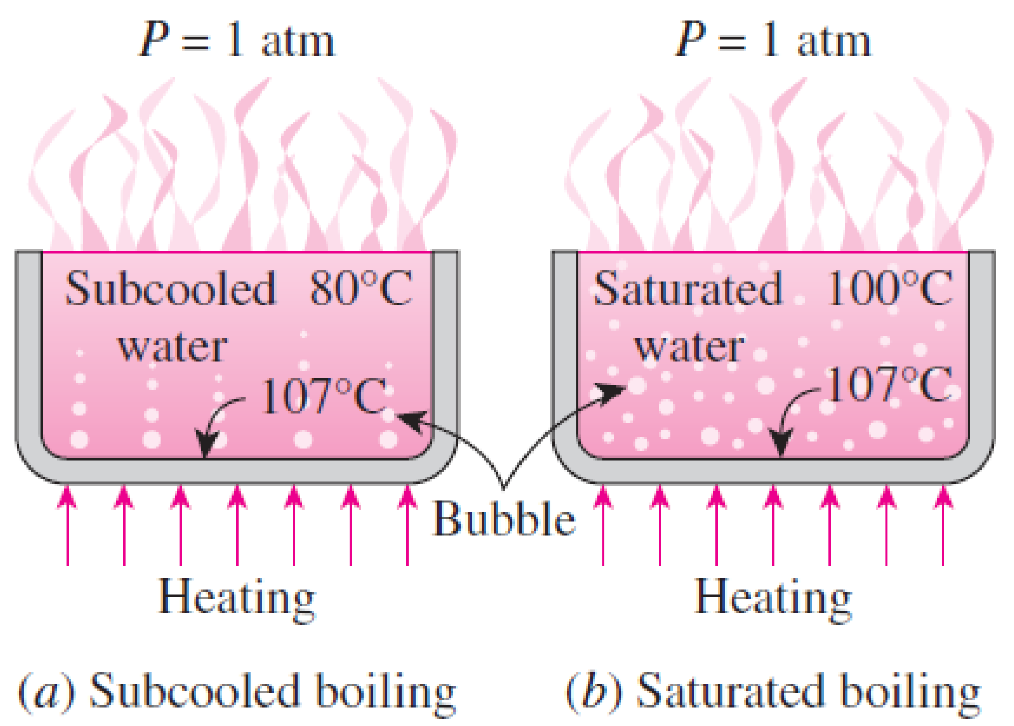


FIGURE 10–4
Classification of boiling on the basis of the presence of bulk liquid temperature.

POOL BOILING

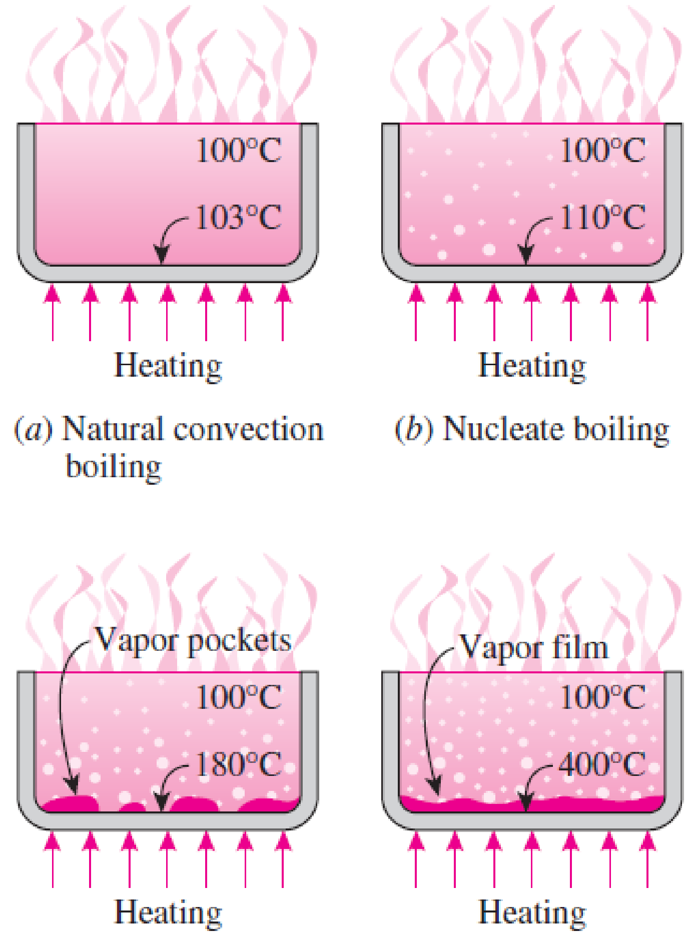
In pool boiling, the fluid is not forced to flow by a mover such as a pump.

Any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy.

Boiling Regimes and the Boiling Curve

$$\dot{q}_{\text{boiling}} = h(T_s - T_{\text{sat}}) = h\Delta T_{\text{excess}}$$

Boiling takes different forms, depending on the $\Delta T_{excess} = T_s - T_{sat}$



(*a*) Natural convection boiling

(*b*) Nucleate boiling

(*c*) Transition boiling

(*d*) Film boiling

FIGURE 10–5

Different boiling regimes in pool boiling.

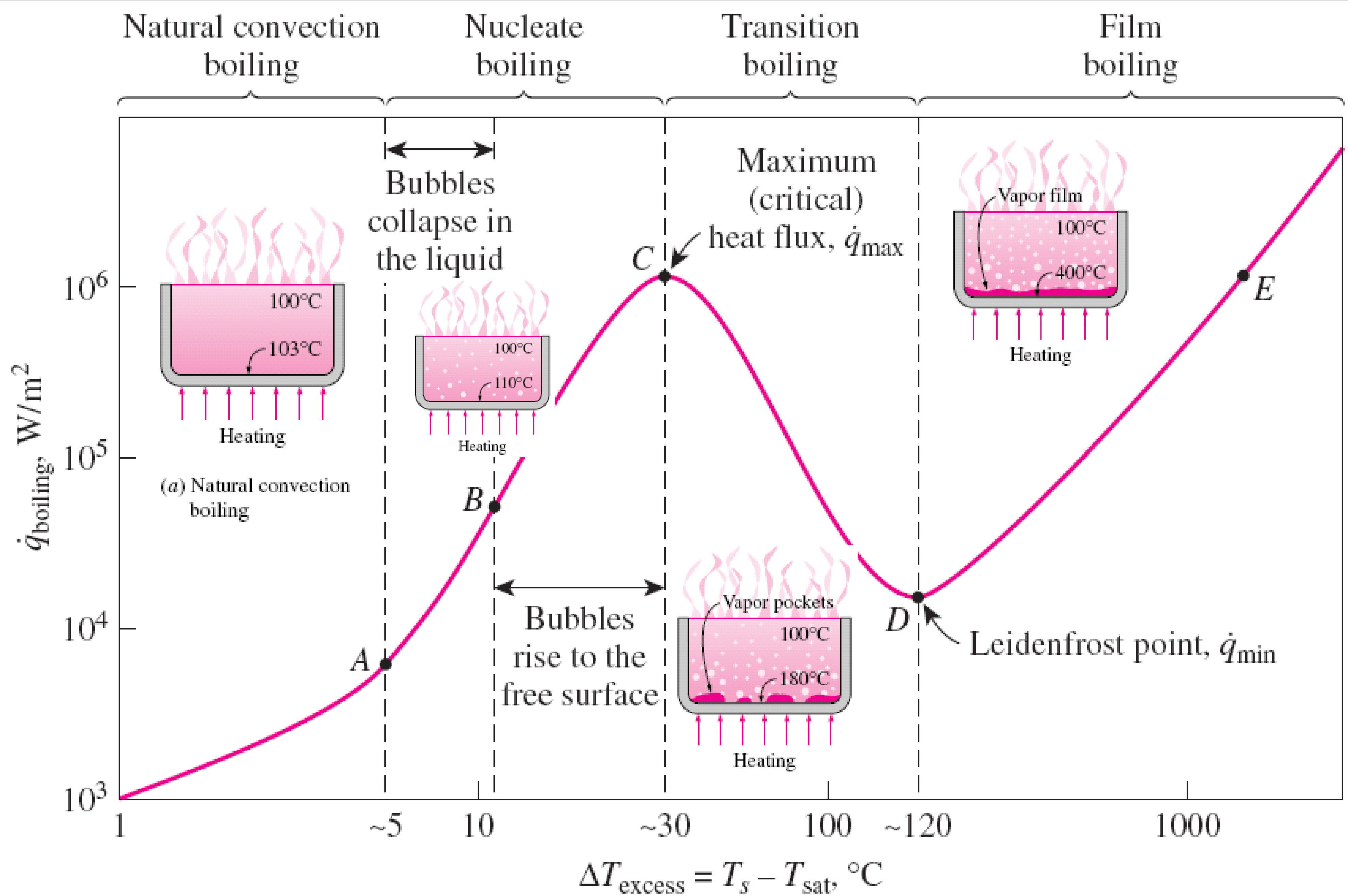
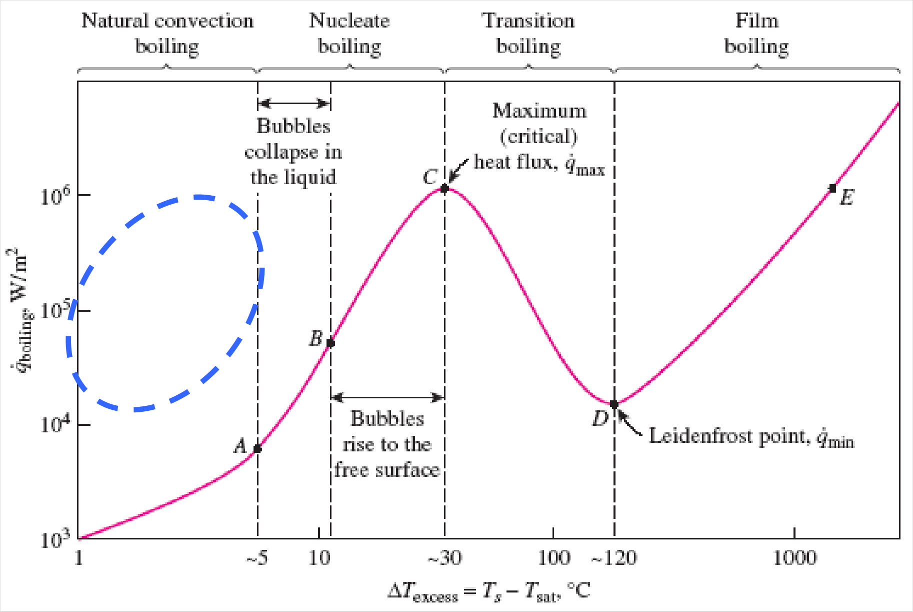


FIGURE 10–6

Typical boiling curve for water at 1 atm pressure.

Natural Convection Boiling (to Point *A* on the Boiling Curve)

- Bubbles do not form on the heating surface until the liquid is heated a few degrees above the saturation temperature (about 2 to 6°C for water)
- The liquid is slightly *superheated* in this case (*metastable* state).
- The fluid motion in this mode of boiling is governed by natural convection currents.
- Heat transfer from the heating surface to the fluid is by natural convection.
- For the conditions of Fig. 10–6, natural convection boiling ends at an excess temperature of about 5°C.



Nucleate Boiling (between Points A and C)

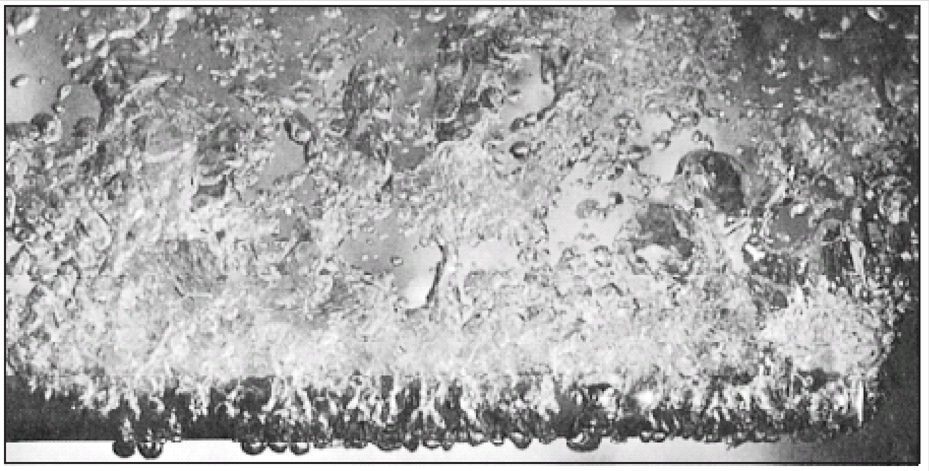
- The bubbles form at an increasing rate at an increasing number of nucleation sites as we move along the boiling curve toward point *C*.

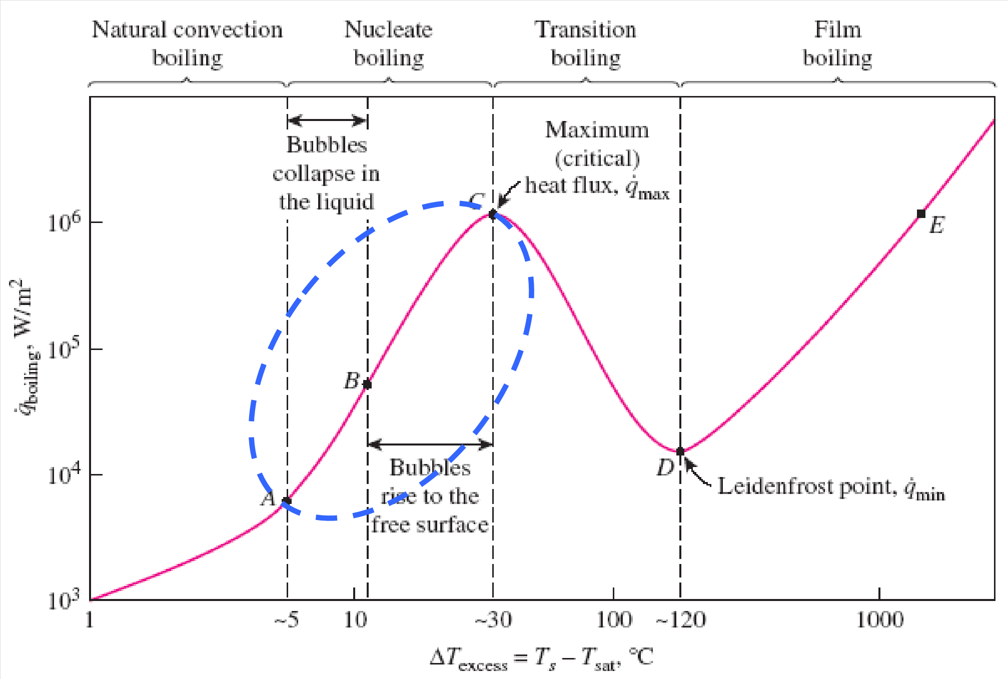
- **Region *A–B*** — *isolated bubbles.*

$$(5°C \leq \Delta T_{excess} \leq 10°C)$$

- **Region *B–C*** — numerous *continuous columns of vapor* in the liquid.

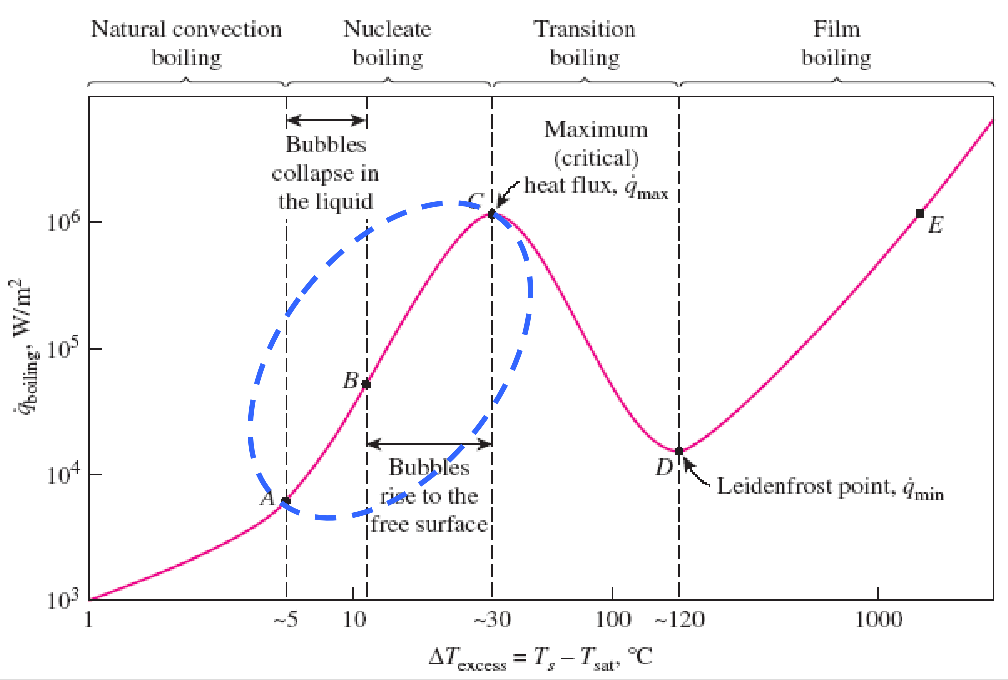
$$(10°C \leq \Delta T_{excess} \leq 30°C)$$





Point *A* is referred to as the ***onset of nucleate boiling (ONB)***.

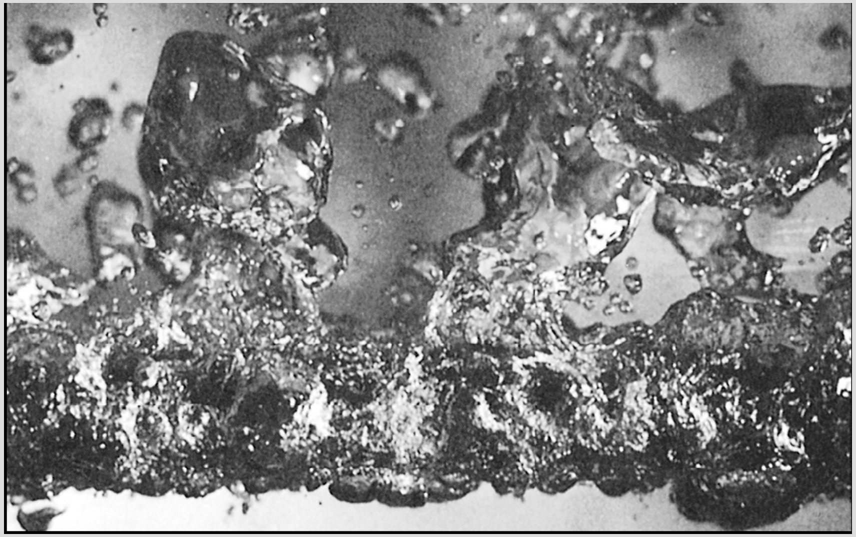
- In region *A–B* the stirring and agitation caused by the entrainment of the liquid to the heater surface is primarily responsible for the increased heat transfer coefficient.
- In region *A–B* the large heat fluxes obtainable in this region are caused by the combined effect of liquid entrainment and evaporation.
- For the entire nucleate boiling range, the heat transfer coefficient ranges from about 2000 to 30,000 W/m^2·K.



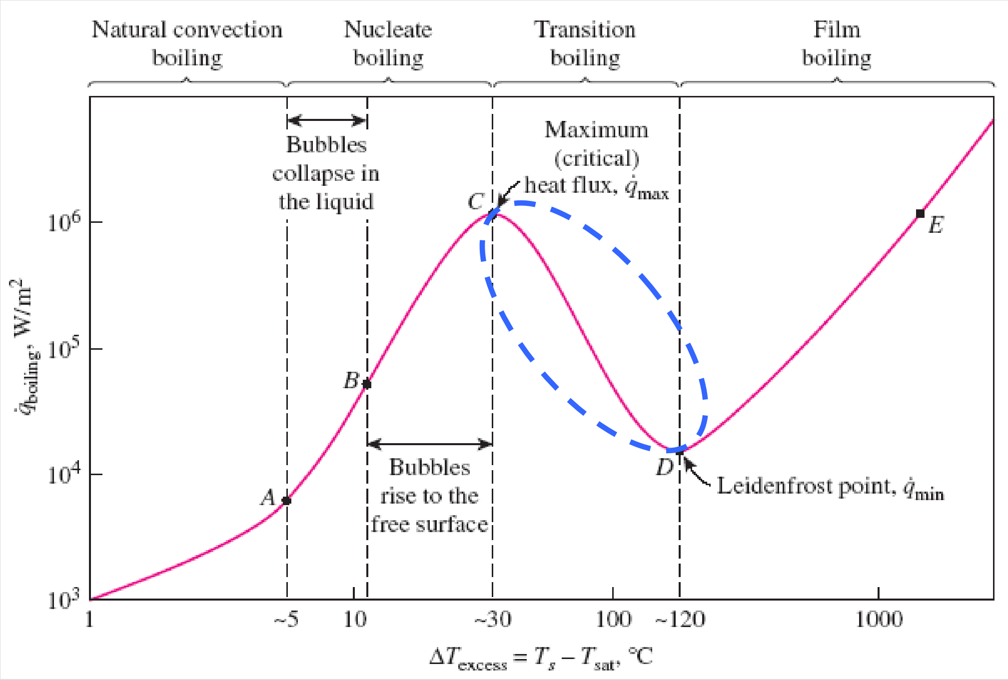
- After point *B* the heat flux increases at a lower rate with increasing ΔT_{excess}, and reaches a maximum at point *C*.
- The heat flux at this point is called the **critical** (or **maximum**) **heat flux**, and is of prime engineering importance.

Transition Boiling (between Points C and D)

- When ΔT_{excess} is increased past point *C*, the heat flux decreases.
- This is because a large fraction of the heater surface is covered by a vapor film, which acts as an insulation.

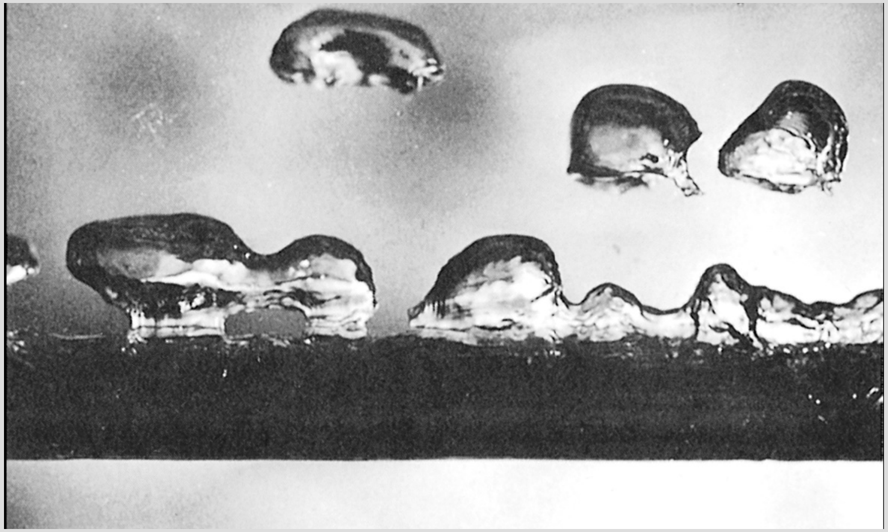


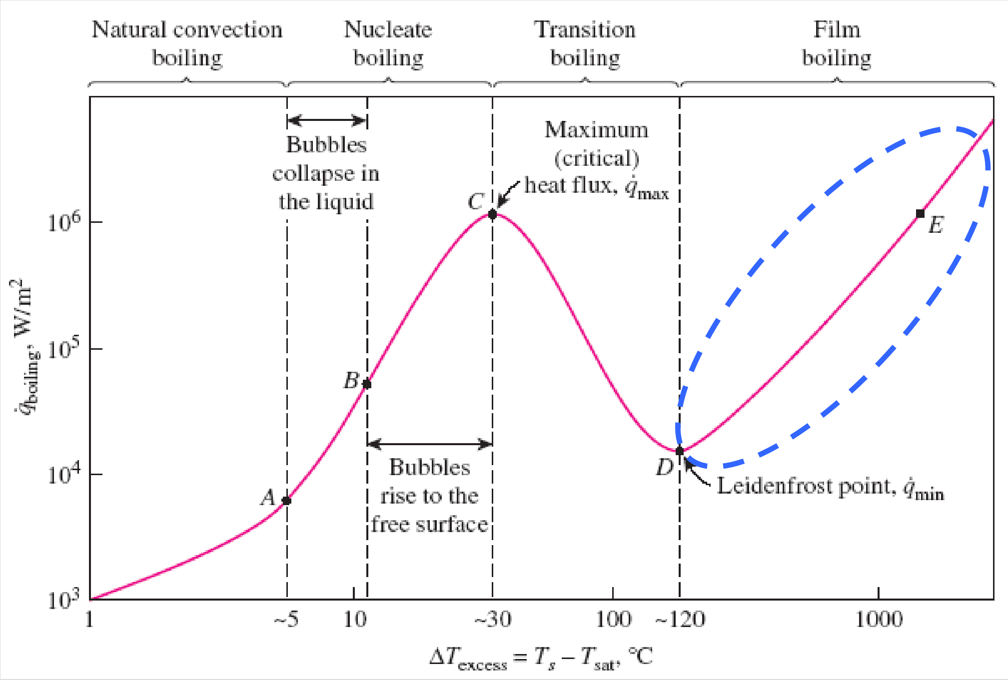
- In the transition boiling regime, both nucleate and film boiling partially occur.
- Operation in the transition boiling regime, which is also called the *unstable film boiling regime*, is avoided in practice.
- For water, transition boiling occurs over the excess temperature range from about 30°C to about 120°C.



Film Boiling (beyond Point *D*

- Beyond point *D* the heater surface is completely covered by a continuous stable vapor film.
- Point *D*, where the heat flux reaches a minimum is called the **Leidenfrost point**.
- The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates in the film boiling region.
- The heat transfer rate increases with increasing excess temperature due to radiation to the liquid.





Burnout Phenomenon

- A typical boiling process does not follow the boiling curve beyond point *C*.
- When the power applied to the heated surface exceeded the value at point *C* even slightly, the surface temperature increased suddenly to point *E*.
- When the power is reduced gradually starting from point *E* the cooling curve follows Fig. 10–8 with a sudden drop in excess temperature when point *D* is reached.

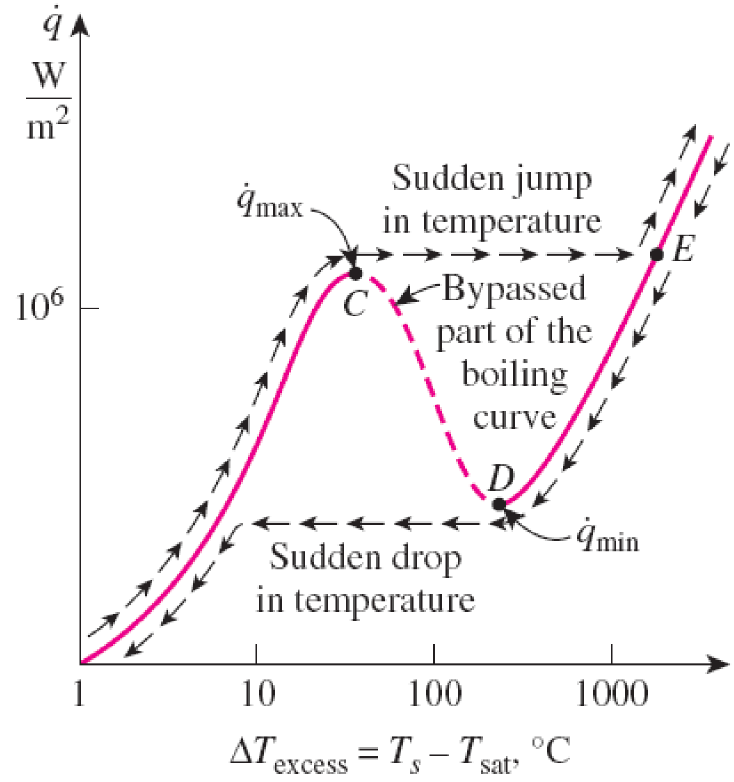


FIGURE 10–8
The actual boiling curve obtained with heated platinum wire in water as the heat flux is increased and then decreased.

Any attempt to increase the heat flux beyond q_{max} will cause the operation point on the boiling curve to jump suddenly from point *C* to point *E*.

However, surface temperature that corresponds to point *E* is beyond the melting point of most heater materials, and *burnout* occurs.

Therefore, point *C* on the boiling curve is also called the **burnout point**, and the heat flux at this point the **burnout heat flux**.

Most boiling heat transfer equipment in practice operate slightly below q_{max} to avoid any disastrous burnout.

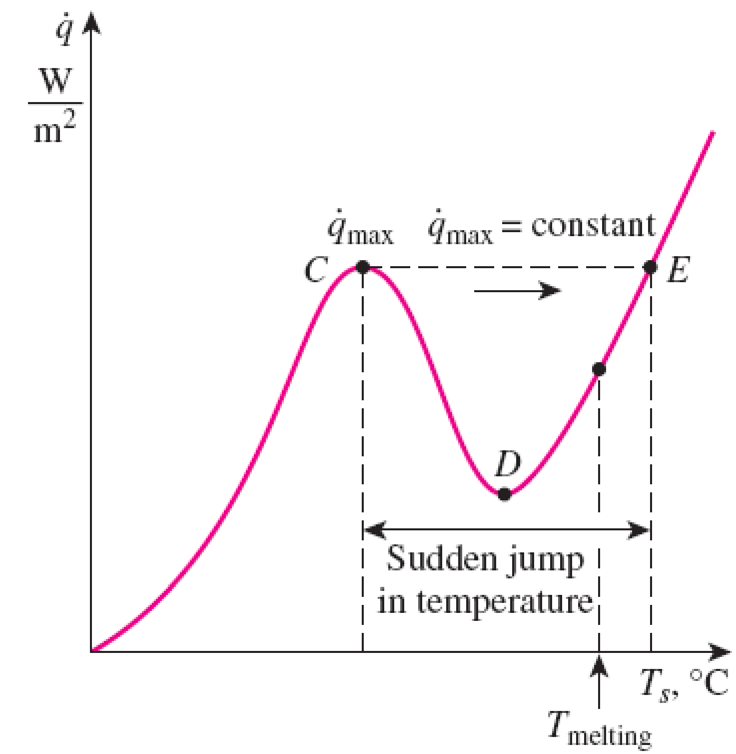


FIGURE 10–9
An attempt to increase the boiling heat flux beyond the critical value often causes the temperature of the heating element to jump suddenly to a value that is above the melting point, resulting in *burnout*.

Heat Transfer Correlations in Pool Boiling

- Boiling regimes differ considerably in their character.
- Different heat transfer relations need to be used for different boiling regimes.
- In the *natural convection boiling* regime heat transfer rates can be accurately determined using natural convection relations.

Nucleate Boiling

- No general theoretical relations for heat transfer in the nucleate boiling regime is available.
- Experimental based correlations are used.
- The rate of heat transfer strongly depends on the nature of nucleation and the type and the condition of the heated surface.

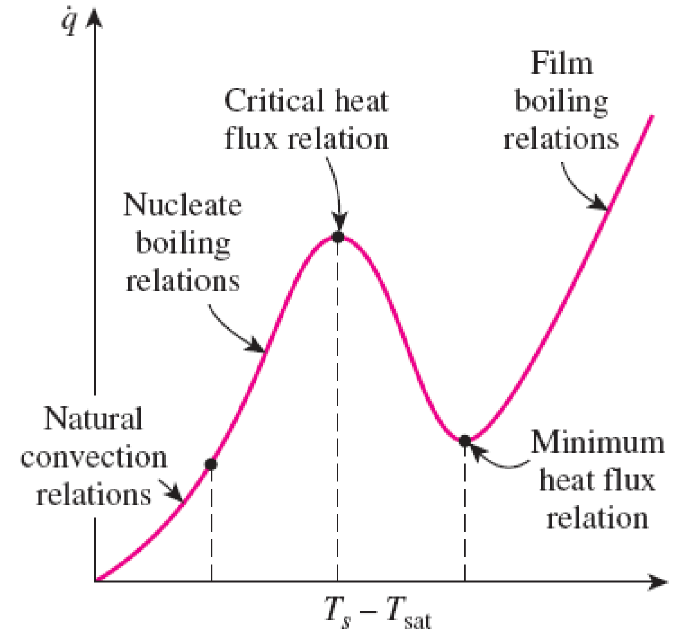


FIGURE 10–10
Different relations are used to determine the heat flux in different boiling regimes.

- For nucleate boiling a widely used correlation proposed in 1952 by Rohsenow:

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \mathrm{Pr}_l^n}\right]^3$$

$\dot{q}_{\text{nucleate}}$ = nucleate boiling heat flux, W/m²
μ_l = viscosity of the liquid, kg/m·s
h_{fg} = enthalpy of vaporization, J/kg
g = gravitational acceleration, m/s²
ρ_l = density of the liquid, kg/m³
ρ_v = density of the vapor, kg/m³
σ = surface tension of liquid–vapor interface, N/m
c_{pl} = specific heat of the liquid, J/kg·°C
T_s = surface temperature of the heater, °C
T_{sat} = saturation temperature of the fluid, °C
C_{sf} = experimental constant that depends on surface–fluid combination
Pr_l = Prandtl number of the liquid
n = experimental constant that depends on the fluid

$$\dot{q} = \left(\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)\left(\frac{\text{J}}{\text{kg}}\right) \times \left(\frac{\frac{\text{m}}{\text{s}^2}\frac{\text{kg}}{\text{m}^3}}{\frac{\text{N}}{\text{m}}}\right)^{1/2}\left(\frac{\frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}{}^\circ\text{C}}{\frac{\text{J}}{\text{kg}}}\right)^3 = \frac{\text{W}}{\text{m}}\left(\frac{1}{\text{m}^2}\right)^{1/2}(1)^3 = \text{W/m}^2$$

FIGURE 10–11
Equation 10–2 gives the boiling heat flux in W/m² when the quantities are expressed in the units specified in their descriptions.

TABLE 10–1

Surface tension of liquid–vapor interface for water

T, °C	σ, N/m*
0	0.0757
20	0.0727
40	0.0696
60	0.0662
80	0.0627
100	0.0589
120	0.0550
140	0.0509
160	0.0466
180	0.0422
200	0,0377
220	0.0331
240	0.0284
260	0.0237
280	0.0190
300	0.0144
320	0.0099
340	0.0056
360	0.0019
374	0.0

*Multiply by 0.06852 to convert to lbf/ft or by 2.2046 to convert to lbm/s².

TABLE 10-2

Surface tension of some fluids (from Suryanarayana, originally based on data from Jasper)

Substance and Temp. Range	Surface Tension, σ, N/m* (T in °C)
Ammonia, –75 to –40°C:	$0.0264 + 0.000223T$
Benzene, 10 to 80°C:	$0.0315 - 0.000129T$
Butane, –70 to –20°C:	$0.0149 - 0.000121T$
Carbon dioxide, –30 to –20°C:	$0.0043 - 0.000160T$
Ethyl alcohol, 10 to 70°C:	$0.0241 - 0.000083T$
Mercury, 5 to 200°C:	$0.4906 - 0.000205T$
Methyl alcohol, 10 to 60°C:	$0.0240 - 0.000077T$
Pentane, 10 to 30°C:	$0.0183 - 0.000110T$
Propane, –90 to –10°C:	$0.0092 - 0.000087T$

*Multiply by 0.06852 to convert to lbf/ft or by 2.2046 to convert to lbm/s².

TABLE 10–3

Values of the coefficient C_{sf} and n for various fluid–surface combinations

Fluid–Heating Surface Combination	C_{sf}	n
Water–copper (polished)	0.0130	1.0
Water–copper (scored)	0.0068	1.0
Water–stainless steel (mechanically polished)	0.0130	1.0
Water–stainless steel (ground and polished)	0.0060	1.0
Water–stainless steel (teflon pitted)	0.0058	1.0
Water–stainless steel (chemically etched)	0.0130	1.0
Water–brass	0.0060	1.0
Water–nickel	0.0060	1.0
Water–platinum	0.0130	1.0
n-Pentane–copper (polished)	0.0154	1.7
n-Pentane–chromium	0.0150	1.7
Benzene–chromium	0.1010	1.7
Ethyl alcohol–chromium	0.0027	1.7
Carbon tetrachloride–copper	0.0130	1.7
Isopropanol–copper	0.0025	1.7

Peak Heat Flux

- The *maximum* (or *critical*) *heat flux* (CHF) in nucleate pool boiling:

$$\dot{q}_{max} = C_{cr}\, h_{fg}[\sigma g \rho^2_v (\rho_l - \rho_v)]^{1/4}$$

C_{cr} is a constant whose value depends on the heater geometry, but generally is about 0.15.

- The CHF is independent of the fluid–heating surface combination, as well as the viscosity, thermal conductivity, and the specific heat of the liquid.
- The CHF increases with pressure up to about one-third of the critical pressure, and then starts to decrease and becomes zero at the critical pressure.
- The CHF is proportional to h_{fg}, and large maximum heat fluxes can be obtained using fluids with a large enthalpy of vaporization, such as water.

TABLE 10–4

Values of the coefficient C_{cr} for use in Eq. 10–3 for maximum heat flux (dimensionless parameter $L^* = L[g(\rho_l - \rho_v)/\sigma]^{1/2}$)

Heater Geometry	C_{cr}	Charac. Dimension of Heater, L	Range of L^*
Large horizontal flat heater	0.149	Width or diameter	$L^* > 27$
Small horizontal flat heater[1]	$18.9K_1$	Width or diameter	$9 < L^* < 20$
Large horizontal cylinder	0.12	Radius	$L^* > 1.2$
Small horizontal cylinder	$0.12L^{*-0.25}$	Radius	$0.15 < L^* < 1.2$
Large sphere	0.11	Radius	$L^* > 4.26$
Small sphere	$0.227L^{*-0.5}$	Radius	$0.15 < L^* < 4.26$

[1] $K_1 = \sigma/[g(\rho_l - \rho_v)A_{heater}]$

Minimum Heat Flux

- Minimum heat flux, which occurs at the Leidenfrost point, is of practical interest since it represents the lower limit for the heat flux in the film boiling regime.
- Zuber derived the following expression for the minimum heat flux for a *large horizontal plate*

$$\dot{q}_{min} = 0.09 \rho_v h_{fg} \left[\frac{\sigma g (\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

- This relation above can be in error by 50% or more.

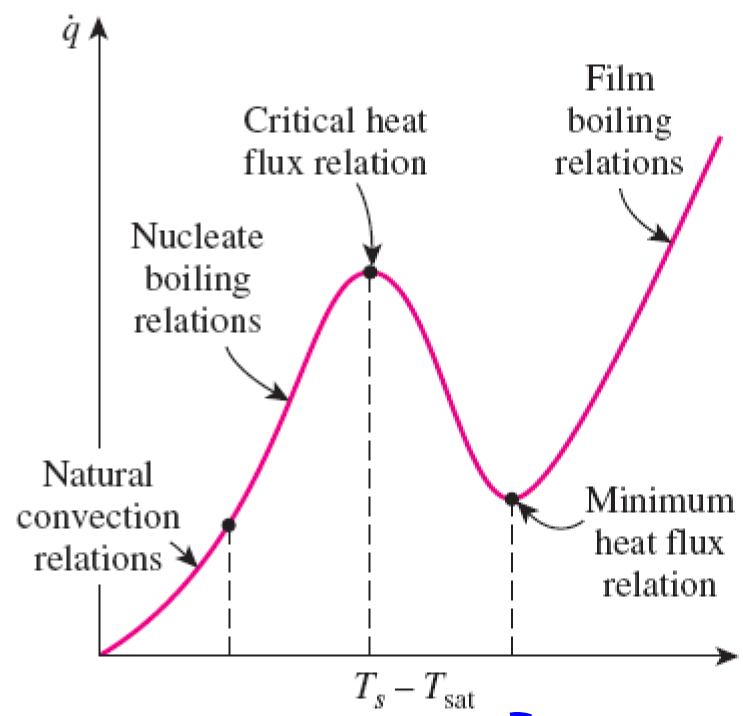


FIGURE 10–10
Different relations are used to determine the heat flux in different boiling regimes.

Transition boiling regime

Operation in the *transition boiling* regime ($30°C \leq \Delta T_{excess} \leq 120°C$) is normally avoided in the design of heat transfer equipment, and thus no major attempt has been made to develop general correlations for boiling heat transfer in this regime. However, the upper (*peak heat flux*, $\dot{q}_{max}$) and the lower (*minimum heat flux*, $\dot{q}_{min}$) limits of this region are of interest to heat transfer equipment designers.

Film Boiling

The heat flux for film boiling on a *horizontal cylinder* or *sphere* of diameter *D* is given by

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{g k_v^3\ \rho_v(\rho_l - \rho_v)[h_{fg} + 0.4c_{pv}(T_s - T_{\text{sat}})]}{\mu_v D(T_s - T_{\text{sat}})}\right]^{1/4}(T_s - T_{\text{sat}})$$

$$C_{\text{film}} = \begin{cases} 0.62 \text{ for horizontal cylinders} \\ 0.67 \text{ for spheres} \end{cases}$$

- At high surface temperatures (typically above 300°C), heat transfer across the vapor film by *radiation* becomes significant and needs to be considered.

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)$$

For $\dot{q}_{\text{rad}} < \dot{q}_{\text{film}}$,

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}}$$

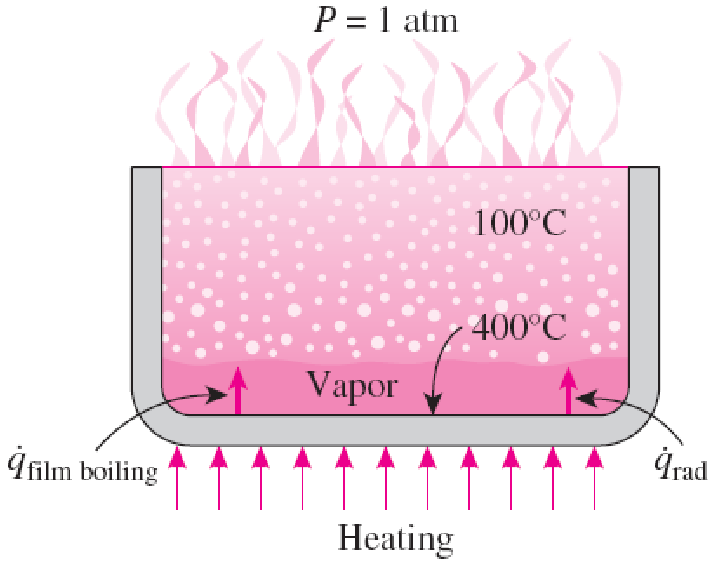


FIGURE 10–12
At high heater surface temperatures, radiation heat transfer becomes significant during film boiling.

Enhancement of Heat Transfer in Pool Boiling

- The rate of heat transfer in the nucleate boiling regime strongly depends on the number of active nucleation sites on the surface, and the rate of bubble formation at each site.
- Therefore, modification that enhances *nucleation* on the heating surface will also enhance *heat transfer* in nucleate boiling.
- *Irregularities* on the heating surface, including roughness and dirt, serve as additional nucleation sites during boiling.
- The effect of surface roughness is observed to decay with time.

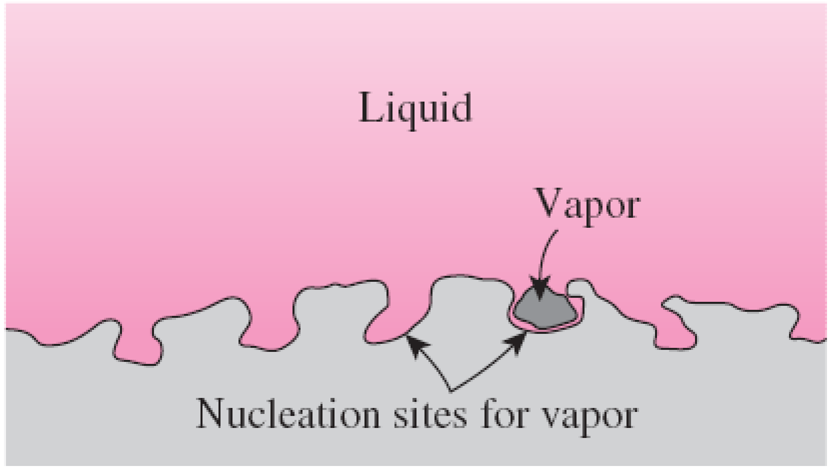


FIGURE 10–13
The cavities on a rough surface act as nucleation sites and enhance boiling heat transfer.

- Surfaces that provide enhanced heat transfer in nucleate boiling *permanently* are being manufactured and are available in the market.
- Heat transfer can be enhanced by a factor of up to 10 during nucleate boiling, and the critical heat flux by a factor of 3.
- The use of finned surfaces is also known to enhance nucleate boiling heat transfer and the maximum heat flux.
- Boiling heat transfer can also be enhanced by other techniques such as *mechanical agitation* and *surface vibration.*
- These techniques are not practical, however, because of the complications involved.

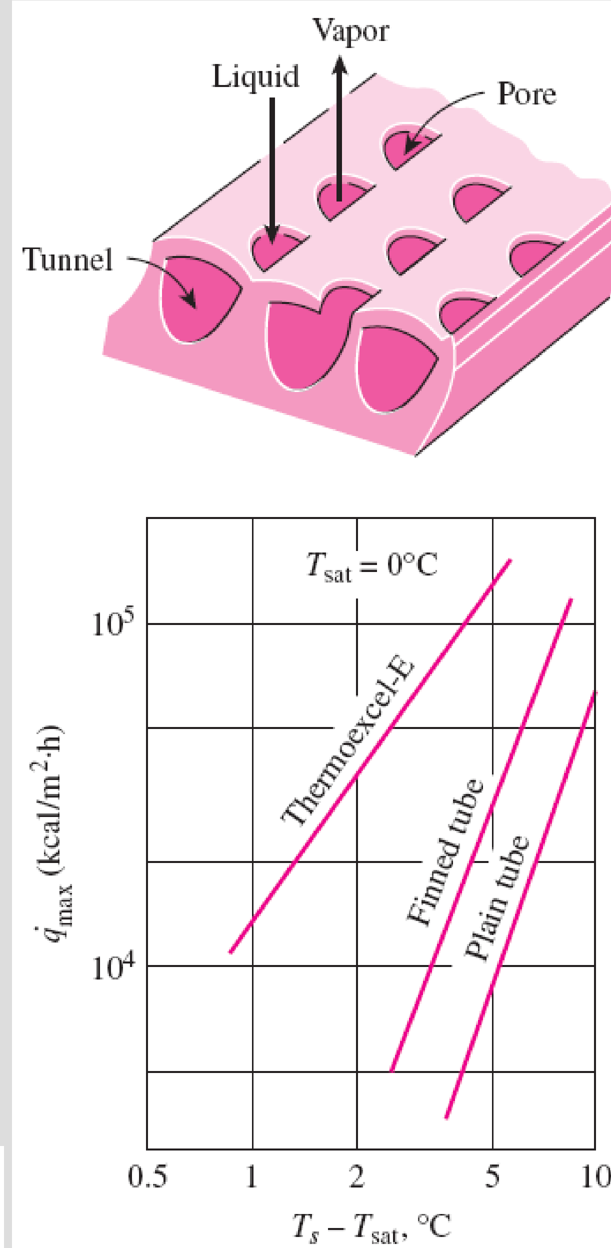


FIGURE 10–14
The enhancement of boiling heat transfer in Freon-12 by a mechanically roughened surface, thermoexcel-E.

FLOW BOILING

- In **flow boiling**, the fluid is forced to move by an external source such as a pump as it undergoes a phase-change process.
- It exhibits the combined effects of convection and pool boiling.
- *External flow boiling* over a plate or cylinder is similar to pool boiling, but the added motion increases both the nucleate boiling heat flux and the maximum heat flux considerably.
- The higher the velocity, the higher the nucleate boiling heat flux and the critical heat flux.
- *Internal flow boiling*, commonly referred to as *two-phase flow*, is much more complicated in nature because there is no free surface for the vapor to escape, and thus both the liquid and the vapor are forced to flow together.

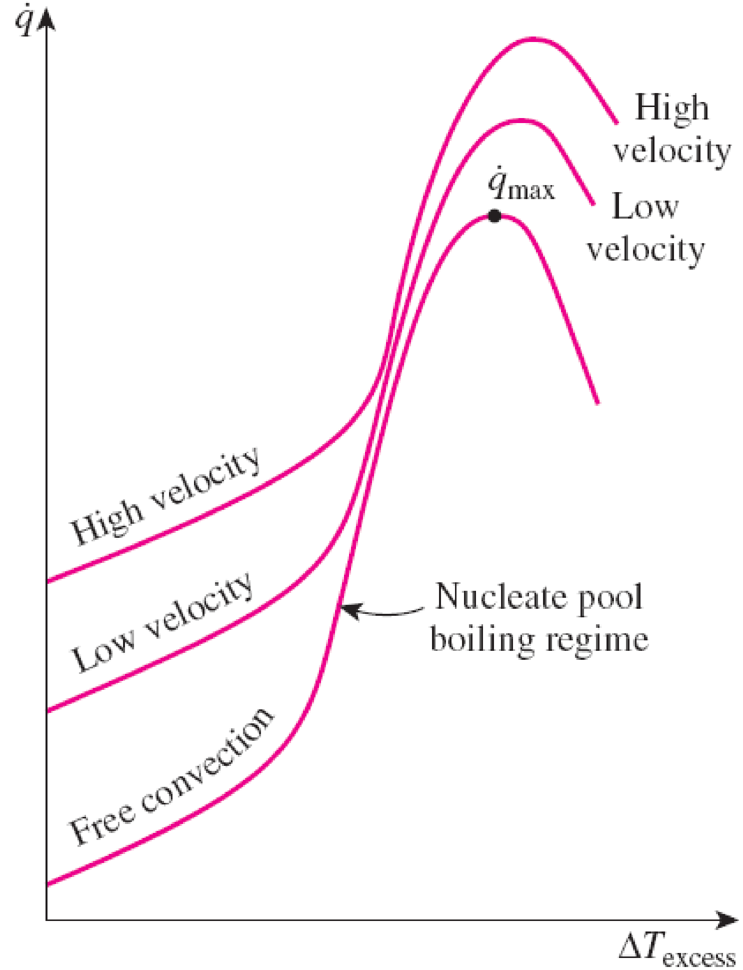


FIGURE 10–18
The effect of forced convection on external flow boiling for different flow velocities.

- The two-phase flow in a tube exhibits different flow boiling regimes, depending on the relative amounts of the liquid and the vapor phases.
- Note that the tube contains a liquid before the bubbly flow regime and a vapor after the mist-flow regime.
- Heat transfer in those two cases can be determined using the appropriate relations for single-phase convection heat transfer.

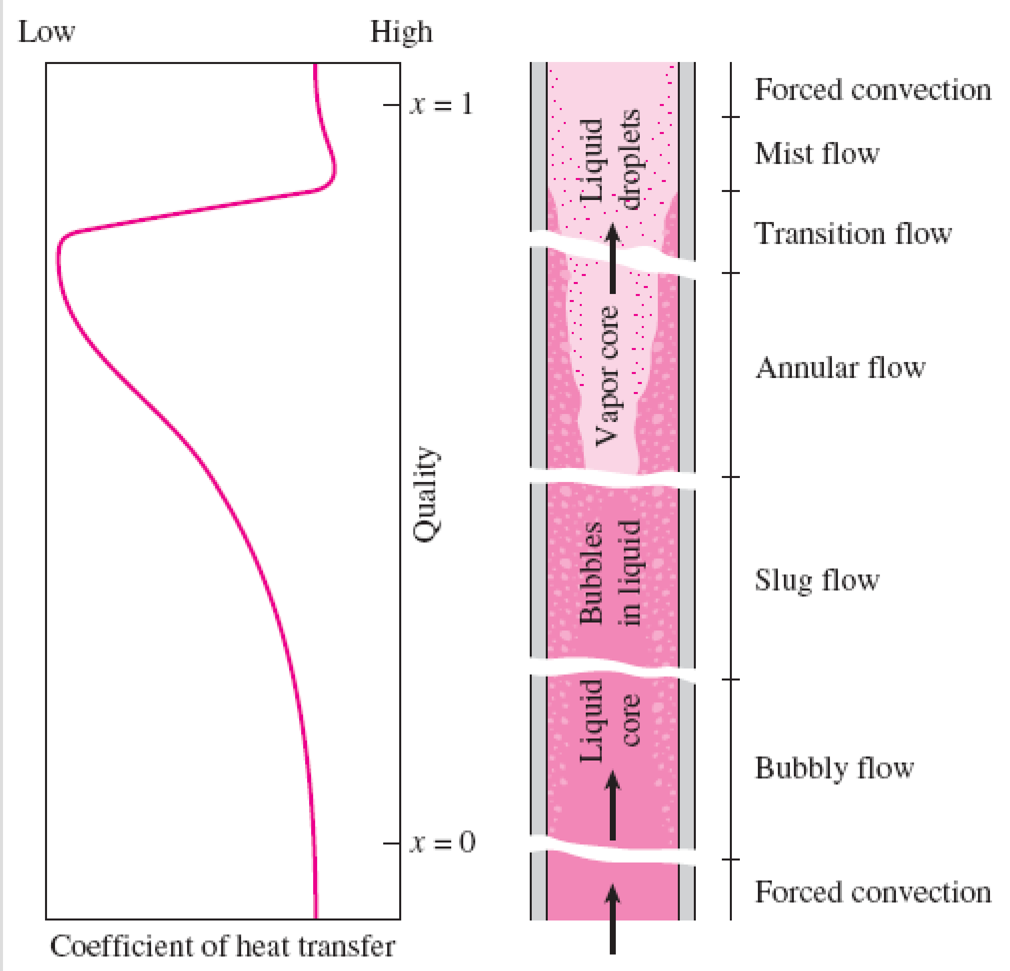
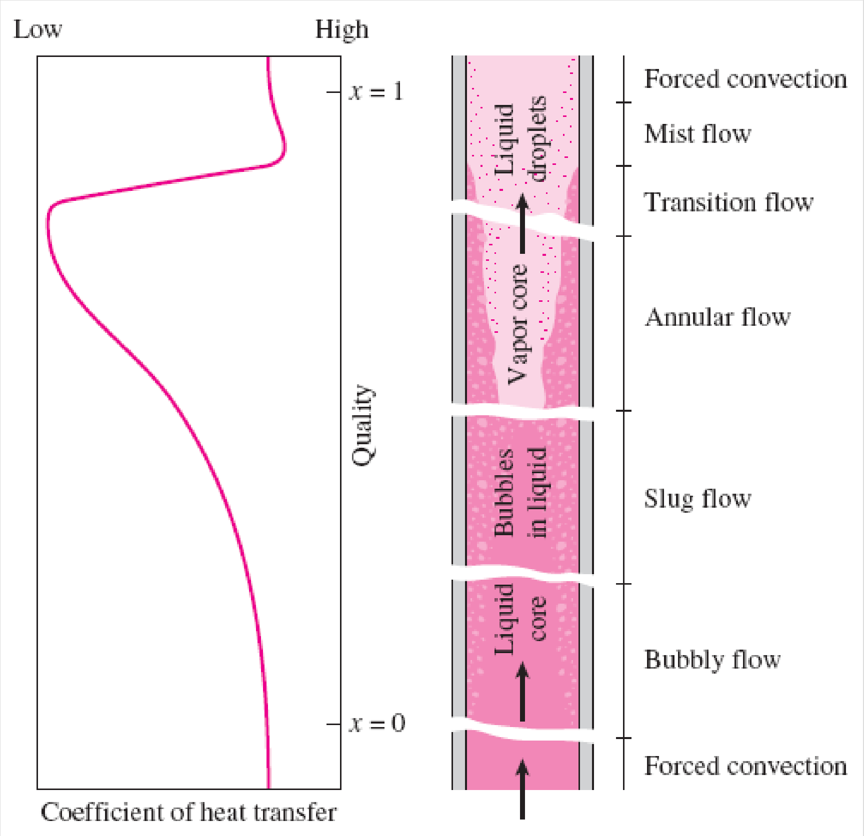


FIGURE 10–19
Different flow regimes encountered in flow boiling in a tube under forced convection.

- Liquid single-phase flow
  - ✓ In the inlet region the liquid is subcooled and heat transfer to the liquid is by *forced convection* (assuming no subcooled boiling).
- Bubbly flow
  - ✓ Individual bubbles
  - ✓ Low mass qualities
- Slug flow
  - ✓ Bubbles coalesce into slugs of vapor.
  - ✓ Moderate mass qualities
- Annular flow
  - ✓ Core of the flow consists of vapor only, and liquid adjacent to the walls.
  - ✓ Very high heat transfer coefficients
- Mist flow
  - ✓ A sharp decrease in the heat transfer coefficient
- Vapor single-phase flow
  - ✓ The liquid phase is completely evaporated and vapor is superheated.



CONDENSATION HEAT TRANSFER

Condensation occurs when the temperature of a vapor is reduced *below* its saturation temperature.

Film condensation

- The condensate wets the surface and forms a liquid film.
- The surface is blanketed by a liquid film which serves as a *resistance* to heat transfer.

Dropwise condensation

- The condensed vapor forms droplets on the surface.
- The droplets slide down when they reach a certain size.
- No liquid film to resist heat transfer.
- As a result, heat transfer rates that are more than 10 times larger than with film condensation can be achieved.

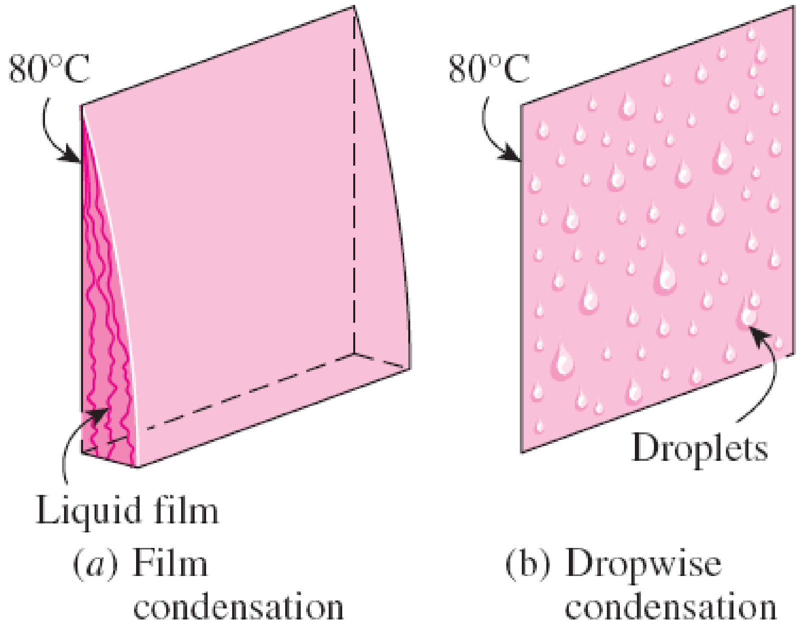


FIGURE 10–20
When a vapor is exposed to a surface at a temperature below T_{sat}, condensation in the form of a liquid film or individual droplets occurs on the surface.

FILM CONDENSATION

- Liquid film starts forming at the top of the plate and flows downward under the influence of gravity.
- δ *increases* in the flow direction x
- Heat in the amount h_{fg} is released during condensation and is *transferred* through the film to the plate surface.
- T_s must be below the saturation temperature for condensation.
- The *temperature* of the condensate is T_{sat} at the interface and decreases gradually to T_s at the wall.

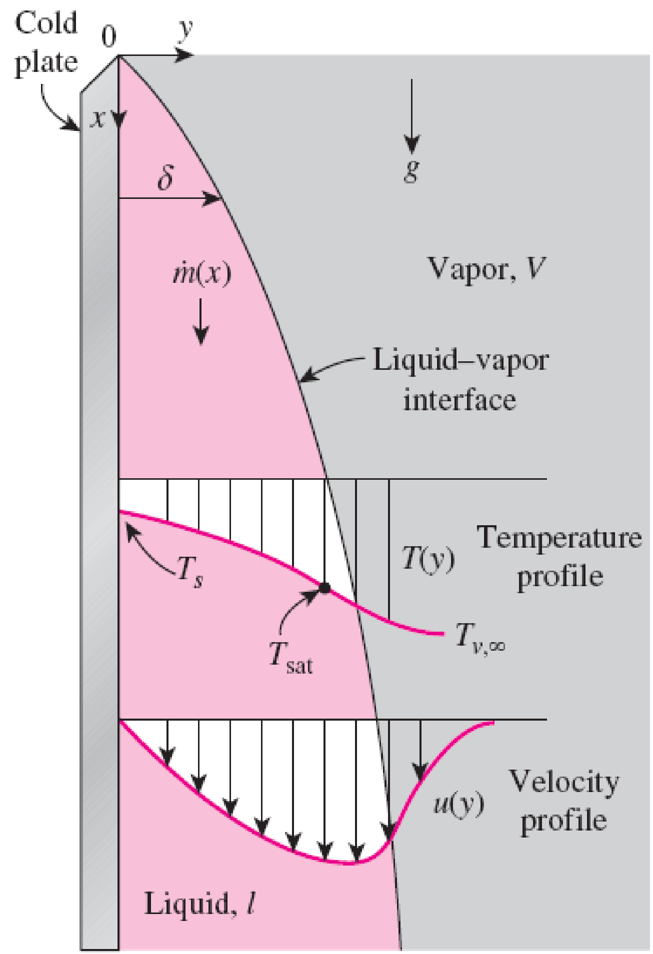


FIGURE 10–21
Film condensation on a vertical plate.

$$\mathrm{Re} = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4\, A_c\, \rho_l V_l}{p \mu_l} = \frac{4\, \rho_l V_l\, \delta}{\mu_l} = \frac{4\dot{m}}{p \mu_l}$$

$D_h = 4A_c/p = 4\delta$ = hydraulic diameter of the condensate flow, m

p = wetted perimeter of the condensate, m

$A_c = -p\delta$ = wetted perimeter $\times$ film thickness, m^2, cross-sectional area of the condensate flow at the lowest part of the flow

ρ_l = density of the liquid, kg/m^3

μ_l = viscosity of the liquid, kg/m·s

V_l = average velocity of the condensate at the lowest part of the flow, m/s

$\dot{m} = \rho_l V_l A_c$ = mass flow rate of the condensate at the lowest part, kg/s

Heat transfer in condensation depends on whether the condensate flow is *laminar* or *turbulent*. The criterion for the flow regime is provided by the Reynolds number.

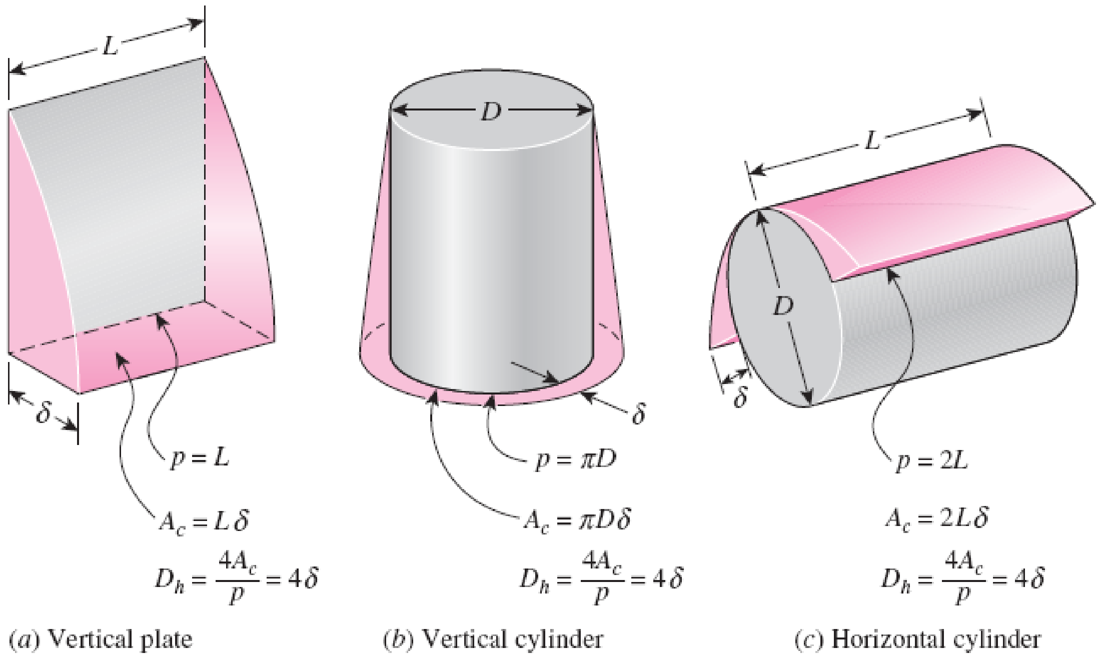


FIGURE 10–22
The wetted perimeter p, the condensate cross-sectional area A_c, and the hydraulic diameter D_h for some common geometries.

When the final state is subcooled liquid instead of saturated liquid:

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s)$$

Modified latent heat of vaporization

For vapor that enters the condenser as **superheated vapor** at a temperature T_v instead of as saturated vapor:

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s) + c_{pv}(T_v - T_{\text{sat}})$$

$$\dot{Q}_{\text{conden}} = hA_s(T_{\text{sat}} - T_s) = \dot{m}h^*_{fg}$$

Rate of heat transfer

$$\text{Re} = \frac{4\dot{Q}_{\text{conden}}}{p\mu_l h^*_{fg}} = \frac{4A_s h(T_{\text{sat}} - T_s)}{p\mu_l h^*_{fg}}$$

This relation is convenient to use to determine the Reynolds number when the condensation heat transfer coefficient or the rate of heat transfer is known.

$$T_f = (T_{\text{sat}} + T_s)/2$$

The properties of the liquid should be evaluated at the *film temperature*

The h_{fg} should be evaluated at T_{sat}

Flow Regimes

- The dimensionless parameter controlling the transition between regimes is the Reynolds number defined as:

$$\text{Re} = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4 A_c \rho_l V_l}{p \mu_l} = \frac{4 \rho_l V_l \delta}{\mu_l} = \frac{4 \dot{m}}{p \mu_l}$$

- Three prime flow regimes:
  - ✓ Re < 30 – Laminar (wave-free)
  - ✓ 30 < Re < 1800 – Laminar (wavy)
  - ✓ Re > 1800 – Turbulent
- The Reynolds number increases in the flow direction.

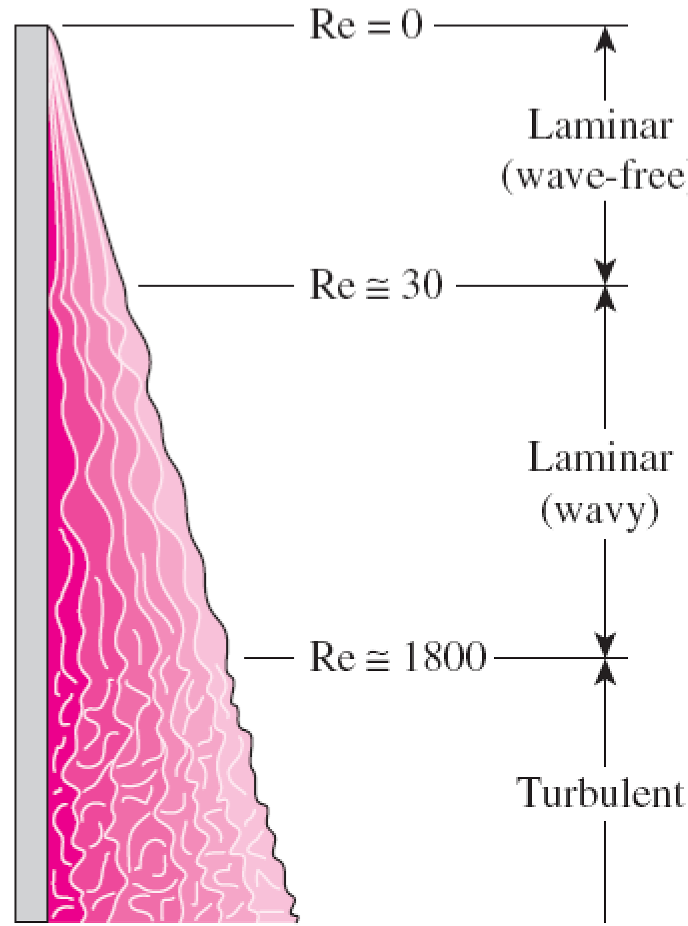


FIGURE 10–23
Flow regimes during film condensation on a vertical plate.

Heat Transfer Correlations for Film Condensation

1 Vertical Plates

Assumptions:

1. Both the plate and the vapor are maintained at *constant temperatures* of T_s and T_{sat}, respectively, and the temperature across the liquid film varies *linearly.*

2. Heat transfer across the liquid film is by pure *conduction*.

3. The velocity of the vapor is low (or zero) so that it exerts *no drag* on the condensate (no viscous shear on the liquid–vapor interface).

4. The flow of the condensate is *laminar* (Re<30) and the properties of the liquid are constant.

5. The acceleration of the condensate layer is negligible.

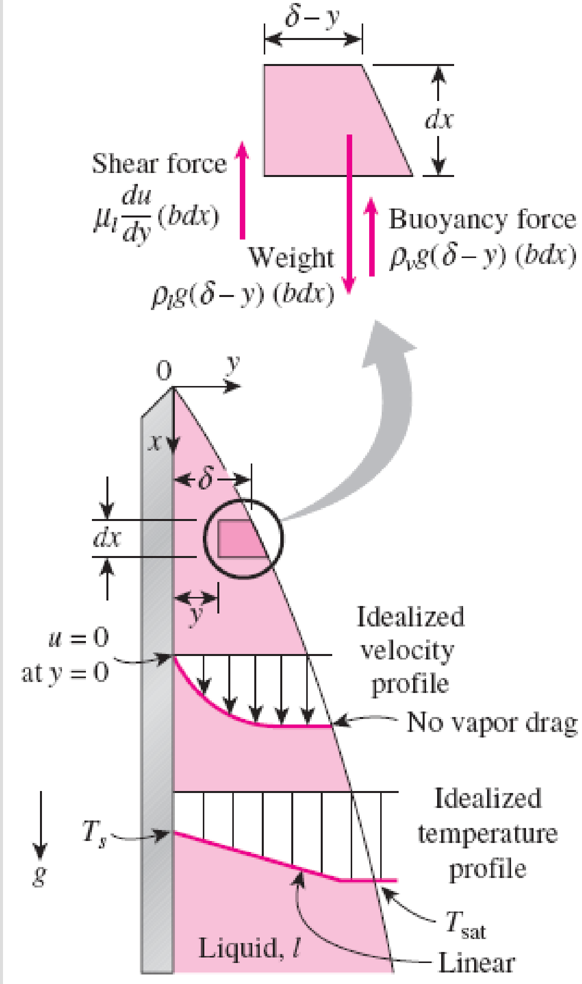


FIGURE 10–24
The volume element of condensate on a vertical plate considered in Nusselt's analysis.

Then Newton's second law of motion for the volume element shown in Fig. 10–24 in the vertical x-direction can be written as

$$\sum F_x = ma_x = 0$$

since the acceleration of the fluid is zero. Noting that the only force acting downward is the weight of the liquid element, and the forces acting upward are the viscous shear (or fluid friction) force at the left and the buoyancy force, the force balance on the volume element becomes

$$F_{\text{downward}\downarrow} = F_{\text{upward}\uparrow}$$

$$\text{Weight} = \text{Viscous shear force} + \text{Buoyancy force}$$

$$\rho_l g(\delta - y)(bdx) = \mu_l \frac{du}{dy}(bdx) + \rho_v\, g(\delta - y)(bdx)$$

Canceling the plate width b and solving for du/dy gives

$$\frac{du}{dy} = \frac{g(\rho_l - \rho_v)g(\rho - y)}{\mu_l}$$

Integrating from $y = 0$ where $u = 0$ (because of the no-slip boundary condition) to $y = y$ where $u = u(y)$ gives

$$u(y) = \frac{g(\rho_l - \rho_v)g}{\mu_l}\left(y\delta - \frac{y^2}{2}\right) \tag{10–12}$$

The mass flow rate of the condensate at a location x, where the boundary layer thickness is δ, is determined from

$$\dot{m}(x) = \int_A \rho_l u(y)dA = \int_{y=0}^{\delta} \rho_l u(y)bdy \tag{10–13}$$

Substituting the $u(y)$ relation from Equation 10–12 into Eq. 10–13 gives

$$\dot{m}\,(x) = \frac{gb\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l} \tag{10–14}$$

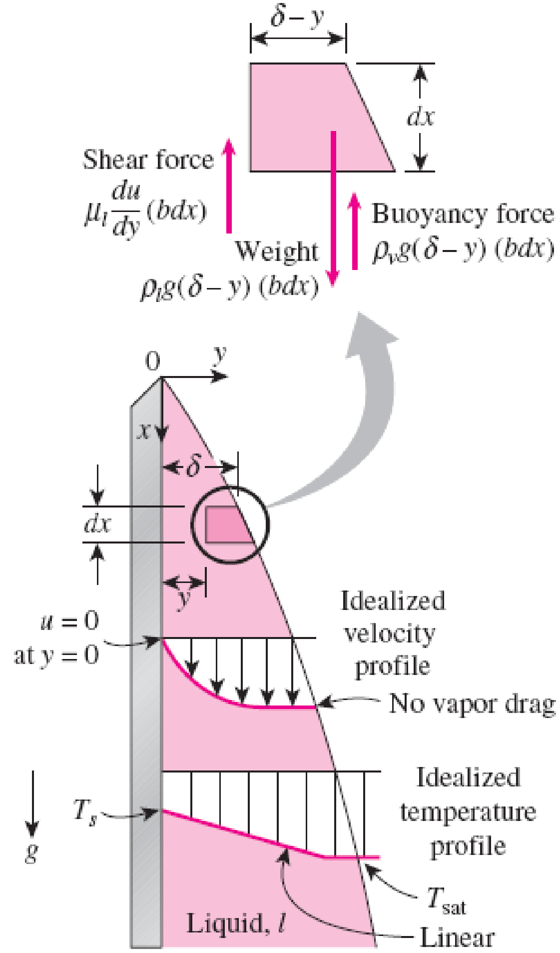


FIGURE 10–24

The volume element of condensate on a vertical plate considered in Nusselt's analysis.

whose derivative with respect to x is

$$\frac{d\dot{m}}{dx} = \frac{gb\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l}\frac{d\delta}{dx} \tag{10–15}$$

which represents the rate of condensation of vapor over a vertical distance dx. The rate of heat transfer from the vapor to the plate through the liquid film is simply equal to the heat released as the vapor is condensed and is expressed as

$$d\dot{Q} = h_{fg}d\dot{m} \;=\; k_l(bdx)\frac{T_{sat} - T_s}{\delta} \rightarrow \frac{d\dot{m}}{dx} = \frac{k_l\, b}{h_{fg}}\frac{T_{sat} - T_s}{\delta} \tag{10–16}$$

Equating Eqs. 10–15 and 10–16 for $d\dot{m}/dx$ to each other and separating the variables give

$$\delta^3 d\delta = \frac{\mu_l\, k_l(T_{sat} - T_s)}{g\rho_l\,(\rho_l - \rho_v)h_{fg}}\,dx \tag{10–17}$$

Integrating from $x = 0$ where $\delta = 0$ (the top of the plate) to $x = x$ where $\delta = \delta(x)$, the liquid film thickness at any location x is determined to be

$$\delta(x) = \left[\frac{4\mu_l\, k_l(T_{sat} - T_s)x}{g\rho_l\,(\rho_l - \rho_v)h_{fg}}\right]^{1/4} \tag{10–18}$$

The heat transfer rate from the vapor to the plate at a location x can be expressed as

$$\dot{q}_x = h_x(T_{sat} - T_s) = k_l\frac{T_{sat} - T_s}{\delta} \rightarrow h_x = \frac{k_l}{\delta(x)} \tag{10–19}$$

Substituting the $\delta(x)$ expression from Eq. 10–18, the local heat transfer coefficient h_x is determined to be

$$h_x = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}\, k_l^3}{4\mu_l\,(T_{sat} - T_s)x}\right]^{1/4} \tag{10–20}$$

The average heat transfer coefficient over the entire plate is determined from its definition by substituting the h_x relation and performing the integration. It gives

$$h = h_{vert} = \frac{1}{L}\int_0^L h_x\, dx = \frac{4}{3}h_{x=L} = 0.943\left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}\, k_l^3}{\mu_l\,(T_{sat} - T_s)L}\right]^{1/4} \tag{10–21}$$

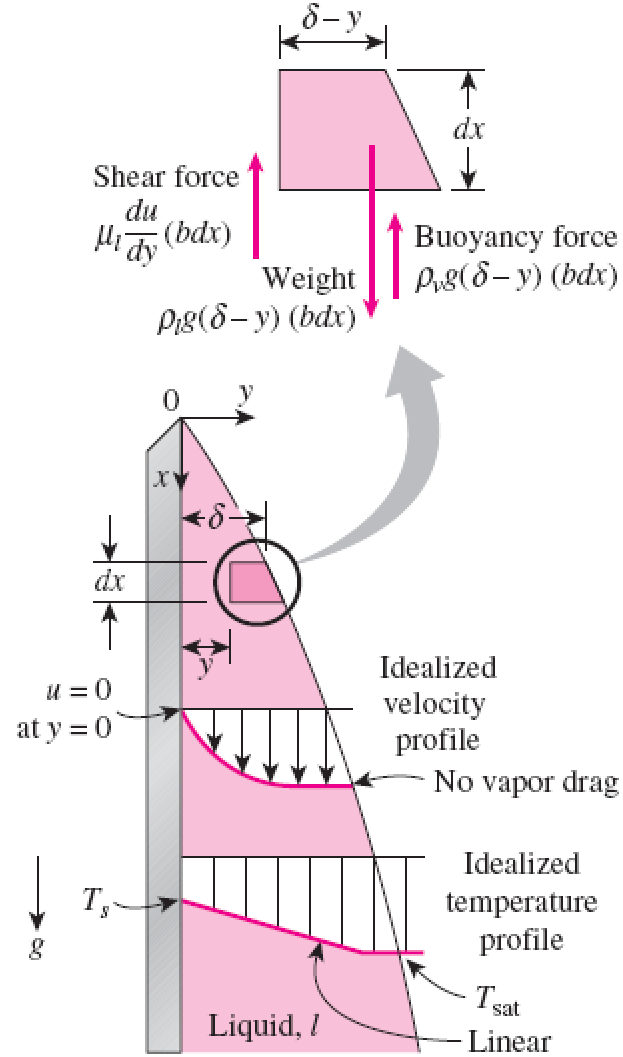


FIGURE 10–24
The volume element of condensate on a vertical plate considered in Nusselt's analysis.

The *average heat transfer coefficient* for laminar film condensation over a vertical flat plate of height L is

$$h_{\text{vert}} = 0.943\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}\,k_l^3}{\mu_l\,(T_{\text{sat}} - T_s)L}\right]^{1/4} \qquad (\text{W/m}^2\cdot\text{K}), \qquad 0 < \text{Re} < 30 \qquad \text{(10-22)}$$

g = gravitational acceleration, m/s²
ρ_l, ρ_v = densities of the liquid and vapor, respectively, kg/m³
μ_l = viscosity of the liquid, kg/m·s
$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s)$ = modified latent heat of vaporization, J/kg
k_l = thermal conductivity of the liquid, W/m·K
L = height of the vertical plate, m
T_s = surface temperature of the plate, °C
T_{sat} = saturation temperature of the condensing fluid, °C

$$h_{\text{vert}} = \left(\frac{\dfrac{\text{m}}{\text{s}^2}\dfrac{\text{kg}}{\text{m}^3}\dfrac{\text{kg}}{\text{m}^3}\dfrac{\text{J}}{\text{kg}}\left(\dfrac{\text{W}}{\text{m}\cdot\text{K}}\right)^3}{\dfrac{\text{kg}}{\text{m}\cdot\text{s}}\cdot\text{K}\cdot\text{m}}\right)^{1/4} = \left[\frac{\text{m}}{\text{s}}\,\frac{1}{\text{m}^6}\,\frac{\text{W}^3}{\text{m}^3\cdot\text{K}^3}\,\frac{\text{J}}{\text{K}}\right] = \left(\frac{\text{W}^4}{\text{m}^8\cdot\text{K}^4}\right)^{1/4} = \text{W/m}^2\cdot\text{K}$$

FIGURE 10–25
Equation 10–22 gives the condensation heat transfer coefficient in W/m²·K when the quantities are expressed in the units specified in their descriptions.

$$\text{Re} \cong \frac{4g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l^2} = \frac{4g\rho_l^2}{3\mu_l^2}\left(\frac{k_l}{h_{x=L}}\right)^3 = \frac{4g}{3\nu_l^2}\left(\frac{k_l}{3h_{\text{vert}}/4}\right)^3$$

$$h_{\text{vert}} \cong 1.47k_l\,\text{Re}^{-1/3}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \qquad \begin{matrix} 0 < \text{Re} < 30 \\ \rho_v \ll \rho_l \end{matrix}$$

$$T_f = (T_{\text{sat}} + T_s)/2$$

All properties of the liquid are to be evaluated at the film temperature. The h_{fg} and ρ_v are to be evaluated at the saturation temperature T_{sat}.

Wavy Laminar Flow on Vertical Plates

The average heat transfer coefficient in wavy laminar condensate flow for $\rho_v \ll \rho_l$ and $30 < \text{Re} < 1800$

$$h_{\text{vert, wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \qquad \begin{array}{l} 30 < \text{Re} < 1800 \\ \rho_v \ll \rho_l \end{array}$$

$$h_{\text{vert, wavy}} = 0.8\,\text{Re}^{0.11}\, h_{\text{vert (smooth)}}$$

A simpler alternative to the relation above

$$\text{Re}_{\text{vert, wavy}} = \left[4.81 + \frac{3.70\, L k_l (T_{\text{sat}} - T_s)}{\mu_l\, h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}, \qquad \rho_v \ll \rho_l$$

Turbulent Flow on Vertical Plates

Turbulent flow of condensate on *vertical plates:*

$$h_{\text{vert, turbulent}} = \frac{\text{Re}\, k_l}{8750 + 58\,\text{Pr}^{-0.5}\left(\text{Re}^{0.75} - 253\right)}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \qquad \begin{array}{l}\text{Re} > 1800\\ \rho_v \ll \rho_l\end{array}$$

$\rho_v \ll \rho_l$

The physical properties of the condensate are again to be evaluated at the film temperature $T_f = (T_{sat} + T_s)/2$.

$$\text{Re}_{\text{vert, turbulent}} = \left[\frac{0.0690\, L k_l\, \text{Pr}^{0.5}\,(T_{\text{sat}} - T_s)}{\mu_l\, h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3} - 151\,\text{Pr}^{0.5} + 253\right]^{4/3}$$

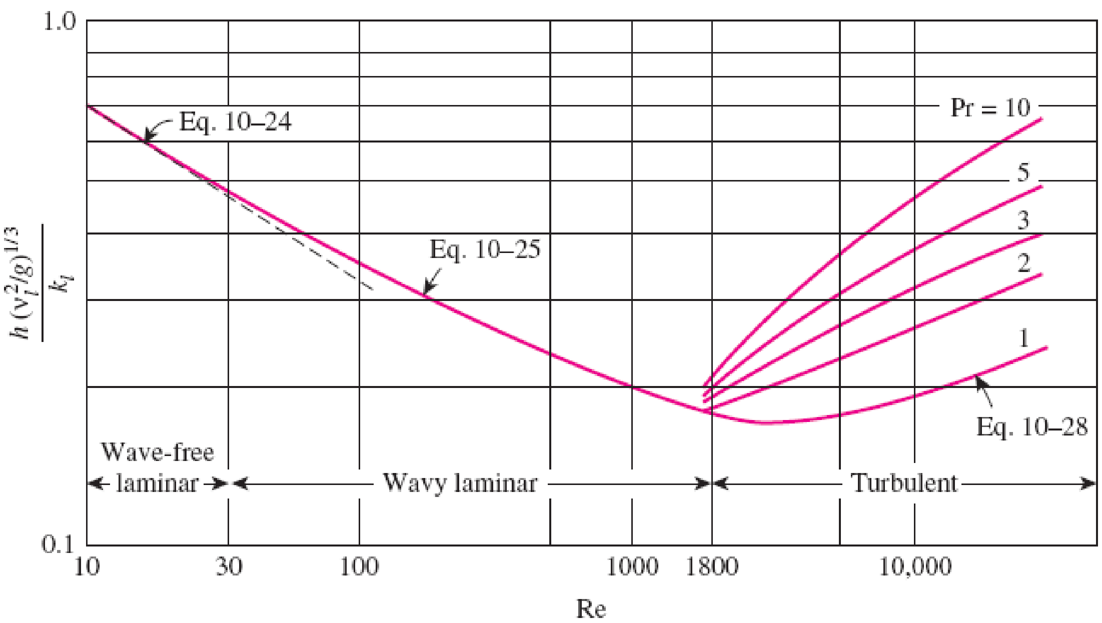


FIGURE 10–26
Nondimensionalized heat transfer coefficients for the wave-free laminar, wavy laminar, and turbulent flow of condensate on vertical plates.

2 Inclined Plates

Equation 10–22 was developed for vertical plates, but it can also be used for laminar film condensation on the upper surfaces of plates that are *inclined* by an angle θ from the *vertical,* by replacing g in that equation by $g \cos\theta$.

$$h_{\text{inclined}} = h_{\text{vert}} (\cos \theta)^{1/4} \qquad \text{(laminar)}$$

$$h_{\text{vert}} = 0.943 \left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg} k_l^3}{\mu_l (T_{\text{sat}} - T_s)L}\right]^{1/4} \qquad (\text{W/m}^2\cdot\text{K}), \qquad 0 < \text{Re} < 30$$

(10-22)

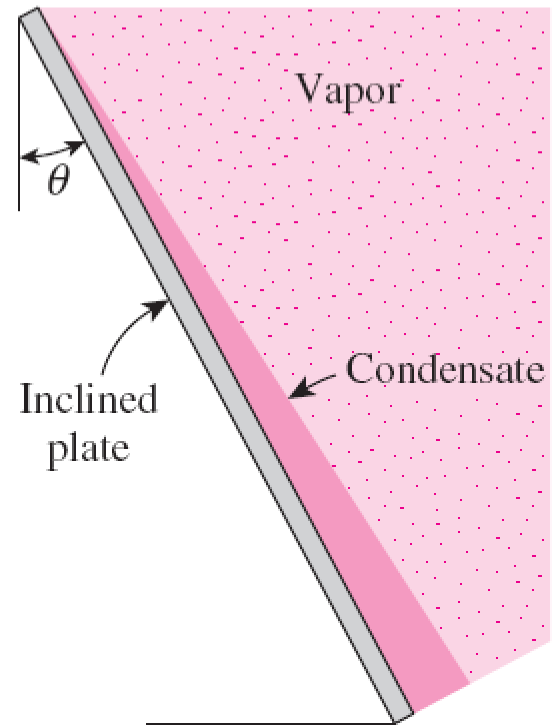


FIGURE 10–27
Film condensation on an inclined plate.

3 Vertical Tubes

Equation 10–22 for vertical plates can also be used to calculate the average heat transfer coefficient for laminar film condensation on the outer surfaces of vertical tubes provided that the tube diameter is large relative to the thickness of the liquid film.

4 Horizontal Tubes and Spheres

The average heat transfer coefficient for film condensation on the outer surfaces of a *horizontal tube* is

$$h_{\text{horiz}} = 0.729 \left[\frac{g\rho_l(\rho_l - \rho_v)\, h^*_{fg}\, k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}\right]^{1/4} \quad (\text{W/m}^2\cdot\text{K})$$

For a ***sphere***, replace the constant 0.729 by 0.815.

A comparison of the heat transfer coefficient relations for a vertical tube of height *L* and a horizontal tube of diameter *D* yields

$$\frac{h_{\text{vert}}}{h_{\text{horiz}}} = 1.29\left(\frac{D}{L}\right)^{1/4}$$

Setting $h_{\text{vert}} = h_{\text{horiz}}$ gives $L = 1.29^4 D = 2.77D$

For a tube whose length is 2.77 times its diameter, the average heat transfer coefficient for laminar film condensation will be the *same* whether the tube is positioned horizontally or vertically.

For $L > 2.77D$, the heat transfer coefficient is higher in the horizontal position.

Considering that the length of a tube in any practical application is several times its diameter, it is common practice to place the tubes in a condenser *horizontally* to maximize the condensation heat transfer coefficient on the outer surfaces of the tubes.

5 Horizontal Tube Banks

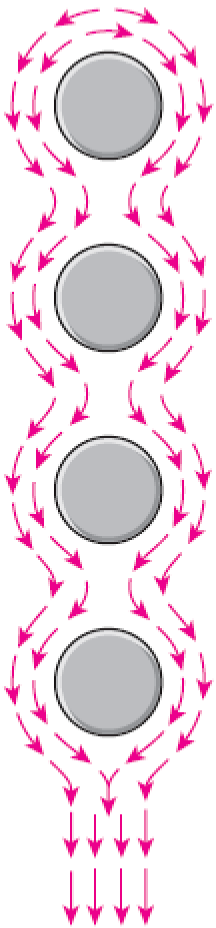


FIGURE 10–28

Film condensation on a vertical tier of horizontal tubes.

The average thickness of the liquid film at the lower tubes is much larger as a result of condensate falling on top of them from the tubes directly above.

Therefore, the average heat transfer coefficient at the lower tubes in such arrangements is smaller.

Assuming the condensate from the tubes above to the ones below drain smoothly, the average film condensation heat transfer coefficient for all tubes in a vertical tier can be expressed as

$$h_{\text{horiz, }N\text{ tubes}} = 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)\,h_{fg}^{*}\,k_l^3}{\mu_l(T_{\text{sat}}-T_s)\,ND}\right]^{1/4} = \frac{1}{N^{1/4}}\,h_{\text{horiz, 1 tube}}$$

$$h_{\text{horiz}} = 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)\,h_{fg}^{*}\,k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4} \quad (\text{W/m}^2\cdot\text{K})$$

This relation does not account for the increase in heat transfer due to the ripple formation and turbulence caused during drainage, and thus generally yields conservative results.

Effect of Vapor Velocity

In the analysis above we assumed the vapor velocity to be small and thus the vapor drag exerted on the liquid film to be negligible, which is usually the case.

However, when the vapor velocity is high, the vapor will “pull” the liquid at the interface along since the vapor velocity at the interface must drop to the value of the liquid velocity.

If the vapor flows downward (i.e., in the same direction as the liquid), this additional force will increase the average velocity of the liquid and thus decrease the film thickness.

This, in turn, will decrease the thermal resistance of the liquid film and thus increase heat transfer.

Upward vapor flow has the opposite effects: the vapor exerts a force on the liquid in the opposite direction to flow, thickens the liquid film, and thus decreases heat transfer.

The Presence of Noncondensable Gases in Condensers

Experimental studies show that the presence of noncondensable gases in the vapor has a detrimental effect on condensation heat transfer.

Even small amounts of a noncondensable gas in the vapor cause significant drops in heat transfer coefficient during condensation.

It is common practice to periodically vent out the noncondensable gases that accumulate in the condensers to ensure proper operation.

Heat transfer in the presence of a noncondensable gas strongly depends on the nature of the vapor flow and the flow velocity.

A *high flow velocity* is more likely to remove the stagnant noncondensable gas from the vicinity of the surface, and thus *improve* heat transfer.

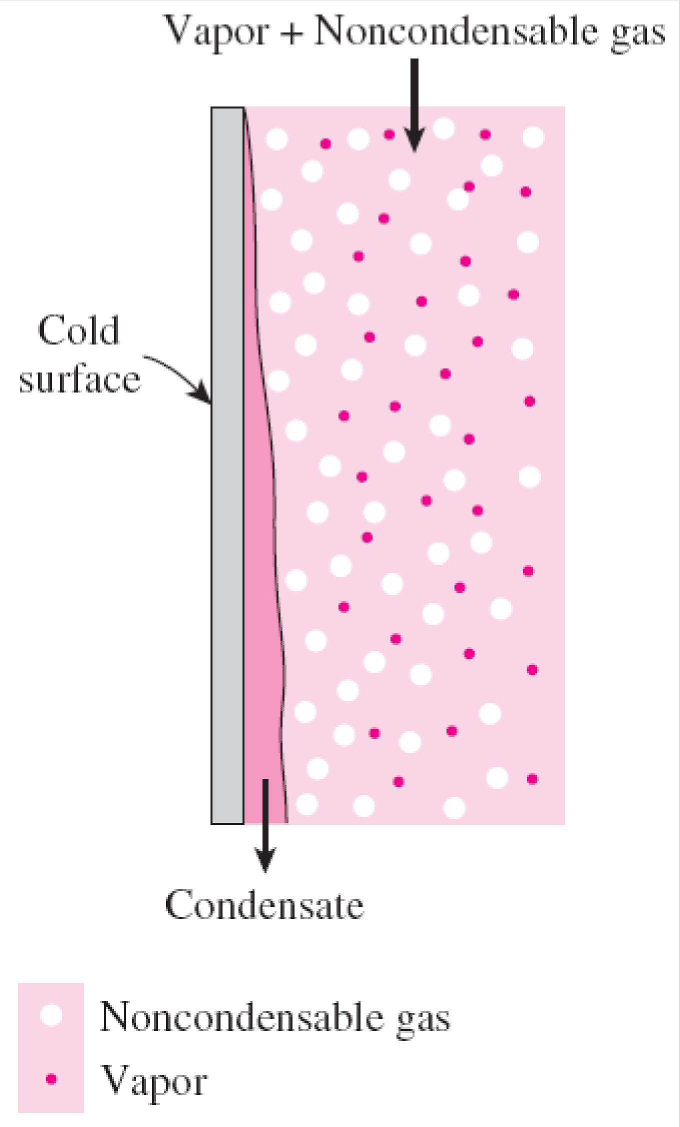


FIGURE 10–29
The presence of a noncondensable gas in a vapor prevents the vapor molecules from reaching the cold surface easily, and thus impedes condensation heat transfer.

FILM CONDENSATION INSIDE HORIZONTAL TUBES

Most condensation processes encountered in refrigeration and air-conditioning applications involve condensation on the *inner surfaces* of horizontal or vertical tubes.

Heat transfer analysis of condensation inside tubes is complicated by the fact that it is strongly influenced by the vapor velocity and the rate of liquid accumulation on the walls of the tubes.

For *low vapor velocities:*

$$h_{\text{internal}} = 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\left(h_{fg}+\frac{3}{8}c_{pl}(T_{\text{sat}}-T_s)\right)\right]^{1/4}$$

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right)_{\text{inlet}} < 35{,}000$$

The Reynolds number of the vapor is to be evaluated at the tube *inlet* conditions using the internal tube diameter as the characteristic length.

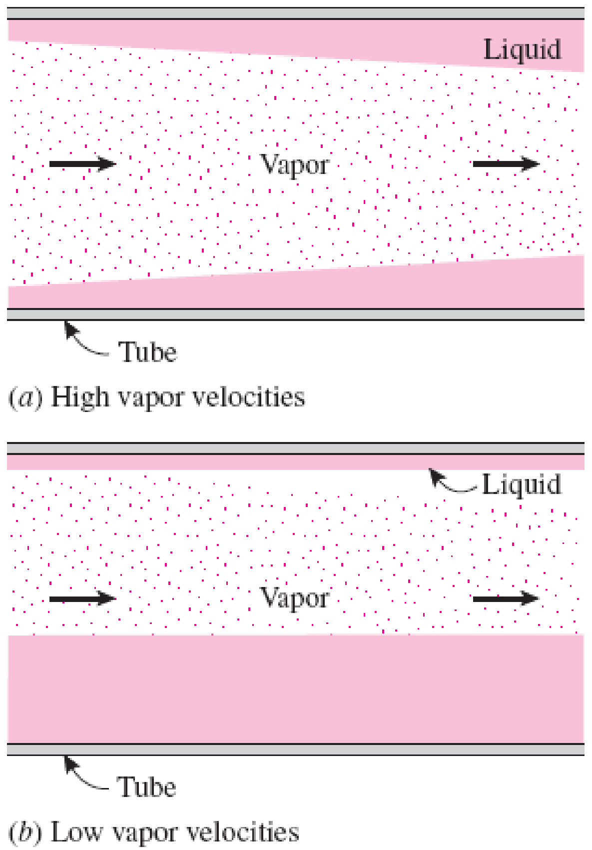


FIGURE 10–34
Condensate flow in a horizontal tube with high and low vapor velocities.

DROPWISE CONDENSATION

Dropwise condensation, characterized by countless droplets of varying diameters on the condensing surface instead of a continuous liquid film and extremely large heat transfer coefficients can be achieved with this mechanism.

The small droplets that form at the nucleation sites on the surface grow as a result of continued condensation, coalesce into large droplets, and slide down when they reach a certain size, clearing the surface and exposing it to vapor. There is no liquid film in this case to resist heat transfer.

As a result, with dropwise condensation, heat transfer coefficients can be achieved that are more than 10 times larger than those associated with film condensation.

The challenge in dropwise condensation is not to achieve it, but rather, to *sustain* it for prolonged periods of time.

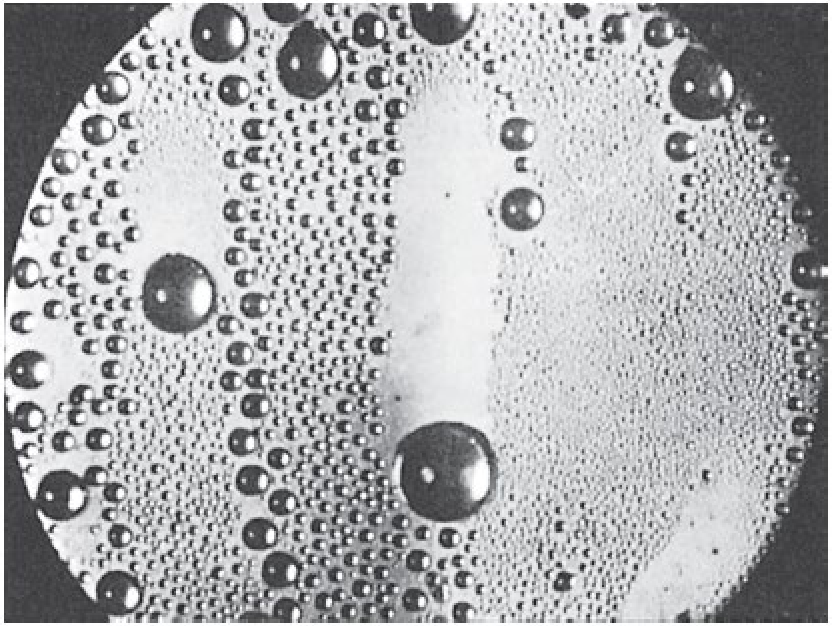


FIGURE 10–35
Dropwise condensation of steam on a vertical surface.

Dropwise condensation of *steam* on *copper surfaces:*

$$h_{\text{dropwise}} = \begin{cases} 51{,}104 + 2044T_{\text{sat}}, & 22°\text{C} < T_{\text{sat}}, 100°\text{C} \\ 255{,}310 & T_{\text{sat}} > 100°\text{C} \end{cases}$$

Summary

- Boiling heat transfer
- Pool boiling
  - ✓ Boiling regimes and the boiling curve
- Flow boiling
- Condensation heat transfer
- Film condensation
- Film condensation inside horizontal tubes
- Dropwise condensation