

Citation for published version:

Xie, M, Zhang, Y, Krasny, MJ, Rhead, A, Bowen, C & Arafa, M 2018, 'Energy harvesting from coupled bending-twisting oscillations in carbon-fibre reinforced polymer laminates', *Mechanical Systems and Signal Processing*, vol. 107, pp. 429-438. <https://doi.org/10.1016/j.ymssp.2018.01.026>

DOI:

[10.1016/j.ymssp.2018.01.026](https://doi.org/10.1016/j.ymssp.2018.01.026)

Publication date:

2018

Document Version

Early version, also known as pre-print

[Link to publication](#)

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Energy harvesting from coupled bending-twisting oscillations in carbon-fibre reinforced polymer laminates

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Highlight

- First demonstration of laminated beam with bend-twist coupling for energy harvesting
- Unique approach to tune the frequency response of the resonant harvesting system
- Novel laminate structure can be tuned to provide a broader-band energy harvester

Abstract

The energy harvesting capability of resonant structures, such as piezoelectric cantilever beams, can be improved by utilizing coupled oscillations that generate favourable strain mode distributions. In this work, we present the first demonstration of the use of a laminated carbon fibre reinforced polymer to create cantilever beams that undergo coupled bending-twisting oscillations for energy harvesting applications. Piezoelectric layers that operate in bending and shear mode are attached to the bend-twist coupled beam surface at locations of maximum bending and torsional strains in the first mode of vibration to fully exploit the strain distribution along the beam. Modelling of this new bend-twist harvesting system is presented, which compares favourably with the experimental results. It is demonstrated that the variety of bend and torsional modes of the harvesters can be utilized to create a harvester that operates over a wider range of frequencies and such device architectures provides a unique way in which to tune the frequency response of the resonant harvesting system.

Keywords

Piezoelectric, Energy harvesting, Bend-twisting coupling, Lumped-parameter model

Nomenclature

C	Damping matrix
F	Force
I	Effective mass moment of inertia
k	Stiffness
k_T	Torsional stiffness
$[K]$	Stiffness matrix
l	Arm length denoting bend-twist coupling
m	Effective mass
$[M]$	Mass matrix
t	Time
T	Kinetic energy
V	Potential energy

x	Transverse displacement of tip cross-section
y	Base motion
Y	Amplitude of base motion
α	Damping coefficient for mass matrix
β	Damping coefficient for stiffness matrix
δ	Nodal degrees of freedom
ϕ	Cross-section rotation under no torsional loads
θ	Cross-section rotation under no torsional loads
ω	Frequency

1. Introduction

The conversion of mechanical vibration into useful electrical energy has been a subject of intensive research for many years for applications related to self-powered sensors and wireless systems to minimize the need for batteries and eliminates the need for battery management. A common architecture for vibration-based energy harvesting devices is a base-excited elastic structure, such as a mass-spring system or a cantilever beam, which is typically used in electromagnetic and piezoelectric devices, respectively. One of the challenges in designing continuous systems, such as beams and plates, for piezoelectric energy harvesting lies in the placement of the power generating materials, which are usually sheets that are bonded or embedded within a host structure. These structures often have unique strain distributions in their vibration modes, and the highly strained parts of the structure that are most effective for energy harvesting occur only in localized zones. The placement of power generation materials only at such discrete locations while leaving other parts uncovered causes the structure to be partially utilized for energy harvesting, which lowers the power density. This is the case, for example, of a cantilever beam undergoing flexural vibration at its first bending mode, where maximum strain occurs near the root. To overcome this difficulty, systems operating across multiple vibration modes have been proposed [1, 2] in order to utilize two or more vibration mode shapes in energy harvesting. The exploitation of more than one vibration mode also enables the device to harvest vibrations over a wider frequency range since in many cases the ambient vibrations to be harvested often span a range of frequencies and amplitudes.

Vibration modes involving mixed deformations, especially bending and twisting motions are particularly appealing in this respect since these motions can be controlled by design. The study of coupled bend-twist oscillations has been of interest to the aeronautical engineering community for decades owing to its application in the vibration analysis of aircraft wings and rotating turbine blades. Recent interest in morphing structures has spurred research in the use of piezoelectric actuators to achieve controlled bend-twist deformations [3-5]. The use of bend-twist oscillations in energy harvesting applications is relatively recent. In this context, reference is made to the work of Abdelkefi *et al.* [6] who designed a unimorph cantilever beam undergoing bending–torsion vibrations consisting of a single piezoelectric layer and two asymmetric tip masses thereby generating a twisting moment from base excitation. This structure was tuned to provide a broader band energy harvester by adjusting the first two global natural frequencies to be relatively close to each other. Reference is also made to the work of Gao *et al.* [7] in which torsional vibration at the second mode of a cantilever beam carrying an eccentric proof mass was employed for energy harvesting through a lead zirconate titanate (PZT) material. The main advantages of this design

were small displacement amplitudes and low natural frequency. Shan *et al.* [8] employed vortex-induced vibrations to design a piezoelectric energy harvester with bending-torsion vibration.

An effective way of designing structures with inherent bend-twist coupling behavior is the use of composite laminates. By tailoring the laminate lay-up these anisotropic structures can be deliberately designed to exhibit interactions between extension, shear, bending or shear [9], which are not present in conventional isotropic materials. In this work, we shall employ for the purpose of energy harvesting a laminated carbon fibre reinforced polymer (CFRP) composite cantilever beam that has a specific laminate lay-up to achieve coupled bending and twisting deformations. This approach is particularly attractive since it removes the need for more complex design configurations; such as the use of an eccentric proof mass or asymmetric tip masses. The ability to tailor the bend-twist coupled laminate architecture also provides scope for a wide design space to tailor the cantilever response to the vibration spectrum being harvested. To utilize larger portions of the beam for power generation, we employ two different types of PZT materials that are attached to the beam's surface. One PZT patch (MFC M8557-P1) responds to uniaxial straining (extension-mode) and is placed close to the beam's root for effective harvesting of the bending mode, whereas the other material (MFC M8557-F1) operates in the shear-mode, and is placed along the beam's mid span where torsional shear strains are higher. This effort is expected to eventually yield devices with greater power densities, and greater bandwidth which are increasingly in high demand for small sensors and other applications where miniaturization is of the essence. Modelling is used to provide an analysis of the frequency response of the harvesting system.

The remainder of this paper is organized into four sections. Section 2 presents a mechanical model of the device. Section 3 details the fabrication of the laminated composite beams. Experimental work is carried out in Section 4. Finally, Section 5 is dedicated to conclusions and recommendations.

2. Fabrication of Composite Bend-Twist Laminate

The cantilever beam used in this work was manufactured by using unidirectional CFRP plies (Cytec MTM 49-3/ T800HB-140-36%) that were manufactured by hand-layup and cured in an autoclave. Figure 1 shows the schematic illustration of the cantilever beam, its dimensions and the position of the piezoelectric patches. The cantilever consists of 10 symmetric plies with the layup of $[0, 30, 0, 45, -45]_s$, where 0° ply is along the span of the beam. This laminate system was chosen since (i) the symmetric nature of the laminate ensures that it will remain flat and warp free after high temperature curing, and (ii) to achieve bend-twist coupling the laminate should be unbalanced. The width of the cantilever beam is 100 mm and the thickness is 1.45mm after curing. The beam has free length of 499 mm with an additional 60 mm for clamping between two aluminium plates.

Two different types of PZT materials are bonded to the beam's surface for energy harvesting. The types chosen in this work are Macro Fibre Composites (MFC) which consist of PZT fibers embedded in an epoxy matrix with interdigitated electrodes. One PZT patch (Smart Material, MFC M8557-P1) responds to uniaxial straining and is placed close to the beam's root (at a distance of 24.5 mm from the beam's fixation, as shown in Figure 1) and in this case the piezoelectric fibres

in the MFC are aligned along the length of the device. The second patch (Smart Material, MFC M8557-F1) operates in the shear-mode where the piezoelectric fibres in the MFC are at 45° to its length axis. This MFC is placed along the beam's mid span (at a distance of 68.2 mm from the root patch, as shown in Figure 1) where torsional shear strains are higher.

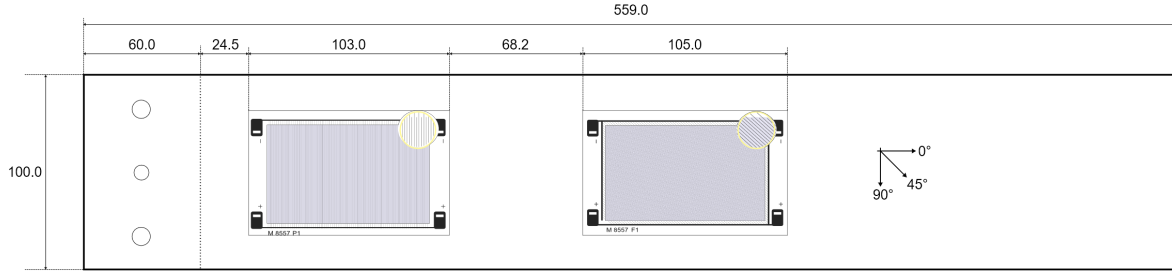


Figure 1. Schematic illustration of the cantilever beam and position of piezo-patches. The cantilever was clamped at left hand side.

The stiffness of the cantilever can be characterized by the static load-deflection test. In this case, the force deflection curve was measured on an Instron 3365 tensile tester with a 100 N load cell by applying a load at the tip of the clamped cantilever while measuring the tip deflection. Figure 2 shows the load - deflection response of the cantilever. The results indicates a linear relationship between force and displacement below a 50 mm deflection (~ 3 N) and the stiffness is a constant of 0.057 N/mm.

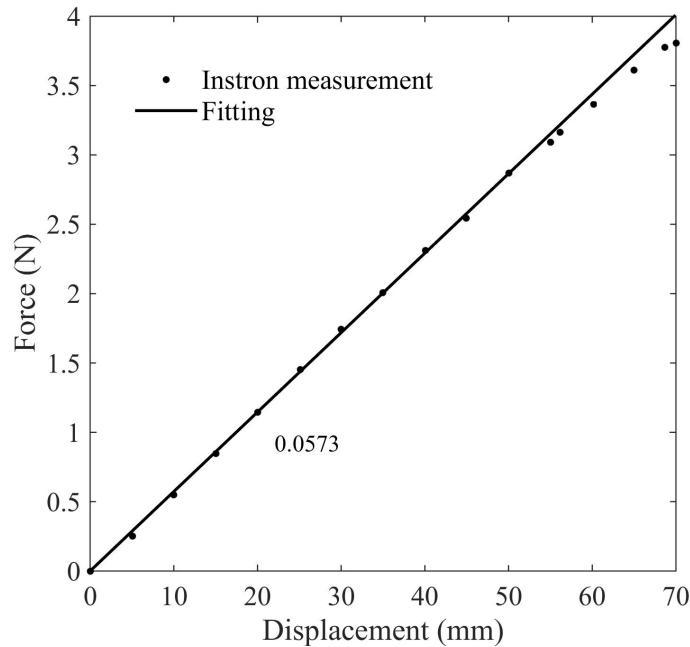


Figure 2. Force deflection curve of cantilever indicated a linear response at deflections below 50mm (~ 3 N).

Due to the bend-twist coupling, the tip of the beam developed an angle of twist θ that increased with the tip displacement, as shown in Figure 3(a). Figure 3(b) demonstrates that under 50 mm,

twisting angle increased linearly with displacement, which correlates with the force-displacement observations. It can be noticed when the displacement was zero, the tip of the beam had small twist. However, both force and angle had nonlinear behaviour above 50 mm displacement and beyond 50 mm twisting became more obvious than bending.

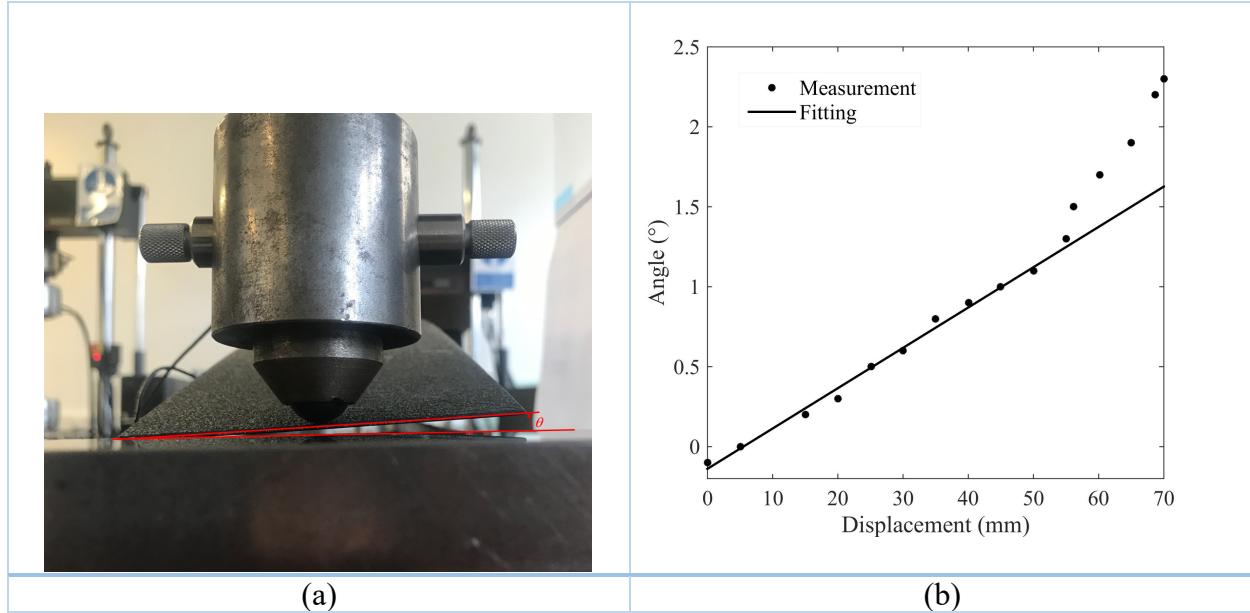


Figure 3. Bend-twist coupling of the cantilever beam. (a) Image of cantilever undergoing bend-twist coupling, θ is the twisting displacement (b) Relationship between twisting angle and deflection (linear below 50mm, slope=0.0252°/mm).

3. Mechanical Model

Figure 4 shows a schematic of the cantilever beam undergoing both bending and twisting deflections. While structures undergoing coupled bend-twist deformations are essentially two dimensional, the aim of this work is to develop a simplified lumped-parameter model to enable the analysis of the system as a two degree of freedom (2-DOF) model. In this way, the transverse displacement x of the centroid of the cross-section at the tip, and the angular displacement θ of the tip section are taken as representative degrees of freedom that designate bending and twisting deformations of the cantilever beam, respectively.

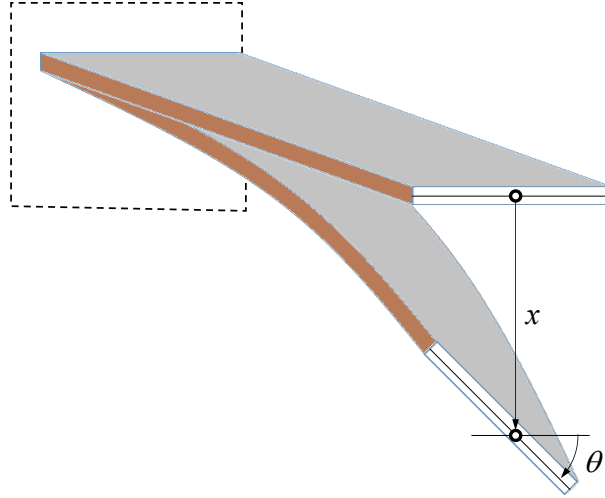


Figure 4. Schematic illustration of a deformed of cantilever beam of a rectangular cross-section undergoing a coupled bend-twist deformation pattern featured by a transverse displacement x and an angle of twist θ of the tip cross section.

The laminated composite beam is modeled as an equivalent lumped-parameter beam section undergoing coupled bend-twist oscillations, as shown in Figure 5. This simplified model provides some physical insight into the problem and allows a study of the effect of the individual design parameters on the system response. The beam is modeled as a 2-DOF mass that undergoes coupled bend-twist oscillations. The bending displacement is denoted by x and resembles the transverse displacement of the beam tip. The twist displacement is denoted by θ and represents the angle of twist of the beam tip cross-section under no external torsional loads. Both degrees of freedom are piezoelectrically coupled and the electrical outputs are connected to resistive loads..

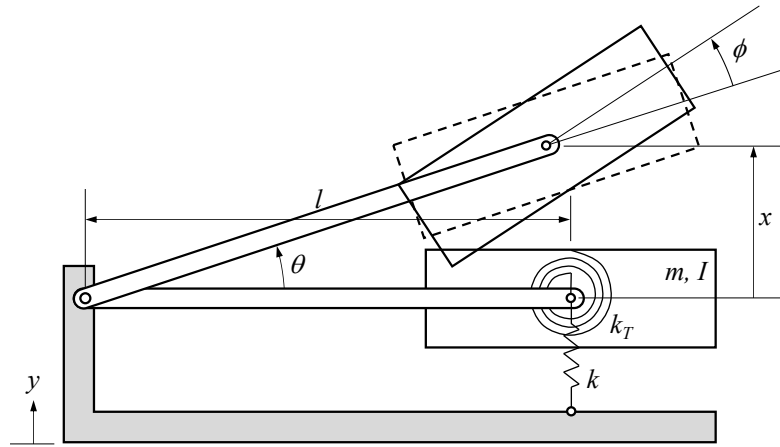


Figure 5. Lumped-parameter model of the bend-twist coupled system.

The total kinetic energy of the system is given by:

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I (\dot{\theta} + \dot{\phi})^2 \quad (1)$$

where m and I are the effective mass and mass moment of inertia, respectively, and the angle ϕ represents the cross-section rotation due to torsional loads. These will typically arise as a result of backward coupling twisting moments generated by the torsional MFC M8557-F1 patches. From geometry, it can be observed that:

$$(x - y) = l\theta \quad (2)$$

where l denotes the arm length that designates the inherent bend-twist coupling. Equation (1) can hence be expressed as:

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \left(\frac{\dot{x}}{l} - \frac{\dot{y}}{l} + \dot{\phi} \right)^2 \quad (3)$$

The total strain energy of the system is given by:

$$V = \frac{1}{2} k (x - y)^2 + \frac{1}{2} k_T \phi^2 \quad (4)$$

where k_T is the torsional stiffness of the beam. Using Lagrange's equations, the equations of motion of the system are obtained as:

$$\begin{bmatrix} m + I/l^2 & I/l \\ I/l & I \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k_T \end{bmatrix} \begin{Bmatrix} x \\ \phi \end{Bmatrix} = \begin{Bmatrix} ky + I\ddot{y}/l^2 \\ I\ddot{y}/l \end{Bmatrix} \quad (5)$$

Equation (5) can be written in a general form as:

$$[M]\{\ddot{\delta}\} + [K]\{\delta\} = \{F\} \quad (6)$$

Damping is arbitrarily introduced through a proportional damping matrix in the form:

$$[C] = \alpha[M] + \beta[K] \quad (7)$$

where α and β are damping coefficients for the mass and stiffness matrices, respectively. Equation (7) describes the mechanical behavior of the beam and can be used to obtain the steady-state response under a sinusoidal base excitation of the form $y(t) = Y \sin \omega t$. Table 1 lists the values of the system parameters, as estimated from the experimental tests on the cantilever beam.

3.1. Parameter Estimation

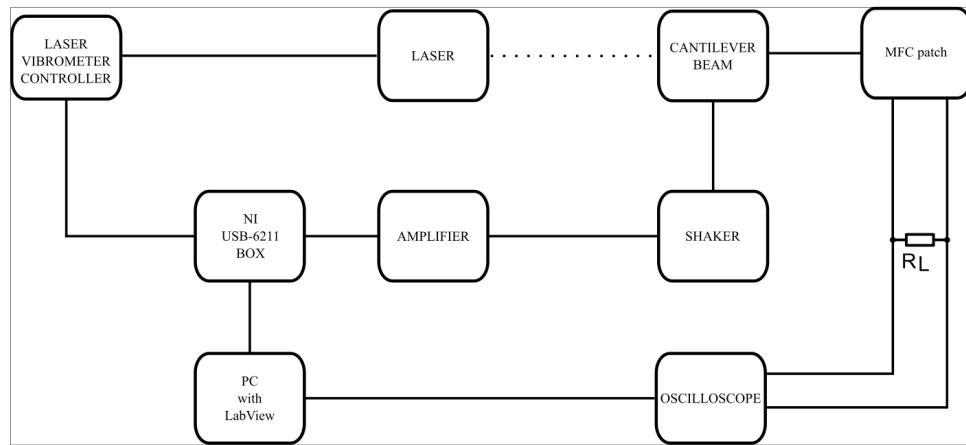
Parameters of the model are obtained from experimental observations and, whenever possible, from equivalent lumped-parameter models. The total mass of the beam was measured to be 147 g, from which the effective mass, m , is estimated to be $0.24 \times 147 = 35.28$ g [10]. The linear stiffness, k , was estimated from the experimental load-deflection test of the cantilever beam and is reported to be 57 N/m (see Section 2). From the same test, the arm length, l , is approximated to be 1.74

m/rad as illustrated in Figure 3(b). The effective mass moment of inertia, I , and torsional stiffness, k_T , were taken as $5 \times 10^{-5} \text{ kgm}^4$ and 4 Nm/rad , respectively. Finally the damping coefficients are assumed to be $\alpha = 3$ and $\beta = 0$.

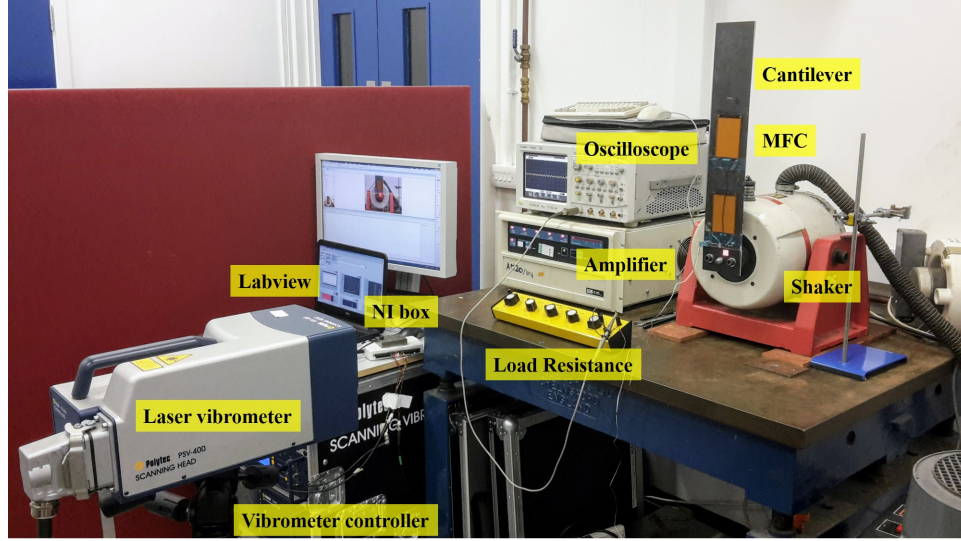
4. Experimental Work

In order to show a cantilever beam undergoing both bending and twisting deflections two standard MFC (Macro Fibre Composite) piezoelectric patches M8557-P1 and M8557-F1 from Smart Materials were bonded to the root and mid span of the cantilever beam, respectively. The MFC device is a flexible lead zirconium titanate (PZT) based sheet of aligned rectangular piezoelectric fibres sandwiched between epoxy layers and interdigitated electrodes with a manufacturer's specified capacitance of 12.84 nF and 13.26 nF for M8557-P1 and M8557-F1, respectively [11]. The M8557-P1 patch is designed to be affected by expanding motion (extension-mode), while M8557-F1 with fibres rotated at 45° to the root was used to respond to the beam's twisting motion (shear-mode).

The cantilever beam was mounted vertically on to an electrodynamic shaker (LDS V455) by two aluminium plates. Figure 6a shows a schematic of the experimental setup and Figure 6b shows an image of the system, along with the cantilever harvester. A laser vibrometer (Polytec PSV-400-M4 with VD-09 decoder) was placed 1m from the cantilever's clamping at which the laser scanning point was positioned. An electrodynamic shaker was controlled by an amplified (LDS PA 1000) signal generated by NI USB-6211 (National Instruments) with a dedicated LabView script. The electrical output from the MFCs was connected to electrical load, set by resistance decade (JJ JUNIOR Resistance Decade from Educational Measurements Ltd). Finally, the signal was gathered by oscilloscope (Agilent Infiniium 54835A with 10M Ω /10pF TEK P6137 oscilloscope probe) controlled by previously mentioned LabView script.



(a)



(b)

Figure 6. Experimental setup. (a) Schematic diagram of experiment setup. (b) Experiment setup showing cantilever beam mounted vertically on to the shaker.

The power output from the MFC patches was determined from the equation,

$$P_{OUT} = U_{RMS}^2 / R_L \quad (8)$$

where U_{RMS} is the root mean square of the voltage output across the load resistor and R_L is electrical resistance of the load resistor; in this case $R_L=10 \text{ k}\Omega$.

Power output generated by piezoelectric materials is highly sensitive to the matching impedance, which is dependent on the excitation frequency. The optimum load impedance fulfils the condition,

$$\omega R_L C_p = 1 \quad (9)$$

which can be transformed to form:

$$R_L = 1/(C_p * 2\pi f) \quad (10)$$

where C_p is a capacitance of the piezoelectric device, $\omega = 2\pi f$ is angular frequency, and f is frequency. For the sweep function in the interesting frequency range between 6-45 Hz, optimum load impedance varies from $2 \text{ M}\Omega$ to $267 \text{ k}\Omega$. In order to minimize the influence of the matching impedance, the value of $10 \text{ k}\Omega$ was chosen which is significantly far away from the optimum load impedances in the examined frequency range.

5. Results and discussion

For energy harvesting characterization, the electrodynamic shaker was excited at a constant 1g (9.8 m/s^2) and the vibration frequency was swept from 5 to 52 Hz to cover a range of vibration modes. Figure 7 shows the measured power output from both bending and torsional MFC patches. The extension-mode MFC attached on the root of the cantilever generated power at a lower frequency of 6.5 Hz only, which reached a peak of 1.15 mW. However, the shear-mode MFC at the mid span had a small power peak at 6.5 Hz and two higher peak powers at 37 and 45 Hz (0.7 mW and 0.4 mW, respectively). Since the MFC on the root harvested the bending and the MFC in the mid-span harvested the twisting of the cantilever beam, the bending mode and twisting mode can be characterized experimentally and Figure 7 indicates that the bending mode is 6.5 Hz and the twisting mode is 37 and/or 45 Hz. The small peak in power at 6.5 Hz from the shear-mode MFC is thought to be due to the bending of the beam.

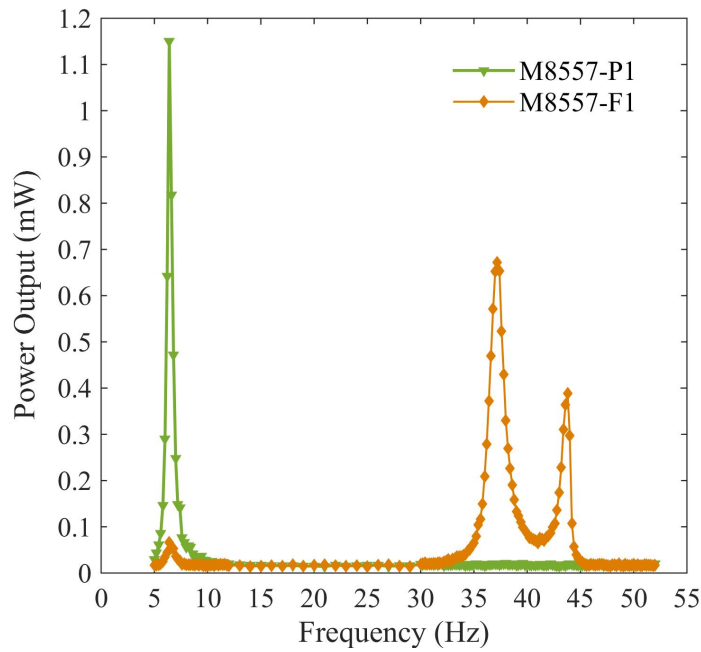


Figure 7. Power output from MFC patches at excitation level of 1g in frequency range of 5-52 Hz. Green(▼) output from M8557-P1 (extension-mode), red (◆) output from M8557-F1 (shear-mode).

Figure 8 shows the prediction of the 2-DOF model as the steady-state response of the tip displacement relative to the base motion (x - y) and angle of twist (ϕ) for the range of frequencies studied experimentally. It can be observed that the 2-DOF model predicts resonance frequencies at 6.4 Hz and 45 Hz, which compare very favorably with the experimental values in Figure 7. The first mode is dominated by transverse deformation, which explains the high electrical output of the extension-mode PZT patch, and the other mode is dominated by twist, which is expected to yield an output from the upper shear-mode PZT patch. The third resonance obtained experimentally at 37 Hz is a mixed mode that combines transverse deformation and twist and could not be captured by the present 2-DOF model, but indicated the potential of the harvesting system to operate at multiple modes.

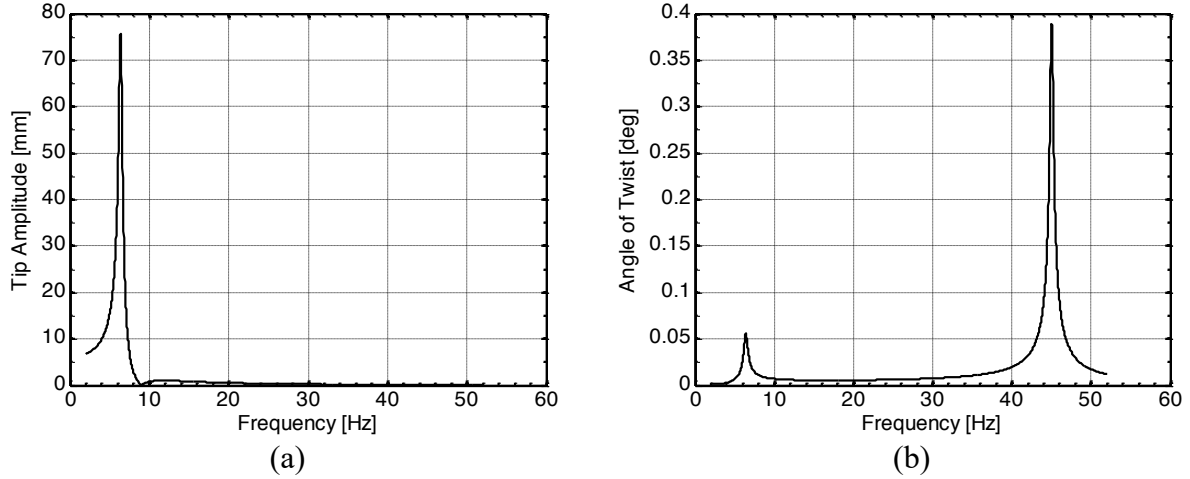


Figure 8. Simulated frequency response of the 2-DOF model at 1g excitation level. (a) Variation of the tip displacement amplitude with respect to the base motion (x - y); (b) angle of twist ϕ .

Figure 9 schematically illustrates the vibration mode shapes observed experimentally by viewing the beam through a strobe light. It can be observed that the deformation pattern in the first mode (7 Hz) is predominantly bending. The ensuing uniaxial strains, which are greatest at the root, leads to a greater electrical output from the extension-mode PZT patch, the lower patch in Figure 6b. The shear-mode PZT patch (upper patch in Figure 6b) is not subjected to significant torsion and hence provides minimal output, as shown in Figure 7. The second mode (37 Hz) was observed to contain primarily twisting motions, superimposed on a bending deformation pattern that is somewhat close to the second bending mode in a homogeneous cantilever beam. In this mode, bending deformations at the root are small, hence output from the lower patch is insignificant, whereas torsional deformations provide a higher output from the upper patch. The third mode (45 Hz) is dominated by twisting, which provides favourable drive conditions for the shear-mode patch and no output from the extension-mode one, which agrees with the output shown in Figure 7.

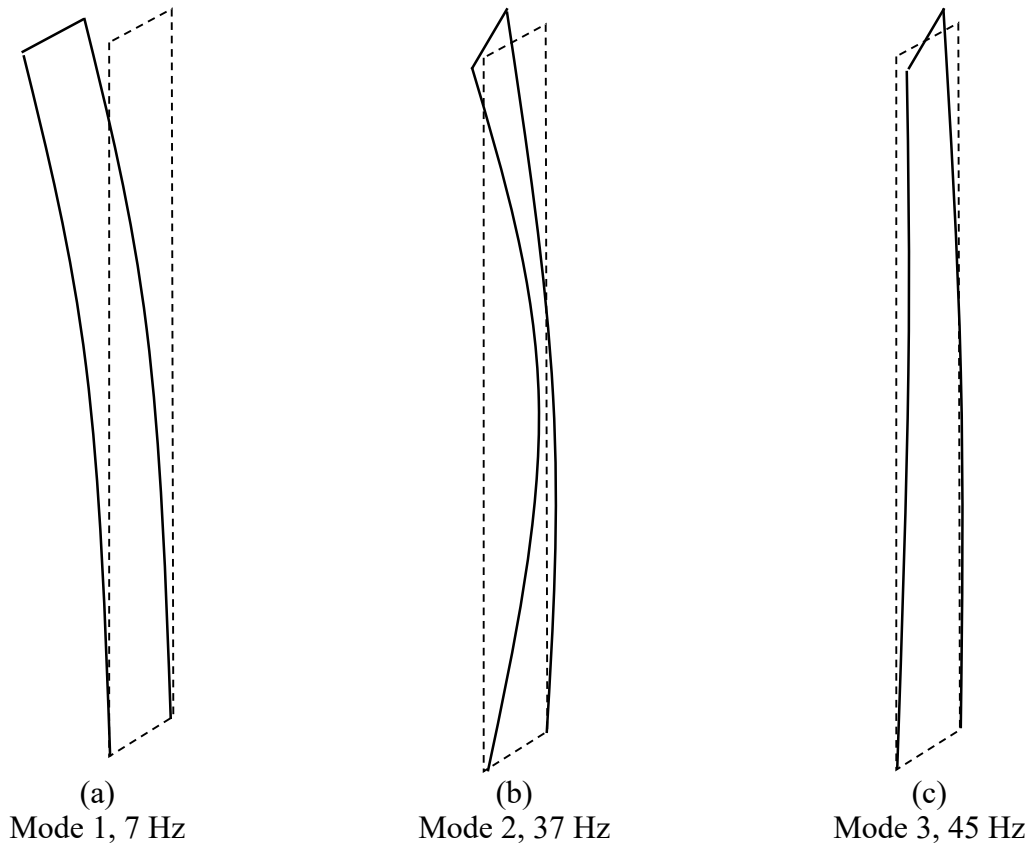


Figure 9. Schematic illustrations of vibration modes observed experimentally.

6. Conclusions

This work has presented the first demonstration of the use of a laminated carbon fibre reinforced polymer to create cantilever beams with coupled bending-twisting oscillations for energy harvesting applications. Modelling of this new bend-twist system is presented, which compares favourably with the experimental results. It is demonstrated that the variety of bend and torsional modes of the harvesters can be utilized to create a harvester that operated in a wider range of frequencies and such device architectures provides a unique approach in which to tune the frequency response of the resonant harvesting system. The advantages of the proposed design is that the structure can be tuned to provide a broader-band energy harvester by adjusting natural frequencies. In addition, torsional modes typically exhibit small displacement amplitudes compared to bending modes; this provides scope for smaller volume device architectures. By exploiting a laminate architecture which inherently exhibits bend-twist coupling the complexity of the system is reduced since there is no need to use an eccentric proof mass or asymmetric tip masses to achieve torsion and the numerous potential lay-ups that exhibit bend-twist coupling provides a large design space to optimise the frequency response of the cantilever. This approach therefore provides new opportunities for the development of broader-band energy harvesting materials and systems.

Acknowledgement

This work was supported by both the European Commission's Marie Skłodowska-Curie Actions (MSCA), through the Marie Skłodowska-Curie Individual Fellowships (IF-EF) (H2020-MSCA-IF-2015- EF-703950-HEAPPs) under Horizon 2020 and the European Research Council under the European Union's Seventh Framework Programme (FP/2007–2013)/ERC Grant Agreement no. 320963 on Novel Energy Materials, Engineering Science and Integrated Systems (NEMESIS).

References

1. Arafa, M., "Multi-Modal Vibration Energy Harvesting using a Trapezoidal Plate," ASME Journal of Vibration and Acoustics 134 (4) (2012) 041010 (6 pages).
2. Sharpes, N., Abdelkefi, A., Abdelmoula, H., Kumar, P., Adler, J. and Priya, S., "Mode shape combination in a two-dimensional vibration energy harvester through mass loading structural modification" Applied Physics Letters 109, 033901 (2016)
3. Chiarelli, M.R., Binante, V., Botturi, S., Massai, A., Kunzmann, J., Colbertaldo, A., and Romano, D.G., "On the active deformations of hybrid specimens" Aircraft Engineering and Aerospace Technology, 88 (5) (2016) 676 – 687
4. Raither, W., Bergamini, A., Gandhi, F., and Ermanni, P., "Adaptive bending-twist coupling in laminated composite plates by controllable shear stress transfer" Composites: Part A 43 (2012) 1709–1716
5. Samur, A., and Erturk, A., "Multifunctional double-bimorph piezoelectric composite for bending-twisting actuation, active stiffness change, and energy harvesting" Proceedings of the 54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Structures, Structural Dynamics, and Materials and Co-located Conferences, April 8-11, 2013, Boston, Massachusetts
6. Abdelkefi, A., Najar, F., Nayfeh, A.H. and Ben Ayed, S., "An energy harvester using piezoelectric cantilever beams undergoing coupled bending–torsion vibrations" Smart Materials and Structures 20 (2011) 115007 (11pp)
7. Gao, S., Zhang, G., Jin, L., Li, P. and Liu, H., "Study on characteristics of the piezoelectric energy-harvesting from the torsional vibration of thin-walled cantilever beams" Microsystems Technologies (DOI 10.1007/s00542-017-3336-6)
8. Shan, X., Deng, J., Song, R. and Xie, T., "A Piezoelectric Energy Harvester with Bending–Torsion Vibration in Low-Speed Water" Applied Science, 2017, 7, 116; doi:10.3390/app7020116
9. York, C.B., "Extension-twist coupled laminates for aero-elastic compliant blade design" Proceedings of the 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, 23-26 April 2012, Honolulu, Hawaii.
10. Rao, S.S., *Mechanical Vibrations*, Pearson, 2016.
11. Smart Material Corp., 2017. Macro Fiber Composite (MFC) brochure [WWW Document]. URL <https://www.smart-material.com/Datasheets.html> (accessed 28.7.2017)