

Local Energy and Planned Ramping Product Joint Market Based on a Distributed Optimization Method

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Abstract—High penetration of renewable energy generation (REG) in the distribution system increases both the power uncertainty at a given interval and the power variation between two intervals. Reserve markets addressing power uncertainty have been widely investigated. However, there is a lack of market mechanisms regarding the power variation of the load and REGs. This paper thus defines a planned ramping (PR) product to follow the net load variation and extends the local energy market to include the trading of PR products. Players are economically compensated for their PR products. Bidding models of dispatchable generators and flexible load aggregators in the joint market are investigated. To solve the market problem in polynomial time, a distributed market clearing method is developed based on the ADMM algorithm. The joint market is tested on a modified IEEE 33-bus system. It verifies that introducing the PR market can encourage flexible loads to provide more PR service to accommodate the net load variation. As such, the ramping cost of dispatchable generators is reduced by 29.09% in the test case. The planned energy curtailment from REG is also reduced. The computational efficiency of the proposed distributed clearing method is validated by comparing it with a centralized method.

Index Terms—Distributed market clearing, flexible load aggregator, local electricity market, planned ramping product, renewable energy generation.

NOMENCLATURE

A. Indexes

i	Index of dispatchable DG.
j	Index of load aggregator.
t	Index of time slot.

B. Parameters

I	Number of dispatchable DGs.
J	Number of load aggregators.
V	Number of time slots in a day.
D	Length of a time slot.

$p_{i,\min}^g, p_{i,\max}^g$	Minimum and maximum output of a DG.
P_t^{ren}	Forecast power of a REG.
$R_{i,\max}^{g,u}, R_{i,\max}^{g,d}$	Maximum ramping capacity of a DG.
L_t	Uncertain load power.
ρ^{ren}	The bidding price of REG.
$\rho_i^{g,-}, \rho_i^{g,+}$	Bidding price DG in real-time market.
$\rho_j^{\text{LA},-}, \rho_j^{\text{LA},+}$	Bidding price of LA in real-time market.
$\pi_{i,t}^{g,u}, \pi_{i,t}^{g,d}$	The bidding prices of upward and downward PR products from a DG.
$\pi_j^{\text{LA},u}, \pi_j^{\text{LA},d}$	The bidding prices for upward and downward PR from a LA.

C. Variables

b_i^g	Operating state of a dispatchable DG.
$b_{k,t}^p$	Operating status of a PAL.
$b_{k,t}^s$	Operating status of a SIL.
$b_{k,\tau}^T$	Starting state of a TSL.
$l_{j,t}^{\text{LA}}$	The load power of a LA.
$l_{k,t}^f$	The power of a flexible load.
$l_{k,t}^T$	The load power of a TSL.
$l_{k,t}^P$	The load power of a PAL.
$l_{k,t}^S$	The load power of a SIL.
$p_{i,t}^g$	Power output of a dispatchable DG.
p_t^{ren}	Accepted power of REG.
$r_{i,t}^{g,u}, r_{i,t}^{g,d}$	Upward and downward PR product of a DG.
$r_{j,t}^{\text{LA},u}, r_{j,t}^{\text{LA},d}$	Upward and downward PR product of a LA.
$\Delta p_{i,t,s}^{g,+}, \Delta p_{i,t,s}^{g,-}$	Regulation power of DG in real-time market.
$\Delta p_{j,t,s}^{\text{LA},+}, \Delta p_{j,t,s}^{\text{LA},-}$	Regulation power of LA in real-time market.
λ_t^e	Clearing price of energy.
λ_t^u, λ_t^d	Clearing price of upward and downward PR product.
$\lambda_{t,s}^{\text{RT},+}, \lambda_{t,s}^{\text{RT},-}$	Clearing price for incremental and decremental power deviation in real-time market.

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I. INTRODUCTION

IN modern power systems, the increasing penetration level of renewable energy generation (REG) requires more and more ramping capacity to accommodate both the power uncertainty at the current time interval and power variation

between two intervals. To balance the uncertainty and follow the variation of REG and the load, ramping capacity is required to be provided by dispatchable generators as well as flexible loads. If there is not enough ramping capacity to follow the variation of net system load, planned curtailment of REG or load will be foreseeable.

In power systems, the ramping capacity is dispatched in two distinguished approaches for different purposes. One purpose to address the power uncertainty and the ramping capacity is deployed in a matter of seconds [1]. This part of the ramping capacity is referred to as reserve ramping (RR) and is procured in the reserve market [1], [2]. The other purpose is to accommodate the forecasted net load variation between two dispatch intervals and is dispatched 5-min or 10-min ahead of real-time operation. This part of the ramping capacity is referred to as planned ramping (PR). Great importance has been attached to the PR in the modern power system by independent system operators (ISOs) in the USA, i.e. California ISO (CAISO) and Midcontinent ISO (MISO) [3]. The introduction of the PR market in the distribution system will encourage local players to provide more PR products and thus improve the system's capability to accept REG.

Reference [1] has distinguished the PR (named as the flexible ramping product) from traditional operating reserve especially from the perspective of response time. Then, the co-optimization model of energy and reserve is extended to include PR. For fuel-fired generators, the ramping process will cause additional operational costs and increase equipment degradation. Reference [4] has investigated the ramping cost of fuel-fired generators and added the PR cost as a penalty to the day-ahead scheduling problem. Reference [5] points out that it is reasonable to pay generators for their PR service to motivate local players to provide more flexible resources. However, the existing energy-only market does not encourage market players to provide PR or compensate for their ramping costs. This will result in a shortage of PR and power curtailment of REG. Accordingly, in modern and future distribution systems with high penetration of REGs, it is necessary to introduce the PR market for both safety and economic concerns.

Flexible loads will be important players in providing PR services in distribution systems. These loads include electric vehicles (EV), thermostatically controlled loads (TCL), electric heat pumps (EHP), heat, ventilation, and air-conditioning (HVAC) [6]–[9]. Due to the characteristics of small scale and dispersed distribution in the system, they generally participate in electricity markets through an intermediate party, e.g., a load aggregator (LA) [10], [11] or virtual power plant (VPP) [12]. To ensure an effective participation of LA in the electricity market, a proper bidding strategy is needed. A price-based self-scheduling MILP model for a DR aggregator was proposed in [13] with the aim to maximize the aggregator's payoff for participation in the day-ahead market. Considering the uncertainties associated with the renewable generation and real-time price; Reference [14] proposed a robustness of the day-ahead (DA) bidding strategy for load aggregators. The strategy is formulated as a CVaR-based risk-averse optimal problem. In [15], LA's bidding strategy is to maximize the expected profit by formulating a stochastic mixed-integer linear

programming problem. In these researches, LAs assume the role of price-takers in the market. In [16] and [17], price-maker based strategies are investigated for LAs using game theoretic approaches. In [16], the EV aggregator's bidding strategy is formulated as a mathematical program with equilibrium constraints (MPEC). A supply function equilibrium is used to model aggregator's strategic bid, which will impact the clearing price at the system level. In [17], the LA's strategy in the DA market is represented as a stochastic Stackelberg Game. The LA acts as a price-maker in the market by bidding strategically and minimizing its energy procurement cost by leveraging the flexibility of its consumers. The LA's interactions with the wholesale market were modeled as a bilevel mixed-integer linear programming (MILP) problem.

However, there are two difficulties for LAs to participate in the PR market: i) A LA can hardly provide explicit ramping parameters and power boundaries like conventional generators, because its ramping parameters depend on the operating parameters and the states of all flexible loads within a group. ii) Numerous and diversified flexible loads, as well as new emerging prosumers, increase the scale and complexity of the market clearing problem. These issues are addressed in this paper by developing a distributed market clearing method based on the alternating direction method of multipliers (ADMM) algorithm. This paper presents original contributions, including:

- Different from the existing local energy market model such as the local transactive energy market [18], [19] and peer-to-peer (P2P) market [20], [21], a local energy and PR product joint market is proposed in this paper. Market players are economically compensated for their PR products. The PR product model of LAs is developed. Then, an optimization problem for the joint market is proposed to minimize the total cost of energy in the DA market, the bid-based cost of PR products, and bid-based cost of balancing power deviation in real-time (RT) market.

- Different from the centralized energy markets where LAs act as price takers [22]–[24] and the game-based market [25], a distributed market clearing method is developed in this study based on the ADMM algorithm. It solves the market problem within polynomial time. The local market operator (LMO) runs a generation dispatch subproblem and updates the prices of the DA energy market, PR product, and RT market. Each LA dispatches its flexible loads in response to these price signals in a distributed manner to minimize its total bill or to maximize its total revenue.

- Load flexibility degrees are investigated from perspectives of time, power, and continuously in this study. Then, different flexible load models are established from the level of electric appliances. Their capability to provide PR service is investigated in this paper.

The rest of this paper is organized as follows. The structure of the energy and PR joint market is presented in Section II. The optimization model and the distributed market clearing method for the joint market are developed in Section III. In Section IV, numerical simulation studies are implemented. The conclusions of this paper are presented in Section V.

II. STRUCTURE OF ENERGY AND PR PRODUCT JOINT MARKET

A. Planned Ramping Product

To cope with the uncertainty and variation of the net load in a given system, the ramping capacity is dispatched in two different approaches for distinguished purposes. One purpose is to address the uncertainty of the net load at the current time interval and the other purpose is used to accommodate the net load variation from time interval t to $t+1$ as illustrated in Fig. 1. The former is referred to as reserve ramping (RR) and will be dispatched instantaneously depending on the differences between the actual net load and the forecasted one. The latter is used to address the forecast net load variation from the current to the next dispatch interval. This service is denoted as planned ramping (PR) (which is named as “locked ramping” in [26]). This service is increasingly important in modern power systems to follow the net load variation caused by the generation of REGs. The PR service can be provided by dispatchable DG and flexible loads.

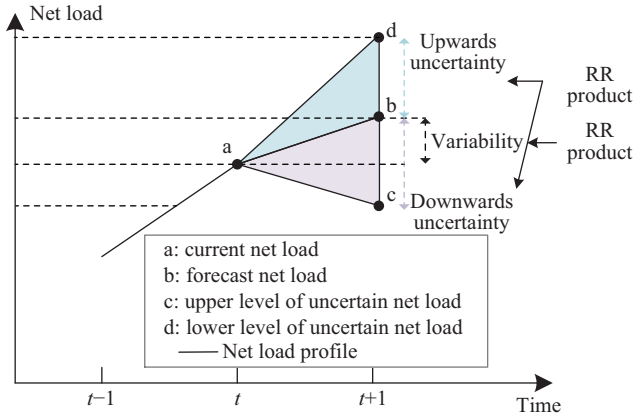


Fig. 1. Illustration of the reserved ramping (RR) and planned ramping (PR).

The local energy market is extended to include PR products in this paper. The structure of the energy and PR product joint market is presented in Section II-B. The market optimization model of the joint market is developed in Section III.

B. Structure of the Energy and PR Product Joint Market

The RP product is procured simultaneously with the energy in a day-ahead local joint market proposed in this paper. In the joint market, players include dispatchable DGs (e.g., diesel engine), REGs, (e.g., PV, wind power), uncertain loads, and flexible loads. LAs of flexible loads and dispatchable DGs are providers of PR products to follow the variation of the net load, namely the uncertain loads minus output of REGs. The joint market is managed by the LMO.

In the distribution system, it is operationally impractical and computationally infeasible to manage numerous and scattered flexible loads centrally by the LMO. To address this issue, a market clearing process is carried out in a distributed manner, based on a distributed optimization algorithm. The market structure is shown in Fig. 2.

Flexible loads participate in the local market through a LA. Load parameters are submitted to the LA. Then, the LA

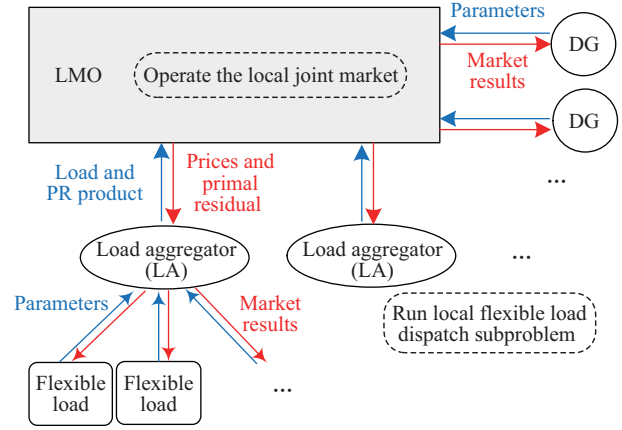


Fig. 2. Structure of the local joint market.

submits its bid, including energy demand and PR product, to the LMO after running a flexible load dispatch subproblem in a distributed manner. The LMO collects the bid information from dispatchable DGs and the forecast data of REGs. Then, it performs a generation dispatch subproblem. The LMO is also responsible to update the clearing prices of energy and PR products in a centralized manner. The joint market clearing problem is presented in Section III-A. The distributed clearing process is realized based on the ADMM algorithm [27]–[30], which can be used to solve large scale convex and non-convex problems. The detailed modeling process is presented in Section III-B.

III. MODELING FOR THE ENERGY AND PR JOINT MARKET

A. Joint Market Model

The LMO operates the local joint market. In the market, a dispatchable DG submits a quadratic or linearized bidding model for its energy supply and two bidding prices for its upwards and downwards PR products to the LMO. A REG submits a bidding price for its generation to the LMO.

Each LA submits two bidding prices for its upwards and downwards PR products to the LMO. The market is organized based on the pool market mode and the clearing prices of energy and PR products are determined by the marginal cost [31].

1) Objective Function

The objective of the local joint market is to minimize the total cost to meet the total energy demand and the PR requirements of the net load in the DA market plus the weighted cost in the real-time (RT) market. The energy cost includes the bid-based cost from dispatchable DGs, and REGs. The PR cost includes the bid-based cost from dispatchable DGs and LAs. The objective function is expressed as:

$$\min_{\{x^g, x_j^{LA} | \forall j\}} f = \sum_{t=1}^V \left[\sum_{i=1}^I ((C_{i,t}^g(p_{i,t}^g) + \pi_i^{g,u} r_{i,t}^{g,u} + \pi_i^{g,d} r_{i,t}^{g,d}) + \rho^{\text{ren}} p_t^{\text{ren}} + \sum_{j=1}^J (\pi_j^{LA,u} r_{j,t}^{LA,u} + \pi_j^{LA,d} r_{j,t}^{LA,d}) + C^{\text{RT}} \right] \quad (1)$$

$$C_{i,t}^g(p_{i,t}^g) = a_i^g + b_i^g p_{i,t}^g + c_i^g p_{i,t}^{g,2}, \forall i, t \quad (2)$$

$$C^{\text{RT}} = \omega_s \sum_{s=1}^S \sum_{t=1}^V \left[\sum_{i=1}^I (\rho_i^{g,-} \Delta p_{i,t,s}^{g,-} + \rho_i^{g,+} \Delta p_{i,t,s}^{g,+}) + \sum_{j=1}^J (\rho_j^{\text{LA},-} \Delta p_{j,t,s}^{\text{LA},-} + \rho_j^{\text{LA},+} \Delta p_{j,t,s}^{\text{LA},+}) \right] \quad (3)$$

where $C_{i,t}^g(p_{i,t}^g)$ is the generation cost of a dispatchable DG as expressed in (2) [31]; $p_{i,t}^g$ is the power output of a dispatchable DG; a_i^g , b_i^g , and c_i^g are coefficients of the quadratic cost function; $r_{i,t}^{g,u}$ and $r_{i,t}^{g,d}$ are the upward and downward PR products provided by a dispatchable DG; $\pi_i^{g,u}$ and $\pi_i^{g,d}$ are its bidding prices of the upward and downward PR products, respectively; p_t^{ren} is the accepted power of REG, ρ^{ren} is the bidding price of REG; $r_{j,t}^{\text{LA},u}$ and $r_{j,t}^{\text{LA},d}$ are the upward and downward PR products provided by a LA; $\pi_j^{\text{LA},u}$ and $\pi_j^{\text{LA},d}$ are its bidding prices of the upward and downward PR products, respectively; V is the number of time slots in the day-ahead market, I is the number of dispatchable DGs, J is the number of LAs, S is the number of real-time scenarios. C^{RT} represents the weighted RT market cost to balance the DA forecast power deviation. ω_s is the weight factor of scenario s . $\rho_i^{g,+}$ and $\rho_i^{g,-}$ are the bidding prices for the increment and decrement power quantities, namely $\Delta p_{i,t,s}^{g,+}$, $\Delta p_{i,t,s}^{g,-}$, respectively, by a DG in the real-time market. When using the vector form, $\mathbf{x}^g = \{p_{i,t}^g, r_{i,t}^{g,u}, r_{i,t}^{g,d}, \Delta p_{i,t,s}^{g,+}, \Delta p_{i,t,s}^{g,-}, p_t^{\text{ren}} | \forall i, t, s\} \in \mathbb{R}^{n_g}$ denotes the vector of generation variables consisting of DGs' outputs, DGs' ramping variables, and REG's outputs; n_g is the number of generation variables. $\rho_j^{\text{LA},+}$ and $\rho_j^{\text{LA},-}$ is the bidding price for the increment and decrement power quantity, namely $\Delta p_{j,t,s}^{\text{LA},+}$, $\Delta p_{j,t,s}^{\text{LA},-}$, respectively, by a LA in the real-time market. When using the vector form, $\mathbf{x}_j^{\text{LA}} = \{l_{j,t}^{\text{LA}}, r_{j,t}^{\text{LA},u}, r_{j,t}^{\text{LA},d}, \Delta p_{j,t,s}^{\text{LA},+}, \Delta p_{j,t,s}^{\text{LA},-} | \forall t, s\} \in \mathbb{R}^{n_{\text{LA}}}$ denotes that the vector consists of the demand variable and PR variables of LA j , n_{LA} is the number of operational variables of a LA.

2) Global Constraints of the Joint Market

The power balance constraint is given in (4):

$$\sum_{i=1}^I p_{i,t}^g + p_t^{\text{ren}} = \sum_{j=1}^J l_{j,t}^{\text{LA}} + L_t, \forall t : \lambda_t^e \quad (4)$$

where L_t represents the uncertain load. $l_{j,t}^{\text{LA}} (t = 1, 2, \dots, V)$ is the load demand variable of LA j . λ_t^e is the dual variable in the dual problem, also known as the marginal price of energy.

Planned ramping requirement constraints:

$$\sum_{i=1}^I r_{i,t}^{g,u} + \sum_{j=1}^J r_{j,t}^{\text{LA},u} = R_t^{\text{req},u}, t \in \mathbf{T}^u : \lambda_t^u \quad (5)$$

$$\sum_{i=1}^I r_{i,t}^{g,d} + \sum_{j=1}^J r_{j,t}^{\text{LA},d} = R_t^{\text{req},d}, t \in \mathbf{T}^d : \lambda_t^d \quad (6)$$

where λ_t^u , λ_t^d are the dual variables in the dual problem as well as the marginal prices of PR products. $\mathbf{T}^u/\mathbf{T}^d$ represents the interval set when the ramping requirement of net system

load is positive/negative, namely:

$$\mathbf{T}^u \triangleq \{t : L_t - p_t^{\text{ren}} > L_{t-1} - p_{t-1}^{\text{ren}}\}$$

$$\mathbf{T}^d \triangleq \{t : L_t - p_t^{\text{ren}} < L_{t-1} - p_{t-1}^{\text{ren}}\}$$

Power balance constraints under possible real-time scenarios are presented in Eqs. (7) and (8), respectively corresponding to positive and negative power deviation of system net load $\Delta p_{t,s}^{\text{net}}$, which is presented by Eq. (9).

$$\sum_{i=1}^I \Delta p_{i,t,s}^{g,+} + \sum_{j=1}^J \Delta p_{j,t,s}^{\text{LA},+} = \Delta p_{t,s}^{\text{net}},$$

$$\text{if } \Delta p_{t,s}^{\text{net}} > 0, \forall t : \lambda_{t,s}^+ \quad (7)$$

$$\sum_{i=1}^I \Delta p_{i,t,s}^{g,-} + \sum_{j=1}^J \Delta p_{j,t,s}^{\text{LA},-} = \Delta p_{t,s}^{\text{net}},$$

$$\text{if } \Delta p_{t,s}^{\text{net}} < 0, \forall t : \lambda_{t,s}^- \quad (8)$$

$$\Delta p_{t,s}^{\text{net}} = L_{t,s} - L_t - (p_{t,s}^{\text{ren}} - p_t^{\text{ren}}), \forall t, s \quad (9)$$

Network constraints in the distribution system:

In a radial network, the power flow on line $i - j$ is the sum of the withdraw power on its downstream nodes and the downstream network losses. As this study aims to consider the impact of network congestion on the local market, the network loss is neglected here. Hence, the power flow of a radial distribution system and the network constraints are modeled as:

$$p_{m,n,t}^{\text{line}} = \sum_{h \in \Omega_{m,n}} p_{h,t}^{\text{node}} \quad (10)$$

$$p_{h,t}^{\text{node}} = L_{h,t} + \sum_{j=1}^J l_{j,t}^{\text{LA}} R_{j,h}^{\text{LA}} - p_{h,t}^{\text{ren}} - \sum_{i=1}^I p_{i,t}^g R_{i,h}^g \quad (11)$$

$$-p_{m,n,\max}^{\text{line}} \leq p_{m,n,t}^{\text{line}} \leq p_{m,n,\max}^{\text{line}}, t \in \mathbf{T}^u : \mu_t^-, \mu_t^+ \quad (12)$$

where, $\Omega_{m,n}$ is the set of downstream nodes of line $m - n$; $p_{h,t}^{\text{node}}$ is the withdraw power from node h ; $p_{m,n,\max}^{\text{line}}$ is the transmission capacity of line $m - n$; $L_{h,t}$ and $p_{h,t}^{\text{ren}}$ is the uncertain load power and REG's outputs connected to node h ; $R_{j,h}^{\text{LA}}$ is a 1/0 parameter representing whether or not LA j is connected to node h ; $R_{i,h}^g$ is a 1/0 parameter representing whether or not dispatchable DG i is connected to node h .

When considering the network losses and voltage control, the linearized power flow model for distribution systems in [32] can be adopted.

3) Local Constraints for DGs

Power output constraints of dispatchable DGs:

$$b_{i,t}^g p_{i,\min}^g \leq p_{i,t}^g \leq b_{i,t}^g p_{i,\max}^g, \forall i, t \quad (13)$$

where $b_{i,t}^g$ refers to the operating state of a dispatchable DG; $p_{i,\min}^g$ and $p_{i,\max}^g$ are the minimum and maximum output of a dispatchable DG.

Power output constraints of DGs under each scenario:

$$b_{i,t}^g p_{i,\min}^g \leq p_{i,t,s}^g \leq b_{i,t}^g p_{i,\max}^g, \forall i, t \quad (14)$$

$$p_{i,t,s}^g = p_{i,t}^g + \Delta p_{i,t,s}^{g,+} - \Delta p_{i,t,s}^{g,-} \quad (15)$$

When there is not enough ramping capacity in the system, the energy of REG may be curtailed, so that it is subjective to the following constraint:

$$0 \leq p_t^{\text{ren}} \leq P_t^{\text{ren}} \quad (16)$$

where P_t^{ren} is the forecast output of REG.

Constraints for ramping products of a dispatchable DG:

$$r_{i,t}^{\text{g,u}} = \max(p_{i,t}^{\text{g}} - p_{i,t-1}^{\text{g}}, 0), \forall i, t \in \mathbf{T}^{\text{u}} \quad (17)$$

$$r_{i,t}^{\text{g,d}} = \max(p_{i,t-1}^{\text{g}} - p_{i,t}^{\text{g}}, 0), \forall i, t \in \mathbf{T}^{\text{d}} \quad (18)$$

$$0 \leq r_{i,t}^{\text{g,u}} \leq R_{i,\max}^{\text{g,u}}, \forall i, t \quad (19)$$

$$0 \leq r_{i,t}^{\text{g,d}} \leq R_{i,\max}^{\text{g,d}}, \forall i, t \quad (20)$$

where $R_{i,\max}^{\text{g,u}}$ and $R_{i,\max}^{\text{g,d}}$ are the maximum upward and downward ramping capability of a dispatchable DG.

Ramping constraints of DGs under each scenario:

$$-R_{i,\max}^{\text{g,d}} \leq p_{i,t,s}^{\text{g,RT}} - p_{i,t-1,s}^{\text{g,RT}} \leq R_{i,\max}^{\text{g,u}}, \forall i, t, s \quad (21)$$

4) Local Constraints for Flexible Loads

Flexible loads involve air conditioners, water heating, laundry machines, wet appliances, and so on. They have different flexibility degrees. In this study, flexible loads are classified into three categories: time-shiftable loads (TSL) [33], power-adjustable loads (PAL) [34], and state-interruptible loads (SIL) [35]. Their operational models are constructed in this study.

Operational constraints of TSLs:

A TSL refers to an appliance that has a fixed working mode (i.e., electricity usage curve is fixed) but its working time is shiftable within a certain time interval: $[T_0, T_{\text{end}}]$. Such a load involves a water heater. The load power of a TSL only depends on its starting time. Once it starts, it cannot be turned off or interrupted before finishing. Its operation can be expressed as the following constraints:

$$l_{k,t}^{\text{T}} = \sum_{\tau=t-T_{\text{d},k}+1}^t b_{k,\tau}^{\text{T}} L_k^{\text{T}} \quad (22)$$

$$\sum_{\tau=T_0,k}^{T_{\text{end},k}-T_{\text{d},k}} b_{k,\tau}^{\text{T}} = 1 \quad (23)$$

where $l_{k,t}^{\text{T}}$ is the power of a TSL at time t ; $T_{\text{d},k}$ is the length of its continuous working time; L_k^{T} is the rated power of the appliance; $b_{k,\tau}^{\text{T}}$ is a binary variable representing its starting state; T_0,k and $T_{\text{end},k}$ refer to the start and end time of its available working interval; Equation (23) indicates the load is started only once.

Operational constraints of PALs:

A PAL refers to an appliance whose working power is adjustable and it is not interruptible during operation. Some thermostatically controlled loads belong to such types of flexible loads. PALs are subjective to the following constraints:

$$b_{k,t}^{\text{P}} L_{k,\min}^{\text{P}} \leq l_{k,t}^{\text{P}} \leq b_{k,t}^{\text{P}} L_{k,\max}^{\text{P}} \quad (24)$$

$$\sum_{t=T_0,k}^{T_{\text{end},k}} l_{k,t}^{\text{P}} D = Q_{k,t}^{\text{P}} \quad (25)$$

$$\sum_{t=T_0,k}^{T_{\text{end},k}-1} \max(b_{k,t}^{\text{P}} - b_{k,t-1}^{\text{P}}, 0) = 1 \quad (26)$$

where D is the length of each time slot; $l_{k,t}^{\text{P}}$ is the load power of a PAL; $L_{k,\min}^{\text{P}}$ and $L_{k,\max}^{\text{P}}$ are the minimum and maximum load power of the PAL; $b_{k,t}^{\text{P}}$ represents the operating status of the PAL; $Q_{k,t}^{\text{P}}$ is its total energy requirement.

Equation (25) indicates that the total energy requirement of a PAL needs to be satisfied. Equation (26) indicates that the load is started only once.

Operational constraints of SILs:

A SIL refers to an appliance that is interruptible during operation and its power is adjustable, such as an EV. The operation of SILs is subjective to the following constraints:

$$b_{k,t}^{\text{S}} L_{k,\min}^{\text{S}} \leq l_{k,t}^{\text{S}} \leq b_{k,t}^{\text{S}} L_{k,\max}^{\text{S}} \quad (27)$$

$$\sum_{t=T_0,k}^{T_{\text{end},k}} l_{k,t}^{\text{S}} D = Q_{k,t}^{\text{S}} \quad (28)$$

where $l_{k,t}^{\text{S}}$ is the load power of a SIL; $L_{k,\min}^{\text{S}}$ and $L_{k,\max}^{\text{S}}$ are the minimum and maximum load power of the SIL; $b_{k,t}^{\text{S}}$ represents the working status of a SIL; $Q_{k,t}^{\text{S}}$ refers to its total energy requirement.

5) Local Constraints for a LA

A LA manages a group of flexible loads. It purchases energy from the local market to satisfy the demand of flexible loads and provides PR products to the local market by dispatching its flexible loads. The operational constraints for a LA are expressed as:

$$l_{j,t}^{\text{LA}} = \sum_{k=1}^K l_{k,t}^{\text{f}}, \forall j, t \quad (29)$$

$$r_{j,t}^{\text{LA,u}} = \max(l_{j,t-1}^{\text{LA}} - l_{j,t}^{\text{LA}}, 0), t \in \mathbf{T}^{\text{u}}, \forall j \quad (30)$$

$$r_{j,t}^{\text{LA,d}} = \max(l_{j,t}^{\text{LA}} - l_{j,t-1}^{\text{LA}}, 0), t \in \mathbf{T}^{\text{d}}, \forall j \quad (31)$$

where K refers to the number of flexible loads managed by a LA; $l_{k,t}^{\text{f}}$ is the load power variable of flexible load k . It can be a TSL, PAL or SIL; For TSLs, the superscript f in $l_{k,t}^{\text{f}}$ in (29)–(31) is replaced by T . The operation of TSLs is subjective to constraints (22) and (23). For PALs, the superscript f in $l_{k,t}^{\text{f}}$ is replaced by P . The operation of PALs is subject to constraints (24)–(26). For SILs, the superscript f in $l_{k,t}^{\text{f}}$ is replaced by the S . The operation of SILs is subjective to constraints (27) and (28).

Different from a dispatchable DG providing the upward ramping product by increasing its output, a LA provide the upward ramping product by decreasing its load power, and vice versa.

The operational constraints for a LA under each scenario:

$$l_{j,t}^{\text{LA,RT}} = \sum_{k=1}^K l_{k,t}^{\text{f,RT}}, \forall j, t, s \quad (32)$$

$$l_{j,t}^{\text{LA,RT}} = l_{j,t}^{\text{LA,+}} + \Delta p_{j,t,s}^{\text{LA,+}} - \Delta p_{j,t,s}^{\text{LA,-}}, \forall j, t \quad (33)$$

where $l_{j,t,s}^{\text{LA,RT}}$ is the load demand variable of LA j under scenario s . In Eq. (32), $l_{k,t,s}^{\text{f,RT}}$ represents the real-time power

of a flexible load and is subjected to the load operational constraints, such as those in Eqs. (22–23), (24–26), or (27–28) depending on its flexibility type.

In summary, the objective function of the joint market model is shown in (1), which is subject to global constraints in (4)–(12), local constraints for DGs in (13)–(21), and local constraints for flexible loads and the LA in (22)–(33). The joint market model is a mixed-integer quadratic programming (MIQP) problem.

B. Distributed Optimization Algorithm for the Local Joint Market

The joint market model can be solved in a distributed manner using ADMM, which is a distributed method. It first decomposes a large scale of optimization problems into a series of small scale subproblems based on the augmented Lagrangian relaxation technique. Then, each subproblem is solved locally and the dual variables introduced in the Lagrangian relaxation function are centrally updated. The interactive information between local agent/aggregator and center operator is shown in Fig. 2. The solving process is executed in an iterative fashion until the convergence criteria is satisfied. The detailed principle and solving process of ADMM are presented in Appendix A.

Three sets of global constraints in (4)–(9) of the joint market model can be represented by the matrix and vector forms as shown in (34). They can be relaxed and added to the objective function using the Lagrangian multiplier method. Let $\lambda_1 \in \mathbb{R}^{n_1}$ ($n_1 = 3V + S \times V$) denote the Lagrangian multiplier vector corresponding to the equality constraint in (4)–(9).

The network constraints are global inequality constraints which can be derived by substituting $p_{h,t}^{\text{node}}$ in (10) by Eq. (11) and substituting them into Eq. (12) to replace $p_{m,n,t}^{\text{line}}$. Then, the matrix and vector form of the global inequality constraints in (12) representing network constraints are shown in (35). The inequality constraints can be transformed into equality constraints as shown in Eq. (36) by introducing a set of slack variables $\mathbf{y} \in \mathbb{R}_+^{2V \times n_{\text{line}}}$. They are also added to the objective function through using the Lagrangian multiplier method. Let $\lambda_2 \in \mathbb{R}^{n_2}$ ($n_2 = 2V \times n_{\text{line}}$) denote the Lagrangian multiplier vector corresponding to the equality constraint (36).

Then, the augmented Lagrangian function of the market clearing problem can be written as (37).

$$\mathbf{A}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} = \mathbf{c} \quad (34)$$

$$\mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} \leq \mathbf{e} \quad (35)$$

$$\mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} = \mathbf{e} \quad (36)$$

$$L(\mathbf{x}^g, p_t^{\text{ren}}, \mathbf{x}_j^{\text{LA}}, \boldsymbol{\lambda}) = f + \lambda_1^T \left(\mathbf{A}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} - \mathbf{c} \right) +$$

$$(\rho/2) \left\| \mathbf{A}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} - \mathbf{c} \right\|_2^2 + \lambda_2^T \left(\mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} - \mathbf{e} \right) + (\rho/2) \left\| \mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} - \mathbf{e} \right\|_2^2 \quad (37)$$

where $\mathbf{A}^g \in \mathbb{R}^{n_1 \times n_g}$ is the submatrix related to the generation variables in the global equality constraints. $\mathbf{A}_j^{\text{LA}} \in \mathbb{R}^{n_1 \times n_{\text{LA}}}$ is the submatrix related to $\mathbf{x}_j^{\text{LA}}$ in the global equality constraints. $\mathbf{c}$ represents the constant vector in the global equality constraints. $\mathbf{E}$ is a unit matrix. $\rho \in \mathbb{R}^+$ is a penalty parameter for the ADMM algorithm. The penalty parameter ρ in this study is initialized to a relatively small value and is updated incrementally in the iteration process. The dual variables λ_1 indicate the DA energy prices, PR prices, and RT market prices under each scenario. The dual variables λ_2 indicate the network congestion prices.

The global Lagrangian function can be decoupled into several local Lagrangian functions corresponding to the LA dispatch subproblem in (38), generation dispatch subproblem in (39), and slack variable optimization problem in (40).

$$L_{j*}^{\text{LA}}(\{\mathbf{x}_j^{\text{LA}} | \forall j\}, \mathbf{x}^g, \lambda_1, \lambda_2) = f_{j*}^{\text{LA}} + (\lambda_1^T \mathbf{A}_{j*}^{\text{LA}} + \lambda_2^T \mathbf{D}_{j*}^{\text{LA}}) \mathbf{x}_{j*}^{\text{LA}} + (\rho/2) \left\| \mathbf{A}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} - \mathbf{c} \right\|_2^2 + (\rho/2) \left\| \mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} - \mathbf{e} \right\|_2^2 \quad \forall j^* \quad (38)$$

$$L^G(\mathbf{x}^g, \mathbf{x}_j^{\text{LA}}, \mathbf{y}, \lambda_1, \lambda_2) = f^g + (\lambda_1^T \mathbf{A}^g + \lambda_2^T \mathbf{D}^g) \mathbf{x}^g + (\rho/2) \left\| \mathbf{A}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} - \mathbf{c} \right\|_2^2 + (\rho/2) \left\| \mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} - \mathbf{e} \right\|_2^2 \quad (39)$$

$$L^{\text{sl}}(\mathbf{x}^g, \mathbf{x}_j^{\text{LA}}, \mathbf{y}, \lambda_1, \lambda_2) = \lambda_2^T \mathbf{E} \mathbf{y} + (\rho/2) \left\| \mathbf{D}^g \mathbf{x}^g + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}} + \mathbf{E} \mathbf{y} - \mathbf{e} \right\|_2^2 \quad (40)$$

where f_{j*}^{LA} is the cost related to LA j^* in (1). f^g is the cost from the generation side in (1); $\mathbf{x}_j^{\text{LA}}$ needs to satisfy the local constraints for flexible loads in (22)–(31); $\mathbf{x}^g$ needs to satisfy the local constraints for DGs in (13)–(20); $\mathbf{y}$ is non-negative variables.

ADMM for the joint market clearing consists of the following iterations:

$$\mathbf{x}_{j*}^{\text{LA}(h+1)} := \arg \min_{\mathbf{x}_{j*}^{\text{LA}}} L_{j*}^{\text{LA}}(\mathbf{x}_{j*}^{\text{LA}}, \{\mathbf{x}_j^{\text{LA}(h)} | \forall j \neq j^*\},$$

$$\mathbf{x}^{\mathbf{g}(h)}, \lambda_1^{(h)}, \lambda_2^{(h)}) \quad (41)$$

$$\mathbf{x}^{\mathbf{g}(h+1)} := \arg \min_{\mathbf{x}^{\mathbf{g}}} L^G(\mathbf{x}^{\mathbf{g}}, \{\mathbf{x}_j^{\text{LA}(h)} | \forall j\}, \lambda_1^{(h)}, \lambda_2^{(h)}) \quad (42)$$

$$\mathbf{y}^{(h+1)} := \arg \min_{\mathbf{y}} L^{\text{sl}}(\mathbf{y}, \mathbf{x}^{\mathbf{g}(h)}, \{\mathbf{x}_j^{\text{LA}(h)} | \forall j\}, \lambda_1^{(h)}, \lambda_2^{(h)}) \quad (43)$$

$$\lambda_1^{(h+1)} := \lambda_1^{(h)} + \rho \left(\mathbf{A}^{\mathbf{g}} \mathbf{x}^{\mathbf{g}(h+1)} + \sum_{j=1}^J \mathbf{A}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}(h+1)} - \mathbf{c} \right) \quad (44)$$

$$\lambda_2^{(h+1)} := \lambda_2^{(h)} + \rho \left(\mathbf{D}^{\mathbf{g}} \mathbf{x}^{\mathbf{g}(h+1)} + \sum_{j=1}^J \mathbf{D}_j^{\text{LA}} \mathbf{x}_j^{\text{LA}(h+1)} + \mathbf{E} \mathbf{y}^{(h+1)} - \mathbf{e} \right) \quad (45)$$

where h the index of the iteration.

The termination criteria may be determined by the primal residual column vector $\Delta \mathbf{c}$ which reflects the primal feasibility after the h -th iteration [27]:

$$\Delta \mathbf{c} = \begin{bmatrix} \mathbf{A}^{\mathbf{g}} \\ \mathbf{D}^{\mathbf{g}} \end{bmatrix} \mathbf{x}^{\mathbf{g}(h)} + \sum_{j=1}^J \begin{bmatrix} \mathbf{A}_j^{\text{LA}} \\ \mathbf{D}_j^{\text{LA}} \end{bmatrix} \mathbf{x}_j^{\text{LA}(h)} + \begin{bmatrix} \mathbf{O} \\ \mathbf{E} \end{bmatrix} - \begin{bmatrix} \mathbf{c} \\ \mathbf{e} \end{bmatrix} \quad (46)$$

Given a feasible tolerance $\varepsilon > 0$, the criterion for the determination is that the maximum element in $\Delta \mathbf{c}$ must be less than a predetermined tolerance ε , where:

$$\|\Delta \mathbf{c}\|_{\infty} < \varepsilon \quad (47)$$

The flowchart of the market clearing process based on the ADMM algorithm is presented in Fig. 3. i) DGs submit their bids to the LMO. The LMO collects or forecasts the uncertain load and initializes the prices of energy and PR products. ii) Each LA runs a local flexible load dispatch subproblem based on the received price signals from the LMO. Then, it submits the flexible load dispatch results to the LMO. iii) The LTO runs the local generation dispatch subproblem. iv) The prices are updated based on the ADMM method by the LMO. v) Check whether the iterative process satisfies the determination criteria. If yes, terminate the iteration and obtain the market clearing results. Otherwise, continue the iteration. It should be noted that the generation dispatch subproblem can also be run by an agent or a generator aggregator. Then, it submits the results to the LMO.

C. Benchmark Models

To verify the value of introducing the PR market into the local market, an energy-only market model is taken as a benchmark model for comparison. In the energy-only market, only energy costs are considered in the market. DGs only submit energy bids to the LMO. LAs only respond to the energy prices in the local market.

To demonstrate the computational efficiency of the distributed market clearing method, the proposed model is compared with a centralized market clearing method, which solves the problem consisting of (1)–(22) as a single-level problem.

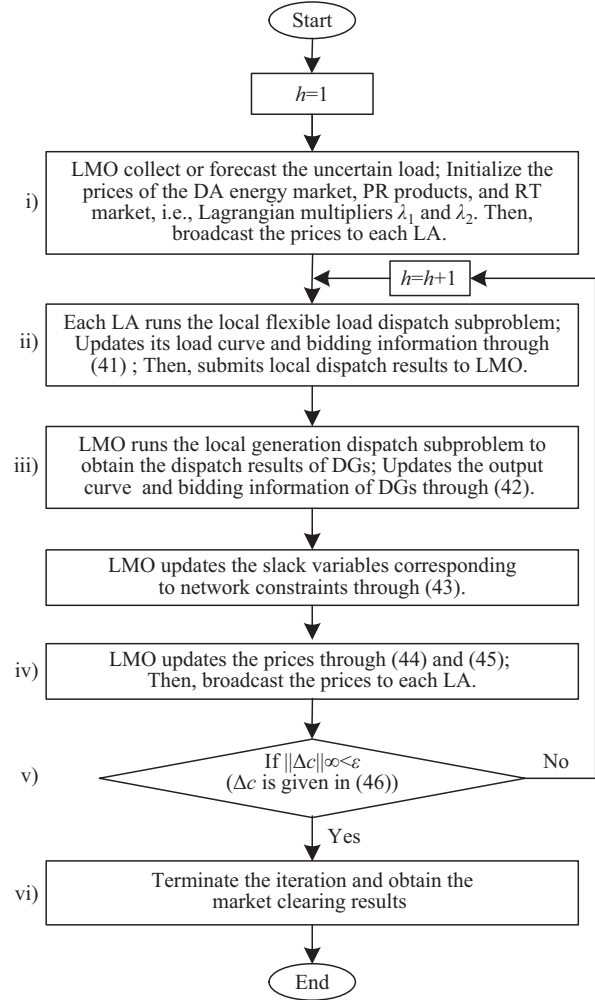


Fig. 3. Flowchart of the joint market clearing process based on ADMM.

IV. SIMULATION RESULTS

A. Test Case Settings

The proposed local energy and PR joint market is tested on a modified IEEE 33-bus system which is shown in Fig. 4. Branch parameters are available in [36]. Parameters of three dispatchable DGs are given in Table I. The three types of flexible loads mentioned in Section III-B, namely TSL, PAL, and SIL, are separately managed by three LAs. Each LA manages 10 flexible loads. Their parameters, including minimum power, maximum power, and available operating intervals, are given in Appendix Table A1. LAs are energy consumers in the DA market and act as price takers. They are providers in the PR market and flexibility providers in the RT market and act as price makers. Thus, LAs will submit the prices for PR and RT power regulation in the PR market and RT market, respectively. Their bidding prices are presented in Table II.

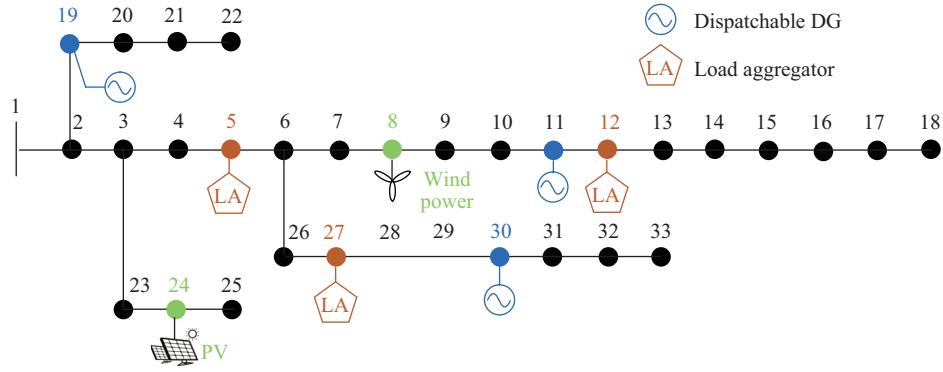


Fig. 4. Single line diagram of modified IEEE 33 bus test system.

TABLE I
DG OPERATIONAL PARAMETERS

Item	Min output (kW)	Max output (kW)	Quadratic function (pence/kWh)			Cost of PR product (pence/kW)	
			a	b	c	upward	downward
DG 1	300	600	0.0005	5.2	82	1.20	1.20
DG 2	250	1200	0.0008	5.3	96	1.05	1.05
DG 3	300	1500	0.001	4.9	78	1.33	1.33

TABLE II
BIDDING PRICES OF LAs

Product	LA of TSL	LA of PAL	LA of SIL
Bidding price for PR	1.02	0.96	0.87
Bidding price for RT positive power deviation	/	0.4	0.35
Bidding price for RT negative power deviation	/	0.4	0.35

B. Results of the Joint Market

The trading volumes of DGs and LAs in the joint market are shown in Fig. 5. The majority of the load demand is satisfied by dispatchable DGs. The energy clearing prices are determined by the marginal costs of dispatchable DGs and have the same trend as the net load profile.

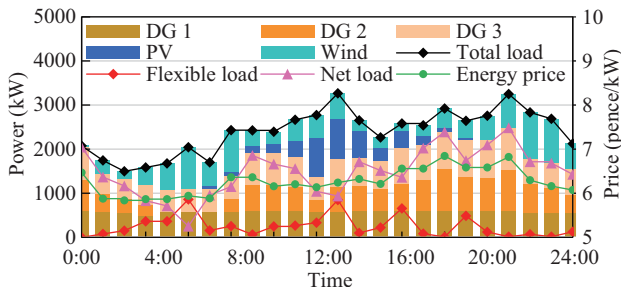


Fig. 5. Market results of the energy trading.

The flexible loads are scheduled to consume energy at low price periods. The profile of flexible loads varies oppositely to the trend of the net load profile. It indicates that flexible loads contribute to reducing the peak-to-valley ratio of the net load profile and providing the ramping products to the system.

C. Market Results of PR Products

The system ramping requirements and PR products from DGs and LAs are presented in Fig. 6.

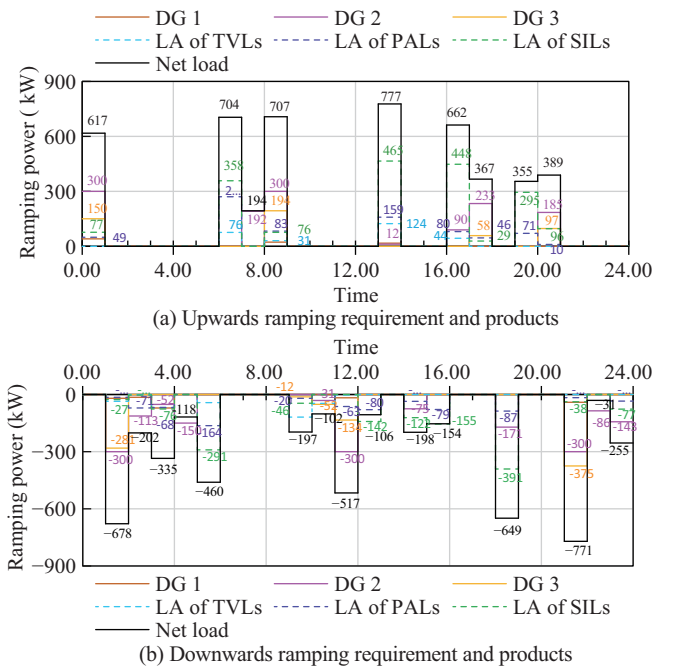


Fig. 6. PR requirement of the net load and PR products from DGs and LAs.

In the joint market, dispatchable DGs and LAs of flexible loads provide PR products to follow the variation of net load in the distribution system. The amounts of PR products from dispatchable DGs and LAs are listed in Table III. As DG 1 runs at full capacity most of the time, it provides very few PR products. DG 2 and DG 3 satisfy a large portion of the system's ramping requirements. LAs also provide a portion of PR products to the system and can obtain certain economical compensation for their service.

The load profiles (each is composed of 10 flexible loads) of three LAs are presented in Fig. 7. Three types of flexible loads provide different amounts of PR products, which depend on their flexibility degrees. SILs are interruptible during operations and provide more PR products than TSLs and PALs. PALs can change their load power during operations

and provide more PR products than TSL, whose load power is unchangeable.

TABLE III
TOTAL FLEXIBLE RAMPING PRODUCTS FROM EACH PLAYER (kW)

Product	Requirement	DG 1	DG 2	DG 3	LA for TSL	LA for PAL	LA for SIL
Ramp-up	4772	81	1311	500	275	770	1844
Ramp-down	4772	163	1721	854	315	733	1366

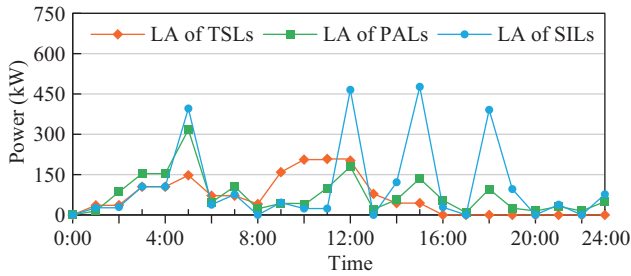


Fig. 7. Load profiles of three LAs managing different types of flexible loads.

The PR products provided by three types of flexible loads are 590, 1502, and 3210 kW, respectively. It indicates that loads with a higher flexibility degree can gain more economic compensation from the PR market.

D. Comparison with the Energy-only Market

The results of the joint market are compared with the energy-only market in Table IV. To facilitate comparison, the amount of PR products from dispatchable DGs and LAs are calculated after the clearing of the energy-only market.

TABLE IV
THE RESULTS OF THE JOINT MARKET AND ENERGY-ONLY MARKET

Market results	Joint market	Energy-only market
LAs' payment for energy (£)	360.19	359.95
PR from LAs (kW)	5259	3345
LAs' profit of ramping product (£)	51.60	/
LAs' total payment bill (£)	308.59	359.95
DGs' generation cost (£)	2393.64	2391.56
PR product from DGs (kW)	4284	6198
DGs' ramping cost (£)	64.52	90.98
DGs' total operation cost (£)	2458.16	2482.54
Provider surplus in the market (£)	1261.3	1221.55
Weighted cost in RT market (£)	11.68	11.71

Flexible loads in the joint market provide 57.18% more PR products than that in the energy-only market. As they can make profits from the PR market, their payment bill (electricity purchasing cost minus profit from the PR product) is reduced by 14.27%, i.e. from 365.12 £ to 308.59 £.

After introducing the PR market, flexible loads provide more PR service to the distribution system. As a result, PR required from dispatchable DGs is reduced by 30.87% compared to the energy-only market. DGs' ramping cost is consequently reduced by 29.09%. The total operational cost of dispatchable DGs is reduced by 0.98%, namely from 2,482.54 £ to 2,458.16 £.

Distribution systems are witnessing an increasing penetration level of REGs, which will further increase the PR requirements. The value of introducing the PR market is further demonstrated in a modified case, where the generation from REGs is doubled and the uncertain load is increased by 50%. In the modified case, the results of the joint market and the energy-only market are compared in Table V. The PR requirement and PR products from LAs in the joint market are compared with the energy-only market in Fig. 8.

TABLE V
THE RESULTS OF THE JOINT MARKET AND ENERGY-ONLY MARKET

Market results	Joint market	Energy-only market
Curtailment of REGs (kWh)	161	590
Ramping product from LAs (kW)	4574	3729
LAs' total payment bill (£)	320.49	363.78
DGs' generation cost (£)	3026.54	3099.32
Ramping product from DGs (kW)	9599	10725
DGs' total operation cost (£)	3129.55	3206.57
Provider surplus in the market (£)	3263.54	3036.45

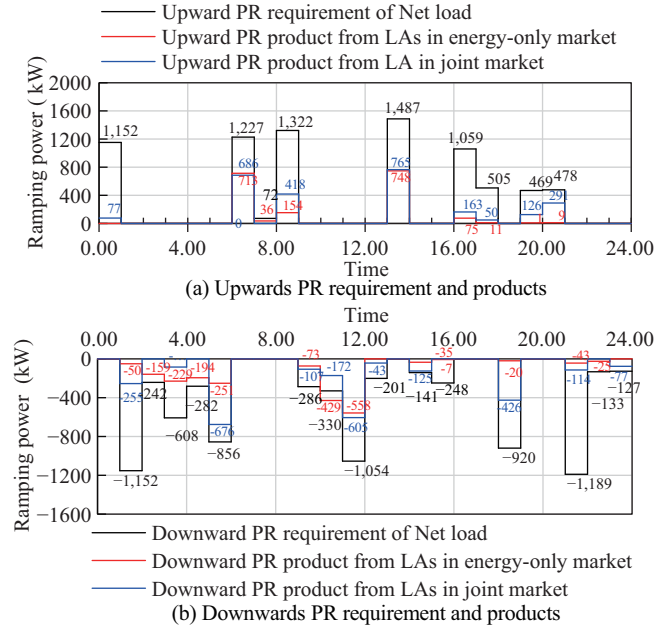


Fig. 8. System PR requirement and LAs' PR products in different markets.

Figure 8 shows that LAs of flexible loads in the joint market provide more PR products than that in the energy-only market. It increases the flexibility of the distribution system. The energy curtailment of REGs is reduced from 590 kWh to 161 kWh. It indicates that encouraging flexible loads to provide PR services can promote the consumption of REGs.

In addition, the generation cost of dispatchable DGs is reduced from 3099.32 £ to 3026.54 £. The total results are presented in Table IV. The total operational cost of dispatchable DGs is reduced by 2.62%. The reason is that more products are provided by flexible loads. The generation from DGs is also reduced due to the increased consumption of REGs.

E. Comparison with the Centralized Optimization Model

The solution time of the distributed market clearing method is compared with the centralized market clearing method on a

computer with the configuration of i7-10700 CUP @2.9 GHz and 16.0 GB RAM. The results of the solution time of both methods are presented in Table VI.

TABLE VI

COMPARISON OF THE SOLUTION TIME BETWEEN THE TWO METHODS (S)

Number of LA	3	6	9	12
Distributed market method	94	128	254	463
Centralized market method	109	475	965	2424

In the base case, considering 3 LAs in the market, the distributed method achieves a comparable performance to the centralized method. However, with the increase of the number of LAs, the solution time of the distributed method increases linearly, namely the method ensures the joint market problem can be solved in polynomial time. In comparison, the solution time of the centralized method increases exponentially. When the number of LAs increases to 12, the solution time of the distributed method is 463 s and that of the centralized method is 2424 s.

To further demonstrate the validity of the distributed method, the optimization results of the distributed clearing method are compared with the centralized clearing method. The market clearing results of both methods are presented in Table VII.

TABLE VII

THE RESULTS OF THE DISTRIBUTED AND CENTRALIZED MARKET METHOD

Market results	Distributed method	Centralized method
Total operation cost (£)	2818.34	2813.48
LAs' payment for energy (£)	360.19	360.72
PR from LAs (kW)	5259	4972
LAs' profit of ramping product (£)	51.60	54.12
LAs' total payment bill (£)	308.59	306.59
DGs' generation cost (£)	2393.64	2391.23
PR product from DGs (kW)	4284	4571
DGs' ramping cost (£)	64.52	62.48
DGs' total operation cost (£)	2458.16	2445.75

The total operational cost of the joint market under the distributed method is 2818.34 £, which is 0.17% higher than the centralized method (i.e., 2813.48 £) which is tolerable in practical applications. The clearing price of the DA energy market under both methods is presented in Fig. 9. It shows that the distributed clearing method can obtain a DA market price profile very close to that obtained by the centralized clearing method, which demonstrates the validity of the proposed method.

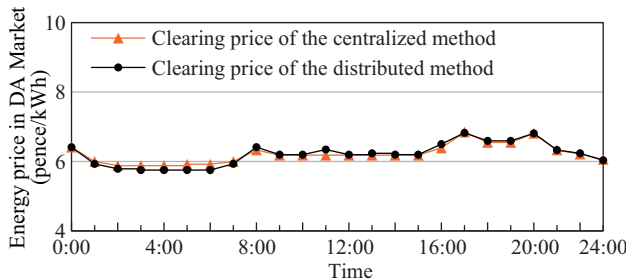


Fig. 9. Clearing price of DA energy market under both methods.

V. CONCLUSIONS

A local energy and PR joint market is developed in distribution systems. The introduction of the PR market encourages flexible loads to provide more PR service to the system. The ramping cost from dispatchable DGs is reduced by 29.09% in the test case and the total operational cost of DGs is reduced by 0.98%. Increased PR products improves the system's capability in accepting REG and the planned power curtailment is reduced compared to the energy-only market.

The difficulty in managing numerous and diversified flexible loads is overcome by the proposed distributed optimization method. This method ensures that the joint market problem can be solved in polynomial time. Flexible loads are dispatched by LA locally rather than by the LMO, which improves customers' information security and reduce the communication infrastructure costs.

The introduction of the PR market can compensate flexible loads more fairly for their ramping services. Loads with higher degrees of flexibility can gain more profits from the PR market. In a future study, energy storage and the new emerging prosumers will also be considered in the joint market.

APPENDIX

A. Principle of ADMM Method

1) ADMM is proposed to solve a large-scale optimization problem as shown by Eq. (A1) in a distributed manner.

$$\begin{aligned} \min \quad & f(\mathbf{x}) + g(\mathbf{z}) \\ \text{s.t.} \quad & A\mathbf{x} + B\mathbf{z} = \mathbf{c} \\ & \mathbf{x} \in C_1, \mathbf{z} \in C_2 \end{aligned} \quad (\text{A1})$$

With variables $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{z} \in \mathbb{R}^m$, where $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$, $\mathbf{c} \in \mathbb{R}^p$. Thus, the objective function is separable over two sets of variables, which are coupled through a set of equality constraints.

2) Augmented Lagrangian relaxation function:

The augmented Lagrangian relaxation function is constructed by relaxing and adding the constraints into the objective function through relaxing variables λ and introducing an extra L_2 norm term $\|A\mathbf{x} + B\mathbf{z} - \mathbf{c}\|_2^2$ to the objective.

$$\begin{aligned} L_p(\mathbf{x}, \mathbf{z}, \lambda) &= f(\mathbf{x}) + g(\mathbf{z}) + \\ & \lambda^T (A\mathbf{x} + B\mathbf{z} - \mathbf{c}) + (\rho/2) \|A\mathbf{x} + B\mathbf{z} - \mathbf{c}\|_2^2 \end{aligned} \quad (\text{A2})$$

where ρ is the penalty parameter. The first line in Eq. (A2) is the standard Lagrangian for the problem. L_2 norm term $\|A\mathbf{x} + B\mathbf{z} - \mathbf{c}\|_2^2$ can facilitate the convergence of the solving process for the problem.

3) ADMM solves the dual problem with iterations:

$$\begin{aligned} \mathbf{x}^{h+1} &:= \arg \min_{\mathbf{x} \in C_1} L_p(\mathbf{x}, \mathbf{z}^h, \lambda^h) \\ \mathbf{z}^{h+1} &:= \arg \min_{\mathbf{z} \in C_2} L_p(\mathbf{x}^h, \mathbf{z}, \lambda^h) \\ \lambda^{h+1} &:= \lambda^h + \rho (A\mathbf{x}^{h+1} + B\mathbf{z}^{h+1} - \mathbf{c}) \end{aligned}$$

4) Convergence

Residual convergence: $\mathbf{r}^h \rightarrow \mathbf{0}$ as $k \rightarrow \infty$, i.e., the iterates approach feasibility, where $\mathbf{r}^h = A\mathbf{x}^{h+1} + B\mathbf{z}^{h+1} - \mathbf{c}$.

B. Table A1. Parameters of Flexible Loads

TABLE A1
PARAMETERS OF FLEXIBLE LOADS

Load no.	Max P, kW	Min P, kW	Total energy, kWh	Interval of availability 00:00–24:00																						Interruptible
1	31	31	155	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
2	51	51	153	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
3	44	44	176	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
4	54	54	162	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
5	38	38	114	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
6	35	35	175	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	33	33	132	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
8	41	41	205	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	36	36	180	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10	34	34	102	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
11	47	10	293	0	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
12	59	11	151	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
13	49	15	175	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
14	46	9	121	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
15	41	9	147	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
16	47	12	175	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
17	55	15	227	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
18	43	13	192	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
19	58	12	160	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
20	52	10	127	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
21	84	13	277	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
22	71	10	327	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1
23	51	12	193	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
24	77	13	228	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1
25	54	9	274	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
26	76	12	114	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
27	54	15	313	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1
28	52	10	274	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
29	68	13	120	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
30	53	11	445	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

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