5 Finite Control Volume Analysis

CHAPTER OPENING PHOTO: Wind turbine farms (this is the Middelgrunden Offshore Wind Farm in Denmark) are becoming more common. Finite control volume analysis can be used to estimate the amount of energy transferred between the moving air and each turbine rotor. *(Photograph courtesy of Siemens Wind Power.)*

Learning Objectives

After completing this chapter, you should be able to:

- select an appropriate finite control volume to solve a fluid mechanics problem.
- apply conservation of mass and energy and Newton's second law of motion to the contents of a finite control volume to get important answers.
- know how velocity changes and energy transfers in fluid flows are related to forces and torques.
- understand why designing for minimum loss of energy in fluid flows is so important.

Many fluid mechanics problems can be solved by using control volume analysis.

To solve many practical problems in fluid mechanics, questions about the behavior of the contents of a finite region in space (a finite control volume) are answered. For example, we may be asked to estimate the maximum anchoring force required to hold a turbojet engine stationary during a test. Or we may be called on to design a propeller to move a boat both forward and backward. Or we may need to determine how much power it would take to move natural gas from one location to another many miles away.

The bases of finite control volume analysis are some fundamental laws of physics, namely, conservation of mass, Newton's second law of motion, and the first and second laws of thermodynamics. While some simplifying approximations are made for practicality, the engineering answers possible with the estimates of this powerful analysis method have proven valuable in numerous instances.

Conservation of mass is the key to tracking flowing fluid. How much enters and leaves a control volume can be ascertained.

Newton's second law of motion leads to the conclusion that forces can result from or cause changes in a flowing fluid's velocity magnitude and/or direction. Moment of force (torque) can result from or cause changes in a flowing fluid's moment of velocity. These forces and torques can be associated with work and power transfer.

The first law of thermodynamics is a statement of conservation of energy. The second law of thermodynamics identifies the loss of energy associated with every actual process. The mechanical energy equation based on these two laws can be used to analyze a large variety of steady, incompressible flows in terms of changes in pressure, elevation, speed, and of shaft work and loss.

Good judgment is required in defining the finite region in space, the control volume, used in solving a problem. What exactly to leave out of and what to leave in the control volume are important considerations. The formulas resulting from applying the fundamental laws to the contents of the control volume are easy to interpret physically and are not difficult to derive and use.

Because a finite region of space, a control volume, contains many fluid particles and even more molecules that make up each particle, the fluid properties and characteristics are often average values. In Chapter 6 an analysis of fluid flow based on what is happening to the contents of an infinitesimally small region of space or control volume through which numerous molecules simultaneously flow (what we might call a point in space) is considered.

5.1 Conservation of Mass—The Continuity Equation

5.1.1 Derivation of the Continuity Equation

A system is defined as a collection of unchanging contents, so the ***conservation of mass*** principle for a system is simply stated as

$$\text{time rate of change of the system mass} = 0$$

or

The amount of mass in a system is constant.

$$\frac{DM_{\text{sys}}}{Dt} = 0 \tag{5.1}$$

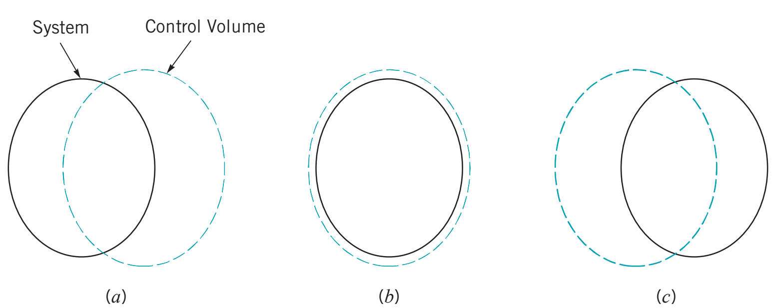
where the system mass, M_{sys}, is more generally expressed as

$$M_{\text{sys}} = \int_{\text{sys}} \rho \, d\forall \tag{5.2}$$

and the integration is over the volume of the system. In words, Eq. 5.2 states that the system mass is equal to the sum of all the density-volume element products for the contents of the system.

For a system and a fixed, nondeforming control volume that are coincident at an instant of time, as illustrated in Fig. 5.1, the Reynolds transport theorem (Eq. 4.19) with B = mass and $b = 1$ allows us to state that

$$\frac{D}{Dt}\int_{\text{sys}} \rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.3}$$



■ **FIGURE 5.1 System and control volume at three different instances of time. (*a*) System and control volume at time $t - \delta t$. (*b*) System and control volume at time t, coincident condition. (*c*) System and control volume at time $t + \delta t$.**

or

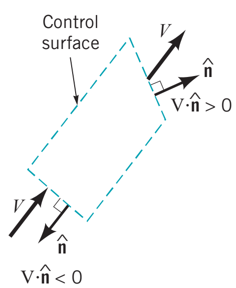
time rate of change of the mass of the coincident system	=	time rate of change of the mass of the contents of the coincident control volume	+	net rate of flow of mass through the control surface

In Eq. 5.3, we express the time rate of change of the system mass as the sum of two control volume quantities, the time rate of change of the mass of the contents of the control volume,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall$$

and the net rate of mass flow through the control surface,

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$



When a flow is steady, all field properties (i.e., properties at any specified point) including density remain constant with time and the time rate of change of the mass of the contents of the control volume is zero. That is,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall = 0$$

The integrand, $\mathbf{V} \cdot \hat{\mathbf{n}} \, dA$, in the mass flowrate integral represents the product of the component of velocity, $\mathbf{V}$, perpendicular to the small portion of control surface and the differential area, dA. Thus, $\mathbf{V} \cdot \hat{\mathbf{n}} \, dA$ is the volume flowrate through dA and $\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$ is the mass flowrate through dA. Furthermore, as shown in the sketch in the margin, the sign of the dot product $\mathbf{V} \cdot \hat{\mathbf{n}}$ is "+" for flow *out* of the control volume and "−" for flow *into* the control volume since $\hat{\mathbf{n}}$ is considered positive when it points out of the control volume. When all of the differential quantities, $\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$, are summed over the entire control surface, as indicated by the integral

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

the result is the net mass flowrate through the control surface, or

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} \tag{5.4}$$

where $\dot{m}$ is the mass flowrate (lbm/s, slug/s or kg/s). If the integral in Eq. 5.4 is positive, the net flow is out of the control volume; if the integral is negative, the net flow is into the control volume.

The continuity equation is a statement that mass is conserved.

The control volume expression for conservation of mass, which is commonly called the ***continuity equation***, for a fixed, nondeforming control volume is obtained by combining Eqs. 5.1, 5.2, and 5.3 to obtain

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0} \tag{5.5}$$

In words, Eq. 5.5 states that to conserve mass the time rate of change of the mass of the contents of the control volume plus the net rate of mass flow through the control surface must equal zero. Actually, the same result could have been obtained more directly by equating the rates of mass flow into and out of the control volume to the rates of accumulation and depletion of mass within the control volume (see Section 3.6.2). It is reassuring, however, to see that the Reynolds transport theorem works for this simple-to-understand case. This confidence will serve us well as we develop control volume expressions for other important principles.

An often-used expression for ***mass flowrate***, $\dot{m}$, through a section of control surface having area A is

$$\boxed{\dot{m} = \rho Q = \rho A V} \tag{5.6}$$

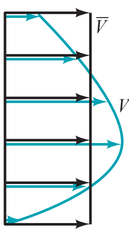
Mass flowrate equals the product of density and volume flowrate.

where ρ is the fluid density, Q is the volume flowrate (ft^3/s or m^3/s), and V is the component of fluid velocity perpendicular to area A. Since

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

application of Eq. 5.6 involves the use of *representative* or average values of fluid density, ρ, and fluid velocity, V. For incompressible flows, ρ is uniformly distributed over area A. For compressible flows, we will normally consider a uniformly distributed fluid density at each section of flow and allow density changes to occur only from section to section. The appropriate fluid velocity to use in Eq. 5.6 is the average value of the component of velocity normal to the section area involved. This average value, $\overline{V}$, defined as

$$\boxed{\overline{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\rho A}} \tag{5.7}$$



is shown in the figure in the margin.

If the velocity is considered uniformly distributed (one-dimensional flow) over the section area, A, then

$$\overline{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\rho A} = V \tag{5.8}$$

and the bar notation is not necessary (as in Example 5.1). When the flow is not uniformly distributed over the flow cross-sectional area, the bar notation reminds us that an average velocity is being used (as in Examples 5.2 and 5.4).

V5.1 Sink flow

5.1.2 Fixed, Nondeforming Control Volume

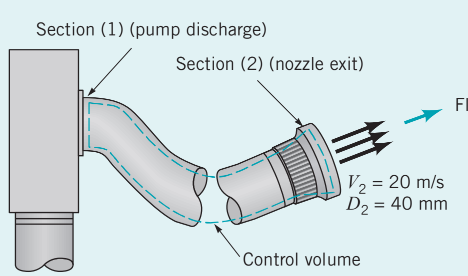
In many applications of fluid mechanics, an appropriate control volume to use is fixed and nondeforming. Several example problems that involve the continuity equation for fixed, nondeforming control volumes (Eq. 5.5) follow.

EXAMPLE 5.1 Conservation of Mass—Steady, Incompressible Flow

GIVEN Water flows steadily through a nozzle at the end of a fire hose as illustrated in Fig. E5.1*a*. According to local regulations, the nozzle exit velocity must be at least 20 m/s as shown in Fig. E5.1*b*.

FIND Determine the minimum pumping capacity, Q, required in m^3/s.

■ FIGURE E5.1*a*



■ FIGURE E5.1*b*

SOLUTION

The pumping capacity sought is the volume flowrate delivered by the fire pump to the hose and nozzle. Since we desire knowledge about the pump discharge flowrate and we have information about the nozzle exit flowrate, we link these two flowrates with the control volume designated with the dashed line in Fig. E5.1*b*. This control volume contains, at any instant, water that is within the hose and nozzle from the pump discharge to the nozzle exit plane.

Equation 5.5 is applied to the contents of this control volume to give

$$\overbrace{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, dV\!\!\!\!-}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0 \qquad \textbf{(1)}$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. Because there is only one inflow [the pump discharge, section (1)] and one outflow [the nozzle exit, section (2)], Eq. (1) becomes

$$\rho_2 A_2 V_2 - \rho_1 A_1 V_1 = 0$$

so that with $\dot{m} = \rho A V$

$$\dot{m}_1 = \dot{m}_2 \qquad \textbf{(2)}$$

Because the mass flowrate is equal to the product of fluid density, ρ, and volume flowrate, Q (see Eq. 5.6), we obtain from Eq. 2

$$\rho_2 Q_2 = \rho_1 Q_1 \qquad \textbf{(3)}$$

Liquid flow at low speeds, as in this example, may be considered incompressible. Therefore

$$\rho_2 = \rho_1 \qquad \textbf{(4)}$$

and from Eqs. 3 and 4

$$Q_2 = Q_1 \qquad \textbf{(5)}$$

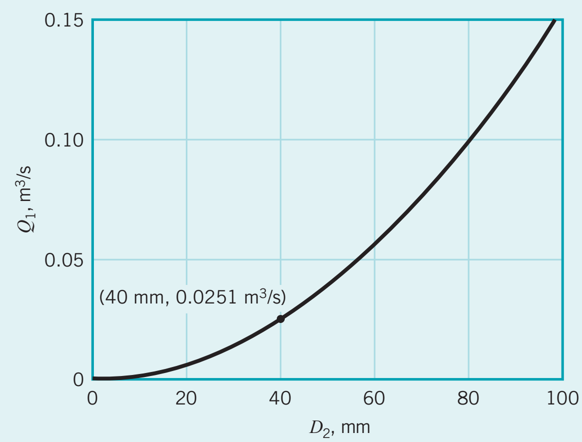
The pumping capacity is equal to the volume flowrate at the nozzle exit. If, for simplicity, the velocity distribution at the nozzle exit plane, section (2), is considered uniform (one-dimensional), then from Eq. 5

$$Q_1 = Q_2 = V_2 A_2$$

$$= V_2 \frac{\pi}{4} D_2^2 = (20 \text{ m/s}) \frac{\pi}{4} \left(\frac{40 \text{ mm}}{1000 \text{ mm/m}} \right)^2$$

$$= 0.0251 \text{ m}^3\text{/s} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for various values of the nozzle exit diameter, D_2, the results shown in Fig. E5.1*c* are obtained. The flowrate is proportional to the exit area, which varies as the diameter squared. Hence, if the diameter were doubled, the flowrate would increase by a factor of four, provided the exit velocity remained the same.

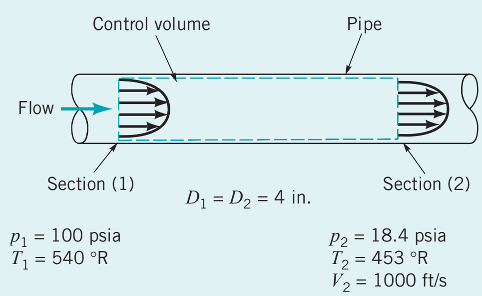


■ FIGURE E5.1*c*

EXAMPLE 5.2 Conservation of Mass—Steady, Compressible Flow

GIVEN Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Fig. E5.2. The uniformly distributed temperature and pressure at each section are given. The average air velocity (nonuniform velocity distribution) at section (2) is 1000 ft/s.

FIND Calculate the average air velocity at section (1).



■ FIGURE E5.2

SOLUTION

The average fluid velocity at any section is that velocity which yields the section mass flowrate when multiplied by the section average fluid density and section area (Eq. 5.7). We relate the flows at sections (1) and (2) with the control volume designated with a dashed line in Fig. E5.2.

Equation 5.5 is applied to the contents of this control volume to obtain

$$\overbrace{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, dV\!\!\!\!-}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. The control surface integral involves mass flowrates at sections (1) and (2) so that from Eq. 5.4 we get

$$\int_{\mathrm{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \dot{m}_2 - \dot{m}_1 = 0$$

or

$$\dot{m}_1 = \dot{m}_2 \qquad (1)$$

and from Eqs. 1, 5.6, and 5.7 we obtain

$$\rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \qquad (2)$$

or since $A_1 = A_2$

$$\overline{V}_1 = \frac{\rho_2}{\rho_1} \overline{V}_2 \qquad (3)$$

Air at the pressures and temperatures involved in this example problem behaves like an ideal gas. The ideal gas equation of state (Eq. 1.8) is

$$\rho = \frac{p}{RT} \qquad (4)$$

Thus, combining Eqs. 3 and 4 we obtain

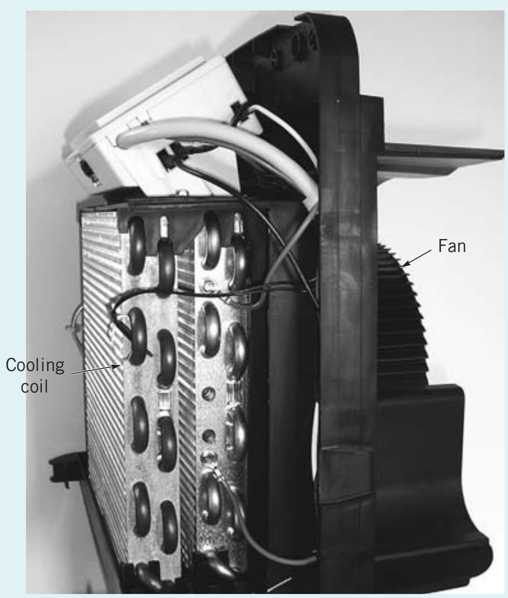
$$\overline{V}_1 = \frac{p_2 T_1 \overline{V}_2}{p_1 T_2} = \frac{(18.4 \text{ psia})(540\ ^\circ\text{R})(1000 \text{ ft/s})}{(100 \text{ psia})(453\ ^\circ\text{R})} = 219 \text{ ft/s} \qquad \textbf{(Ans)}$$

COMMENT We learn from this example that the continuity equation (Eq. 5.5) is valid for compressible as well as incompressible flows. Also, nonuniform velocity distributions can be handled with the average velocity concept. Significant average velocity changes can occur in pipe flow if the fluid is compressible.

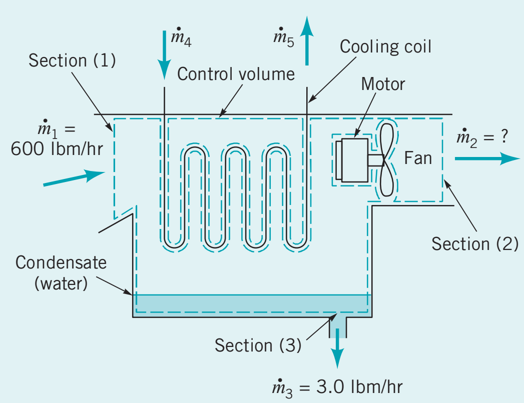
EXAMPLE 5.3 Conservation of Mass—Two Fluids

GIVEN The inner workings of a dehumidifier are shown in Fig. E5.3*a*. Moist air (a mixture of dry air and water vapor) enters the dehumidifier at the rate of 600 lbm/hr. Liquid water drains out of the dehumidifier at a rate of 3.0 lbm/hr. A simplified sketch of the process is provided in Fig. E5.3*b*.

FIND Determine the mass flowrate of the dry air and the water vapor leaving the dehumidifier.



■ FIGURE E5.3*a*



■ FIGURE E5.3*b*

SOLUTION

The unknown mass flowrate at section (2) is linked with the known flowrates at sections (1) and (3) with the control volume designated with a dashed line in Fig. E5.3*b*. The contents of the control volume are the air and water vapor mixture and the condensate (liquid water) in the dehumidifier at any instant.

Not included in the control volume are the fan and its motor, and the condenser coils and refrigerant. Even though the flow in the vicinity of the fan blade is unsteady, it is unsteady in a cyclical way. Thus, the flowrates at sections (1), (2), and (3) appear steady and the time rate of change of the mass of the contents of the control volume may be considered equal to zero on a time-average basis. The application of **Eqs. 5.4** and **5.5** to the control volume contents results in

$$\int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

or

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 600 \text{ lbm/hr} - 3.0 \text{ lbm/hr}$$
$$= 597 \text{ lbm/hr} \qquad \textbf{(Ans)}$$

COMMENT Note that the continuity equation (Eq. 5.5) can be used when there is more than one stream of fluid flowing through the control volume.

The answer is the same with a control volume which includes the cooling coils to be within the control volume. The continuity equation becomes

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 + \dot{m}_4 - \dot{m}_5 \qquad \textbf{(1)}$$

where $\dot{m}_4$ is the mass flowrate of the cooling fluid flowing into the control volume, and $\dot{m}_5$ is the flowrate out of the control volume through the cooling coil. Since the flow through the coils is steady, it follows that $\dot{m}_4 = \dot{m}_5$. Hence, Eq. 1 gives the same answer as obtained with the original control volume.

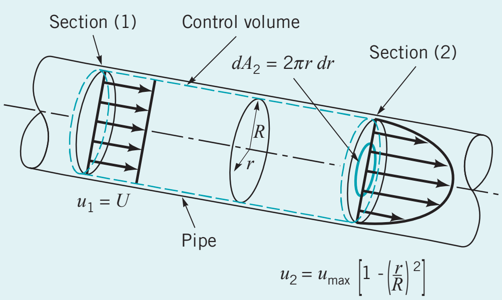
EXAMPLE 5.4 Conservation of Mass—Nonuniform Velocity Profile

GIVEN Incompressible, laminar water flow develops in a straight pipe having radius R as indicated in Fig. E5.4*a*. At section (1), the velocity profile is uniform; the velocity is equal to a constant value U and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of $u_{\max}$ at the centerline.

FIND

(a) How are U and $u_{\max}$ related?

(b) How are the average velocity at section (2), $\overline{V}_2$, and $u_{\max}$ related?



■ FIGURE E5.4*a*

SOLUTION

(a) An appropriate control volume is sketched (dashed lines) in Fig. E5.4*a*. The application of Eq. 5.5 to the contents of this control volume yields

$$\overbrace{\cancel{\frac{\partial}{\partial t}\int_{\text{cv}} \rho\, d\V}}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = 0 \qquad \textbf{(1)}$$

At the inlet, section (1), the velocity is uniform with $V_1 = U$ so that

$$\int_{(1)} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -\rho_1 A_1 U \qquad \textbf{(2)}$$

At the outlet, section (2), the velocity is not uniform. However, the net flowrate through this section is the sum of flows through numerous small washer-shaped areas of size $dA_2 = 2\pi r\, dr$ as shown by the shaded area element in Fig. E5.4*b*. On each of these infinitesimal areas the fluid velocity is denoted as u_2. Thus, in the limit of infinitesimal area elements, the summation is replaced by an integration and the outflow through section (2) is given by

$$\int_{(2)} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \rho_2 \int_0^R u_2 2\pi r\, dr \qquad \textbf{(3)}$$

By combining Eqs. 1, 2, and 3 we get

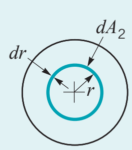
$$\rho_2 \int_0^R u_2 2\pi r\, dr - \rho_1 A_1 U = 0 \qquad \textbf{(4)}$$

Since the flow is considered incompressible, $\rho_1 = \rho_2$. The parabolic velocity relationship for flow through section (2) is used in Eq. 4 to yield

$$2\pi u_{\max} \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr - A_1 U = 0 \qquad \textbf{(5)}$$

Integrating, we get from Eq. 5

$$2\pi u_{\max} \left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)_0^R - \pi R^2 U = 0$$



■ FIGURE E5.4*b*

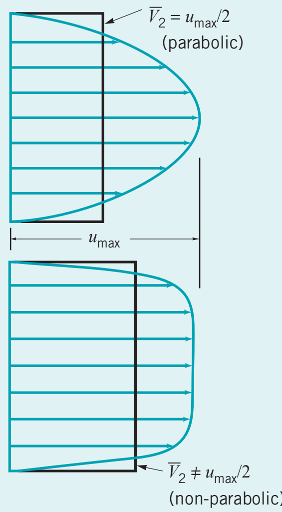
or

$$u_{max} = 2U \qquad \textbf{(Ans)}$$

(b) Since this flow is incompressible, we conclude from Eq. 5.7 that U is the average velocity at all sections of the control volume. Thus, the average velocity at section (2), $\overline{V}_2$, is one-half the maximum velocity, u_{max}, there or

$$\overline{V}_2 = \frac{u_{max}}{2} \qquad \textbf{(Ans)}$$

COMMENT The relationship between the maximum velocity at section (2) and the average velocity is a function of the "shape" of the velocity profile. For the parabolic profile assumed in this example, the average velocity, $u_{max}/2$, is the actual "average" of the maximum velocity at section (2), $u_2 = u_{max}$, and the minimum velocity at that section, $u_2 = 0$. However, as shown in Fig. E5.4*c*, if the velocity profile is a different shape (non-parabolic), the average velocity is not necessarily one half of the maximum velocity.

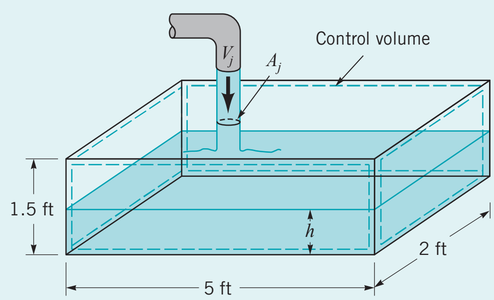


■ FIGURE E5.4c

EXAMPLE 5.5 Conservation of Mass—Unsteady Flow

GIVEN A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicated in Fig. E5.5*a*.

FIND Estimate the time rate of change of the depth of water in the tub, $\partial h/\partial t$, in inches per minute at any instant.



■ FIGURE E5.5a

SOLUTION

We use the fixed, nondeforming control volume outlined with a dashed line in Fig. E5.5*a*. This control volume includes in it, at any instant, the water accumulated in the tub, some of the water flowing from the faucet into the tub, and some air. Application of Eqs. 5.4 and 5.5 to these contents of the control volume results in

$$\frac{\partial}{\partial t}\int_{\substack{\text{air}\\\text{volume}}} \rho_{air}\, d\maltese_{air} + \frac{\partial}{\partial t}\int_{\substack{\text{water}\\\text{volume}}} \rho_{water}\, d\maltese_{water} - \dot{m}_{water} + \dot{m}_{air} = 0 \quad \textbf{(1)}$$

Recall that the mass, *dm,* of fluid contained in a small volume $d\maltese$ is $dm = \rho\, d\maltese$. Hence, the two *integrals* in Eq. 1 represent the total amount of air and water in the control volume, and the sum of the first two *terms* is the time rate of change of mass within the control volume.

Note that the time rate of change of air mass and water mass are each not zero. Recognizing, however, that the air mass must be conserved, we know that the time rate of change of the mass of air in the control volume must be equal to the rate of air mass flow out of the control volume. For simplicity, we disregard any water evaporation that occurs. Thus, applying Eqs. 5.4 and 5.5 to the air only and to the water only, we obtain

$$\frac{\partial}{\partial t}\int_{\substack{\text{air}\\\text{volume}}} \rho_{air}\, d\maltese_{air} + \dot{m}_{air} = 0$$

for air, and

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\\text{volume}}} \rho_{water}\, d\maltese_{water} = \dot{m}_{water} \qquad \textbf{(2)}$$

for water. The volume of water in the control volume is given by

$$\int_{\substack{\text{water}\\\text{volume}}} \rho_{water}\, d\maltese_{water} = \rho_{water}\left[h(2\text{ ft})(5\text{ ft}) + (1.5\text{ ft} - h)A_j\right] \qquad \textbf{(3)}$$

where A_j is the cross-sectional area of the water flowing from the faucet into the tub. Combining Eqs. 2 and 3, we obtain

$$\rho_{water}\,(10\text{ ft}^2 - A_j)\frac{\partial h}{\partial t} = \dot{m}_{water}$$

and, thus, since $\dot{m} = \rho Q$,

$$\frac{\partial h}{\partial t} = \frac{Q_{water}}{(10\text{ ft}^2 - A_j)}$$

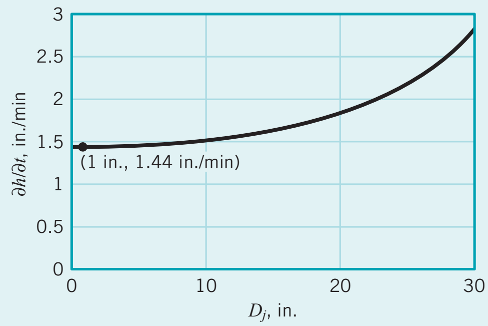
For $A_j \ll 10\ \text{ft}^2$ we can conclude that

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10\ \text{ft}^2)}$$

or

$$\frac{\partial h}{\partial t} = \frac{(9\ \text{gal/min})(12\ \text{in./ft})}{(7.48\ \text{gal/ft}^3)(10\ \text{ft}^2)} = 1.44\ \text{in./min} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for the same flowrate but with various water jet diameters, D_j, the results shown in Fig. E5.5*b* are obtained. With the flowrate held constant, the value of $\partial h/\partial t$ is nearly independent of the jet diameter for values of the diameter less than about 10 in.



■ **FIGURE E5.5*b***

The preceding example problems illustrate some important results of applying the conservation of mass principle to the contents of a fixed, nondeforming control volume. The dot product $\mathbf{V} \cdot \hat{\mathbf{n}}$ is "+" for flow out of the control volume and "−" for flow into the control volume. Thus, mass flowrate out of the control volume is "+" and mass flowrate in is "−." When the flow is steady, the time rate of change of the mass of the contents of the control volume

The appropriate sign convention must be followed.

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall$$

is zero and the net amount of mass flowrate, $\dot{m}$, through the control surface is therefore also zero

$$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0 \qquad \textbf{(5.9)}$$

V5.2 Shop vac filter

If the steady flow is also incompressible, the net amount of volume flowrate, Q, through the control surface is also zero:

$$\sum Q_{\text{out}} - \sum Q_{\text{in}} = 0 \qquad \textbf{(5.10)}$$

An unsteady, but cyclical flow can be considered steady on a time-average basis. When the flow is unsteady, the instantaneous time rate of change of the mass of the contents of the control volume is not necessarily zero and can be an important variable. When the value of

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall$$

is "+," the mass of the contents of the control volume is increasing. When it is "−," the mass of the contents of the control volume is decreasing.

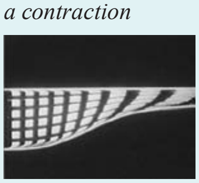
When the flow is uniformly distributed over the opening in the control surface (one-dimensional flow),

$$\dot{m} = \rho A V$$

where V is the uniform value of the velocity component normal to the section area A. When the velocity is nonuniformly distributed over the opening in the control surface,

$$\dot{m} = \rho A \overline{V} \qquad \textbf{(5.11)}$$

V5.3 Flow through a contraction



where $\overline{V}$ is the average value of the component of velocity normal to the section area A as defined by Eq. 5.7.

For steady flow involving only one stream of a specific fluid flowing through the control volume at sections (1) and (2),

$$\dot{m} = \rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2 \qquad \textbf{(5.12)}$$

and for incompressible flow,

$$Q = A_1 \overline{V}_1 = A_2 \overline{V}_2 \qquad \textbf{(5.13)}$$

For steady flow involving more than one stream of a specific fluid or more than one specific fluid flowing through the control volume,

$$\sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$$

The variety of example problems solved above should give the correct impression that the fixed, nondeforming control volume is versatile and useful.

F l u i d s i n t h e N e w s

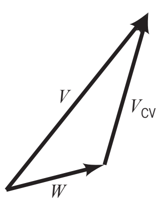
New 1.6 GPF standards Toilets account for approximately 40% of all indoor household water use. To conserve water, the new standard is 1.6 gallons of water per flush (gpf). Old toilets use up to 7 gpf; those manufactured after 1980 use 3.5 gpf. Neither are considered low-flush toilets. A typical 3.2 person household in which each person flushes a 7-gpf toilet 4 times a day uses 32,700 gallons of water each year; with a 3.5-gpf toilet the amount is reduced to 16,400 gallons. Clearly the new 1.6-gpf toilets will save even more water. However, designing a toilet that flushes properly with such a small amount of water is not simple. Today there are two basic types involved: those that are gravity powered and those that are pressure powered. Gravity toilets (typical of most currently in use) have rather long cycle times. The water starts flowing under the action of gravity and the swirling vortex motion initiates the siphon action which builds to a point of discharge. In the newer pressure-assisted models, the *flowrate* is large but the cycle time is short and the amount of water used is relatively small. (See Problem 5.32.)

5.1.3 Moving, Nondeforming Control Volume

Some problems are most easily solved by using a moving control volume.

It is sometimes necessary to use a nondeforming control volume attached to a moving reference frame. Examples include control volumes containing a gas turbine engine on an aircraft in flight, the exhaust stack of a ship at sea, and the gasoline tank of an automobile passing by.

As discussed in Section 4.4.6, when a moving control volume is used, the fluid velocity relative to the moving control volume (relative velocity) is an important flow field variable. The relative velocity, **W**, is the fluid velocity seen by an observer moving with the control volume. The control volume velocity, $\mathbf{V}_{\text{cv}}$, is the velocity of the control volume as seen from a fixed coordinate system. The absolute velocity, **V**, is the fluid velocity seen by a stationary observer in a fixed coordinate system. These velocities are related to each other by the vector equation



$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{\text{cv}} \tag{5.14}$$

as illustrated by the figure in the margin. This is the same as Eq. 4.22, introduced earlier.

For a system and a moving, nondeforming control volume that are coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) for a moving control volume leads to

$$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\maltese + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA \tag{5.15}$$

From Eqs. 5.1 and 5.15, we can get the control volume expression for conservation of mass (the continuity equation) for a moving, nondeforming control volume, namely,

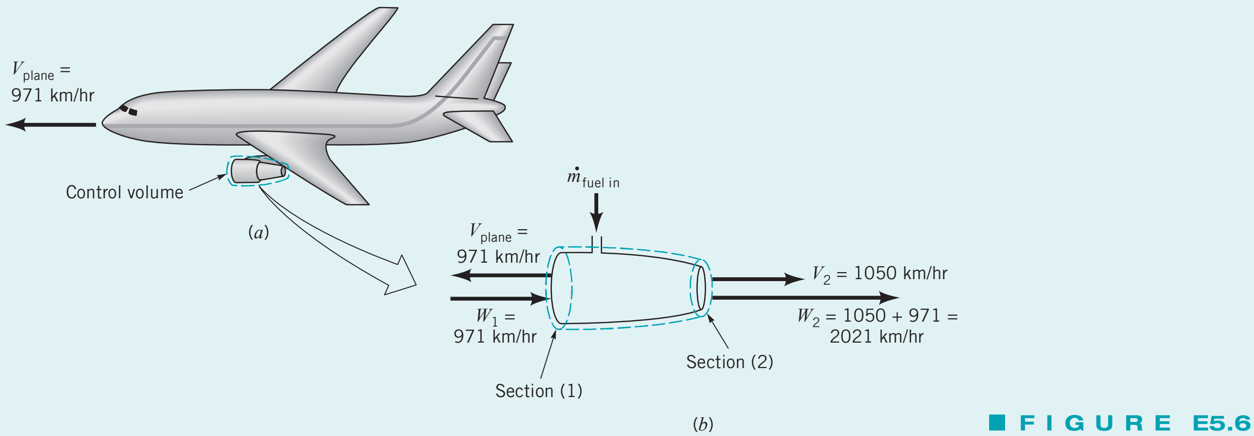
$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\maltese + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0} \tag{5.16}$$

Some examples of the application of Eq. 5.16 follow.

EXAMPLE 5.6 Conservation of Mass—Compressible Flow with a Moving Control Volume

GIVEN An airplane moves forward at a speed of 971 km/hr as shown in Fig. E5.6a. The frontal intake area of the jet engine is 0.80 m^2 and the entering air density is 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2, and the exhaust gas density is 0.515 kg/m^3.

FIND Estimate the mass flowrate of fuel into the engine in kg/hr.



■ FIGURE E5.6

SOLUTION

The control volume, which moves with the airplane (see Fig. E5.6b), surrounds the engine and its contents and includes all fluids involved at an instant. The application of Eq. 5.16 to these contents of the control volume yields

$$\frac{\partial}{\partial t}\underbrace{\int_{cv} \rho\, d\mathcal{V}}_{\text{0 (flow relative to moving control volume is considered steady on a time-average basis)}} + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}}\, dA = 0 \qquad \textbf{(1)}$$

Assuming one-dimensional flow, we evaluate the surface integral in Eq. 1 and get

$$-\dot{m}_{\substack{\text{fuel}\\ \text{in}}} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

or

$$\dot{m}_{\substack{\text{fuel}\\ \text{in}}} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1 \qquad \textbf{(2)}$$

We consider the intake velocity, W_1, relative to the moving control volume, as being equal in magnitude to the speed of the airplane, 971 km/hr. The exhaust velocity, W_2, also needs to be measured relative to the moving control volume. Since a fixed observer noted that the exhaust gases were moving away from the engine at a speed of 1050 km/hr, the speed of the exhaust gases relative to the moving control volume, W_2, is determined as follows by using Eq. 5.14

$$V_2 = W_2 + V_{\text{plane}}$$

or

$$W_2 = V_2 - V_{\text{plane}} = 1050 \text{ km/hr} - (-971 \text{ km/hr}) = 2021 \text{ km/hr}$$

and is shown in Fig. E5.6*b*.

From Eq. 2,

$$\dot{m}_{\substack{\text{fuel}\\ \text{in}}} = (0.515 \text{ kg/m}^3)(0.558 \text{ m}^2)(2021 \text{ km/hr})(1000 \text{ m/km}) - (0.736 \text{ kg/m}^3)(0.80 \text{ m}^2)(971 \text{ km/hr})(1000 \text{ m/km})$$
$$= (580{,}800 - 571{,}700) \text{ kg/hr}$$

$$\dot{m}_{\substack{\text{fuel}\\ \text{in}}} = 9100 \text{ kg/hr} \qquad \textbf{(Ans)}$$

COMMENT Note that the fuel flowrate was obtained as the difference of two large, nearly equal numbers. Precise values of W_2 and W_1 are needed to obtain a modestly accurate value of $\dot{m}_{\substack{\text{fuel}\\ \text{in}}}$.

EXAMPLE 5.7 Conservation of Mass—Relative Velocity

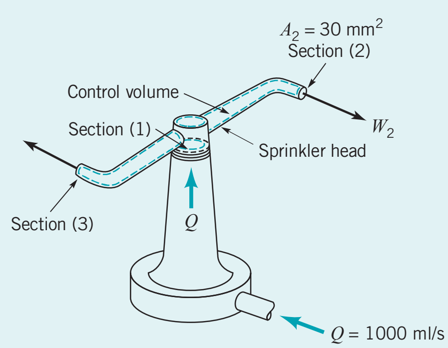
GIVEN Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.7. The exit area of each of the two nozzles is 30 mm^2.

FIND Determine the average speed of the water leaving the nozzle, relative to the nozzle, if

(a) the rotary sprinkler head is stationary,

(b) the sprinkler head rotates at 600 rpm, and

(c) the sprinkler head accelerates from 0 to 600 rpm.



■ FIGURE E5.7

Solution

(a) We specify a control volume that contains the water in the rotary sprinkler head at any instant. This control volume is nondeforming, but it moves (rotates) with the sprinkler head.
The application of Eq. 5.16 to the contents of this control volume for situation **(a)**, **(b)**, or **(c)** of the problem results in the same expression, namely

flow is steady or the control volume is filled with an incompressible fluid

$$\cancelto{0}{\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\mathcal{V}} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

or

$$\sum \rho_{\text{out}} A_{\text{out}} W_{\text{out}} - \sum \rho_{\text{in}} A_{\text{in}} W_{\text{in}} = 0 \qquad \textbf{(1)}$$

The time rate of change of the mass of water in the control volume is zero because the flow is steady and the control volume is filled with water.

Because there is only one inflow [at the base of the rotating arm, section (1)] and two outflows [the two nozzles at the tips of the arm, sections (2) and (3), each have the same area and fluid velocity], Eq. 1 becomes

$$\rho_2 A_2 W_2 + \rho_3 A_3 W_3 - \rho_1 A_1 W_1 = 0 \qquad \textbf{(2)}$$

Hence, for incompressible flow with $\rho_1 = \rho_2 = \rho_3$, Eq. 2 becomes

$$A_2 W_2 + A_3 W_3 - A_1 W_1 = 0$$

With $Q = A_1 W_1$, $A_2 = A_3$, and $W_2 = W_3$ it follows that

$$W_2 = \frac{Q}{2A_2}$$

or

$$W_2 = \frac{(1000 \text{ ml/s})(0.001 \text{ m}^3\text{/liter})(10^6 \text{ mm}^2\text{/m}^2)}{(1000 \text{ ml/liter})(2)(30 \text{ mm}^2)}$$

$$= 16.7 \text{ m/s} \qquad \textbf{(Ans)}$$

(b), (c) The value of W_2 is independent of the speed of rotation of the sprinkler head and represents the average velocity of the water exiting from each nozzle with respect to the nozzle for cases **(a)**, **(b)**, and **(c)**.

COMMENT The velocity of water discharging from each nozzle, when viewed from a stationary reference (i.e., V_2), will vary as the rotation speed of the sprinkler head varies since from Eq. 5.14,

$$V_2 = W_2 - U$$

where $U = \omega R$ is the speed of the nozzle and ω and R are the angular velocity and radius of the sprinkler head, respectively.

When a moving, nondeforming control volume is used, the dot product sign convention used earlier for fixed, nondeforming control volume applications is still valid. Also, if the flow within the moving control volume is steady, or steady on a time-average basis, the time rate of change of the mass of the contents of the control volume is zero. Velocities seen from the control volume reference frame (relative velocities) must be used in the continuity equation. Relative and absolute velocities are related by a vector equation (Eq. 5.14), which also involves the control volume velocity.

5.1.4 Deforming Control Volume

Care is needed to ensure that absolute and relative velocities are used correctly.

Occasionally, a deforming control volume can simplify the solution of a problem. A deforming control volume involves changing volume size and control surface movement. Thus, the Reynolds transport theorem for a moving control volume can be used for this case, and Eqs. 4.23 and 5.1 lead to

$$\boxed{\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0} \qquad \textbf{(5.17)}$$

The time rate of change term in Eq. 5.17,

$$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\mathcal{V}$$

is usually nonzero and must be carefully evaluated because the extent of the control volume varies with time. The mass flowrate term in Eq. 5.17,

$$\int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$

must be determined with the relative velocity, $\mathbf{W}$, the velocity referenced to the control surface. Since the control volume is deforming, the control surface velocity is not necessarily uniform and identical to the control volume velocity, $\mathbf{V}_{cv}$, as was true for moving, nondeforming control volumes. For the deforming control volume,

The velocity of the surface of a deforming control volume is not the same at all points on the surface.

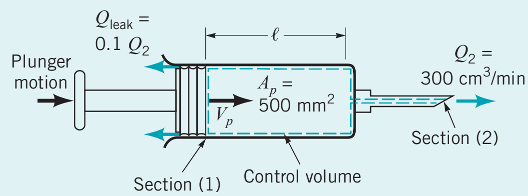
$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cs} \tag{5.18}$$

where $\mathbf{V}_{cs}$ is the velocity of the control surface as seen by a fixed observer. The relative velocity, $\mathbf{W}$, must be ascertained with care wherever fluid crosses the control surface. Two example problems that illustrate the use of the continuity equation for a deforming control volume, Eq. 5.17, follow.

EXAMPLE 5.8 Conservation of Mass—Deforming Control Volume

GIVEN A syringe (Fig. E5.8) is used to inoculate a cow. The plunger has a face area of 500 mm². The liquid in the syringe is to be injected steadily at a rate of 300 cm³/min. The leakage rate past the plunger is 0.10 times the volume flowrate out of the needle.

FIND With what speed should the plunger be advanced?



■ FIGURE E5.8

SOLUTION

The control volume selected for solving this problem is the deforming one illustrated in Fig. E5.8. Section (1) of the control surface moves with the plunger. The surface area of section (1), A_1, is considered equal to the circular area of the face of the plunger, A_p, although this is not strictly true, since leakage occurs. The difference is small, however. Thus,

$$A_1 = A_p \tag{1}$$

Liquid also leaves the needle through section (2), which involves fixed area A_2. The application of Eq. 5.17 to the contents of this control volume gives

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V} + \dot{m}_2 + \rho Q_{leak} = 0 \tag{2}$$

Even though Q_{leak} and the flow through section area A_2 are steady, the time rate of change of the mass of liquid in the shrinking control volume is not zero because the control volume is getting smaller. To evaluate the first term of Eq. 2, we note that

$$\int_{cv} \rho \, d\mathcal{V} = \rho(\ell A_1 + \mathcal{V}_{needle}) \tag{3}$$

where ℓ is the changing length of the control volume (see Fig. E5.8) and $\mathcal{V}_{needle}$ is the volume of the needle. From Eq. 3, we obtain

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\mathcal{V} = \rho A_1 \frac{\partial \ell}{\partial t} \tag{4}$$

Note that

$$-\frac{\partial \ell}{\partial t} = V_p \tag{5}$$

where V_p is the speed of the plunger sought in the problem statement. Combining Eqs. 2, 4, and 5 we obtain

$$-\rho A_1 V_p + \dot{m}_2 + \rho Q_{leak} = 0 \tag{6}$$

However, from Eq. 5.6, we see that

$$\dot{m}_2 = \rho Q_2 \tag{7}$$

and Eq. 6 becomes

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \tag{8}$$

Solving Eq. 8 for V_p yields

$$V_p = \frac{Q_2 + Q_{leak}}{A_1} \tag{9}$$

Since $Q_{leak} = 0.1Q_2$, Eq. 9 becomes

$$V_p = \frac{Q_2 + 0.1Q_2}{A_1} = \frac{1.1Q_2}{A_1}$$

and

$$V_p = \frac{(1.1)(300 \text{ cm}^3/\text{min})}{(500 \text{ mm}^2)}\left(\frac{1000 \text{ mm}^3}{\text{cm}^3}\right)$$
$$= 660 \text{ mm/min} \quad \textbf{(Ans)}$$

EXAMPLE 5.9 Conservation of Mass—Deforming Control Volume

GIVEN Consider Example 5.5.

FIND Solve the problem of Example 5.5 using a deforming control volume that includes only the water accumulating in the bathtub.

Solution

For this deforming control volume, Eq. 5.17 leads to

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\ \text{volume}}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0 \tag{1}$$

The first term of Eq. 1 can be evaluated as

$$\frac{\partial}{\partial t}\int_{\substack{\text{water}\\ \text{volume}}} \rho \, d\mathcal{V} = \frac{\partial}{\partial t}[\rho h(2 \text{ ft})(5 \text{ ft})]$$

$$= \rho \, (10 \text{ ft}^2)\frac{\partial h}{\partial t} \tag{2}$$

The second term of Eq. 1 can be evaluated as

$$\int_{\text{cs}} \rho \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = -\rho\left(V_j + \frac{\partial h}{\partial t}\right)A_j \tag{3}$$

where A_j and V_j are the cross-sectional area and velocity of the water flowing from the faucet into the tube. Thus, from Eqs. 1, 2, and 3 we obtain

$$\frac{\partial h}{\partial t} = \frac{V_j A_j}{(10 \text{ ft}^2 - A_j)} = \frac{Q_{\text{water}}}{(10 \text{ ft}^2 - A_j)}$$

or for $A_j \ll 10 \text{ ft}^2$

$$\frac{\partial h}{\partial t} = \frac{9(\text{gal/min})(12 \text{ in./ft})}{(7.48 \text{ gal/ft}^3)(10 \text{ ft}^2)} = 1.44 \text{ in./min} \qquad \textbf{(Ans)}$$

COMMENT Note that these results using a deforming control volume are the same as that obtained in Example 5.5 with a fixed control volume.

The conservation of mass principle is easily applied to the contents of a control volume. The appropriate selection of a specific kind of control volume (for example, fixed and nondeforming, moving and nondeforming, or deforming) can make the solution of a particular problem less complicated. In general, where fluid flows through the control surface, it is advisable to make the control surface perpendicular to the flow. In the sections ahead we learn that the conservation of mass principle is primarily used in combination with other important laws to solve problems.

5.2 Newton's Second Law—The Linear Momentum and Moment-of-Momentum Equations

V5.4 Smokestack plume momentum

5.2.1 Derivation of the Linear Momentum Equation

Newton's second law of motion for a system is

time rate of change of the linear momentum of the system	=	sum of external forces acting on the system

Since momentum is mass times velocity, the momentum of a small particle of mass $\rho d\mathcal{V}$ is $\mathbf{V}\rho d\mathcal{V}$. Thus, the momentum of the entire system is $\int_{\text{sys}} \mathbf{V}\rho d\mathcal{V}$ and Newton's law becomes

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\mathcal{V} = \sum \mathbf{F}_{\text{sys}} \tag{5.19}$$

Any reference or coordinate system for which this statement is true is called *inertial*. A fixed coordinate system is inertial. A coordinate system that moves in a straight line with constant velocity and is thus without acceleration is also inertial. We proceed to develop the control volume formula for this important law. When a control volume is coincident with a system at an instant of time, the forces acting on the system and the forces acting on the contents of the coincident control volume (see Fig. 5.2) are instantaneously identical, that is,

Forces acting on a flowing fluid can change its velocity magnitude and/or direction.

$$\sum \mathbf{F}_{\text{sys}} = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{coincident control volume}}} \tag{5.20}$$

Furthermore, for a system and the contents of a coincident control volume that is fixed and nondeforming, the Reynolds transport theorem [Eq. 4.19 with b set equal to the velocity (i.e., momentum per unit mass), and B_{sys} being the system momentum] allows us to conclude that

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho \, d\mathcal{V} = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\mathcal{V} + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.21}$$

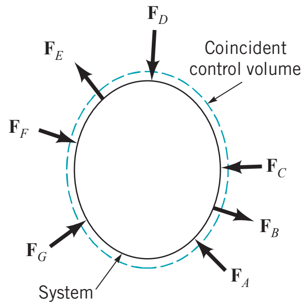


FIGURE 5.2 External forces acting on system and coincident control volume.

or

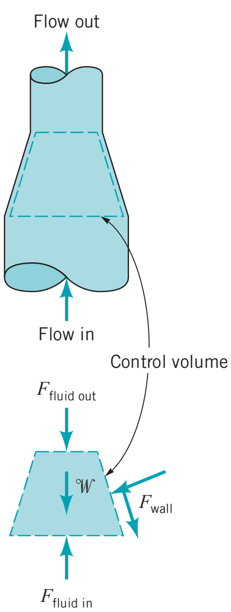
time rate of change of the linear momentum of the system	=	time rate of change of the linear momentum of the contents of the control volume	+	net rate of flow of linear momentum through the control surface

V5.5 Marine propulsion

Equation 5.21 states that the time rate of change of system linear momentum is expressed as the sum of the two control volume quantities: the time rate of change of the *linear momentum of the contents of the control volume*, and the net rate of *linear momentum flow through the control surface*. As particles of mass move into or out of a control volume through the control surface, they carry linear momentum in or out. Thus, linear momentum flow should seem no more unusual than mass flow.

For a control volume that is fixed (and thus inertial) and nondeforming, Eqs. 5.19, 5.20, and 5.21 provide an appropriate mathematical statement of Newton's second law of motion as

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}} \tag{5.22}$$



We call Eq. 5.22 the ***linear momentum equation***.

In our application of the linear momentum equation, we initially confine ourselves to fixed, nondeforming control volumes for simplicity. Subsequently, we discuss the use of a moving but inertial, nondeforming control volume. We do not consider deforming control volumes and accelerating (noninertial) control volumes. If a control volume is noninertial, the acceleration components involved (for example, translation acceleration, Coriolis acceleration, and centrifugal acceleration) require consideration.

The forces involved in Eq. 5.22 are body and surface forces that act on what is contained in the control volume as shown in the sketch in the margin. The only body force we consider in this chapter is the one associated with the action of gravity. We experience this body force as weight, $\mathscr{W}$. The surface forces are basically exerted on the contents of the control volume by material just outside the control volume in contact with material just inside the control volume. For example, a wall in contact with fluid can exert a reaction surface force on the fluid it bounds. Similarly, fluid just outside the control volume can push on fluid just inside the control volume at a common interface, usually an opening in the control surface through which fluid flow occurs. An immersed object can resist fluid motion with surface forces.

The linear momentum terms in the momentum equation deserve careful explanation. We clarify their physical significance in the following sections.

V5.6 Force due to a water jet

5.2.2 Application of the Linear Momentum Equation

The linear momentum equation for an inertial control volume is a vector equation (Eq. 5.22). In engineering applications, components of this vector equation resolved along orthogonal coordinates, for example, x, y, and z (rectangular coordinate system) or r, θ, and x (cylindrical coordinate system), will normally be used. A simple example involving steady, incompressible flow is considered first.

EXAMPLE 5.10 Linear Momentum—Change in Flow Direction

GIVEN As shown in Fig. E5.10*a*, a horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ ft/s, strikes a vane, and is turned through an angle θ.

FIND Determine the anchoring force needed to hold the vane stationary if gravity and viscous effects are negligible.

SOLUTION

We select a control volume that includes the vane and a portion of the water (see Figs. E5.10*b*, *c*) and apply the linear momentum equation to this fixed control volume. The only portions of the control surface across which fluid flows are section (1) (the entrance) and section (2) (the exit). Hence, the x and z components of Eq. 5.22 become

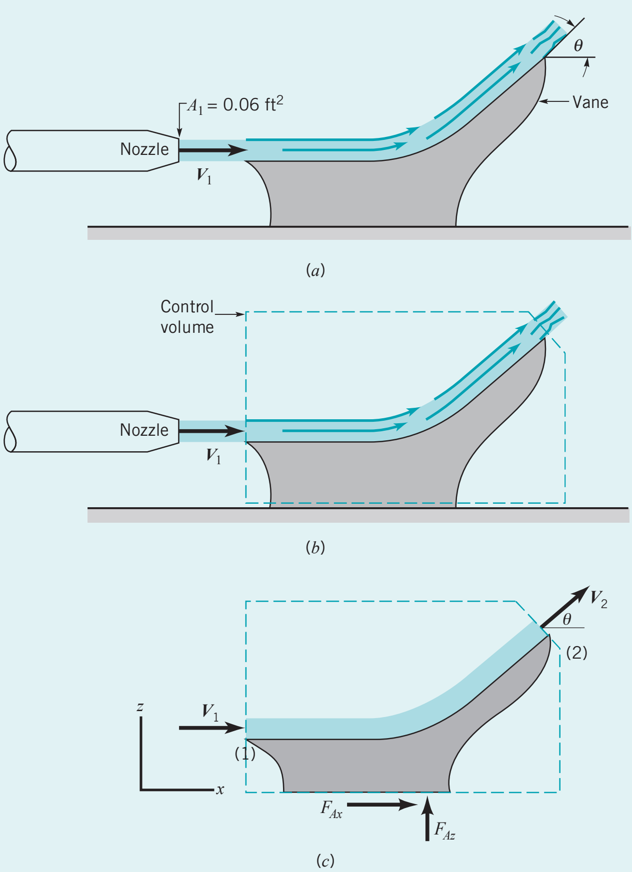
$$\overbrace{\frac{\partial}{\partial t}\int_{\text{cv}} u\,\rho\, d\mathcal{V}}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} u\,\rho\,\mathbf{V}\cdot\hat{\mathbf{n}}\,dA = \sum F_x$$

and

$$\overbrace{\frac{\partial}{\partial t}\int_{\text{cv}} w\,\rho\, d\mathcal{V}}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} w\,\rho\,\mathbf{V}\cdot\hat{\mathbf{n}}\,dA = \sum F_z$$

or

$$u_2\rho A_2V_2 - u_1\rho A_1V_1 = \sum F_x \tag{1}$$



■ FIGURE E5.10

and

$$w_2\rho A_2V_2 - w_1\rho A_1V_1 = \sum F_z \tag{2}$$

where $\mathbf{V} = u\hat{\mathbf{i}} + w\hat{\mathbf{k}}$, and ΣF_x and ΣF_z are the net x and z components of force acting on the contents of the control volume. Depending on the particular flow situation being considered and the coordinate system chosen, the x and z components of velocity, u and w, can be positive, negative, or zero. In this example the flow is in the positive directions at both the inlet and the outlet.

The water enters and leaves the control volume as a free jet at atmospheric pressure. Hence, there is atmospheric pressure surrounding the entire control volume, and the net pressure force on the control volume surface is zero. If we neglect the weight of the water and vane, the only forces applied to the control volume contents are the horizontal and vertical components of the anchoring force, F_{Ax} and F_{Az}, respectively.

With negligible gravity and viscous effects, and since $p_1 = p_2$, the speed of the fluid remains constant so that $V_1 = V_2 = 10$ ft/s (see the Bernoulli equation, Eq. 3.7). Hence, at section (1), $u_1 = V_1$, $w_1 = 0$, and at section (2), $u_2 = V_1\cos\theta$, $w_2 = V_1\sin\theta$.

By using this information, Eqs. 1 and 2 can be written as

$$V_1\cos\theta\,\rho\,A_2V_1 - V_1\,\rho\,A_1V_1 = F_{Ax} \tag{3}$$

and

$$V_1\sin\theta\,\rho\,A_2V_1 - 0\,\rho\,A_1V_1 = F_{Az} \tag{4}$$

Equations 3 and 4 can be simplified by using conservation of mass, which states that for this incompressible flow $A_1V_1 = A_2V_2$, or $A_1 = A_2$ since $V_1 = V_2$. Thus

$$F_{Ax} = -\rho A_1V_1^2 + \rho A_1V_1^2\cos\theta = -\rho A_1V_1^2\,(1-\cos\theta) \tag{5}$$

and

$$F_{Az} = \rho A_1V_1^2\sin\theta \tag{6}$$

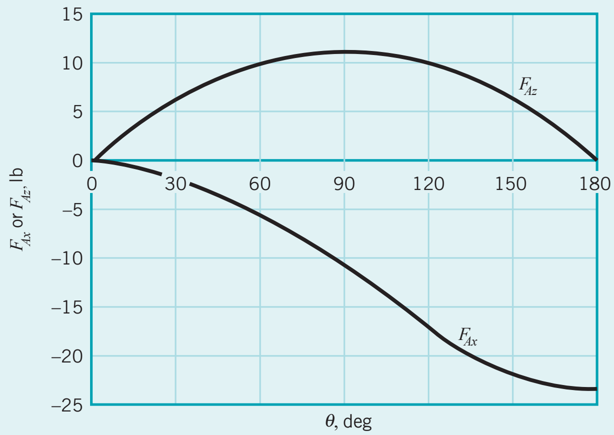
With the given data we obtain

$$\begin{aligned} F_{Ax} &= -(1.94\text{ slugs/ft}^3)(0.06\text{ ft}^2)(10\text{ ft/s})^2(1-\cos\theta) \\ &= -11.64(1-\cos\theta)\text{ slugs}\cdot\text{ft/s}^2 \\ &= -11.64(1-\cos\theta)\text{ lb} \end{aligned} \qquad \textbf{(Ans)}$$

and

$$\begin{aligned} F_{Az} &= (1.94\text{ slugs/ft}^3)(0.06\text{ ft}^2)(10\text{ ft/s})^2\sin\theta \\ &= 11.64\sin\theta\text{ lb} \end{aligned} \qquad \textbf{(Ans)}$$

COMMENTS The values of F_{Ax} and F_{Az} as a function of θ are shown in Fig. E5.10*d*. Note that if $\theta = 0$ (i.e., the vane does not turn the water), the anchoring force is zero. The inviscid fluid merely slides along the vane without putting any force on it. If $\theta = 90°$, then $F_{Ax} = -11.64$ lb and $F_{Az} = 11.64$ lb. It is necessary to push on the vane (and, hence, for the vane to push on the water)



■ FIGURE E5.10*d*

to the left (F_{Ax} is negative) and up in order to change the direction of flow of the water from horizontal to vertical. This momentum change requires a force. If $\theta = 180°$, the water jet is turned back on itself. This requires no vertical force ($F_{Az} = 0$), but the horizontal force ($F_{Ax} = -23.3$ lb) is two times that required if $\theta = 90°$. This horizontal fluid momentum change requires a horizontal force only.

Note that the anchoring force (Eqs. 5, 6) can be written in terms of the mass flowrate, $\dot{m} = \rho A_1 V_1$, as

$$F_{Ax} = -\dot{m}V_1(1 - \cos\theta)$$

and

$$F_{Az} = \dot{m}V_1 \sin\theta$$

In this example exerting a force on a fluid flow resulted in a change in its direction only (i.e., change in its linear momentum).

Fluids in the News

Where the plume goes Commercial airliners have wheel brakes very similar to those on highway vehicles. In fact, antilock brakes now found on most new cars were first developed for use on airplanes. However, when landing, the major braking force comes from the engine rather than the wheel brakes. Upon touchdown, a piece of engine cowling translates aft and blocker doors drop down, directing the engine airflow into a honeycomb structure called a cascade. The cascade reverses the direction of the high-speed engine exhausts by nearly 180° so that it flows forward. As predicted by the *momentum equation*, the air passing through the engine produces a substantial braking force—the reverse thrust. Designers must know the flow pattern of the exhaust plumes to eliminate potential problems. For example, the plumes of hot exhaust must be kept away from parts of the aircraft where repeated heating and cooling could cause premature fatigue. Also, the plumes must not re-enter the engine inlet, or blow debris from the runway in front of the engine, or envelop the vertical tail. (See Problem 5.67.)

EXAMPLE 5.11 Linear Momentum—Weight, Pressure, and Change in Speed

GIVEN As shown in Fig. E5.11*a*, water flows through a nozzle attached to the end of a laboratory sink faucet with a flowrate of 0.6 liters/s. The nozzle inlet and exit diameters are 16 and 5 mm, respectively, and the nozzle axis is vertical. The mass of the nozzle is 0.1 kg. The pressure at section (1) is 464 kPa.

FIND Determine the anchoring force required to hold the nozzle in place.

SOLUTION

The anchoring force sought is the reaction force between the faucet and nozzle threads. To evaluate this force we select a control volume that includes the entire nozzle and the water contained in the nozzle at an instant, as is indicated in Figs. E5.11*a* and E5.11*b*. All of the vertical forces acting on the contents of this control volume are identified in Fig. E5.11*b*. The action of atmospheric pressure cancels out in every direction and is not shown. Gage pressure forces do not cancel out in the vertical direction and are shown. Application of the vertical or z direction component of Eq. 5.22 to the contents of this control volume leads to

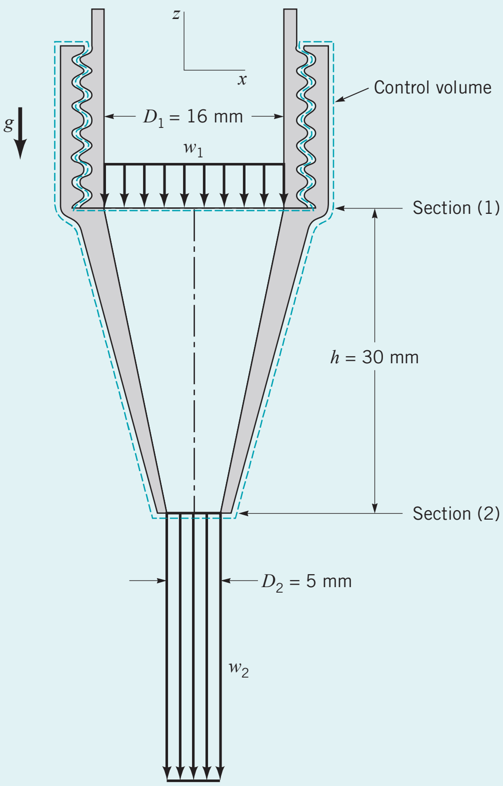
$$\overbrace{\cancel{\frac{\partial}{\partial t}\int_{\text{cv}} w\rho \, d\mathcal{V}}}^{0 \text{ (flow is steady)}} + \int_{\text{cs}} w\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = F_A - \mathcal{W}_n - p_1A_1 - \mathcal{W}_w + p_2A_2 \qquad (1)$$

where w is the z direction component of fluid velocity, and the various parameters are identified in the figure.

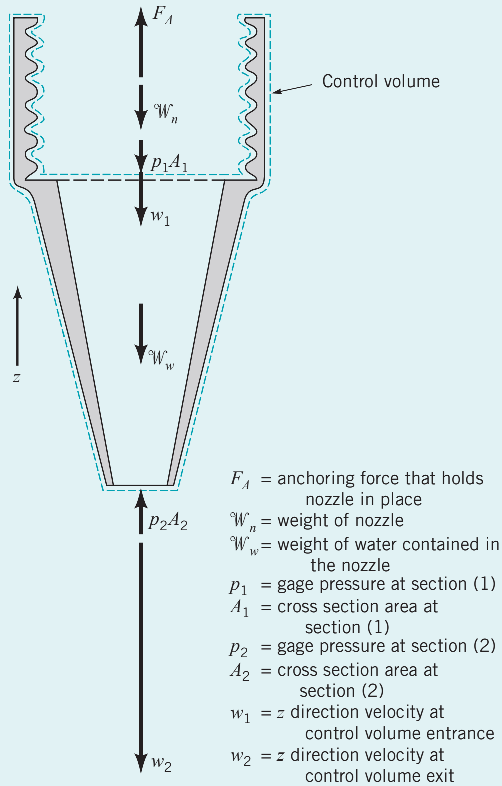
Note that the positive direction is considered "up" for the forces. We will use this same sign convention for the fluid velocity, w, in Eq. 1. In Eq. 1, the dot product, $\mathbf{V}\cdot\hat{\mathbf{n}}$, is "+" for flow out of the control volume and "−" for flow into the control volume. For this particular example

$$\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \pm|w|\, dA \qquad (2)$$

with the "+" used for flow out of the control volume and "−" used for flow in. To evaluate the control surface integral in Eq. 1, we need to assume a distribution for fluid velocity, w, and fluid density, ρ. For simplicity, we assume that w is uniformly distributed or constant, with magnitudes of w_1 and w_2 over cross-sectional areas A_1 and A_2. Also, this flow is incompressible so the



■ FIGURE E5.11*a*



■ FIGURE E5.11*b*

fluid density, ρ, is constant throughout. Proceeding further we obtain for Eq. 1

$$(-\dot{m}_1)(-w_1) + \dot{m}_2(-w_2) = F_A - \mathcal{W}_n - p_1A_1 - \mathcal{W}_w + p_2A_2 \quad \textbf{(3)}$$

where $\dot{m} = \rho AV$ is the mass flowrate.

Note that $-w_1$ and $-w_2$ are used because both of these velocities are "down." Also, $-\dot{m}_1$ is used because it is associated with flow into the control volume. Similarly, $+\dot{m}_2$ is used because it is associated with flow out of the control volume. Solving Eq. 3 for the anchoring force, F_A, we obtain

$$F_A = \dot{m}_1w_1 - \dot{m}_2w_2 + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad \textbf{(4)}$$

From the conservation of mass equation, Eq. 5.12, we obtain

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad \textbf{(5)}$$

which when combined with Eq. 4 gives

$$F_A = \dot{m}(w_1 - w_2) + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad \textbf{(6)}$$

It is instructive to note how the anchoring force is affected by the different actions involved. As expected, the nozzle weight, $\mathcal{W}_n$, the water weight, $\mathcal{W}_w$, and gage pressure force at section (1), p_1A_1, all increase the anchoring force, while the gage pressure force at section (2), p_2A_2, acts to decrease the anchoring force. The change in the vertical momentum flowrate, $\dot{m}(w_1 - w_2)$, will, in this instance, decrease the anchoring force because this change is negative $(w_2 > w_1)$.

To complete this example we use quantities given in the problem statement to quantify the terms on the right-hand side of Eq. 6.

From Eq. 5.6,

$$\dot{m} = \rho w_1 A_1 = \rho Q = (999 \text{ kg/m}^3)(0.6 \text{ liter/s})(10^{-3} \text{ m}^3\text{/liter}) = 0.599 \text{ kg/s} \quad \textbf{(7)}$$

and

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\pi(D_1^2/4)} = \frac{(0.6 \text{ liter/s})(10^{-3} \text{ m}^3\text{/liter})}{\pi(16 \text{ mm})^2/4(1000^2 \text{ mm}^2\text{/m}^2)} = 2.98 \text{ m/s} \quad \textbf{(8)}$$

Also from Eq. 5.6,

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\pi(D_2^2/4)} = \frac{(0.6 \text{ liter/s})(10^{-3} \text{ m}^3\text{/liter})}{\pi(5 \text{ mm})^2/4(1000^2 \text{ mm}^2\text{/m}^2)} = 30.6 \text{ m/s} \quad \textbf{(9)}$$

The weight of the nozzle, $\mathcal{W}_n$, can be obtained from the nozzle mass, m_n, with

$$\mathcal{W}_n = m_n g = (0.1 \text{ kg})(9.81 \text{ m/s}^2) = 0.981 \text{ N} \quad \textbf{(10)}$$

The weight of the water in the control volume, $\mathcal{W}_w$, can be obtained from the water density, ρ, and the volume of water, $\forall_w$, in the truncated cone of height h. That is,

$$\mathcal{W}_w = \rho \mathcal{V}_w g$$

where

$$\begin{aligned}\mathcal{V}_w &= \tfrac{1}{12}\pi h(D_1^2 + D_2^2 + D_1 D_2)\\ &= \frac{1}{12}\pi \frac{(30\text{ mm})}{(1000\text{ mm/m})}\\ &\quad \times \left[\frac{(16\text{ mm})^2 + (5\text{ mm})^2 + (16\text{ mm})(5\text{ mm})}{(1000^2\text{ mm}^2/\text{m}^2)}\right]\\ &= 2.84 \times 10^{-6}\text{ m}^3\end{aligned}$$

Thus,

$$\begin{aligned}\mathcal{W}_w &= (999\text{ kg/m}^3)(2.84 \times 10^{-6}\text{ m}^3)(9.81\text{ m/s}^2)\\ &= 0.0278\text{ N} \qquad \textbf{(11)}\end{aligned}$$

The gage pressure at section (2), p_2, is zero since, as discussed in Section 3.6.1, when a subsonic flow discharges to the atmosphere as in the present situation, the discharge pressure is essentially atmospheric. The anchoring force, F_A, can now be determined from Eqs. 6 through 11 with

$$\begin{aligned}F_A &= (0.599\text{ kg/s})(2.98\text{ m/s} - 30.6\text{ m/s}) + 0.981\text{ N}\\ &\quad + (464\text{ kPa})(1000\text{ Pa/kPa})\frac{\pi(16\text{ mm})^2}{4(1000^2\text{ mm}^2/\text{m}^2)}\\ &\quad + 0.0278\text{ N} - 0\end{aligned}$$

or

$$\begin{aligned}F_A &= -16.5\text{ N} + 0.981\text{ N} + 93.3\text{ N} + 0.0278\text{ N}\\ &= 77.8\text{ N} \qquad \textbf{(Ans)}\end{aligned}$$

Since the anchoring force, F_A, is positive, it acts upward in the z direction. The nozzle would be pushed off the pipe if it were not fastened securely.

COMMENT The control volume selected above to solve problems such as these is not unique. The following is an alternate solution that involves two other control volumes—one containing only the nozzle and the other containing only the water in the nozzle. These control volumes are shown in Figs. E5.11*c* and E5.11*d* along with the vertical forces acting on the contents of each control volume. The new force involved, R_z, represents the interaction between the water and the conical inside surface of the nozzle. It includes the net pressure and viscous forces at this interface.

Application of Eq. 5.22 to the contents of the control volume of Fig. E5.11*c* leads to

$$F_A = \mathcal{W}_n + R_z - p_{\text{atm}}(A_1 - A_2) \qquad \textbf{(12)}$$

The term $p_{\text{atm}}(A_1 - A_2)$ is the resultant force from the atmospheric pressure acting upon the exterior surface of the nozzle (i.e., that portion of the surface of the nozzle that is not in contact with the water). Recall that the pressure force on a curved surface (such as the exterior surface of the nozzle) is equal to the pressure times the projection of the surface area on a plane perpendicular to the axis of the nozzle. The projection of this area on a plane perpendicular to the z direction is $A_1 - A_2$. The effect of the atmospheric pressure on the internal area (between the nozzle and the water) is already included in R_z which represents the net force on this area.

Similarly, for the control volume of Fig. E5.11*d* we obtain

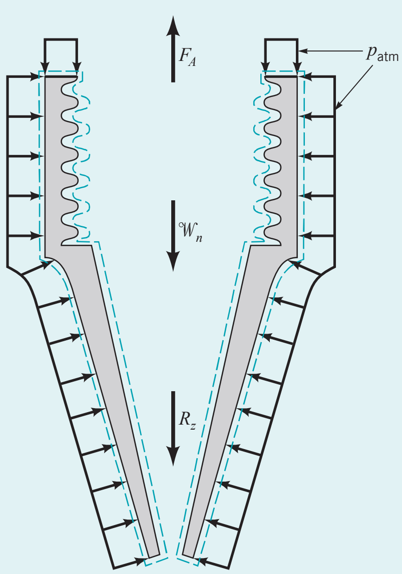
$$\begin{aligned}R_z &= \dot{m}(w_1 - w_2) + \mathcal{W}_w + (p_1 + p_{\text{atm}})A_1\\ &\quad - (p_2 - p_{\text{atm}})A_2\end{aligned} \qquad \textbf{(13)}$$

where p_1 and p_2 are gage pressures. From Eq. 13 it is clear that the value of R_z depends on the value of the atmospheric pressure, p_{atm}, since $A_1 \neq A_2$. That is, we must use absolute pressure, not gage pressure, to obtain the correct value of R_z. From Eq. 13 we can easily identify which forces acting on the flowing fluid change its velocity magnitude and thus linear momentum.

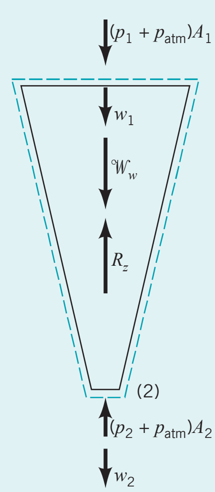
By combining Eqs. 12 and 13 we obtain the same result for F_A as before (Eq. 6):

$$F_A = \dot{m}(w_1 - w_2) + \mathcal{W}_n + p_1 A_1 + W_w - p_2 A_2$$

Note that although the force between the fluid and the nozzle wall, R_z, is a function of p_{atm}, the anchoring force, F_A, is not. That is, we were correct in using gage pressure when solving for F_A by means of the original control volume shown in Fig. E5.11*b*.

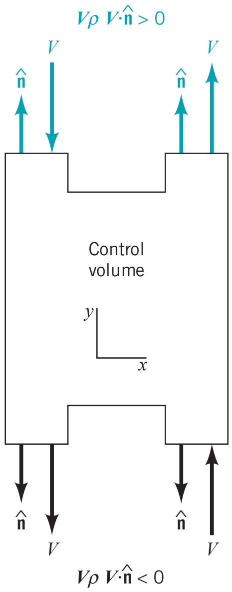


■ FIGURE E5.11*c*



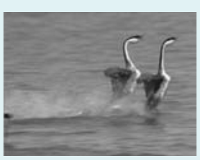
■ FIGURE E5.11*d*

Several important generalities about the application of the linear momentum equation (Eq. 5.22) are apparent in the example just considered.



1. When the flow is uniformly distributed over a section of the control surface where flow into or out of the control volume occurs, the integral operations are simplified. Thus, one-dimensional flows are easier to work with than flows involving nonuniform velocity distributions.
2. Linear momentum is directional; it can have components in as many as three orthogonal coordinate directions. Furthermore, along any one coordinate, the linear momentum of a fluid particle can be in the positive or negative direction and thus be considered as a positive or a negative quantity. In Example 5.11, only the linear momentum in the z direction was considered (all of it was in the negative z direction and was hence treated as being negative).
3. The flow of positive or negative linear momentum *into* a control volume involves a negative $\mathbf{V}\cdot\hat{\mathbf{n}}$ product. Momentum flow *out* of the control volume involves a positive $\mathbf{V}\cdot\hat{\mathbf{n}}$ product. The correct algebraic sign ($+$ or $-$) to assign to momentum flow ($\mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA$) will depend on the sense of the velocity ($+$ in positive coordinate direction, $-$ in negative coordinate direction) and the $\mathbf{V}\cdot\hat{\mathbf{n}}$ product ($+$ for flow out of the control volume, $-$ for flow into the control volume). This is shown in the figure in the margin. In Example 5.11, the momentum flow into the control volume past section (1) was a positive ($+$) quantity while the momentum flow out of the control volume at section (2) was a negative ($-$) quantity.
4. The time rate of change of the linear momentum of the contents of a nondeforming control volume (i.e., $\partial/\partial t \int_{\text{cv}} \mathbf{V}\rho\, d\mathcal{V}$) is zero for steady flow. The momentum problems considered in this text all involve steady flow.

V5.7 Running on water



5. If the control surface is selected so that it is perpendicular to the flow where fluid enters or leaves the control volume, the surface force exerted at these locations by fluid outside the control volume on fluid inside will be due to pressure. Furthermore, when subsonic flow exits from a control volume into the atmosphere, atmospheric pressure prevails at the exit cross section. In Example 5.11, the flow was subsonic and so we set the exit flow pressure at the atmospheric level. The continuity equation (Eq. 5.12) allowed us to evaluate the fluid flow velocities w_1 and w_2 at sections (1) and (2).
6. The forces due to atmospheric pressure acting on the control surface may need consideration as indicated by Eq. 13 in Example 5.11 for the reaction force between the nozzle and the fluid. When calculating the anchoring force, F_A, the forces due to atmospheric pressure on the control surface cancel each other (for example, after combining Eqs. 12 and 13 the atmospheric pressure forces are no longer involved) and gage pressures may be used.
7. The external forces have an algebraic sign, positive if the force is in the assigned positive coordinate direction and negative otherwise.

A control volume diagram is similar to a free-body diagram.

8. Only external forces acting on the contents of the control volume are considered in the linear momentum equation (Eq. 5.22). If the fluid alone is included in a control volume, reaction forces between the fluid and the surface or surfaces in contact with the fluid [wetted surface(s)] will need to be in Eq. 5.22. If the fluid and the wetted surface or surfaces are within the control volume, the reaction forces between fluid and wetted surface(s) do not appear in the linear momentum equation (Eq. 5.22) because they are internal, not external forces. The anchoring force that holds the wetted surface(s) in place is an external force, however, and must therefore be in Eq. 5.22.
9. The force required to anchor an object will generally exist in response to surface pressure and/or shear forces acting on the control surface, to a change in linear momentum flow through the control volume containing the object, and to the weight of the object and the fluid contained in the control volume. In Example 5.11 the nozzle anchoring force was required mainly because of pressure forces and partly because of a change in linear momentum flow associated with accelerating the fluid in the nozzle. The weight of the water and the nozzle contained in the control volume influenced the size of the anchoring force only slightly.

Fluids in the News

Motorized surfboard When Bob Montgomery, a former professional surfer, started to design his motorized surfboard (called a jet board), he discovered that there were many engineering challenges to the design. The idea is to provide surfing to anyone, no matter where they live, near or far from the ocean. The rider stands on the device like a surfboard and steers it like a surfboard by shifting his/her body weight. A new, sleek, compact 45-horsepower engine and pump was designed to fit within the surfboard hull. Thrust is produced in response to the change in *linear momentum* of the water stream as it enters through the inlet passage and exits through an appropriately designed nozzle. Some of the fluid dynamic problems associated with designing the craft included one-way valves so that water does not get into the engine (at both the intake or exhaust ports), buoyancy, hydrodynamic lift, drag, thrust, and hull stability. (See Problem 5.68.)

To further demonstrate the use of the linear momentum equation (Eq. 5.22), we consider another one-dimensional flow example before moving on to other facets of this important equation.

EXAMPLE 5.12 Linear Momentum—Pressure and Change in Flow Direction

GIVEN Water flows through a horizontal, 180° pipe bend as illustrated in Fig. E5.12*a*. The flow cross-sectional area is constant at a value of 0.1 ft^2 through the bend. The magnitude of the flow velocity everywhere in the bend is axial and 50 ft/s. The absolute pressures at the entrance and exit of the bend are 30 psia and 24 psia, respectively.

FIND Calculate the horizontal (x and y) components of the anchoring force required to hold the bend in place.

SOLUTION

Since we want to evaluate components of the anchoring force to hold the pipe bend in place, an appropriate control volume (see dashed line in Fig. E5.12*a*) contains the bend and the water in the bend at an instant. The horizontal forces acting on the contents of this control volume are identified in Fig. E5.12*b*. Note that the weight of the water is vertical (in the negative z direction) and does not contribute to the x and y components of the anchoring force. All of the horizontal normal and tangential forces exerted on the fluid and the pipe bend are resolved and combined into the two resultant components, F_{Ax} and F_{Ay}. These two forces act on the control volume contents, and thus for the x direction, Eq. 5.22 leads to

$$\int_{\text{cs}} u\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = F_{Ax} \tag{1}$$

At sections (1) and (2), the flow is in the y direction and therefore $u = 0$ at both cross sections. There is no x direction momentum flow into or out of the control volume and we conclude from Eq. 1 that

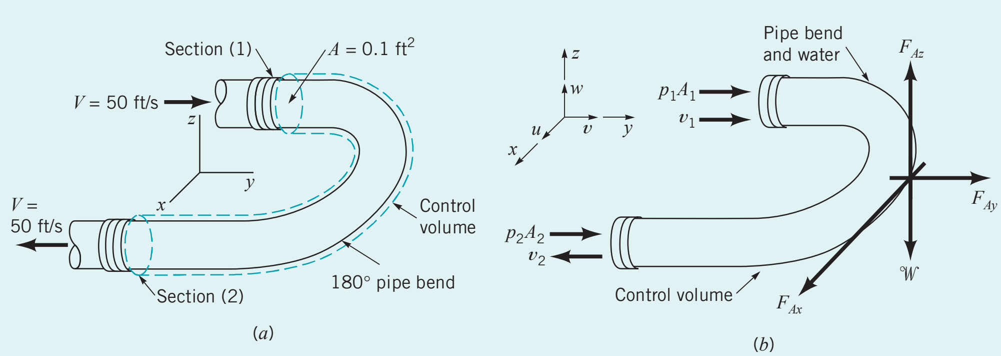
$$F_{Ax} = 0 \qquad \textbf{(Ans)}$$

For the y direction, we get from Eq. 5.22

$$\int_{\text{cs}} v\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = F_{Ay} + p_1A_1 + p_2A_2 \tag{2}$$

For one-dimensional flow, the surface integral in Eq. 2 is easy to evaluate and Eq. 2 becomes

$$(+v_1)(-\dot{m}_1) + (-v_2)(+\dot{m}_2) = F_{Ay} + p_1A_1 + p_2A_2 \tag{3}$$



■ FIGURE E5.12

Note that the y component of velocity is positive at section (1) but is negative at section (2). Also, the mass flowrate term is negative at section (1) (flow in) and is positive at section (2) (flow out). From the continuity equation (Eq. 5.12), we get

$$\dot{m} = \dot{m}_1 = \dot{m}_2 \tag{4}$$

and thus Eq. 3 can be written as

$$-\dot{m}(v_1 + v_2) = F_{Ay} + p_1A_1 + p_2A_2 \tag{5}$$

Solving Eq. 5 for F_{Ay} we obtain

$$F_{Ay} = -\dot{m}(v_1 + v_2) - p_1A_1 - p_2A_2 \tag{6}$$

From the given data we can calculate the mass flowrate, $\dot{m}$, from Eq. 5.6 as

$$\dot{m} = \rho_1 A_1 v_1 = (1.94 \text{ slugs/ft}^3)(0.1 \text{ ft}^2)(50 \text{ ft/s}) = 9.70 \text{ slugs/s}$$

For determining the anchoring force, F_{Ay}, the effects of atmospheric pressure cancel and thus gage pressures for p_1 and p_2 are appropriate. By substituting numerical values of variables into Eq. 6, and using the fact that 1 lb = 1 slug · ft/s² we get

$$\begin{aligned} F_{Ay} = &-(9.70 \text{ slugs/s})(50 \text{ ft/s} + 50 \text{ ft/s}) \\ &- (30 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ &- (24 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \end{aligned}$$

$$F_{Ay} = -970 \text{ lb} - 220 \text{ lb} - 134 \text{ lb} = -1324 \text{ lb} \qquad \textbf{(Ans)}$$

The negative sign for F_{Ay} is interpreted as meaning that the y component of the anchoring force is actually in the negative y direction, not the positive y direction as originally indicated in Fig. E5.12*b*.

COMMENT As with Example 5.11, the anchoring force for the pipe bend is independent of the atmospheric pressure. However, the force that the bend puts on the fluid inside of it, R_y, depends on the atmospheric pressure. We can see this by using a control volume which surrounds only the fluid within the bend as shown in Fig. E5.12*c*. Application of the momentum equation to this situation gives

$$R_y = -\dot{m}(v_1 + v_2) - p_1A_1 - p_2A_2$$

where p_1 and p_2 must be in terms of absolute pressure because the force between the fluid and the pipe wall, R_y, is the complete pressure effect (i.e., absolute pressure). We see that forces exerted on the flowing fluid result in a change in its velocity direction (a change in linear momentum).

Thus, we obtain

$$\begin{aligned} R_y = &-(9.70 \text{ slugs/s})(50 \text{ ft/s} + 50 \text{ ft/s}) \\ &- (30 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ &- (24 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ = &-1748 \text{ lb} \end{aligned} \tag{7}$$

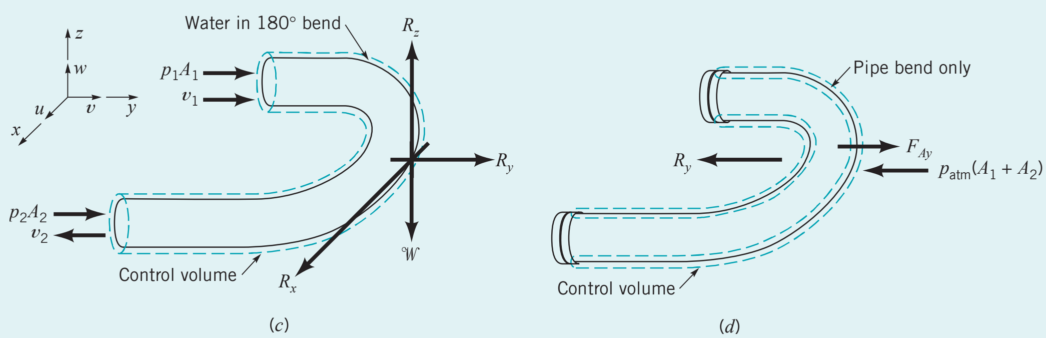
We can use the control volume that includes just the pipe bend (without the fluid inside it) as shown in Fig. E5.12*d* to determine F_{Ay}, the anchoring force component in the y direction necessary to hold the bend stationary. The y component of the momentum equation applied to this control volume gives

$$F_{Ay} = R_y + p_{\text{atm}}(A_1 + A_2) \tag{8}$$

where R_y is given by Eq. 7. The $p_{\text{atm}}(A_1 + A_2)$ term represents the net pressure force on the outside portion of the control volume. Recall that the pressure force on the inside of the bend is accounted for by R_y. By combining Eqs. 7 and 8 and using the fact that $p_{\text{atm}} = 14.7 \text{ lb/in.}^2 (144 \text{ in.}^2/\text{ft}^2) = 2117 \text{ lb/ft}^2$, we obtain

$$F_{Ay} = -1748 \text{ lb} + 2117 \text{ lb/ft}^2 (0.1 \text{ ft}^2 + 0.1 \text{ ft}^2) = -1324 \text{ lb}$$

in agreement with the original answer obtained using the control volume of Fig. E5.12*b*.



■ FIGURE E5.12 cont.

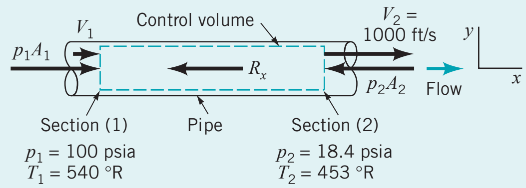
V5.8 Fire hose

In Examples 5.10 and 5.12 the force exerted on a flowing fluid resulted in a change in flow direction only. This force was associated with constraining the flow, with a vane in Example 5.10, and with a pipe bend in Example 5.12. In Example 5.11 the force exerted on a flowing fluid resulted in a change in velocity magnitude only. This force was associated with a converging nozzle. Anchoring forces are required to hold a vane or conduit stationary. They are most easily estimated with a control volume that contains the vane or conduit and the flowing fluid involved. Alternately, two separate control volumes can be used, one containing the vane or conduit only and one containing the flowing fluid only.

EXAMPLE 5.13 Linear Momentum—Pressure, Change in Speed, and Friction

GIVEN Air flows steadily between two cross sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Fig. E5.13, where the uniformly distributed temperature and pressure at each cross section are given. If the average air velocity at section (2) is 1000 ft/s, we found in Example 5.2 that the average air velocity at section (1) must be 219 ft/s. Assume uniform velocity distributions at sections (1) and (2).

FIND Determine the frictional force exerted by the pipe wall on the air flow between sections (1) and (2).



■ FIGURE E5.13

SOLUTION

The control volume of Example 5.2 is appropriate for this problem. The forces acting on the air between sections (1) and (2) are identified in Fig. E5.13. The weight of air is considered negligibly small. The reaction force between the wetted wall of the pipe and the flowing air, R_x, is the frictional force sought. Application of the axial component of Eq. 5.22 to this control volume yields

$$\int_{\text{cs}} u\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -R_x + p_1A_1 - p_2A_2 \qquad \textbf{(1)}$$

The positive x direction is set as being to the right. Furthermore, for uniform velocity distributions (one-dimensional flow), Eq. 1 becomes

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = -R_x + p_1A_1 - p_2A_2 \qquad \textbf{(2)}$$

From conservation of mass (Eq. 5.12) we get

$$\dot{m} = \dot{m}_1 = \dot{m}_2 \qquad \textbf{(3)}$$

so that Eq. 2 becomes

$$\dot{m}(u_2 - u_1) = -R_x + A_2(p_1 - p_2) \qquad \textbf{(4)}$$

Solving Eq. 4 for R_x, we get

$$R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1) \qquad \textbf{(5)}$$

The equation of state gives

$$\rho_2 = \frac{p_2}{RT_2} \qquad \textbf{(6)}$$

and the equation for area A_2 is

$$A_2 = \frac{\pi D_2^2}{4} \qquad \textbf{(7)}$$

Thus, from Eqs. 3, 6, and 7

$$\dot{m} = \left(\frac{p_2}{RT_2}\right)\left(\frac{\pi D_2^2}{4}\right) u_2$$

English Engineering (EE) units are often used for this kind of flow. The gas constant, R, for air in EE units is

$$\text{R} = \frac{1716(\text{ft} \cdot \text{lb})/(\text{slug} \cdot {}^\circ\text{R})}{32.174(\text{lbm/slug})} = 53.3(\text{ft} \cdot \text{lb})/(\text{lbm} \cdot {}^\circ\text{R})$$

$$\text{Hence, } \dot{m} = \frac{(18.4 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{[53.3(\text{ft} \cdot \text{lb})/(\text{lbm} \cdot {}^\circ\text{R})]\,(453\ {}^\circ\text{R})} \times \frac{\pi(4 \text{ in.})^2}{4(144 \text{ in.}^2/\text{ft}^2)}(1000 \text{ ft/s}) = 9.57 \text{ lbm/s} \qquad \textbf{(8)}$$

Thus, from Eqs. 5 and 8

$$\begin{aligned} R_x &= \frac{\pi(4 \text{ in.})^2}{4}(100 \text{ psia} - 18.4 \text{ psia}) \\ &\quad - (9.57 \text{ lbm})(1000 \text{ ft/s} - 219 \text{ ft/s})/ \\ &\quad\ 32.174(\text{lbm} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2) \\ &= 1025 \text{ lb} - 232 \text{ lb} \end{aligned}$$

or

$$R_x = 793 \text{ lb} \qquad \textbf{(Ans)}$$

COMMENT For this compressible flow, the pressure difference drives the motion which results in a frictional force, R_x, and an acceleration of the fluid (i.e., a velocity magnitude increase). For a similar incompressible pipe flow, a pressure difference results in fluid motion with a frictional force only (i.e., no change in velocity magnitude).

EXAMPLE 5.14 Linear Momentum—Weight, Pressure, Friction, and Nonuniform Velocity Profile

GIVEN Consider the flow of Example 5.4 to be vertically upward.

FIND Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

SOLUTION

A control volume (see dashed lines in Fig. E5.14) that includes only fluid from section (1) to section (2) is selected. The forces acting on the fluid in this control volume are identified in Fig. E5.14. The application of the axial component of Eq. 5.22 to the fluid in this control volume results in

$$\int_{cs} w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = p_1 A_1 - R_z - \mathscr{W} - p_2 A_2 \qquad \textbf{(1)}$$

where R_z is the resultant force of the wetted pipe wall on the fluid. Further, for uniform flow at section (1), and because the flow at section (2) is out of the control volume, Eq. 1 becomes

$$(+w_1)(-\dot{m}_1) + \int_{A_2} (+w_2)\rho(+w_2\, dA_2) = p_1 A_1 - R_z - \mathscr{W} - p_2 A_2 \qquad \textbf{(2)}$$

The positive direction is considered up. The surface integral over the cross-sectional area at section (2), A_2, is evaluated by using the parabolic velocity profile obtained in Example 5.4, $w_2 = 2w_1[1 - (r/R)^2]$, as

$$\int_{A_2} w_2 \rho w_2\, dA_2 = \rho \int_0^R w_2^2\, 2\pi r\, dr = 2\pi\rho \int_0^R (2w_1)^2 \left[1 - \left(\frac{r}{R}\right)^2\right]^2 r\, dr$$

or

$$\int_{A_2} w_2 \rho w_2\, dA_2 = 4\pi\rho w_1^2 \frac{R^2}{3} \qquad \textbf{(3)}$$

Combining Eqs. 2 and 3 we obtain

$$-w_1^2\rho\pi R^2 + \tfrac{4}{3} w_1^2 \rho\pi R^2 = p_1 A_1 - R_z - \mathscr{W} - p_2 A_2 \qquad \textbf{(4)}$$

Solving Eq. 4 for the pressure drop from section (1) to section (2), $p_1 - p_2$, we obtain

$$p_1 - p_2 = \frac{\rho w_1^2}{3} + \frac{R_z}{A_1} + \frac{\mathscr{W}}{A_1} \qquad \textbf{(Ans)}$$

COMMENT We see that the drop in pressure from section (1) to section (2) occurs because of the following:

1. The change in momentum flow between the two sections associated with going from a uniform velocity profile to a parabolic velocity profile, $\rho w_1^2/3$
2. Pipe wall friction, R_z
3. The weight of the water column, $\mathscr{W}$; a hydrostatic pressure effect.

If the velocity profiles had been identically parabolic at sections (1) and (2), the momentum flowrate at each section would have been identical, a condition we call "fully developed" flow. Then, the pressure drop, $p_1 - p_2$, would be due only to pipe wall friction and the weight of the water column. If in addition to being fully developed, the flow involved negligible weight effects (for example, horizontal flow of liquids or the flow of gases in any direction) the drop in pressure between any two sections, $p_1 - p_2$, would be a result of pipe wall friction only.

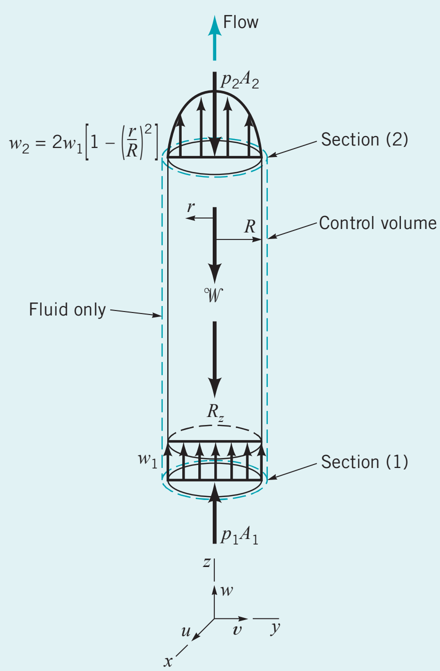
Note that although the average velocity is the same at section (1) as it is at section (2) ($\overline{V}_1 = \overline{V}_2 = w_1$), the momentum flux across section (1) is not the same as it is across section (2). If it were, the left-hand side of Eq. (4) would be zero. For this nonuniform flow the momentum flux can be written in terms of the average velocity, $\overline{V}$, and the *momentum coefficient*, β, as

$$\beta = \frac{\int w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA}{\rho \overline{V}^2 A}$$

Hence the momentum flux can be written as

$$\int_{cs} w\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = -\beta_1 w_1^2 \rho\pi R^2 + \beta_2 w_1^2 \rho\pi R^2$$

where $\beta_1 = 1$ ($\beta = 1$ for uniform flow) and $\beta_2 = 4/3$ ($\beta > 1$ for any nonuniform flow).



■ FIGURE E5.14

EXAMPLE 5.15 Linear Momentum—Thrust

GIVEN A static thrust stand as sketched in Fig. E5.15 is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity = 200 m/s; exhaust gas velocity = 500 m/s; intake cross-sectional area = 1 m^2; intake static pressure = −22.5 kPa = 78.5 kPa (abs); intake static temperature = 268 K; exhaust static pressure = 0 kPa= 101 kPa (abs).

FIND Estimate the nominal anchoring force for which to design.

SOLUTION

The cylindrical control volume outlined with a dashed line in Fig. E5.15 is selected. The external forces acting in the axial direction are also shown. Application of the momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\int_{cs} u\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = p_1A_1 + F_{th} - p_2A_2 - p_{atm}(A_1 - A_2) \quad (1)$$

where the pressures are absolute. Thus, for one-dimensional flow, Eq. 1 becomes

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 + F_{th} \quad (2)$$

The positive direction is to the right. The conservation of mass equation (Eq. 5.12) leads to

$$\dot{m} = \dot{m}_1 = \rho_1A_1u_1 = \dot{m}_2 = \rho_2A_2u_2 \quad (3)$$

Combining Eqs. 2 and 3 and using gage pressure we obtain

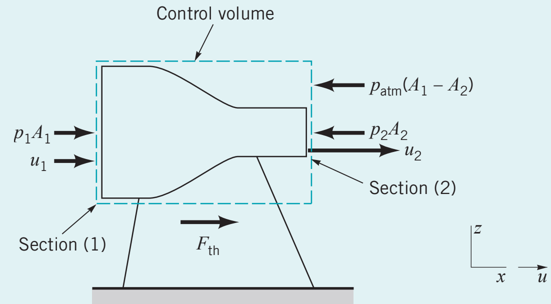
$$\dot{m}(u_2 - u_1) = p_1A_1 - p_2A_2 + F_{th} \quad (4)$$

Solving Eq. 4 for the thrust force, F_{th}, we obtain

$$F_{th} = -p_1A_1 + p_2A_2 + \dot{m}(u_2 - u_1) \quad (5)$$

We need to determine the mass flowrate, $\dot{m}$, to calculate F_{th}, and to calculate $\dot{m} = \rho_1A_1u_1$, we need ρ_1. From the ideal gas equation of state

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(78.5\text{ kPa})(1000\text{ Pa/kPa})[1(\text{N/m}^2)/\text{Pa}]}{(286.9\text{ J/kg}\cdot\text{K})(268\text{ K})(1\text{ N}\cdot\text{m/J})} = 1.02\text{ kg/m}^3$$



■ FIGURE E5.15

Thus,

$$\dot{m} = \rho_1A_1u_1 = (1.02\text{ kg/m}^3)(1\text{ m}^2)(200\text{ m/s}) = 204\text{ kg/s} \quad (6)$$

Finally, combining Eqs. 5 and 6 and substituting given data with $p_2 = 0$, we obtain

$$F_{th} = -(1\text{ m}^2)(-22.5\text{ kPa})(1000\text{ Pa/kPa})[1(\text{N/m}^2)/\text{Pa}] + (204\text{ kg/s})(500\text{ m/s} - 200\text{ m/s})[1\text{ N/(kg}\cdot\text{m/s}^2)]$$

or

$$F_{th} = 22{,}500\text{ N} + 61{,}200\text{ N} = 83{,}700\text{ N} \quad \textbf{(Ans)}$$

COMMENT The force of the thrust stand on the engine is directed toward the right. Conversely, the engine pushes to the left on the thrust stand (or aircraft).

Fluids in the News

Bow thrusters In the past, large ships required the use of tugboats for precise maneuvering, especially when docking. Nowadays, most large ships (and many moderate to small ones as well) are equipped with bow thrusters to help steer in close quarters. The units consist of a mechanism (usually a ducted propeller mounted at right angles to the fore/aft axis of the ship) that takes water from one side of the bow and ejects it as a water jet on the other side. The *momentum flux* of this jet produces a starboard or port force on the ship for maneuvering. Sometimes a second unit is installed in the stern. Initially used in the bows of ferries, these versatile control devices have became popular in offshore oil servicing boats, fishing vessels, and larger ocean-going craft. They permit unassisted maneuvering alongside of oilrigs, vessels, loading platforms, fishing nets, and docks. They also provide precise control at slow speeds through locks, narrow channels, and bridges, where the rudder becomes very ineffective. (See Problem 5.69.)

EXAMPLE 5.16 Linear Momentum—Nonuniform Pressure

GIVEN A sluice gate across a channel of width b is shown in the closed and open positions in Figs. E5.16*a* and E5.16*b*.

FIND Is the anchoring force required to hold the gate in place larger when the gate is closed or when it is open?

SOLUTION

We will answer this question by comparing expressions for the horizontal reaction force, R_x, between the gate and the water when the gate is closed and when the gate is open. The control volume used in each case is indicated with dashed lines in Figs. E5.16*a* and E5.16*b*.

When the gate is closed, the horizontal forces acting on the contents of the control volume are identified in Fig. E5.16*c*. Application of Eq. 5.22 to the contents of this control volume yields

$$\int_{\text{cs}} u\rho \cancelto{0\ \text{(no flow)}}{\mathbf{V}\cdot\hat{\mathbf{n}}}\, dA = \tfrac{1}{2}\gamma H^2 b - R_x \tag{1}$$

Note that the hydrostatic pressure force, $\gamma H^2 b/2$, is used. From Eq. 1, the force exerted on the water by the gate (which is equal to the force necessary to hold the gate stationary) is

$$R_x = \tfrac{1}{2}\gamma H^2 b \tag{2}$$

which is equal in magnitude to the hydrostatic force exerted on the gate by the water.

When the gate is open, the horizontal forces acting on the contents of the control volume are shown in Fig. E5.16*d*. Application of Eq. 5.22 to the contents of this control volume leads to

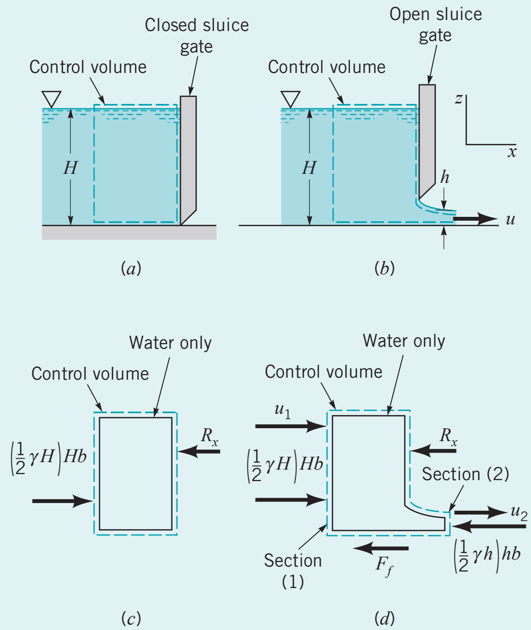
$$\int_{\text{cs}} u\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \tfrac{1}{2}\gamma H^2 b - R_x - \tfrac{1}{2}\gamma h^2 b - F_f \tag{3}$$

Note that because the water at sections (1) and (2) is flowing along straight, horizontal streamlines, the pressure distribution at those locations is hydrostatic, varying from zero at the free surface to γ times the water depth at the bottom of the channel (see Chapter 3, Section 3.4). Thus, the pressure forces at sections (1) and (2) (given by the pressure at the centroid times the area) are $\gamma H^2 b/2$ and $\gamma h^2 b/2$, respectively. Also, the frictional force between the channel bottom and the water is specified as F_f. The surface integral in Eq. 3 is nonzero only where there is flow across the control surface. With the assumption of uniform velocity distributions,

$$\int_{\text{cs}} u\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = (u_1)\rho(-u_1)Hb + (+u_2)\rho(+u_2)hb \tag{4}$$

Thus, Eqs. 3 and 4 combine to form

$$-\rho u_1^2 Hb + \rho u_2^2 hb = \tfrac{1}{2}\gamma H^2 b - R_x - \tfrac{1}{2}\gamma h^2 b - F_f \tag{5}$$



■ FIGURE E5.16

If $H \gg h$, the upstream velocity, u_1, is much less than u_2 so that the contribution of the incoming momentum flow to the control surface integral can be neglected and from Eq. 5 we obtain

$$R_x = \tfrac{1}{2}\gamma H^2 b - \tfrac{1}{2}\gamma h^2 b - F_f - \rho u_2^2 hb \tag{6}$$

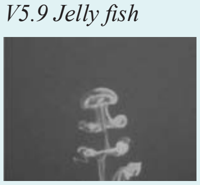
By using the continuity equation, $\dot{m} = \rho bHu_1 = \rho bhu_2$, Eq. (6) can be rewritten as

$$R_x = \tfrac{1}{2}\gamma H^2 b - \tfrac{1}{2}\gamma h^2 b - F_f - \dot{m}(u_2 - u_1) \tag{7}$$

Hence, since $u_2 > u_1$, by comparing the expressions for R_x (Eqs. 2 and 7) we conclude that the reaction force between the gate and the water (and therefore the anchoring force required to hold the gate in place) is smaller when the gate is open than when it is closed. **(Ans)**

The linear momentum equation can be written for a moving control volume.

V5.9 Jelly fish



All of the linear momentum examples considered thus far have involved stationary and nondeforming control volumes which are thus inertial because there is no acceleration. A nondeforming control volume translating in a straight line at constant speed is also inertial because there is no acceleration. For a system and an inertial, moving, nondeforming control volume that are both coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) leads to

$$\frac{D}{Dt}\int_{\text{sys}} \mathbf{V}\rho\, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho \mathbf{W}\cdot\hat{\mathbf{n}}\, dA \tag{5.23}$$

When we combine Eq. 5.23 with Eqs. 5.19 and 5.20, we get

$$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho\, d\forall + \int_{\text{cs}} \mathbf{V}\rho \mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}} \tag{5.24}$$

When the equation relating absolute, relative, and control volume velocities (Eq. 5.14) is used with Eq. 5.24, the result is

$$\frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{W} + \mathbf{V}_{\text{cv}})\rho\, d\forall + \int_{\text{cs}} (\mathbf{W} + \mathbf{V}_{\text{cv}})\rho \mathbf{W}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}} \tag{5.25}$$

For a constant control volume velocity, $\mathbf{V}_{\text{cv}}$, and steady flow in the control volume reference frame,

$$\frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{W} + \mathbf{V}_{\text{cv}})\rho \, d\mathcal{V} = 0 \tag{5.26}$$

Also, for this inertial, nondeforming control volume

$$\int_{\text{cs}} (\mathbf{W} + \mathbf{V}_{\text{cv}})\rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA = \int_{\text{cs}} \mathbf{W}\rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA + \mathbf{V}_{\text{cv}} \int_{\text{cs}} \rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA \tag{5.27}$$

For steady flow (on an instantaneous or time-average basis), Eq. 5.15 gives

$$\int_{\text{cs}} \rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0 \tag{5.28}$$

Combining Eqs. 5.25, 5.26, 5.27, and 5.28, we conclude that the linear momentum equation for an inertial, moving, nondeforming control volume that involves steady (instantaneous or time-average) flow is

The linear momentum equation for a moving control volume involves the relative velocity.

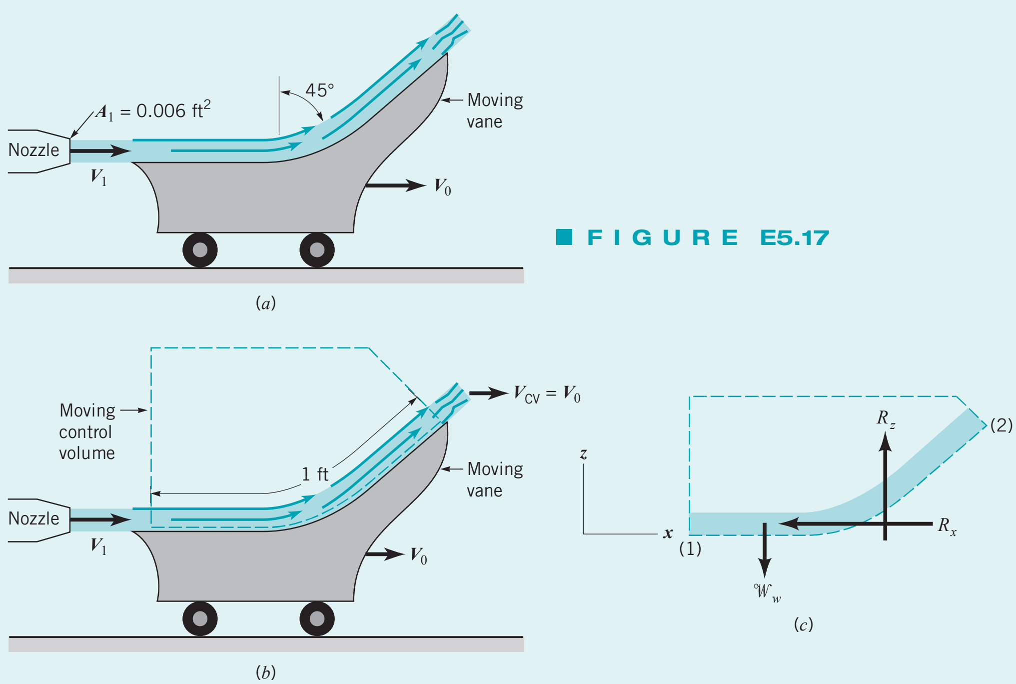
$$\boxed{\int_{\text{cs}} \mathbf{W}\rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}} \tag{5.29}$$

Example 5.17 illustrates the use of Eq. 5.29.

EXAMPLE 5.17 Linear Momentum—Moving Control Volume

GIVEN A vane on wheels moves with constant velocity $\mathbf{V}_0$ when a stream of water having a nozzle exit velocity of $\mathbf{V}_1$ is turned 45° by the vane as indicated in Fig. E5.17*a*. Note that this is the same moving vane considered in Section 4.4.6 earlier. The speed of the water jet leaving the nozzle is 100 ft/s, and the vane is moving to the right with a constant speed of 20 ft/s.

FIND Determine the magnitude and direction of the force, **F**, exerted by the stream of water on the vane surface.



■ FIGURE E5.17

SOLUTION

To determine the magnitude and direction of the force, **F**, exerted by the water on the vane, we apply Eq. 5.29 to the contents of the moving control volume shown in Fig. E5.17*b*. The forces acting on the contents of this control volume are indicated in Fig. E5.17*c*. Note that since the ambient pressure is atmospheric, all pressure forces cancel each other out. Equation 5.29 is applied to the contents of the moving control volume in component directions. For the x direction (positive to the right), we get

$$\int_{\text{cs}} W_x \rho \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = -R_x$$

or

$$(+W_1)(-\dot{m}_1) + (+W_2 \cos 45°)(+\dot{m}_2) = -R_x \qquad \textbf{(1)}$$

where

$$\dot{m}_1 = \rho_1 W_1 A_1 \qquad \text{and} \qquad \dot{m}_2 = \rho_2 W_2 A_2.$$

For the vertical or z direction (positive up) we get

$$\int_{\text{cs}} W_z \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = R_z - \mathcal{W}_w$$

or

$$(+W_2 \sin 45°)(+\dot{m}_2) = R_z - \mathcal{W}_w \qquad \textbf{(2)}$$

We assume for simplicity that the water flow is frictionless and that the change in water elevation across the vane is negligible. Thus, from the Bernoulli equation (Eq. 3.7) we conclude that the speed of the water relative to the moving control volume, W, is constant or

$$W_1 = W_2$$

The relative speed of the stream of water entering the control volume, W_1, is

$$W_1 = V_1 - V_0 = 100 \text{ ft/s} - 20 \text{ ft/s} = 80 \text{ ft/s} = W_2$$

The water density is constant so that

$$\rho_1 = \rho_2 = 1.94 \text{ slugs/ft}^3$$

Application of the conservation of mass principle to the contents of the moving control volume (Eq. 5.16) leads to

$$\dot{m}_1 = \rho_1 W_1 A_1 = \rho_2 W_2 A_2 = \dot{m}_2$$

Combining results we get

$$R_x = \rho W_1^2 A_1 (1 - \cos 45°)$$

or

$$R_x = (1.94 \text{ slugs/ft}^3)(80 \text{ ft/s})^2(0.006 \text{ ft}^2)(1 - \cos 45°)$$
$$= 21.8 \text{ lb}$$

Also,

$$R_z = \rho W_1^2 (\sin 45°) A_1 + \mathcal{W}_w$$

where

$$\mathcal{W}_w = \rho g A_1 \ell$$

Thus,

$$R_z = (1.94 \text{ slugs/ft}^3)(80 \text{ ft/s})^2(\sin 45°)(0.006 \text{ ft}^2) + (62.4 \text{ lb/ft}^3)(0.006 \text{ ft}^2)(1 \text{ ft})$$
$$= 52.6 \text{ lb} + 0.37 \text{ lb} = 53 \text{ lb}$$

Combining the components we get

$$R = \sqrt{R_x^2 + R_z^2} = [(21.8 \text{ lb})^2 + (53 \text{ lb})^2]^{1/2} = 57.3 \text{ lb}$$

The angle of **R** from the x direction, α, is

$$\alpha = \tan^{-1} \frac{R_z}{R_x} = \tan^{-1} (53 \text{ lb}/21.8 \text{ lb}) = 67.6°$$

The force of the water on the vane is equal in magnitude but opposite in direction from **R**; thus it points to the right and down at an angle of 67.6° from the x direction and is equal in magnitude to 57.3 lb. **(Ans)**

COMMENT The force of the fluid on the vane in the x-direction, $R_x = 21.8$ lb, is associated with x-direction motion of the vane at a constant speed of 20 ft/s. Since the vane is not accelerating, this x-direction force is opposed mainly by a wheel friction force of the same magnitude. From basic physics we recall that the power this situation involves is the product of force and speed. Thus,

$$\mathcal{P} = R_x V_0$$
$$= \frac{(21.8 \text{ lb})(20 \text{ ft/s})}{550(\text{ft} \cdot \text{lb})/(\text{hp} \cdot \text{s})}$$
$$= 0.79 \text{ hp}$$

All of this power is consumed by friction.

It is clear from the preceding examples that a flowing fluid can be forced to

1. change direction
2. speed up or slow down
3. have a velocity profile change
4. do only some or all of the above
5. do none of the above

A net force on the fluid is required for achieving any or all of the first four above. The forces on a flowing fluid balance out with no net force for the fifth.

Typical forces considered in this book include

(a) pressure
(b) friction
(c) weight

and involve some type of constraint such as a vane, channel, or conduit to guide the flowing fluid. A flowing fluid can cause a vane, channel or conduit to move. When this happens, power is produced.

The selection of a control volume is an important matter. For determining anchoring forces, consider including fluid and its constraint in the control volume. For determining force between a fluid and its constraint, consider including only the fluid in the control volume.

5.2.3 Derivation of the Moment-of-Momentum Equation[2]

In many engineering problems, the moment of a force with respect to an axis, namely, *torque*, is important. Newton's second law of motion has already led to a useful relationship between forces and linear momentum flow. The linear momentum equation can also be used to solve problems involving torques. However, by forming the moment of the linear momentum and the resultant force associated with each particle of fluid with respect to a point in an inertial coordinate system, we will develop a ***moment-of-momentum equation*** that relates *torques* and *angular momentum flow* for the contents of a control volume. When torques are important, the moment-of-momentum equation is often more convenient to use than the linear momentum equation.

The angular momentum equation is derived from Newton's second law.

Application of Newton's second law of motion to a particle of fluid yields

$$\frac{D}{Dt}(\mathbf{V}\rho\,\delta\forall) = \delta\mathbf{F}_{\text{particle}} \tag{5.30}$$

where $\mathbf{V}$ is the particle velocity measured in an inertial reference system, ρ is the particle density, $\delta\forall$ is the infinitesimally small particle volume, and $\delta\mathbf{F}_{\text{particle}}$ is the resultant external force acting on the particle. If we form the moment of each side of Eq. 5.30 with respect to the origin of an inertial coordinate system, we obtain

$$\mathbf{r}\times\frac{D}{Dt}(\mathbf{V}\rho\,\delta\forall) = \mathbf{r}\times\delta\mathbf{F}_{\text{particle}} \tag{5.31}$$

where $\mathbf{r}$ is the position vector from the origin of the inertial coordinate system to the fluid particle (Fig. 5.3). We note that

$$\frac{D}{Dt}[(\mathbf{r}\times\mathbf{V})\rho\,\delta\forall] = \frac{D\mathbf{r}}{Dt}\times\mathbf{V}\rho\,\delta\forall + \mathbf{r}\times\frac{D(\mathbf{V}\rho\,\delta\forall)}{Dt} \tag{5.32}$$

and

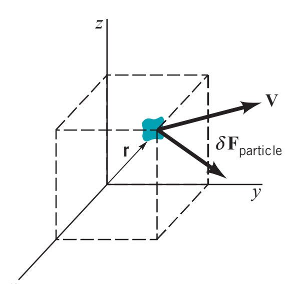
$$\frac{D\mathbf{r}}{Dt} = \mathbf{V} \tag{5.33}$$

Thus, since

$$\mathbf{V}\times\mathbf{V} = 0 \tag{5.34}$$

by combining Eqs. 5.31, 5.32, 5.33, and 5.34, we obtain the expression

$$\frac{D}{Dt}[(\mathbf{r}\times\mathbf{V})\rho\,\delta\forall] = \mathbf{r}\times\delta\mathbf{F}_{\text{particle}} \tag{5.35}$$



■ FIGURE 5.3 Inertial coordinate system.

[2]This section may be omitted, along with Sections 5.2.4 and 5.3.5, without loss of continuity in the text material. However, these sections are recommended for those interested in Chapter 12.

Equation 5.35 is valid for every particle of a system. For a system (collection of fluid particles), we need to use the sum of both sides of Eq. 5.35 to obtain

$$\int_{\text{sys}} \frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V}] = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.36}$$

where

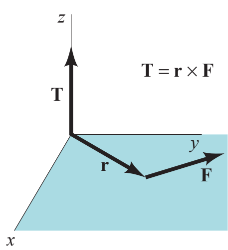
$$\sum \mathbf{r} \times \delta \mathbf{F}_{\text{particle}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.37}$$

We note that

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V} = \int_{\text{sys}} \frac{D}{Dt}[(\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V}] \tag{5.38}$$

since the sequential order of differentiation and integration can be reversed without consequence. (Recall that the material derivative, $D(\ )/Dt$, denotes the time derivative following a given system; see Section 4.2.1.) Thus, from Eqs. 5.36 and 5.38 we get

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V} = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \tag{5.39}$$



or

the time rate of change of the moment-of-momentum of the system	=	sum of external torques acting on the system

The sketch in the margin illustrates what torque, $\mathbf{T} = \mathbf{r} \times \mathbf{F}$, is. For a control volume that is instantaneously coincident with the system, the torques acting on the system and on the control volume contents will be identical:

$$\sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{cv}} \tag{5.40}$$

Further, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19) leads to

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V} + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.41}$$

or

time rate of change of the moment-of-momentum of the system	=	time rate of change of the moment-of-momentum of the contents of the control volume	+	net rate of flow of the moment-of-momentum through the control surface

For a control volume that is fixed (and therefore inertial) and nondeforming, we combine Eqs. 5.39, 5.40, and 5.41 to obtain the moment-of-momentum equation:

For a system, the rate of change of moment-of-momentum equals the net torque.

$$\boxed{\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho \, d\mathcal{V} + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum (\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the} \\ \text{control volume}}}} \tag{5.42}$$

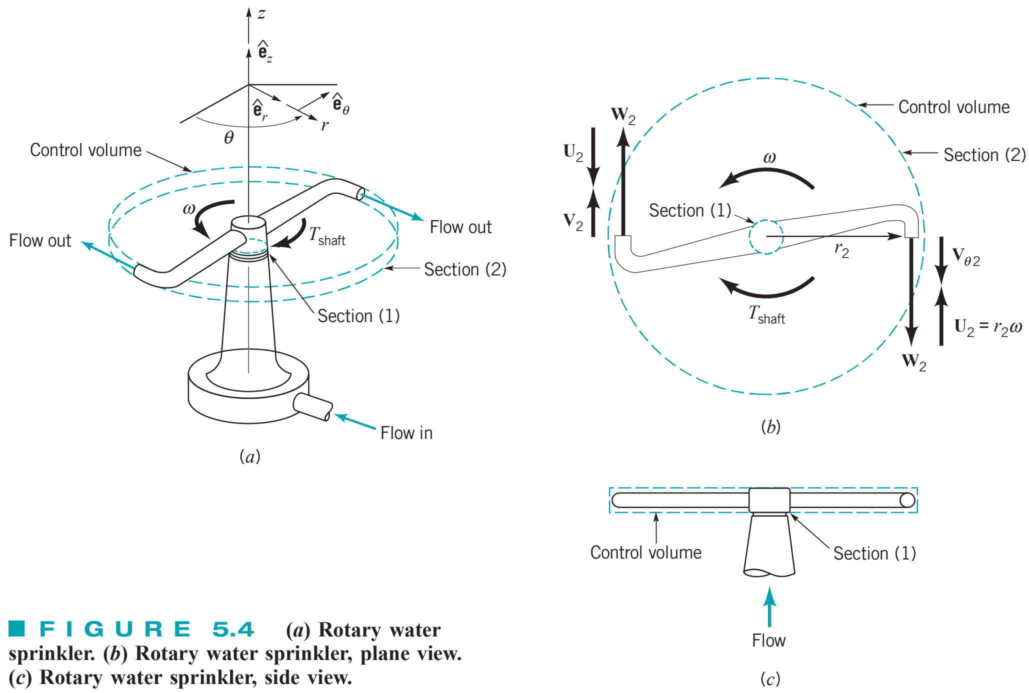
An important category of fluid mechanical problems that is readily solved with the help of the moment-of-momentum equation (Eq. 5.42) involves machines that rotate or tend to rotate around a single axis. Examples of these machines include rotary lawn sprinklers, ceiling fans, lawn mower blades, wind turbines, turbochargers, and gas turbine engines. As a class, these devices are often called turbomachines.

5.2.4 Application of the Moment-of-Momentum Equation[3]

We simplify our use of Eq. 5.42 in several ways:

1. We assume that flows considered are one-dimensional (uniform distributions of average velocity at any section).

[3]This section may be omitted, along with Sections 5.2.3 and 5.3.5, without loss of continuity in the text material. However, these sections are recommended for those interested in Chapter 12.



■ **FIGURE 5.4** **(*a*) Rotary water sprinkler. (*b*) Rotary water sprinkler, plane view. (*c*) Rotary water sprinkler, side view.**

V5.10 Rotating lawn sprinkler

2. We confine ourselves to steady or steady-in-the-mean cyclical flows. Thus,

$$\frac{\partial}{\partial t}\int_{\text{cv}} (\mathbf{r} \times \mathbf{V})\rho \, d\forall = 0$$

at any instant of time for steady flows or on a time-average basis for cyclical unsteady flows.

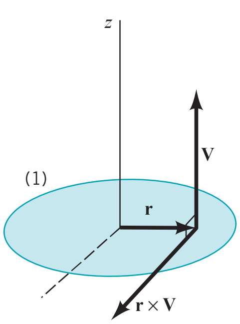
3. We work only with the component of Eq. 5.42 resolved along the axis of rotation.

Change in moment of fluid velocity around an axis can result in torque and rotation around that same axis.

Consider the rotating sprinkler sketched in Fig. 5.4. Because the direction and magnitude of the flow through the sprinkler from the inlet [section (1)] to the outlet [section (2)] of the arm changes, the water exerts a torque on the sprinkler head causing it to tend to rotate or to actually rotate in the direction shown, much like a turbine rotor. In applying the moment-of-momentum equation (Eq. 5.42) to this flow situation, we elect to use the fixed and nondeforming control volume shown in Fig. 5.4. This disk-shaped control volume contains within its boundaries the spinning or stationary sprinkler head and the portion of the water flowing through the sprinkler contained in the control volume at an instant. The control surface cuts through the sprinkler head's solid material so that the shaft torque that resists motion can be clearly identified. When the sprinkler is rotating, the flow field in the stationary control volume is cyclical and unsteady, but steady in the mean. We proceed to use the axial component of the moment-of-momentum equation (Eq. 5.42) to analyze this flow.

The integrand of the moment-of-momentum flow term in Eq. 5.42,

$$\int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$



can be nonzero only where fluid is crossing the control surface. Everywhere else on the control surface this term will be zero because $\mathbf{V} \cdot \hat{\mathbf{n}} = 0$. Water enters the control volume axially through the hollow stem of the sprinkler at section (1). At this portion of the control surface, the component of $\mathbf{r} \times \mathbf{V}$ resolved along the axis of rotation is zero because as illustrated by the figure in the margin, $\mathbf{r} \times \mathbf{V}$ lies in the plane of section (1), perpendicular to the axis of rotation. Thus, there is no axial moment-of-momentum flow in at section (1). Water leaves the control volume through each of the two nozzle openings at section (2). For the exiting flow, the magnitude of the axial component of $\mathbf{r} \times \mathbf{V}$ is $r_2 V_{\theta 2}$, where r_2 is the radius from the axis of rotation to the nozzle centerline and $V_{\theta 2}$ is the value of the tangential component of the velocity of the flow exiting each nozzle as observed from a frame of reference attached to the fixed and nondeforming control volume. The fluid velocity measured relative to a fixed control surface is an absolute velocity, **V**. The velocity of the nozzle exit flow as viewed from the nozzle is called the relative velocity, **W**. The absolute and relative velocities, **V** and **W**, are related by the vector relationship

$$\mathbf{V} = \mathbf{W} + \mathbf{U} \tag{5.43}$$

where **U** is the velocity of the moving nozzle as measured relative to the fixed control surface.

The cross product and the dot product involved in the moment-of-momentum flow term of Eq. 5.42,

$$\int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho\mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

can each result in a positive or negative value. For flow into the control volume, $\mathbf{V} \cdot \hat{\mathbf{n}}$ is negative. For flow out, $\mathbf{V} \cdot \hat{\mathbf{n}}$ is positive. The correct algebraic sign to assign the axis component of $\mathbf{r} \times \mathbf{V}$ can be ascertained by using the right-hand rule. The positive direction along the axis of rotation is the direction the thumb of the right hand points when it is extended and the remaining fingers are curled around the rotation axis in the positive direction of rotation as illustrated in Fig. 5.5. The direction of the axial component of $\mathbf{r} \times \mathbf{V}$ is similarly ascertained by noting the direction of the cross product of the radius from the axis of rotation, $r\hat{\mathbf{e}}_r$, and the tangential component of absolute velocity, $V_\theta \hat{\mathbf{e}}_\theta$. Thus, for the sprinkler of Fig. 5.4, we can state that

The algebraic sign of $\mathbf{r} \times \mathbf{V}$ is obtained by the right-hand rule.

$$\left[\int_{\text{cs}} (\mathbf{r} \times \mathbf{V})\rho\mathbf{V} \cdot \hat{\mathbf{n}}\, dA\right]_{\text{axial}} = (-r_2 V_{\theta 2})(+\dot{m}) \tag{5.44}$$

where, because of mass conservation, $\dot{m}$ is the total mass flowrate through both nozzles. As was demonstrated in Example 5.7, the mass flowrate is the same whether the sprinkler rotates or not. The correct algebraic sign of the axial component of $\mathbf{r} \times \mathbf{V}$ can be easily remembered in the following way: if $\mathbf{V}_\theta$ and **U** are in the same direction, use $+$; if $\mathbf{V}_\theta$ and **U** are in opposite directions, use $-$.

The torque term $[\sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}}]$ of the moment-of-momentum equation (Eq. 5.42) is analyzed next. Confining ourselves to torques acting with respect to the axis of rotation only, we conclude that the shaft torque is important. The net torque with respect to the axis of rotation associated with normal forces exerted on the contents of the control volume will be very small if not zero. The net axial torque due to fluid tangential forces is also negligibly small for the control volume of Fig. 5.4. Thus, for the sprinkler of Fig. 5.4

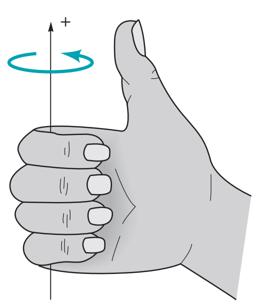
$$\sum \left[(\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the}\\ \text{control volume}}}\right]_{\text{axial}} = \mathbf{T}_{\text{shaft}} \tag{5.45}$$

Note that we have entered T_{shaft} as a positive quantity in Eq. 5.45. This is equivalent to assuming that T_{shaft} is in the same direction as rotation.

For the sprinkler of Fig. 5.4, the axial component of the moment-of-momentum equation (Eq. 5.42) is, from Eqs. 5.44 and 5.45

$$-r_2 V_{\theta 2}\dot{m} = T_{\text{shaft}} \tag{5.46}$$

We interpret T_{shaft} being a negative quantity from Eq. 5.46 to mean that the shaft torque actually opposes the rotation of the sprinkler arms as shown in Fig. 5.4. The shaft torque, T_{shaft}, opposes rotation in all turbine devices.



■ FIGURE 5.5 **Right-hand rule convention.**

We could evaluate the ***shaft power,*** $\dot{W}_{\text{shaft}}$, associated with ***shaft torque,*** T_{shaft}, by forming the product of T_{shaft} and the rotational speed of the shaft, ω. [We use the notation that W = work, $(\dot{\ })$ = $d(\ )/dt$, and thus $\dot{W}$ = power.] Thus, from Eq. 5.46 we get

Power is equal to angular velocity times torque.

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega = -r_2 V_{\theta 2}\dot{m}\,\omega \tag{5.47}$$

Since $r_2\omega$ is the speed of each sprinkler nozzle, U, we can also state Eq. 5.47 in the form

$$\dot{W}_{\text{shaft}} = -U_2 V_{\theta 2}\dot{m} \tag{5.48}$$

Shaft work per unit mass, w_{shaft}, is equal to $\dot{W}_{\text{shaft}}/\dot{m}$. Dividing Eq. 5.48 by the mass flowrate, $\dot{m}$, we obtain

$$w_{\text{shaft}} = -U_2 V_{\theta 2} \tag{5.49}$$

V5.11 Impulse-type lawn sprinkler

Negative shaft work as in Eqs. 5.47, 5.48, and 5.49 is work out of the control volume, that is, work done by the fluid on the rotor and thus its shaft.

The principles associated with this sprinkler example can be extended to handle most simplified turbomachine flows. The fundamental technique is not difficult. However, the geometry of some turbomachine flows is quite complicated.

Example 5.18 further illustrates how the axial component of the moment-of-momentum equation (Eq. 5.46) can be used.

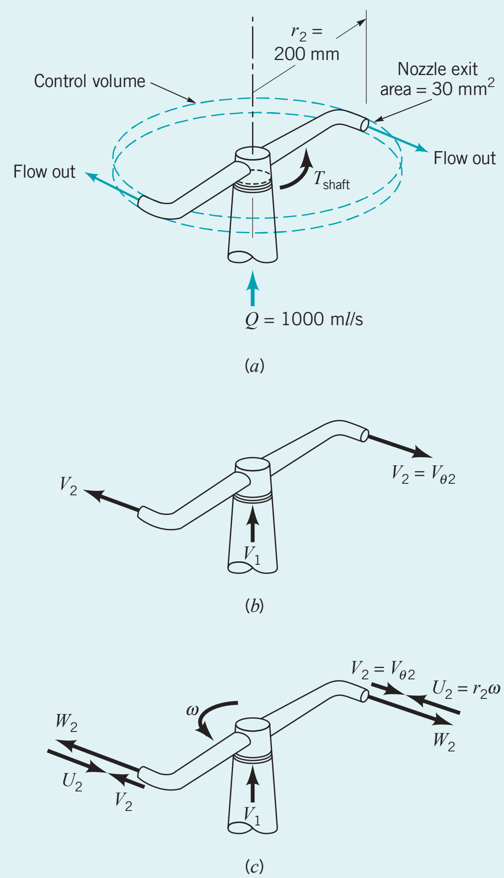
EXAMPLE 5.18 Moment-of-Momentum—Torque

GIVEN Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.18*a*. The exit area of each of the two nozzles is 30 mm^2 and the flow leaving each nozzle is in the tangential direction. The radius from the axis of rotation to the centerline of each nozzle is 200 mm.

FIND **(a)** Determine the resisting torque required to hold the sprinkler head stationary.

(b) Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min.

(c) Determine the speed of the sprinkler if no resisting torque is applied.



■ **FIGURE E5.18**

SOLUTION

To solve parts **(a)**, **(b)**, and **(c)** of this example we can use the same fixed and nondeforming, disk-shaped control volume illustrated in Fig. 5.4. As indicated in Fig. E5.18*a*, the only axial torque considered is the one resisting motion, T_{shaft}.

(a) When the sprinkler head is held stationary as specified in part **(a)** of this example problem, the velocities of the fluid entering and leaving the control volume are shown in Fig. E5.18*b*. Equation 5.46 applies to the contents of this control volume. Thus,

$$T_{\text{shaft}} = -r_2 V_{\theta 2}\dot{m} \tag{1}$$

Since the control volume is fixed and nondeforming and the flow exiting from each nozzle is tangential,

$$V_{\theta 2} = V_2 \tag{2}$$

Equations 1 and 2 give

$$T_{\text{shaft}} = -r_2 V_2 \dot{m} \tag{3}$$

In Example 5.7, we ascertained that $V_2 = 16.7$ m/s. Thus, from Eq. 3 with

$$\dot{m} = Q\rho = \frac{(1000 \text{ ml/s})(10^{-3} \text{ m}^3/\text{liter})(999 \text{ kg/m}^3)}{(1000 \text{ ml/liter})}$$
$$= 0.999 \text{ kg/s}$$

we obtain

$$T_{\text{shaft}} = -\frac{(200 \text{ mm})(16.7 \text{ m/s})(0.999 \text{ kg/s})[1 (\text{N/kg})/(\text{m/s}^2)]}{(1000 \text{ mm/m})}$$

or

$$T_{\text{shaft}} = -3.34 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$

(b) When the sprinkler is rotating at a constant speed of 500 rpm, the flow field in the control volume is unsteady but cyclical. Thus, the flow field is steady in the mean. The velocities of the flow entering and leaving the control volume are as indicated in Fig. E5.18*c*. The absolute velocity of the fluid leaving each nozzle, V_2, is from Eq. 5.43,

$$V_2 = W_2 - U_2 \tag{4}$$

where

$$W_2 = 16.7 \text{ m/s}$$

as determined in Example 5.7. The speed of the nozzle, U_2, is obtained from

$$U_2 = r_2\omega \tag{5}$$

Application of the axial component of the moment-of-momentum equation (Eq. 5.46) leads again to Eq. 3. From Eqs. 4 and 5,

$$V_2 = 16.7 \text{ m/s} - r_2\omega$$
$$= 16.7 \text{ m/s} - \frac{(200 \text{ mm})(500 \text{ rev/min})(2\pi \text{ rad/rev})}{(1000 \text{ mm/m})(60 \text{ s/min})}$$

or

$$V_2 = 16.7 \text{ m/s} - 10.5 \text{ m/s} = 6.2 \text{ m/s}$$

Thus, using Eq. 3, with $\dot{m} = 0.999$ kg/s (as calculated previously), we get

$$T_{\text{shaft}} = -\frac{(200 \text{ mm})(6.2 \text{ m/s})\, 0.999 \text{ kg/s}\, [1 (\text{N/kg})/(\text{m/s}^2)]}{(1000 \text{ mm/m})}$$

or

$$T_{\text{shaft}} = -1.24 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$

COMMENT Note that the resisting torque associated with sprinkler head rotation is much less than the resisting torque that is required to hold the sprinkler stationary.

(c) When no resisting torque is applied to the rotating sprinkler head, a maximum constant speed of rotation will occur as demonstrated below. Application of Eqs. 3, 4, and 5 to the contents of the control volume results in

$$T_{\text{shaft}} = -r_2(W_2 - r_2\omega)\dot{m} \tag{6}$$

For no resisting torque, Eq. 6 yields

$$0 = -r_2(W_2 - r_2\omega)\dot{m}$$

Thus,

$$\omega = \frac{W_2}{r_2} \tag{7}$$

In Example 5.4, we learned that the relative velocity of the fluid leaving each nozzle, W_2, is the same regardless of the speed of rotation of the sprinkler head, ω, as long as the mass flowrate of the fluid, $\dot{m}$, remains constant. Thus, by using Eq. 7 we obtain

$$\omega = \frac{W_2}{r_2} = \frac{(16.7 \text{ m/s})(1000 \text{ mm/m})}{(200 \text{ mm})} = 83.5 \text{ rad/s}$$

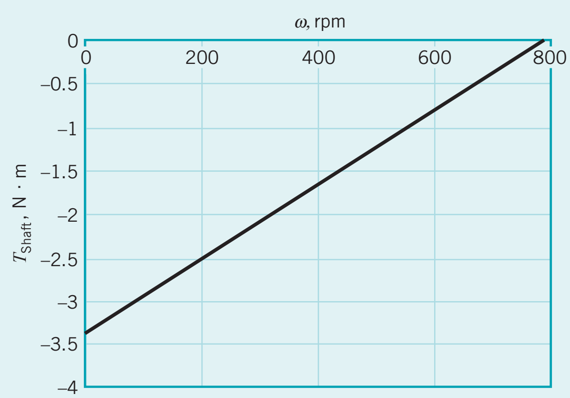
or

$$\omega = \frac{(83.5 \text{ rad/s})(60 \text{ s/min})}{2\,\pi \text{ rad/rev}} = 797 \text{ rpm} \qquad \textbf{(Ans)}$$

For this condition ($T_{\text{shaft}} = 0$), the water both enters and leaves the control volume with zero angular momentum.

COMMENT Note that forcing a change in direction of a flowing fluid, in this case with a sprinkler, resulted in rotary motion and a useful "sprinkling" of water over an area.

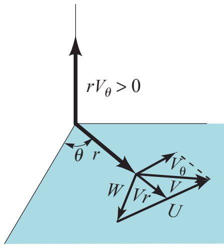
By repeating the calculations for various values of the angular velocity, ω, the results shown in Fig. E5.18*d* are obtained. It is seen that the magnitude of the resisting torque associated with rotation is less than the torque required to hold the rotor stationary. Even in the absence of a resisting torque, the rotor maximum speed is finite.

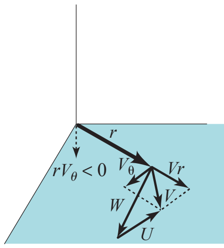


■ FIGURE E5.18*d*

When the moment-of-momentum equation (Eq. 5.42) is applied to a more general, one-dimensional flow through a rotating machine, we obtain

$$\boxed{T_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}} V_{\theta\text{out}})} \tag{5.50}$$





by applying the same kind of analysis used with the sprinkler of Fig. 5.4. The "−" is used with mass flowrate into the control volume, $\dot{m}_{\text{in}}$, and the "+" is used with mass flowrate out of the control volume, $\dot{m}_{\text{out}}$, to account for the sign of the dot product, $\mathbf{V} \cdot \hat{\mathbf{n}}$, involved. Whether "+" or "−" is used with the rV_θ product depends on the direction of $(\mathbf{r} \times \mathbf{V})_{\text{axial}}$. A simple way to determine the sign of the rV_θ product is to compare the direction of V_θ and the blade speed, U. As shown in the margin, if V_θ and U are in the same direction, then the rV_θ product is positive. If V_θ and U are in opposite directions, the rV_θ product is negative. The sign of the shaft torque is "+" if T_{shaft} is in the same direction along the axis of rotation as ω, and "−" otherwise.

The shaft power, $\dot{W}_{\text{shaft}}$, is related to shaft torque, T_{shaft}, by

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega \tag{5.51}$$

Thus, using Eqs. 5.50 and 5.51 with a "+" sign for T_{shaft} in Eq. 5.50, we obtain

$$\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}}\omega V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}}\omega V_{\theta\text{out}}) \tag{5.52}$$

or since $r\omega = U$

$$\boxed{\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm U_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}} V_{\theta\text{out}})} \tag{5.53}$$

When shaft torque and shaft rotation are in the same (opposite) direction, power is into (out of) the fluid.

The "+" is used for the UV_θ product when U and V_θ are in the same direction; the "−" is used when U and V_θ are in opposite directions. Also, since $+T_{\text{shaft}}$ was used to obtain Eq. 5.53, when $\dot{W}_{\text{shaft}}$ is positive, power is into the fluid (for example, a pump), and when $\dot{W}_{\text{shaft}}$ is negative, power is out of the fluid (for example, a turbine).

The shaft work per unit mass, w_{shaft}, can be obtained from the shaft power, $\dot{W}_{\text{shaft}}$, by dividing Eq. 5.53 by the mass flowrate, $\dot{m}$. By conservation of mass,

$$\dot{m} = \dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

From Eq. 5.53, we obtain

$$\boxed{w_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta\text{in}}) + (\pm U_{\text{out}} V_{\theta\text{out}})} \tag{5.54}$$

The application of Eqs. 5.50, 5.53, and 5.54 is demonstrated in Example 5.19. More examples of the application of Eqs. 5.50, 5.53, and 5.54 are included in Chapter 12.

EXAMPLE 5.19 Moment-of-Momentum—Power

GIVEN An air fan has a bladed rotor of 12-in. outside diameter and 10-in. inside diameter as illustrated in Fig. E5.19*a*. The height of each rotor blade is constant at 1 in. from blade inlet to outlet. The flowrate is steady, on a time-average basis, at 230 ft^3/min and the absolute velocity of the air at blade inlet, $\mathbf{V}_1$, is radial. The blade discharge angle is 30° from the tangential direction. The rotor rotates at a constant speed of 1725 rpm.

FIND Estimate the power required to run the fan.

SOLUTION

We select a fixed and nondeforming control volume that includes the rotating blades and the fluid within the blade row at an instant, as shown with a dashed line in Fig. E5.19*a*. The flow within this control volume is cyclical, but steady in the mean. The only torque we consider is the driving shaft torque, T_{shaft}. This torque is provided by a motor. We assume that the entering and leaving flows are each represented by uniformly distributed velocities and flow properties. Since shaft power is sought, Eq. 5.53 is appropriate. Application of Eq. 5.53 to the contents of the control volume in Fig. E5.19 gives

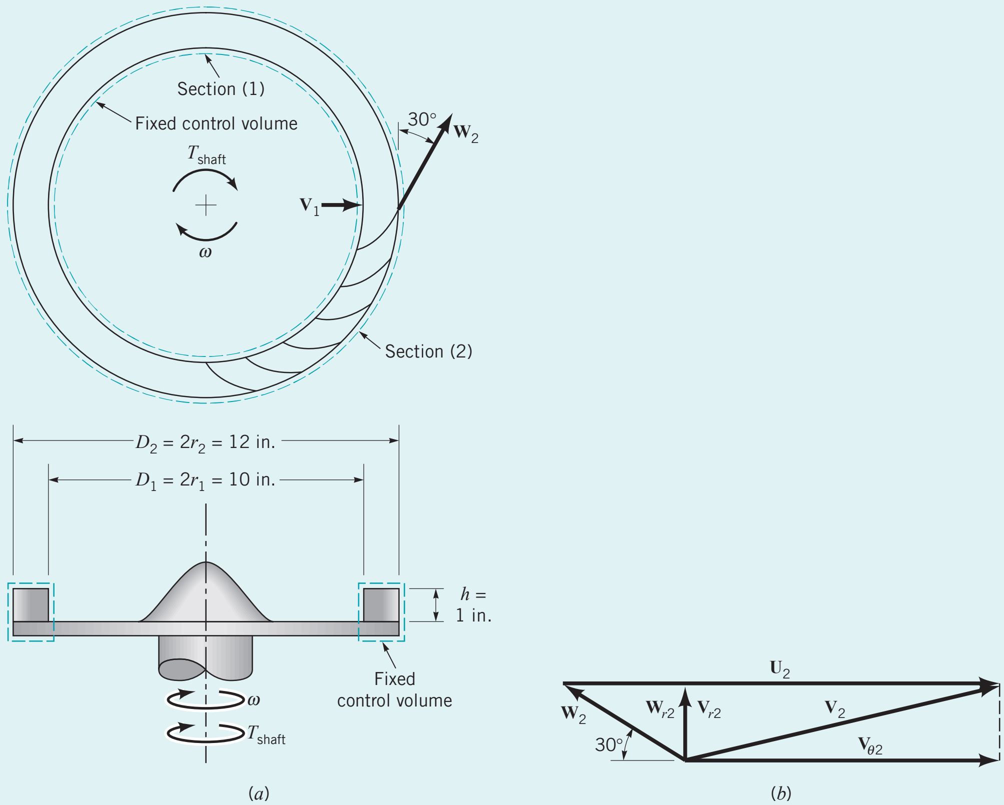
$$\dot{W}_{\text{shaft}} = -\dot{m}_1(\pm U_1 \overbrace{V_{\theta 1}}^{0\ (\mathbf{V}_1 \text{ is radial})}) + \dot{m}_2(\pm U_2 V_{\theta 2}) \tag{1}$$

From Eq. 1 we see that to calculate fan power, we need mass flowrate, $\dot{m}$, rotor exit blade velocity, U_2, and fluid tangential velocity at blade exit, $V_{\theta 2}$. The mass flowrate, $\dot{m}$, is easily obtained from Eq. 5.6 as

$$\dot{m} = \rho Q = \frac{(2.38 \times 10^{-3}\ \text{slug/ft}^3)(230\ \text{ft}^3/\text{min})}{(60\ \text{s/min})}$$

$$= 0.00912\ \text{slug/s} \tag{2}$$

Often, problems involving fans are solved using English Engineering units. Since 1slug = 32.174 lbm, we could have used as the density of air $\rho_{\text{air}} = (2.38 \times 10^{-3}\ \text{slug/ft}^3)(32.174\text{lbm/slug}) = 0.0766\ \text{lbm/ft}^3$.



■ FIGURE E5.19

Then

$$\dot{m} = \frac{(0.0766 \text{ lbm/ft}^3)(230 \text{ ft}^3\text{/min})}{(60 \text{ s/min})} = 0.294 \text{ lbm/s}$$

The rotor exit blade speed, U_2, is

$$U_2 = r_2\omega = \frac{(6 \text{ in.})(1725 \text{ rpm})(2\pi \text{ rad/rev})}{(12 \text{ in./ft})(60 \text{ s/min})} = 90.3 \text{ ft/s} \quad (3)$$

To determine the fluid tangential speed at the fan rotor exit, $V_{\theta 2}$, we use Eq. 5.43 to get

$$\mathbf{V}_2 = \mathbf{W}_2 + \mathbf{U}_2 \quad (4)$$

The vector addition of Eq. 4 is shown in the form of a "velocity triangle" in Fig. E5.19*b*. From Fig. E5.19*b*, we can see that

$$V_{\theta 2} = U_2 - W_2 \cos 30° \quad (5)$$

To solve Eq. 5 for $V_{\theta 2}$ we need a value of W_2, in addition to the value of U_2 already determined (Eq. 3). To get W_2, we recognize that

$$W_2 \sin 30° = V_{r2} \quad (6)$$

where V_{r2} is the radial component of either $\mathbf{W}_2$ or $\mathbf{V}_2$. Also, using Eq. 5.6, we obtain

$$\dot{m} = \rho A_2 V_{r2} \quad (7)$$

or since

$$A_2 = 2\,\pi r_2 h \quad (8)$$

where h is the blade height, Eqs. 7 and 8 combine to form

$$\dot{m} = \rho 2\pi r_2 h V_{r2} \quad (9)$$

Taking Eqs. 6 and 9 together we get

$$W_2 = \frac{\dot{m}}{\rho 2\pi r_2 h \sin 30°} = \frac{\rho Q}{\rho 2\pi r_2 h \sin 30°} = \frac{Q}{2\pi r_2 h \sin 30°} \quad (10)$$

Substituting known values into Eq. 10, we obtain

$$W_2 = \frac{(230 \text{ ft}^3\text{/min})(12 \text{ in./ft})(12 \text{ in./ft})}{(60 \text{ s/min})2\pi(6 \text{ in.})(1 \text{ in.}) \sin 30°} = 29.3 \text{ ft/s}$$

By using this value of W_2 in Eq. 5 we get

$$V_{\theta 2} = U_2 - W_2 \cos 30° = 90.3 \text{ ft/s} - (29.3 \text{ ft/s})(0.866) = 64.9 \text{ ft/s}$$

Equation 1 can now be used to obtain

$$\dot{W}_{\text{shaft}} = \dot{m}\, U_2 V_{\theta 2} = \frac{(0.00912 \text{ slug/s})(90.3 \text{ ft/s})(64.9 \text{ ft/s})}{[1 (\text{slug} \cdot \text{ft/s}^2)\text{/lb}][550 (\text{ft} \cdot \text{lb})/(\text{hp} \cdot \text{s})]}$$

with BG units.

With EE units

$$\dot{W}_{\text{shaft}} = \frac{(0.294 \text{ lbm/s})(90.3 \text{ ft/s})(64.9 \text{ ft/s})}{[32.174 (\text{lbm} \cdot \text{ft})/(\text{lb/s}^2)][550 (\text{ft} \cdot \text{lb})/(\text{hp} \cdot \text{s})]}$$

In either case

$$\dot{W}_{\text{shaft}} = 0.097 \text{ hp} \quad \textbf{(Ans)}$$

COMMENT Note that the "+" was used with the $U_2V_{\theta 2}$ product because U_2 and $V_{\theta 2}$ are in the same direction. This result, 0.097 hp, is the power that needs to be delivered through the fan shaft for the given conditions. Ideally, all of this power would go into the flowing air. However, because of fluid friction, only some of this power will produce useful effects (e.g., movement and pressure rise) on the air. How much useful effect depends on the efficiency of the energy transfer between the fan blades and the fluid.

5.3 First Law of Thermodynamics—The Energy Equation

5.3.1 Derivation of the Energy Equation

The ***first law of thermodynamics*** for a system is, in words

time rate of increase of the total stored energy of the system	=	net time rate of energy addition by heat transfer into the system	+	net time rate of energy addition by work transfer into the system

In symbolic form, this statement is

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = \left(\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}}\right)_{\text{sys}} + \left(\sum \dot{W}_{\text{in}} - \sum \dot{W}_{\text{out}}\right)_{\text{sys}}$$

The first law of thermodynamics is a statement of conservation of energy.

or

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = (\dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{in}}{\text{net}}})_{\text{sys}} \tag{5.55}$$

Some of these variables deserve a brief explanation before proceeding further. The total stored energy per unit mass for each particle in the system, e, is related to the internal energy per unit mass, $\check{u}$, the kinetic energy per unit mass, $V^2/2$, and the potential energy per unit mass, gz, by the equation

$$e = \check{u} + \frac{V^2}{2} + gz \tag{5.56}$$

The net ***rate of heat transfer*** into the system is denoted with $\dot{Q}_{\text{net in}}$, and the net rate of work transfer into the system is labeled $\dot{W}_{\text{net in}}$. Heat transfer and work transfer are considered "+" going into the system and "−" coming out.

Equation 5.55 is valid for inertial and noninertial reference systems. We proceed to develop the control volume statement of the first law of thermodynamics. For the control volume that is coincident with the system at an instant of time

$$(\dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{in}}{\text{net}}})_{\text{sys}} = (\dot{Q}_{\underset{\text{in}}{\text{net}}} + \dot{W}_{\underset{\text{in}}{\text{net}}})_{\substack{\text{coincident}\\ \text{control volume}}} \tag{5.57}$$

Furthermore, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19 with the parameter b set equal to e) allows us to conclude that

$$\frac{D}{Dt}\int_{\text{sys}} e\rho \, d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\forall + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}} \, dA \tag{5.58}$$

or in words,

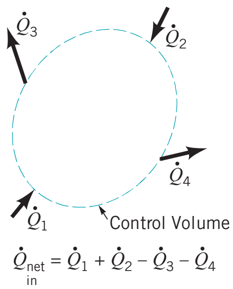
the time rate of increase of the total stored energy of the system	=	the time rate of increase of the total stored energy of the contents of the control volume	+	the net rate of flow of the total stored energy out of the control volume through the control surface

Combining Eqs. 5.55, 5.57, and 5.58 we get the control volume formula for the first law of thermodynamics:

The energy equation involves stored energy and heat and work transfer.

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\forall + \int_{\text{cs}} e\rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA = (\dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{net}\\ \text{in}}})_{\text{cv}}} \tag{5.59}$$

The total stored energy per unit mass, e, in Eq. 5.59 is for fluid particles entering, leaving, and within the control volume. Further explanation of the heat transfer and work transfer involved in this equation follows.



The heat transfer rate, $\dot{Q}$, represents all of the ways in which energy is exchanged between the control volume contents and surroundings because of a temperature difference. Thus, radiation, conduction, and/or convection are possible. As shown by the figure in the margin, heat transfer into the control volume is considered positive, heat transfer out is negative. In many engineering applications, the process is *adiabatic*; the heat transfer rate, $\dot{Q}$, is zero. The net heat transfer rate, $\dot{Q}_{\text{net in}}$, can also be zero when $\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} = 0$.

The work transfer rate, $\dot{W}$, also called *power*, is positive when work is done on the contents of the control volume by the surroundings. Otherwise, it is considered negative. Work can be transferred across the control surface in several ways. In the following paragraphs, we consider some important forms of work transfer.

In many instances, work is transferred across the control surface by a moving shaft. In rotary devices such as turbines, fans, and propellers, a rotating shaft transfers work across that portion of the control surface that slices through the shaft. Even in reciprocating machines like positive displacement internal combustion engines and compressors that utilize piston-in-cylinder arrangements, a rotating crankshaft is used. Since work is the dot product of force and related displacement, rate of work (or power) is the dot product of force and related displacement per unit time. For a rotating shaft, the power transfer, $\dot{W}_{\text{shaft}}$, is related to the shaft torque that causes the rotation, T_{shaft}, and the angular velocity of the shaft, ω, by the relationship

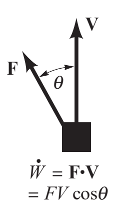
$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\omega$$

When the control surface cuts through the shaft material, the shaft torque is exerted by shaft material at the control surface. To allow for consideration of problems involving more than one shaft we use the notation

$$\dot{W}_{\substack{\text{shaft}\\ \text{net in}}} = \sum \dot{W}_{\substack{\text{shaft}\\ \text{in}}} - \sum \dot{W}_{\substack{\text{shaft}\\ \text{out}}} \tag{5.60}$$

Work transfer can also occur at the control surface when a force associated with fluid normal stress acts over a distance. Consider the simple pipe flow illustrated in Fig. 5.6 and the control volume shown. For this situation, the fluid normal stress, σ, is simply equal to the negative of fluid pressure, p, in all directions; that is,

$$\sigma = -p \tag{5.61}$$



This relationship can be used with varying amounts of approximation for many engineering problems (see Chapter 6).

The power transfer, $\dot{W}$, associated with a force $\mathbf{F}$ acting on an object moving with velocity $\mathbf{V}$ is given by the dot product $\mathbf{F}\cdot\mathbf{V}$. This is illustrated by the figure in the margin. Hence, the power transfer associated with normal stresses acting on a single fluid particle, $\delta\dot{W}_{\text{normal stress}}$, can be evaluated as the dot product of the normal stress force, $\delta\mathbf{F}_{\text{normal stress}}$, and the fluid particle velocity, $\mathbf{V}$, as

$$\delta\dot{W}_{\text{normal stress}} = \delta\mathbf{F}_{\text{normal stress}}\cdot\mathbf{V}$$

If the normal stress force is expressed as the product of local normal stress, $\sigma = -p$, and fluid particle surface area, $\hat{\mathbf{n}}\,\delta A$, the result is

$$\delta\dot{W}_{\text{normal stress}} = \sigma\hat{\mathbf{n}}\,\delta A\cdot\mathbf{V} = -p\hat{\mathbf{n}}\,\delta A\cdot\mathbf{V} = -p\mathbf{V}\cdot\hat{\mathbf{n}}\,\delta A$$

For all fluid particles on the control surface of Fig. 5.6 at the instant considered, power transfer due to fluid normal stress, $\dot{W}_{\text{normal stress}}$, is

$$\dot{W}_{\substack{\text{normal}\\ \text{stress}}} = \int_{\text{cs}} \sigma\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \int_{\text{cs}} -p\mathbf{V}\cdot\hat{\mathbf{n}}\, dA \tag{5.62}$$

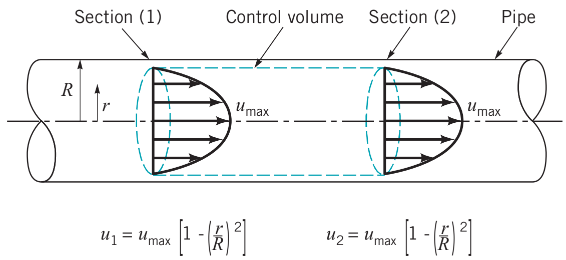


FIGURE 5.6 Simple, fully developed pipe flow.

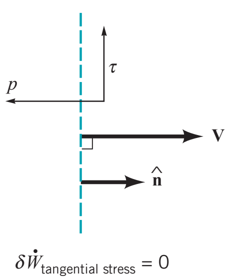
Note that the value of $\dot{W}_{\text{normal stress}}$ for particles on the wetted inside surface of the pipe is zero because $\mathbf{V} \cdot \hat{\mathbf{n}}$ is zero there. Thus, $\dot{W}_{\text{normal stress}}$ can be nonzero only where fluid enters and leaves the control volume. Although only a simple pipe flow was considered, Eq. 5.62 is quite general and the control volume used in this example can serve as a general model for other cases.

Work is transferred by rotating shafts, normal stresses, and tangential stresses.

Work transfer can also occur at the control surface because of tangential stress forces. Rotating shaft work is transferred by tangential stresses in the shaft material. For a fluid particle, shear stress force power, $\delta\dot{W}_{\text{tangential stress}}$, can be evaluated as the dot product of tangential stress force, $\delta\mathbf{F}_{\text{tangential stress}}$, and the fluid particle velocity, $\mathbf{V}$. That is,

$$\delta\dot{W}_{\text{tangential stress}} = \delta\mathbf{F}_{\text{tangential stress}} \cdot \mathbf{V}$$

For the control volume of Fig. 5.6, the fluid particle velocity is zero everywhere on the wetted inside surface of the pipe. Thus, no tangential stress work is transferred across that portion of the control surface. Furthermore, if we select the control surface so that it is perpendicular to the fluid particle velocity, then the tangential stress force is also perpendicular to the velocity. Therefore, the tangential stress work transfer is zero on that part of the control surface. This is illustrated in the figure in the margin. Thus, in general, we select control volumes like the one of Fig. 5.6 and consider fluid tangential stress power transfer to be negligibly small.



Using the information we have developed about power, we can express the first law of thermodynamics for the contents of a control volume by combining Eqs. 5.59, 5.60, and 5.62 to obtain

$$\frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\Psi + \int_{\text{cs}} e\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} - \int_{\text{cs}} p\mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{5.63}$$

When the equation for total stored energy (Eq. 5.56) is considered with Eq. 5.63, we obtain the ***energy equation***:

$$\boxed{\frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\Psi + \int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}} \tag{5.64}$$

5.3.2 Application of the Energy Equation

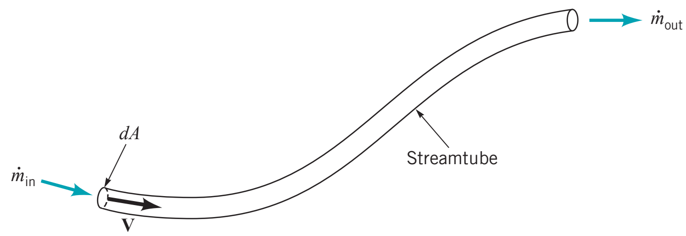
In Eq. 5.64, the term $\partial/\partial t \int_{\text{cv}} e\rho \, d\Psi$ represents the time rate of change of the total stored energy, e, of the contents of the control volume. This term is zero when the flow is steady. This term is also zero in the mean when the flow is steady in the mean (cyclical).

In Eq. 5.64, the integrand of

$$\int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

can be nonzero only where fluid crosses the control surface ($\mathbf{V} \cdot \hat{\mathbf{n}} \neq 0$). Otherwise, $\mathbf{V} \cdot \hat{\mathbf{n}}$ is zero and the integrand is zero for that portion of the control surface. If the properties within parentheses, $\check{u}$, p/ρ, $V^2/2$, and gz, are all assumed to be uniformly distributed over the flow cross-sectional areas involved, the integration becomes simple and gives

$$\int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum_{\substack{\text{flow}\\ \text{out}}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m} - \sum_{\substack{\text{flow}\\ \text{in}}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\dot{m} \tag{5.65}$$



■ FIGURE 5.7
Streamtube flow.

Furthermore, if there is only one stream entering and leaving the control volume, then

$$\int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA =$$

$$\left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{out}} \dot{m}_{\text{out}} - \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{in}} \dot{m}_{\text{in}} \tag{5.66}$$

Uniform flow as described above will occur in an infinitesimally small diameter streamtube as illustrated in Fig. 5.7. This kind of streamtube flow is representative of the steady flow of a particle of fluid along a pathline. We can also idealize actual conditions by disregarding nonuniformities in a finite cross section of flow. We call this one-dimensional flow and although such uniform flow rarely occurs in reality, the simplicity achieved with the one-dimensional approximation often justifies its use. More details about the effects of nonuniform distributions of velocities and other fluid flow variables are considered in Section 5.3.4 and in Chapters 8, 9, and 10.

If shaft work is involved, the flow must be unsteady, at least locally (see Refs. 1 and 2). The flow in any fluid machine that involves shaft work is unsteady within that machine. For example, the velocity and pressure at a fixed location near the rotating blades of a fan are unsteady. However, upstream and downstream of the machine, the flow may be steady. Most often shaft work is associated with flow that is unsteady in a recurring or cyclical way. On a time-average basis for flow that is one-dimensional, cyclical, and involves only one stream of fluid entering and leaving the control volume, Eq. 5.64 can be simplified with the help of Eqs. 5.9 and 5.66 to form

$$\boxed{\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \left(\frac{p}{\rho}\right)_{\text{out}} - \left(\frac{p}{\rho}\right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}} \tag{5.67}$$

We call Eq. 5.67 the *one-dimensional energy equation for steady-in-the-mean flow*. Note that Eq. 5.67 is valid for incompressible and compressible flows. Often, the fluid property called *enthalpy*, $\check{h}$, where

$$\check{h} = \check{u} + \frac{p}{\rho} \tag{5.68}$$

The energy equation is sometimes written in terms of enthalpy.

is used in Eq. 5.67. With enthalpy, the one-dimensional energy equation for steady-in-the-mean flow (Eq. 5.67) is

$$\boxed{\dot{m}\left[\check{h}_{\text{out}} - \check{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}} \tag{5.69}$$

Equation 5.69 is often used for solving compressible flow problems. Examples 5.20 and 5.21 illustrate how Eqs. 5.67 and 5.69 can be used.

EXAMPLE 5.20 Energy—Pump Power

GIVEN A pump delivers water at a steady rate of 300 gal/min as shown in Fig. E5.20. Just upstream of the pump [section (1)] where the pipe diameter is 3.5 in., the pressure is 18 psi. Just downstream of the pump [section (2)] where the pipe diameter is 1 in., the pressure is 60 psi. The change in water elevation across the pump is zero. The rise in internal energy of water, $\check{u}_2 - \check{u}_1$, associated with a temperature rise across the pump is 93 ft · lb/lbm. The pumping process is considered to be adiabatic.

FIND Determine the power (hp) required by the pump.

SOLUTION

We include in our control volume the water contained in the pump between its entrance and exit sections. Application of Eq. 5.67 to the contents of this control volume on a time-average basis yields

$$\dot{m}\left[\check{u}_2 - \check{u}_1 + \left(\frac{p}{\rho}\right)_2 - \left(\frac{p}{\rho}\right)_1 + \frac{V_2^2 - V_1^2}{2} + \underbrace{g(z_2 - z_1)}_{0\text{ (no elevation change)}}\right]$$
$$= \underbrace{\dot{Q}_{\substack{\text{net}\\\text{in}}}}_{0\text{ (adiabatic flow)}} + \dot{W}_{\substack{\text{shaft}\\\text{net in}}} \qquad \textbf{(1)}$$

We can solve directly for the power required by the pump, $\dot{W}_{\text{shaft net in}}$, from Eq. 1, after we first determine the mass flowrate, $\dot{m}$, the speed of flow into the pump, V_1, and the speed of the flow out of the pump, V_2. All other quantities in Eq. 1 are given in the problem statement. From Eq. 5.6, we get

$$\dot{m} = \rho Q = \frac{(1.94 \text{ slugs/ft}^3)(300 \text{ gal/min})(32.174 \text{ lbm/slug})}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})}$$
$$= 41.8 \text{ lbm/s} \qquad \textbf{(2)}$$

Also from Eq. 5.6,

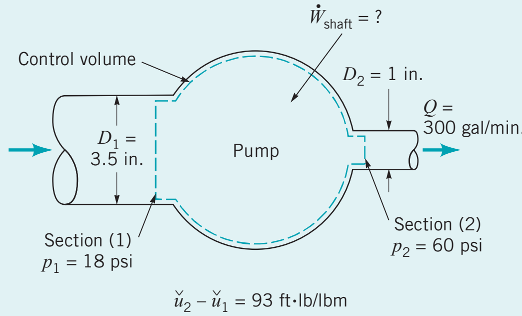
$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4}$$

so

$$V_1 = \frac{Q}{A_1} = \frac{(300 \text{ gal/min})4(12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi(3.5 \text{ in.})^2}$$
$$= 10.0 \text{ ft/s} \qquad \textbf{(3)}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{(300 \text{ gal/min})4\,(12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi(1 \text{ in.})^2}$$
$$= 123 \text{ ft/s} \qquad \textbf{(4)}$$



■ FIGURE E5.20

Substituting the values of Eqs. 2, 3, and 4 and values from the problem statement into Eq. 1 we obtain

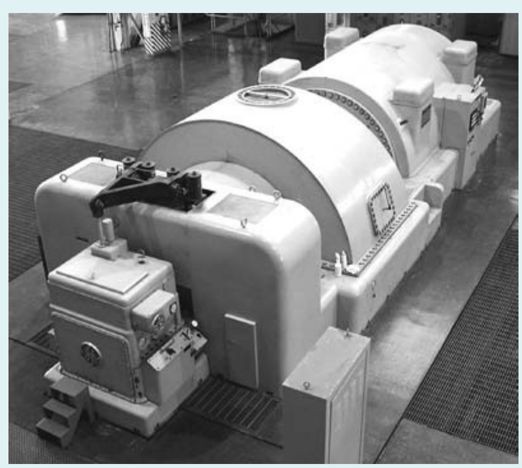
$$\dot{W}_{\substack{\text{shaft}\\\text{net in}}} = (41.8 \text{ lbm/s})\Bigg[(93 \text{ ft}\cdot\text{lb/lbm}) + \frac{(60 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)(32.174 \text{ lbm/slug})} - \frac{(18 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)(32.174 \text{ lbm/slug})} + \frac{(123 \text{ ft/s})^2 - (10.0 \text{ ft/s})^2}{2[32.174\,(\text{lbm}\cdot\text{ft})/(\text{lb}\cdot\text{s}^2)]}\Bigg]$$
$$\times \frac{1}{[550(\text{ft}\cdot\text{lb/s})/\text{hp}]} = 32.2 \text{ hp} \qquad \textbf{(Ans)}$$

COMMENT Of the total 32.2 hp, internal energy change accounts for 7.09 hp, the pressure rise accounts for 7.37 hp, and the kinetic energy increase accounts for 17.8 hp.

EXAMPLE 5.21 Energy—Turbine Power per Unit Mass of Flow

GIVEN A steam turbine generator unit used to produce electricity is shown in Fig. E5.21*a*. Assume the steam enters a turbine with a velocity of 30 m/s and enthalpy, $\check{h}_1$, of 3348 kJ/kg (see Fig. E5.21*b*). The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. The flow through the turbine is adiabatic, and changes in elevation are negligible.

FIND Determine the work output involved per unit mass of steam through-flow.

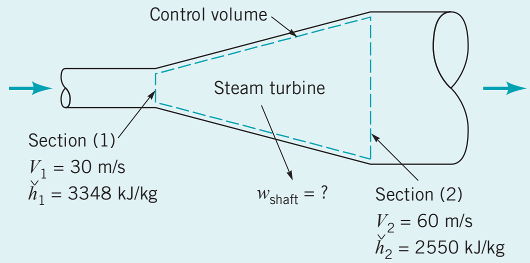


■ FIGURE E5.21*a*

Solution

We use a control volume that includes the steam in the turbine from the entrance to the exit as shown in Fig. E5.21*b*. Applying Eq. 5.69 to the steam in this control volume we get

$$\dot{m}\left[\check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} + \underbrace{g(z_2 - z_1)}_{0\text{ (elevation change is negligible)}}\right] = \underbrace{\dot{Q}_{\substack{\text{net}\\ \text{in}}}}_{0\text{ (adiabatic flow)}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} \qquad \textbf{(1)}$$



■ FIGURE E5.21*b*

The work output per unit mass of steam through-flow, $w_{\text{shaft net in}}$, can be obtained by dividing Eq. 1 by the mass flow rate, $\dot{m}$, to obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\dot{m}} = \check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} \qquad \textbf{(2)}$$

Since $w_{\text{shaft net out}} = -w_{\text{shaft net in}}$, we obtain

$$w_{\substack{\text{shaft}\\ \text{net out}}} = \check{h}_1 - \check{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

or

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348\text{ kJ/kg} - 2550\text{ kJ/kg} + \frac{[(30\text{ m/s})^2 - (60\text{ m/s})^2][1\text{ J/(N·m)}]}{2[1\text{ (kg·m)/(N·s}^2)](1000\text{ J/kJ})}$$

Thus,

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348\text{ kJ/kg} - 2550\text{ kJ/kg} - 1.35\text{ kJ/kg}$$

$$= 797\text{ kJ/kg} \qquad \textbf{(Ans)}$$

COMMENT Note that in this particular example, the change in kinetic energy is small in comparison to the difference in enthalpy involved. This is often true in applications involving steam turbines. To determine the power output, $\dot{W}_{\text{shaft}}$, we must know the mass flowrate, $\dot{m}$.

If the flow is steady throughout, one-dimensional, and only one fluid stream is involved, then the shaft work is zero and the energy equation is

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \left(\frac{p}{\rho}\right)_{\text{out}} - \left(\frac{p}{\rho}\right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} \qquad \textbf{(5.70)}$$

V5.12 Pelton wheel turbine

We call Eq. 5.70 the *one-dimensional*, *steady flow energy equation*. This equation is valid for incompressible and compressible flows. For compressible flows, enthalpy is most often used in the one-dimensional, steady flow energy equation and, thus, we have

$$\dot{m}\left[\check{h}_{\text{out}} - \check{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} \qquad \textbf{(5.71)}$$

An example of the application of Eq. 5.70 follows.

EXAMPLE 5.22 Energy—Temperature Change

GIVEN The 420-ft waterfall shown in Fig. E5.22*a* involves steady flow from one large body of water to another.

FIND Determine the temperature change associated with this flow.

Solution

To solve this problem we consider a control volume consisting of a small cross-sectional streamtube from the nearly motionless surface of the upper body of water to the nearly motionless surface of the lower body of water as is sketched in Fig. E5.22*b*. We need to determine $T_2 - T_1$. This temperature change is related to the change of internal energy of the water, $\check{u}_2 - \check{u}_1$, by the relationship

$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{\check{c}} \qquad \textbf{(1)}$$

■ **FIGURE E5.22a**
[Photograph of Akaka Falls (Hawaii) courtesy of Scott and Margaret Jones.]

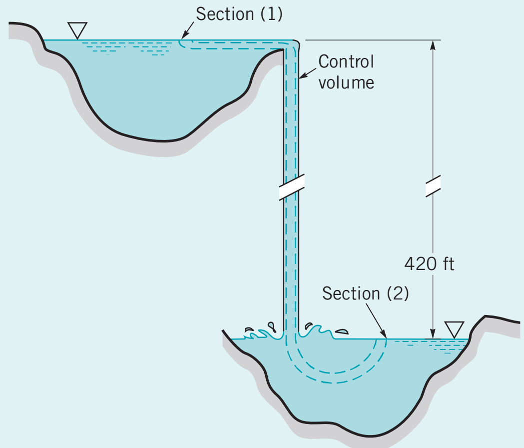
where $\check{c} = 1$ Btu/(lbm · °R) is the specific heat of water. The application of Eq. 5.70 to the contents of this control volume leads to

$$\dot{m}\left[\check{u}_2 + \check{u}_1 + \left(\frac{p}{\rho}\right)_2 - \left(\frac{p}{\rho}\right)_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$= \dot{Q}_{\underset{\text{in}}{\text{net}}} \tag{2}$$

We assume that the flow is adiabatic. Thus $\dot{Q}_{\text{net in}} = 0$. Also,

$$\left(\frac{p}{\rho}\right)_1 = \left(\frac{p}{\rho}\right)_2 \tag{3}$$



■ **FIGURE E5.22b**

because the flow is incompressible and atmospheric pressure prevails at sections (1) and (2). Furthermore,

$$V_1 = V_2 = 0 \tag{4}$$

because the surface of each large body of water is considered motionless. Thus, Eqs. 1 through 4 combine to yield

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{\check{c}}$$

so that with

$$\check{c} = [1 \text{ Btu/(lbm} \cdot {}^\circ\text{R)}]\,(778 \text{ ft} \cdot \text{lb/Btu})$$
$$= [778 \text{ ft} \cdot \text{lb/(lbm} \cdot {}^\circ\text{R)}]$$

$$T_2 - T_1 = \frac{(32.2 \text{ ft/s}^2)(420 \text{ ft})}{[778 \text{ ft} \cdot \text{lb/(lbm} \cdot {}^\circ\text{R)}][32.2 \text{ (lbm} \cdot \text{ft)/(lb} \cdot \text{s}^2)]}$$
$$= 0.540\ {}^\circ\text{R} \qquad \textbf{(Ans)}$$

COMMENT Note that it takes a considerable change of potential energy to produce even a small increase in temperature.

A form of the energy equation that is most often used to solve incompressible flow problems is developed in the next section.

5.3.3 Comparison of the Energy Equation with the Bernoulli Equation

When the one-dimensional energy equation for steady-in-the-mean flow, Eq. 5.67, is applied to a flow that is steady, Eq. 5.67 becomes the one-dimensional, steady-flow energy equation, Eq. 5.70. The only difference between Eq. 5.67 and Eq. 5.70 is that shaft power, $\dot{W}_{\text{shaft net in}}$, is zero if the flow is steady throughout the control volume (fluid machines involve locally unsteady flow). If in addition to being steady, the flow is incompressible, we get from Eq. 5.70

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\underset{\text{in}}{\text{net}}} \tag{5.72}$$

Dividing Eq. 5.72 by the mass flowrate, $\dot{m}$, and rearranging terms we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\underset{\text{in}}{\text{net}}}) \tag{5.73}$$

where

$$q_{\underset{\text{in}}{\text{net}}} = \frac{\dot{Q}_{\text{net in}}}{\dot{m}}$$

is the heat transfer rate per mass flowrate, or heat transfer per unit mass. Note that Eq. 5.73 involves energy per unit mass and is applicable to one-dimensional flow of a single stream of fluid between two sections or flow along a streamline between two sections.

If the steady, incompressible flow we are considering also involves negligible viscous effects (frictionless flow), then the Bernoulli equation, Eq. 3.7, can be used to describe what happens between two sections in the flow as

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} \tag{5.74}$$

where $\gamma = \rho g$ is the specific weight of the fluid. To get Eq. 5.74 in terms of energy per unit mass, so that it can be compared directly with Eq. 5.73, we divide Eq. 5.74 by density, ρ, and obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} \tag{5.75}$$

A comparison of Eqs. 5.73 and 5.75 prompts us to conclude that

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\underset{\text{in}}{\text{net}}} = 0 \tag{5.76}$$

when the steady incompressible flow is frictionless. For steady incompressible flow with friction, we learn from experience (second law of thermodynamics) that

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\underset{\text{in}}{\text{net}}} > 0 \tag{5.77}$$

In Eqs. 5.73 and 5.75, we can consider the combination of variables

$$\frac{p}{\rho} + \frac{V^2}{2} + gz$$

Minimizing loss is the central goal of fluid mechanical design.

as equal to *useful* or *available energy*. Thus, from inspection of Eqs. 5.73 and 5.75, we can conclude that $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}$ represents the ***loss*** of useful or available energy that occurs in an incompressible fluid flow because of friction. In equation form we have

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\underset{\text{in}}{\text{net}}} = \text{loss} \tag{5.78}$$

For a frictionless flow, Eqs. 5.73 and 5.75 tell us that loss equals zero.

It is often convenient to express Eq. 5.73 in terms of loss as

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - \text{loss} \tag{5.79}$$

An example of the application of Eq. 5.79 follows.

EXAMPLE 5.23 Energy—Effect of Loss of Available Energy

GIVEN As shown in Fig. E5.23*a*, air flows from a room through two different vent configurations: a cylindrical hole in the wall having a diameter of 120 mm and the same diameter cylindrical hole in the wall but with a well-rounded entrance. The room pressure is held constant at 1.0 kPa above atmospheric pressure. Both vents exhaust into the atmosphere. As discussed in Section 8.4.2, the loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is $0.5V_2^2/2$ where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.

FIND Compare the volume flowrates associated with the two different vent configurations.

SOLUTION

We use the control volume for each vent sketched in Fig. E5.23*a*. What is sought is the flowrate, $Q = A_2V_2$, where A_2 is the vent exit cross-sectional area, and V_2 is the uniformly distributed exit velocity. For both vents, application of Eq. 5.79 leads to

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \underbrace{\cancel{gz_2} = \frac{p_1}{\rho} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}}_{0\ (\text{no elevation change})} - {}_1\text{loss}_2 \qquad 0\,(V_1 \approx 0) \tag{1}$$

where ${}_1\text{loss}_2$ is the loss between sections (1) and (2). Solving Eq. 1 for V_2 we get

$$V_2 = \sqrt{2\left[\left(\frac{p_1 - p_2}{\rho}\right) - {}_1\text{loss}_2\right]} \tag{2}$$

Since

$${}_1\text{loss}_2 = K_L \frac{V_2^2}{2} \tag{3}$$

where K_L is the loss coefficient ($K_L = 0.5$ and 0.05 for the two vent configurations involved), we can combine Eqs. 2 and 3 to get

$$V_2 = \sqrt{2\left[\left(\frac{p_1 - p_2}{\rho}\right) - K_L \frac{V_2^2}{2}\right]} \tag{4}$$

Solving Eq. 4 for V_2 we obtain

$$V_2 = \sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \tag{5}$$

Therefore, for flowrate, Q, we obtain

$$Q = A_2V_2 = \frac{\pi D_2^2}{4}\sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \tag{6}$$

For the rounded entrance cylindrical vent, Eq. 6 gives

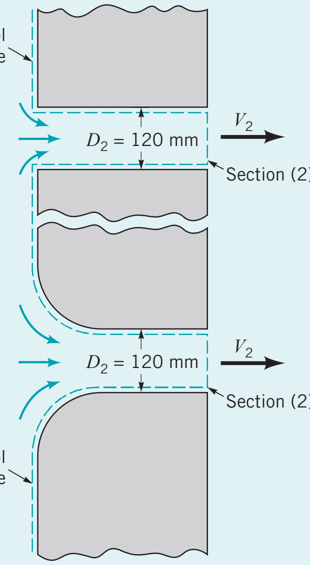
$$Q = \frac{\pi(120\text{ mm})^2}{4(1000\text{ mm/m})^2} \times \sqrt{\frac{(1.0\text{ kPa})(1000\text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23\text{ kg/m}^3)[(1 + 0.05)/2][1(\text{N}\cdot\text{s}^2)/(\text{kg}\cdot\text{m})]}}$$

or

$$Q = 0.445\text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

For the cylindrical vent, Eq. 6 gives us

$$Q = \frac{\pi(120\text{ mm})^2}{4(1000\text{ mm/m})^2} \times \sqrt{\frac{(1.0\text{ kPa})(1000\text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23\text{ kg/m}^3)[(1 + 0.5)/2][1(\text{N}\cdot\text{s}^2)/(\text{kg}\cdot\text{m})]}}$$

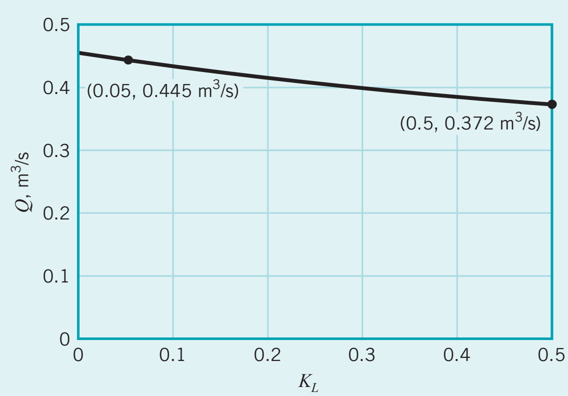


■ FIGURE E5.23*a*

or

$$Q = 0.372\text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

COMMENT By repeating the calculations for various values of the loss coefficient, K_L, the results shown in Fig. E5.23*b* are obtained. Note that the rounded entrance vent allows the passage of more air than does the cylindrical vent because the loss associated with the rounded entrance vent is less than that for the cylindrical one. For this flow the pressure drop, $p_1 - p_2$, has two purposes: (1) overcome the loss associated with the flow, and (2) produce the kinetic energy at the exit. Even if there were no loss (i.e., $K_L = 0$), a pressure drop would be needed to accelerate the fluid through the vent.



■ FIGURE E5.23*b*

An important group of fluid mechanics problems involves one-dimensional, incompressible, steady-in-the-mean flow with friction and shaft work. Included in this category are constant density flows through pumps, blowers, fans, and turbines. For this kind of flow, Eq. 5.67 becomes

$$\dot{m}\left[\check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{shaft}\\\text{net in}}} \tag{5.80}$$

Dividing Eq. 5.80 by mass flowrate and using the work per unit mass, $w_{\substack{\text{shaft}\\\text{net in}}} = \dot{W}_{\substack{\text{shaft}\\\text{net in}}}/\dot{m}$, we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\\text{net in}}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\\text{in}}}) \qquad \textbf{(5.81)}$$

The mechanical energy equation can be written in terms of energy per unit mass.

If the flow is steady throughout, Eq. 5.81 becomes identical to Eq. 5.73, and the previous observation that $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}$ equals the loss of available energy is valid. Thus, we conclude that Eq. 5.81 can be expressed as

$$\boxed{\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\\text{net in}}} - \text{loss}} \qquad \textbf{(5.82)}$$

V5.13 Energy transfer

This is a form of the energy equation for steady-in-the-mean flow that is often used for incompressible flow problems. It is sometimes called the *mechanical energy* equation or the *extended Bernoulli* equation. Note that Eq. 5.82 involves energy per unit mass (ft · lb/slug = ft^2/s^2 or N · m = m^2/s^2).

According to Eq. 5.82, when the shaft work is into the control volume, as for example with a pump, a larger amount of loss will result in more shaft work being required for the same rise in available energy. Similarly, when the shaft work is out of the control volume (for example, a turbine), a larger loss will result in less shaft work out for the same drop in available energy. Designers spend a great deal of effort on minimizing losses in fluid flow components. The following examples demonstrate why losses should be kept as small as possible in fluid systems.

EXAMPLE 5.24 Energy—Fan Work and Efficiency

GIVEN An axial-flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow upstream of the fan involves negligible speed.

FIND Determine how much of the work to the air actually produces useful effects, that is, fluid motion and a rise in available energy. Estimate the fluid mechanical efficiency of this fan.

SOLUTION

We select a fixed and nondeforming control volume as is illustrated in Fig. E5.24. The application of Eq. 5.82 to the contents of this control volume leads to

0 (atmospheric pressures cancel) 0 ($V_1 \approx 0$)

$$w_{\substack{\text{shaft}\\\text{net in}}} - \text{loss} = \left(\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + \cancel{gz_2}\right) - \left(\cancel{\frac{p_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) \qquad \textbf{(1)}$$

0 (no elevation change)

where $w_{\text{shaft net in}} - \text{loss}$ is the amount of work added to the air that produces a useful effect. Equation 1 leads to

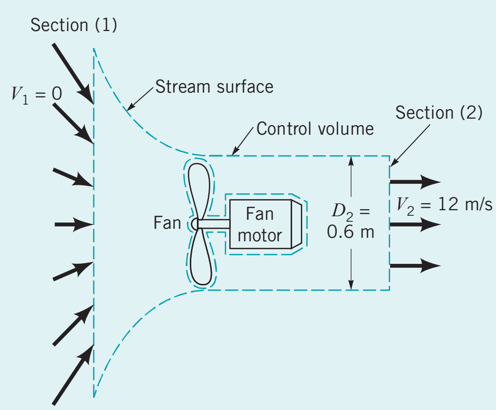
$$w_{\substack{\text{shaft}\\\text{net in}}} - \text{loss} = \frac{V_2^2}{2} = \frac{(12\text{ m/s})^2}{2[1(\text{kg·m})/(\text{N·s}^2)]}$$
$$= 72.0\text{ N·m/kg} \qquad \textbf{(2)} \quad \textbf{(Ans)}$$

A reasonable estimate of *efficiency,* η, would be the ratio of amount of work that produces a useful effect, Eq. 2, to the amount of work delivered to the fan blades. That is

$$\eta = \frac{w_{\substack{\text{shaft}\\\text{net in}}} - \text{loss}}{w_{\substack{\text{shaft}\\\text{net in}}}} \qquad \textbf{(3)}$$

To calculate the efficiency, we need a value of $w_{\text{shaft net in}}$, which is related to the power delivered to the blades, $\dot{W}_{\text{shaft net in}}$. We note that

$$w_{\substack{\text{shaft}\\\text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\\text{net in}}}}{\dot{m}} \qquad \textbf{(4)}$$



■ FIGURE E5.24

where the mass flowrate, $\dot{m}$, is (from Eq. 5.6)

$$\dot{m} = \rho A V = \rho \frac{\pi D_2^2}{4} V_2 \tag{5}$$

For fluid density, ρ, we use 1.23 kg/m^3 (standard air) and, thus, from Eqs. 4 and 5 we obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{(\rho \pi D_2^2/4)V_2} = \frac{(0.4 \text{ kW})[1000 \text{ (Nm)/(skW)}]}{(1.23 \text{ kg/m}^3)[(\pi)(0.6 \text{ m})^2/4](12 \text{ m/s})}$$

or

$$w_{\substack{\text{shaft}\\ \text{net in}}} = 95.8 \text{ N·m/kg} \tag{6}$$

From Eqs. 2, 3, and 6 we obtain

$$\eta = \frac{72.0 \text{ N·m/kg}}{95.8 \text{ N·m/kg}} = 0.752 \qquad \textbf{(Ans)}$$

COMMENT Note that only 75% of the power that was delivered to the air resulted in useful effects, and, thus, 25% of the shaft power is lost to air friction.

Fluids in the News

Curtain of air An air curtain is produced by blowing air through a long rectangular nozzle to produce a high-velocity sheet of air, or a "curtain of air." This air curtain is typically directed over a doorway or opening as a replacement for a conventional door. The air curtain can be used for such things as keeping warm air from infiltrating dedicated cold spaces, preventing dust and other contaminates from entering a clean environment, and even just keeping insects out of the workplace, still allowing people to enter or exit. A disadvantage over conventional doors is the added *power requirements* to operate the air curtain, although the advantages can outweigh the disadvantage for various industrial applications. New applications for current air curtain designs continue to be developed. For example, the use of air curtains as a means of road tunnel fire security is currently being investigated. In such an application, the air curtain would act to isolate a portion of the tunnel where fire has broken out and not allow smoke and fumes to infiltrate the entire tunnel system. (See Problem 5.123.)

If Eq. 5.82, which involves energy per unit mass, is multiplied by fluid density, ρ, we obtain

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} + \rho w_{\substack{\text{shaft}\\ \text{net in}}} - \rho(\text{loss}) \tag{5.83}$$

V5.14 Water plant aerator

where $\gamma = \rho g$ is the specific weight of the fluid. Equation 5.83 involves *energy per unit volume* and the units involved are identical with those used for pressure (ft · lb/ft^3 = lb/ft^2 or N · m/m^3 = N/m^2).

If Eq. 5.82 is divided by the acceleration of gravity, g, we get

$$\boxed{\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L} \tag{5.84}$$

where

$$h_s = w_{\text{shaft net in}}/g = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\dot{m}g} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\gamma Q} \tag{5.85}$$

The energy equation written in terms of energy per unit weight involves heads.

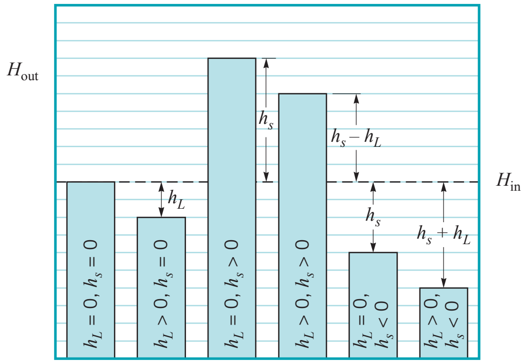
is the ***shaft work head*** and h_L = loss/g is the ***head loss***. Equation 5.84 involves *energy per unit weight* (ft · lb/lb = ft or N · m/N = m). In Section 3.7, we introduced the notion of "head," which is energy per unit weight. Units of length (for example, ft, m) are used to quantify the amount of head involved. If a turbine is in the control volume, h_s is negative because it is associated with shaft work out of the control volume. For a pump in the control volume, h_s is positive because it is associated with shaft work into the control volume.

We can define a total head, H, as follows

$$H = \frac{p}{\gamma} + \frac{V^2}{2g} + z$$

Then Eq. 5.84 can be expressed as

$$H_{\text{out}} = H_{\text{in}} + h_s - h_L$$



■ **FIGURE 5.8 Total-head change in fluid flows.**

Some important possible values of H_{out} in comparison to H_{in} are shown in Fig. 5.8. Note that h_L (head loss) always reduces the value of H_{out}, except in the ideal case when it is zero. Note also that h_L lessens the effect of shaft work that can be extracted from a fluid. When $h_L = 0$ (ideal condition) the shaft work head, h_s, and the change in total head are the same. This head change is sometimes called "ideal head change." The corresponding ideal shaft work head is the minimum required to achieve a desired effect. For work out, it is the maximum possible. Designers usually strive to minimize loss. In Chapter 12 we learn of one instance when minimum loss is sacrificed for survivability of fish coursing through a turbine rotor.

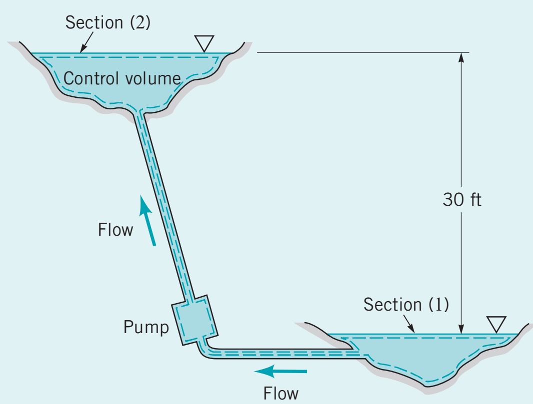
EXAMPLE 5.25 Energy—Head Loss and Power Loss

GIVEN The pump shown in Fig. E5.25*a* adds 10 horsepower to the water as it pumps water from the lower lake to the upper lake. The elevation difference between the lake surfaces is 30 ft and the head loss is 15 ft.

FIND Determine

(a) the flowrate and

(b) the power loss associated with this flow.



■ **FIGURE E5.25*a***

SOLUTION

(a) The energy equation (Eq. 5.84) for this flow is

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L \quad \textbf{(1)}$$

where points 2 and 1 (corresponding to "out" and "in" in Eq. 5.84) are located on the lake surfaces. Thus, $p_2 = p_1 = 0$ and $V_2 = V_1 = 0$ so that Eq. 1 becomes

$$h_s = h_L + z_2 - z_1 \quad \textbf{(2)}$$

where $z_2 = 30$ ft, $z_1 = 0$, and $h_L = 15$ ft. The pump head is obtained from Eq. 5.85 as

$$\begin{aligned} h_s &= \dot{W}_{\text{shaft net in}}/\gamma Q \\ &= (10 \text{ hp})(550 \text{ ft·lb/s/hp})/(62.4 \text{ lb/ft}^3)\, Q \\ &= 88.1/Q \end{aligned}$$

where h_s is in ft when Q is in ft^3/s.

Hence, from Eq. 2,

$$88.1/Q = 15 \text{ ft} + 30 \text{ ft}$$

or

$$Q = 1.96 \text{ ft}^3/\text{s} \quad \textbf{(Ans)}$$

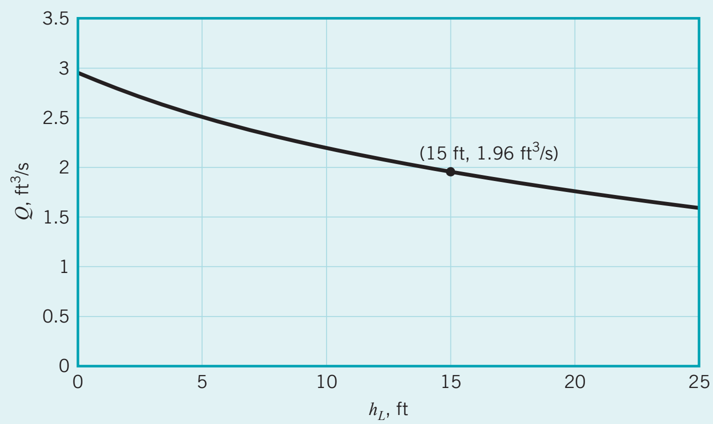
COMMENT Note that in this example the purpose of the pump is to lift the water (a 30-ft head) and overcome the head loss (a 15-ft head); it does not, overall, alter the water's pressure or velocity.

(b) The power lost due to friction can be obtained from Eq. 5.85 as

$$\dot{W}_{\text{loss}} = \gamma Q h_L = (62.4 \text{ lb/ft}^3)(1.96 \text{ ft}^3\text{/s})(15 \text{ ft})$$
$$= 1830 \text{ ft·lb/s } (1 \text{ hp/550 ft·lb/s})$$
$$= 3.33 \text{ hp} \qquad \textbf{(Ans)}$$

COMMENTS The remaining 10 hp − 3.33 hp = 6.67 hp that the pump adds to the water is used to lift the water from the lower to the upper lake. This energy is not "lost," but it is stored as potential energy.

By repeating the calculations for various head losses, h_L, the results shown in Fig. E5.25*b* are obtained. Note that as the head loss increases, the flowrate decreases because an increasing portion of the 10 hp supplied by the pump is lost and, therefore, not available to lift the fluid to the higher elevation.



■ FIGURE E5.25*b*

A comparison of the energy equation and the Bernoulli equation has led to the concept of loss of available energy in incompressible fluid flows with friction. In Chapter 8, we discuss in detail some methods for estimating loss in incompressible flows with friction. In Section 5.4 and Chapter 11, we demonstrate that loss of available energy is also an important factor to consider in compressible flows with friction.

Fluids in the News

Smart shocks Vehicle shock absorbers are dampers used to provide a smooth, controllable ride. When going over a bump, the relative motion between the tires and the vehicle body displaces a piston in the shock and forces a viscous fluid through a small orifice or channel. The viscosity of the fluid produces a *head loss* that dissipates energy to dampen the vertical motion. Current shocks use a fluid with fixed viscosity. However, recent technology has been developed that uses a synthetic oil with millions of tiny iron balls suspended in it. These tiny balls react to a magnetic field generated by an electric coil on the shock piston in a manner that changes the fluid viscosity, going anywhere from essentially no damping to a solid almost instantly. A computer adjusts the current to the coil to select the proper viscosity for the given conditions (i.e., wheel speed, vehicle speed, steering-wheel angle, lateral acceleration, brake application, and temperature). The goal of these adjustments is an optimally tuned shock that keeps the vehicle on a smooth, even keel while maximizing the contact of the tires with the pavement for any road conditions. (See Problem 5.107.)

5.3.4 Application of the Energy Equation to Nonuniform Flows

The forms of the energy equation discussed in Sections 5.3.2 and 5.3.3 are applicable to one-dimensional flows, flows that are approximated with uniform velocity distributions where fluid crosses the control surface.

If the velocity profile at any section where flow crosses the control surface is not uniform, inspection of the energy equation for a control volume, Eq. 5.64, suggests that the integral

$$\int_{cs} \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

The kinetic energy coefficient is used to account for nonuniform flows.

will require special attention. The other terms of Eq. 5.64 can be accounted for as already discussed in Sections 5.3.2 and 5.3.3.

For one stream of fluid entering and leaving the control volume, we can define the relationship

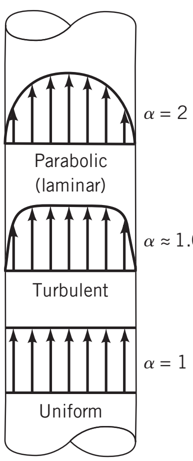
$$\int_{cs} \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \dot{m}\left(\frac{\alpha_{out}\overline{V}^2_{out}}{2} - \frac{\alpha_{in}\overline{V}^2_{in}}{2}\right)$$

where α is the ***kinetic energy coefficient*** and $\overline{V}$ is the average velocity defined earlier in Eq. 5.7. From the above we can conclude that

$$\frac{\dot{m}\alpha\overline{V}^2}{2} = \int_A \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

for flow through surface area A of the control surface. Thus,

$$\alpha = \frac{\displaystyle\int_A (V^2/2)\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA}{\dot{m}\overline{V}^2/2} \tag{5.86}$$



It can be shown that for any velocity profile, $\alpha \geq 1$, with $\alpha = 1$ only for uniform flow. Some typical velocity profile examples for flow in a conventional pipe are shown in the sketch in the margin. Therefore, for nonuniform velocity profiles, the energy equation on an energy per unit mass basis for the incompressible flow of one stream of fluid through a control volume that is steady in the mean is

$$\boxed{\frac{p_{out}}{\rho} + \frac{\alpha_{out}\overline{V}^2_{out}}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in}\overline{V}^2_{in}}{2} + gz_{in} + w_{\substack{shaft\\ net\ in}} - \text{loss}} \tag{5.87}$$

On an energy per unit volume basis we have

$$p_{out} + \frac{\rho\alpha_{out}\overline{V}^2_{out}}{2} + \gamma z_{out} = p_{in} + \frac{\rho\alpha_{in}\overline{V}^2_{in}}{2} + \gamma z_{in} + \rho w_{\substack{shaft\\ net\ in}} - \rho(\text{loss}) \tag{5.88}$$

and on an energy per unit weight or head basis we have

$$\boxed{\frac{p_{out}}{\gamma} + \frac{\alpha_{out}\overline{V}^2_{out}}{2g} + z_{out} = \frac{p_{in}}{\gamma} + \frac{\alpha_{in}\overline{V}^2_{in}}{2g} + z_{in} + \frac{w_{\substack{shaft\\ net\ in}}}{g} - h_L} \tag{5.89}$$

The following examples illustrate the use of the kinetic energy coefficient.

EXAMPLE 5.26 Energy—Effect of Nonuniform Velocity Profile

GIVEN The small fan shown in Fig. E5.26 moves air at a mass flowrate of 0.1 kg/min. Upstream of the fan, the pipe diameter is 60 mm, the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 30 mm, the flow is turbulent, the velocity profile is quite uniform, and the kinetic energy coefficient, α_2, is equal to 1.08. The rise in static pressure across the fan is 0.1 kPa and the fan motor draws 0.14 W.

FIND Compare the value of loss calculated: **(a)** assuming uniform velocity distributions, **(b)** considering actual velocity distributions.

Solution

Application of Eq. 5.87 to the contents of the control volume shown in Fig. E5.26 leads to

$$\frac{p_2}{\rho} + \frac{\alpha_2 \overline{V}_2^2}{2} + \underbrace{\cancel{gz_2} = \frac{p_1}{\rho} + \frac{\alpha_1 \overline{V}_1^2}{2} + \cancel{gz_1}}_{\text{0 (change in } gz \text{ is negligible)}}$$
$$- \text{loss} + w_{\substack{\text{shaft}\\ \text{net in}}} \tag{1}$$

or solving Eq. 1 for loss we get

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \frac{\alpha_1 \overline{V}_1^2}{2} - \frac{\alpha_2 \overline{V}_2^2}{2} \tag{2}$$

To proceed further, we need values of $w_{\text{shaft net in}}$, $\overline{V}_1$, and $\overline{V}_2$. These quantities can be obtained as follows. For shaft work

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\text{power to fan motor}}{\dot{m}}$$

or

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{(0.14 \text{ W})[(1 \text{ N}\cdot\text{m/s})/\text{W}]}{0.1 \text{ kg/min}}(60 \text{ s/min})$$
$$= 84.0 \text{ N}\cdot\text{m/kg} \tag{3}$$

For the average velocity at section (1), $\overline{V}_1$, from Eq. 5.11 we obtain

$$\overline{V}_1 = \frac{\dot{m}}{\rho A_1}$$
$$= \frac{\dot{m}}{\rho(\pi D_1^2/4)} \tag{4}$$
$$= \frac{(0.1 \text{ kg/min})(1 \text{ min}/60 \text{ s})(1000 \text{ mm/m})^2}{(1.23 \text{ kg/m}^3)[\pi(60 \text{ mm})^2/4]}$$
$$= 0.479 \text{ m/s}$$

For the average velocity at section (2), $\overline{V}_2$,

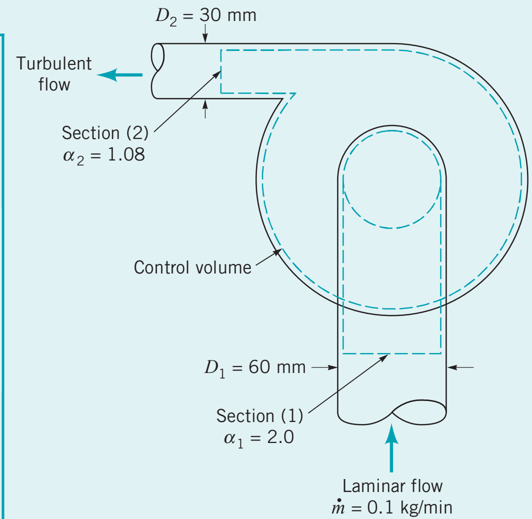
$$\overline{V}_2 = \frac{(0.1 \text{ kg/min})(1 \text{ min}/60 \text{ s})(1000 \text{ mm/m})^2}{(1.23 \text{ kg/m}^3)[\pi(30 \text{ mm})^2/4]}$$
$$= 1.92 \text{ m/s} \tag{5}$$

(a) For the assumed uniform velocity profiles ($\alpha_1 = \alpha_2 = 1.0$), Eq. 2 yields

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \frac{\overline{V}_1^2}{2} - \frac{\overline{V}_2^2}{2} \tag{6}$$

Using Eqs. 3, 4, and 5 and the pressure rise given in the problem statement, Eq. 6 gives

$$\text{loss} = 84.0\,\frac{\text{N}\cdot\text{m}}{\text{kg}} - \frac{(0.1 \text{ kPa})(1000 \text{ Pa/kPa})(1 \text{ N/m}^2/\text{Pa})}{1.23 \text{ kg/m}^3}$$
$$+ \frac{(0.479 \text{ m/s})^2}{2[1 \text{ (kg}\cdot\text{m)/(N}\cdot\text{s}^2)]} - \frac{(1.92 \text{ m/s})^2}{2[1 \text{ (kg}\cdot\text{m)/(N}\cdot\text{s}^2)]}$$



■ FIGURE E5.26

or

$$\text{loss} = 84.0 \text{ N}\cdot\text{m/kg} - 81.3 \text{ N}\cdot\text{m/kg}$$
$$+ 0.115 \text{ N}\cdot\text{m/kg} - 1.84 \text{ N}\cdot\text{m/kg}$$
$$= 0.975 \text{ N}\cdot\text{m/kg} \qquad \textbf{(Ans)}$$

(b) For the actual velocity profiles ($\alpha_1 = 2$, $\alpha_2 = 1.08$), Eq. 1 gives

$$\text{loss} = w_{\substack{\text{shaft}\\ \text{net in}}} - \left(\frac{p_2 - p_1}{\rho}\right) + \alpha_1 \frac{\overline{V}_1^2}{2} - \alpha_2 \frac{\overline{V}_2^2}{2} \tag{7}$$

If we use Eqs. 3, 4, and 5 and the given pressure rise, Eq. 7 yields

$$\text{loss} = 84 \text{ N}\cdot\text{m/kg} - \frac{(0.1 \text{ kPa})(1000 \text{ Pa/kPa})(1 \text{ N/m}^2/\text{Pa})}{1.23 \text{ kg/m}^3}$$
$$+ \frac{2(0.479 \text{ m/s})^2}{2[1 \text{ (kg}\cdot\text{m)/(N}\cdot\text{s}^2)]} - \frac{1.08(1.92 \text{ m/s})^2}{2[1 \text{ (kg}\cdot\text{m)/(N}\cdot\text{s}^2)]}$$

or

$$\text{loss} = 84.0 \text{ N}\cdot\text{m/kg} - 81.3 \text{ N}\cdot\text{m/kg}$$
$$+ 0.230 \text{ N}\cdot\text{m/kg} - 1.99 \text{ N}\cdot\text{m/kg}$$
$$= 0.940 \text{ N}\cdot\text{m/kg} \qquad \textbf{(Ans)}$$

COMMENT The difference in loss calculated assuming uniform velocity profiles and actual velocity profiles is not large compared to $w_{\text{shaft net in}}$ for this fluid flow situation.

EXAMPLE 5.27 Energy—Effect of Nonuniform Velocity Profile

GIVEN Consider the flow situation of Example 5.14.

FIND Apply Eq. 5.87 to develop an expression for the fluid pressure drop that occurs between sections (1) and (2). By comparing the equation for pressure drop obtained presently with the result of Example 5.14, obtain an expression for loss between sections (1) and (2).

SOLUTION

Application of Eq. 5.87 to the flow of Example 5.14 (see Fig. E5.14) leads to

$$\frac{p_2}{\rho} + \frac{\alpha_2 \overline{w}_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{\alpha_1 \overline{w}_1^2}{2} + gz_1 - \text{loss} + \underbrace{w_{\substack{\text{shaft}\\ \text{net in}}}}_{0\ \text{(no shaft work)}} \tag{1}$$

Solving Eq. 1 for the pressure drop, $p_1 - p_2$, we obtain

$$p_1 - p_2 = \rho\left[\frac{\alpha_2 \overline{w}_2^2}{2} - \frac{\alpha_1 \overline{w}_1^2}{2} + g(z_2 - z_1) + \text{loss}\right] \tag{2}$$

Since the fluid velocity at section (1), w_1, is uniformly distributed over cross-sectional area A_1, the corresponding kinetic energy coefficient, α_1, is equal to 1.0. The kinetic energy coefficient at section (2), α_2, needs to be determined from the velocity profile distribution given in Example 5.14. Using Eq. 5.86 we get

$$\alpha_2 = \frac{\int_{A_2} \rho w_2^3 \, dA_2}{\dot{m}\overline{w}_2^2} \tag{3}$$

Substituting the parabolic velocity profile equation into Eq. 3 we obtain

$$\alpha_2 = \frac{\rho \int_0^R (2w_1)^3[1 - (r/R)^2]^3 2\pi r \, dr}{(\rho A_2 \overline{w}_2)\overline{w}_2^2}$$

From conservation of mass, since $A_1 = A_2$

$$w_1 = \overline{w}_2 \tag{4}$$

Then, substituting Eq. 4 into Eq. 3, we obtain

$$\alpha_2 = \frac{\rho 8 \overline{w}_2^3 2\pi \int_0^R [1 - (r/R)^2]^3 r \, dr}{\rho \pi R^2 \overline{w}_2^3}$$

or

$$\alpha_2 = \frac{16}{R^2}\int_0^R [1 - 3(r/R)^2 + 3(r/R)^4 - (r/R)^6] r \, dr = 2 \tag{5}$$

Now we combine Eqs. 2 and 5 to get

$$p_1 - p_2 = \rho\left[\frac{2.0\overline{w}_2^2}{2} - \frac{1.0\overline{w}_1^2}{2} + g(z_2 - z_1) + \text{loss}\right] \tag{6}$$

However, from conservation of mass $\overline{w}_2 = \overline{w}_1 = \overline{w}$ so that Eq. 6 becomes

$$p_1 - p_2 = \frac{\rho \overline{w}^2}{2} + \rho g(z_2 - z_1) + \rho(\text{loss}) \tag{7}$$

The term associated with change in elevation, $\rho g(z_2 - z_1)$, is equal to the weight per unit cross-sectional area, $\mathcal{W}/A$, of the water contained between sections (1) and (2) at any instant,

$$\rho g(z_2 - z_1) = \frac{\mathcal{W}}{A} \tag{8}$$

Thus, combining Eqs. 7 and 8 we get

$$p_1 - p_2 = \frac{\rho \overline{w}^2}{2} + \frac{\mathcal{W}}{A} + \rho(\text{loss}) \tag{9}$$

The pressure drop between sections (1) and (2) is due to:

1. The change in kinetic energy between sections (1) and (2) associated with going from a uniform velocity profile to a parabolic velocity profile.
2. The weight of the water column, that is, hydrostatic pressure effect.
3. Viscous loss.

Comparing Eq. 9 for pressure drop with the one obtained in Example 5.14 (i.e., the answer of Example 5.14) we obtain

$$\frac{\rho \overline{w}^2}{2} + \frac{\mathcal{W}}{A} + \rho(\text{loss}) = \frac{\rho \overline{w}^2}{3} + \frac{R_z}{A} + \frac{\mathcal{W}}{A} \tag{10}$$

or

$$\text{loss} = \frac{R_z}{\rho A} - \frac{\overline{w}^2}{6} \qquad \textbf{(Ans)}$$

COMMENT We conclude that while some of the pipe wall friction force, R_z, resulted in loss of available energy, a portion of this friction, $\rho A\overline{w}^2/6$, led to the velocity profile change.

5.3.5 Combination of the Energy Equation and the Moment-of-Momentum Equation[4]

If Eq. 5.82 is used for one-dimensional incompressible flow through a turbomachine, we can use Eq. 5.54, developed in Section 5.2.4 from the moment-of-momentum equation (Eq. 5.42), to evaluate

[4]This section may be omitted without loss of continuity in the text material. This section should not be considered without prior study of Sections 5.2.3 and 5.2.4. All of these sections are recommended for those interested in Chapter 12.

shaft work. This application of both Eqs. 5.54 and 5.82 allows us to ascertain the amount of loss that occurs in incompressible turbomachine flows as is demonstrated in Example 5.28.

EXAMPLE 5.28 Energy—Fan Performance

GIVEN Consider the fan of Example 5.19.

FIND Show that only some of the shaft power into the air is converted into useful effects. Develop a meaningful efficiency equation and a practical means for estimating lost shaft energy.

SOLUTION

We use the same control volume used in Example 5.19. Application of Eq. 5.82 to the contents of this control volume yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss} \qquad \textbf{(1)}$$

As in Example 5.26, we can see with Eq. 1 that a "useful effect" in this fan can be defined as

$$\text{useful effect} = w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}$$

$$= \left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) \qquad \textbf{(2)} \quad \textbf{(Ans)}$$

In other words, only a portion of the shaft work delivered to the air by the fan blades is used to increase the available energy of the air; the rest is lost because of fluid friction.

A meaningful efficiency equation involves the ratio of shaft work converted into a useful effect (Eq. 2) to shaft work into the air, $w_{\text{shaft net in}}$. Thus, we can express efficiency, η, as

$$\eta = \frac{w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}}{w_{\substack{\text{shaft}\\ \text{net in}}}} \qquad \textbf{(3)}$$

However, when Eq. 5.54, which was developed from the moment-of-momentum equation (Eq. 5.42), is applied to the contents of the control volume of Fig. E5.19, we obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = +U_2 V_{\theta 2} \qquad \textbf{(4)}$$

Combining Eqs. 2, 3, and 4, we obtain

$$\eta = \{[(p_2/\rho) + (V_2^2/2) + gz_2] - [(p_1/\rho) + (V_1^2/2) + gz_1]\}/U_2 V_{\theta 2} \qquad \textbf{(5)} \quad \textbf{(Ans)}$$

Equation 5 provides us with a practical means to evaluate the efficiency of the fan of Example 5.19.

Combining Eqs. 2 and 4, we obtain

$$\text{loss} = U_2 V_{\theta 2} - \left[\left(\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) - \left(\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1\right)\right] \qquad \textbf{(6)} \quad \textbf{(Ans)}$$

COMMENT Equation 6 provides us with a useful method of evaluating the loss due to fluid friction in the fan of Example 5.19 in terms of fluid mechanical variables that can be measured.

5.4 Second Law of Thermodynamics—Irreversible Flow[5]

The second law of thermodynamics formalizes the notion of loss.

The second law of thermodynamics affords us with a means to formalize the inequality

$$\check{u}_2 - \check{u}_1 - q_{\substack{\text{net}\\ \text{in}}} \geq 0 \qquad \textbf{(5.90)}$$

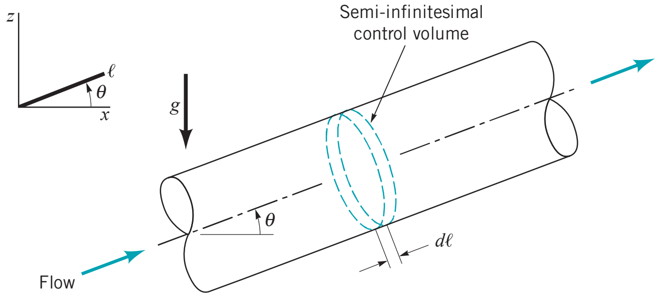
for steady, incompressible, one-dimensional flow with friction (see Eq. 5.73). In this section we continue to develop the notion of loss of useful or available energy for flow with friction. Minimization of loss of available energy in any flow situation is of obvious engineering importance.

5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation

If we apply the one-dimensional, steady flow energy equation, Eq. 5.70, to the contents of a control volume that is infinitesimally thin as illustrated in Fig 5.8, the result is

$$\dot{m}\left[d\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,(dz)\right] = \delta\dot{Q}_{\substack{\text{net}\\ \text{in}}} \qquad \textbf{(5.91)}$$

[5]This entire section may be omitted without loss of continuity in the text material.



■ FIGURE 5.9 Semi-infinitesimal control volume.

For all pure substances including common engineering working fluids, such as air, water, oil, and gasoline, the following relationship is valid (see, for example, Ref. 3).

$$T\,ds = d\check{u} + p\,d\left(\frac{1}{\rho}\right) \tag{5.92}$$

where T is the absolute temperature and s is the *entropy* per unit mass.

Combining Eqs. 5.91 and 5.92 we get

$$\dot{m}\left[T\,ds - p\,d\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta\dot{Q}_{\substack{\text{net}\\\text{in}}}$$

or, dividing through by $\dot{m}$ and letting $\delta q_{\substack{\text{net}\\\text{in}}} = \delta\dot{Q}_{\substack{\text{net}\\\text{in}}}/\dot{m}$, we obtain

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz = -(T\,ds - \delta q_{\substack{\text{net}\\\text{in}}}) \tag{5.93}$$

5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics

A general statement of the second law of thermodynamics is

$$\frac{D}{Dt}\int_{\text{sys}} s\rho\,d\forall \geq \sum\left(\frac{\delta\dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{sys}} \tag{5.94}$$

or in words,

The second law of thermodynamics involves entropy, heat transfer, and temperature.

the time rate of increase of the entropy of a system	$\geq$	sum of the ratio of net heat transfer rate into system to absolute temperature for each particle of mass in the system receiving heat from surroundings

The right-hand side of Eq. 5.94 is identical for the system and control volume at the instant when system and control volume are coincident; thus,

$$\sum\left(\frac{\delta\dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{sys}} = \sum\left(\frac{\delta\dot{Q}_{\substack{\text{net}\\\text{in}}}}{T}\right)_{\text{cv}} \tag{5.95}$$

With the help of the Reynolds transport theorem (Eq. 4.19) the system time derivative can be expressed for the contents of the coincident control volume that is fixed and nondeforming. Using Eq. 4.19, we obtain

$$\frac{D}{Dt}\int_{\text{sys}} s\rho\,d\forall = \frac{\partial}{\partial t}\int_{\text{cv}} s\rho\,d\forall + \int_{\text{cs}} s\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA \tag{5.96}$$

For a fixed, nondeforming control volume, Eqs. 5.94, 5.95, and 5.96 combine to give

$$\frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\forall + \int_{\text{cs}} s\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \geq \sum \left(\frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T} \right)_{\text{cv}} \tag{5.97}$$

At any instant for steady flow

$$\frac{\partial}{\partial t}\int_{\text{cv}} s\rho \, d\forall = 0 \tag{5.98}$$

If the flow consists of only one stream through the control volume and if the properties are uniformly distributed (one-dimensional flow), Eqs. 5.97 and 5.98 lead to

$$\dot{m}(s_{\text{out}} - s_{\text{in}}) \geq \sum \frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T} \tag{5.99}$$

For the infinitesimally thin control volume of Fig. 5.8, Eq. 5.99 yields

$$\dot{m} \, ds \geq \sum \frac{\delta \dot{Q}_{\substack{\text{net}\\ \text{in}}}}{T} \tag{5.100}$$

If all of the fluid in the infinitesimally thin control volume is considered as being at a uniform temperature, T, then from Eq. 5.100 we get

The relationship between entropy and heat transfer rate depends on the process involved.

$$T \, ds \geq \delta q_{\substack{\text{net}\\ \text{in}}}$$

or

$$T \, ds - \delta q_{\substack{\text{net}\\ \text{in}}} \geq 0 \tag{5.101}$$

The equality is for any reversible (frictionless) process; the inequality is for all irreversible (friction) processes.

5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics

Combining Eqs. 5.93 and 5.101, we conclude that

$$-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g \, dz \right] \geq 0 \tag{5.102}$$

The equality is for any steady, reversible (frictionless) flow, an important example being flow for which the Bernoulli equation (Eq. 3.7) is applicable. The inequality is for all steady, irreversible (friction) flows. The actual amount of the inequality has physical significance. It represents the extent of loss of useful or available energy which occurs because of irreversible flow phenomena including viscous effects. Thus, Eq. 5.102 can be expressed as

$$\boxed{-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g \, dz \right] = \delta(\text{loss}) = (T \, ds - \delta q_{\substack{\text{net}\\ \text{in}}})} \tag{5.103}$$

The irreversible flow loss is zero for a frictionless flow and greater than zero for a flow with frictional effects. Note that when the flow is frictionless, Eq. 5.103 multiplied by density, ρ, is identical to Eq. 3.5. Thus, for steady frictionless flow, Newton's second law of motion (see Section 3.1) and the first and second laws of thermodynamics lead to the same differential equation,

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2} \right) + g \, dz = 0 \tag{5.104}$$

If some shaft work is involved, then the flow must be at least locally unsteady in a cyclical way and the appropriate form of the energy equation for the contents of an infinitesimally thin control volume can be developed starting with Eq. 5.67. The resulting equation is

$$\boxed{-\left[\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g\,dz\right] = \delta(\text{loss}) - \delta w_{\substack{\text{shaft}\\\text{net in}}}} \tag{5.105}$$

Equations 5.103 and 5.105 are valid for incompressible and compressible flows. If we combine Eqs. 5.92 and 5.103, we obtain

$$d\check{u} + p d\left(\frac{1}{\rho}\right) - \delta q_{\substack{\text{net}\\\text{in}}} = \delta(\text{loss}) \tag{5.106}$$

For incompressible flow, $d(1/\rho) = 0$ and, thus, from Eq. 5.106,

$$d\check{u} - \delta q_{\substack{\text{net}\\\text{in}}} = \delta(\text{loss}) \tag{5.107}$$

Applying Eq. 5.107 to a finite control volume, we obtain

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\substack{\text{net}\\\text{in}}} = \text{loss}$$

which is the same conclusion we reached earlier (see Eq. 5.78) for incompressible flows.

For compressible flow, $d(1/\rho) \neq 0$, and thus when we apply Eq. 5.106 to a finite control volume we obtain

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} + \int_{\text{in}}^{\text{out}} p d\left(\frac{1}{\rho}\right) - q_{\substack{\text{net}\\\text{in}}} = \text{loss} \tag{5.108}$$

indicating that $u_{\text{out}} - u_{\text{in}} - q_{\text{net in}}$ is not equal to loss.

5.4.4 Application of the Loss Form of the Energy Equation

Zero loss is associated with the Bernoulli equation.

Steady flow along a pathline in an incompressible and frictionless flow field provides a simple application of the loss form of the energy equation (Eq. 5.105). We start with Eq. 5.105 and integrate it term by term from one location on the pathline, section (1), to another one downstream, section (2). Note that because the flow is frictionless, loss = 0. Also, because the flow is steady throughout, $w_{\text{shaft net in}} = 0$. Since the flow is incompressible, the density is constant. The control volume in this case is an infinitesimally small diameter streamtube (Fig. 5.7). The resultant equation is

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \tag{5.109}$$

which is identical to the Bernoulli equation (Eq. 3.7) already discussed in Chapter 3.

If the frictionless and steady pathline flow of the fluid particle considered above was compressible, application of Eq. 5.105 would yield

$$\int_1^2 \frac{dp}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{V_1^2}{2} + gz_1 \tag{5.110}$$

To carry out the integration required, $\int_1^2 (dp/\rho)$, a relationship between fluid density, ρ, and pressure, p, must be known. If the frictionless compressible flow we are considering is adiabatic and involves the flow of an ideal gas, it is shown in Section 11.1 that

$$\frac{p}{\rho^k} = \text{constant} \tag{5.111}$$

where $k = c_p/c_v$ is the ratio of gas specific heats, c_p and c_v, which are properties of the fluid. Using Eq. 5.111 we get

$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) \tag{5.112}$$

Thus, Eqs. 5.110 and 5.112 lead to

$$\frac{k}{k-1}\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1}\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \tag{5.113}$$

Note that this equation is identical to Eq. 3.24. An example application of Eqs. 5.109 and 5.113 follows.

EXAMPLE 5.29 Energy—Comparison of Compressible and Incompressible Flow

GIVEN Air steadily expands adiabatically and without friction from stagnation conditions of 100 psia and 520 °R to 14.7 psia.

FIND Determine the velocity of the expanded air assuming (a) incompressible flow, (b) compressible flow.

SOLUTION

(a) If the flow is considered incompressible, the Bernoulli equation, Eq. 5.109, can be applied to flow through an infinitesimal cross-sectional streamtube, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). From Eq. 5.109 we get

0 (1 is the stagnation state)

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2} = \frac{p_1}{\rho} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1} \tag{1}$$

0 (changes in gz are negligible for air flow)

or

$$V_2 = \sqrt{2\left(\frac{p_1 - p_2}{\rho}\right)}$$

We can calculate the density at state (1) by assuming that air behaves like an ideal gas,

$$\rho = \frac{p_1}{RT_1} = \frac{(100\text{ psia})(144\text{ in.}^2/\text{ft}^2)}{(1716\text{ ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})(520\ {}^\circ\text{R})} = 0.0161\text{ slug/ft}^3 \tag{2}$$

Thus,

$$V_2 = \sqrt{\frac{2(100\text{ psia} - 14.7\text{ psia})(144\text{ in.}^2/\text{ft}^2)}{(0.016\text{ slug/ft}^3)[1\ (\text{lb}\cdot\text{s}^2)/(\text{slug}\cdot\text{ft})]}} = 1240\text{ ft/s} \qquad \textbf{(Ans)}$$

The assumption of incompressible flow is not valid in this case since for air a change from 100 psia to 14.7 psia would undoubtedly result in a significant density change.

(b) If the flow is considered compressible, Eq. 5.113 can be applied to the flow through an infinitesimal cross-sectional control volume, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). We obtain

0 (1 is the stagnation state)

$$\frac{k}{k-1}\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + \cancel{gz_2} = \frac{k}{k-1}\frac{p_1}{\rho_1} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1} \tag{3}$$

0 (changes in gz are negligible for air flow)

or

$$V_2 = \sqrt{\frac{2k}{k-1}\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right)} \tag{4}$$

Given in the problem statement are values of p_1 and p_2. A value of ρ_1 was calculated earlier (Eq. 2). To determine ρ_2 we need to make use of a property relationship for reversible (frictionless) and adiabatic flow of an ideal gas that is derived in Chapter 11; namely,

$$\frac{p}{\rho^k} = \text{constant} \tag{5}$$

where $k = 1.4$ for air. Solving Eq. 5 for ρ_2 we get

$$\rho_2 = \rho_1\left(\frac{p_2}{p_1}\right)^{1/k}$$

or

$$\rho_2 = (0.0161\text{ slug/ft}^3)\left[\frac{14.7\text{ psia}}{100\text{ psia}}\right]^{1/1.4} = 0.00409\text{ slug/ft}^3$$

Then, from Eq. 4, with $p_1 = 100\text{ lb/in.}^2(144\text{ in.}^2/\text{ft}^2) = 14{,}400\text{ lb/ft}^2$ and $p_2 = 14.7\text{ lb/in.}^2(144\text{ in.}^2/\text{ft}^2) = 2117\text{ lb/ft}^2$,

$$V_2 = \sqrt{\frac{(2)(1.4)}{1.4-1}\left(\frac{14{,}400\text{ lb/ft}^2}{0.0161\text{ slug/ft}^3} - \frac{2117\text{ lb/ft}^2}{0.00409\text{ slug/ft}^3}\right)}$$
$$= 1620\ (\text{lb}\cdot\text{ft/slug})^{1/2}[(1\text{ slug}\cdot\text{ft/s}^2)/\text{lb}]^{1/2}$$

or

$$V_2 = 1620\text{ ft/s} \qquad \textbf{(Ans)}$$

COMMENT A considerable difference exists between the air velocities calculated assuming incompressible and compressible flow. In Section 3.8.1, a discussion of when a fluid flow may be appropriately considered incompressible is provided. Basically, when flow speed is less than a third of the speed of sound in the fluid involved, incompressible flow may be assumed with only a small error.

5.5 Chapter Summary and Study Guide

In this chapter the flow of a fluid is analyzed by using important principles including conservation of mass, Newton's second law of motion, and the first and second laws of thermodynamics as applied to control volumes. The Reynolds transport theorem is used to convert basic system-orientated laws into corresponding control volume formulations.

The continuity equation, a statement of the fact that mass is conserved, is obtained in a form that can be applied to any flow—steady or unsteady, incompressible or compressible. Simplified forms of the continuity equation enable tracking of fluid everywhere in a control volume, where it enters, where it leaves, and within. Mass or volume flowrates of fluid entering or leaving a control volume and rate of accumulation or depletion of fluid within a control volume can be estimated.

The linear momentum equation, a form of Newton's second law of motion applicable to flow of fluid through a control volume, is obtained and used to solve flow problems. Net force results from or causes changes in linear momentum (velocity magnitude and/or direction) of fluid flowing through a control volume. Work and power associated with force can be involved.

The moment-of-momentum equation, which involves the relationship between torque and changes in angular momentum, is obtained and used to solve flow problems dealing with turbines (energy extracted from a fluid) and pumps (energy supplied to a fluid).

The steady-state energy equation, obtained from the first law of thermodynamics (conservation of energy), is written in several forms. The first (Eq. 5.69) involves power terms. The second form (Eq. 5.82 or 5.84) is termed the mechanical energy equation or the extended Bernoulli equation. It consists of the Bernoulli equation with extra terms that account for energy losses due to friction in the flow, as well as terms accounting for the work of pumps or turbines in the flow.

conservation of mass
continuity equation
mass flowrate
linear momentum equation
moment-of-momentum equation
shaft power
shaft torque
first law of thermodynamics
heat transfer rate
energy equation
loss
shaft work head
head loss
kinetic energy coefficient

The following checklist provides a study guide for this chapter. When your study of the entire chapter and end-of-chapter exercises has been completed you should be able to

- write out meanings of the terms listed here in the margin and understand each of the related concepts. These terms are particularly important and are set in ***italic, bold, and color*** type in the text.
- select an appropriate control volume for a given problem and draw an accurately labeled control volume diagram.
- use the continuity equation and a control volume to solve problems involving mass or volume flowrate.
- use the linear momentum equation and a control volume, in conjunction with the continuity equation as necessary, to solve problems involving forces related to linear momentum change.
- use the moment-of-momentum equation to solve problems involving torque and related work and power due to angular momentum change.
- use the energy equation, in one of its appropriate forms, to solve problems involving losses due to friction (head loss) and energy input by pumps or extraction by turbines.
- use the kinetic energy coefficient in the energy equation to account for nonuniform flows.

Some of the important equations in this chapter are given below.

Conservation of mass	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0$	**(5.5)**
Mass flowrate	$\dot{m} = \rho Q = \rho A V$	**(5.6)**
Average velocity	$\overline{V} = \dfrac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\rho A}$	**(5.7)**
Steady flow mass conservation	$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0$	**(5.9)**
Moving control volume mass conservation	$\frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$	**(5.16)**

Deforming control volume mass conservation	$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho \, d\forall + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$	**(5.17)**
Force related to change in linear momentum	$\frac{\partial}{\partial t}\int_{\text{cv}} \mathbf{V}\rho \, d\forall + \int_{\text{cs}} \mathbf{V}\rho\mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$	**(5.22)**
Moving control volume force related to change in linear momentum	$\int_{\text{cs}} \mathbf{W}\rho\mathbf{W} \cdot \hat{\mathbf{n}} \, dA = \sum \mathbf{F}_{\substack{\text{contents of the}\\ \text{control volume}}}$	**(5.29)**
Vector addition of absolute and relative velocities	$\mathbf{V} = \mathbf{W} + \mathbf{U}$	**(5.43)**
Shaft torque from force	$\sum \left[(\mathbf{r} \times \mathbf{F})_{\substack{\text{contents of the}\\ \text{control volume}}} \right]_{\text{axial}} = \mathbf{T}_{\text{shaft}}$	**(5.45)**
Shaft torque related to change in moment-of-momentum (angular momentum)	$T_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}} V_{\theta\text{out}})$	**(5.50)**
Shaft power related to change in moment-of-momentum (angular momentum)	$\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm U_{\text{in}} V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}} V_{\theta\text{out}})$	**(5.53)**
First law of thermodynamics (Conservation of energy)	$\frac{\partial}{\partial t}\int_{\text{cv}} e\rho \, d\forall + \int_{\text{cs}} \left(\check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$	**(5.64)**
Conservation of power	$\dot{m}\left[\check{h}_{\text{out}} - \check{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\substack{\text{net}\\ \text{in}}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}}$	**(5.69)**
Conservation of mechanical energy	$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\substack{\text{shaft}\\ \text{net in}}} - \text{loss}$	**(5.82)**

References

1. Eck, B., *Technische Stromungslehre*, Springer-Verlag, Berlin, Germany, 1957.
2. Dean, R. C., "On the Necessity of Unsteady Flow in Fluid Machines," *ASME Journal of Basic Engineering* 81D; 24–28, March 1959.
3. Moran, M. J., and Shapiro, H. N., *Fundamentals of Engineering Thermodynamics*, 6th Ed., Wiley, New York, 2008.

Review Problems

Go to Appendix G for a set of review problems with answers. Detailed solutions can be found in *Student Solution Manual and Study Guide for Fundamentals of Fluid Mechanics*, by Munson et al. (© 2009 John Wiley and Sons, Inc.).

Problems

Note: Unless otherwise indicated, use the values of fluid properties found in the tables on the inside of the front cover. Problems designated with an (*) are intended to be solved with the aid of a programmable calculator or a computer. Problems designated with a (†) are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide the necessary data. There is not a unique answer to these problems.

Answers to the even-numbered problems are listed at the end of the book. Access to the videos that accompany problems can be obtained through the book's web site, www.wiley.com/college/munson. The lab-type problems can also be accessed on this web site.

Section 5.1.1 Derivation of the Continuity Equation

5.1 Explain why the mass of the contents of a system is constant with time.

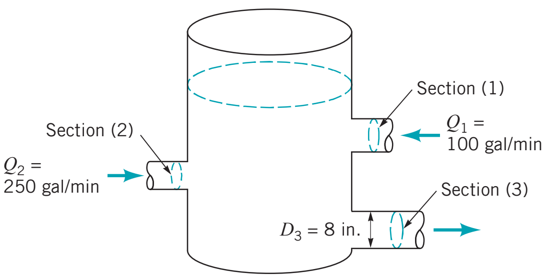
5.2 Explain how the mass of the contents of a control volume can vary with time or not.

5.3 Explain the concept of a coincident control volume and system and why it is useful.

5.4 Obtain a photograph/image of a situation for which the conservation of mass law is important. Briefly describe the situation and its relevance.

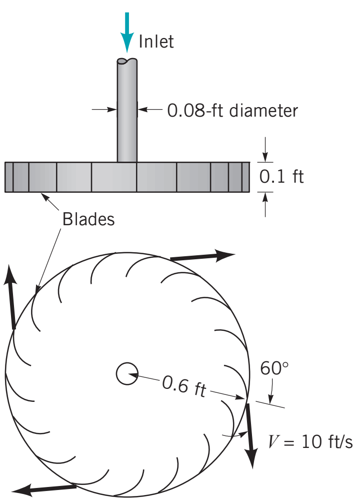
Section 5.1.2 Fixed, Nondeforming Control Volume—Uniform Velocity Profile or Average Velocity.

5.5 Water enters a cylindrical tank through two pipes at rates of 250 and 100 gal/min (see Fig. P5.5). If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through an 8-in. inside-diameter pipe.



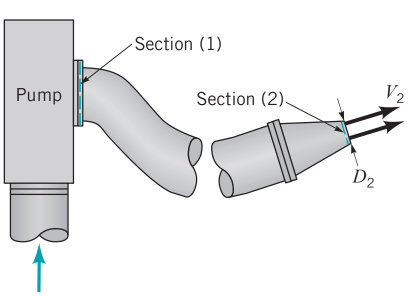
■ FIGURE P5.5

5.6 Water flows out through a set of thin, closely spaced blades as shown in Fig. 5.6 with a speed of $V = 10$ ft/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.



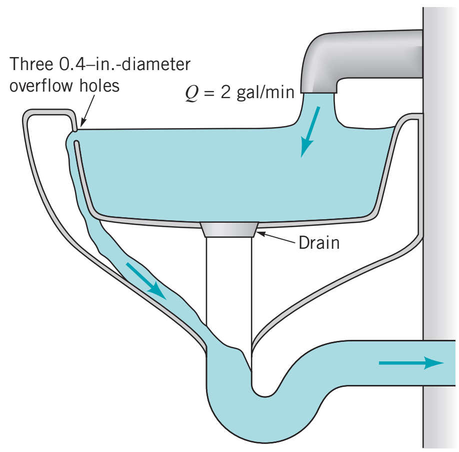
■ FIGURE P5.6

5.7 The pump shown in Fig. P5.7 produces a steady flow of 10 gal/s through the nozzle. Determine the nozzle exit diameter, D_2, if the exit velocity is to be $V_2 = 100$ ft/s.



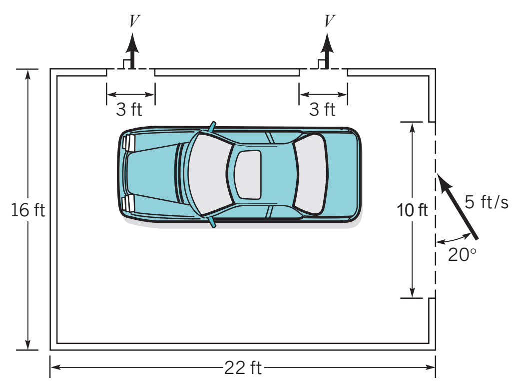
■ FIGURE P5.7

5.8 Water flows into a sink as shown in Video V5.1 and Fig. P5.8 at a rate of 2 gallons per minute. Determine the average velocity through each of the three 0.4-in.-diameter overflow holes if the drain is closed and the water level in the sink remains constant.



■ FIGURE P5.8

5.9 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.9. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.

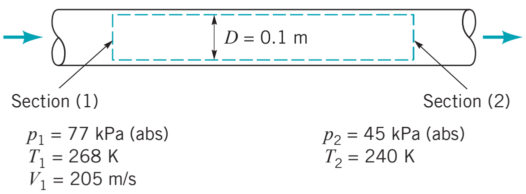


■ FIGURE P5.9

5.10 The human circulatory system consists of a complex branching pipe network ranging in diameter from the aorta (largest) to the capillaries (smallest). The average radii and the number of these vessels is shown in the table below. Does the average blood velocity increase, decrease, or remain constant as it travels from the aorta to the capillaries?

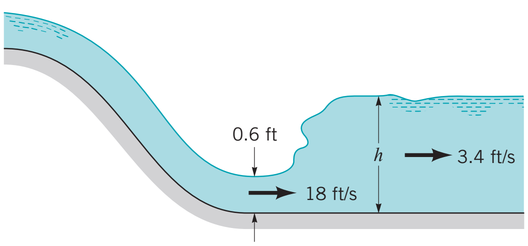
Vessel	Average Radius, mm	Number
Aorta	12.5	1
Arteries	2.0	159
Arterioles	0.03	1.4×10^7
Capillaries	0.006	3.9×10^9

5.11 Air flows steadily between two cross sections in a long, straight section of 0.1-m inside diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.11. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).



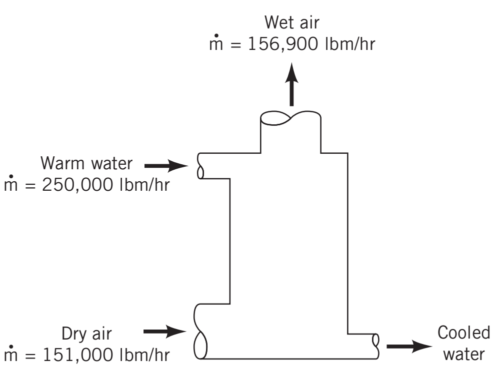
■ **FIGURE P5.11**

5.12 A hydraulic jump (see **Video V10.10**) is in place downstream from a spillway as indicated in Fig. P5.12. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



■ **FIGURE P5.12**

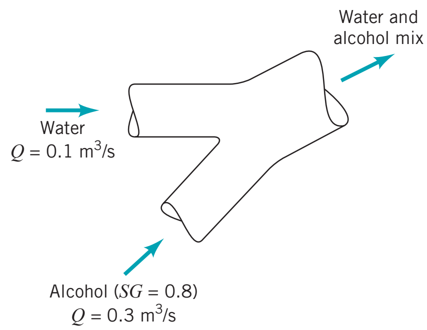
5.13 An evaporative cooling tower (see Fig. P5.13) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lbm/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lbm/hr. If the rate of wet air flow out of the tower is 156,900 lbm/hr, determine the rate of water evaporation in lbm/hr and the rate of cooled water flow in lbm/hr.



■ **FIGURE P5.13**

5.14 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross-sectional area is 3.5 ft^2, estimate the density of the exhaust gases in lbm/ft^3.

5.15 Water at 0.1 m^3/s and alcohol (SG=0.8) at 0.3 m^3/s are mixed in a y-duct as shown in Fig. 5.15. What is the average density of the mixture of alcohol and water?

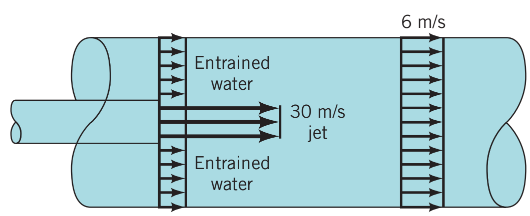


■ **FIGURE P5.15**

5.16 Freshwater flows steadily into an open 55-gal drum initially filled with seawater. The freshwater mixes thoroughly with the seawater and the mixture overflows out of the drum. If the freshwater flowrate is 10 gal/min, estimate the time in seconds required to decrease the difference between the density of the mixture and the density of freshwater by 50%.

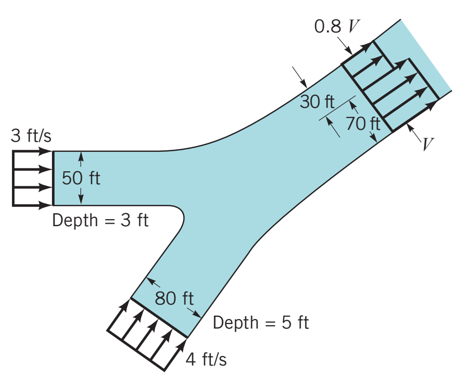
Section 5.1.2 Fixed, Nondeforming Control Volume—Nonuniform Velocity Profile

5.17 A water jet pump (see Fig. P5.17) involves a jet cross-sectional area of 0.01 m^2, and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross-sectional area associated with the jet and entrained streams is 0.075 m^2. These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross-sectional area of 0.075 m^2. Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.



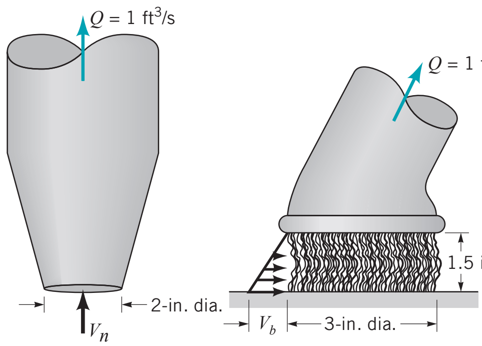
■ **FIGURE P5.17**

5.18 Two rivers merge to form a larger river as shown in Fig. P5.18. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of V.



■ **FIGURE P5.18**

5.19 Various types of attachments can be used with the shop vac shown in **Video V5.2**. Two such attachments are shown in Fig. P5.19—a nozzle and a brush. The flowrate is 1 ft^3/s. **(a)** Determine the average velocity through the nozzle entrance, V_n. **(b)** Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to V_b along the length of the bristles as shown in the figure. Determine the value of V_b.



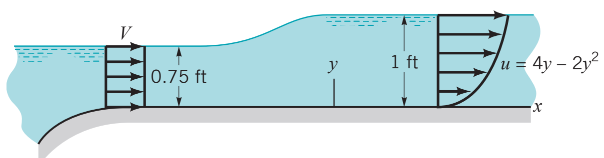
■ FIGURE P5.19

5.20 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\bar{u}$, to centerline velocity, u_c, for **(a)** $n = 4$, **(b)** $n = 6$, **(c)** $n = 8$, **(d)** $n = 10$. Compare the different velocity profiles.

5.21 As shown in Fig. P5.21, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.

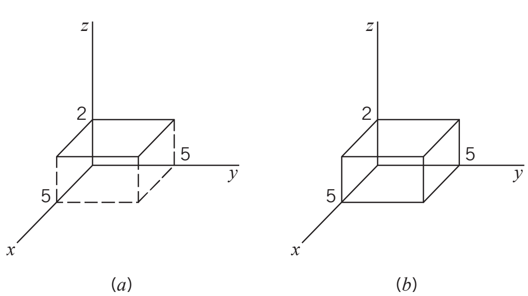


■ FIGURE P5.21

5.22 A water flow situation is described by the velocity field equation

$$\mathbf{V} = (3x + 2)\,\hat{\mathbf{i}} + (2y - 4)\,\hat{\mathbf{j}} - 5z\hat{\mathbf{k}}\ \text{ft/s}$$

where x, y, and z are in feet. **(a)** Determine the mass flowrate through the rectangular area in the plane corresponding to $z = 2$ feet having corners at $(x, y, z) = (0, 0, 2)$, $(5, 0, 2)$, $(5, 5, 2)$, and $(0, 5, 2)$ as shown in Fig P5.22*a*. **(b)** Show that mass is conserved in the control volume having corners at $(x, y, z) = (0, 0, 2)$, $(5, 0, 2)$, $(5, 5, 2)$, $(0, 5, 2)$, $(0, 0, 0)$, $(5, 0, 0)$, $(5, 5, 0)$, and $(0, 5, 0)$, as shown in Fig. P5.22*b*.

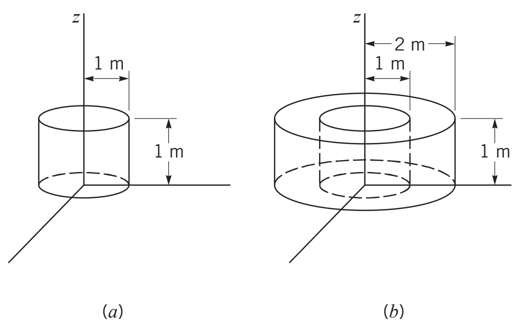


■ FIGURE P5.22

5.23 An incompressible flow velocity field (water) is given as

$$\mathbf{V} = -\frac{1}{r}\,\hat{\mathbf{e}}_r + \frac{1}{r}\,\hat{\mathbf{e}}_\theta\ \text{m/s}$$

where r is in meters. **(a)** Calculate the mass flowrate through the cylindrical surface at $r = 1$ m from $z = 0$ to $z = 1$ m as shown in Fig.P5.23*a*. **(b)** Show that mass is conserved in the annular control volume from $r = 1$ m to $r = 2$ m and $z = 0$ to $z = 1$ m as shown in Fig. P5.23*b*.

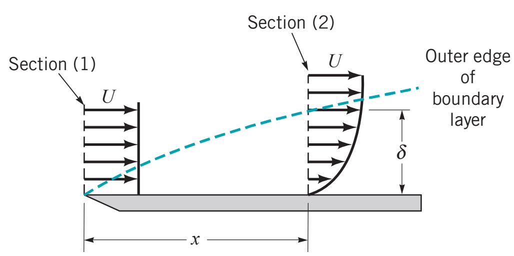


■ FIGURE P5.23

5.24 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.24. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

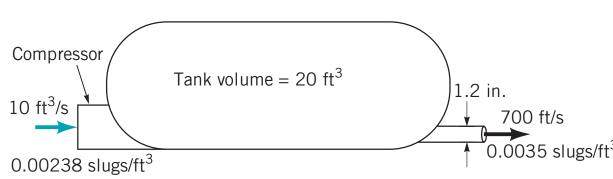
develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.



■ FIGURE P5.24

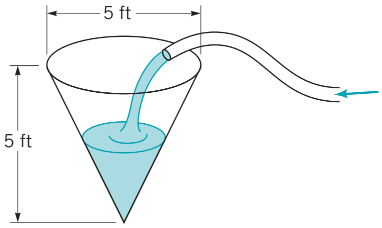
Section 5.1.2 Fixed, Nondeforming Control Volume—Unsteady Flow

5.25 Air at standard conditions enters the compressor shown in Fig. P5.25 at a rate of 10 ft^3/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 $slugs/ft^3$ and a uniform speed of 700 ft/s. **(a)** Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. **(b)** Determine the average time rate of change of air density within the tank.



■ FIGURE P5.25

5.26 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.26) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.



■ FIGURE P5.26

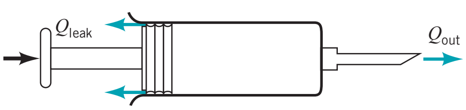
†**5.27** Estimate the maximum flowrate of rainwater (during a heavy rain) that you would expect from the downspout connected to the gutters of your house. List all assumptions and show all calculations.

Section 5.1.3 Moving, Nondeforming Control Volume

5.28 For an automobile moving along a highway, describe the control volume you would use to estimate the flowrate of air across the radiator. Explain how you would estimate the velocity of that air.

Section 5.1.4 Deforming Control Volume

5.29 A hypodermic syringe (see Fig. P5.29) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks past the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.



■ FIGURE P5.29

5.30 The Hoover Dam (see Video V2.4) backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8000 cfs, how many feet per 24-hour day will the lake level rise?

5.31 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to **(a)** keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, and **(b)** reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?

5.32 (See Fluids in the News article **"New 1.6 gpf standards,"** Section 5.1.2.) When a toilet is flushed, the water depth, h, in the tank as a function of time, t, is as given in the table. The size of the rectangular tank is 19 in. by 7.5 in. **(a)** Determine the volume of water used per flush, gpf. **(b)** Plot the flowrate for $0 \leq t \leq 6$ s.

t (s)	h (in.)
0	5.70
0.5	5.33
1.0	4.80
2.0	3.45
3.0	2.40
4.0	1.50
5.0	0.75
6.0	0

Section 5.2.1 Derivation of the Linear Momentum Equation

5.33 What is fluid linear momentum and the "flow" of linear momentum?

5.34 Explain the physical meaning of each of the terms of the linear momentum equation (Eq. 5.22).

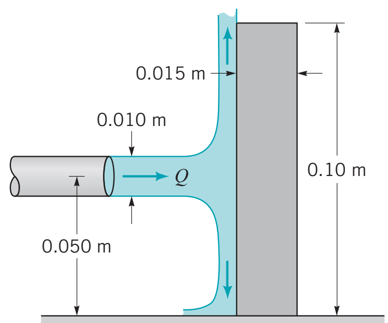
5.35 What is an inertial control volume?

5.36 Distinguish between body and surface forces.

5.37 Obtain a photograph/image of a situation in which the linear momentum of a fluid changes during flow from one location to another. Explain briefly how force is involved.

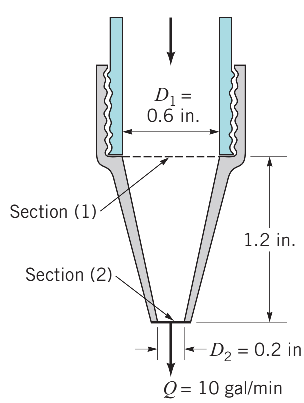
Section 5.2.2 Application of the Linear Momentum Equation (Also see Lab Problems 5.140, 5.141, 5.142, and 5.143.)

5.38 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.6 and Fig. P5.38. Determine the minimum volume flowrate needed to tip the block.



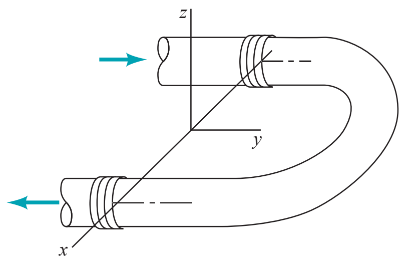
■ FIGURE P5.38

5.39 Determine the anchoring force required to hold in place the conical nozzle attached to the end of the laboratory sink faucet shown in Fig. P5.39 when the water flowrate is 10 gal/min. The nozzle weight is 0.2 lb. The nozzle inlet and exit inside diameters are 0.6 and 0.2 in., respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 1.2 in. The pressure at section (1) is 68 psi.



■ FIGURE P5.39

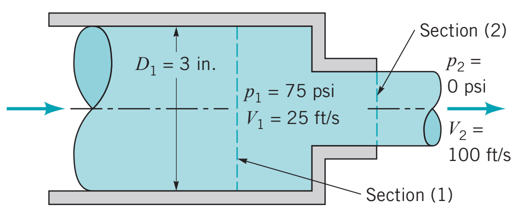
5.40 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.40. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s.



■ FIGURE P5.40

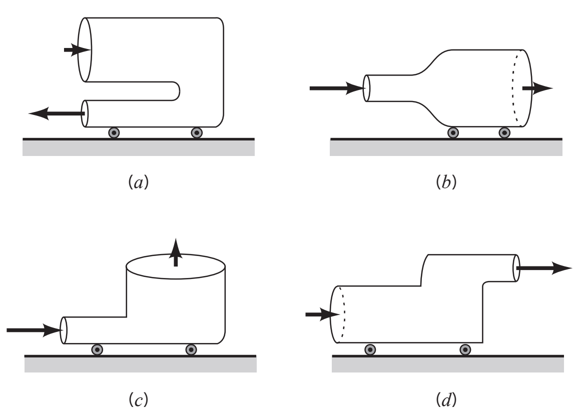
The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

5.41 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.41 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.



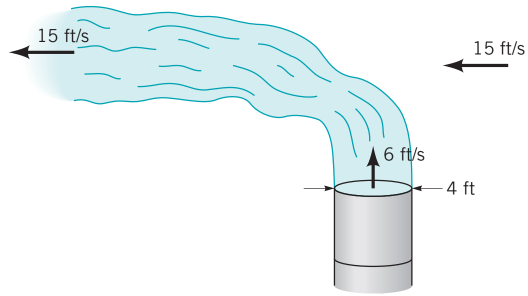
■ FIGURE P5.41

5.42 The four devices shown in Fig. P5.42 rest on frictionless wheels, are restricted to move in the x direction only, and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.



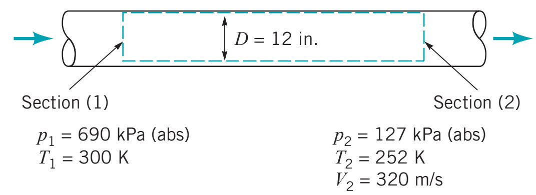
■ FIGURE P5.42

5.43 Exhaust (assumed to have the properties of standard air) leaves the 4-ft-diameter chimney shown in **Video V5.4** and Fig. P5.43 with a speed of 6 ft/s. Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind, 15 ft/s. Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



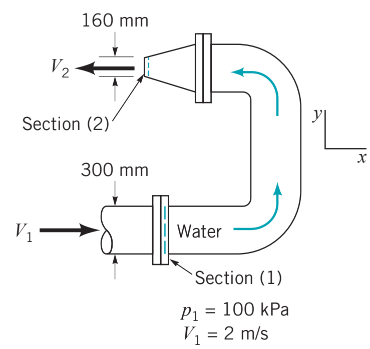
■ FIGURE P5.43

5.44 Air flows steadily between two cross sections in a long, straight section of 12-in.-inside diameter pipe. The static temperature and pressure at each section are indicated in Fig P5.44. If the average air velocity at section (2) is 320 m/s, determine the average air velocity at section (1). Determine the frictional force exerted by the pipe wall on the air flowing between sections (1) and (2). Assume uniform velocity distributions at each section.



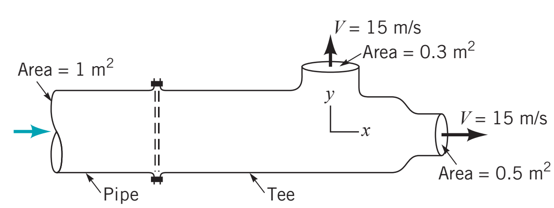
■ FIGURE P5.44

5.45 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.45 in place. Atmospheric pressure is 100 kPa(abs). The gage pressure at section (1) is 100 kPa. At section (2), the water exits to the atmosphere.



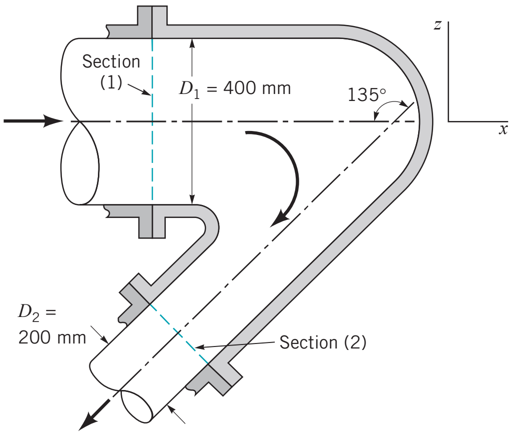
■ FIGURE P5.45

5.46 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.46. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.



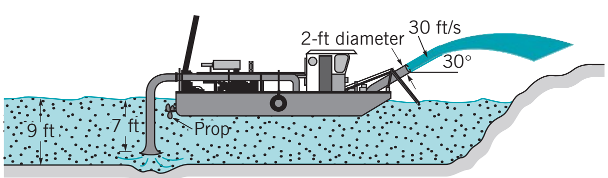
■ FIGURE P5.46

5.47 A converging elbow (see Fig. P5.47) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m^3 between sections (1) and (2). The water volume flowrate is 0.4 m^3/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.



■ FIGURE P5.47

5.48 The hydraulic dredge shown in Fig. P5.48 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.



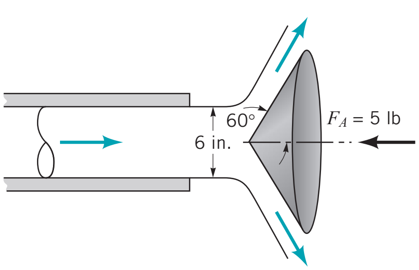
■ FIGURE P5.48

5.49 A static thrust stand is to be designed for testing a specific jet engine. Knowing the following conditions for a typical test,

$$\begin{aligned} \text{intake air velocity} &= 700 \text{ ft/s} \\ \text{exhaust gas velocity} &= 1640 \text{ ft/s} \\ \text{intake cross section area} &= 10 \text{ ft}^2 \\ \text{intake static pressure} &= 11.4 \text{ psia} \\ \text{intake static temperature} &= 480\ ^\circ\text{R} \\ \text{exhaust gas pressure} &= 0 \text{ psi} \end{aligned}$$

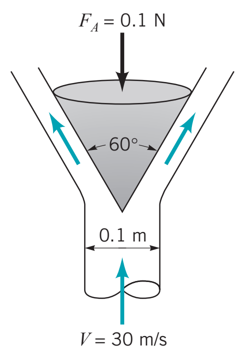
estimate a nominal thrust to design for.

5.50 A horizontal, circular cross-sectional jet of air having a diameter of 6 in. strikes a conical deflector as shown in Fig. P5.50. A horizontal anchoring force of 5 lb is required to hold the cone in place. Estimate the nozzle flowrate in ft^3/s. The magnitude of the velocity of the air remains constant.



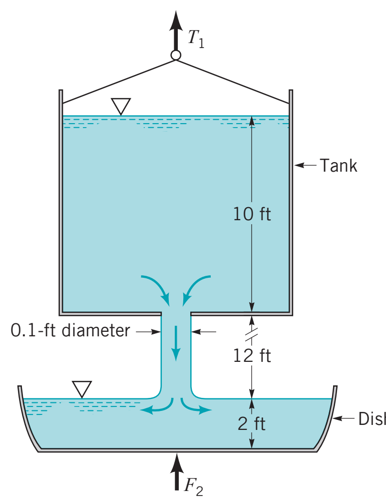
■ FIGURE P5.50

5.51 A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in Fig. P5.51. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.



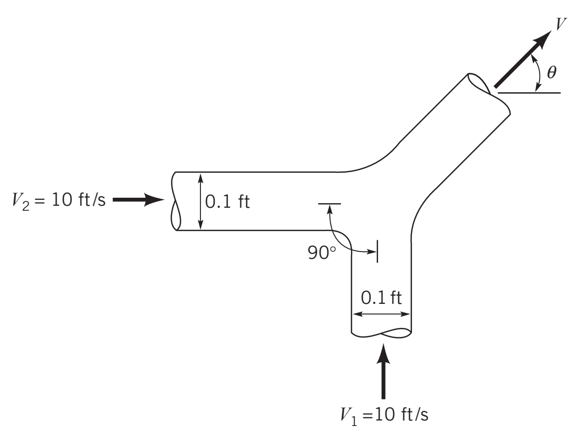
■ FIGURE P5.51

5.52 Water flows from a large tank into a dish as shown in Fig. P5.52. **(a)** If at the instant shown the tank and the water in it weigh W_1 lb, what is the tension, T_1, in the cable supporting the tank? **(b)** If at the instant shown the dish and the water in it weigh W_2 lb, what is the force, F_2, needed to support the dish?



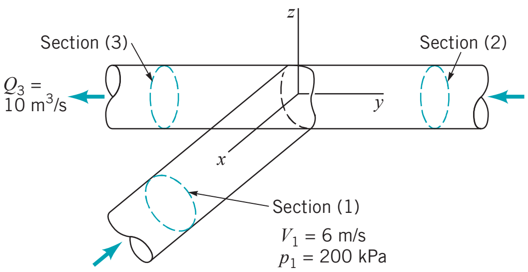
■ FIGURE P5.52

5.53 Two water jets of equal size and speed strike each other as shown in Fig. P5.53. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



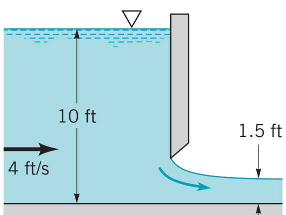
■ FIGURE P5.53

5.54 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.54, estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.



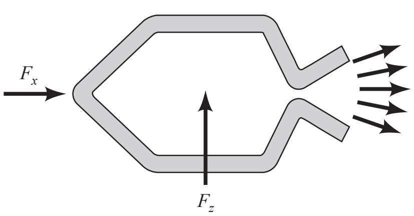
■ FIGURE P5.54

5.55 Determine the magnitude of the horizontal component of the anchoring force required to hold in place the sluice gate shown in Fig. 5.55. Compare this result with the size of the horizontal component of the anchoring force required to hold in place the sluice gate when it is closed and the depth of water upstream is 10 ft.



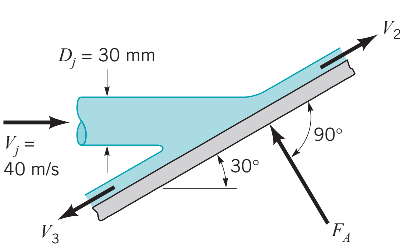
■ FIGURE P5.55

5.56 The rocket shown in Fig. P5.56. is held stationary by the horizontal force, F_x, and the vertical force, F_z. The velocity and pressure of the exhaust gas are 5000 ft/s and 20 psia at the nozzle exit, which has a cross section area of 60 in.². The exhaust mass flowrate is constant at 21 lbm/s. Determine the value of the restraining force F_x. Assume the exhaust flow is essentially horizontal.



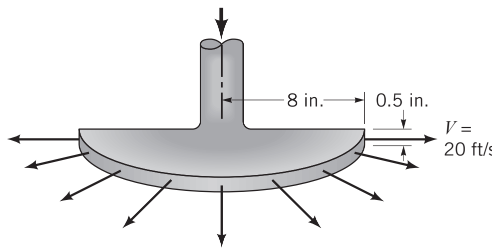
■ FIGURE P5.56

5.57 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.57. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: **(a)** the magnitude of F_A, the anchoring force required to hold the plate stationary; **(b)** the fraction of mass flow along the plate surface in each of the two directions shown; **(c)** the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.



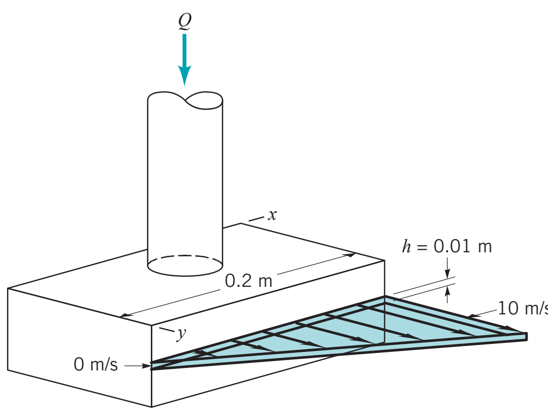
■ FIGURE P5.57

5.58 Water is sprayed radially outward over 180° as indicated in Fig. P5.58. The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.



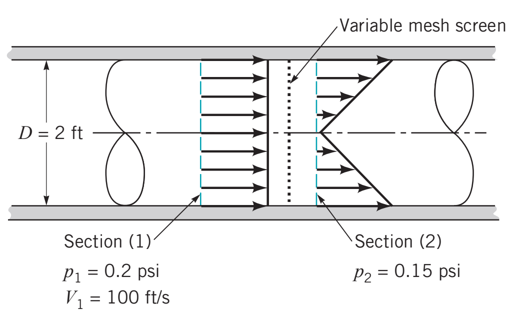
■ FIGURE P5.58

5.59 A sheet of water of uniform thickness (h = 0.01 m) flows from the device shown in Fig. P5.59. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the y component of anchoring force necessary to hold this device stationary.



■ FIGURE P5.59

5.60 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig. P5.60 in the air flow through a



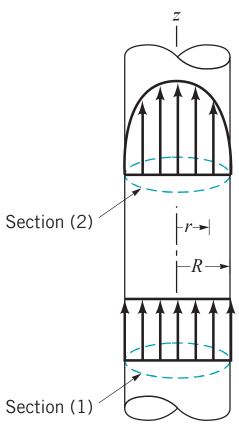
■ FIGURE P5.60

2-ft-diameter circular cross section duct. The static pressures upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

5.61 Water flows vertically upward in a circular cross-sectional pipe as shown in Fig. P5.61. At section (1), the velocity profile over the cross-sectional area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c\left(\frac{R-r}{R}\right)^{1/7}\hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe radius, and r = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

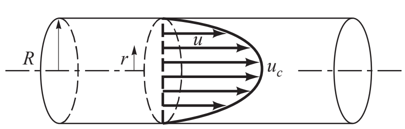


■ FIGURE P5.61

5.62 In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic. That is,

$$u = u_c\left[1-\left(\frac{r}{R}\right)^2\right]$$

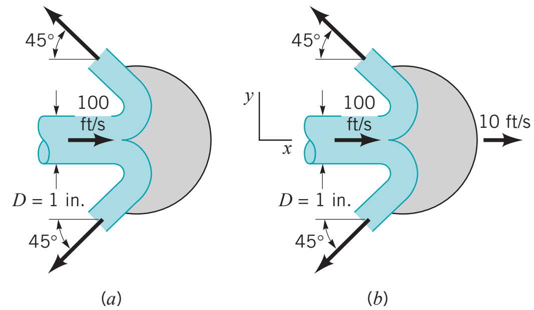
as is illustrated in Fig. P5.62. Compare the axial direction momentum flowrate calculated with the average velocity, $\bar{u}$, with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.



■ FIGURE P5.62

†5.63 Water from a garden hose is sprayed against your car to rinse dirt from it. Estimate the force that the water exerts on the car. List all assumptions and show calculations.

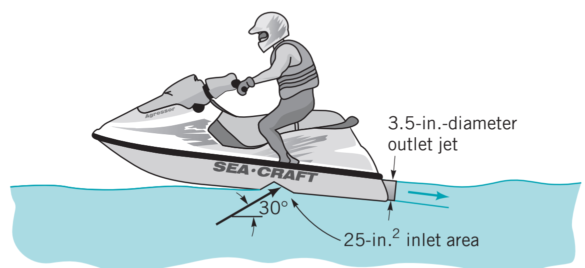
5.64 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.64 and **Video V5.6**. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to **(a)** hold the vane stationary, **(b)** confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.



■ FIGURE P5.64

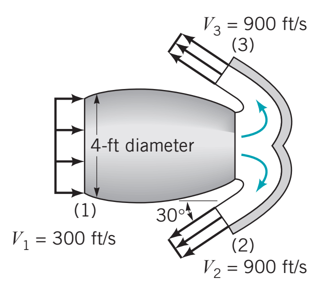
5.65 How much power is transferred to the moving vane of Problem 5.64?

5.66 The thrust developed to propel the jet ski shown in **Video V9.11** and Fig. P5.66 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



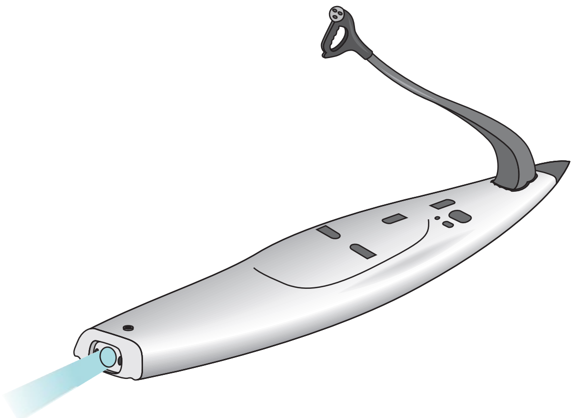
■ FIGURE P5.66

5.67 (See Fluids in the News article titled **"Where the plume goes,"** Section 5.2.2.) Air flows into the jet engine shown in Fig. P5.67 at a rate of 9 slugs/s and a speed of 300 ft/s. Upon landing, the engine exhaust exits through the reverse thrust mechanism with a speed of 900 ft/s in the direction indicated. Determine the reverse thrust applied by the engine to the airplane. Assume the inlet and exit pressures are atmospheric and that the mass flowrate of fuel is negligible compared to the air flowrate through the engine.



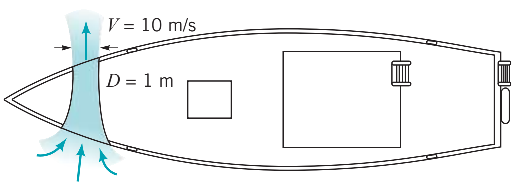
■ FIGURE P5.67

5.68 (See Fluids in the News article titled **"Motorized surfboard,"** Section 5.2.2.) The thrust to propel the powered surfboard shown in Fig. P5.68 is a result of water pumped through the board that exits as a high-speed 2.75-in.-diameter jet. Determine the flowrate and the velocity of the exiting jet if the thrust is to be 300 lb. Neglect the momentum of the water entering the pump.



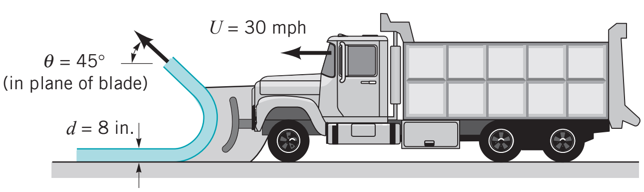
■ FIGURE P5.68

5.69 (See Fluids in the News article titled **"Bow thrusters,"** Section 5.2.2). The bow thruster on the boat shown in Fig. P5.69 is used to turn the boat. The thruster produces a 1-m-diameter jet of water with a velocity of 10 m/s. Determine the force produced by the thruster. Assume that the inlet and outlet pressures are zero and that the momentum of the water entering the thruster is negligible.



■ FIGURE P5.69

5.70 A snowplow mounted on a truck clears a path 12 *ft* through heavy wet snow, as shown in Figure P5.70. The snow is 8 in. deep and its density is 10 lbm/ft^3. The truck travels at 30 mph. The snow is discharged from the plow at an angle of 45° from the direction of travel and 45° above the horizontal, as shown in Figure P5.70. Estimate the force required to push the plow.



■ FIGURE P5.70

Section 5.2.3 Derivation of the Moment-of-Momentum Equation

5.71 What is fluid moment-of-momentum (angular momentum) and the "flow" of moment-of-momentum (angular momentum)?

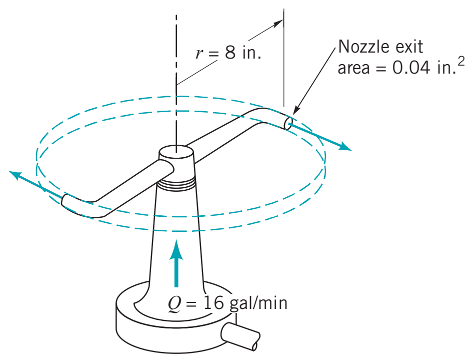
5.72 Describe the orthogonal components of the moment-of-momentum equation (Eq. 5.42) and comment on the direction of each.

5.73 Describe a few examples (include photographs/images) of turbines where the force/torque of a flowing fluid leads to rotation of a shaft.

5.74 Describe a few examples (include photographs/images) of pumps where a fluid is forced to move by "blades" mounted on a rotating shaft.

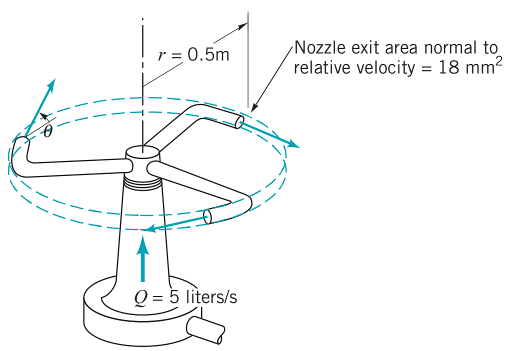
Section 5.2.4 Application of the Moment-of-Momentum Equation

5.75 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.75. The exit cross-sectional area of each of the two nozzles is 0.04 in.^{2}, and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.



■ FIGURE P5.75

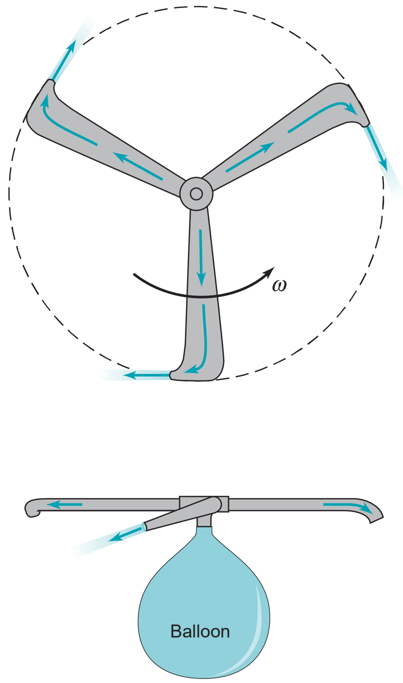
5.76 Five liters/s of water enter the rotor shown in **Video V5.10** and Fig. P5.76 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm^2. How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and **(a)** $\theta = 0°$, **(b)** $\theta = 30°$, **(c)** $\theta = 60°$?



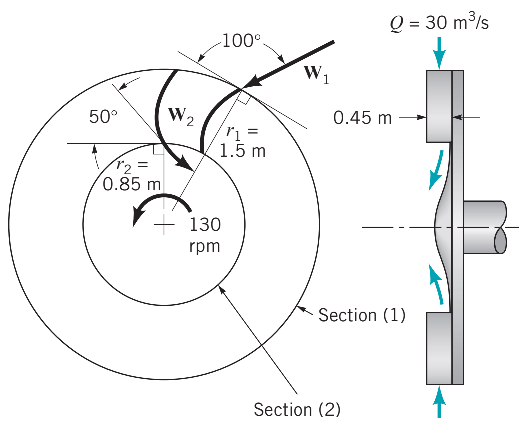
■ FIGURE P5.76

5.77 Shown in Fig. P5.77 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out through small nozzles at the tips of the blades. Explain physically how this flow can cause the rotation necessary to rotate the blades to produce the needed lifting force.

5.78 A simplified sketch of a hydraulic turbine runner is shown in Fig. P5.78. Relative to the rotating runner, water enters at section (1) (cylindrical cross section area A_1 at r_1=1.5 m) at an angle of 100° from the tangential direction and leaves at section (2) (cylindrical cross section area A_2 at r_2=0.85 m) at an angle of 50° from the tangential direction. The blade height at sections (1) and (2) is 0.45 m and the volume flowrate through the turbine is 30 m^3/s. The runner speed is 130 rpm in the direction shown. Determine the shaft power developed.

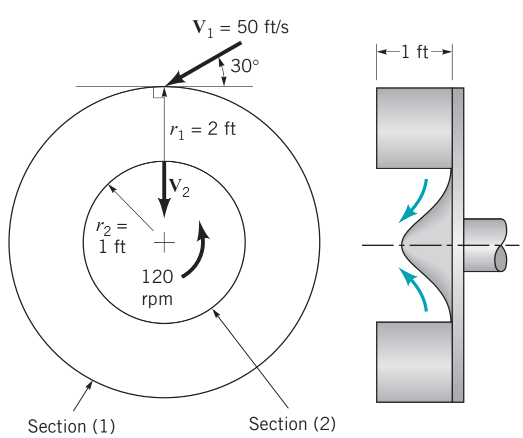


■ FIGURE P5.77



■ FIGURE P5.78

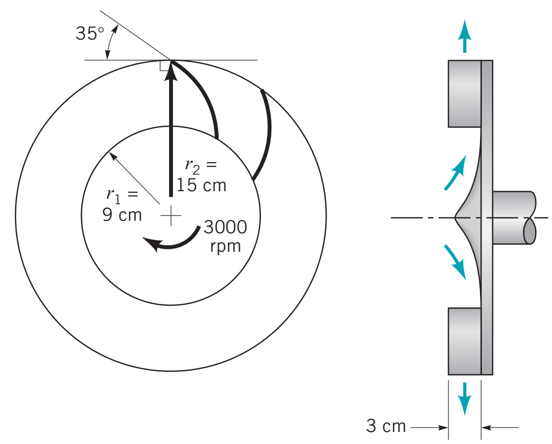
5.79 A water turbine with radial flow has the dimensions shown in Fig.P5.79.The absolute entering velocity is 50 ft/s, and it makes an



■ FIGURE P5.79

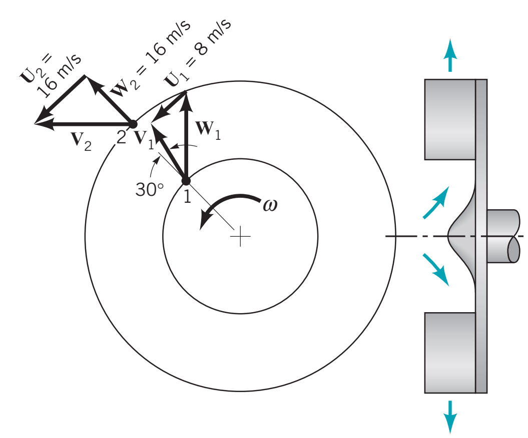
angle of 30° with the tangent to the rotor. The absolute exit velocity is directed radially inward. The angular speed of the rotor is 120 rpm. Find the power delivered to the shaft of the turbine.

5.80 Shown in Fig. P5.80 are front and side views of a centrifugal pump rotor or impeller. If the pump delivers 200 liters/s of water and the blade exit angle is 35° from the tangential direction, determine the power requirement associated with flow leaving at the blade angle. The flow entering the rotor blade row is essentially radial as viewed from a stationary frame.



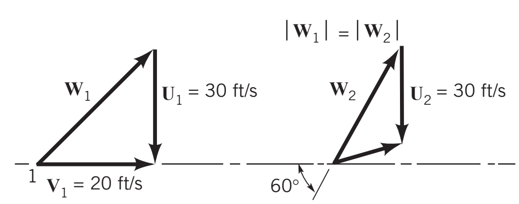
■ FIGURE P5.80

5.81 The velocity triangles for water flow through a radial pump rotor are as indicated in Fig. P5.81. **(a)** Determine the energy added to each unit mass (kg) of water as it flows through the rotor. **(b)** Sketch an appropriate blade section.



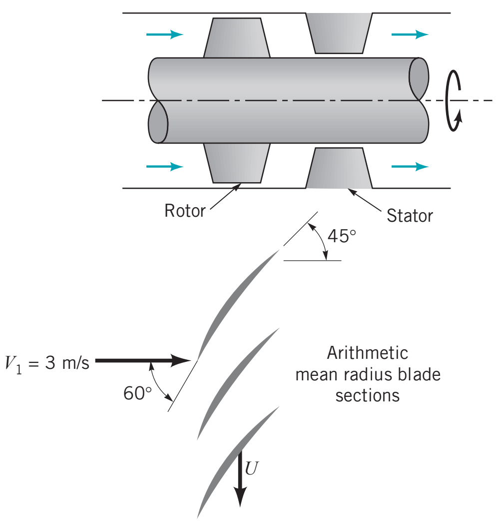
■ FIGURE P5.81

5.82 An axial flow turbomachine rotor involves the upstream (1) and downstream (2) velocity triangles shown in Fig.P5.82. Is this turbomachine a turbine or a fan? Sketch an appropriate blade section and determine energy transferred per unit mass of fluid.



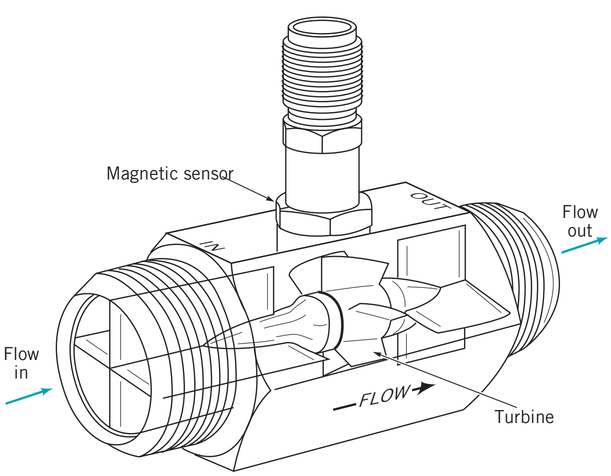
■ FIGURE P5.82

5.83 An axial flow gasoline pump (see Fig. P5.83) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are 60° and 45° from the axial direction. The pump annulus passage cross-sectional area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline? Is this an actual or ideal amount?



■ **FIGURE P5.83**

5.84 Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.84. Show how rotor angular velocity is proportional to average fluid velocity.



■ **FIGURE P5.84** **(Courtesy of EG&G Flow Technology, Inc.)**

5.85 By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed.

Section 5.3.1 Derivation of the Energy Equation

5.86 Distiguish between shaft work and other kinds of work associated with a flowing fluid.

5.87 Define briefly what heat transfer is. What is an adiabatic flow? Give several practical examples of nearly adiabatic flows.

Section 5.3.2 Application of the Energy Equation – No Shaft Work and Section 5.3.3 Comparison of the Energy Equation with the Bernoulli Equation

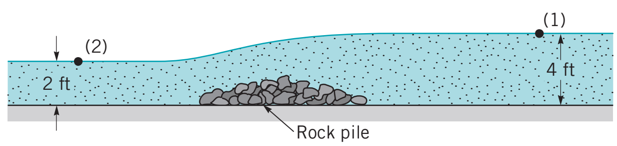
5.88 What is enthalpy and why is it useful for energy considerations in fluid mechanics?

5.89 Cite a few examples of evidence of loss of available energy in actual fluid flows. Why does loss occur?

5.90 Is zero heat transfer a necessary condition for application of the Bernoulli equation (Eq. 5.75)?

5.91 A 1000-m-high waterfall involves steady flow from one large body to another. Detemine the temperature rise associated with this flow.

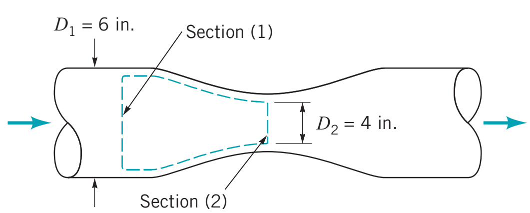
5.92 A 100-ft-wide river with a flowrate of 2400 ft^3/s flows over a rock pile as shown in Fig. P5.92. Determine the direction of flow and the head loss associated with the flow across the rock pile.



■ **FIGURE P5.92**

5.93 Air steadily expands adiabatically and without friction from stagnation conditions of 690 kpa (abs) and 290 K to a static pressure of 101 kpa (abs). Determine the velocity of the expanded air assuming: **(a)** incompressible flow; **(b)** compressible flow.

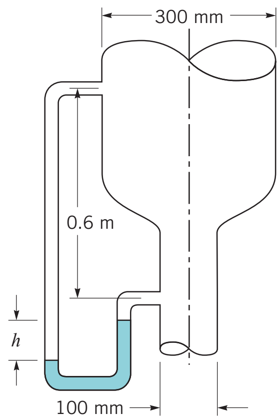
5.94 A horizontal Venturi flow meter consists of a converging–diverging conduit as indicated in Fig. P5.94. The diameters of cross sections (1) and (2) are 6 and 4 in. The velocity and static pressure are uniformly distributed at cross sections (1) and (2). Determine the volume flowrate (ft^3/s) through the meter if $p_1 - p_2 = 3$ psi, the flowing fluid is oil ($\rho = 56$ lbm/ft^3), and the loss per unit mass from (1) to (2) is negligibly small.



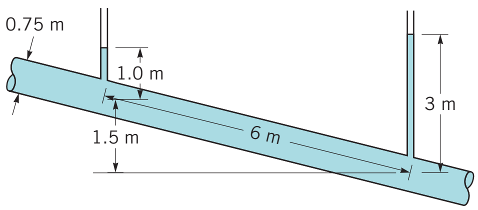
■ **FIGURE P5.94**

5.95 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.95. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

5.96 An incompressible liquid flows steadily along the pipe shown in Fig. P5.96. Determine the direction of flow and the head loss over the 6-m length of pipe.

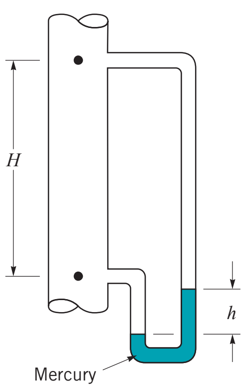


■ **FIGURE P5.95**



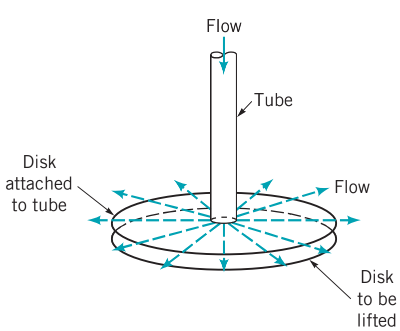
■ **FIGURE P5.96**

5.97 Water flows through a vertical pipe, as is indicated in Fig. P5.97. Is the flow up or down in the pipe? Explain.



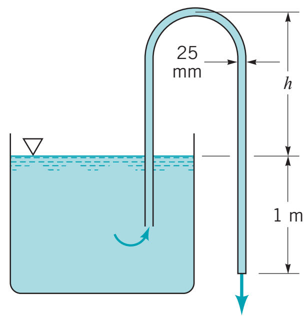
■ **FIGURE P5.97**

5.98 A circular disk can be lifted up by blowing on it with the device shown in Fig. P5.98. Explain why this happens.



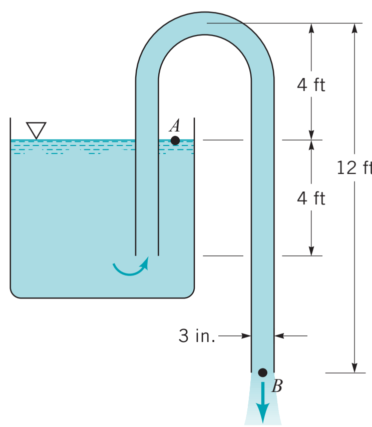
■ **FIGURE P5.98**

5.99 A siphon is used to draw water at 20°C from a large container as indicated in Fig. P5.99. Does changing the elevation, h, of the siphon centerline above the water level in the tank vary the flowrate through the siphon? Explain. What is the maximum allowable value of h?



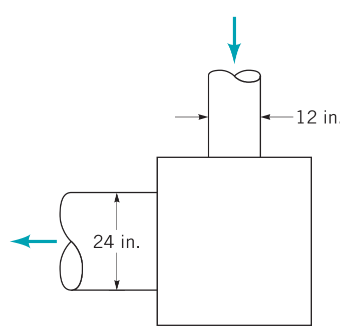
■ **FIGURE P5.99**

5.100 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.100. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.



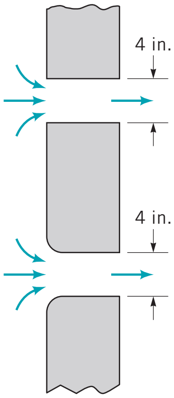
■ **FIGURE P5.100**

5.101 Water flows through a valve (see Fig.P5.101) at the rate of 1000 lbm/s. The pressure just upstream of the valve is 90 psi and the pressure drop across the valve is 50 psi. The inside diameters of the valve inlet and exit pipes are 12 and 24 in. If the flow through the valve occurs in a horizontal plane determine the loss in available energy across the valve.



■ **FIGURE P5.101**

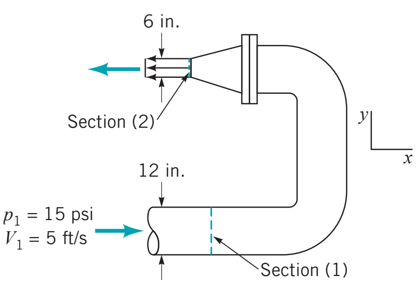
5.102 Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 4 in. and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Fig. P5.102). The room is held at a constant pressure of 1.5 psi above atmospheric. Both vents exhaust into the atmosphere. The loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is $0.5V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.



■ FIGURE P5.102

5.103 A gas expands through a nozzle from a pressure of 300 psia to a pressure of 5 psia. The enthalpy change involved, $\check{h}_1 - \check{h}_2$, is 150 Btu/lbm. If the expansion is adiabatic but with frictional effects and the inlet gas speed is negligibly small, determine the exit gas velocity.

5.104 For the 180° elbow and nozzle flow shown in Fig. P5.104, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

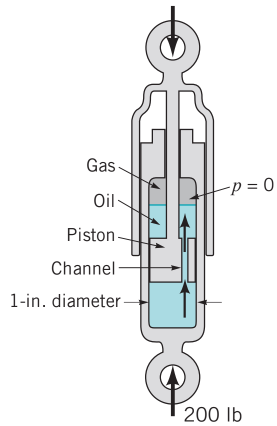


■ FIGURE P5.104

5.105 An automobile engine will work best when the back pressure at the interface of the exhaust manifold and the engine block is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

†5.106 Explain how, in terms of the loss of available energy involved, a home sink water faucet valve works to vary the flow from the shutoff condition to maximum flow. Explain how you would estimate the size of the overflow drain holes needed in the sink of **Video V5.1** (**Video V3.9** may be helpful).

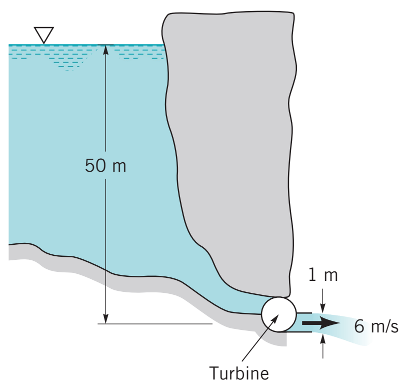
5.107 (See Fluids in the News article titled **"Smart shocks,"** Section 5.3.3.) A 200-lb force applied to the end of the piston of the shock absorber shown in Fig. P5.107 causes the two ends of the shock absorber to move toward each other with a speed of 5 ft/s. Determine the head loss associated with the flow of the oil through the channel. Neglect gravity and any friction force between the piston and cylinder walls.



■ FIGURE P5.107

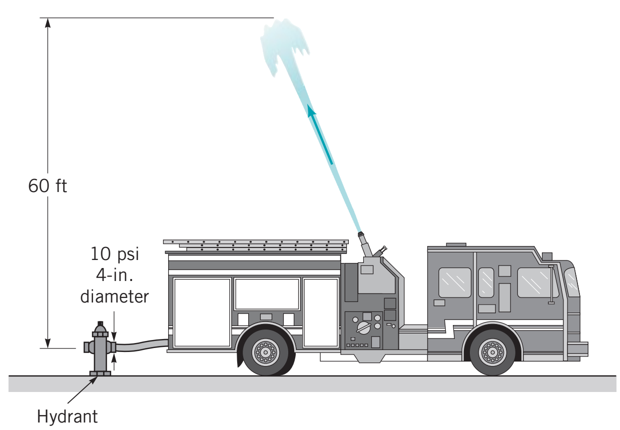
Section 5.3.2 Application of the Energy Equation–With Shaft Work

5.108 What is the maximum possible power output of the hydroelectric turbine shown in Fig.P5.108?



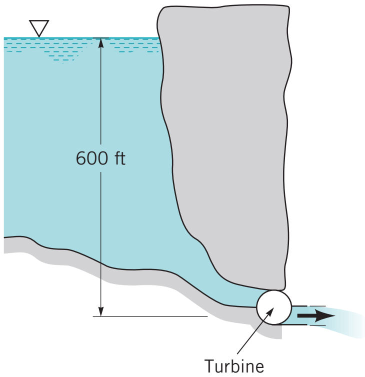
■ FIGURE P5.108

5.109 The pumper truck shown in Fig. P5.109 is to deliver 1.5 ft^3/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in.-diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.



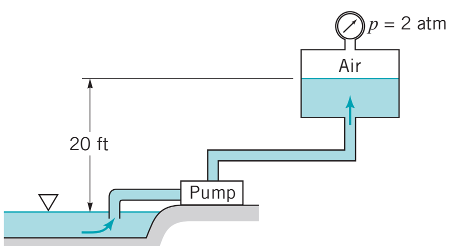
■ FIGURE P5.109

5.110 The hydroelectric turbine shown in Fig. P5.110 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.110

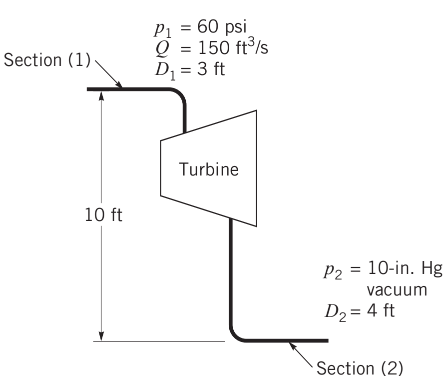
5.111 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.111 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.



■ FIGURE P5.111

5.112 A hydraulic turbine is provided with 4.25 m^3/s of water at 415 kPa. A vacuum gage in the turbine discharge 3 m below the turbine inlet centerline reads 250 mm Hg vacuum. If the turbine shaft output power is 1100 kW, calculate the power loss through the turbine. The supply and discharge pipe inside diameters are identically 80 mm.

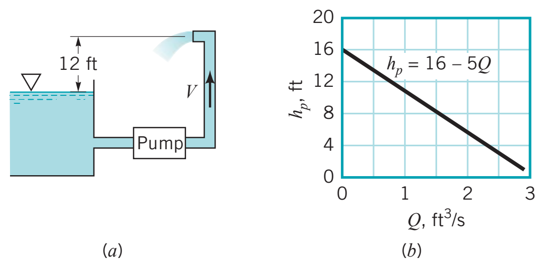
5.113 Water is supplied at 150 ft^3/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.113. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).



■ FIGURE P5.113

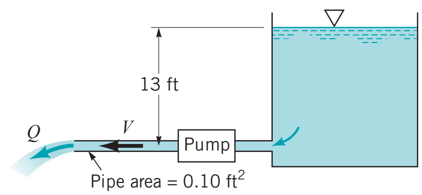
5.114 A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, estimate the actual and ideal stagnation temperature rise through the compressor. Estimate the ratio of ideal to actual temperature rise to obtain an approximate value of the efficiency.

5.115 Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.115*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.115*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.



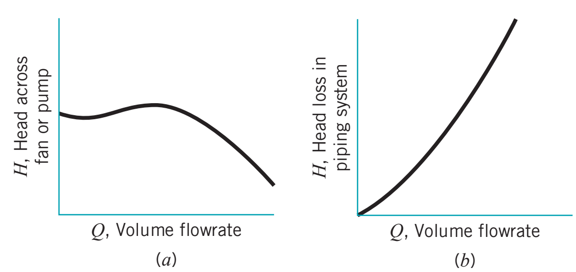
■ FIGURE P5.115

5.116 Water is pumped from the large tank shown in Fig. P5.116. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.



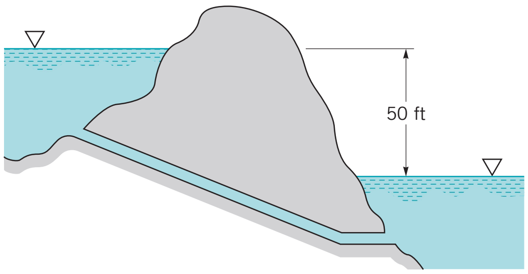
■ FIGURE P5.116

5.117 When a fan or pump is tested at the factory, head curves (head across the fan or pump versus volume flowrate) are often produced. A generic fan or pump head curve is shown in Fig.P5.117*a*. For any piping system, the drop in pressure or head involved because of loss can be estimated as a function of volume flowrate. A generic piping system loss curve is shown in Fig.P5.117*b*. When the pump or fan and piping system associated with the two curves of Fig.P5.117 are combined, what will the flowrate be? Why? How can the flowrate through this combined system be varied?



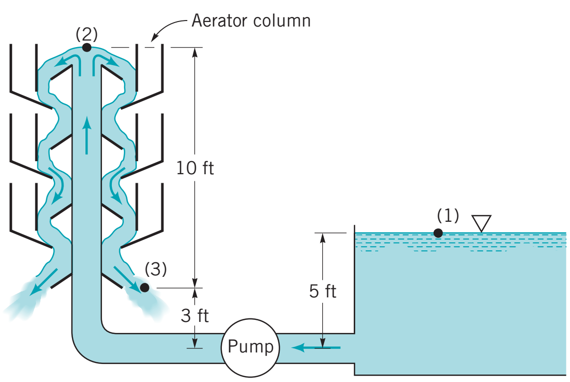
■ FIGURE P5.117

5.118 Water flows by gravity from one lake to another as sketched in Fig. P5.118 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.



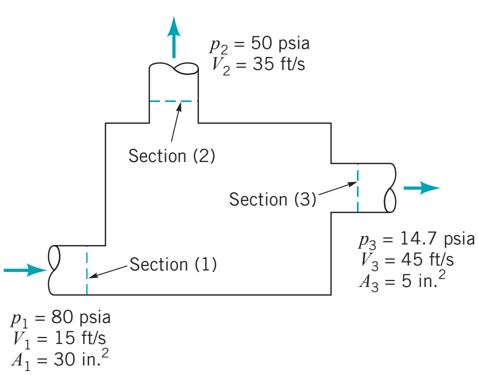
■ FIGURE P5.118

5.119 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.14** and Fig. P5.119 at a rate of 3.0 ft^3/s. (a) Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. (b) Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



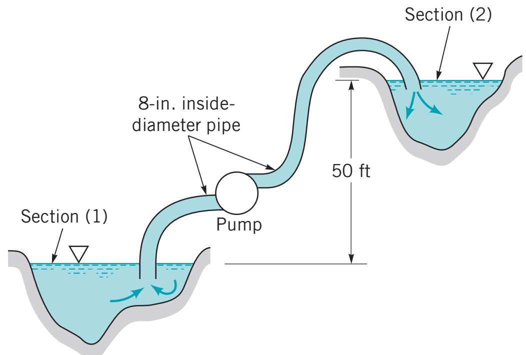
■ FIGURE P5.119

5.120 A liquid enters a fluid machine at section (1) and leaves at sections (2) and (3) as shown in Fig. P5.120. The density of the fluid is constant at 2 $slugs/ft^3$. All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. P5.120, determine the amount of shaft power involved.



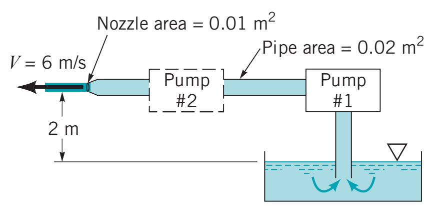
■ FIGURE P5.120

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss of available energy associated with 2.5 ft^3/s being pumped from sections (1) to (2) is loss $= 61\overline{V}^2/2$ ft^2/s^2, where $\overline{V}$ is the average velocity of water in the 8-in. inside diameter piping involved. Determine the amount of shaft power required.



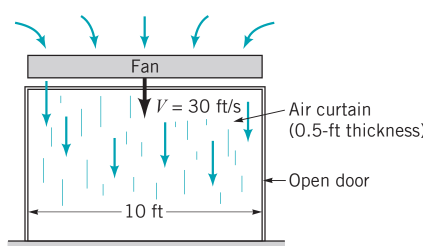
■ FIGURE P5.121

5.122 Water is to be pumped from the large tank shown in Fig. P5.122 with an exit velocity of 6 m/s. It was determined that the original pump (pump 1) that supplies 1 kW of power to the water did not produce the desired velocity. Hence, it is proposed that an additional pump (pump 2) be installed as indicated to increase the flowrate to the desired value. How much power must pump 2 add to the water? The head loss for this flow is $h_L = 250Q^2$, where h_L is in m when Q is in m^3/s.



■ FIGURE P5.122

5.123 (See Fluids in the News article titled **"Curtain of air,"** Section 5.3.3.) The fan shown in Fig. P5.123 produces an air curtain to separate a loading dock from a cold storage room. The air curtain is a jet of air 10 ft wide, 0.5 ft thick moving with speed $V = 30$ ft/s. The loss associated with this flow is loss $= K_L V^2/2$, where $K_L = 5$. How much power must the fan supply to the air to produce this flow?

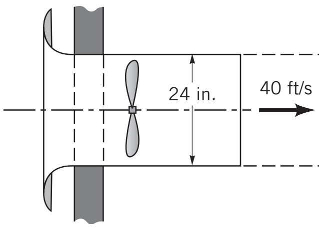


■ FIGURE P5.123

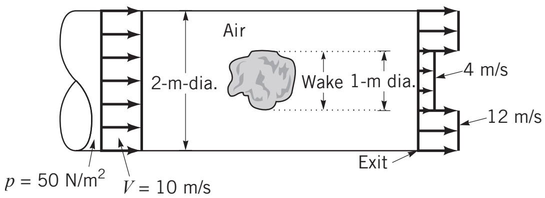
Section 5.3.2 Application of the Energy Equation—Combined with Linear momentum

5.124 If a $\frac{3}{4}$-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.124, estimate **(a)** the efficiency of the fan and **(b)** the thrust of the supporting member on the conduit enclosing the fan.

5.125 Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.125. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m^2, respectively. At the



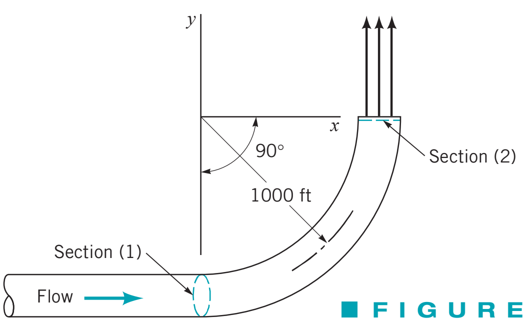
■ FIGURE P5.124



■ FIGURE P5.125

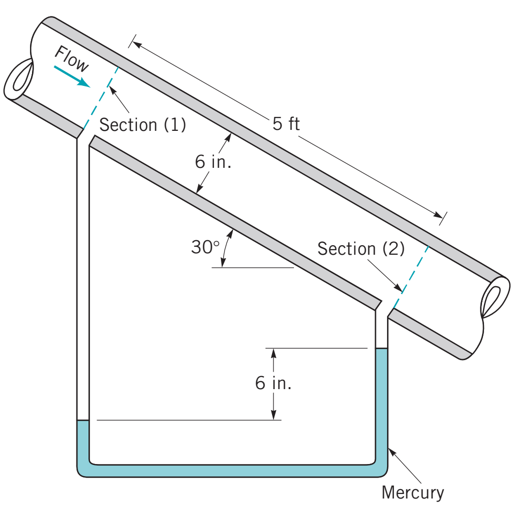
pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.

5.126 Water flows through a 2-ft-diameter pipe arranged horizontally in a circular arc as shown in Fig. P5.126. If the pipe discharges to the atmosphere ($p = 14.7$ psia) determine the x and y components of the resultant force exerted by the water on the piping between sections (1) and (2). The steady flowrate is 3000 ft^3/min. The loss in pressure due to fluid friction between sections (1) and (2) is 60 psi.



■ FIGURE P5.126

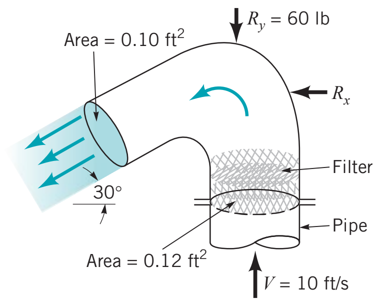
5.127 Water flows steadily down the inclined pipe as indicated in Fig. P5.127. Determine the following: **(a)** the difference in pressure



■ FIGURE P5.127

$p_1 - p_2$, **(b)** the loss between sections (1) and (2), **(c)** the net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

5.128 Water flows steadily in a pipe and exits as a free jet through an end cap that contains a filter as shown in Fig. P5.128. The flow is in a horizontal plane. The axial component, R_y, of the anchoring force needed to keep the end cap stationary is 60 lb. Determine the head loss for the flow through the end cap.

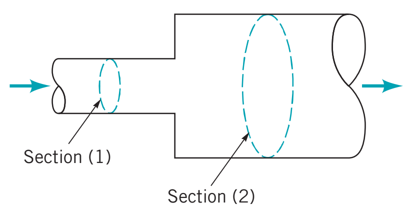


■ FIGURE P5.128

5.129 When fluid flows through an abrupt expansion as indicated in Fig. P5.129, the loss in available energy across the expansion, loss_{ex}, is often expressed as

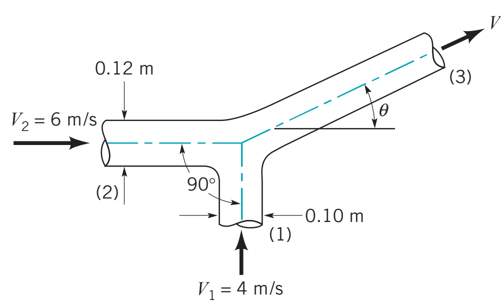
$$\text{loss}_{\text{ex}} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.



■ FIGURE P5.129

5.130 Two water jets collide and form one homogeneous jet as shown in Fig. P5.130. **(a)** Determine the speed, V, and direction, θ, of the combined jet. **(b)** Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.130

Section 5.3.4 Application of the Energy Equation to Nonuniform Flows

5.131 Water flows vertically upward in a circular cross-sectional pipe. At section (1), the velocity profile over the cross-sectional area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c\left(\frac{R-r}{R}\right)^{1/7}\hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).

5.132 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for **(a)** $n = 5$, **(b)** $n = 6$, **(c)** $n = 7$, **(d)** $n = 8$, **(e)** $n = 9$, **(f)** $n = 10$.

5.133 A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient, α_2, is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: **(a)** assuming uniform velocity distributions, **(b)** considering actual velocity distributions.

Section 5.3.5 Combination of the Energy Equation and the Moment-of-Momentum Equation

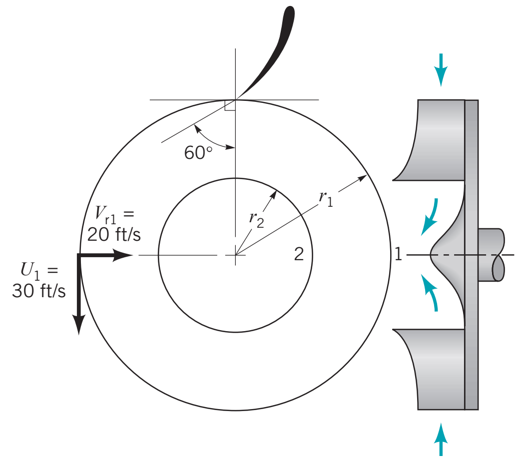
5.134 Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity, V_θ, of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

5.135 Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm, and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

5.136 Water enters an axial-flow turbine rotor with an absolute velocity tangential component, V_θ, of 15 ft/s. The corresponding blade velocity, U, is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

5.137 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.

5.138 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.



■ FIGURE P5.137

Section 5.4 Second Law of Thermodynamics—Irreversible Flow

5.139 Why do all actual fluid flows involve loss of available energy?

■ Lab Problems

5.140 This problem involves the force that a jet of air exerts on a flat plate as the air is deflected by the plate. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

5.141 This problem involves the pressure distribution produced on a flat plate that deflects a jet of air. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

5.142 This problem involves the force that a jet of water exerts on a vane when the vane turns the jet through a given angle. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

5.143 This problem involves the force needed to hold a pipe elbow stationary. To proceed with this problem, go to Appendix H which is located on the book's web site, www.wiley.com/college/munson.

■ Life Long Learning Problems

5.144 What are typical efficiencies associated with swimming and how can they be improved?

5.145 Explain how local ionization of flowing air can accelerate it. How can this be useful?

5.146 Discuss the main causes of loss of available energy in a turbo-pump and how they can be minimized. What are typical turbo-pump efficiencies?

5.147 Discuss the main causes of loss of available energy in a turbine and how they can be minimized. What are typical turbine efficiencies?

■ FE Exam Problems

Sample FE (Fundamentals of Engineering) exam questions for fluid mechanics are provided on the book's web site, www.wiley.com/college/munson.