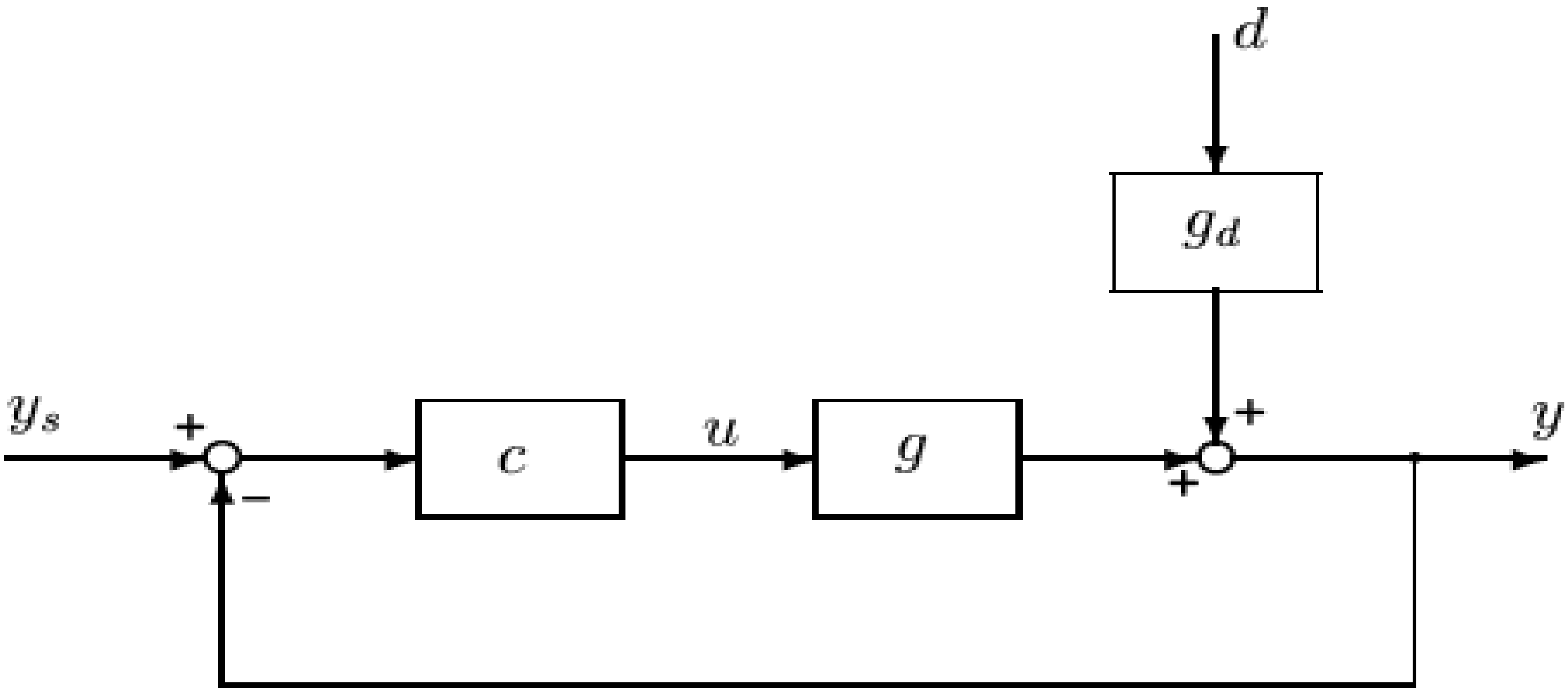
CONTROLLER



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On-Off Controllers

- Simple
- Cheap
- Used In residential heating and domestic refrigerators
- Limited use in process control due to continuous cycling of controlled variable $\Rightarrow$ excessive wear on control valve.

On-Off Controllers (continued)

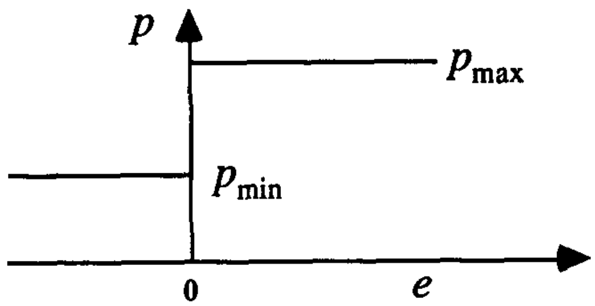
Synonyms:

"two-position" or "bang-bang" controllers.

$$p(t) = \begin{cases} p_{max} & if \quad e > 0 \\ p_{min} & if \quad e < 0 \end{cases}$$ **ideal case**

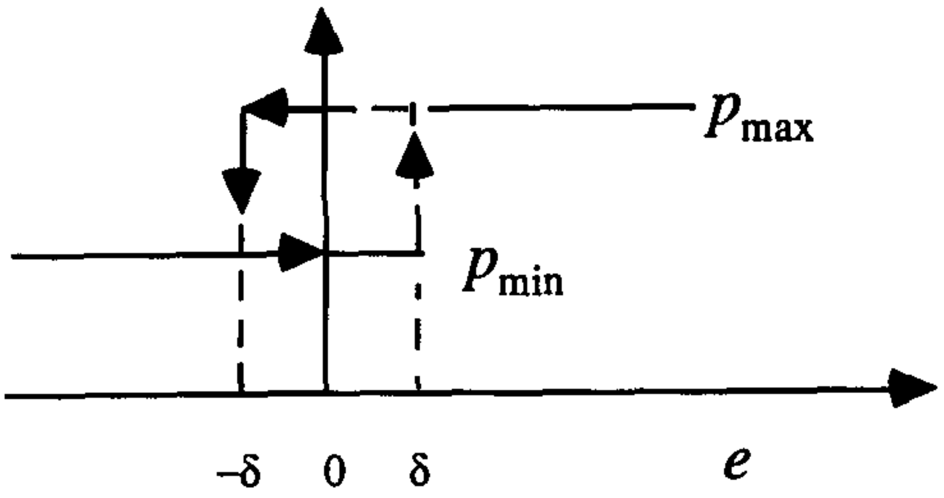
p_{max} **is the "on" value**

p_{min} **is the "off" value**



Controller output has two possible values.

Practical case (dead band)



$$p(t) = \begin{cases} P_{max} & for \quad e > \delta \\ P_{min} & for \quad e < -\delta \end{cases}$$

Feedback Controllers

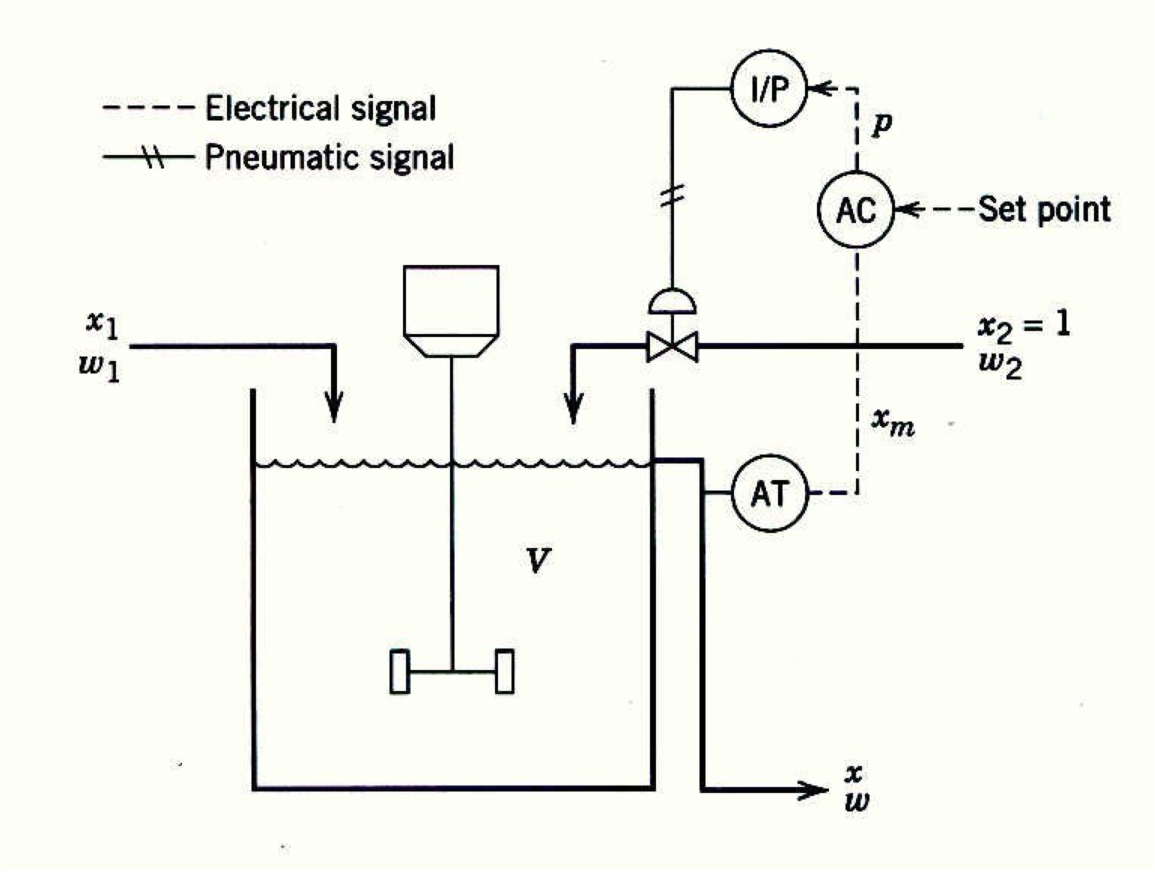
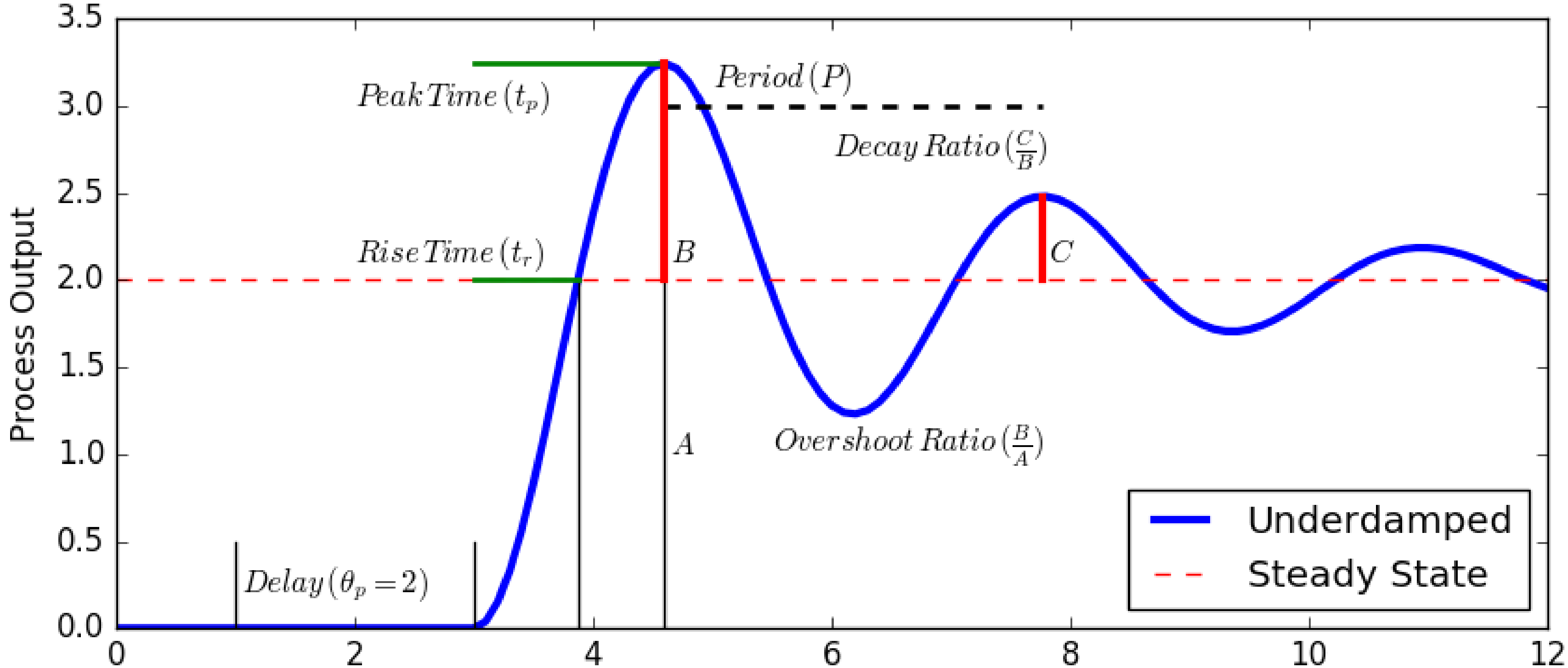
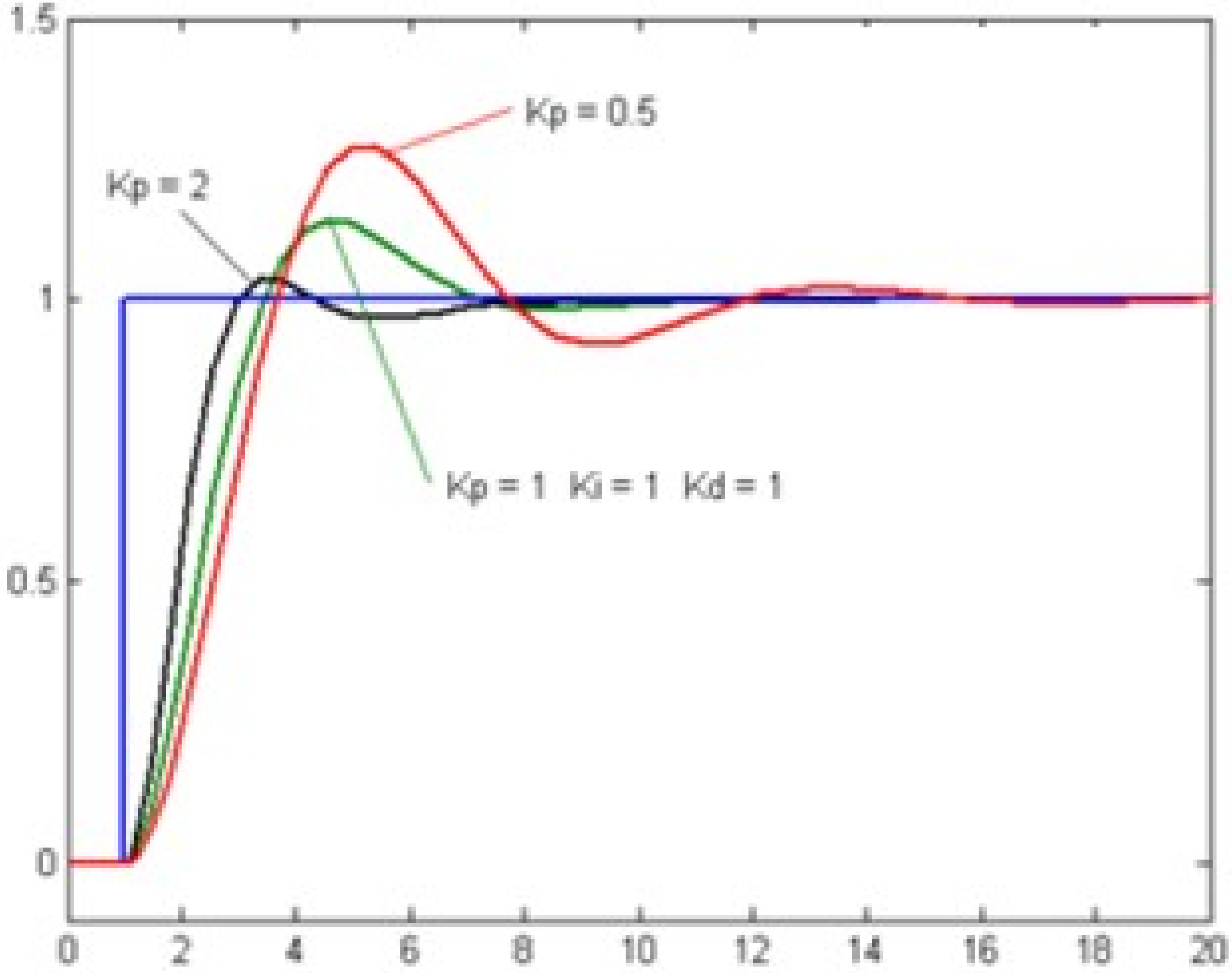


Figure 8.1 Schematic diagram for a stirred-tank blending system.

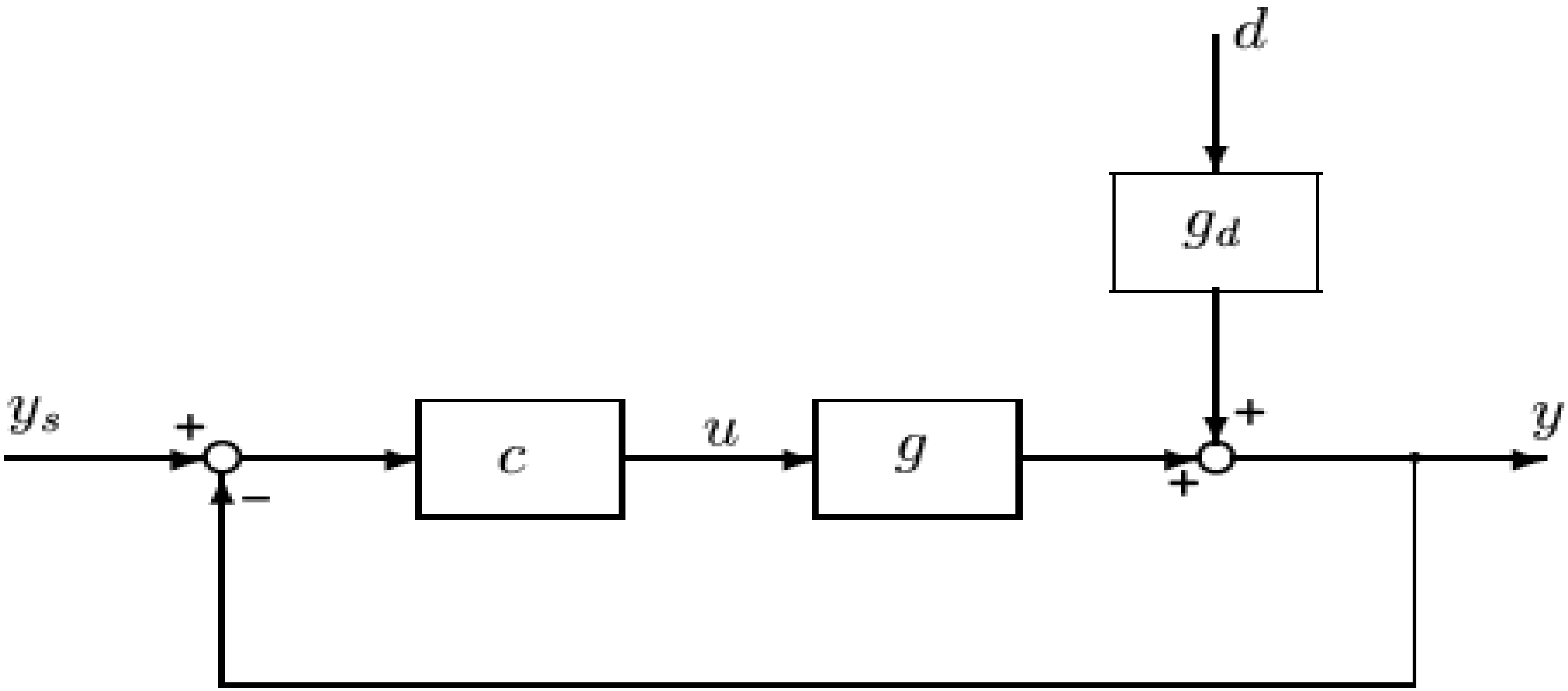


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Process Output
0
2
4
6
8
10
12
$Peak\,Time\,(t_p)$
$Period\,(P)$
$Decay\,Ratio\,(\frac{C}{B})$
$Rise\,Time\,(t_r)$
B
C
A
$Overshoot\,Ratio\,(\frac{B}{A})$
$Delay\,(\theta_p = 2)$
Underdamped
Steady State



1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20
Kp = 0.5
Kp = 2
Kp = 1 Ki = 1 Kd = 1

PID CONTROLLER



PID Control Algorithm

$$c(t) = c_0 + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t)\,dt + \tau_D \frac{de(t)}{dt} \right]$$

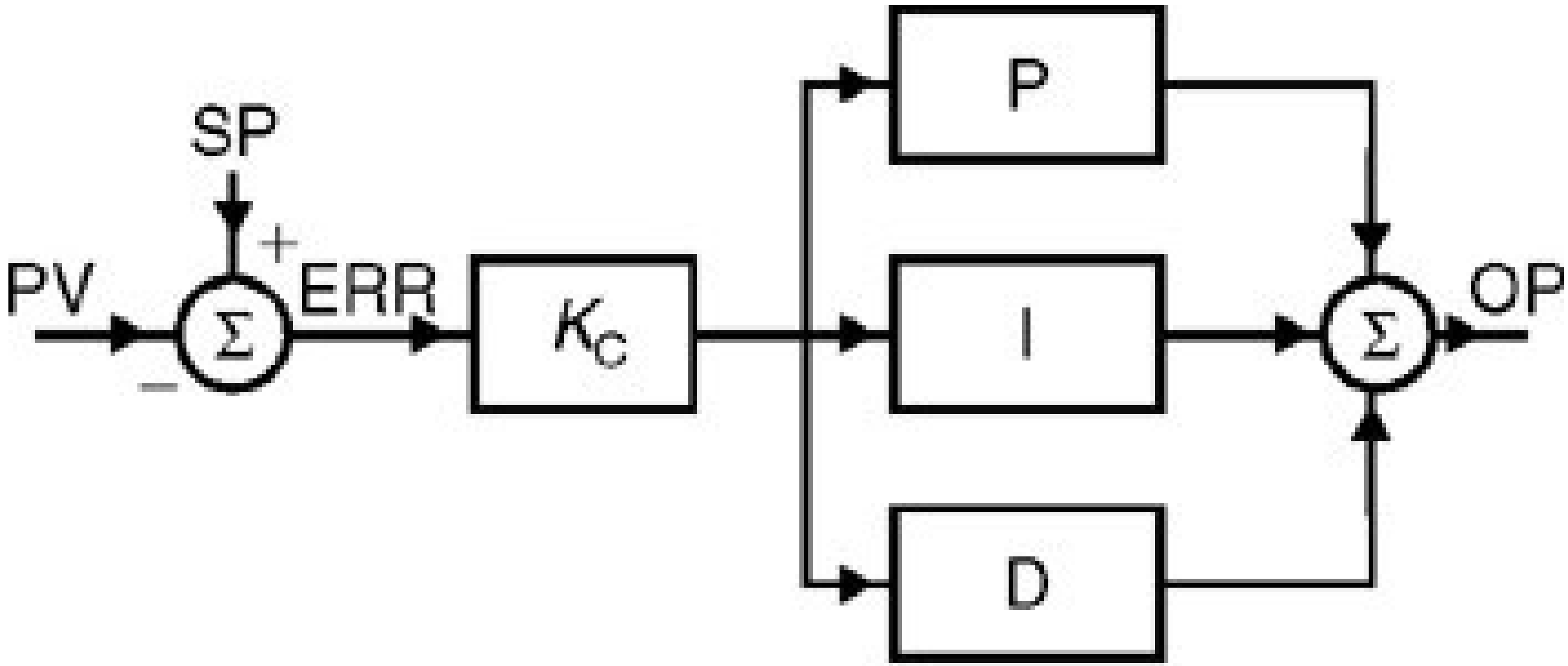
where $e(t) = y_{sp} - y_s(t)$

Definition of Terms

- $e(t)$- the error from setpoint [$e(t)=y_{sp}-y_s$].
- K_c- the controller gain is a tuning parameter and largely determines the controller aggressiveness.
- τ_I- the reset time is a tuning parameter and determines the amount of integral action.
- τ_D- the derivative time is a tuning parameter and determines the amount of derivative action.

Transfer Function for a PID Controller

$$G_c(s) = \frac{C(s)}{E(s)} = K_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]$$



$$OP = K_C \times \left(T_{DER} + \frac{1}{T_{INT}} + 1 \right)$$

$$OP = \text{Gain} \times (\text{D-control} + \text{I-control} + \text{P-control})$$

Basic Control Modes

Next we consider the three basic control modes starting with the simplest mode, *proportional control*.

Proportional Control

In feedback control, the objective is to reduce the error signal to zero where

$$e(t) = y_{sp}(t) - y_m(t) \qquad (8\text{-}1)$$

and

$e(t)$ = error signal

$y_{sp}(t)$ = set point

$y_m(t)$ = measured value of the controlled variable (or equivalent signal from the sensor/transmitter)

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Although Eq. 8-1 indicates that the set point can be time-varying, in many process control problems it is kept constant for long periods of time.

For proportional control, the controller output is proportional to the error signal,

$$p(t) = \bar{p} + K_c e(t) \qquad (8\text{-}2)$$

where:

$p(t) =$ controller output

$\bar{p} =$ bias (steady-state) value

$K_c =$ controller gain (usually dimensionless)

The key concepts behind proportional control are the following:

1. The controller gain can be adjusted to make the controller output changes as sensitive as desired to deviations between set point and controlled variable;
2. the sign of K_c can be chosed to make the controller output increase (or decrease) as the error signal increases.

For proportional controllers, bias $\bar{p}$ can be adjusted, a procedure referred to as *manual reset*.

Some controllers have a proportional band setting instead of a controller gain. The *proportional band PB* (in %) is defined as

$$PB \triangleq \frac{100\%}{K_c} \qquad (8\text{-}3)$$

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In order to derive the transfer function for an ideal proportional controller (without saturation limits), define a deviation variable $p'(t)$ as

$$p'(t) \triangleq p(t) - \bar{p} \tag{8-4}$$

Then Eq. 8-2 can be written as

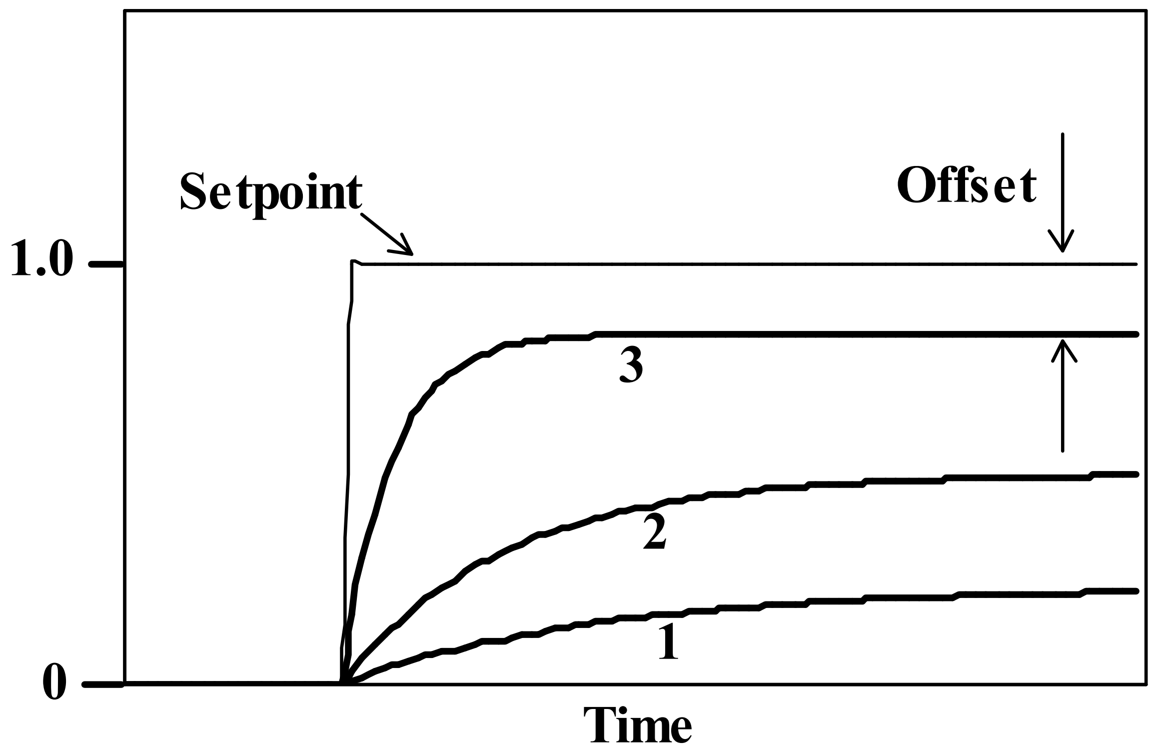
$$p'(t) = K_c e(t) \tag{8-5}$$

The transfer function for proportional-only control:

$$\frac{P'(s)}{E(s)} = K_c \tag{8-6}$$

An inherent disadvantage of proportional-only control is that a steady-state error occurs after a set-point change or a sustained disturbance.

Offset Resulting from P-only Control



Integral Control

For integral control action, the controller output depends on the integral of the error signal over time,

$$p(t) = \bar{p} + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^* \qquad (8\text{-}7)$$

where τ_I, an adjustable parameter referred to as the integral time or reset time, has units of time.

Integral control action is widely used because it provides an important practical advantage, the elimination of offset. Consequently, integral control action is normally used in conjunction with proportional control as the *proportional-integral (PI)* controller:

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^*\right) \qquad (8\text{-}8)$$

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The corresponding transfer function for the PI controller in Eq. 8-8 is given by

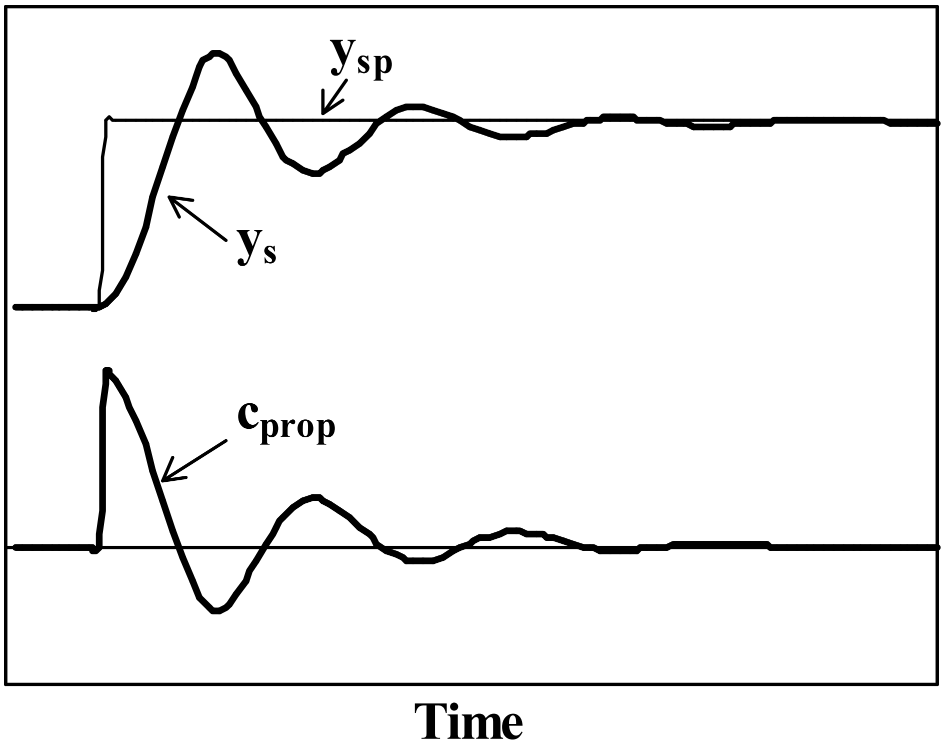
$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right) \qquad (8\text{-}9)$$

Some commercial controllers are calibrated in terms of $1/\tau_I$ (repeats per minute) rather than τ_I (minutes, or minutes per repeat).

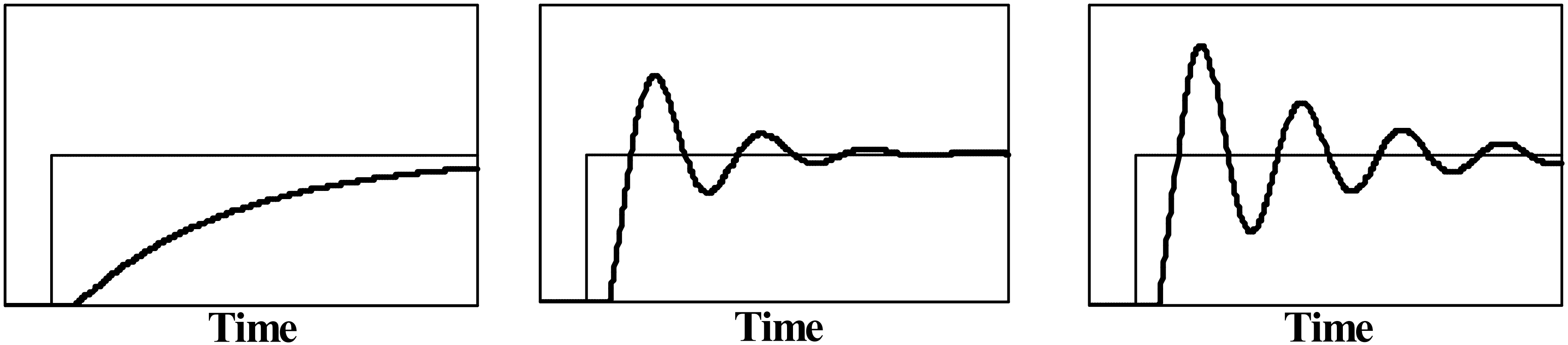
Reset Windup

- An inherent disadvantage of integral control action is a phenomenon known as *reset windup* or *integral windup*.
- Recall that the integral mode causes the controller output to change as long as $e(t^*) \neq 0$ in Eq. 8-8.

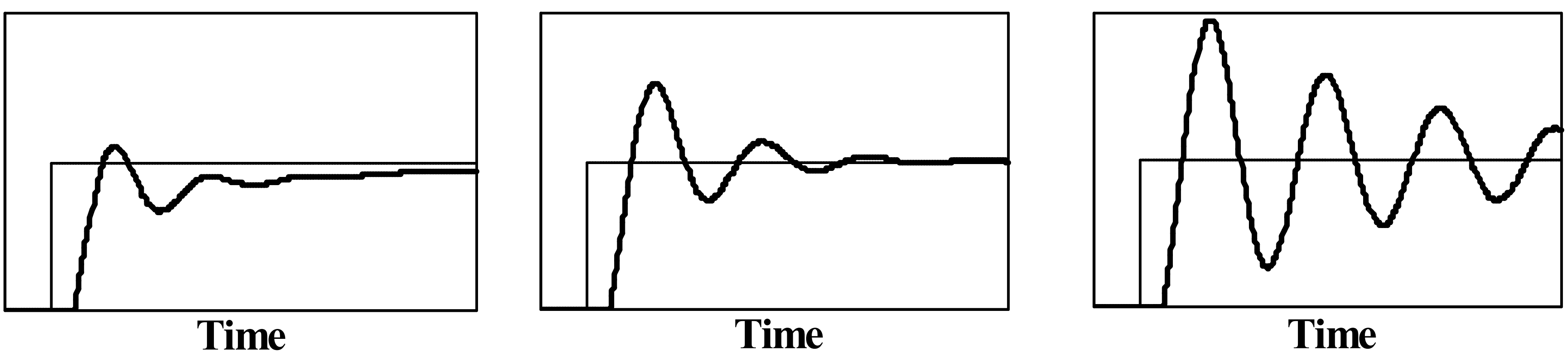
- When a sustained error occurs, the integral term becomes quite large and the controller output eventually saturates.
- Further buildup of the integral term while the controller is saturated is referred to as reset windup or *integral windup*.



Effect of Variations in K_c



Effect of Variations in t_I



Derivative Control

The function of derivative control action is to anticipate the future behavior of the error signal by considering its rate of change.

- The anticipatory strategy used by the experienced operator can be incorporated in automatic controllers by making the controller output proportional to the rate of change of the error signal or the controlled variable.

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- Thus, for *ideal* derivative action,

$$p(t) = \bar{p} + \tau_D \frac{de(t)}{dt} \qquad (8\text{-}10)$$

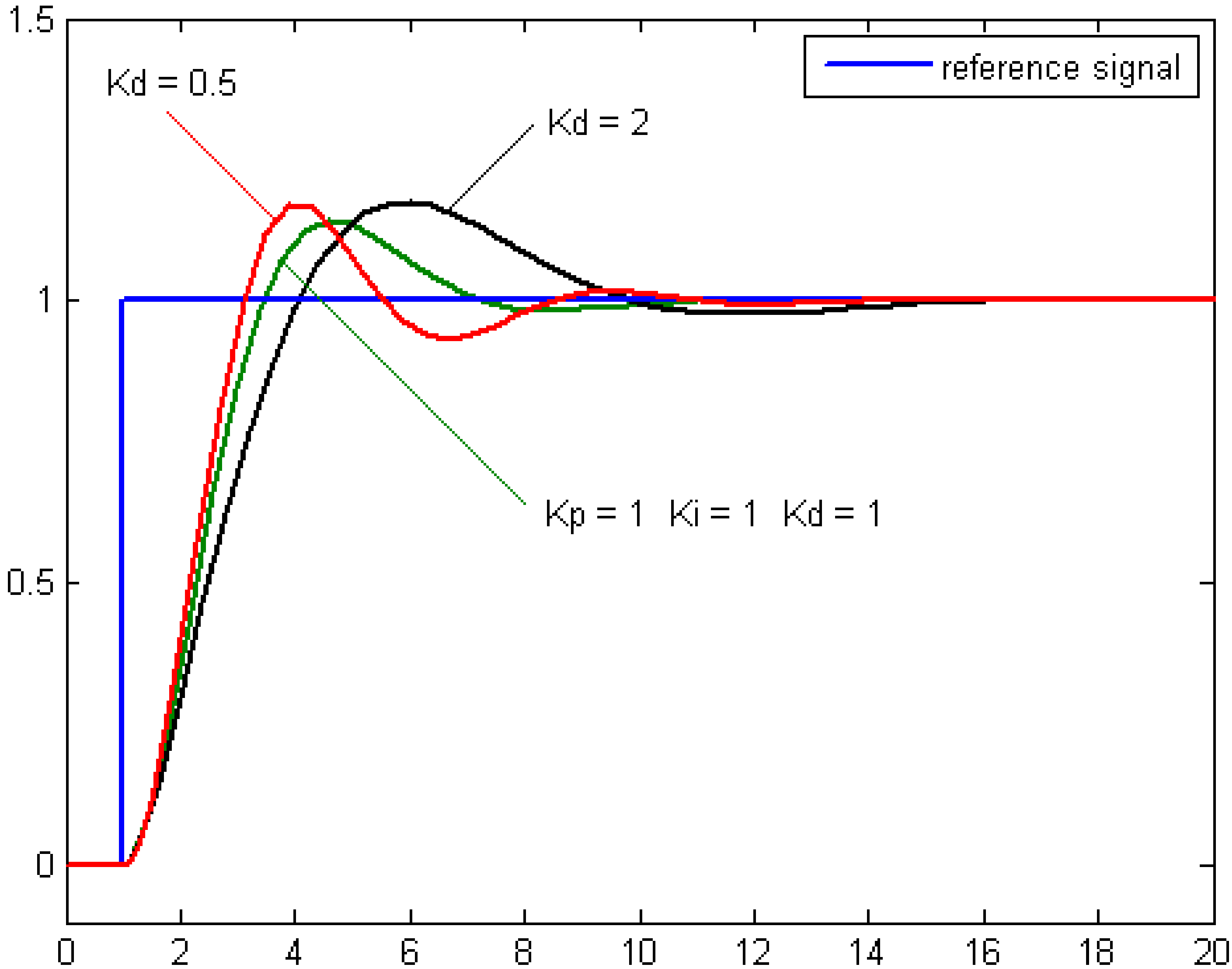
where τ_D, the derivative time, has units of time.

For example, an ideal PD controller has the transfer function:

$$\frac{P'(s)}{E(s)} = K_c(1 + \tau_D s) \qquad (8\text{-}11)$$

- By providing anticipatory control action, the derivative mode tends to stabilize the controlled process.

- Unfortunately, the ideal proportional-derivative control algorithm in Eq. 8-10 is *physically unrealizable* because it cannot be implemented exactly.



reference signal
Kd = 0.5
Kd = 2
Kp = 1 Ki = 1 Kd = 1
1.5
1
0.5
0
0 2 4 6 8 10 12 14 16 18 20

Proportional-Integral-Derivative (PID) Control

Now we consider the combination of the proportional, integral, and derivative control modes as a PID controller.

- Many variations of PID control are used in practice.
- Next, we consider the three most common forms.

Parallel Form of PID Control

The *parallel form* of the PID control algorithm (without a derivative filter) is given by

$$p(t) = \bar{p} + K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^* + \tau_D\frac{de(t)}{dt}\right] \qquad (8\text{-}13)$$

The corresponding transfer function is:

$$\frac{P'(s)}{E(s)} = K_c\left[1+\frac{1}{\tau_I s}+\tau_D s\right] \tag{8-14}$$

Series Form of PID Control

Historically, it was convenient to construct early analog controllers (both electronic and pneumatic) so that a PI element and a PD element operated in series.

Commercial versions of the series-form controller have a derivative filter that is applied to either the derivative term, as in Eq. 8-12, or to the PD term, as in Eq. 8-15:

$$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\alpha\tau_D s+1}\right) \tag{8-15}$$

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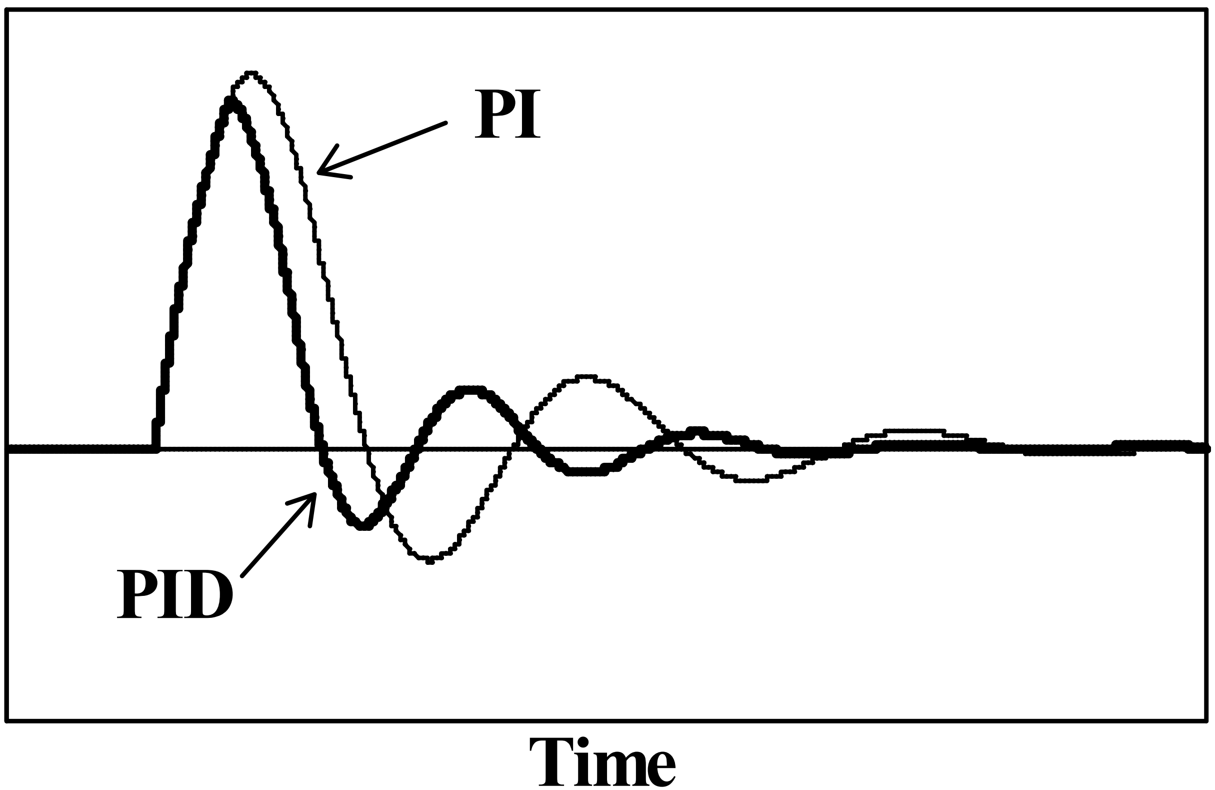
Expanded Form of PID Control

In addition to the well-known series and parallel forms, the *expanded form* of PID control in Eq. 8-16 is sometimes used:

$$p(t) = \bar{p} + K_c e(t) + K_I \int_0^t e(t^*)\, dt^* + K_D \frac{de(t)}{dt} \tag{8-16}$$



Controller Comparison

P - Simplest controller to tune (K_c).
- Offset with sustained disturbance or setpoint change.

PI - More complicated to tune (K_c, τ_I) .
- Better performance than P
- No offset
- Most popular FB controller

PID - Most complicated to tune (K_c, τ_I, τ_D) .
- Better performance than PI
- No offset
- Derivative action may be affected by noise

Typical Response of Feedback Control Systems

Consider response of a controlled system after a sustained disturbance occurs (e.g., step change in the disturbance variable)

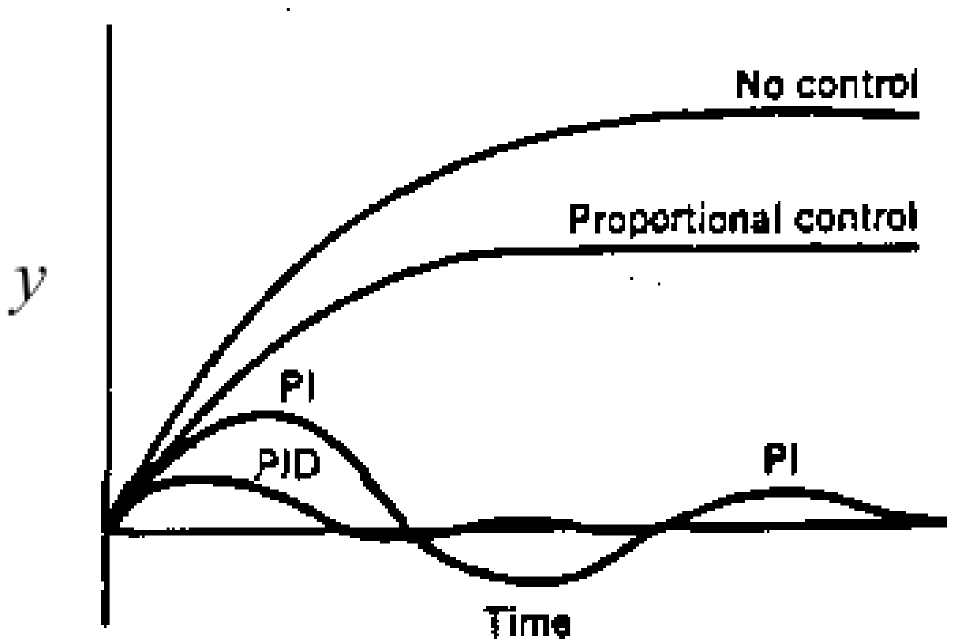


Figure 8.12. Typical process responses with feedback control.

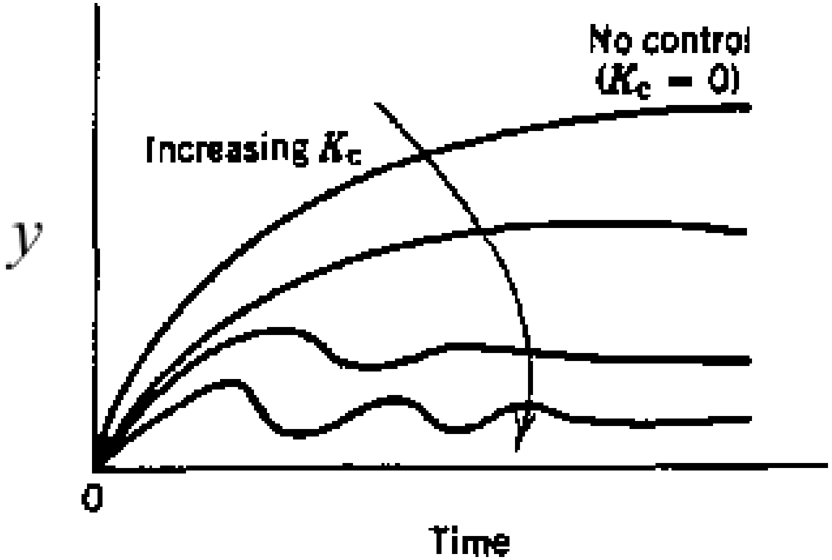
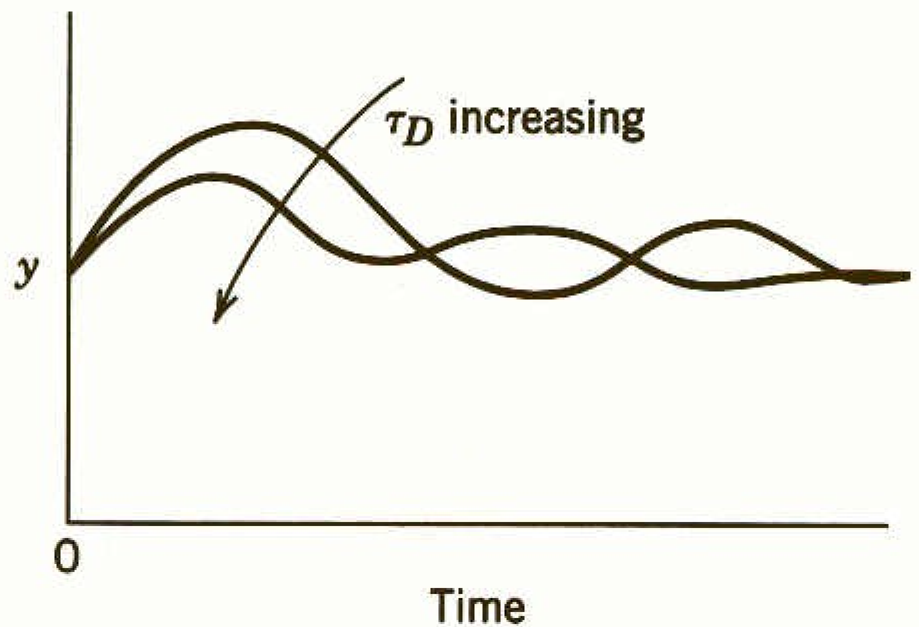


Figure 8.13. Proportional control: effect of controller gain.

Figure 8.15. PID control: effect of derivative time.



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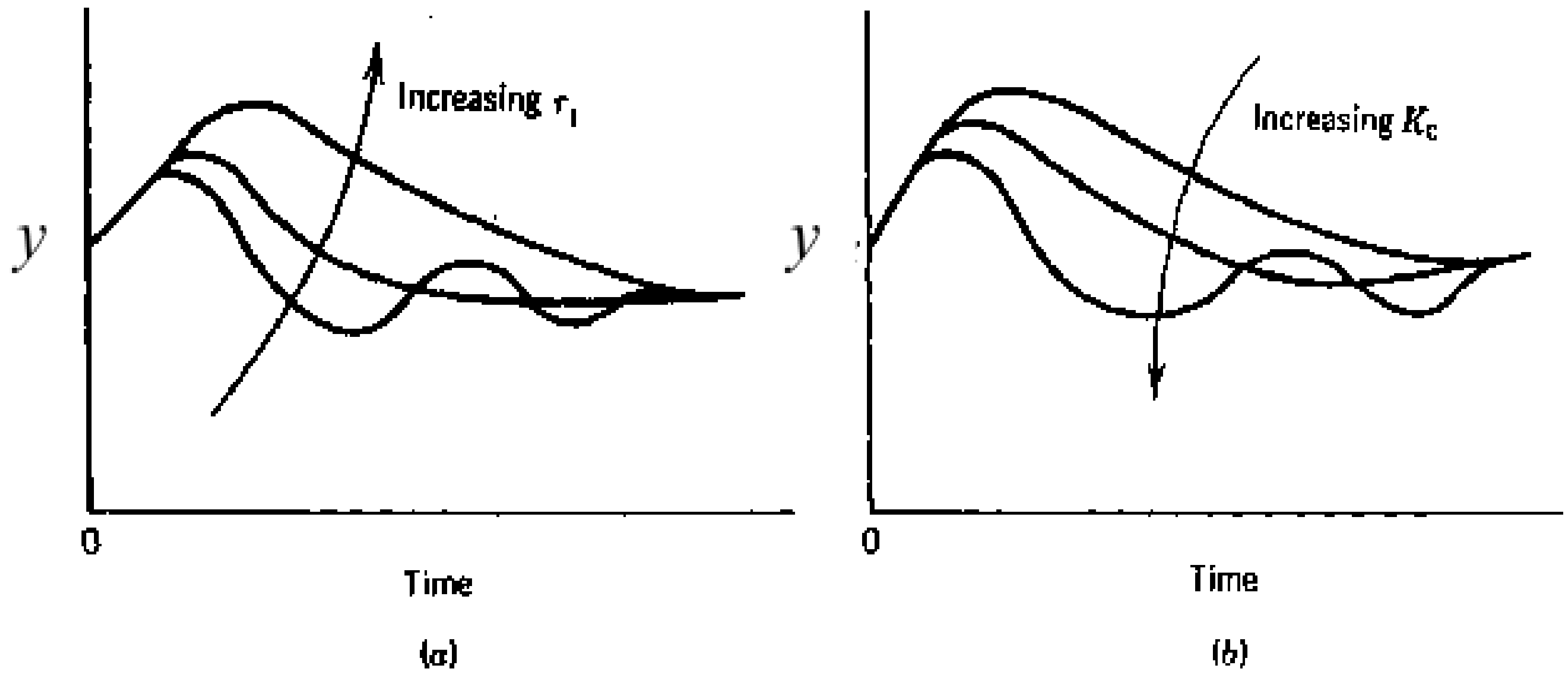

Figure 8.14. PI control: (a) effect of reset time (b) effect of controller gain.

Controller Tuning: A Motivational Example

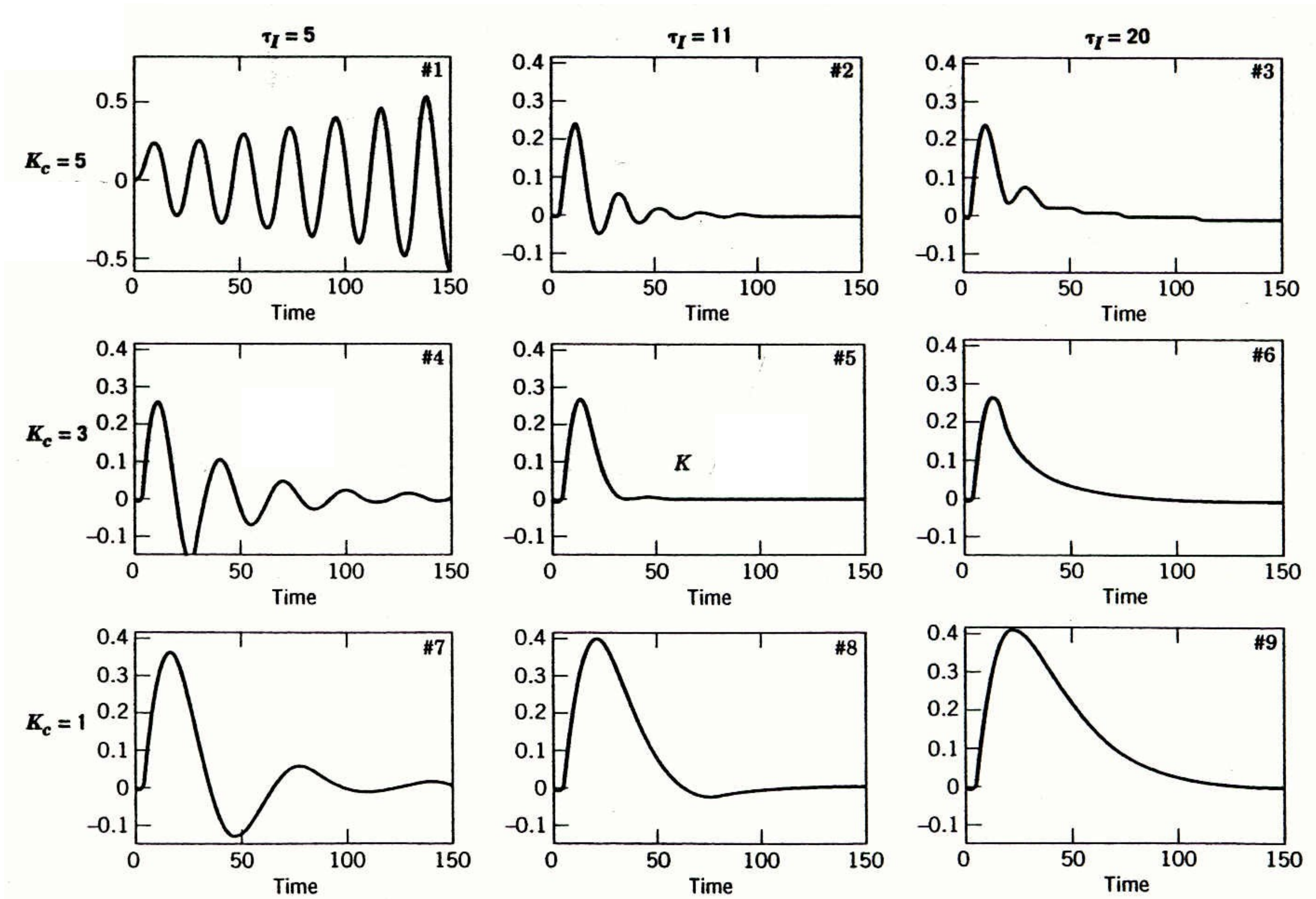

Fig. 12.1. Unit-step disturbance responses for the candidate controllers (FOPTD Model: K = 1, $\theta = 4$, $\tau = 20$).