Topic 1: Basics of Power Systems

ECE 5332: Communications and Control for Smart

Spring 2012

Power Systems

- The Four Main Elements in Power Systems:
  - Power Production / Generation
  - Power Transmission
  - Power Distribution
  - Power Consumption / Load

- Of course, we also need monitoring and control systems.

Power Systems

- Power Production:
  - Different Types:
    - Traditional
    - Renewable
  - Capacity, Cost, Carbon Emission
  - Step-up Transformers

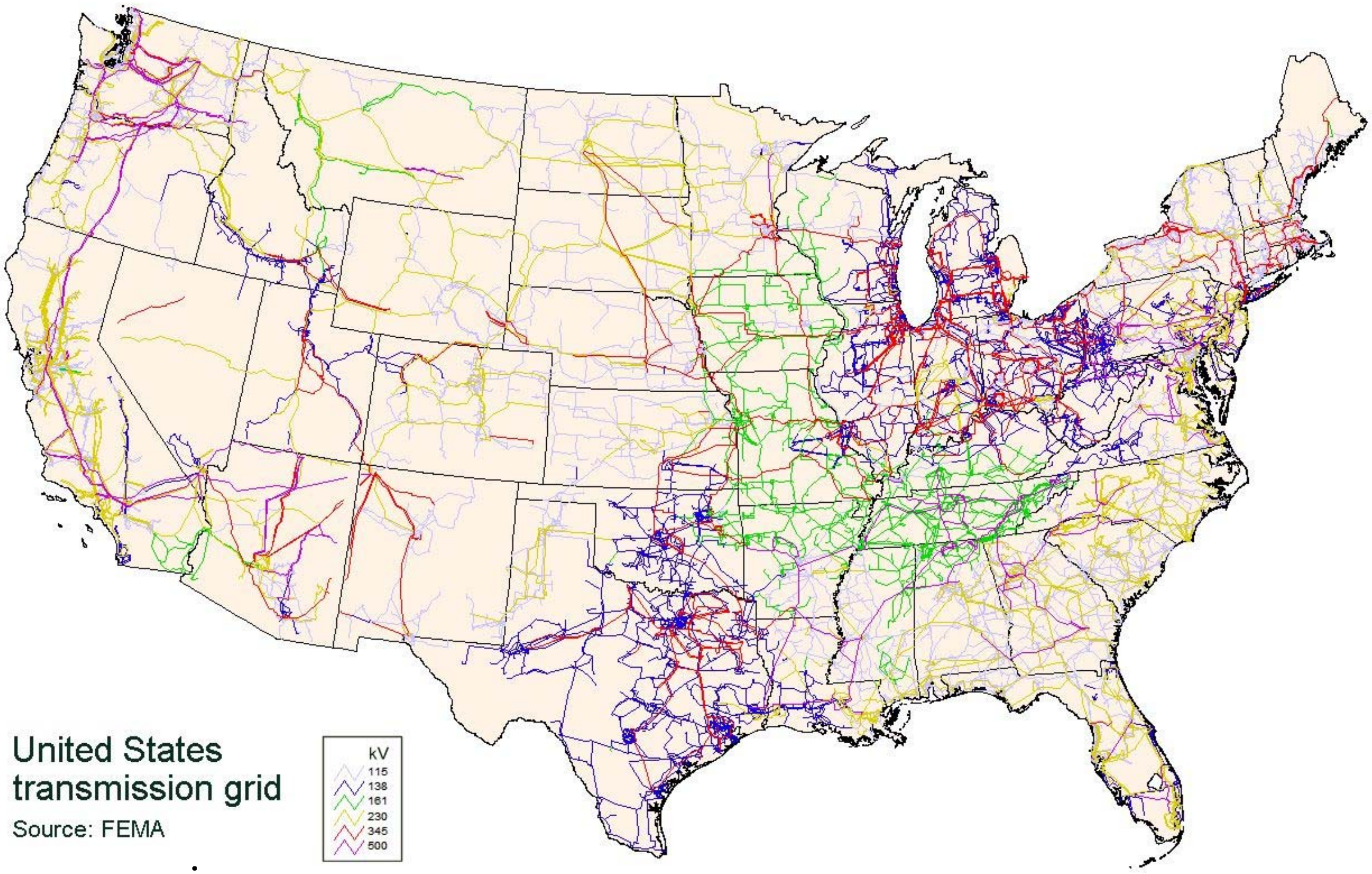
Power Systems

- Power Transmission:
  - High Voltage (HV) Transmission Lines
  - Several Hundred Miles

  - Switching Stations
    - Transformers
    - Circuit Breakers

Power Systems

- The Power Transmission Grid in the United States:



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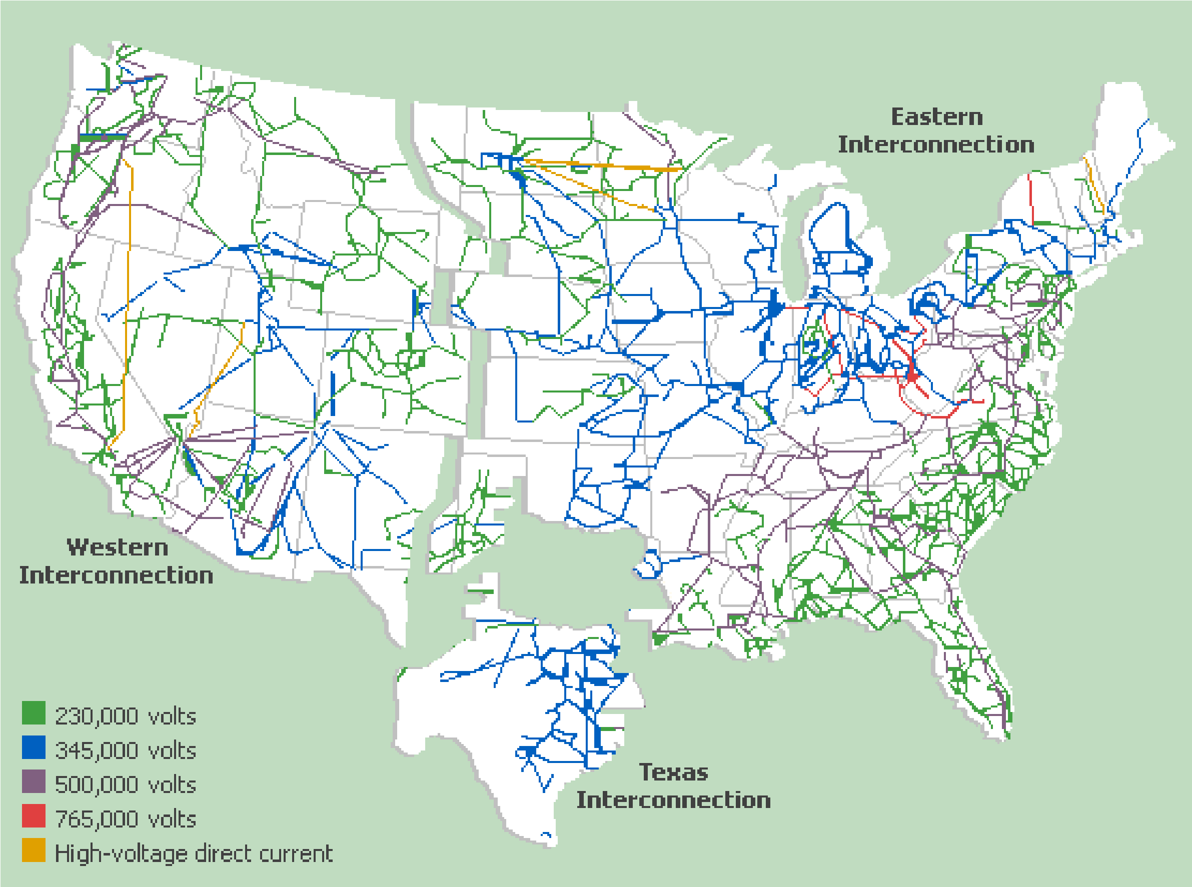
Power Systems

- Major Inter-connections in the United States:



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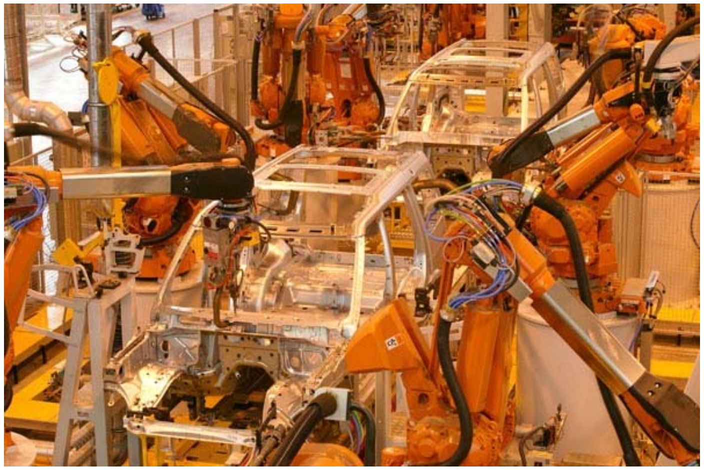
Power Systems

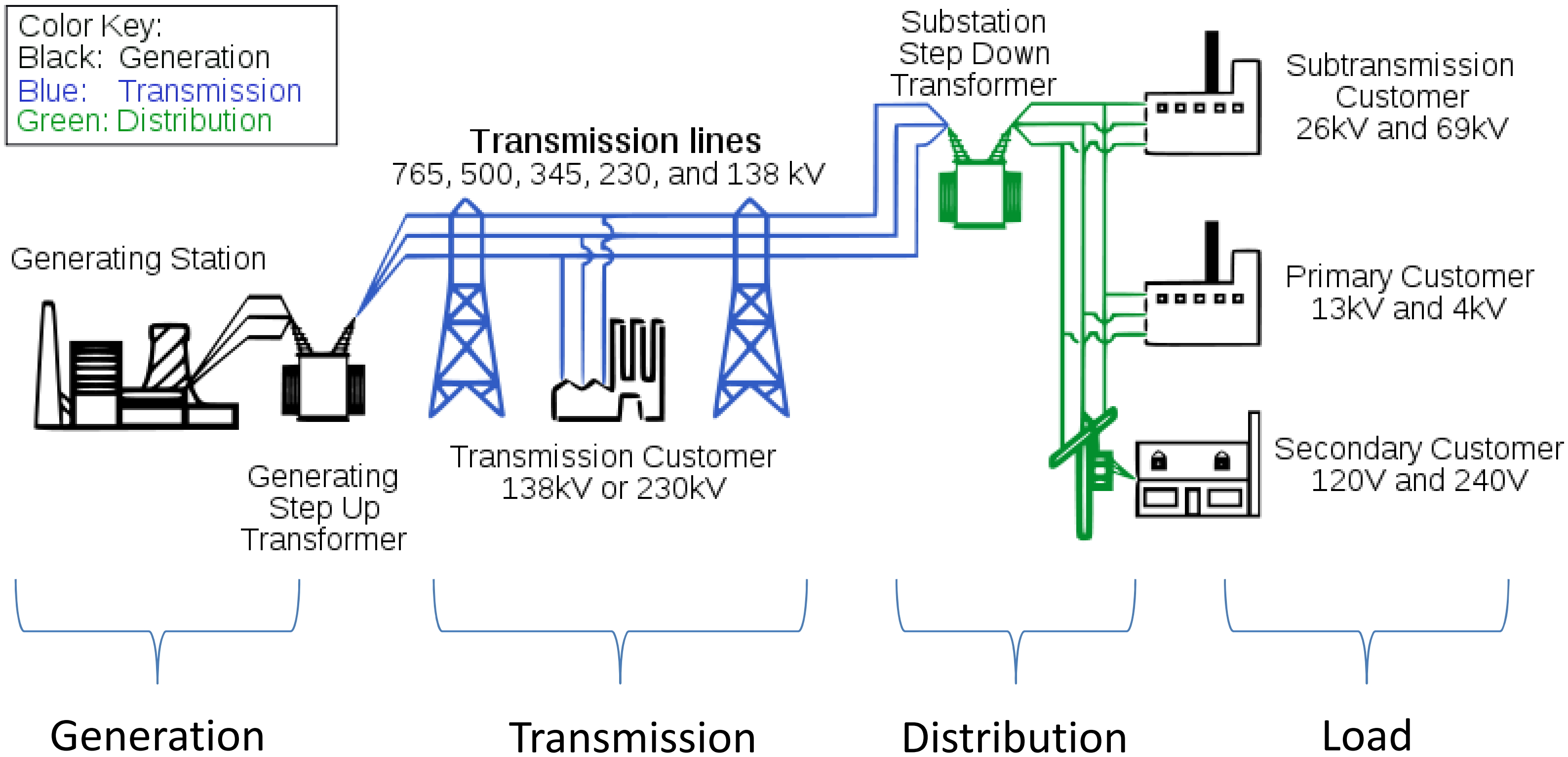
- Power Distribution:
  - Medium Voltage (MV) Transmission Lines (< 50 kV)
  - Power Deliver to Load Locations
  - Interface with Consumers / Metering
  - Distribution Sub-stations
    - Step-Down Transformers
    - Distribution Transformers

Power Systems

- Power Consumption:
  - Industrial
  - Commercial
  - Residential
  - Demand Response
    - Controllable Load
    - Non-Controllable



Power Systems



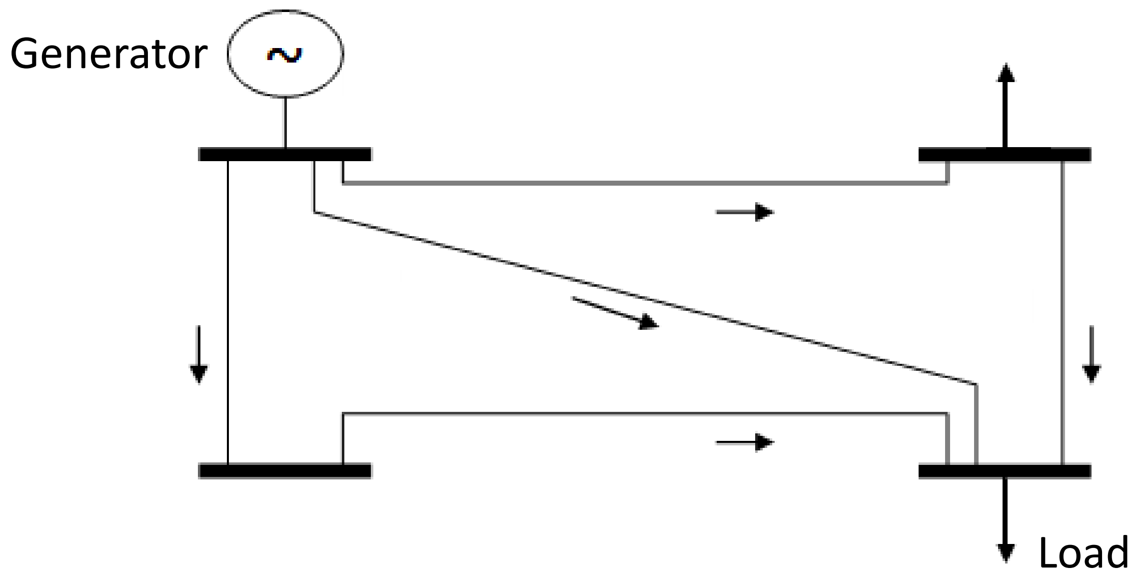
Power Systems

- Power System Control:
  - Data Collection: Sensors, PMUs, etc.
  - Decision Making: Controllers
  - Actuators: Circuit Breakers, etc.

Power Grid Graph Representation

Nodes: Buses

Links: Transmission Lines



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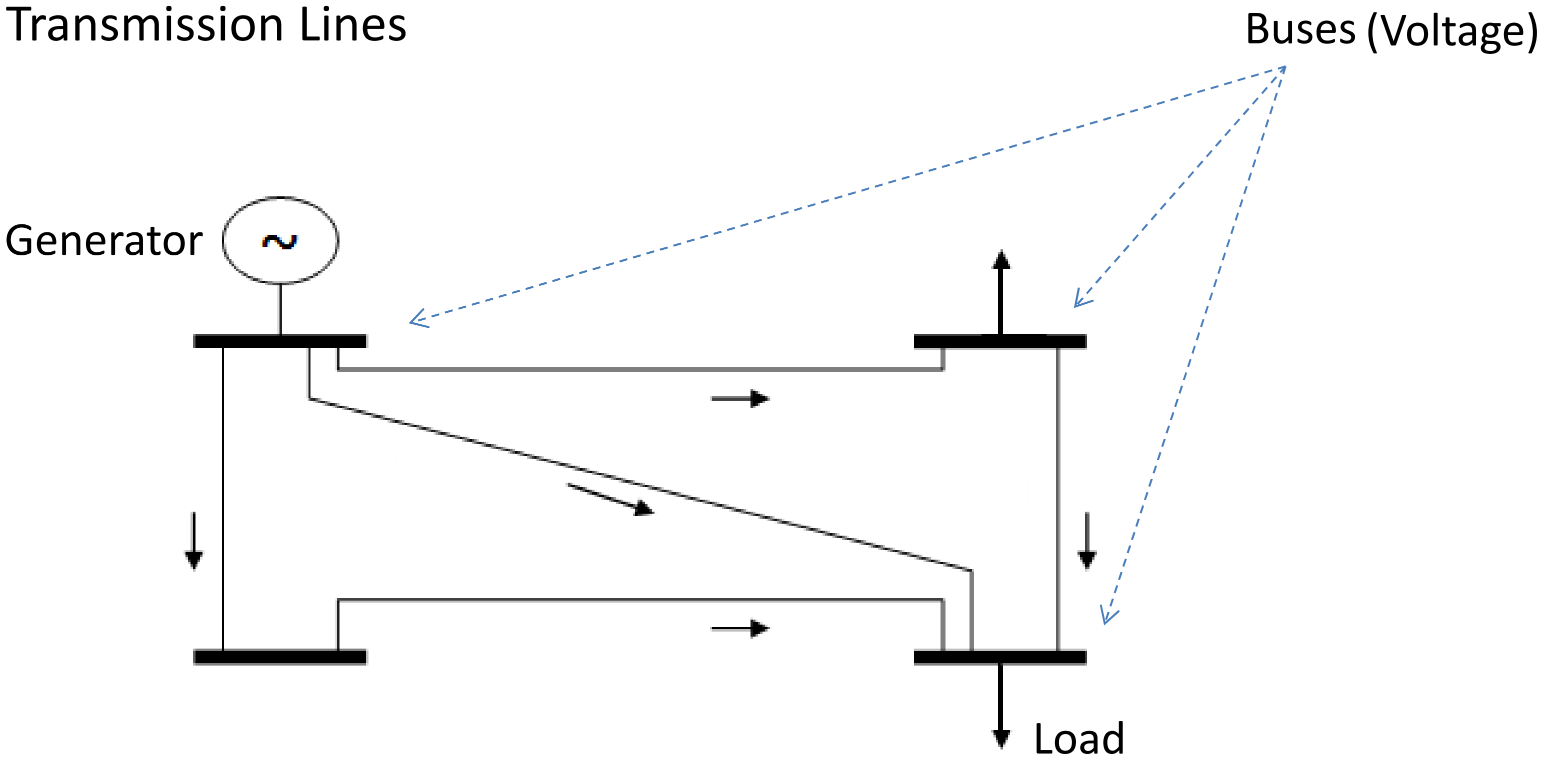

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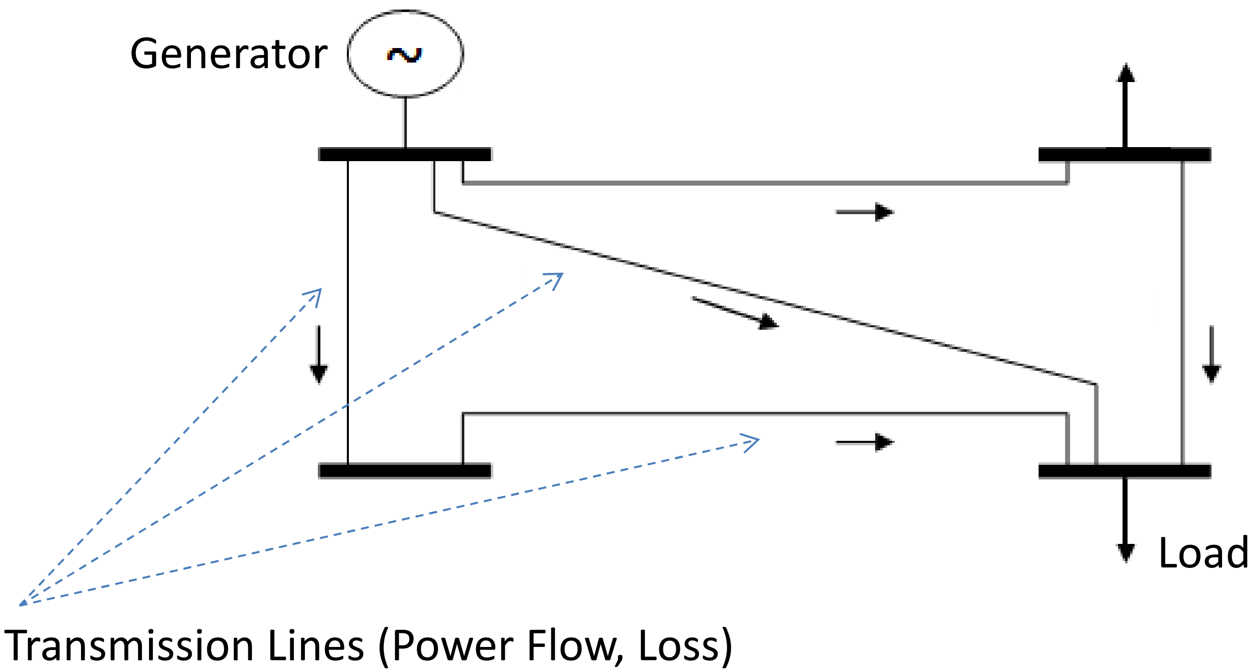

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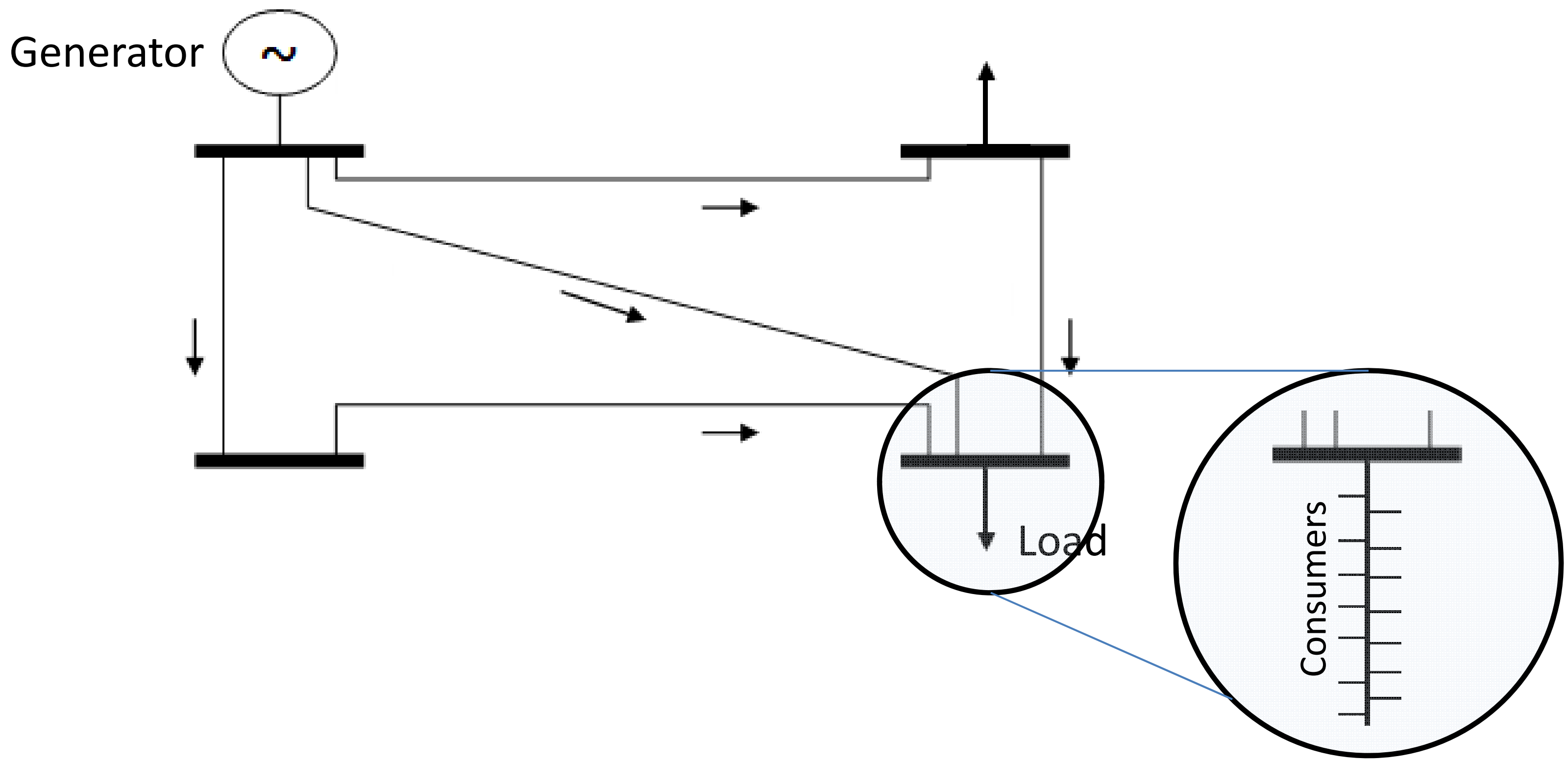

Power Grid Graph Representation

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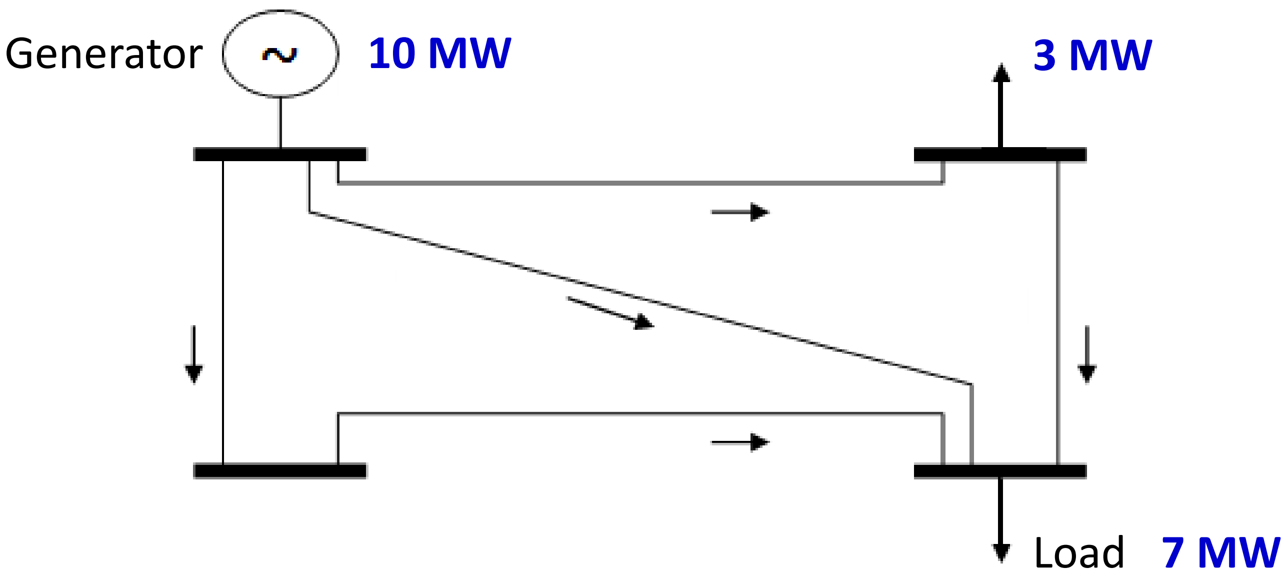


Transmission Line Admittance

- Admittance y is defined as the inverse of impedance z:
  - $z = r + j\,x$ (r: Resistance, x: Reactance)
  - $y = g + j\,b$ (g: Conductance, b: Susceptance)
  - $y = 1 / z$
  - Parameter g is usually positive
  - Parameter b:
    - Positive: Capacitor
    - Negative: Inductor

Transmission Line Admittance

- For the transmission line connecting bus i to bus k:
  - Addmitance: y_{ik}
  - Example:

$$y_{ik} = 1 - j\,4 \quad \text{(per unit)}$$

  - Note that, y_{ii} is denoted by y_i and indicates:
    - Susceptance for any shunt element (capacitor) to ground at bus i.

Y-Bus Matrix

- We define:
  - Y_{bus} = [Y_{ij}] where
    - Diagonal Elements: $$Y_{ii} = y_i + \sum_{k=1, k \neq i}^{N} y_{ik}$$
    - Off-diagonal Elements: $$Y_{ij} = -y_{ij}$$
  - Note that Y_{bas} matrix depends on the power grid topology and the admittance of all transmission lines.
  - N is the number of busses in the grid.

Y-Bus Matrix

- Example: For a grid with 4-buses, we have:

$$Y_{bus} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix}$$

- After separating the real and imaginary parts:

$$Y_{bus} = G + j\,B$$

Bus Voltage

- Let V_i denote the voltage at bus i:

- Note that, V_i is a phasor, with magnitude and angle.

$$V_i = |V_i| \angle \theta_i$$

- In most operating scenarios we have:

$$|V_i| \approx |V_j| \qquad\qquad \theta_i \neq \theta_j$$

Power Flow Equations

- Let S_i denote the power injection at bus i:

$$S_i = P_i + j\,Q_i$$

Active Power   Reactive Power

- Generation Bus: $P_i > 0$
- Load Bus: $P_i < 0$ (negative power injection)

Power Flow Equations

- Using Kirchhoff laws, AC Power Flow Equations become:

$$P_k = \sum_{j=1}^{N} |V_k||V_j| \left(G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j) \right)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j| \left(G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j) \right)$$

- Do we know all notations here?

- If we know enough variables, we can obtain the rest of variables by solving a system of nonlinear equations.

Power Flow Equations

- The AC Power Flow Equations are complicated to solve.

- Next, we try to simplify the equations in three steps.

- **Step 1**: For most networks, G << B. Thus, we set G = 0:

$$\Rightarrow \quad P_k = \sum_{j=1}^{N} |V_k||V_j|\left(B_{kj}\sin(\theta_k - \theta_j)\right)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(-B_{kj}\cos(\theta_k - \theta_j)\right)$$

Power Flow Equations

- **Step 2**: For most neighboring buses: $\left|\theta_i - \theta_j\right| \leq 10^\circ \text{ to } 15^\circ$.

- As a result, we have: $$\begin{cases} Sin\,(\theta_k - \theta_j) \approx \theta_k - \theta_j \\ Cos\,(\theta_k - \theta_j) \approx 1 \end{cases}$$

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(B_{kj}(\theta_k - \theta_j)\right)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(-B_{kj}\right)$$

Power Flow Equations

- **Step 3**: In per-unit, $|V_i|$ is very close to 1.0 (0.95 to 1.05).

- As a result, we have: $|V_i||V_j| \approx 1$.

$$\Longrightarrow \quad P_k = \sum_{j=1}^{N} B_{kj}(\theta_k - \theta_j)$$

$$Q_k = \sum_{j=1}^{N}\left(-B_{kj}\right) = -B_{kk} - \sum_{\substack{j=1,\\ j\neq k}}^{N} B_{kj} = -b_k$$

- P_k has a linear model and Q_k is almost fixed.

Power Flow Equations

- **Step 3**: In per-unit, $|V_i|$ is very close to 1.0 (0.95 to 1.05).

- As a result, we have: $|V_i||V_j| \approx 1$.

DC Power Flow Equations

$$P_k = \sum_{j=1}^{N} B_{kj}(\theta_k - \theta_j)$$

$$Q_k = \sum_{j=1}^{N} \left(-B_{kj}\right) = -B_{kk} - \sum_{\substack{j=1, \\ j \neq k}}^{N} B_{kj} = -b_k$$

- P_k has a linear model and Q_k is almost fixed.

Power Flow Equations

- Given the power injection values at all buses, we can use

$$P_k = \sum_{j=1}^{N} B_{kj}(\theta_k - \theta_j)$$

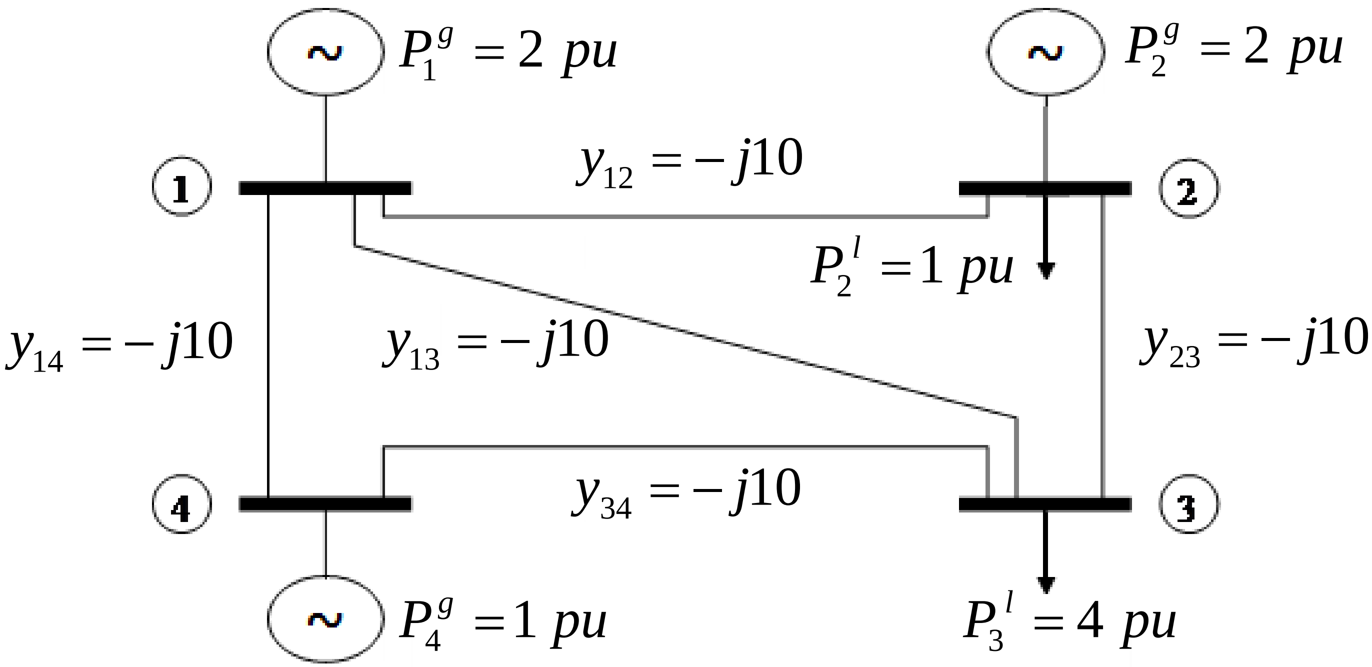
to obtain the voltage angles at all buses.

- Let P_{ij} denote the power flow from bus i to bus j, we have:

$$P_{ij} = B_{ij}(\theta_i - \theta_j)$$

Power Flow Equations

- Example: Obtain power flow values in the following grid:



Power Flow Equations

- First, we obtain the Y-bus matrix:

$$Y_{bus} = j\begin{bmatrix} b_1+b_{12}+b_{13}+b_{14} & -b_{12} & -b_{13} & -b_{14} \\ -b_{21} & b_2+b_{21}+b_{23}+b_{24} & -b_{23} & -b_{24} \\ -b_{31} & -b_{32} & b_3+b_{31}+b_{32}+b_{34} & -b_{34} \\ -b_{41} & -b_{42} & -b_{43} & b_4+b_{41}+b_{42}+b_{43} \end{bmatrix}$$

$$= j\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} = j\,B = j\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}$$

Power Flow Equations

- Next, we write the (active) power flow equations:

$$P_1 = (B_{12} + B_{13} + B_{14})\theta_1 - B_{12}\theta_2 - B_{13}\theta_3 - B_{14}\theta_4$$
$$P_2 = -B_{21}\theta_1 + (B_{21} + B_{23} + B_{24})\theta_2 - B_{23}\theta_3 - B_{24}\theta_4$$
$$P_3 = -B_{31}\theta_1 - B_{32}\theta_2 + (B_{31} + B_{32} + B_{34})\theta_3 - B_{34}\theta_4$$
$$P_4 = -B_{41}\theta_1 - B_{42}\theta_2 - B_{43}\theta_3 + (B_{41} + B_{42} + B_{43})\theta_4$$

- This can be written as:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} B_{12}+B_{13}+B_{14} & -B_{12} & -B_{13} & -B_{14} \\ -B_{21} & B_{21}+B_{23}+B_{24} & -B_{23} & -B_{24} \\ -B_{31} & -B_{32} & B_{31}+B_{32}+B_{34} & B_{34} \\ -B_{41} & -B_{42} & -B_{43} & B_{41}+B_{42}+B_{43} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Power Flow Equations

- From the last two slides, we finally obtain:

$$\begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

- Therefore, the voltage angles are obtained as:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}^{-1} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

Power Flow Equations

- However, the last matrix in the previous slide is singular!
- Therefore, we cannot take the inverse.
- The system of equations would have infinite solutions.
- The problem is that the four angles are not independent.
- What matters is the angular/phase difference.
- We choose one bus (e.g., bus 1) as reference bus: $\theta_1 = 0$.

Power Flow Equations

- We should also remove the corresponding rows/columns:

$$\begin{bmatrix} 2 \\ 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

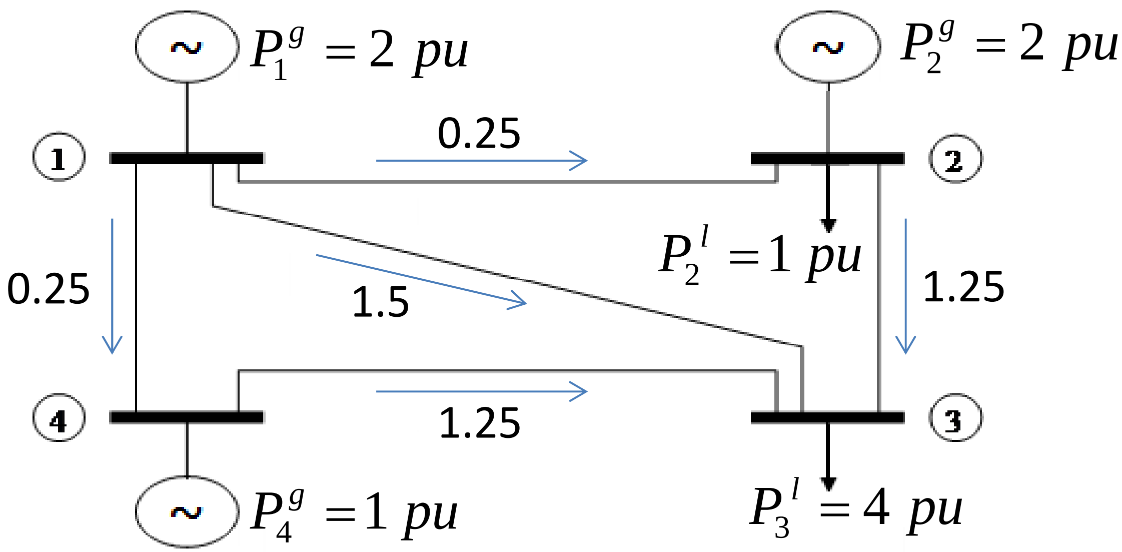
- The angular differences (with respect to θ_1):

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.025 \\ -0.15 \\ -0.025 \end{bmatrix} \Rightarrow \begin{aligned} \theta_2 - \theta_1 &= -0.025 \\ \theta_3 - \theta_1 &= -0.15 \\ \theta_4 - \theta_1 &= -0.025 \end{aligned}$$

Power Flow Equations

- Finally, the power flow values are calculated as:

$$P_{12} = B_{12}(\theta_1 - \theta_2) = 10(0 - -0.025) = 0.25$$
$$P_{13} = B_{13}(\theta_1 - \theta_3) = 10(0 - -0.15) = 1.5$$
$$P_{14} = B_{14}(\theta_1 - \theta_4) = 10(0 - -0.025) = 0.25$$
$$P_{23} = B_{23}(\theta_2 - \theta_3) = 10(-0.025 - -0.15) = 1.25$$
$$P_{34} = B_{34}(\theta_3 - \theta_4) = 10(-0.15 - -0.025) = -1.25$$



Power Flow Equations

- What if the generator connected to bus 1 is renewable?

- What if the capacity of transmission link (1,3) is 1 *pu*?

- What if we can apply demand response to load bus 3?

- What if one of the transmission lines fails?

Economic Dispatch Problem

- In the example we discussed earlier, we had:

Power Supply = Power Load

- In particular, we had:

$$P_1^g + P_2^g + P_4^g = P_2^l + P_3^l$$

- However, generation levels P_1^g, P_2^g, and P_4^g assumed given.

- Q: What if the generators have different generation costs?

Economic Dispatch Problem

- For thermal power plants, generation cost is quadratic:

$$\text{Generation Cost} = C(P) = a_1 + a_2 \times P + a_3 \times P^2$$

- Example: a grid with three power plants:

$$C_1(P_1) = 561 + 7.92 \times P_1 + 0.001562 \times (P_1)^2 \qquad 150\text{ MW} \le P_1 \le 600\text{ MW}$$

$$C_2(P_2) = 310 + 7.85 \times P_2 + 0.001940 \times (P_2)^2 \qquad 100\text{ MW} \le P_2 \le 400\text{ MW}$$

$$C_3(P_3) = 78 + 7.97 \times P_3 + 0.004820 \times (P_3)^2 \qquad 50\text{ MW} \le P_3 \le 200\text{ MW}$$

- Each power plant has some min and max generation levels.

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- Each power plant has some min and max generation levels.

Economic Dispatch Problem

- We should select P_1, P_2, and P_3 to:
  - Meet total load P_{load} = 850 MW
  - Minimize the total cost of generation

- Economic Dispatch Problem:

$$\begin{aligned} \underset{P_1, P_2, P_3}{\text{minimize}} \quad & \mathrm{C}(P_1) + \mathrm{C}(P_2) + \mathrm{C}(P_3) \\ \text{subject to} \quad & 150 \le P_1 \le 600 \\ & 100 \le P_2 \le 400 \\ & 50 \le P_3 \le 200 \\ & P_1 + P_2 + P_3 = 850 \end{aligned}$$

Economic Dispatch Problem

- Is the formulated problem a convex program? Why?

- Convex programs can be solved efficiently.

- An useful software is CVX for Matlab (http://cvxr.com/cvx).

- The optimal economic dispatch solution:

P_1 = 393.2 MW

P_2 = 334.6 MW

P_3 = 122.2 MW

Q: Do they satisfy all constraints?

Economic Dispatch Problem

- Is the formulated problem a convex program? Why?
- Convex programs can be solved efficiently.
- An useful software is CVX for Matlab (http://cvxr.com/cvx).
- The optimal economic dispatch solution:

P_1 = 393.2 MW

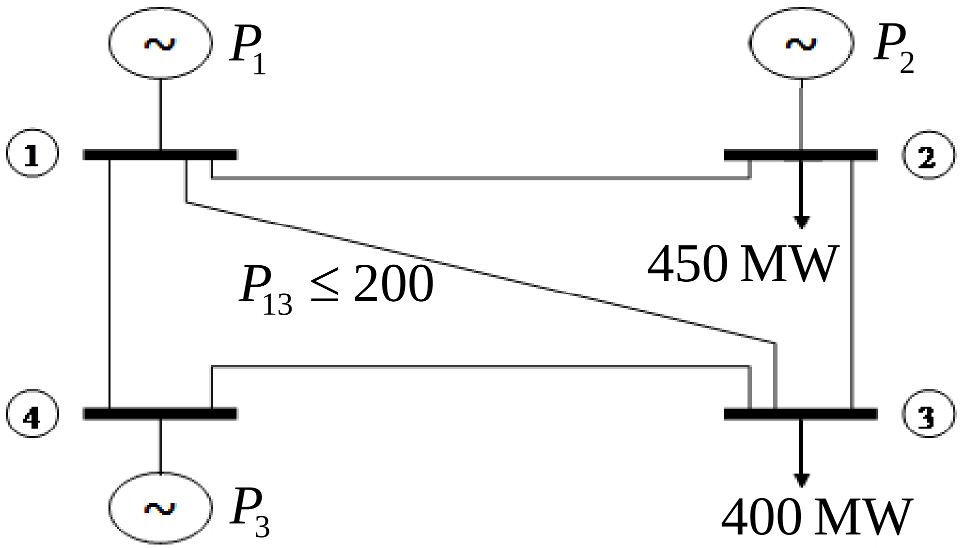
P_2 = 334.6 MW

P_3 = 122.2 MW

Minimum Cost = 3916.6 + 3153.8 + 1123.9
= **8194.3**

Economic Dispatch Problem

- What if we have to satisfy topology constraints?

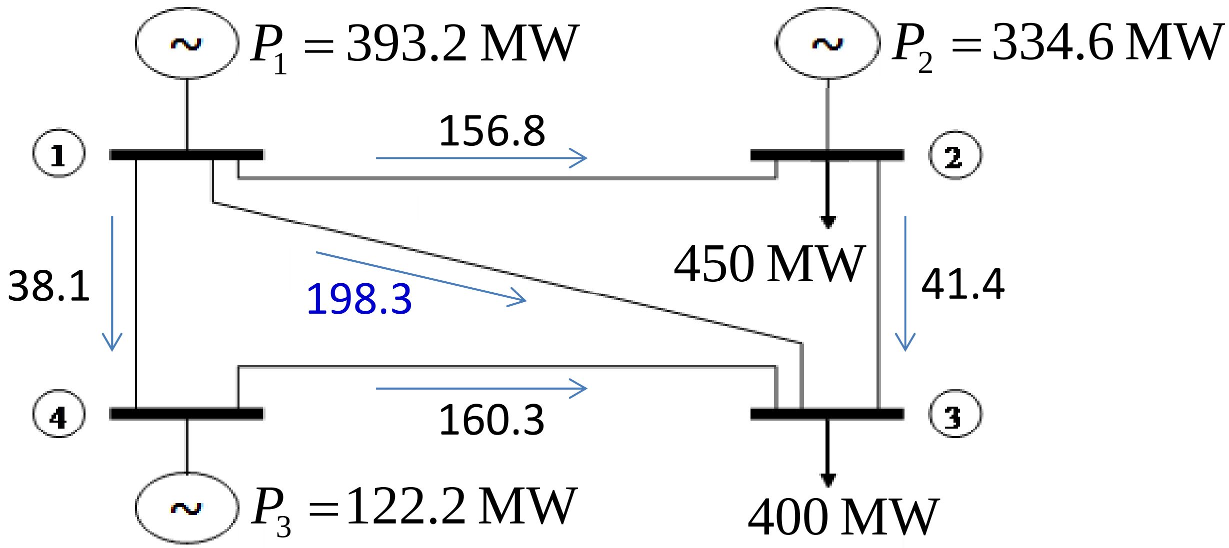


$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} P_2 - 450 \\ -400 \\ P_3 \end{bmatrix}$$

$$P_{13} = B_{13}(\theta_1 - \theta_3) = -10\,\theta_3 \leq 20 \quad \Rightarrow \quad \theta_3 \geq -20$$

Economic Dispatch Problem

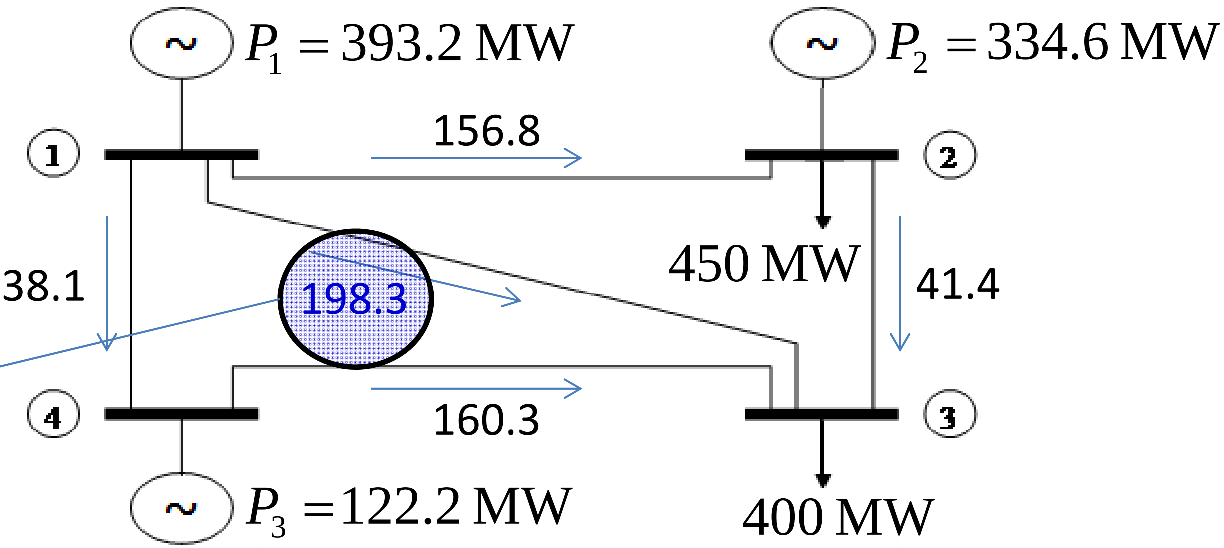
- The same optimal solutions are still valid:



$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} -15.685 \\ -19.830 \\ -3.805 \end{bmatrix} \qquad \theta_3 \geq -20 \qquad P_{13} \leq 200$$

Economic Dispatch Problem

- The same optimal solutions are still valid:



What if $P_{13} \leq 170$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} -15.685 \\ -19.830 \\ -3.805 \end{bmatrix} \qquad \theta_3 \geq -20 \qquad P_{13} \leq 200$$

Economic Dispatch Problem

- Then the economic dispatch problem becomes:

$$\underset{P_1, P_2, P_3, \theta_1, \theta_2, \theta_3}{\text{minimize}} \quad C(P_1) + C(P_2) + C(P_3)$$

$$\text{subject to} \quad 150 \le P_1 \le 600$$

$$100 \le P_2 \le 400$$

$$50 \le P_3 \le 200$$

$$P_1 + P_2 + P_3 = 850$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} P_2 - 450 \\ -400 \\ P_3 \end{bmatrix}$$

$$\theta_3 \ge -17$$

Economic Dispatch Problem

- Then the economic dispatch problem becomes:

$$\underset{P_1, P_2, P_3, \theta_1, \theta_2, \theta_3}{\text{minimize}} \quad C(P_1) + C(P_2) + C(P_3)$$

$$\text{subject to} \quad 150 \leq P_1 \leq 600$$

$$100 \leq P_2 \leq 400$$

$$50 \leq P_3 \leq 200$$

$$P_1 + P_2 + P_3 = 850$$

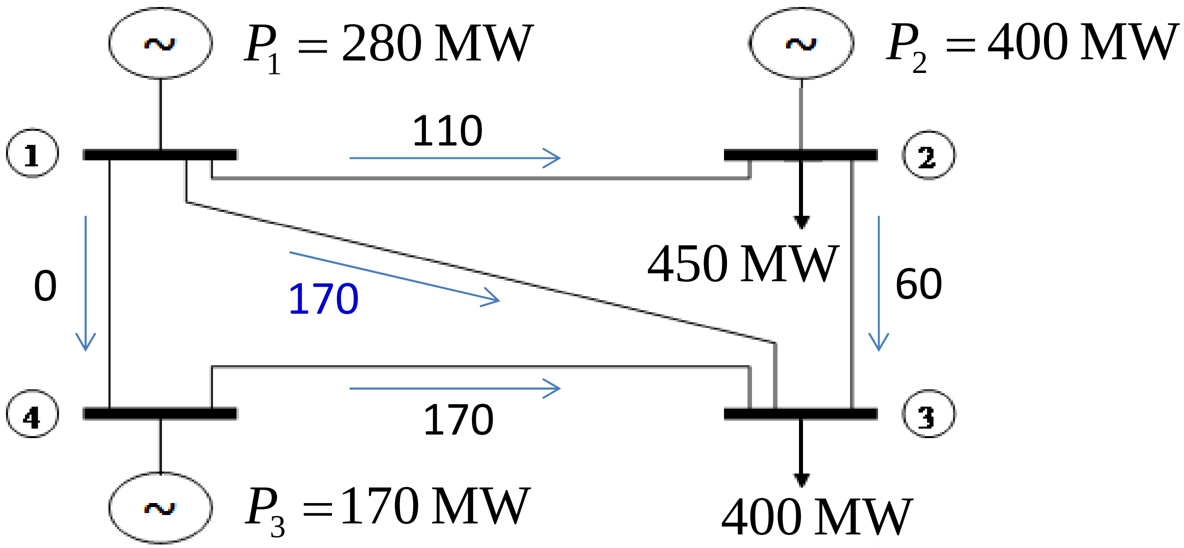
$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} P_2 - 450 \\ -400 \\ P_3 \end{bmatrix}$$

$$\theta_3 \geq -17$$

Still a Convex Program?

Economic Dispatch Problem

- The new optimal solutions are obtained as:

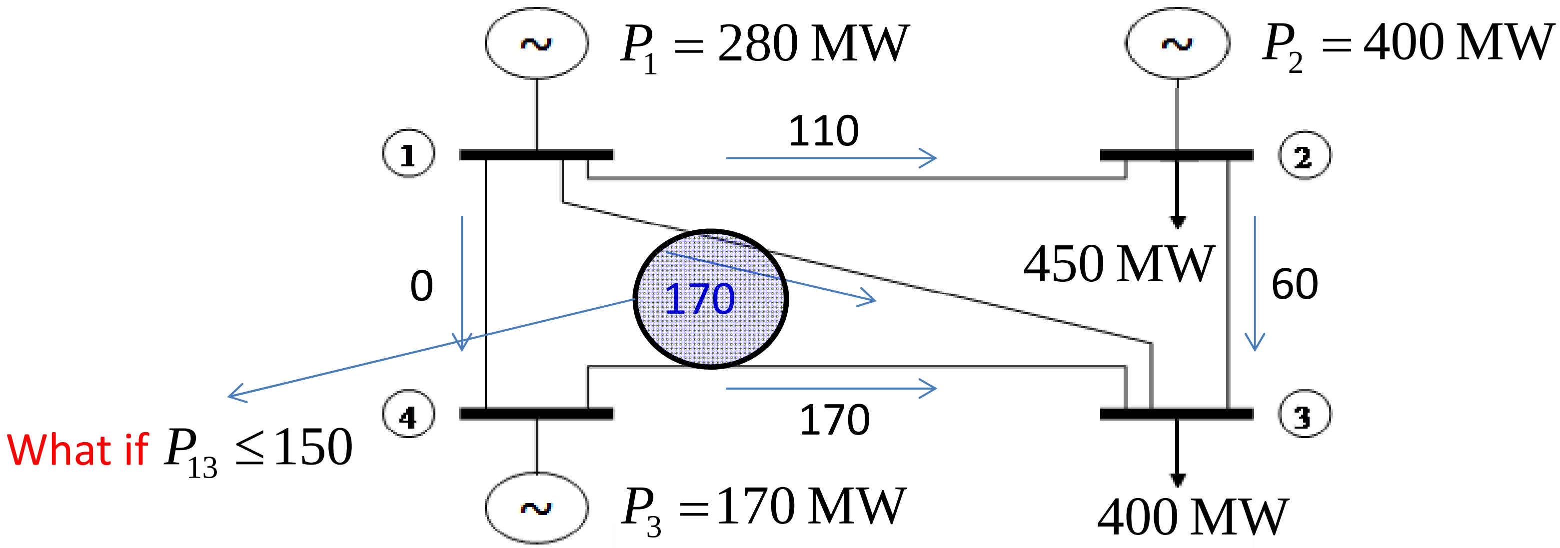


- The total generation cost becomes: **\$8,233.66 > \$8,194.3**

- Here, we had to sacrifice "cost" for "implementation".

Economic Dispatch Problem

- The new optimal solutions are obtained as:



- The total generation cost becomes: **\$8,233.66 > \$8,194.3**

- Here, we had to sacrifice "cost" for "implementation".

Unit Commitment

- Economic Dispatch is solved a few hours ahead of operation.

- On the other hand, we need to decide about the choice of power plants that we want to turn on for the next day.

- This is done by solving the Unit Commitment problem.

- We particularly decide on which slow-starting power plants we should turn on during the next day given various constraints.

- The mathematical concepts are similar to the E-D problem.

References

- W. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, John Wiley & Sons, 2nd Ed., 1996.

- J. McCalley and L. Tesfatsion, "Power Flow Equations", *Lecture Notes*, EE 458, Department of Electrical and Computer Engineering, Iowa State University, Spring 2010.