FILTRATION

Filtration

– A process in which water (or wastewater) passes through porous medium by which non-settleable solids are separated from water (or wastewater).

Non-settleable solids: e.g., Humic acid ~ 0.001 µm, microorganisms ~1 µm, coagulant precipitates ~100 µm

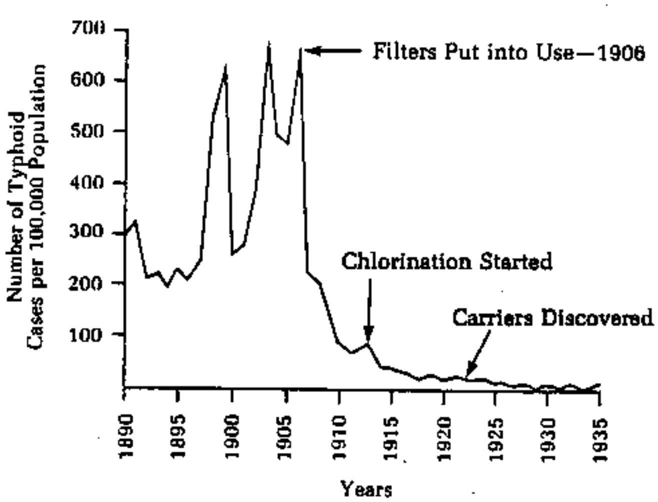
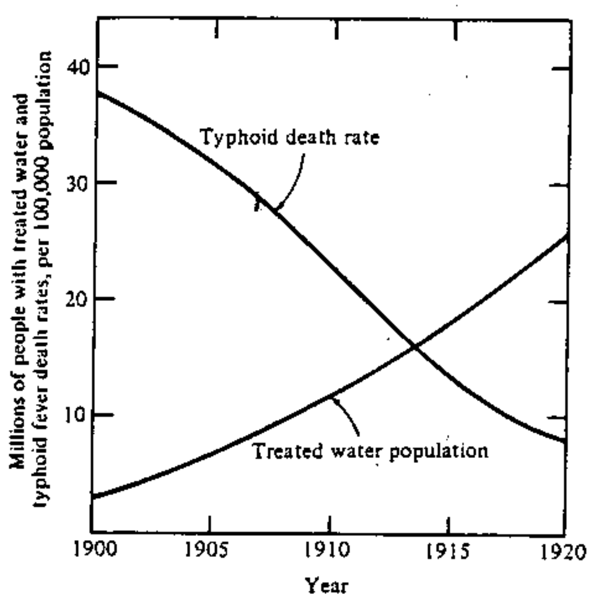


FIGURE 3-1
Typhoid fever cases per 100,000 population from 1890 to 1935, Philadelphia.



Typhoid fever and treated water supplies from 1900 to 1920 (From Vesilind).

(Guillen and Hoek, 2010)

Two basic filter types:

1) Media (depth) filters
2) Membrane (sieving) filters

Media (depth) filtration

– an established technology
– are most widely used in water and wastewater treatment
– rely on cheap, natural media such as sand, anthracite, crushed magnetite, garnet, etc.

Membrane filtration

– primarily used for industrial, biological, and analytical separations.
– offers an absolute barrier to pathogens (protozoa, bacteria, and viruses) depending on the pore size of the membrane.

(3^{rd} DC 228; 4^{th} DC 283)

Types of particles to be removed by Media (depth) filters:

a) Coagulant particles escaped from a sedimentation tank (e.g., Al $(OH)_3$)
b) Inorganic particles (e.g., asbestos fibers, clay, silt)
c) Microbiological particles (e.g., virus, cysts, bacteria, algae, planktons)
d) Organic compounds

- A filter bed is made up originally of fine sand over a layer of supporting gravel.
- With either plain sedimentation of the combination of coagulant flocculation and sedimentation, it is not normally possible to achieve adequate clarity of water.

Expected Performance (in terms of Turbidity):

Lake	→ Coagulation Flocculation Sedimentation	→ Filtration
100 TU	10 TU	< 1 TU
	1 - 10 TU	0.3 TU (3rd DC 228)

- Turbid lake water at up to 100 TU is reduced to approximately 10 TU by coagulation/flocculation and sedimentation. Filtration further decreases turbidity to less than 1 TU.

Note:
- Jackson Candle Turbidity Unit (JTU)
- Nephelometric Turbidity Unit (NTU)
- Formazin Turbidity Unit (FTU)
- 1 TU ≈ 1 JTU ≈ 1 FTU ≈ 1 NTU ≈ 1 mg/L SiO_2
- Safe Drinking Water Act (SDWA) requires 0.5 NTU in 95% of monthly measurements.

Media (depth) Filter

<u>Schematic of a rapid sand filter</u>

Typical gravity filter box in Figure 3-40 (3rd DC 231); Fig 4-43 (4th DC 285)

The filter consists of: a) Media, b) Support media, c) Collection system, and d) Water Troughs

a) Media
- On top of the support media is a layer of graded sand.
  - Sand depth varies between 0.5 and 0.75 m.
  - If a dual media filter is used, the sand is about 0.3 m thick, the coal is about 0.45 m thick

b) Support media

- layers of graded gravel (small on top, large on bottom) traditionally have been used for the support.
- designed to keep the media (sand) in the filter and prevent it from leaving with the filtered water.

c) Collection system - methods of collecting the filtered water
- Underdrain blocks
- Perforated pipes

d) Water Troughs
- Collect the backwash water used to clean the filter.
- A wastewater trough is ~0.7 to 1 m above the top of the sand.
- Placed high enough above the sand layer so that sand will not be carried out with the backwash water.
- Generally, a total water depth of 1.8 to 3 m is allowed above the sand layer for water to build up above the filter.

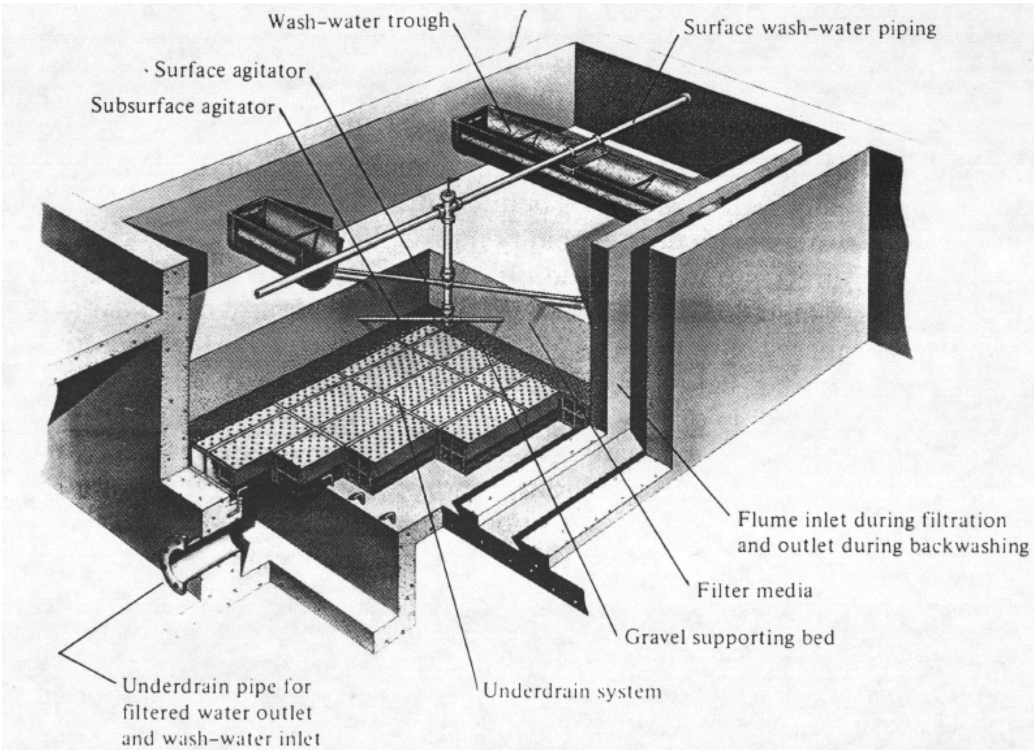


Figure 10.30 Cutaway view of a gravity filter. (Courtesy of The F. B. Leopold Co. Inc., a subsidiary of Mueller Co.)

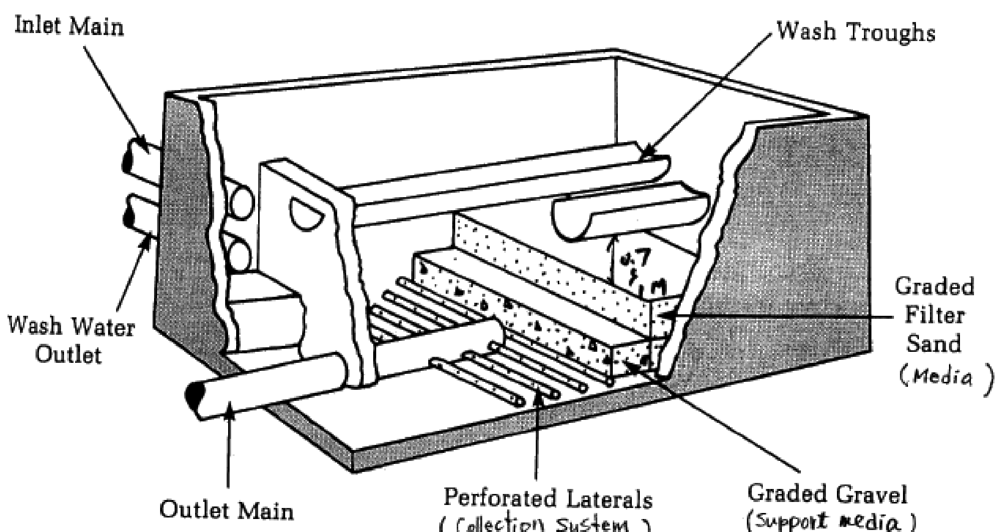


FIGURE 3-40

Typical cross section of a rapid sand filter. (*Source:* American Water Works Association, *Water Treatment Plant Design,* 1969.)

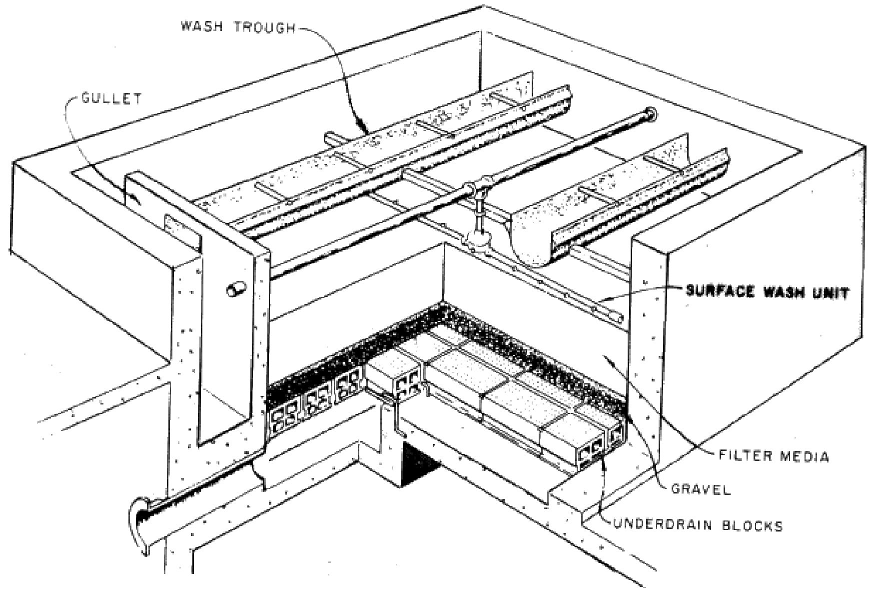


Figure 7.1 Typical gravity filter. (*Source: F. B. Leopold Co.*)

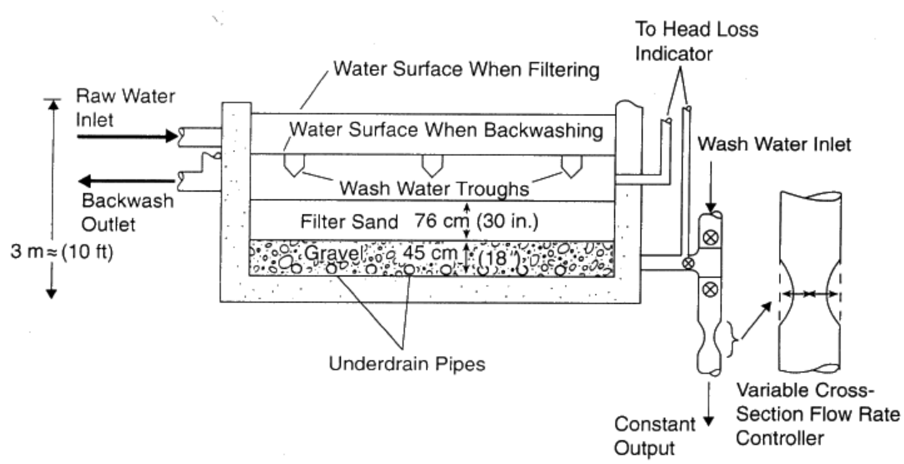
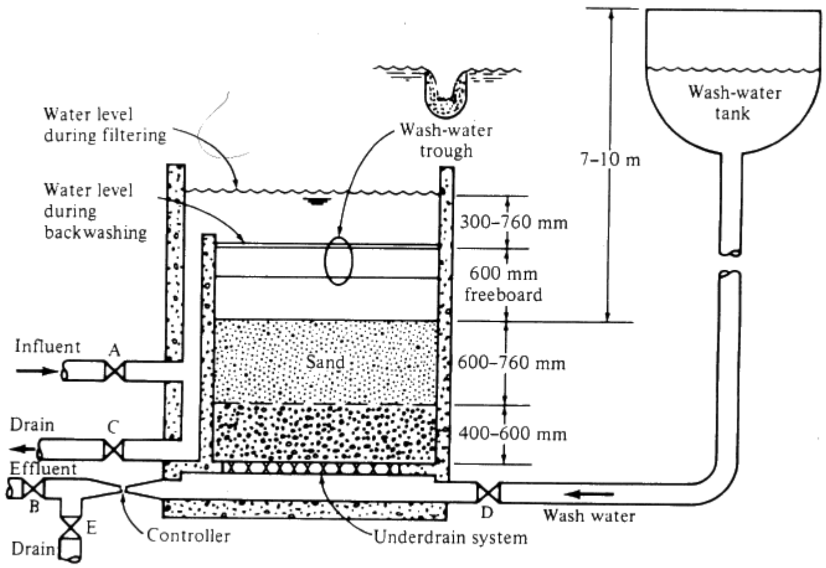


Figure 11–6 Cross section of rapid sand filter. *Source:* Adapted from Linsley and Franzini (1992).


Water level
during filtering
Wash-water
trough
Wash-water
tank
7–10 m
Water level
during
backwashing
300–760 mm
600 mm
freeboard
Influent
A
Sand
600–760 mm
Drain
C
400–600 mm
Effluent
B
E
Drain
Controller
Underdrain system
D
Wash water

Mechanisms of Filtration

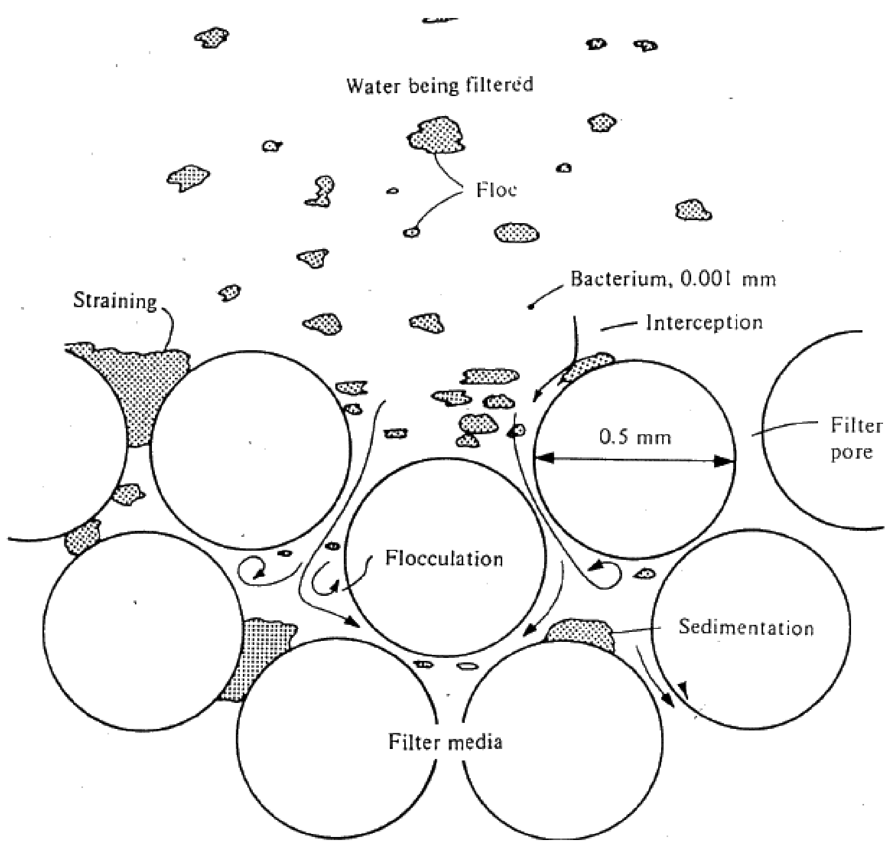
- the mechanisms involved in removing suspended solids in a granular media filter are complex, consisting of:

1) Removal Mechanisms
   - ways of removal of particles in filtration

2) Transport Mechanisms

- How particles get to the surface of the particle of filter media?

a. Straining
   - straining of the particles larger than the pore openings
b. Interception
   - a filter particle intercepts a streamline and held a suspended particle by flow forces
   - Particle flow in the stream line. Then, the stream line gets cut off by the particle of filter bed.
c. Flocculation
   - flocculation occurs when the particles are brought into closer contact within the filter Electrostatic force (van der Waals' force)
d. Sedimentation
   - Particle goes out of the stream line by particle weight and flow force
e. Adsorption
f. Diffusion
   - Concentration gradient between more suspended material and concentration suspension on the surface bed.
g. Hydrodynamics and Inertia
   - Removal due to turbulence mixing in a filter, impingement of particle on surface of particle.



Schematic diagram illustrating straining, flocculation, and sedimentation in a granular-media filter.

Designing Considerations

1) Media type and size (by experience)
2) Head losses

- through the clean filter (Kozeny eqn, Fair & Hatch eqn, Carman & Kozeny eqn, and Rose eqn)
- through clogged filter (empirical)
- through an expanded bed for a given degree of expansion

3) Method of backwashing and cleaning
4) Backwash velocity
5) Pressure required for bed expansion

2. Mathematical Models

a. Backwash flow rate and bed expansion can be calculated -- good
b. Pressure drop (head loss) within the bed is "iffy" but is done -- fair
c. % removal has not been successfully predicted explicitly -- poor

- Because of these reasons, filter design needs pilot plant studies.

Types of Media (depth) Filters

- Two basic types of filter based on hydraulic classification

Classification by:

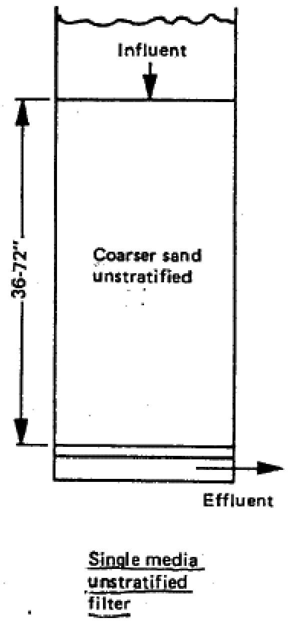
1) Filtration Rate, Hydraulic Loading Rate - flow rate / unit area of the filter
   a) Slow Sand Filter (2.9 - 7.6 $m^3/d{\cdot}m^2$)
   b) Rapid Sand Filter (120 $m^3/d{\cdot}m^2$)
   c) High Rate Filter (800 $m^3/d{\cdot}m^2$)

2) Flow pattern
   a) Down-flow
   b) Up-flow
   c) Bi-flow

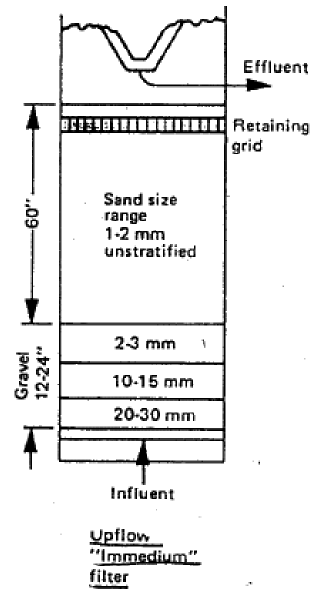
Media Used:	S.G.
Anthracite coal	1.5 – 1.6
Sand	2.65
Gravel	2.6
Garnet	4.0

3) Media type
   a) Sand (single media)
   b) Sand + Anthracite coal (dual media, mixed media)
   c) Sand + Coal + Garnet (Tri-media, mixed media)
   d) Carbon (Granular activated carbon)
   e) Body feed filters (diatomaceous earth)

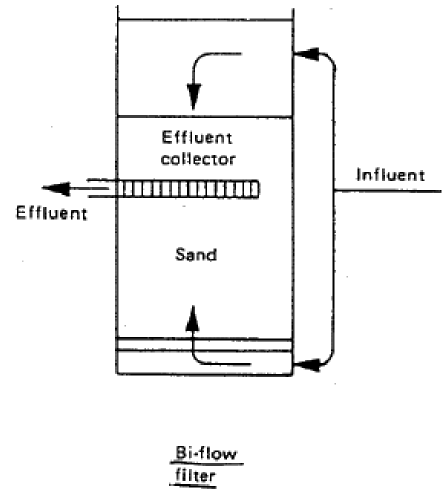
Single media, unstratified filter, Down-flow



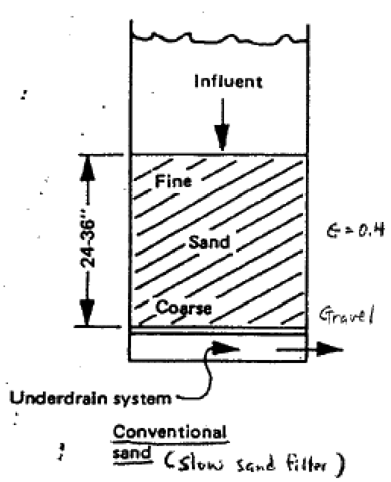
Single media filter, Up-flow



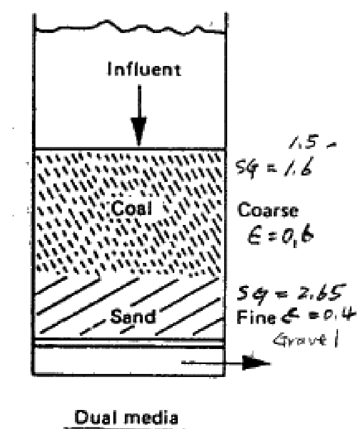
Bi-flow filter



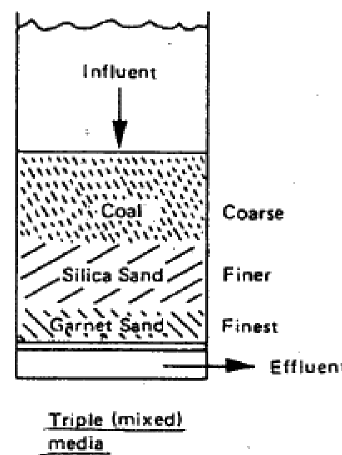
Conventional sand filter, Down-flow



Dual media filter, Down-flow



Triple media filter, Down-flow



Schematic diagrams of filter types/configurations for granular media filtration

Filter types

Slow Sand Filter

- first used in Britain in the 19th century (1800s)
- Loading (Filtration rate), 0.01-0.1 gpm/ft^2 (2.9-7.6 $m^3/d.m^2$)
- Require a large area of land (e.g., 0.5 acre/bed)
- long filter cycle (1 month is common)
- When the pore openings in the filter become too clogged (top 75 mm), it is necessary to stop the application of water and remove the upper layers of sand manually for cleaning.
- Labor intensive
- For smaller municipalities
- Cheaper to built
- Simpler to operate
- Do better job of removing bacteria.

Rapid Sand Filter

- Introduced in 1900s

- filiration rate, 2 -8 gpm/ft^2 (120 $m^3/d.m^2$)
- Graded (layered) sand within the bed.
- A fine sand layer on top, a coarse sand layer on the bottom
- Short filter cycle (24 hrs)
- The max. surface area is 100 m^2.
- The most common type used in water treatment.

High Rate Filter (800 $m^3/d/m^2$)

- 800 $m^3/d.m^2$
- Deep-bed monomedia filters (1.5 - 2.5 m deep)
- Anthracite coal (1.0 - 1.5 mm diameter)

Body Feed Filters

- In order to improve filtration, effluent quality and filter run, add filter media to the filtration process.
- It helps to make good filter cake, permeability is increased, creates less head loss per volume filtered.
- Diatomaceous earth or perlite

Diatomaceous-earth filters

Applications:

- filtration of recycled swimming pool water
- small water treatment systems, but potential for large systems.

Filtration rate: 1 - 5 gpm/ft^2

e.g., Jerome Park Reservoir, NY, demonstration plant

Filtration cycle:

Precoat → Filtration → Removal of the filter cake & body feed addition

Characteristics and Operation of Slow and Rapid Sand Filters

CHARACTERISTICS	SLOW SAND FILTER	RAPID SAND FILTER
1. Filtration Rate	0.01 - 0.1 gpm/ft^2	2 - 8 gpm/ft^2
2. Size (Area) of Bed	Large area (0.5 acre)	Small (0.01 - 0.1 acre)
3. Depth of Bed	12 - 18" gravel 42" sand (usually reduced to 24" by scraping)	12 - 18" gravel 18 - 30" sand
4. Grain size distribution of sand through the filter	Unstratified	Stratified
5. Loss of head through the filter	0.2' H_2O initial 4.0' H_2O final	1.0' H_2O initial 5.0' H_2O final
6. Length of run between cleaning	Long, 20 - 60 days (30 days common)	Short, 12-40 hours (24 hours common)
7. Penetration of suspended matter	superficial	deep throughout
8. Method of cleaning	(1) scrape off surface layer of sand (2") (2) wash surface by traveling washer	(1) scour by mechanical rake, air or water (2) remove dislodged matter from filter by upward flow or backwashing
9. Amount of wash water used in cleaning sand	0.2 - 0.6% of water filtered	1 - 6% of water filtered (4% common)
10. Construction Cost	Lower	Higher
11. Cost of operation	Lower Simple to operate Labor intensive	Higher
12. Application	Smaller municipalities	Larger municipalities
13. Others	Do better job of removing bacteria	

Filtration Rate, Loading Rate, v_a - flow rate / unit area of the filter

$$v_a = \frac{Q}{A_s}$$

where v_a = loading rate, $m^3/d.m^2$
= face velocity = approaching velocity, m/d
Q = flow rate onto filter surface, m^3/d
A_S = surface area of filter, m^2

Surface area, A_S

$$A_S = \frac{Q}{v_a}$$

Example 3-25 (3rd DC 230); Example 4-25 (4th DC 284)

As part of their proposed new treatment plant, Urbana is going to install rapid sand filters after their sedimentation tanks. The design loading rate to the filter is 200 $m^3/d{\cdot}m^2$.

a) How much filter surface area should be provided for their design flow rate of 0.5 m^3/s ?
b) If the surface area per filter box is to be limited to 50 m^2 (preferred), how many filter boxes are required?

Note: The recommended maximum loading rate < 235 m/d.
The maximum surface area/filter = 100 m^2/filter

(Solution)

The surface area required

$$A_S = \frac{Q}{v_a} = \frac{(0.5\,m^3/s)(86{,}400\,s/d)}{200\,m^3/d.m^2} = 216\,m^2$$

If the preferred maximum surface area of the tank = 50 m^2, the number of filters required is

Number = 216 m^2 / (50 m^2 / unit) = 4.32 units

We need to round to an integer. Normally, we build an even number of filters to make construction easier and to reduce costs.

In this case we would propose to build 4 filters.

Check to see that the design loading does not exceed our guideline values of the maximum loading rate < 235 m/d.

$$v_a = \frac{Q}{A_s} = \frac{(0.5\, m^3 / s)(86{,}400\, s / d)}{(4\, filters)(50\, m^2 / filtetr)} = 216\, m / d$$ OK

Note: The recommended maximum loading rate < 235 m/d.
The maximum surface area/filter = 100 m^2/filter

If it is required that the filter capacity be sufficient to handle the design flow rate with **one filter out of service**, we must check the loading with three filters in service.

$$v_a = \frac{Q}{A_s} = \frac{(0.5\, m^3 / s)(86{,}400\, s / d)}{(3\, filters)(50\, m^2 / filtetr)} = 288\, m / d$$ Not OK

Four slightly larger filters would be constructed to meet the required loading (because maximum surface area/filter = 100 m^2/filter).

$$A_s = \frac{Q}{v_a} = \frac{(0.5\, m^3 / s)(86{,}400\, s / d)}{(3\, filters)(235\, m / d)} = 61\, m^2 / filter$$

We construct 4 filters having A_S of 61 m^2 / filter

Note: When one filter is out of service, the plant still meets the requirements:
The recommended maximum loading rate < 235 m/d.
The maximum surface area/filter = 100 m^2/filter

Designing Sand Filters

(1) Objectives:
- a) Maximum amount
- b) Highest quality
- c) Lowest cost

- An objective is the maximum amount of water of the highest quality for the lowest cost.

$$Filtration\ Rate \propto \frac{Driving\ Force}{Resistance}$$

Driving force:
- a) Gravity
- b) Pressure (mainly industrial)

Resistance:
- a) Frictions
- b) Viscosity

Note:

Smaller media size results in a product water with lower turbidity (better quality), but results in higher pressure losses in the filter and shorter operating cycles between cleanings (less products).

Filter Media and Type

Grain Size Characteristics

Sieve analysis (Grain size analysis)

1) The size distribution or variation of a sample of granular material is determined by sieving the sample through a series of standard sieves (screens).

2) The grain size analysis begins by placing the sieve screens in ascending order with the largest opening on top and the smallest openings on the bottom.

3) A sand sample is placed on the top sieve and the stack is shaken for a prescribed amount of time.

4) At the end of the shaking period, the mass of material retained on each sieve is determined.

5) The cumulative mass is recorded and converted into percentages by mass equal to or less than the size of separation of the overlying sieve.

6) Then the **cumulative frequency distribution** is plotted on **a logarithmic-probability paper**.

   - Logarithmic-probability paper assures an almost straight-line plot.

http://en.wikipedia.org/wiki/Sieve_analysis

Table 3-19 (3rd DC 233); Table 4-19 (4th DC 287)

Sieve designation number	Size of opening (mm)
200	0.074
140	0.105
100	0.149
70	0.210
50	0.297
40	0.42
30	0.59
20	0.84
18	1.00
16	1.19
12	1.68
8	2.38
6	3.36
4	4.76

Source: Excerpted from G.A. Fair and J.C. Geyer, Water Supply and Wastewater Disposal, New York: Wiley, pp. 664-670, 1954.

To find the percent of aggregate passing through each sieve, first find the percent retained in each sieve. To do so, the following equation is used,

$$\%\,Retained = \frac{W_{Sieve}}{W_{Total}}(100)$$

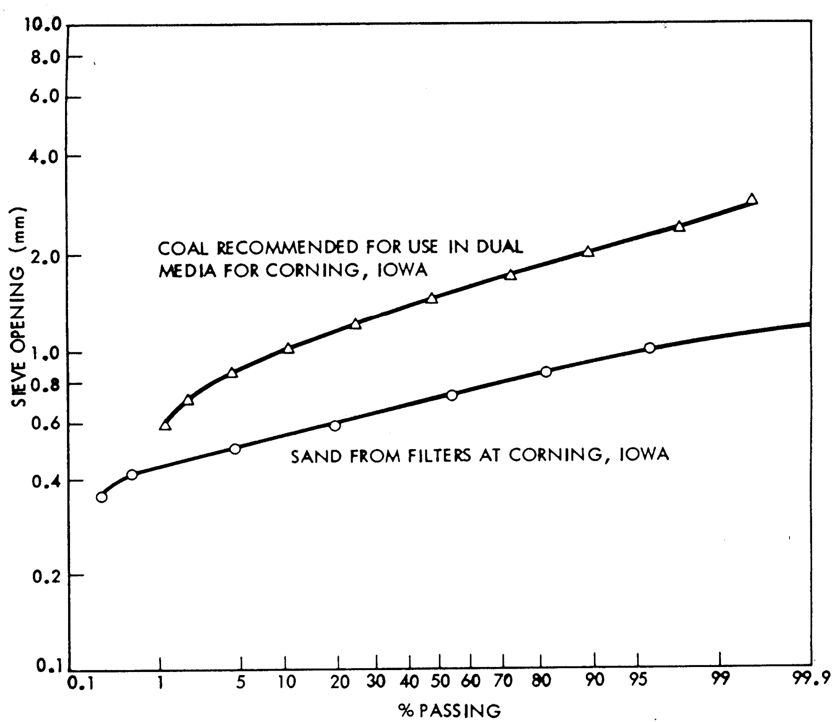
where W_{Sieve} is the weight of aggregate in the sieve and W_{Total} is the total weight of the aggregate.

The next step is to find the cumulative percent of aggregate retained in each sieve. To do so, add up the total amount of aggregate that is retained in each sieve and the amount in the previous sieves.

The cumulative percent passing of the aggregate is found by subtracting the percent retained from 100%.

%Cumulative Passing = 100%
- %Cumulative Retained.

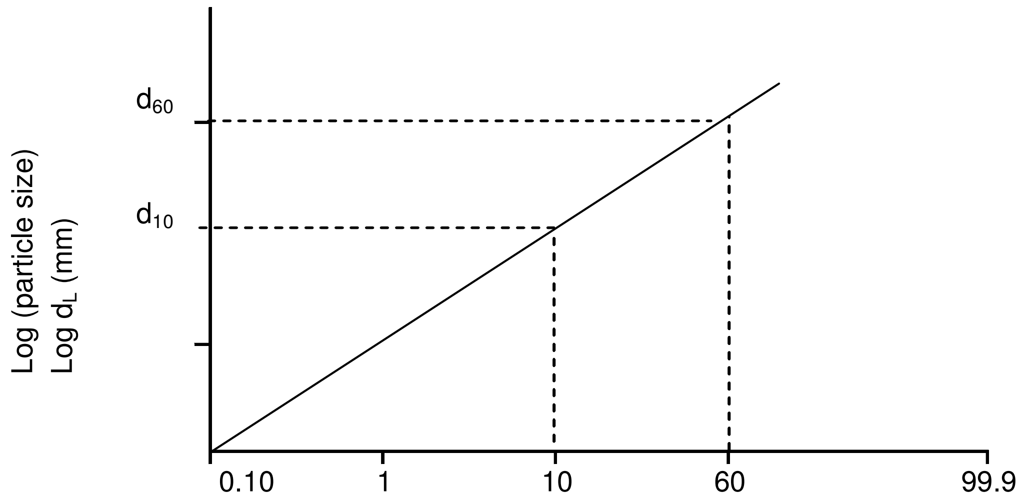
The values are then plotted on a graph with cumulative percent passing on the y axis and logarithmic sieve size on the x axis.



Typical sieve analysis of filter medium presented on a log-probability plot.

Media Parameters (Coefficients)

a. Use a *log-probability paper* and plot wt% vs. particle size (mm). (Cumulative mass percent passing)



b.

Probability (% less than stated size)
Cumulative mass percent passing

Effective size, **E**, d_e, P_{10}, d_{10}, 10-percentile

E = $d_{\mathbf{10}}$ = diameter of 10% less than this size (P_{10}, DC)

Significance of d_{10}:

- 50% of the surface area occurs on particles less than 10% in size.

- Recommended, E = 0.45 mm
  For silica sand, E = 0.35 - 0.55 mm (1.0 mm max)

- Smaller effective sizes result in product water with lower turbidity, but in higher pressure losses in the filter and shorter operating cycles between cleanings.

c. **Uniformity coefficient, UC, U**

$$UC = \frac{d_{60}}{d_{10}} = \frac{\text{60-percentile}}{\text{10-percentile}}$$

- Recommended, UC = 1.5 (1.3 - 1.7)

Significance of U:

- This ratio covers the range in size of half the sand.

d. **Porosity, ϵ**

$$\epsilon = \frac{\text{Void Volume}}{\text{Total Volume}}$$

- Recommended: 0.4 for sand
  0.6 for coal

Example 3-26 (3rd DC, p. 234); Example 4-26 (4th DC 288)

For the size frequencies by weight and by count of the sample of sand listed below, find the effective size, E, and uniformity coefficient, U.

Sieve designation number	Analysis of stock sand (Cumulative mass % passing)
140	0.2
100	0.9
70	4.0
50	9.9
40	21.8
30	39.4
20	59.8
16	74.4
12	91.5
8	96.8
6	99.0

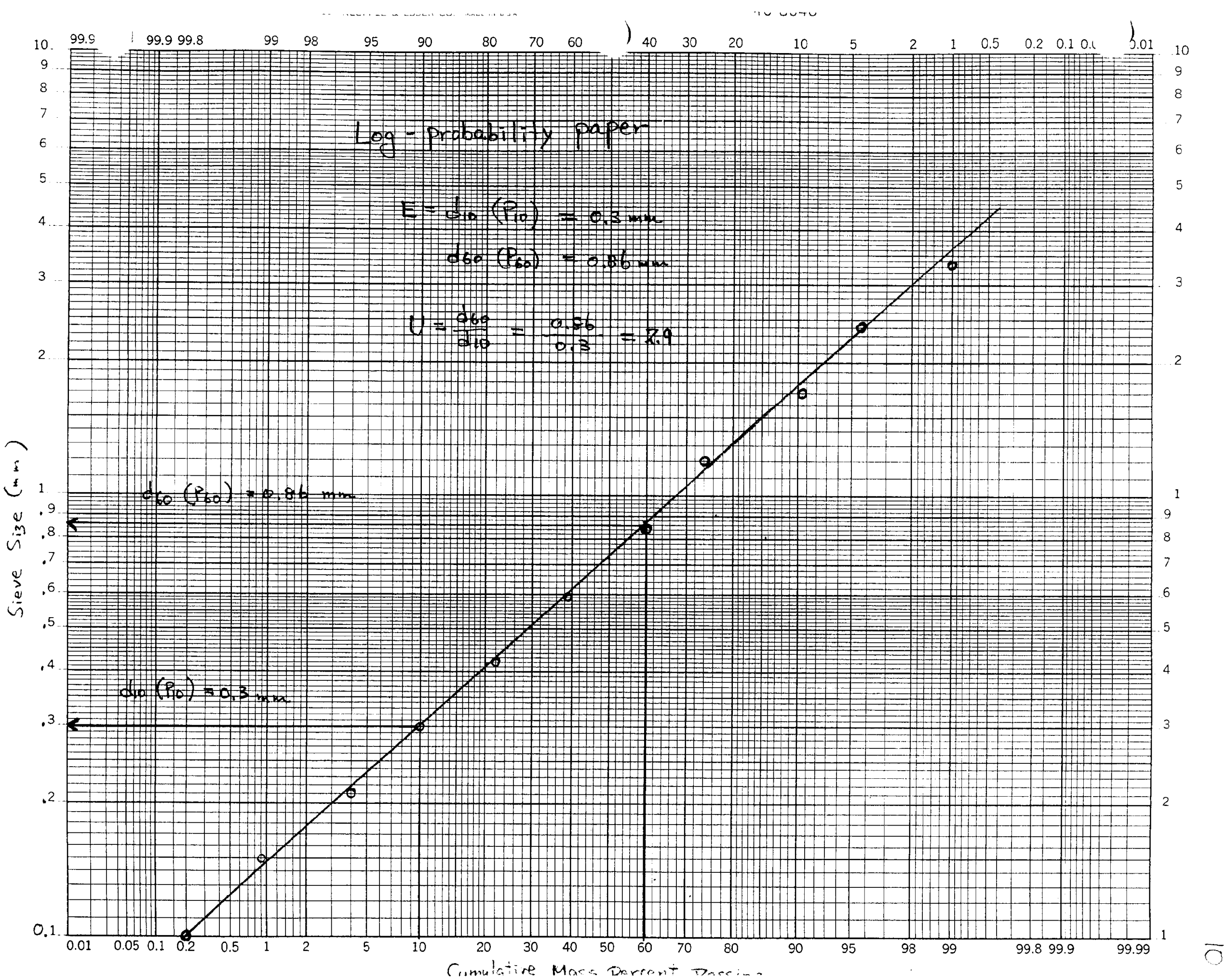
<u>Step</u>

1. Obtain size of opening (mm) from Table 3-19 (3rd DC 233) or Table 4-19 (4th DC 287)

(1)	(2)	(3)
Sieve designation number	Size of opening (mm)	Analysis of stock sand (Cumulative mass % passing)
140	0.105	0.2
100	0.149	0.9
70	0.210	4.0
50	0.297	9.9
40	0.42	21.8
30	0.59	39.4
20	0.84	59.8
16	1.19	74.4
12	1.68	91.5
8	2.38	96.8
6	3.36	99.0

Source: Excerpted from G.A. Fair and J.C. Geyer, Water Supply and Wastewater Disposal, New York: Wiley, pp. 664-670, 1954.

2. Plot the data on a log-probability paper



3. From the plot, obtain:

$E = d_{10} = P_{10} = 0.3$ mm

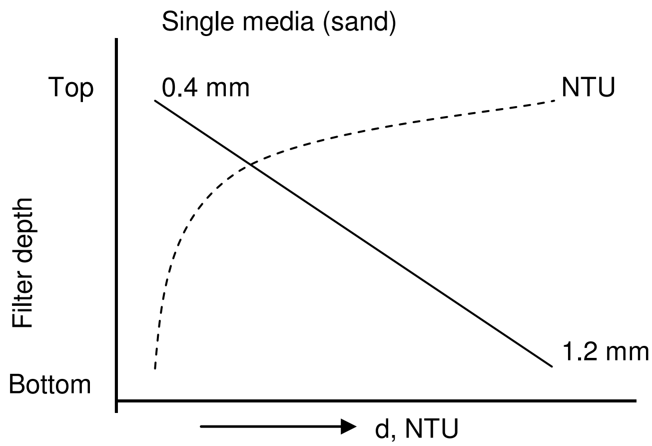
$d_{60} = P_{60} = 0.86$ mm

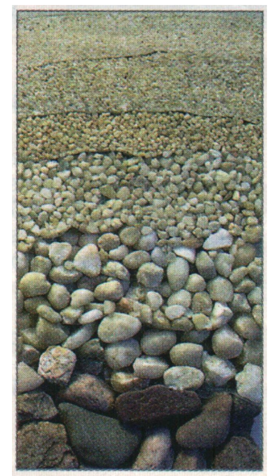
4. Calculate $U = d_{60} / d_{10} = 0.86/0.3 = 2.9$

Dual Media Filter, Mixed Media Filter, or Granular-Media Filter

<u>Media Size, Turbidity</u>

Single Media (Sand)

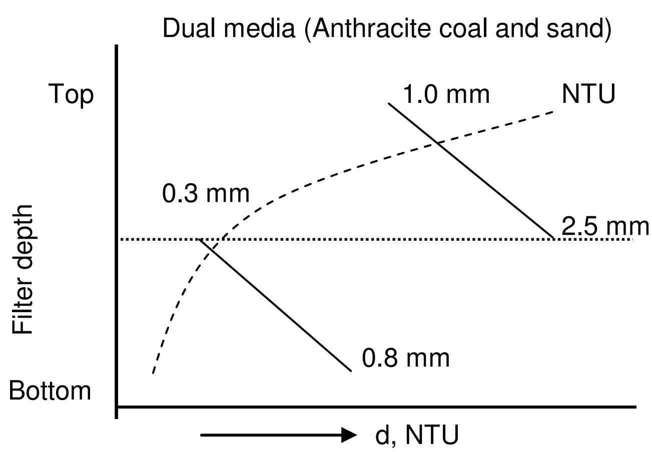


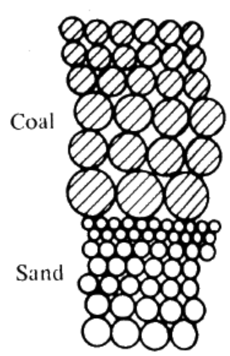


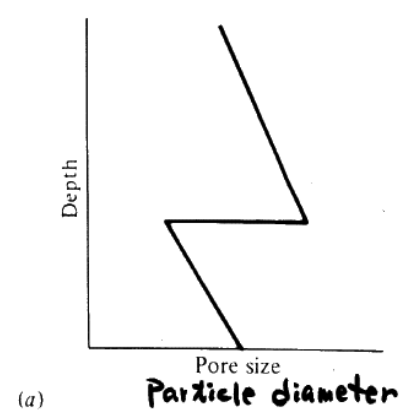
Diameter of particle, d (mm) ————
Turbidity (NTU) - - - - - - - - -

e = 0.4 d_{10} = 0.45 mm UC = 1.5

Dual Media (Anthracite Coal and Sand)



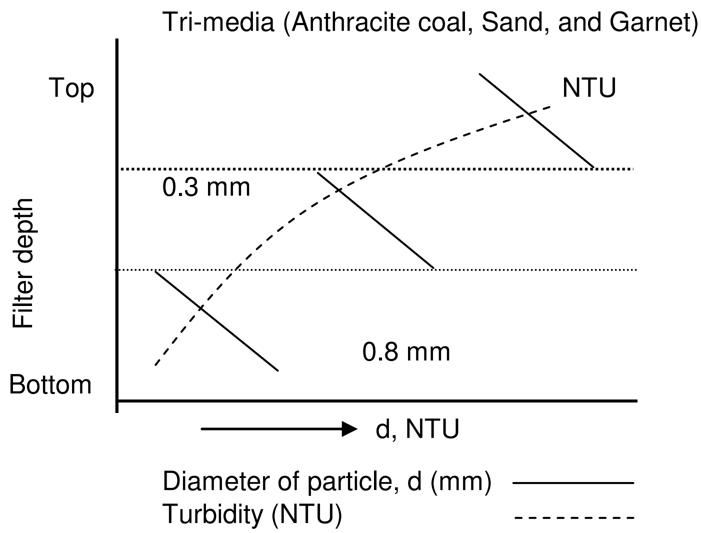




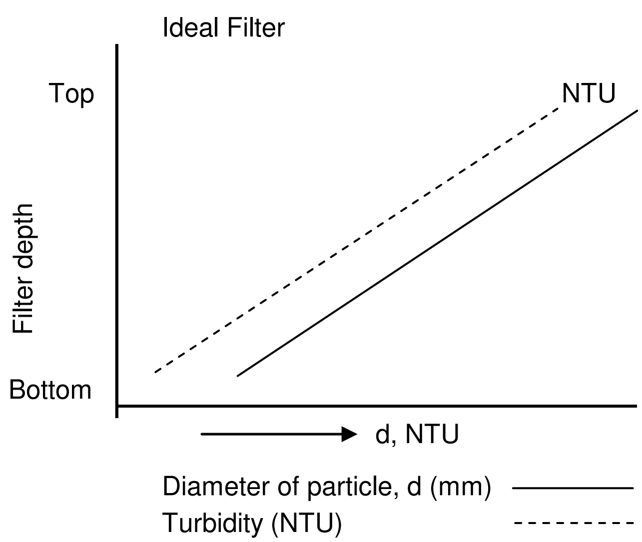
Diameter of particle, d (mm) ————
Turbidity (NTU) - - - - - - - - -

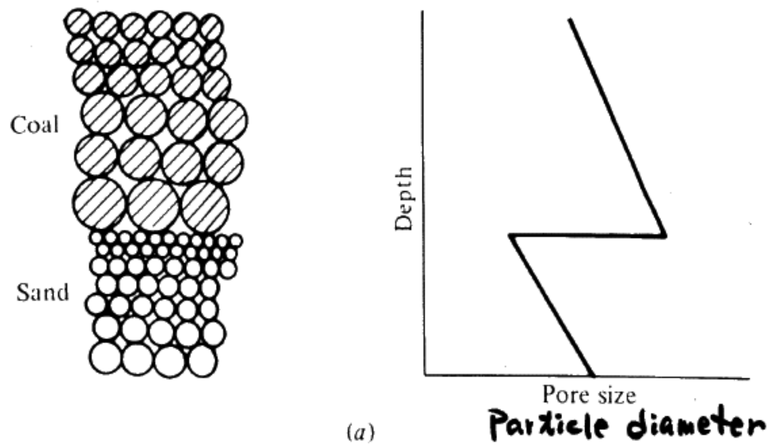
Coal:	e = 0.6	SG = 1.5	UC = 1.5
Sand:	e = 0.4	SG = 2.6	UC = 1.5

Tri-Media (Anthracite Coal, Sand, and Garnet)

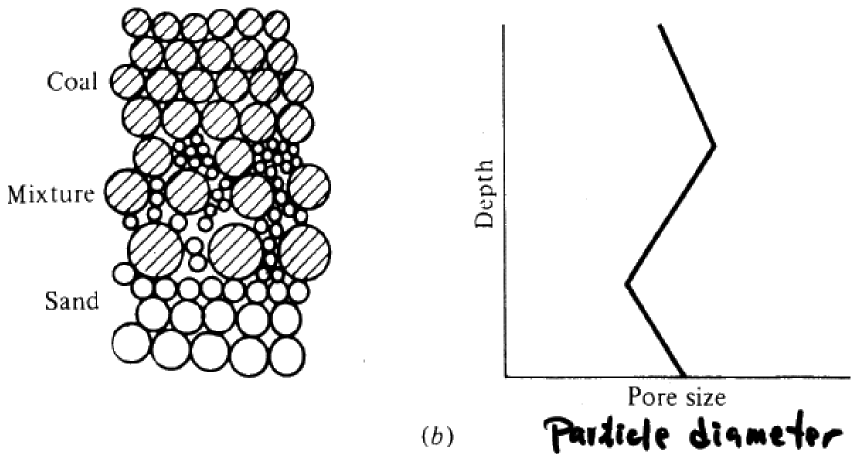


Coal: SG = 1.5
Sand: SG = 2.6
Garnet: SG = 4.0





(a)



(b)

Dual Media, Mixed Media Filter, Granular-Media Filter

- filtration through a layered bed of granular media (usually a coarse anthracite coal underlain by a finer sand) is the most common system of removing colloidal impurities in water.
- developed in early 1940s

a. Gives more loading, 4 gpm/ft^2
(up to 300 $m^3/d.m^2$ DC, p. 229)

b. Provide longer filter runs because it filters from the bottom up

c. utilize more of the filter depth for particle removal.

d. Good quality effluent due to having been polished at bottom filter by the small particles

e. Backwashing and maintenance are more difficult
- backwashing may need big pumps, etc.

Dual (mixed) Media

- Dual media filters are generally superior to Single media (sand) filter due to inverse bed and coal characteristics

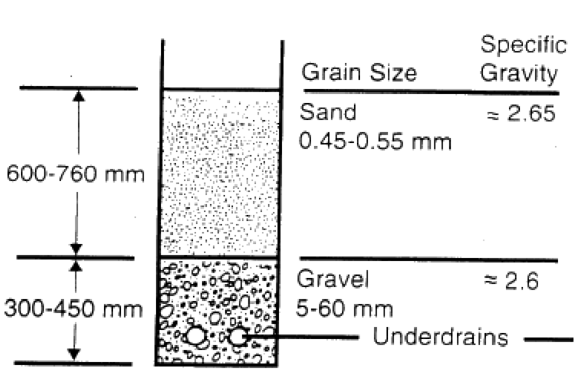
Advantages:

- low head loss
- less wash water
- better effluent quality
- longer operation period
- higher loading

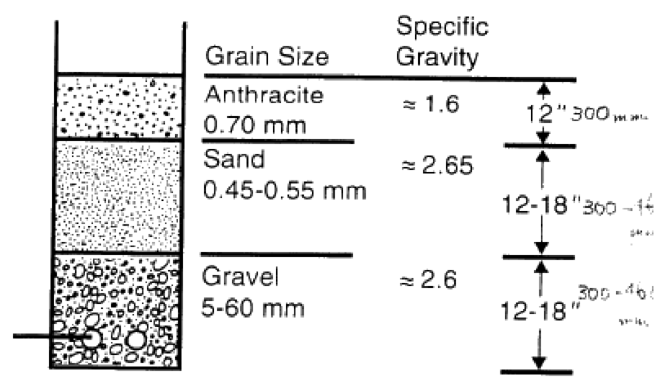
Disadvantages:

- high capital and maintenance cost
- more difficult to operate

Design of filters



(a) Traditional Rapid Sand Filter Bed



(b) Dual Media Anthracite-Sand Filter Bed

Head losses

(DC, p. 236) Notes:
If initial head loss > 0.6 m, the loading rate is too high or the sand has too large a proportion of fine grain sizes

----
(VH, p. 376-378)
Head loss through a clean granular-media is generally
Head loss before backwash 8 - 10 ft (2.4 - 3.0 m)

The minimum filter box depth = 3 m

- must be at least as deep as the highest design head loss.
- The trough bottom should be at least 0.15 m above the expanded bed to prevent loss of filter material.

The **Kozeny** equation:

- To determine head loss in a homogeneous granular- media bed

$$\frac{h}{\ell} = \frac{J\ \nu (1-\varepsilon)^2\ V\ S^2}{g\varepsilon^3 d^2}$$

where

h = head loss, ft of H_2O or m of H_2O
ℓ = depth of filter, ft or m
J = constant, dimensionless
= 6 for filtration in the laminar flow region
ν = kinetic viscosity, ft^2/s or m^2/s
g = acceleration of gravity, ft/sec^2 or m/sec^2
= $32.2\ ft/s^2 = 9.81\ m/s^2$
ϵ = porosity of the stationary filter bed, dimensionless
V = superficial (approach) velocity of water above the bed, ft/sec or m/sec
S = shape factor, dimensionless
= 6.0 for spherical grains (e.g., sand)
= 7.5 for angular grains (e.g., anthracite)
d = mean grain diameter, ft or m

Example: Using the Kozeny equation, calculate the initial head loss through a filter with 0.3 m of uniform sand having a porosity of 0.42 and a grain diameter of 0.5 mm. Assume spherical particles and a water temperature of 10℃. The filtration rate (Surface Loading rate) is $2.7 \times 10^{-3}\ m^3/s \cdot m^2$.

Given:
Filter depth, l = 0.3 m
Porocity, ϵ = 0.42 (42%)
Grain diameter, d = 0.5 mm = 5.0×10^{-4} m
Shape factor for spherical particles, S = 6.0

Filtration rate (Surface Loading rate, Superficial Velocity, Approach velocity),
v = Q/A = 2.7 x 10^{-3} m/s

Acceleration of gravity, g = 9.807 m/s^2 (32.2 ft/s^2)
Kinematic viscosity, ν = 1.31 x 10^{-6} m^2/s at 10℃ from the Table of Properties of Water.

Using the Kozeny equation

$$\frac{h}{\ell} = \frac{J\ \nu (1-\varepsilon)^2\ V\ S^2}{g\varepsilon^3 d^2}$$

$$\frac{h}{\ell} = \frac{(6)\ (1.310x10^{-6}\ m^2/s)\ (1-0.42)^2\ (2.7x10^{-3}\ m/s)\ (6.0)^2}{(9.807\ m/s^2)\ (0.42)^3\ (5x10^{-4}\ m)^2}$$
$$= 1.4\, m/m$$

$$h = (1.4\ m/m)(0.3\ m) = 0.42\ m$$

Initial head loss = 0.42 m

The Fair and Hatch equation

- To determine headlloss through each layer of a clean stratified filter bed.

$$\frac{h}{\ell} = \frac{36\ k\ \nu (1-\varepsilon)^2\ V}{g\ \varepsilon^3\ \psi^2} \quad \sum_{i=1}^{n} \frac{P_i}{d_i^{\,2}}$$

where
k = costant, dimensionless = 5.0
ψ = sphericity of grains, ratio of surface area of equal volume spheres to the actual surface area of grains, dimensionless
= 0.8 - 0.7, rounded to angular for filter sand
= 0.7 - 0.4, angular to jagged for anthracite
P_i = fraction of total weight of filter grains in any layer *i*, dimensionless
d_i = geometric mean diameter of grains in layer *i,* ft or m.

Example 10.13 (VH, p. 338): Calculate the head loss through a clean sand filter with a gradation defined by the sieve analysis given at a filtration rate of 2.7 L/m^2-s (0.0027 m/s). The bed has a depth of 0.70 m with a porosity of 0.45 and the grains of sand have a sphericity of 0.75. A water temperature is 10℃.

Sieve Designation Number (1)	Size of Opening S (mm) (2)	Fraction of Sand Retained Pi (4)
12	1.68	
16	1.19	0.02
20	0.84	0.25
30	0.59	0.47
40	0.42	0.24
50	0.297	0.02

(Solution)

Sieve Analysis and Computation of Head Loss

Sieve Designation Number (1)	Size of Opening S (mm) (2)	Geometric Mean Diameter di (mm) (3)	Fraction of Sand Retained Pi (4)	$p_i/(d_i)^2$ $(mm)^{-2}$ (5)
12	1.68			
16	1.19	1.41	0.02	0.01
20	0.84	1	0.25	0.25
30	0.59	0.7	0.47	0.96
40	0.42	0.5	0.24	0.96
50	0.297	0.35	0.02	0.16
Total			1.00	2.34

Note:

(3) $(1.68 \times 1.19)^{1/2} = 1.41$

(4) from the sieve analysis

(5) $0.02/(1.41)^2 = 0.01$

1) Table 10.5 Sieve Analysis and computation of head loss.

$$\sum \frac{Pi}{{d_i}^2} = \frac{2.34}{mm^2}\frac{(10^3 mm)^2}{m^2} = \frac{2.34\ x\ 10^6}{m^2}$$

Obtain ν at 10°C from Appendix Table A.9 (VH, p. 864)

ν = kinetic (kinematic) viscosity = 1.306×10^{-6} m²/s

V = superficial velocity of water = filtration rate

= 2.7 L/m^2-s = 0.0027 m/s

ℓ = 0.70 m

ψ = 0.75

g = 9.807 m/s^2

Substituting into Fair and Hatch equation (10.41) VH, p. 337

$$\frac{h}{\ell} = \frac{36\,k\,\nu\,(1-\varepsilon)^2 V}{g\,\varepsilon^3\,\psi^2} \sum_{i=1}^{n} \frac{P_i}{d_i^{\,2}}$$

$$\frac{h}{\ell} = \frac{(36)(5.0)\left(1.306x10^{-6}\ m^2/s\right)(1-0.45)^2(0.0027\ m/s)}{\left(9.807\ m/s^2\right)(0.45)^3(0.75)^2}\left(\frac{2.34x10^6}{m^2}\right)$$

$$\frac{h}{\ell} = 0.89\ m/m$$

$$h = (0.89\ m/m)\ \ell = (0.89\ m/m)\ (0.7\ m) = 0.62\ m$$

Filter Hydraulics (Head Losses through Filter Media)

The **Rose** equation (3rd DC 235; 4th DC 289)

- Calculates the head loss (loss of pressure) through a clean stratified-sand filter with uniform porosity.

$$\frac{h_L}{\ell} = \frac{1.067(v_a)^2}{(\phi)(g)(\varepsilon)^4} \sum_{i=1}^{n} \frac{(C_D)(f)}{d}$$

where

h_L = frictional head loss through the filter, m
v_a = approach velocity, m/s
ℓ = depth of filter sand, m (= D)
f = mass fraction of sand particles of diameter d
d = diameter of sand grains, m
ϕ = shape factor, dimensionless
g = acceleration due to gravity, m/s^2
ϵ = porosity, dimensionless
C_D = drag coefficient, dimensionless

Drag coefficient, C_D

For 0.5 ≤ R < 10^4, the drag coefficient for spheres,

$$C_D = \frac{24}{R} + \frac{3}{R^{1/2}} + 0.34$$

where R = Reynolds number

For R < 0.5,

$$C_D = \frac{24}{R}$$

Reynolds number, R

$$R = \frac{v_s\ \phi d}{\nu} = \frac{v_a\ \phi d}{\nu}$$

where

d = particle diameter, m

ν = kinetic viscosity, m^2/sec

Φ = shape factor

v_s = v_a = approach velocity, m/s

= filtration velocity = filtration rate = Q/A

Depth of Expanded Bed

$$\ell_e = (1-\varepsilon)(\ell)\sum_{i=1}^{n}\frac{f}{(1-\varepsilon_e)}$$ or

$$D_e = (1-\varepsilon)(D)\sum_{i=1}^{n}\frac{f}{(1-\varepsilon_e)}$$

where

D_e = ℓ_e = depth of the expanded bed, m

D = ℓ = depth of the media

ϵ = porosity of the bed

ϵ_e = porosity of the expanded bed

f = mass fraction of sand with expanded porosity

The porosity of the expanded bed for a given particle

a) For laminar conditions,

$$\varepsilon_e = (\frac{v_b}{v_s})^{0.22}$$

where

v_b = velocity of backwash, m/s

v_s = settling velocity, m/s

b) For turbulent conditions (backwash) [Richardson and Zaki],

$$\varepsilon_e = (\frac{v_b}{v_s})^{0.22R^{0.1}}$$

where R = the Raynold number and defined as:

$$R = \frac{v_s d_{60\%}}{\nu}$$

where

$d_{60\%}$ = 60-percentile diameter, m

Example 3-27 (3^{rd} DC 238); Example 4-27 (4^{th} DC 291)

Estimate the clean filter loss in Urbana's proposed new sand filter (Example 4-24 4^{th} DC) using the sand described in Example 4-26 (4^{th} DC) and determine if it is reasonable. Use the following assumptions: specific gravity of sand is 2.65, the shape factor (ϕ) is 0.82, the bed porosity (ϵ) is 0.45, the water temperature is 10℃, and the depth of sand (L) is 0.5 m.

(Solution)

Given:
S.G. of sand = 2.65; shape factor, ϕ = 0.82; bed porosity, ϵ = 0.45;
depth of sand, L = 0.5 m; water temperature, T = 10℃
Filtration rate, Q/A = v_a = 216 $m^3/d\text{-}m^2$ (from previous example problem)

1) Geometric mean of diameter of sand grain

Sieve No.	Size of opening (mm)	Geometric mean diameter (mm)	(m)
8	2.38		
12	1.68	1.9996	0.002000
16	1.19	1.4139	0.001414
20	0.84	0.9998	0.001000
30	0.59	0.7040	0.000704
40	0.42	0.4978	0.000498
50	0.297	0.3532	0.000353
70	0.21	0.2497	0.000250
100	0.149	0.1769	0.000177
140	0.105	0.1251	0.000125

For example,
Sieve No. 8-12:
Sieve No. 8 = 2.38 mm
Sieve No. 12 = 1.68 mm

$$\sqrt{2.38\ x\ 1.68} = 1.9996\ mm = 0.002\ m$$

(4)

2) Compute Reynolds number, R

$$R = \frac{v_a\ \phi\ d}{\nu}$$

where

d = 0.002 m (for line 1)
ϕ = shape factor = 0.82
ν = kinetic viscosity = 1.307 x 10^{-6} m^2/sec @T = 10℃

Note that filtration rate = filtration velocity, v_a

$$v_a = \frac{216\ m^3}{d \bullet m^2} \frac{d}{86{,}400\ s} = 0.0025\ m/s$$

$$R = \frac{v_a\ \phi\ d}{\nu} = \frac{(0.0025\ m/s)(0.82)(0.002\ m)}{1.307x10^{-6}\ m^2/s} = 3.137 \qquad (5)$$

3) Compute Drag coefficient, C_D

For $0.5 < R < 10^4$, the drag coefficient for spheres,

$$C_D = \frac{24}{R} + \frac{3}{R^{1/2}} + 0.34 = \frac{24}{3.137} + \frac{3}{(3.137)^{1/2}} + 0.34 = 9.6846 \qquad (6)$$

Note For Re < 0.5

$$C_D = \frac{24}{R}$$

4) Calculate the value of $(C_D)(f) / d$ and $\sum (C_D)(f) / d$

$$\frac{(C_D)(f)}{d} = \frac{(9.6846)(0.053)}{0.002} = 256.64 \qquad (7)$$

(1) Sieve No.	(2) Retained %	(3) Retained f (fraction)	(4) d (m)	(5) R	(6) CD	(7) (CD)(f)/d
8-12	5.3	0.053	0.002000	3.13633	9.686251	256.737
12-16	17.1	0.171	0.001414	2.21772	13.17644	1593.551
16-20	14.6	0.146	0.001000	1.56816	18.04019	2634.394
20-30	20.4	0.204	0.000704	1.10419	24.93034	7224.249
30-40	17.6	0.176	0.000498	0.78078	34.47361	12188.46
40-50	11.9	0.119	0.000353	0.55396	47.69486	16069.99
50-70	5.9	0.059	0.000250	0.39171	61.26961	14474.69
70-100	3.1	0.031	0.000177	0.27745	86.5028	15159.65
100-140	0.7	0.007	0.000125	0.19619	122.3334	6846.292
			Total (CD)(f)/d =		Total	76448.01

$$\sum_{i=1}^{n} \frac{(C_D)(f)}{d} = 76{,}448$$

5) Compute the head loss using Rose En (3-105)

$$h_L = \frac{1.067(v_a)^2(\ell)}{(\phi)(g)(\varepsilon)^4}\sum_{i=1}^{n}\frac{(C_D)(f)}{d}$$

$$= \frac{1.067(0.0025)^2(0.5)}{(0.82)(9.8)(0.45)^4}(7.6448x10^4) = 0.77\ m$$

Unit:

$$\frac{(-)\left(\frac{m}{s}\right)^2(m)}{(-)\left(\frac{m}{s^2}\right)(-)^4}\frac{(-)(-)}{m} = m$$

Filter Cycle (for Rapid sand filters)

Clean filter --> Dirty filter --> Backwash --> Settling --> Clean filter

- In time, the pore openings in the filter, particularly those at the surface, become clogged, and the filter must be cleaned by backwashing.

- Filtration may be stopped when you observe:
  a. Low rate of filtration
  b. Passage of excess turbidity through the bed
  c. Air binding - reduces the rate of filtration

Note:

$$Filtration\ Rate \propto \frac{Driving\ Force}{Resistance}$$

Air binding
- As head loss increases across the bed, the lower portion of the filter is under a partial vacuum. This negative head permits the release of dissolved gasses, which tend to fill the pores of the filter, causing air binding and reducing the rate of filtration.

Our interest:
a. filtration time
b. down time (rinse, backwash, rinse)

Filter Backwashing Frequency
a. Normaly carried out 1/day
b. The frequency may be controlled by:
   - headloss through the bed
   - run time
   - turbidity of effluent

Backwash Rate

Backwash rate = Q/ A = 15 - 20 gpm/ft^2

Backwash Rise Rate

Backwash Rise Rate (ft/min) = 2 - 2.7 ft/min

$$\text{Backwash rise rate} = \frac{Q}{A}$$

range from

$$= \frac{(15\ \text{gal/min})\ (\text{ft}^3/7.48\ \text{gal})}{\text{ft}^2} = 2.0\ \text{ft / min}$$

to

$$= \frac{(20\ \text{gal/min})\ (\text{ft}^3/7.48\ \text{gal})}{\text{ft}^2} = 2.67\ \text{ft/min}$$

<u>Backwashing length</u>

a. Backwashing takes about 5-10 minutes.

<u>Backwash volume</u>

- The water required for backwashing is generally about 2 - 4 % of the water produced.

<u>Filter expansion</u>

- The bed of filter media is expanded hydrolically about 50%

Useful Formula:

1. Filter Surface Area, A (ft^2)

$$A = \frac{Q}{Q/A}$$

where Q = flow rate to filter
Q/A = design hydraulic surface loading rate

2. Total volume of backwash water used, V_b (gal/d)

$$V_b = \begin{pmatrix} Backwash \\ Rate \end{pmatrix} (A) \left(\frac{Backwash\ Length}{Cycle} \right) \left(\frac{Number\ of\ cycle}{d} \right)$$

$$\left(\frac{gal}{d} \right) = \left(\frac{gal / \min}{ft^2} \right) \left(ft^2 \right) \left(\frac{\min}{cycle} \right) \left(\frac{cycles}{d} \right)$$

where
Backwash rate in gal/min-ft^2
Filter Surface Area A in ft^2
Backwash length in min

3. The net water production of the filters (%)

$$\%\ net\ water\ production = \frac{V_P - V_B}{V_P}(100)$$

where V_P = volume of water produced
V_B = volume used for backwash

(Example)

Flow rate to the filter Q = 10 MGD
Use rapid sand filters,
Surface loading rate = 2 gpm/ft^2 = Q/A
Filter run length = 24 hrs
Backwash rate = 20 gpm/ft^2 for 10 min.

Determine:
a. Surface area of the filter (ft^2)?

b. Total volume of backwash water used, V_b (gal/day)?
c. The "net" water production of the filters (%)?

(Solution)

a. Surface area of the filter? $A = Q/(Q/A)$

$$A = \frac{\dfrac{10 \times 10^6 \text{ gal}}{\text{day}} \cdot \dfrac{1 \text{ day}}{1440 \text{ min}}}{\dfrac{2 \text{ gal/min}}{\text{ft}^2}} = 3472 \text{ ft}^2 = 3500 \text{ ft}^2$$

b. Total volume of backwash water used, V_b (gal/day)

$$V_b = \left(\frac{Backwash}{Rate}\right)(A)\left(\frac{Backwash\ Length}{Cycle}\right)\left(\frac{Number\ of\ cycle}{d}\right)$$

$$= \frac{20 \text{ gal}}{\text{min}/\text{ft}^2}(3500 \text{ ft}^2)\ \frac{10 \text{ min}}{\text{cycle}}\ \frac{1 \text{ cycle}}{\text{day}}$$

= 700,000 gpd = 0.7 MGD

c. The "net" water production of the filters (%)?

$$\%\ net\ water\ production = \frac{V_P - V_B}{V_P}(100)$$

where V_P = volume of water produced
V_B = volume used for backwash

$$= \frac{(10 - 0.7) \text{ MGD}}{10 \text{ MGD}} \times 100$$

= 93.5 %

Critical Backwash Velocity, v_c

- Backwash rate necessary to expand the bed completely

$v_c = v_s\, \epsilon^{4.5}$

where

v_c =	critical velocity, fps
v_s =	terminal settling velocity, fps (Type I)
ϵ =	bed porosity = void volume / total bed volume

$\epsilon = 1 - (\rho_b / \rho_s)$

where

ρ_b =	oven-dried mass of the sample at 10°C per its field volume (g/mL)
ρ_s =	particle density = 2.65 g/mL

Terminal Settling Velocity, v_s (fps)

$$v_s = \left[\left(\frac{4}{3}\right)\left(\frac{g}{18.5}\right)\left(\frac{\rho_f}{\mu_f}\right)^{0.6}(SG_P - 1)(d_p)^{1.6}\right]^{0.714}$$

where

v_s =	terminal settling velocity, ft/sec
d_p =	diameter of filter particle, ft
ρ_f =	density of water, $lb.sec^2/ft^4$
μ_f =	absolute viscosity of water, $lb.sec/ft^2$
g =	acceleration of gravity, ft/sec^2 = 32.2 ft/sec^2

SG_p = specific gravity of sand, g/cm^3 = 2.65

* Complete expansion occur when v_c reached for heaviest particle.

Example: Compute the critical backwash velocity, v_c

Given:

SG_p = 2.65
d_p = 3.28 x 10^{-3} ft
ρ_f = 1.936 lb.sec^2/ft^4
μ_f = 2.735E-5 lb.sec/ft^2
g = 32.2 ft/sec^2
ϵ = 42%

(Solution)

1. First compute the terminal settling velocity, v_s , ft/sec

$$v_s = \left[\left(\frac{4}{3}\right)\left(\frac{g}{18.5}\right)\left(\frac{\rho_f}{\mu_f}\right)^{0.6}(SG_P - 1)\left(d_p\right)^{1.6}\right]^{0.714}$$

$$v_s = \left[\left(\frac{4}{3}\right)\left(\frac{32.2}{18.5}\right)\left(\frac{1.936}{2.735x10^{-5}}\right)^{0.6}(2.65-1)\left(3.28x10^{-3}\right)^{1.6}\right]^{0.714}$$

v_s = 0.45 fps

2. Compute the critical backwash velocity, v_c

$v_c = v_s\, \epsilon^{4.5} = (0.45)(0.42)^{4.5} = 9.14 \times 10^{-3}$ fps

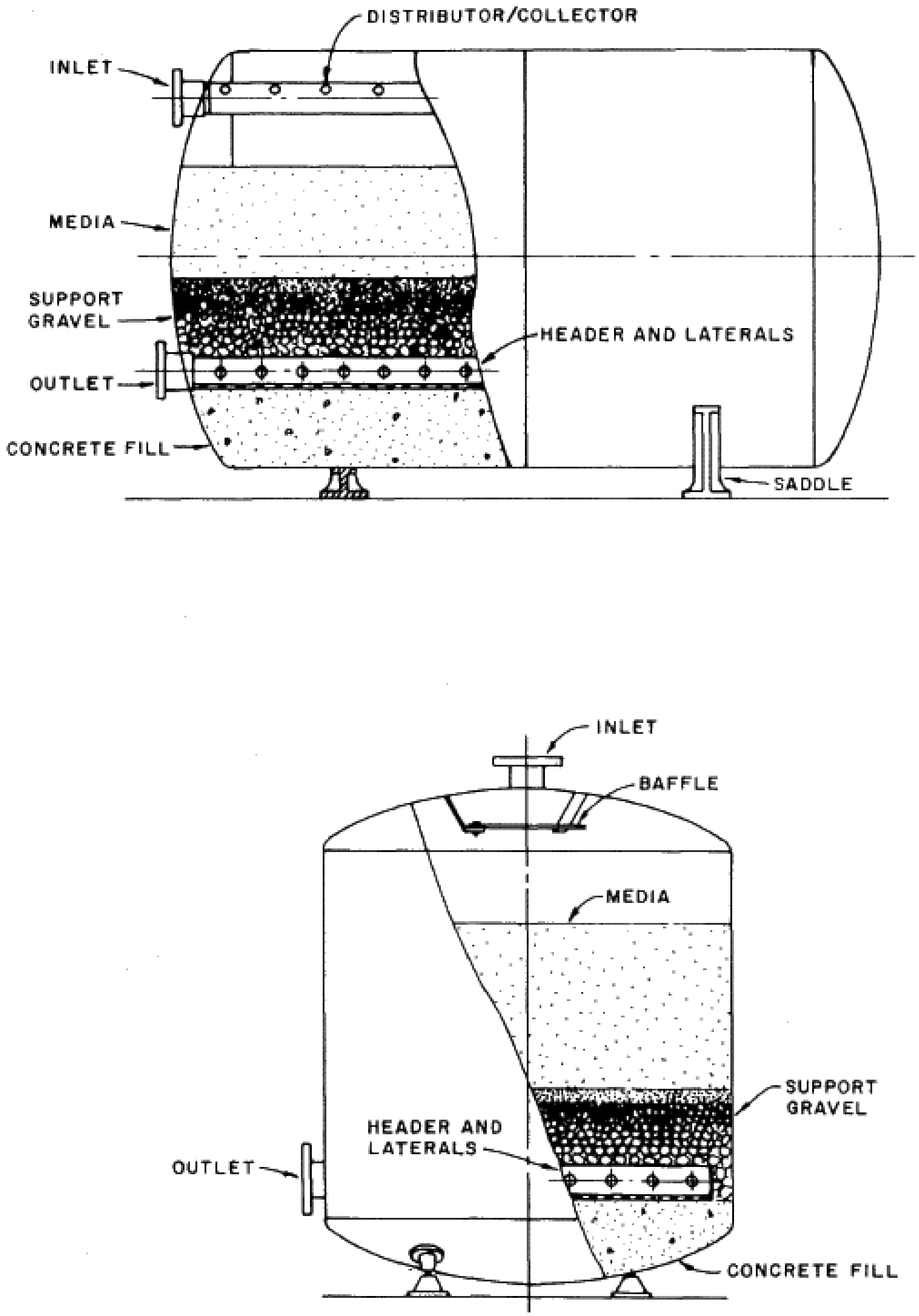


Figure 7.2 Typical horizontal and vertical pressure filters. (*Source: Roberts Filter Manufacturing Co.*)

- Homework is due one week from today!

- Show Nomura Water Treatment Plant